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THE LONARS-AIDED DOPPLER SOLUTION -  
A NEW METHOD FOR PRECISE POSITIONING AT SEA

by

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ABSTRACT

LONARS is a precision position-fixing system based primarily on the use of LORAN-C Radio Navigation. This system is used to precisely position test platforms in a 2500 square mile ocean area off the Florida coast. To meet future stringent accuracy requirements, APL and DMA personnel undertook an at-sea calibration of LONARS during April 1980.

The single pass Doppler solution using the Magnavox Geociever and DMAHTC precise ephemeris was chosen as the calibration standard. The LONARS system was used to model ship's drift during each Doppler pass. Included in the paper is an analysis of significant error sources in the Doppler fix, the utility of shore-ship single pass translocation, <sup>and</sup> a summary of operational problems encountered during the at-sea calibration. <sup>The estimated</sup> accuracy of LONARS-Aided Doppler was found to be 10 meters one-sigma.

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## INTRODUCTION

During the first half of 1980, a task to calibrate Loran-C over a 2500 square mile area off the coast of Cape Canaveral Florida was jointly undertaken by the Applied Physics Laboratory of the Johns Hopkins University (APL/JHU) and the Defense Mapping Agency Hydrographic Topographic Center, (DAMHTC) under the sponsorship of the U. S. Navy. The geodetic reference coordinates for this calibration were based on the Transit navigation satellite system. The Loran-C data was obtained by LONARS (Loran Navigation and Receiving System) developed by APL/JHU for the U. S. Navy.

A description of this calibration effort that focuses primarily on Loran-C issues has recently been reported by Fehlner and Jerardi [1]. The following discussion will be focused on the geodetic reference supplied by the Transit system after a brief introduction to the LONARS system.

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<sup>1</sup> A short description of the Loran-C system is given in Appendix A.

Footnote is on page 3

## THE LONARS SYSTEM

LONARS is a precision navigation system based on Loran-C<sup>1</sup>. The superior performance of LONARS is achieved by a novel application of Robust statistics to the tracking filters within the LONARS receiver [ 2 ]. Figure 1 displays a shipboard LONARS system. The core of the LONARS system is a Hewlett Packard 21MX-E minicomputer. The robust tracking filters are implemented in software within the 21MX-E.

### CALIBRATION CONCEPT

In order to fully realize the potential of the LONARS signal processing algorithms, a system calibration is required. The purpose of the LONARS calibration is the development and validation of a procedure to convert loran time differences to geodetic coordinates. This conversion process is composed of two components, a geodetic component and a propagation component. The geodetic component is the computation of geodetic arc lengths, from which range differences may be readily computed. It was determined that the geodetic component represented no particular problem, given the coordinate of the end points and that any of several arc length algorithms could be used. The Andoyer-Lambert [ 3 ] algorithm was used for the model development due to its simplicity. The propagation component is a model which accounts for Loran-C groundwave propagation in order to derive range differences from loran time differences.

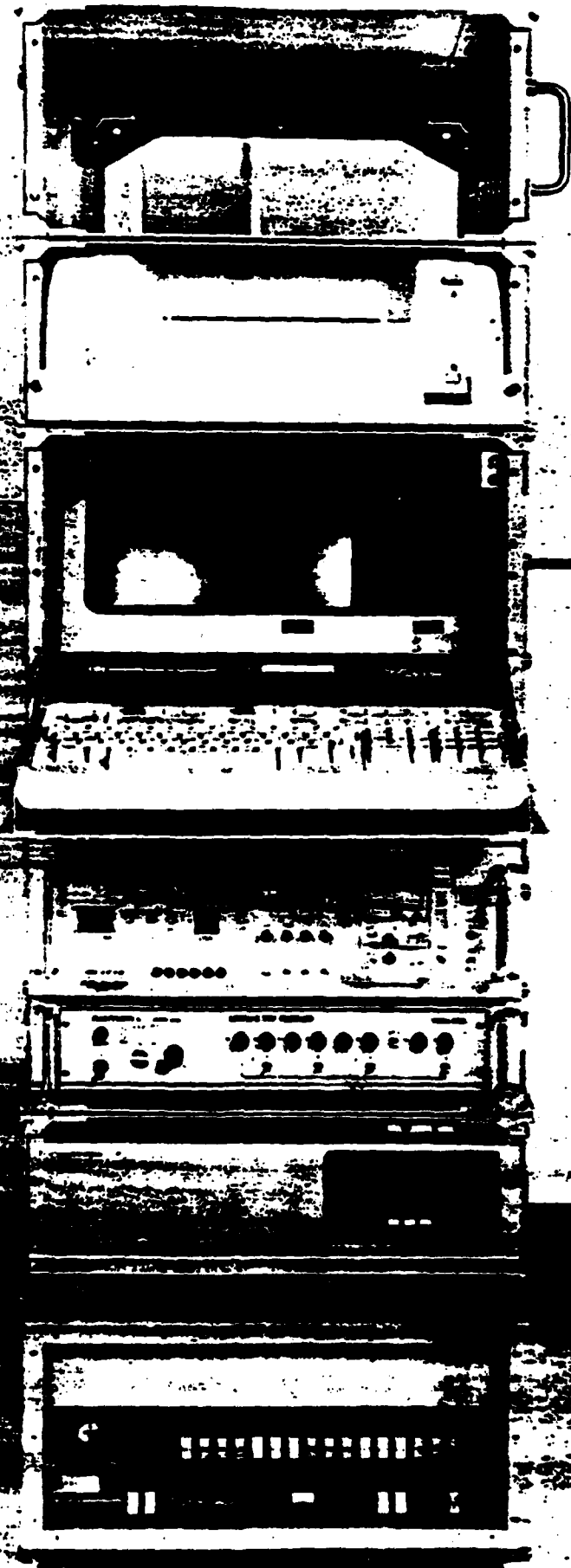


Fig 1



The general plan for any loran calibration is therefore defined as obtaining a coordinated set of loran time differences and corresponding geodetic coordinates. From the geodetic coordinates the range differences can be computed. These range differences and time differences are then the inputs to a regression procedure to determine various propagation parameters.

From statistical design considerations a uniformly spaced data set offers the most flexibility and accuracy for the subsequent regression analysis. Figure 2 graphically displays the overall data acquisition plan. The small solid circles represent the primary calibration data to be used for model development. The small solid squares represent a secondary data set to be used for model validation. A simple rectangular "site-code" grid is used for identification.

The quality of the geodetic coordinates is of vital importance in any such calibration. A clear choice for the reference system is a current state-of-the-art Transit Integral Doppler point position system, which could be adapted to handle platform motion. DMAHTC's DOPL79 program was thus chosen.

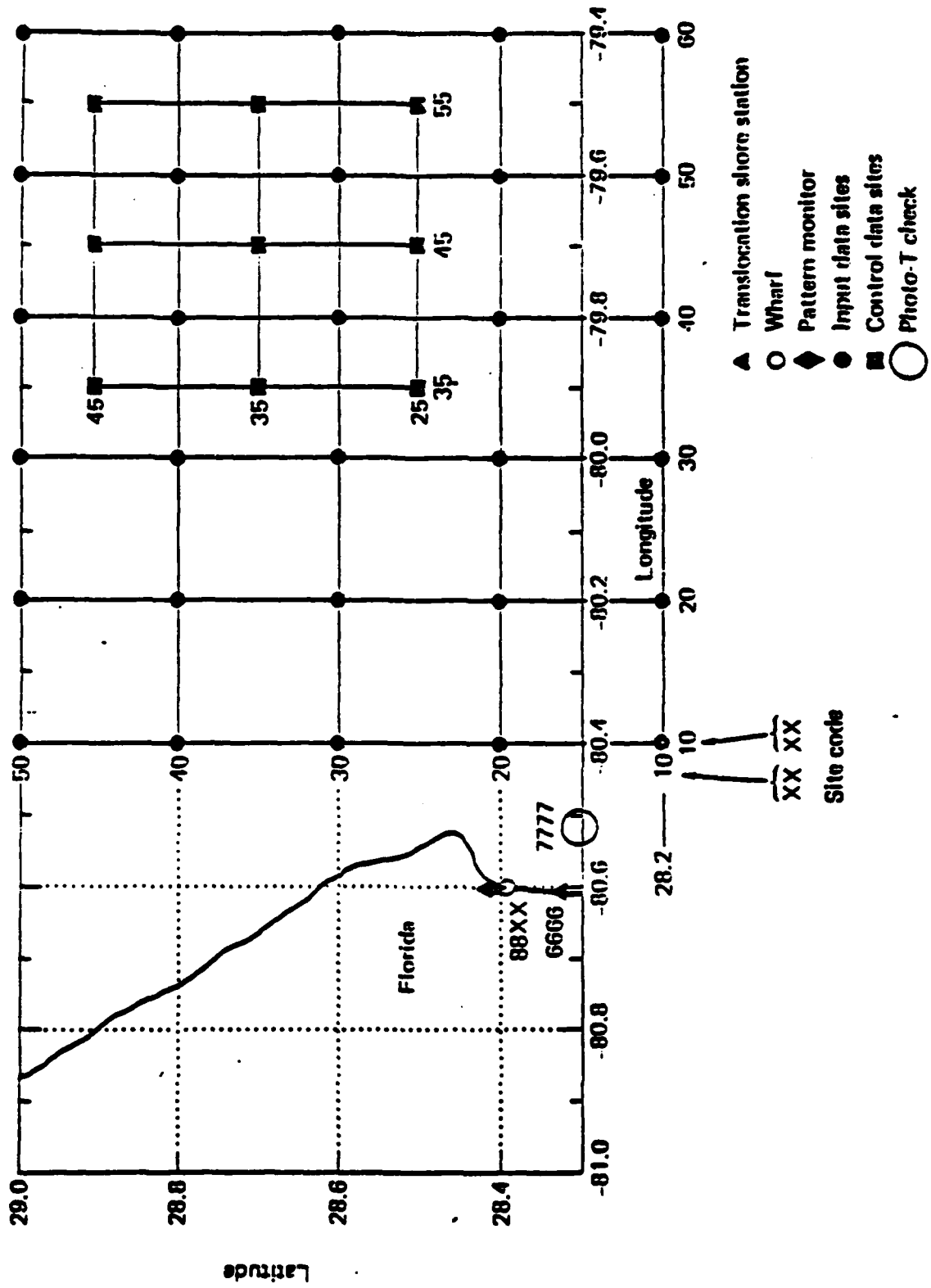


Exhibit-5 Loran calibration area showing planned survey sites.

Figure 2

## EVOLUTION OF DOPL79

The Defense Mapping Agency Hydrographic Topographic Center (DMAHTC) has had a <sup>single</sup> solitary point position reduction capability since 1971. The original software DOPPLR [4] was developed in 1970 as part of the effort in the geodetic community to achieve station position solutions of geodetic quality from the Doppler tracking of Transit Satellites (Navy Navigation Satellites, NNS), using the then newly developed Geceiver. Extensive testing of DOPPLR <sup>ca</sup> occurred during the Department of Defense Geceiver Test Program [5] which concluded that the <sup>single</sup> solitary point positioning mode of operation (NWL Precise Ephemeris held fixed) would be the primary approach to reduction of Geceiver data within DoD. The report assigned an accuracy of 1 meter for each component at the one sigma confidence level for a balanced set of 30 - 50 Transit passes. The stated accuracy was arrived at through comparisons with the High Precision Geodimeter Traverse in the United States.

As a result of great interest in achieving submeter positioning accuracies, an updated and recoded version of the original program began production use in January, 1979. The new version, called DOPL79, carried a number of minor model changes, addition of tropospheric refraction scaling parameter, changes in the data editing function, and a more accurate ephemeris interpolation procedure [6]. Although no external comparisons have been done, the one sigma precision estimate for 30 - 50 TRANSIT passes, assuming properly functioning standard equipment, <sup>is</sup> .70 meters for each axis. This number is based on reductions of

several thousand passes from a number of semi - permanent tracking stations.

Program DOPL79 is the last processing stage of a complex of Fortran 66 computer programs, together called the Doppler Geodetic Point Positioning (DGPP) system residing on the Univac 1100/81 computer at DMAHTC. The DGPP system will process a variety of receiving equipment formats into nominal 30 second Doppler count data, access the appropriate Precise Ephemeris (PE) spans which reside on removable disc packs, and perform the adjustment. Raw data on magnetic tapes created on an Interdata mini computer is the normal input mode for DGPP. The PE has since May, 1975 been computed in a routine production fashion at DMAHTC.

## CALIBRATION DATA FLOW

Prior to this calibration the geodetic accuracy of LONARS was dominated by errors with very high spatial correlation. This fact allowed LONARS data to be used to compensate for platform motion in the Transit solution. In order to minimize the effect of correlated error<sup>s</sup> within the Transit system a translocation technique was used.

Figure 3 indicates the overall data flow to support this scheme. The Doppler data from the ship and shore sites was forwarded to DMAHTC via Autodin where it was handled independently through the reformatting and preprocessing steps. The LONARS data was processed by APL/JHU and a tape file of ship's position was supplied to DMAHTC. A file of loran time differences was also created for the actual calibration computation. As figure 3 indicates the final doppler adjustment program requires three data inputs—fixed site doppler related data, ship doppler related data and LONARS ship motion data.

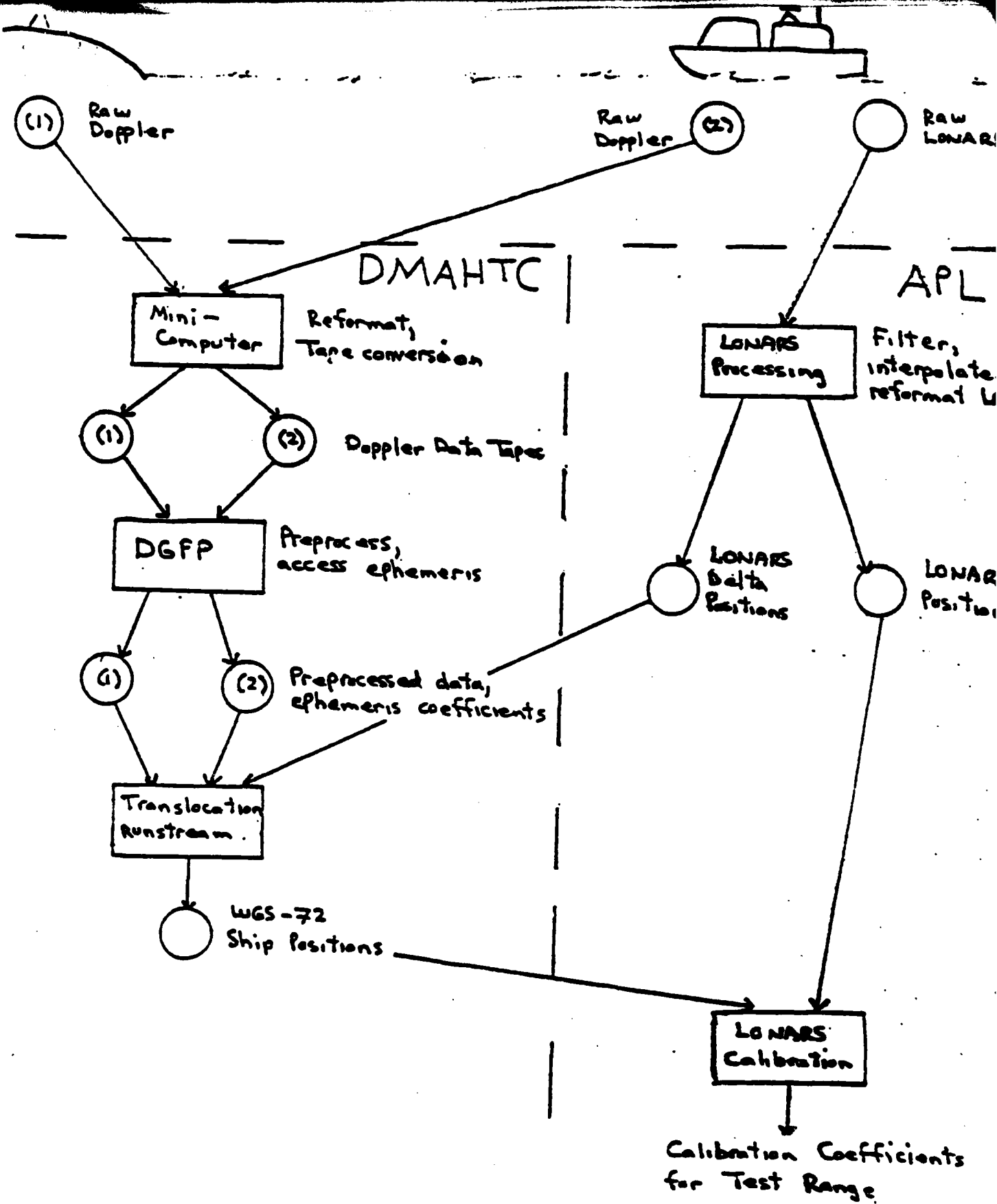


FIG (3) LONARS PROJECT DATA FLOW

## MODIFICATIONS TO DOPL79 SOFTWARE

The motion compensated, height constrained single pass solution with ship - shore data point matching was used to determine the relative position of the ship at a chosen epoch with respect to the shore station. This involved the following steps (See Fig. 4).

- a. Perform a single pass solution for the shore station with 2.5 sigma data editing.
- b. Perform a motion compensated single pass solution for the ship with 2.5 sigma data editing.
- c. Determine the non-deleted observations common to both solutions.
- d. Repeat a. and b. except disable data edit function and only allow common points from c. into the solutions.
- e. Difference cartesian coordinates from d. and apply to the shore station reference coordinates to obtain the final ships coordinates.

To perform the listed steps in a more or less automated fashion required modifications to existing software. A breakout of DOPL79 into functional modules is shown in Fig. 5 with modifications for this project shown in parentheses.

### SINGLE PASS PROCESSING CAPABILITY

Production multipass processing in DOPL79 involves a batch fit to all nonrejected data, and the navigation solution is not required. This capability was coded as an additional functional module (SINGLE) in the program. A two-dimensional solution in either the horizontal plane or the Guier plane may be output.

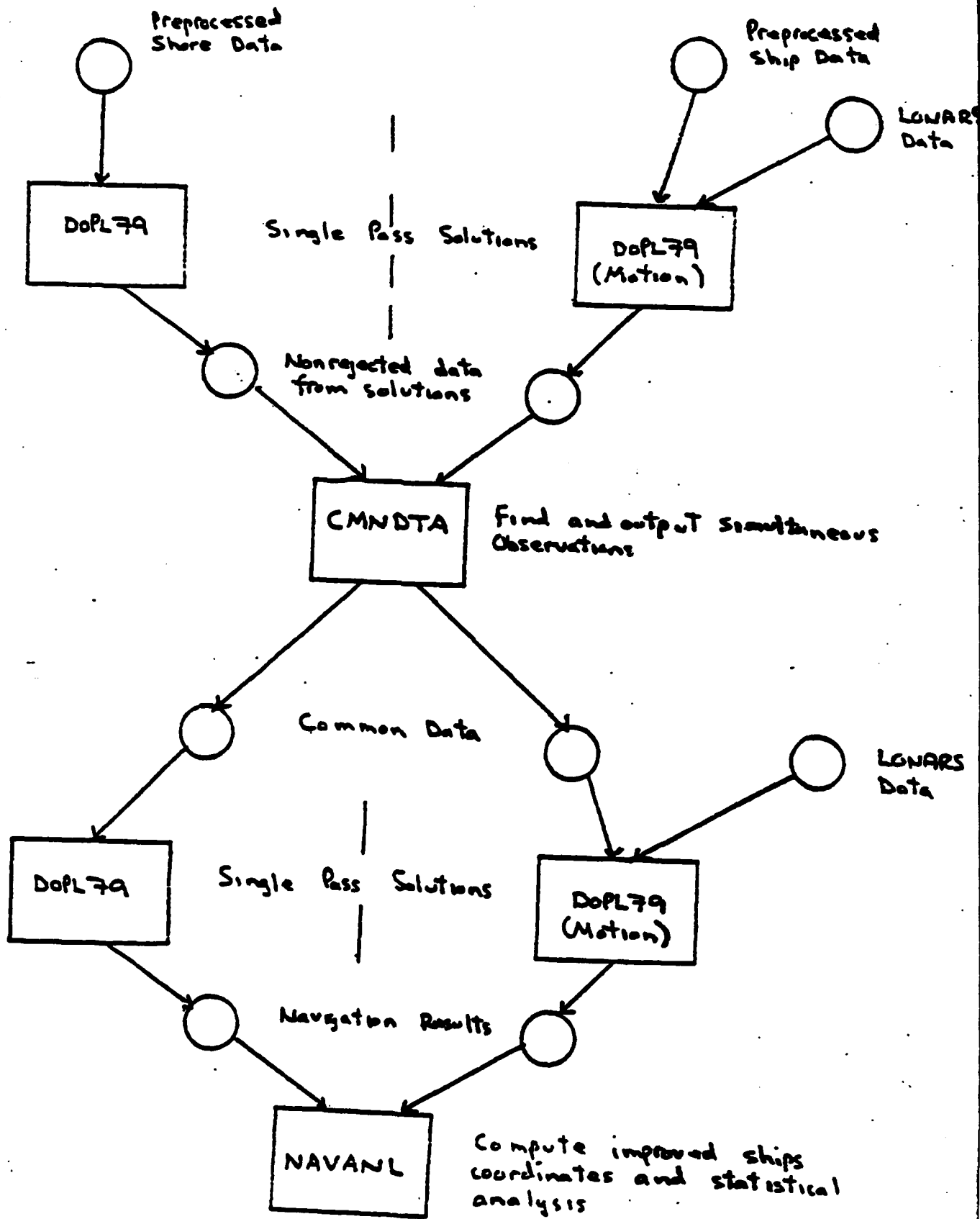


FIG. (4) LONARS PROJECT TRANSLOCATION RUNSTREAM



DoPL79

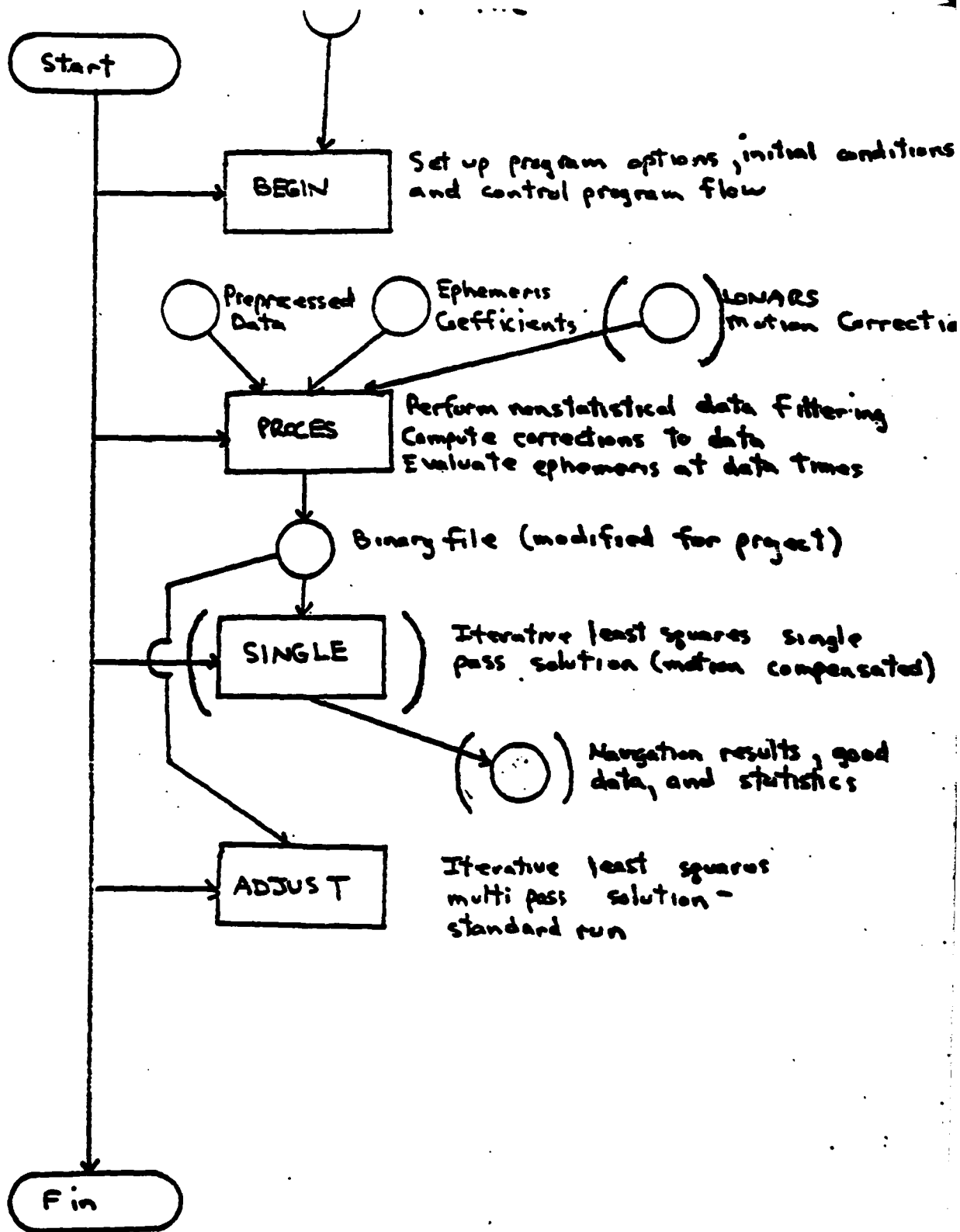


FIG (5) DoPL79 PROGRAM FLOW WITH MODIFICATIONS FOR PROJECT IN PARENTHESES

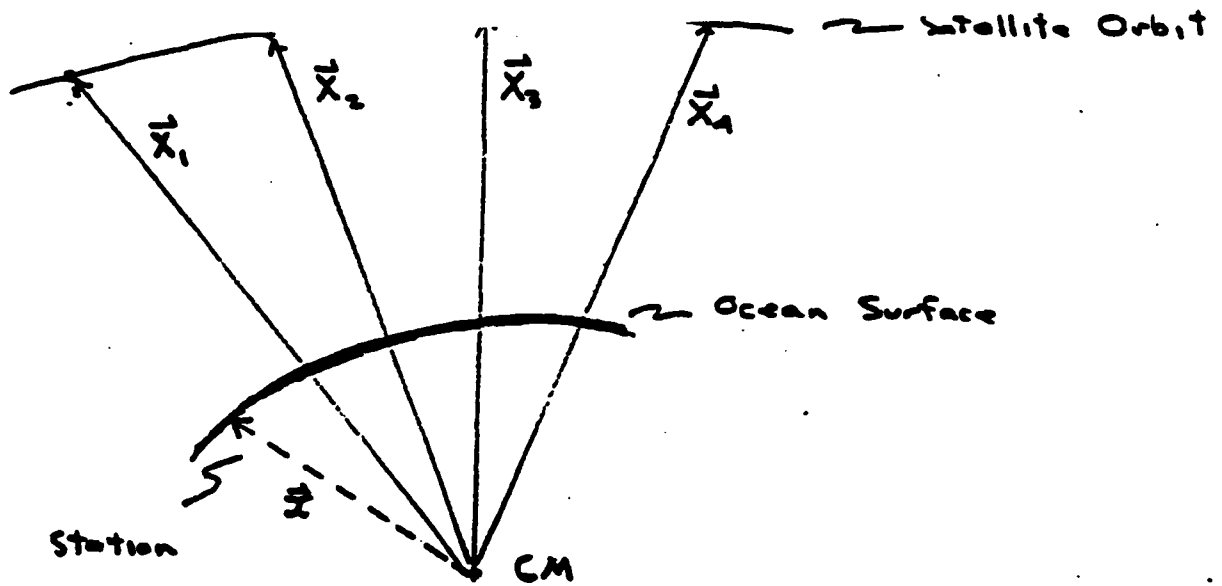
The measurement model used was identical to the multipass case, except the time delay parameter was assumed known and the tropospheric scaling parameter was not determined. The model is given in more detail in Appendix B.

#### SHIP MOTION COMPENSATION

To solve for ships coordinates at a particular epoch, the change in ships position with respect to its position at epoch must be known a priori over the duration of a Transit pass. Including these "delta position" corrections into the normal computation of slant range difference will correct the data for the effect of a drifting platform, leaving only the signal due to a fixed site (Fig. 6). Note that the nature of the measurement model requires that the position corrections be known only at the end times of each Doppler counting interval.

The LONARS position measurements were smoothed and interpolated for Transit emit times at APL. Satellite alerts generated at DMAHTC provided the appropriate time spans. An epoch was chosen several minutes before scheduled rise time and LONARS delta positions (LDP's) were generated with respect to that epoch. This epoch became the epoch time of the Doppler fix, and the LONARS derived horizontal coordinates became the initial horizontal coordinates in the solution. The ellipsoid height at which the solution was constrained was obtained by adding the known MSL height of the antenna to the NASA GEM 10-B geoid height, interpolated from a grid generated for the survey area.

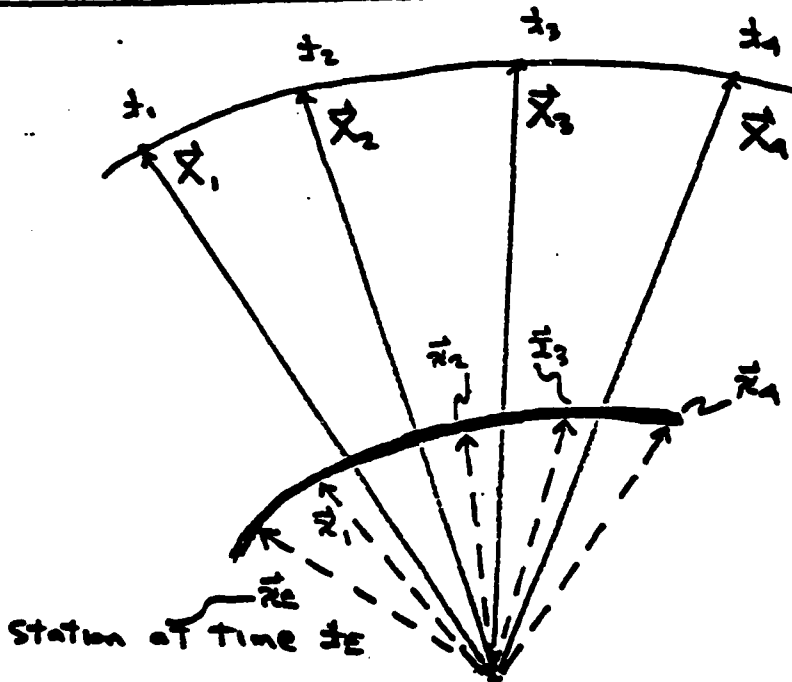
The LDP's were input to DOPL79 as an additional input file in a pass by pass matchup with the standard Doppler data and PE



• Standard Solution model

$$i^{\text{th}} \text{ Range Difference} = |\vec{x}_i - \vec{r}| - |\vec{x}_{i-1} - \vec{r}|$$

Solve for  $\vec{r}$



• Moving platform model

$$i^{\text{th}} \text{ Range Difference} = |\vec{x}_i - \vec{r}_i| - |\vec{x}_{i-1} - \vec{r}_{i-1}|$$

$$= |\vec{x}_i - (\vec{r}_E + \Delta \vec{r}_i)| - |\vec{x}_{i-1} - (\vec{r}_E + \Delta \vec{r}_{i-1})|$$

where  $\Delta \vec{r}_i$  is the  $i^{\text{th}}$  LORANS Delta Position (LDP) correction

Solve for  $\vec{r}_E$

coefficients. In module SINGLE, the observation equation and the data partials were modified to accommodate the LDP's. The following mathematical approximations were made due to the small drift over a pass (less than 3 Km) or the small instantaneous velocity (less than 3 meters/sec) of the ship:

- a. The ship's velocity contribution to the equipment delay range rate terms ( $\dot{\rho}$ ) was ignored.
- b. The LDP's were evaluated at the satellite emit times instead of the station receive times.
- c. The initial coordinates were used in all tropospheric model computations.

These modifications are detailed in Appendix B.

#### TRANSLOCATION RUNSTREAM

Because no orbit corrections were applied, the proper name for the method used is simultaneous single pass point positioning with common data enforced. Once the processing sequence mentioned on page 8 was automated, computations generally proceeded smoothly. Normally 15-20 pairs of passes were processed at a time. The shore station reference coordinates were the result of a 40 pass DOPL79 solution observed before the start of the at-sea campaign. The program performing (e) provided data residual correlation analysis and quality control flags for each pair of passes. Among the quantities monitored were data residual RMS, two-frequency ionospheric corrections, shore station navigation errors, number of times loss of lock occurred, and amount of common data.

## PREMISSION STUDIES

Prior to the field data collection operation a number of small studies were conducted in order to understand the effect of various errors sources. These studies also allow the modified software to be tested under more controlled conditions.

### COMMON AND NON COMMON ERROR SOURCES

An enumeration of the error sources in a single pass Doppler fix would include the following:

- a. Satellite ephemeris
- b. Satellite antenna electrical center
- c. Satellite oscillator
- d. Higher order ionospheric refraction
- e. Residual tropospheric refraction
- f. Constrained height of solution
- g. Preamp, receiver noise
- h. Tracking antenna electrical center
- i. Local environment - RFI and multipath effects
- j. Unmodeled station motion during a pass

Simultaneous Doppler tracking by two stations in geometrically similar positions with respect to the satellite allows a more accurate relative positioning because terms (a. - e.) tend to be of the same magnitude and sign, and have a cancelling effect. Error sources (g. - j.) are independent by station and define the theoretical limit of accuracy attainable using translocation methods.

Note that for a ship - shore tracking pair item j. is a non cancelling error. The LDP corrections turned out to be the

single largest error source in this project, some what limiting the normal improvement allowed by translocation.

#### SINGLE PASS TRANSLOCATION TESTING WITH REAL DATA

Some premission testing with existing land based data sets was done to exercise the single pass software and to develop insights into PE/DOPL79 single pass accuracies. In all runs, a 5 degree data end point cut off was used, and reference coordinates were taken from the multipass solution results. Unweighted RMS navigation errors for individual stations and between stations are given in Table 1.

#### PASS EDITING AND TRANSLOCATION ACCURACY

Two Ohio, USA data sets tracked at separations of .5 degrees in latitude and .8 degrees in longitude had 16 common passes. With no selective pass editing, a slight improvement in relative precision over individual station precision is seen (Table 1, Test 1). If certain pass pairs are deleted based on output statistics from each solution, improved relative results are obtained (Table 1, Test 2). While only around half of the data was used, the repeatability is at the several meter level. Edit criteria were developed empirically and included quantities such as solution data variance, but were not based on the size of the individual navigations.

#### COLLOCATION TESTING

Two standard geociever sets (separate antenna, preamp, and oscillator) tracked 33 common passes at the DMAHTC Herndon, Va. Electronics Lab in February, 1980. Antenna separation was roughly

Test	Station	Passes Input/Used	$\theta, m$	$\lambda, m$	$\Delta\theta, m$	$\Delta\lambda, m$
1	Ohio 1	16/14	3.0	5.2	1.3	4.9
	Ohio 2		2.4	3.9		
2	Ohio 1	16/8	3.9	4.0	1.2	2.0
	Ohio 2		3.1	3.6		
3	30682A	33/29	1.3	1.7	0.4	0.7
	30682B		1.3	1.9		
4	Ohio 1	16/8	3.9	4.0	1.6	7.9
	Ohio 2		3.5	6.3		
5	Ohio 1	16/8	4.2	5.0	1.2	2.9
	Ohio 2		3.5	6.3		

TABLE (1) TRANSLOCATION TESTING WITH REAL DATA

↑ meters. The relative positioning results (Table 1, Test 3) indicate that the noise contribution of the tracking equipment to translocation accuracies is below the 1 meter level. The low single station errors are due to collocation with TRANET STATION 407, which is used to reduce the PE.

#### MONTE CARLO STUDIES WITH SYNTHETIC DATA

Twenty five passes from Station 1 (Ohio) were used to simulate data and model errors which would be encountered in the reduction of the ships Doppler data. Two of the errors, wave motion and linear growth in ships position, are related to the LDP corrections. The third error studied was constraint of the solution at a height which may be inconsistent with the PE system. RMS of navigation errors were examined as in the previous section.

#### UNCOMPENSATED WAVE MOTION

Data was generated from a sine wave with period of 15 seconds and 2 meters peak to peak. The perturbation was applied separately in the North, East and Up directions. RMS navigation errors were always 3 meters or less, and the solution did not appear overly sensitive to this effect.

#### LINEAR GROWTH IN STATION POSITION

A ramp error in station location of 1 meter/minute was applied as an LDP error separately in the North and East directions. Longitude navigations were found to be very sensitive to perturbations in the North direction, with an RMS navigation error of over 20 meters clearly exceeding the error budget. Other combinations were all less than 7 meters.



## HEIGHT CONSTRAINT

For a ship at sea the station height is the most natural coordinate to be considered completely known in the Transit navigation solution. To study the propagation of height error into the navigation, the Ohio translocation results (Table 1, Test 2) were taken as a standard. The run was repeated with the height of Station 1 constrained at 5 meters above its optimal (multi-pass) value as shown in Table 1, Test 4. Note that the relative latitude precision is about the same, but the relative longitude precision degrades by a factor of 4. A second run was made with both station heights increased by 5 meters, and the relative longitude was similar to the standard run (Table 1, Test 5).

These results imply that the differential height error in a translocating pair is a critical factor. It is apparent that some inconsistency may result in using a PE-derived ellipsoid height for the shore station and a gravimetric (geoid + MSL) height for the ship. Thus the NASA GEM 10-B geoid model was used to generate heights for both shore and ship reductions.

## ERROR STUDY CONCLUSIONS

The solution longitude shows the greatest sensitivity to the aforementioned errors. It was decided that passes below 15 degrees at Time of Closest Approach (TCA) would not be tracked to reduce the effect of propagation errors and wave motion. To alleviate <sup>adverse</sup> error propagation into longitude due to translocation geometry, motion errors, and differential height errors,

passes with TCA above 70 degrees were not considered. Also an attempt was made to balance a pass east of station with a pass west of station in a given area to average through geometry dependent errors.

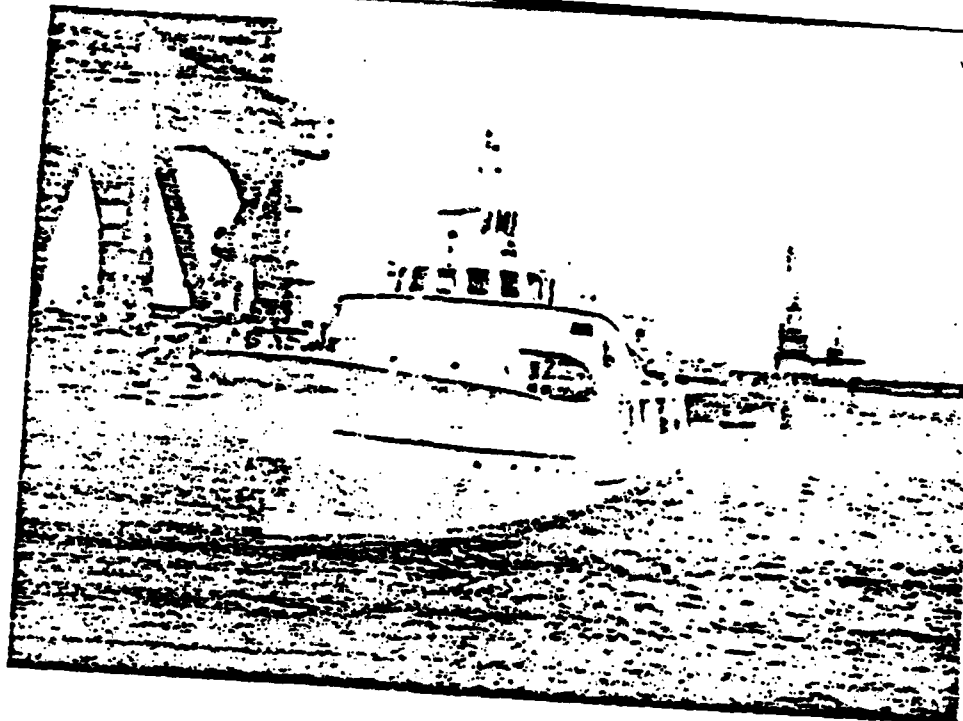
It was concluded that the error budget of 15 meters could be met if the GEM 10-B geoid was differentially accurate to 1-2 meters and if motion errors were .5 meter/minute or less.

#### DATA COLLECTION

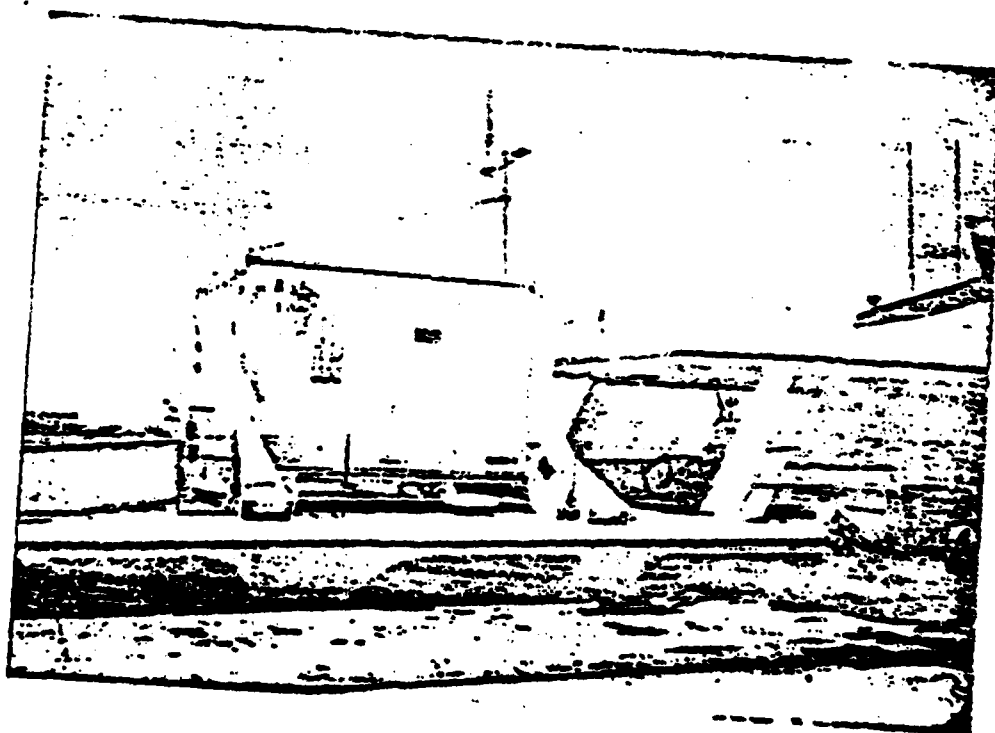
The data collection effort was conducted during April of 1980, using the Research Vessel EL TORO (see Figure 8). The instrumentation (Geoceivers and LONARS) was housed in an equipment module located on the aft-upper deck of the EL TORO (see figure 8).

Figure 9 indicates the actual data collected. The Transit data collected was from satellites 30130 (59), 30140 (60) and 30190 (60). As indicated in Figure 9, five Transit passes were taken at a point denoted "Photo-T area". In this area, precision Photo-Theodelite tracking was used during the initial checkout. This checkout allowed for both equipment performance evaluation and to prove-in the data collection procedures.

The essential feature of the data gathering procedure was that the vessel was dead-in-water during the Transit passes. This drift process allowed for simpler models to be used to describe the platform motion during the Transit passes. Pre-mission analysis indicated that if the vessel were "driven" during the pass, the various "controlled" ship tracks might prove difficult to accommodate.



*Figure 2* EL TORO.



Equipment module mounted on EL TORO.

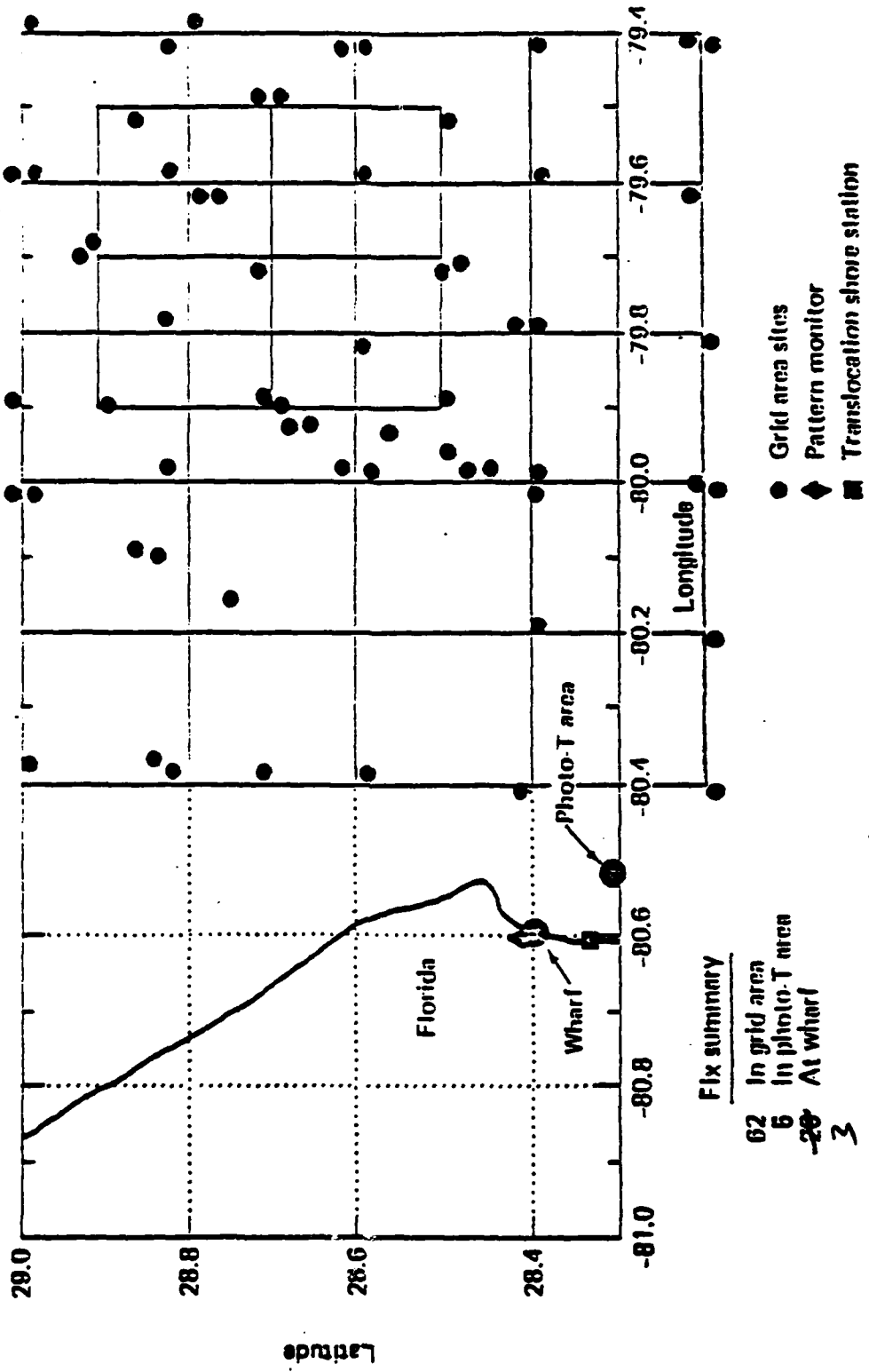


Exhibit I. oron calibration area showing survey sites visited.  
 Figure 9

## DOPPLER DATA ANALYSIS

A total of 68 valid passes were taken between 15 April and 25 April 1980 by the ship while at sea. During that same period 86 passes were recorded at the shore station. Three passes were recorded by the ship while moored to the wharf.

It appears that less than 5% of the Doppler fixes taken were bad due to ephemeris quality or receiver malfunction. Histograms of single pass RMS data residuals and navigations for the shore station are given in Fig. 10. Note that the shore navigations are generally of excellent quality, with mean errors of 2.3 meters in latitude and 3.3 meters in longitude. The shipboard RMS data residuals (Fig. 11), with a mean of .90 meters, are nearly 5 times greater on average than those of the shore station, reflecting the error effects discussed earlier. The magnitude of this superimposed noise effectively disabled the outlier rejection process that would occur at lower noise levels. Correlation coefficients computed for the ship and shore Doppler residuals were generally less than  $\pm .2$ . The ship residuals consistently showed greater structure. Several of the shipboard passes recorded during periods of high sea state were rejected because their data residuals were significantly higher than shown in Fig. 11.

The 5 Photo-T and 3 dockside passes allow for an independent evaluation of the overall system performance. Figure 12 is a plot of the position errors from these two sets. The dockside fixes used the LONARS data for platform motion compensation, even though the actual ship's velocity was zero. A number of

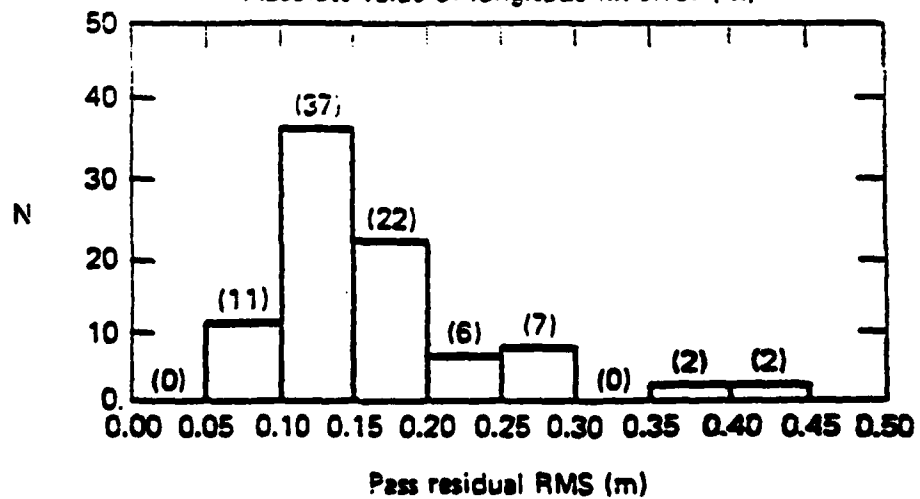
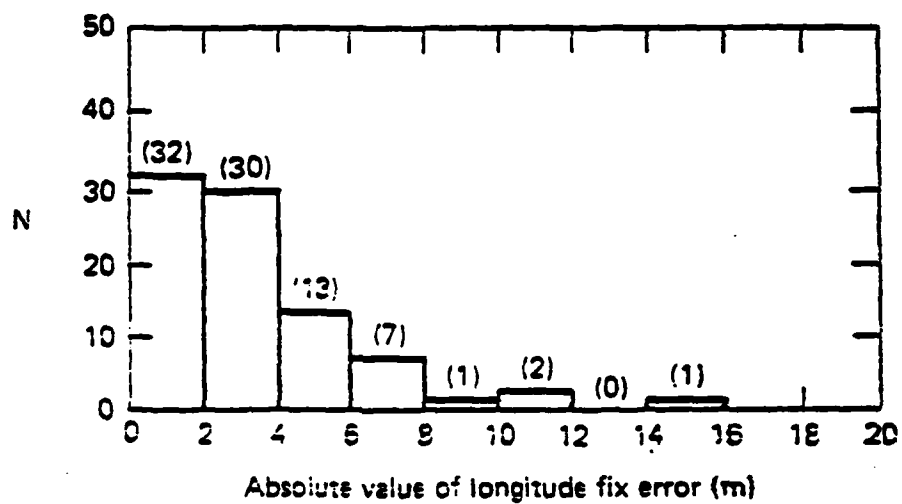
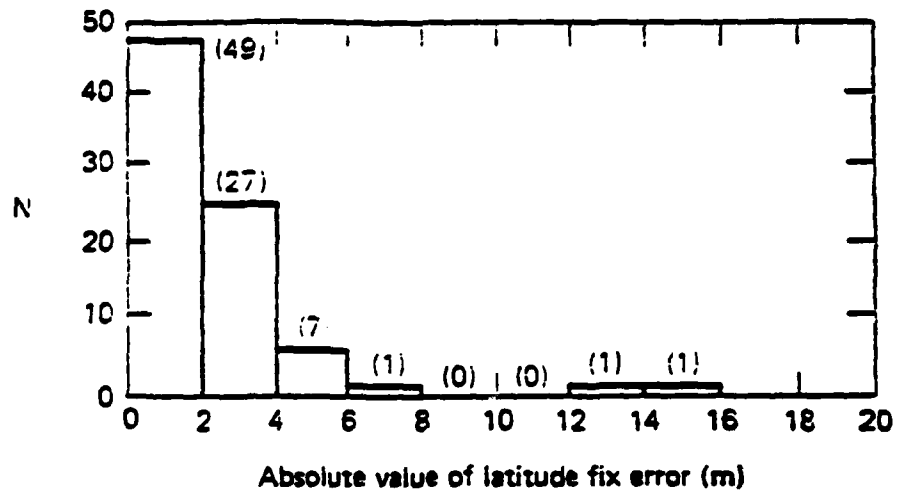


Exhibit Shore station single pass statistics, 13 through 25 April.

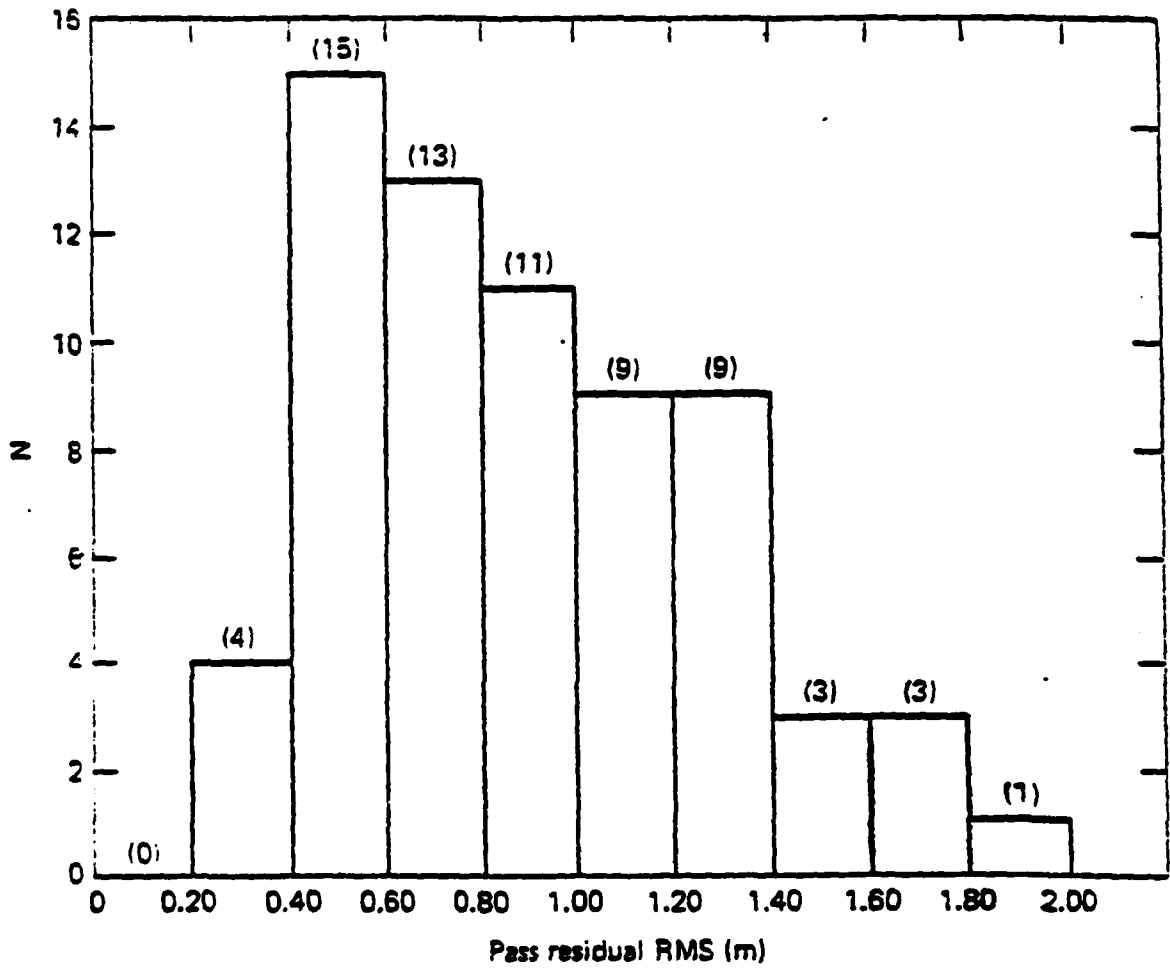


Exhibit // Mobile station single pass statistics, days 13 through 25 April.

dockside passes were lost due to a equipment malfunction, that went unnoticed for several days. We were thus left with only 8 independent samples. Even this small sample size indicated the validity of the premission studies and the full scale data collection proceeded.

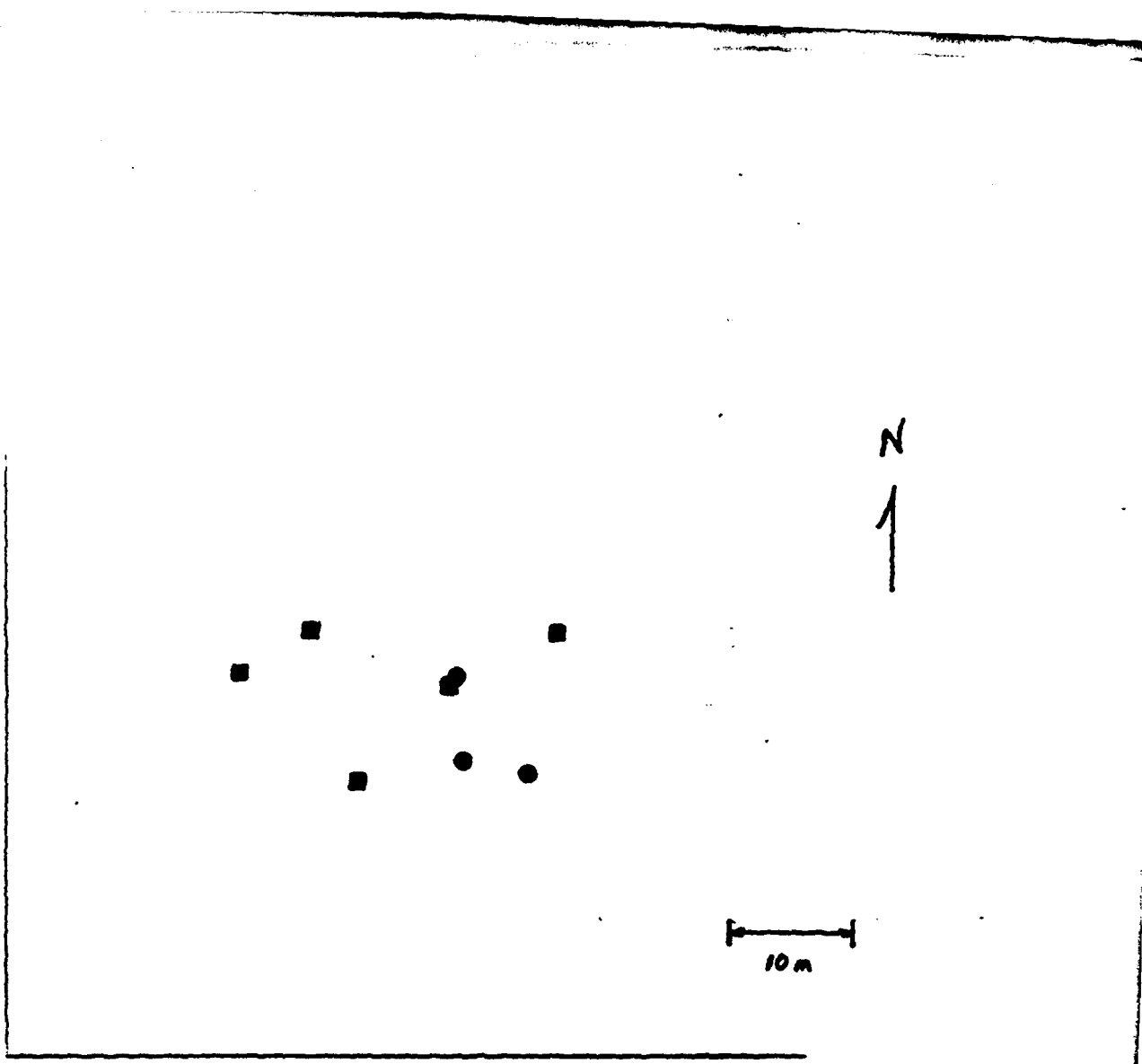
#### CALIBRATION RESULTS

A detailed discussion of the Loran-C propagation model that was developed is given in Reference 1. We will summarize here the major findings. A single ground-wave propagation velocity could be used for all 3 propagation paths (Malone, Jupiter and Carolina Beach). The value of this sea-water (4-5 mhos/meter) propagation velocity is  $299.569 \pm 0.012$  meters/microseconds. Since the Jupiter and Carolina Beach paths are essentially total over sea-water, this leaves only an offset parameter (emission delay, see Appendix A) to be determined.

The propagation from Malone is a mixed path, part land and part sea. The effect of this mixed path can be accommodated by a correction term dependent on the azimuth to the Malone transmitter. The rationale of the correction term is based on the fact the over the area of interest the <sup>R</sup>fraction of land path to total path varies linearly with azimuth. As above there is also an offset term (emission delay) required.

There are several facets of this mixed path that have not been explored (e.g. coastal refraction). The overall LONARS system accuracy using the above model is estimated to be better than 16 meters radial over the entire area. Since the requirements were met there has been little support for further study of this mixed path propagation.





- Photo-T
- Dachende

Figure 12

As a byproduct of the LONARS calibration data analysis we obtained estimates of LONARS aided Doppler fix accuracy. The error distribution is elliptical with the Longitude about 10 meters 1 sigma and Latitude about 5 meters 1 sigma.

#### CONCLUSIONS

From the point of view of "surveying" at sea, we feel that 10 meters accuracy per pass can be readily achieved with current equipment and software. In order to substantially reduce this, further development is needed. The Longitude error magnitude clearly indicated where the effort is required. Addition instrumentation for roll, pitch and heave (vertical velocity) will likely be required.

The use of "Translocation" did not significantly reduce the position errors but this procedure provided an excellent quality control tool for detecting errors in the ephemeris data. For this reason translocation is recommended even when using the Precise Ephemeris.

## APPENDIX A

### THE LORAN-C SYSTEM

Loran-C is a low frequency (100 khz), pulsed, hyperbolic navigation system. The geographic arrangement Southeast U.S. Loran-C chain is given in Fig. A.1. A chain is composed of 3 or more stations. One station is designated as the master and all other stations are designated as secondaries. A chain is identified by its Group Repetition Interval (GRI) which is the period (in tens of microseconds) between pulse groups that each station transmits. The Southeast U. S. chain has a GRI of 7980 which means the stations transmit periodically with a period of 79800 microseconds.

Each station transmits a group of 8 pulses that are separated by one millisecond. This group of pulses allows a phase coding process to be used so that the master and secondaries to be uniquely identified. The overall chain signal format is by time sequencing the stations so that no two signals are received simultaneously.

The transmission pattern is as follows:

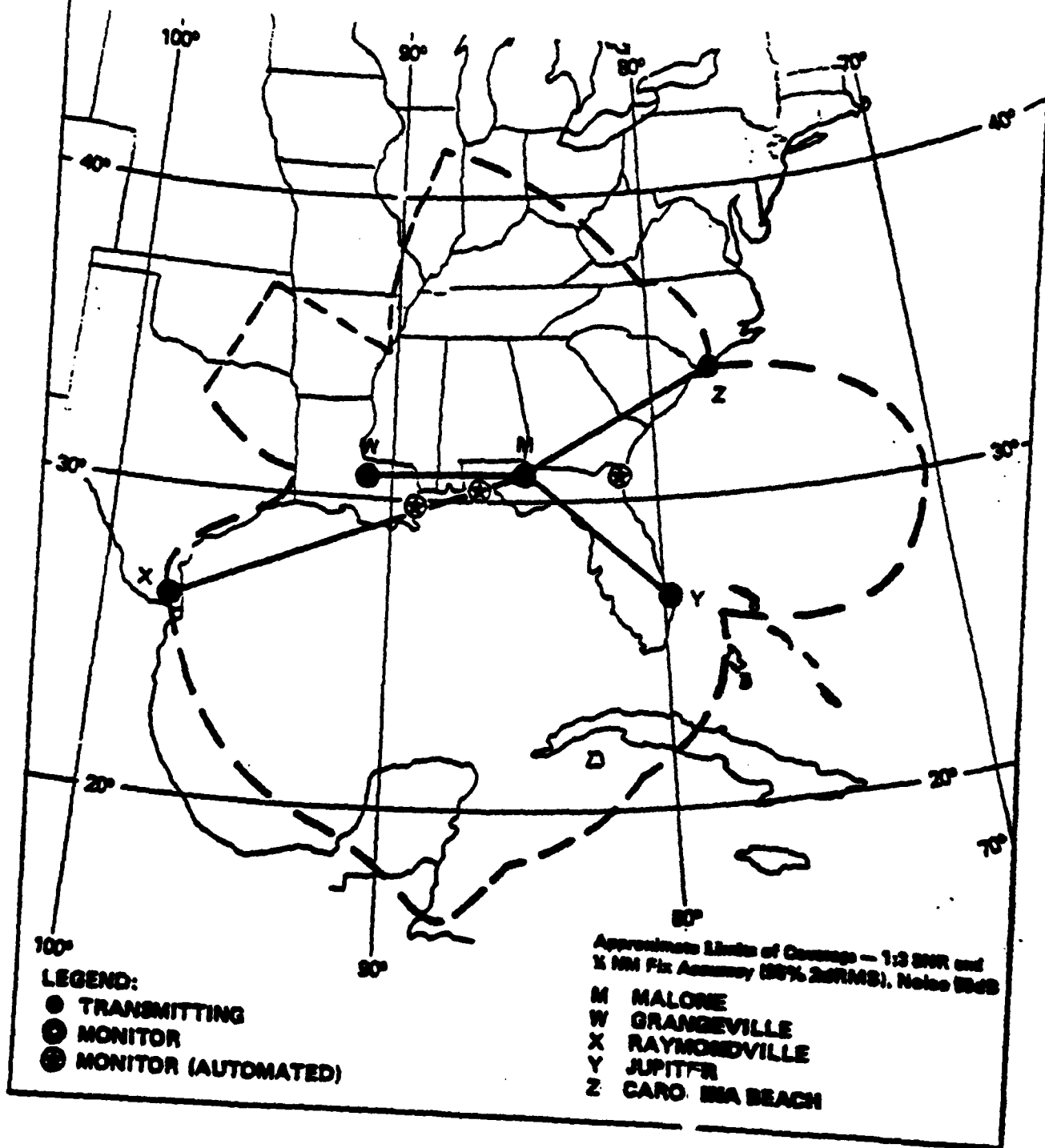
1. The master transmits its coded group of 8 pulses.
2. At a fixed time after the master transmission (known as emission delay) the first secondary transmits its coded group of 8 pulses.
3. Sequentially the other secondaries transmit each at a unique emission delay as in 2.
4. At the GRI the master again transmits and the entire process is repeated.

A receiver in the service area tracks (i.e. measures the phase with respect to a local clock) the various stations in a

# LORAN-C

## SOUTHEAST U.S. CHAIN

### GRI 7980



chain. This process yield the times of arrival (TOA's) of the various stations with respect to a local clock. From three such TOA's two time differences (TDs) can be formed by using one of the TOA's as a reference TOA. These time differences are the fundamental Loran-C coordinates. The lines of constant time difference are hyperbolas with the stations as foci.

## APPENDIX B

### TRANSITPOSITION FIXING OF A MOVING PLATFORM A

#### B.1 Standard (Stationary Receiver) Solution

A single pass solution modeling of Geceiver Doppler observations in the reduction program DOPL79 (program file DOPPLR-test) can be described by the following observation equation:

$$O_i \approx C_i = DR_i/\gamma + \Delta FAT_i + ION_i + CORR_i + DR_i * TD/\gamma$$

Where

$O_i$  = iTH observed Doppler count

$C_i$  = iTH computed Doppler count

$DR_i$  = iTH station to satellite range difference, computed from the iTH and i + 1TH station to satellite ranges.  $DR_i$  implicitly contains the station location parameters  $\bar{X}$ .

Thus

$$DR_i = R_{i+1} - R_i$$

And

$$R_i = \sqrt{(X_{SATi} - X)^2 + (Y_{SATi} - Z)^2 + (Z_{SATi} - Z)^2}$$

$\gamma$  = wave length of transmitter Doppler signal  
 $\Delta F$  = satellite-station frequency offset parameter  
 $\Delta T$  = ith integration interval at the satellite  
 $ION_i$  = ith two frequency ionospheric correction  
 $CORR_i$  = a set of correction terms applied to the ith observation including the following:

correction

$Cl_i$  = corrects ith count for propagation times of the Doppler signal  
 $PC_i$  = corrects ith count for residual difference between ground clock and satellite clock intervals  
 $ERC_i$  = corrects ith count for earth rotation correction effect  
 $DR_i$  = same as  $DR_i$  except applies to instantaneous range rate instead of range  
 $TD$  = equipment delay, assumed known

The three cartesian coordinates and frequency offset are carried as unknowns in a linearized, iterative, constrained least squares solution. The priori constraint (Guier plane\* or station height) is applied to the normal matrix as a weight matrix in cartesian space. Convergence is satisfied when the current coordinates change less than 0.01 meter between iterations. Outlier data point stripping is done at 2.5 times rms of residuals.

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\* W. H. Guier, Studies on Doppler Residuals - I: Dependence on Satellite Orbit Error and Station Position Error, APL/JHU: TG-503, June 1963.

In the standard solution the following terms are computed from an approximate initial coordinate  $\bar{X}^0$ :

$$\text{TROP}_i = \text{TROP}_i (\bar{X}^0)$$

$$\text{PC}_i = \text{PC}_i (\bar{X}^0)$$

The values of  $\bar{X}^0$  in error by more than 10 kilometers will show decimeter level changes in the fix position. For the constrained height solution the weight matrix is recomputed if  $\bar{X}^0$  is more than 100 meters off.

## B.2 Modifications for a Moving Platform

The fundamental difference is that  $\bar{X}$  becomes  $\bar{X}(t)$  where  $t$  is time. Thus any computation involving station position becomes time dependent. The LONARS delta position inputs are correction terms  $\Delta\bar{X}_i$  at time  $T_i$  to the station position  $\bar{X}_E$  at a certain epoch  $T_E$ . The constant  $\bar{X}$  is replaced by

$$\bar{X}_i = \bar{X}_E + \Delta\bar{X}_i$$

The station location parameters are now  $\bar{X}_E$ .

- a. The range is then

$$R_i = \sqrt{(X_{\text{SAT}i} - X_i)^2 + (Y_{\text{SAT}i} - Y_i)^2 + (Z_{\text{SAT}i} - Z_i)^2}$$

- b. The partial derivative with respect to station location is updated with  $\bar{X}_i$
- c. The terms  $Cl_i$  and  $ERC_i$  are computed with  $\bar{X}_i$
- d. The contribution of ground station instantaneous velocity to the  $\dot{D}R_i$  terms is ignored.



- e. The terms  $TROP_i$  and  $PC_i$  are evaluated with the initial epoch position  $\vec{X}_E^0$  only:

$$TROP_i = TROP_i (\vec{X}_E^0)$$

$$PC_i = PC_i (\vec{X}_E^0)$$

- f.  $\Delta\vec{X}_i$  are evaluated at satellite emit times instead of station receive times.

For a vessel freely drifting on the ocean surface with  $\vec{X}_E^0$  derived from the LONARS system, the approximations made in d. e., and f. are valid.

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