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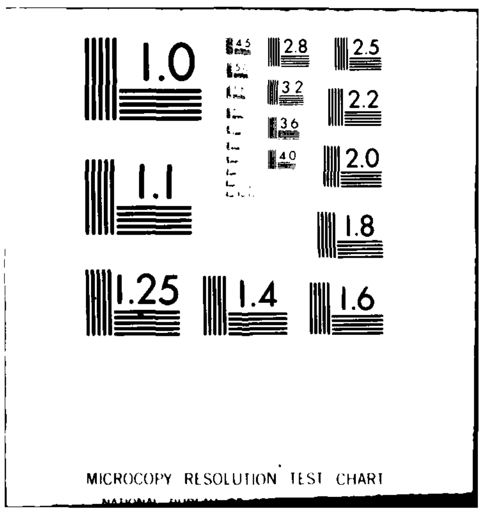

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OPTIMIZATION OF AUTO-PILOT EQUATIONS
FOR RAPID ESTIMATION OF HELICOPTER CONTROL SETTINGS

Interim Technical Report No.1

by
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and
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OPTIMIZATION OF AUTO-PILOT EQUATIONS FOR RAPID ESTIMATION OF HELICOPTER CONTROL SETTINGS

ABSTRACT

An automatic feedback system, based on continuous monitoring of control loads, is used to find the control settings that are required to obtain a given flight condition of a helicopter rotor. A program is developed that searches automatically for the optimum gains and time constants of the system. Satisfactory results are achieved for given conditions as an example.

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OPTIMIZATION OF AUTO-PILOT EQUATIONS FOR RAPID ESTIMATION OF HELICOPTER CONTROL SETTINGS

1. INTRODUCTION

Previous work in the area of helicopter stability and vibrations has shown that an accurate knowledge of the helicopter control settings is a necessary prerequisite to the determination of blade damping or rotor loads. The mathematical formulation of this problem involves solution of a set of non-linear differential equations for the periodic, equilibrium solution. This,in itself, is fairly straightforward; but the problem is complicated by the fact that the unknown control settings appear as forcing functions (and sometimes as coefficients) in the equations. These control settings must be chosen so as to satisfy certain integral constraints on the solution, namely that the helicopter be flying in trim at the desired flight condition.

In reference 1, a solution to the above problem is formulated whereby a set of control equations (called an "automatic pilot") is used to bring the controls to appropriate values simultaneously with the solution of the blade equations. The coefficients of this controller (two gains and two time constants) are chosen by trial and error to give the most rapid convergence to the desired settings.

Although the results in reference (1) are very satisfactory, there are several aspects of the problem which merit further study. First, the choice of parameters in (1) was made on the basis only of a qualitative assessment of when the controls had converged. A more quantitative (and automated) approach is needed before the method can be extended to general problems. Second, several errors have been found in the equations of motion of (1). Thus, the results need to be verified for an accurate set of equations. Third, the results in (1) do not include a study of convergence or optimality conditions (local minima, etc.). Therefore, a more complete analysis is warranted.

In this paper, the corrected equations are studied in more detail in terms of convergence properties. Whereas reference (1) concentrates on loss of stability in extreme conditions (stall, high advance ratio, low torsional frequency), the present work concentrates on the controller characteristics in the normal operating range. To this end, a program is developed that searches automatically for the optimum gains and time constants of the system.

2. MATHEMATICAL DESCRIPTION

2.1. Rotor Equations

The physical and mathematical models used here are the same as in reference (1) with the exception that certain algebraic errors in (1) have been corrected. The physical model, given in Figure 1, shows a single section of a slender, rigid, inelastic blade, which is hinged in the torsional and out-of-plane directions at the center of rotation with restraints, K_θ and K_β . The blade is assumed to flap with angle β, and to feather with angle θ . Fixed coordinates are defined with the Z-direction along the rotor shaft and with the X-direction opposite to the direction of flight. The blade rotates about the shaft in the XY plane with constant angular velocity Ω . The rotor shaft angle ψ, which is measured from down wind, is in radians. The blade position with respect to the fixed coordinate system is thus defined by the three angles ψ, β , and θ, which uniquely define the blade position.

As the result of the derivation and simplication, we have

$$\overset{**}{\beta} + p^2\beta = \bar{F}_\beta + (p^2 - 1)\beta_{pc} \qquad (1)$$

$$\overset{**}{\theta} + \theta = \bar{M}_\theta - (\omega_\theta^2 - 1)(\theta - \theta_0 - \theta_s \sin 2t) \qquad (2)$$

The aerodynamic force F_β and the moment about the pitch hinge M_θ are obtained from piecewise linear, quasi-steady strip theory,

$$F_\beta = \frac{1}{2}\rho c d a U_T^2(\theta - U_P/U_T) + \frac{1}{8}\rho a c^2 d U_T(\dot{\theta} + \Omega\beta - \dot{\phi}) \qquad (3)$$

$$M_\theta = -\frac{1}{32}\rho a c^3 d U_T^2(\dot{\theta} + \Omega\beta - \dot{\phi}) + \frac{1}{2}\rho a c^2 d U_T^2 C_m \qquad (4)$$

Combining (1),(2),(3), and (4), we have

$$\begin{aligned}
&\ddot{\beta} + \frac{\gamma}{8}(1+\mu\sin\psi)\dot{\beta} + [p^2 + (1+\mu\sin\psi)(\frac{\gamma}{8}\mu\cos\psi - \\
&\quad \frac{1}{4}\gamma\bar{c})]\beta - \frac{\gamma}{8}(1+\mu\sin\psi)^2\theta = \frac{\gamma}{8}[\frac{\bar{c}}{4}(1+\mu\sin\psi) \\
&\quad (\dot{\theta} - \dot{\phi}) - (1+\mu\sin\psi)\lambda] + (p^2-1)\beta_{pc} \qquad (5)
\end{aligned}$$

$$\begin{aligned}
&\ddot{\theta} + \frac{\bar{c}^2\gamma}{128 e^2}(1+\mu\sin\psi)\dot{\theta} + \omega_\theta^2\theta - \frac{\gamma\bar{c}^2}{8e^2}(1+\mu\sin\psi)C_m\theta \\
&\quad -(\omega_\theta^2-1)(\theta_0 + \theta_s\sin\psi + \theta_c\cos\psi) + \frac{\bar{c}^2\gamma}{128 e^2}(1+\mu\sin\psi)\beta \\
&\quad = [\frac{\bar{c}\gamma}{8e^2}(1+\mu\sin\psi)\dot{\beta} - \frac{\bar{c}^2\gamma}{8e^2}(1+\mu\sin\psi)\mu\cos\psi\,\beta + \\
&\quad \frac{\bar{c}^2\gamma}{128 e^2}(1+\mu\sin\psi)\dot{\phi} + \frac{\bar{c}^2\gamma}{8e^2}(1+\mu\sin\psi)\lambda]\,C_m \qquad (6)
\end{aligned}$$

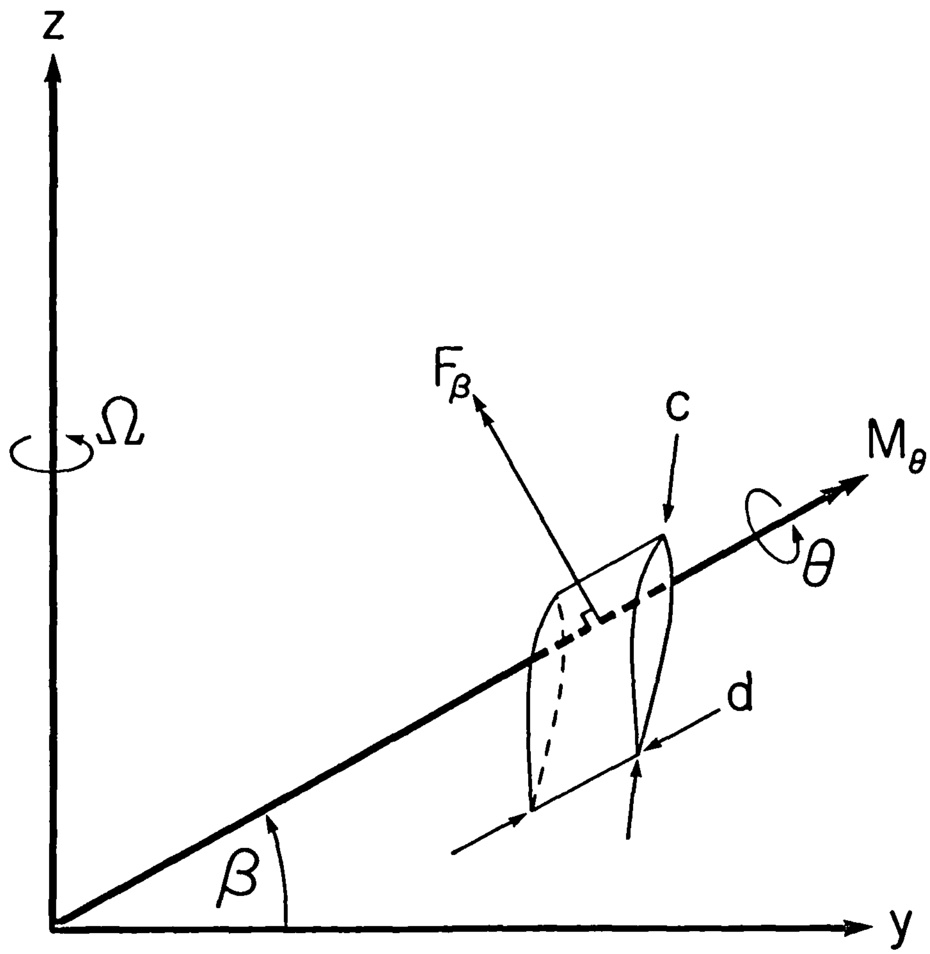

Figure 1. Schematic of Blade Model

2.2. Auto-pilot Equations

In order to simulate the trimmed flight of a helicopter rotor, the rotor is maintained at a fixed value of the thrust coefficient with forward speed. Furthermore, the cyclic pitch is adjusted to suppress first harmonic cyclic flapping ($\beta_c = \beta_s = 0$) and, therefore, to eliminate rotor hub moments.

The following auto-pilot equations, taken from reference (1) represent the strategy whereby the controls ($\theta_c, \theta_s, \theta_c$) are adjusted in order to reach the desired thrust ($T=T_c$) and moments ($PM=PM_c=0$, $RM=RM_c=0$). The cross-coupling, B, accounts for the couping between roll and pitch in a helicopter rotor.

$$\tau_c \dot{\theta}_c + \theta_c = A_c (T_c - T) \tag{7}$$

$$\tau_1 \ddot{\theta}_s + \theta_s = A_1 [(PM - PM_c) + B(RM - RM_c)] \tag{8}$$

$$\tau_1 \ddot{\theta}_c + \theta_c = A_1 [(RM - RM_c) + B(PM - PM_c)] \tag{9}$$

The final form of the auto-pilot equations are found from evaluation of the instantaneous thrust and moments (T,PM,RM) in terms of the flapping angle, β.

$$\dot{\theta}_c = -\frac{\theta_c}{\tau} - \frac{A_c p^2}{\tau_c \sigma}\beta + \frac{A_c \bar{C}_c}{\tau_c} \tag{10}$$

$$\ddot{\theta}_s = -\frac{\dot{\theta}_s}{\tau_1} - \frac{A_1(p^2-1)}{\tau_1 \sigma}\left[\frac{\varepsilon(p^2-1)\sin\psi}{\sigma} + \frac{\varepsilon}{\beta}\bar{C}_L - \cos\psi\right]\beta + A_1\frac{\bar{C}_M}{\tau_1} \tag{11}$$

$$\ddot{\theta}_c = -\frac{\dot{\theta}_c}{\tau_1} - \frac{A_1(p^2-1)}{\tau_1 \sigma}\left[\frac{\varepsilon(p^2-1)\cos\psi}{\sigma} + \frac{\varepsilon}{\beta}\bar{C}_M + \sin\psi\right]\beta - A_1\frac{\bar{C}_L}{\tau_1} \tag{12}$$

2.3. Method of Solution

For the whole system, we combine all the above equations, and write them in matrix form,

$$\begin{Bmatrix} \beta \\ \ddot{\theta} \\ \beta \\ \theta \end{Bmatrix} + \frac{\gamma}{\varepsilon}(1+\mu\sin\psi)\begin{bmatrix} 1 & -\frac{\bar{\gamma}}{4} & \frac{p^2}{\frac{\gamma}{\varepsilon}(1+\mu\sin\psi)} + \mu\cos\psi - 2\bar{C} & -(1+\mu\sin\psi) \\ -\frac{\bar{\gamma} C_m}{\varepsilon} & -\frac{\bar{\gamma}^2}{16\varepsilon^2} & \frac{\bar{\gamma}^2}{\varepsilon^2}\left(\frac{1}{16} + \mu\cos\psi\right) & \frac{\omega_c^2}{\frac{\gamma}{\varepsilon}(1+\mu\sin\psi)} - \frac{\bar{\gamma}^2}{\varepsilon^2}C_m \\ -\frac{1}{\frac{\gamma}{\varepsilon}(1+\mu\sin\psi)} & 0 & 0 & 0 \\ 0 & \frac{1}{\frac{\gamma}{\varepsilon}(1+\mu\sin\psi)} & 0 & 0 \end{bmatrix}\begin{Bmatrix} \dot{\beta} \\ \dot{\theta} \\ \beta \\ \theta \end{Bmatrix} =$$

$$
\left\{\begin{array}{l}
-\frac{\lambda\gamma}{\varepsilon}(1+\mu\sin\psi)-\frac{\gamma\bar{c}}{32}\phi(1+\mu\sin\psi)+(p^2-1)\beta_{pc} \\
\frac{\gamma\bar{c}^2}{\varepsilon c^2}C_m\lambda(1+\mu\sin\psi)+\frac{\gamma\bar{c}^2}{128\varepsilon^2}C_m\phi(1+\mu\sin\psi)+(\omega_c^2-1)(\theta_c+\theta_s\sin\psi+\theta_c\cos\psi) \\
0 \\
0
\end{array}\right\}
\qquad (13)
$$

$$
\begin{Bmatrix} \dot{\theta}_c \\ \dot{\theta}_s \\ \dot{\theta}_c \\ \dot{\theta}_o \\ \dot{\theta}_s \\ \dot{\theta}_c \end{Bmatrix}
+
\begin{bmatrix}
\frac{1}{\tau_c} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\tau_1} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{\tau_1} & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0
\end{bmatrix}
\begin{Bmatrix} \theta_o \\ \theta_s \\ \theta_c \\ \theta_o \\ \theta_s \\ \theta_c \end{Bmatrix}
=
$$

$$
\left\{\begin{array}{l}
-\frac{A_c p^2\beta}{\tau_c\gamma}+\frac{A_c\bar{C}_T}{\tau_c} \\
-\frac{A_1(p^2-1)}{\tau_1\gamma}\left[\frac{\varepsilon(p^2-1)\sin\psi}{\gamma}+\frac{\varepsilon}{\beta}\bar{C}_L-\cos\psi\right]\beta+A_1\frac{\bar{C}_M}{\tau_1} \\
-\frac{A_1(p^2-1)}{\tau_1\gamma}\left[\frac{\varepsilon(p^2-1)\cos\psi}{\gamma}+\frac{\varepsilon}{\beta}\bar{C}_M+\sin\psi\right]\beta-A_1\frac{\bar{C}_L}{\tau_1} \\
0 \\
0 \\
0
\end{array}\right\}
\qquad (14)
$$

The following blade parameters and flight conditions are used in equations (13) and (14) for the numerical examples to follow.

$\mu = 0.3$
$\gamma = 5.0$
$e = 0.05$
$\bar{c} = 0.1$

$C_T/\sigma a=0.0050/0.314=0.0159$

$C_L/\sigma a=0.0$

$C_M/\sigma a=0.0$

$P_s=0.0$

$\alpha_c=0.2$

$\lambda=\left\{\frac{1}{2}\left[(\mu^4+C_T^2)^{\frac{1}{2}}-\mu^2\right]\right\}^{\frac{1}{2}}=8.33012\text{x}10^{-3}$

$\beta_{pc}=0.0$

$\omega_\beta^2=3.5$, $P=1.12$, $\dot{\phi}$ neglected

We select β, $\dot{\beta}$, θ, $\dot{\theta}$, θ_0, $\dot{\theta}_0$, θ_c, $\dot{\theta}_c$, θ_s, and $\dot{\theta}_s$ as the state variables and rewrite equations (13) and (14) in first-order state variable form.

$$
\left.
\begin{aligned}
&\dot{Y}(1)=Y(2)\\
&\dot{Y}(2)=-[1.2544+(0.3\cos x-0.2)(0.625+0.1875\sin x)]Y(1)\\
&\qquad -(0.625+0.1875\sin x)Y(2)\\
&\qquad +(0.625+0.1875\sin x)(1+0.3\sin x)Y(3)\\
&\qquad +0.025(0.625+0.1875\sin x)Y(4)\\
&\qquad -(0.00520632+0.00156189\sin x)\\
&\dot{Y}(3)=Y(4)\\
&\dot{Y}(4)=-(0.625+0.1875\sin x)(0.25)Y(1)\\
&\qquad -12.25Y(3)\\
&\qquad -(0.625+0.1875\sin x)(0.25)Y(4)\\
&\qquad +11.25Y(5)\\
&\qquad +11.25\sin x\ {*}Y(9)\\
&\qquad +11.25\cos x\ {*}Y(7)\\
&\dot{Y}(5)=Y(6)\\
&\dot{Y}(6)=-(A_0/T_0)*0.25088Y(1)-Y(6)/T_0+T_0(0.0159)\\
&\dot{Y}(7)=Y(8)\\
&\dot{Y}(8)=-(A_1/T_1)(0.05088)(0.4070\cos x+\sin x)Y(1)\\
&\qquad -Y(8)/T_1\\
&\dot{Y}(9)=Y(10)\\
&\dot{Y}(10)=-(A_1/T_1)(0.05088)(0.40704\sin x-\cos x)Y(1)\\
&\qquad -Y(10)/T_1
\end{aligned}
\right\}
\qquad (15)
$$

where $x=\psi$

$Y(1)=\beta$

$Y(2)=\dot{\beta}$

$Y(3)=\theta$

$Y(4)=\dot{\theta}$

$Y(5)=\theta_0$

Y(6)= $\dot{\theta}$

Y(7)= θ_L

Y(8)= $\dot{\theta}_L$

Y(9)= θ_s

Y(10)= $\dot{\theta}_s$

We solve the first-order differential equation set (15) by the Runge-Kutta program provided by IBM Scientific Subroutine Package (SSP). Figure 2 shows a typical response of a stable system with all zero initial conditions.

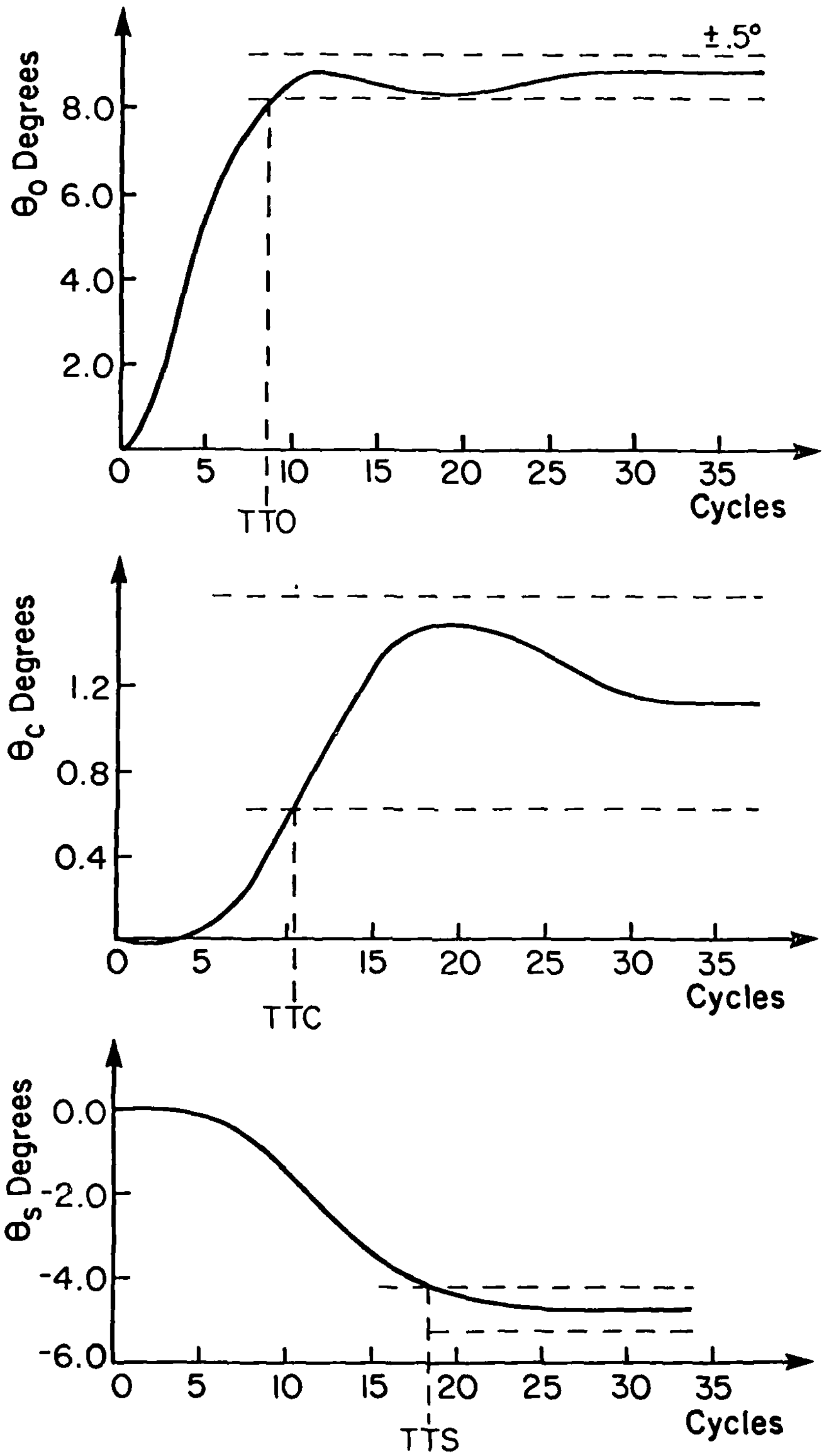

Figure 2

Typical Time History

3. OPTIMALITY CRITERIA AND SEARCH FOR OPTIMUM POINTS

3.1. Optimality Criteria

For a system to be optimal, some kinds of criteria must be used to decide on the utility of the solution.

In our case, for a stable system, the angles θ_o , θ_c , θ_s will ultimately reach the final, stable position. It is our desire that these final values are reached in as short a time as possible. Thus, we choose as a cost function the time required (i.e. the number of rotor revolutions required) for all controls to be within $\pm 0.5^\circ$ of their final values. To do this we designate TT_c , TT_c , and TT_s as the respective times for θ_o , θ_c , and θ_s to converge to within $\pm 0.5^\circ$ of a final value; and we designate the largest of these three as Tmax, Figure 2. Then, we use the minimum of Tmax as the optimality criterion.

3.2. Search for Optimum Points

As we have seen in previous sections, it would be quite difficult to obtain explicit expression for TT_o , TT_c , TT_s , and Tmax. Furthermore, analytical derivatives of Tmax with respect to A_o, A_1, T_c, and T_1 are not readily available, so we use the derivative-free method in our search program. That is, we evaluate functions only; and no derivatives are involved.

The controller used here has four parameters that must be considered: A_o, A_1, T_o, T_1. The procedure is as follows,

(1) A base point is chosen and Tmax is evaluated.

(2) Local searches are made by stepping A_c a distance 0.2 to each side (M1 direction in Table 1) and by evaluating Tmax to see if a lower Tmax is obtained.

(3) If there is no Tmax decrease, we do the same to M2 direction, and then to M3, M4, ..., until M42 direction, or until a decrease is found, see Table 1. We select 0.2 as increment for A_1, 0.3π for T_o, 0.2π for T_1 .

(4) If there is a Tmax decrease, we then use the new point as a base

Increments of Parameters

Direction	A_0	A_1	$T_0*\pi$	$T_1*\pi$	
M1	0.2	0.0	0.0	0.0	
M2	0.0	0.2	0.0	0.0	Change one parameter at a time.
M3	0.0	0.0	0.3	0.0	
M4	0.0	0.0	0.0	0.2	
M5	0.2	0.2	0.0	0.0	
M6	0.2	-0.2	0.0	0.0	
M7	0.2	0.0	0.3	0.0	
M8	0.2	0.0	-0.3	0.0	
M9	0.2	0.0	0.0	0.2	
M10	0.2	0.0	0.0	-0.2	Change two parameters at a time.
M11	0.0	0.2	0.3	0.0	
M12	0.0	0.2	-0.3	0.0	
M13	0.0	0.2	0.0	0.2	
M14	0.0	0.2	0.0	-0.2	
M15	0.0	0.0	0.3	0.2	
M16	0.0	0.0	0.3	-0.2	
M17	0.2	0.2	0.3	0.0	
M18	0.2	0.2	-0.3	0.0	
M19	0.2	-0.2	0.3	0.0	
M20	-0.2	0.2	0.3	0.0	
M21	0.2	0.2	0.0	0.2	
M22	0.2	0.2	0.0	-0.2	
M23	0.2	-0.2	0.0	0.2	Change three parameters at a time.
M24	-0.2	0.2	0.0	0.2	
M25	0.2	0.0	0.3	0.2	
M26	0.2	0.0	0.3	-0.2	
M27	0.2	0.0	-0.3	0.2	
M28	-0.2	0.0	0.3	0.2	
M29	0.0	0.2	0.3	0.2	
M30	0.0	0.2	0.3	-0.2	
M31	0.0	0.2	-0.3	0.2	
M32	0.0	-0.2	0.3	0.2	
M33	0.2	0.2	0.3	0.2	
M34	0.2	0.2	0.3	-0.2	
M35	0.2	0.2	-0.3	0.2	
M36	0.2	-0.2	0.3	0.2	
M37	-0.2	0.2	0.3	0.2	Change four parameters at a time.
M38	0.2	0.2	-0.3	-0.2	
M39	0.2	-0.2	0.3	-0.2	
M40	-0.2	0.2	0.3	-0.2	
M41	0.2	-0.2	-0.3	0.2	
M42	-0.2	0.2	-0.3	0.2	

Table 1 Searching Directions

point and go back to (2). If there is no decrease in Tmax, then we assume that a local minimum point is found.

A flow diagram illustrating the above procedure is given in Figure 3.

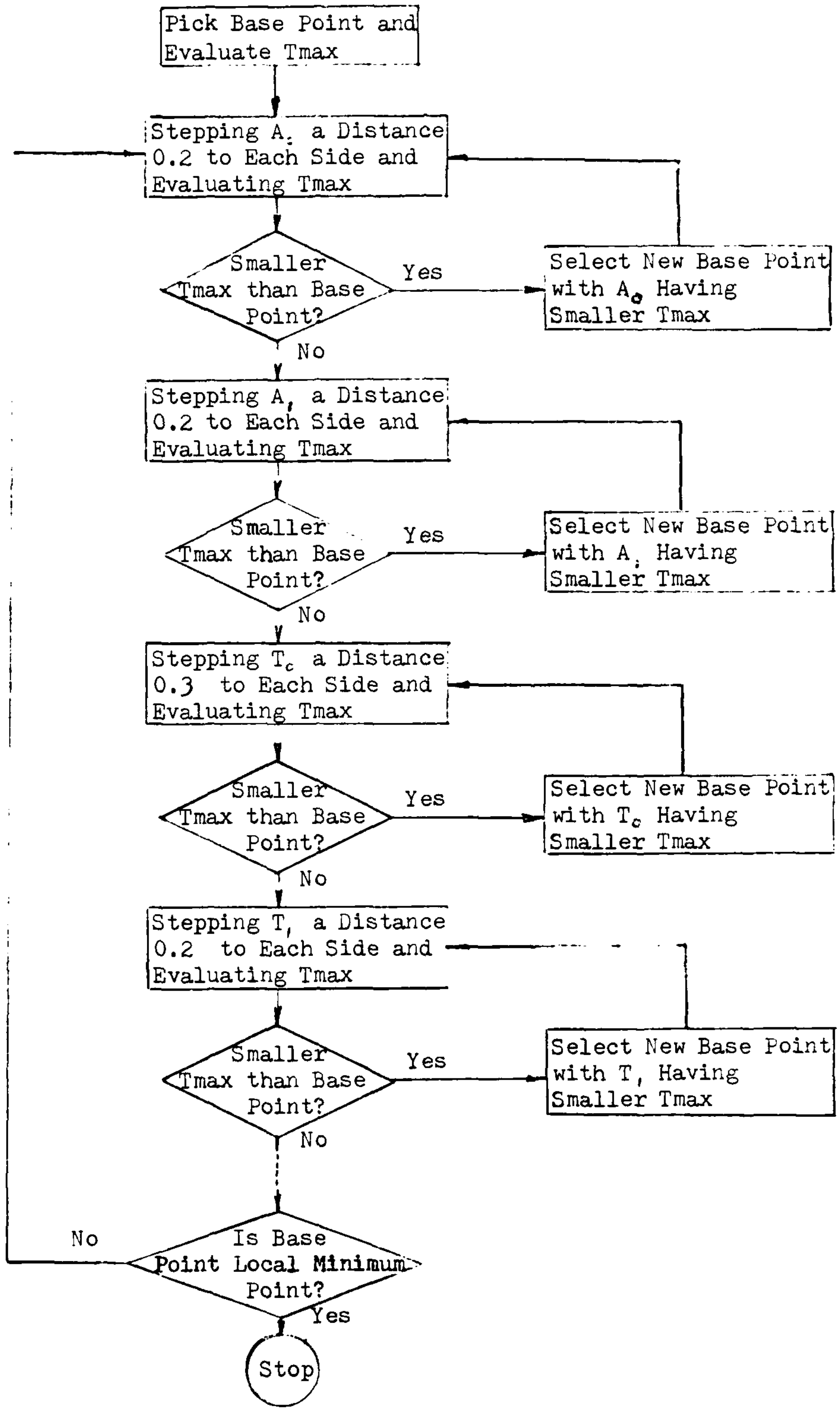

Figure 3 Flow Diagram

4. RESULTS AND DISCUSSION

4.1. Stepping Distance

Although, in principle, a smaller stepping distance implies a more accurate convergence, from an engineering point of view, it is often more efficient to take larger steps such that a meaningful change in cost function can be found. Thus, in this case, we have used steps 0.2 for A_0 and A_1 , 0.3π for T_0 , and 0.2π for T_1 .

4.2. Choice of Stable Points as Starting Points

For the starting points, the system must be stable, otherwise Tmax will be equal to infinity (in our program Tmax is approximately equal to 40 cycles), and the search program will fail.

4.3. Local Minimum and Global Minimum

Since local minima are potentially possible, several different starting points are considered. Should these converge to several different local minima, the global minimum can be chosen from the local ones.

4.4. Optimality Criteria

The minimum Tmax is selected as the optimality criterion in our case. As we can see in Figure 4 and 5, however, the difference in Tmax between two local optima is only 1.28 cycles. On the other hand, one optimum has larger oscillations (in the steady-state) than the other. Therefore, it might be good in future studies to consider these oscillations in the selection of optimality criteria.

4.5. Optimal Control Setting

With blade parameters and flight conditions as shown in pages 7 and 8, and with the minimum Tmax criterion, the optimal control setting is as follow,

$$A_0 = 2.6$$
$$A_1 = 3.6$$
$$T_0 = 2.1\pi$$
$$T_1 = 0.6\pi$$

and

$$TT_c = 4.85 \text{ cycles}$$
$$TT_L = 5.29 \text{ cycles}$$

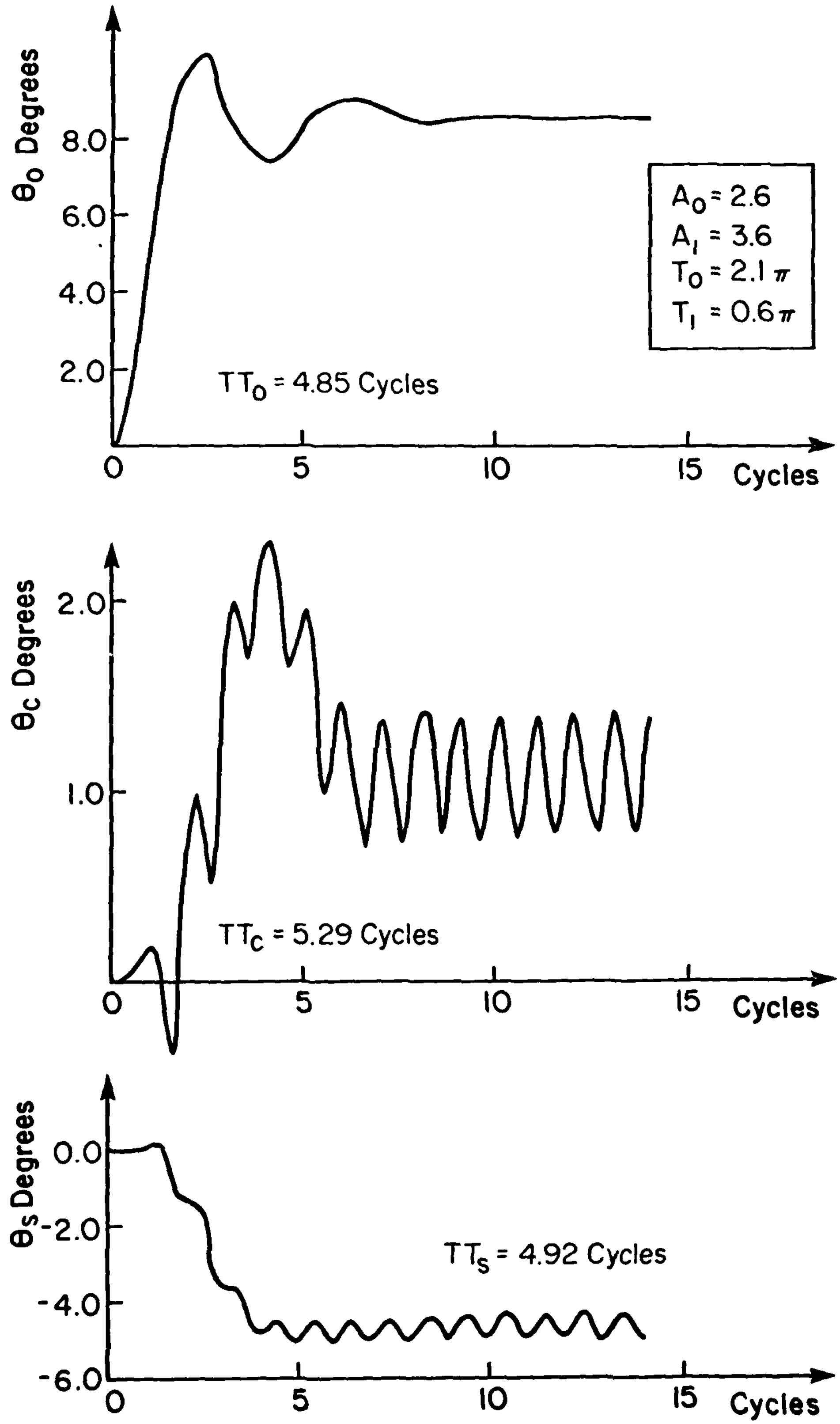

Figure 4. Time History

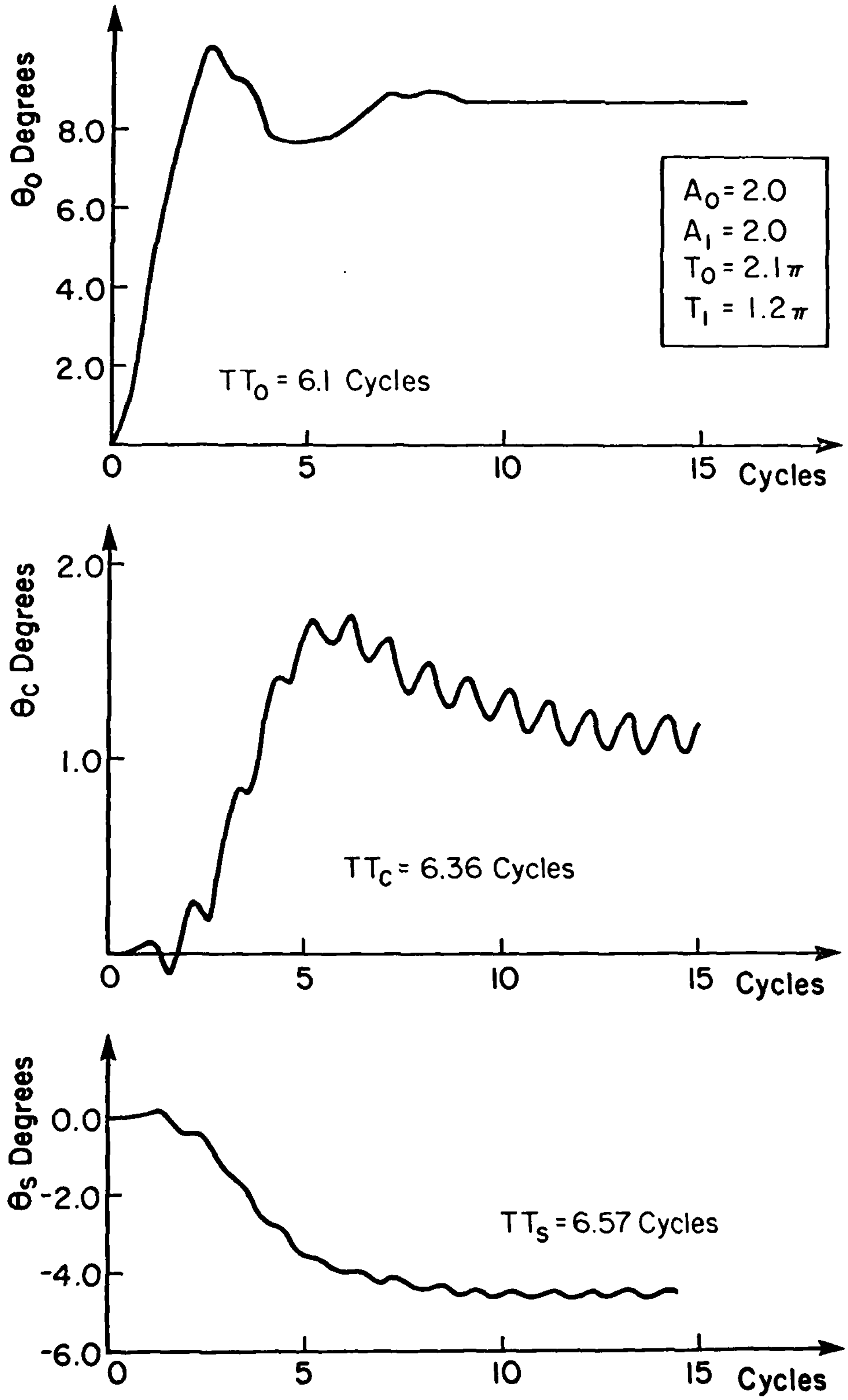

Figure 5. Time History

TT_s = 4.92 cycles

Tmax = TT_c = 5.29 cycles

$\theta_s(\infty)$ = 8.5627 degree

$\theta_c(\infty)$ = 1.1347 degree

$\theta_1(\infty)$ =-4.5841 degree

The optimum results (Figure 4) can be compared with another local optimal (Figure 5). For the optimum settings, in Figure 4, the low cyclic time constant (0.6 π) causes $\pm 0.5^\circ$ oscillations which are on the boundary of the accepted level. In Figure 5, a larger time constant is chosen (T_1 = 1.2 π). This reduces the oscillations to $\pm 0.1^\circ$, but also necessitates lower gains (A_0, A_1 reduced from 2.6,3.6 to 2.0, 2.0). The lower gains imply that the over-all convergence of the mean is slowed, making the mean the critical criterion for convergence. It should be noted that, for an analysis with more than one blade (we only have one), the oscillations would greatly decrease since the rotor would filter out once-per-rev from the controller. This would alter the optimum in Figure 4 (were $\pm 0.5^\circ$ is critical) but would not greatly change the optimum in Figure 5 since the oscillations are not driving the solution.

In conclusion, we can say that the gains and time constants found in Reference (1) are very close to the optimum found here. On the other hand, the research in this report shows that there are two local optima with nearly equal convergence. A comparison of these indicates that an improved optimality criterion might be obtained from a more stringent penalty on steady-state oscillations.

5. ACKNOWLEDGEMENT

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6. NOMENCLATURE

A	area of the blade section
a	slope of the lift curve
A_o, A_1	gains
B	coupling
c	blade chord, m
$\bar{c}$	nondimensionalized blade chord, $\frac{c}{r}$
C_ℓ	blade lift coefficient
$\bar{C}_L$	normalized blade moment coefficient, $\frac{C_L}{\sigma a}$
C_m	blade moment coefficient
$\bar{C}_m$	normalized blade moment coefficient, $\frac{C_m}{\sigma a}$
C_s	root moment coefficient
$\bar{C}_s$	normalized root moment coefficient, $\frac{C_s}{\sigma a}$
C_T	blade thrust coefficient
$\bar{C}_T$	normalized blade thrust coefficient, $\frac{C_T}{\sigma a}$
d	length of blade section
e	inertial ratio, $\sqrt{\frac{I_x}{I_y + mr^2}}$
F_β	lift force
$\bar{F}_\beta$	nondimensionalized lift force, $\frac{F_\beta r}{\Omega^2(I_y + mr^2)}$
h	constant term in algorithmic Runge-Kutta method
I	total inertia, $I_y + mr^2$
I_x, I_y	blade section inertias, kg-m^2
J	total number of periods of integration

k	reduced frequency, $\frac{1}{2}\frac{c}{r}$
K_o, K_1	rate gains, such that $\frac{1}{K}$ is number of radians to full application of linear control predictions
K	spring stiffness at the center of rotation
K_β	flapping spring constant
K_θ	torsional spring constant
b	number of blades
ℓ	lift on blade
L	rolling moment at hub
m	mass of the blade section, number of controls
M	pitching moment at hub
M_θ	moment about the pitch axis
$\overline{M}_\theta$	nondimensionalized moment about the pitch, $\frac{M_\theta}{I_x \Omega^2}$
N	number of second-order degrees of freedom
P	dimensionless rotating flapping frequency
P_s	slope of the pitch moment coefficient
PM_o	pitch moment desired
RM_o	roll moment desired
r	blade radial coordinate
S	steady root moment on hub in rotating system
t	time

T	thrust on the blade
U	total velocity of blade section relative to air
U_p	vertical component of air speed
U_T	horizontal component of air speed
v_x, v_y, v_z	components of v at the blade's c.g.
V	vertical component of wind speed
v	velocity of the blade's center of gravity
α	angle of attack
α_c	critical angle of attack
$\dot{\alpha}$	angular velocity of blade at center of gravity
$\dot{\alpha}_x, \dot{\alpha}_y, \dot{\alpha}_z$	components of $\dot{\alpha}$
β	flapping angle, rad., $\beta_o + \beta_s \sin\psi + \beta_c \cos\psi$
$\beta_o, \beta_s, \beta_c$	components of flapping
γ	lock number, $\frac{4\rho acdr^3}{Iy+mr^2}$
Θ	total pitch angle, rad, $\Theta_e + \Theta_o + \Theta_s \sin\psi + \Theta_c \cos\psi$
Θ_e	elastic portion of pitch angle
$\Theta_o, \Theta_c, \Theta_s$	collective and cyclic pitch
λ	inflow ratio
μ	advance ratio, $\frac{U}{\Omega r}$
ρ	density of air
σ	rotor solidity, $\frac{bc}{\pi r}$
τ_o, τ_1	time constants

ϕ	inflow angle
ψ	rotor azimuth angle, $\psi = \Omega t$
Ω	angular speed at the axes of rotation
ω_θ	pitch frequency
$(\dot{\ })$	$\frac{d}{dt}(\ )$
$(^{*})$	$\frac{d}{d\psi}(\ ) = \frac{1}{\Omega}\frac{d}{dt}(\ )$

7. BIBLIOGRAPHY

1. Kim,Byung S., Control Setting for a Trimmed, Stalled Rotor by an Automatic Feedback System , Master of Science Thesis, Washington University, December 1980.

2. Kuester,James L.,and Mize,Joe H., Optimization Techniques with Fortran ,McGraw-Hill Book Company,1973.

3. Shinners,Stanley M., Modern Control System Theory and Application , Addison-Wesley Publishing Company,1978.

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