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EXPERIMENTAL TESTS OF THE ASTROMETRIC PRECISION OBTAINABLE WITH--ETC(U)
JAN 82 C H TOWNES, E C SUTTON, S SUBRAMANIAN N00014-80-C-0383

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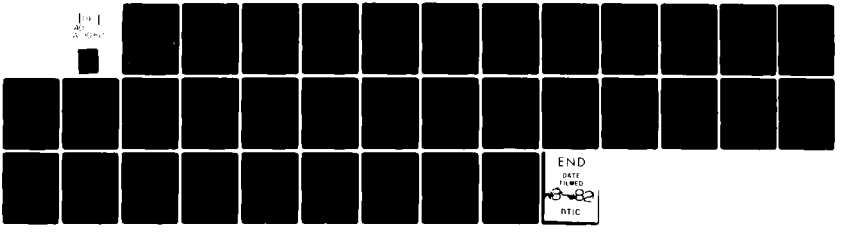

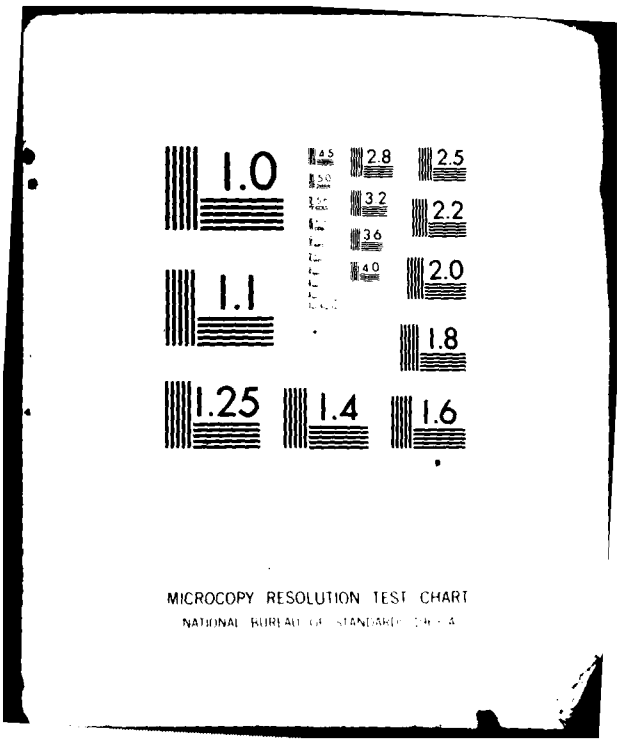
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The purpose of this project was to test whether the relative positions of stars could be measured with an infrared spatial interferometer to high accuracy--that is, 0.1 seconds of arc or better. For such a test, a number of measurements were made on the relative hour angles of three stars, o Ceti, α Orionis, and R Leonis, each separated by about 50°. About ten measurements of o Ceti and α Ori were made each night for 10 nights during a two-week period. The RMS deviation between measurements on a single night was

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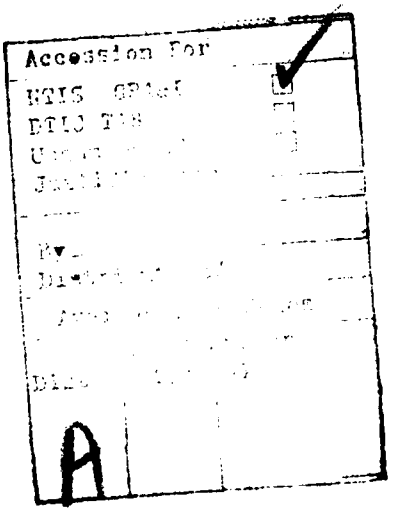
approximately .07 arc seconds and the RMS variation from night to night among the 10 different nights was 0.08 arc seconds. Similar but less extensive results were obtained on R Leonis. This night-to-night positional accuracy is somewhat better than is normally achieved with the best optical techniques, and clearly of value for the Navy's task of assembling accurate stellar positions. The errors encountered appear to be largely due to mechanical instability of the telescopes used. These telescopes were not designed for astrometry, and had both bearing variations and temperature distortions which are estimated to be comparable with the errors found. Errors due to atmospheric seeing appeared to be substantially smaller than these. We believe this series of tests successfully achieved its objective and demonstrated the usefulness of infrared spatial interferometry for very precise astrometric measurements over large angular separations.

Publications

"Multiple Telescope Infrared Interferometry", C.H. Townes and E.C. Sutton, Proceedings of ESO Conference on Scientific Importance of High Resolution at Infrared and Optical Wavelengths, 24-28 March 1981, Garching, pp. 199-223.

"Interferometric Measurements of Stellar Positions in the Infrared", E.C. Sutton, S. Subramanian, and C.H. Townes, submitted to Astronomy and Astrophysics (copy attached).



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Interferometric Measurements of Stellar Positions in the Infrared

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Summary

Differential positional measurements have been made at wavelengths near 11 μm on stellar sources using an interferometric technique. The nightly precision of these measurements is approximately 0.08 arcsec, which is slightly better than the typical astrometric errors obtained at visible wavelengths using photographic zenith tubes. The present interferometric measurements are thought to be dominated by instrumental errors, indicating that a significant improvement is possible in the precision of astrometric measurements. The limitation imposed by long term irregularities in the earth's atmosphere is estimated to be on the order of 0.01 arcsec.

Key words: astrometry - seeing - instruments - interferometry

I. Introduction

Ground-based measurements of stellar positions are seriously limited by the earth's atmosphere, but detailed information on the effects of the atmosphere is rather sparse. The work reported here consists of measurements of the phase stability of the atmosphere at wavelengths near 11 μm. This work was undertaken to determine the practicality of phase-sensitive infrared interferometry, such as aperture synthesis mapping and high precision astrometry. At the same time these measurements provide information on the atmospheric limitations on classical visual astrometry and allow an evaluation of the fundamental limitations on ground-based astrometric measurements.

Although techniques of narrow-field optical astrometry are capable of measuring differential stellar positions for parallax and double-star work to a precision of about 0.01 arcsec, the accuracy of measurements of absolute positions or differences between more widely separated stars is considerably worse. The best results of wide-field astrometry are probably those obtained with photographic zenith tubes. The positional precision of a typical 80 second zenith tube observation of a single star is approximately 0.13 arcsec (Markowitz, 1960; McCarthy, 1980). However, this precision does not improve with repeated measurements as rapidly as the inverse of the square-root of the integration time, due to the presence of systematic long-term errors. The repeatability of nightly mean positions, based on observations of 20 stars per night, has been estimated to be about 0.10 arcsec for the zenith tubes of the U.S. Naval Observatory (McCarthy, 1980), only a small improvement over the precision of a single measurement. This residual error has generally been thought to be due

to long-period structures in the earth's atmosphere, often referred to as refraction anomalies. As such it would represent the ultimate limit of precision for optical, ground-based, wide-field astrometric systems.

The infrared positional measurements reported here show a night-to-night repeatability of 0.08 arcsec. A large fraction of this error is estimated to be due to instrumental effects which in principle could be eliminated but which are difficult to avoid with the present interferometer. Thus the error contributed by long-term atmospheric structure is significantly less than 0.08 arcsec, which in turn is somewhat less than the typical errors reported for zenith tube observations. Although the effects of the atmosphere may be somewhat different in the infrared in the visible, this supports Rafferty's (1980) conclusion that much of the error in zenith tube observations has also been instrumental and not atmospheric in origin.

The precision of astrometric measurements is ultimately limited by the amount of structure present in the earth's atmosphere, principally variations in atmospheric density and water vapor content. Meteorological evidence suggests that the long-term variations, those lasting throughout most of a night's observations, are able to produce astrometric errors of about 0.01 arcsec. These long term variations ultimately limit the precision which can be achieved either by a series of zenith tube observations or by interferometric measurements of a string of successive sources, such as reported here. Interferometric measurements which observe several sources widely separated in the sky at several times during the night should be able to do better than even this limitation, since such a technique allows for a direct determination of the residual atmospheric structure.

II. Techniques of Measurement

A. The Heterodyne Interferometer

The positional measurements described here were obtained using a two-element infrared heterodyne interferometer (Sutton, 1979; Storey, 1979). The heterodyne receivers, employing CO_2 laser local oscillators and HgCdTe photodiode mixers, were mounted at the Coude foci of the twin 81 cm McMath auxiliary solar telescopes at Kitt Peak National Observatory. The interferometer formed by these two telescopes, which are separated by 5.5 meters in a nearly east-west direction, has a minimum lobe spacing of approximately 0.4 arcsec at a wavelength of 11.106 μm. Previous applications of these heterodyne systems include single-telescope high-resolution molecular spectroscopy (Betz and McLaren, 1980) and interferometric measurements of the sizes and shapes of circumstellar dust envelopes (Sutton et al., 1977, 1978, 1979).

In order to use the interferometer for precise positional measurements, it was necessary to insure the stability of both a time reference and the propagation time delay through the telescopes. The time standard of the interferometer was determined by a quartz crystal oscillator with a long-term stability of 1 part in 10^9. The associated clock circuitry was set each night to an accuracy of ±0.5 msec to the timing signal broadcast by the National Bureau of Standards on radio station WWV. The absolute accuracy of the time signal, which is dependent on the propagation delay of the radio signal, is not of great importance since a small offset in absolute time is equivalent to a slight tip in the hour angle of the interferometer baseline. Its stability, which is important, was maintained by the quartz oscillator to better than 5×10^{-5} sec (7×10^{-4} sec of arc)

for 12 hours of integration. Checks were made continually to insure that no timing errors as large as 1 millisec occurred during the observations.

In addition to the stability of the clock circuitry, it was necessary to insure that the optical path lengths through the interferometer were highly stable. Each leg of the interferometer consists of a complex, folded optical path involving reflections off five telescope mirrors with a total path length of over 100 meters. This path length must remain fixed or known to an accuracy of better than 10^{-8} in order to insure that the infrared signal does not undergo phase shifts as large as $\lambda/10$, which would correspond to a positional error of 0.04 $\hat{}$". This accuracy was maintained using a separate interferometric system similar to that described by Storey (1979) to monitor and compare the internal optical path lengths in the two legs of the interferometer. Any unbalanced phase shifts, due principally to thermal drifts in the mirror mountings, were automatically corrected through adjustment of the phase of one of the laser local oscillators. This compensating system, while monitoring the vast majority of the optical system, was not able to monitor the last $\sim$1 meter of path at the top of each telescope just before the heliostat mirrors and was not able to determine the positions of the heliostat mirrors themselves. Possible phase errors arising from this part of the optical path, including flexure of the heliostat mounts and motions due to irregularities in the telescope bearings, will be discussed below in Section C.

B. Analysis of the Data

The data were analyzed by comparing the phase of the observed interference signal with that calculated from the formula

$$\phi_{calc}(t) = 2\pi\frac{B}{\lambda}\{\sin\delta \sin\delta_b + \cos\delta \cos\delta_b \cos[h_b - h(t)]\}. \quad (1)$$

The baseline length B and hour angle h_b were determined independently for each night's observations by requiring that the phase difference

$$\Delta\phi(t) = \phi_{calc}(t) - \phi_{obs}(t) \quad (2)$$

be as independent of time as possible for all of the sources observed. The baseline declination δ_b could not be determined independently of the length B except through use of the first, time-independent term in equation (1). Since unambiguous determination of δ_b would require observations of a large number of sources with different declinations and such extensive data were not available, the value of δ_b was left fixed at a value which had been measured through ordinary surveying techniques. The resulting uncertainty in δ_b had the effect of introducing an unknown offset in ϕ_{calc} and $\Delta\phi$, an offset which was constant for any given source but which would vary between sources at different declinations. An additional declination-dependent phase offset is caused by the failure of the axes of rotation of the heliostat mirrors to intersect precisely.

The source declination δ and hour angle h(t) were calculated from catalog positions and included corrections for the leading terms of precession, nutation, aberration, and proper motion. No correction for atmospheric refraction is needed for an interferometer with a horizontal baseline using the approximation of a plane-parallel atmosphere. The actual curvature of the atmosphere produces a phase shift of much less than 2π except at extreme hour angles. The refractive effect of the slight vertical component of the baseline is similarly negligible.

Due to the uncertainty in the baseline declination δ_b and due to the fact that only a few hours of observations were available for each source on a given night, absolute source positions were not determined from the data. Instead, observations of different sources were compared, and the repeatability of the differences between two sources was used to determine the precision with which positions can be measured. The phase difference between the observations of two different sources at the same hour angle is given by

$$\Delta\phi_1[h(t)]-\Delta\phi_2[h(t)] = \phi_{calc,1}[h(t)]-\phi_{obs,1}[h(t)]- \phi_{calc,2}[h(t)]+\phi_{obs,2}[h(t)]. \quad (3)$$

For a nearly east-west baseline and for observations near the meridian, this quantity depends on the difference in right-ascension errors $\Delta\alpha_1$ and $\Delta\alpha_2$ for the two sources plus a term due to the uncertainty in δ_b:

$$\Delta\phi_1(h\approx 0)-\Delta\phi_2(h\approx 0) \approx \frac{2\pi B}{\lambda}[\Delta\sin\delta_b(\sin\delta_1-\sin\delta_2) + \cos\delta_b(\cos\delta_1\ \Delta\alpha_1-\cos\delta_2\ \Delta\alpha_2)]. \quad (4)$$

Away from the meridian the sensitivity to right-ascension errors is reduced as the cosine of the hour angle and the phase difference becomes sensitive to errors in declination. Since most of the data were obtained fairly near the meridian and since the phase in that region of the sky is less susceptible to systematic effects, the analysis will be concentrated on determining the differences in right-ascension between the various sources.

C. Systematic Errors

The significance of the results is influenced by a number of systematic effects, one of the most important of which is the thermal stability of the

baseline. The interferometer baseline is defined by a rigid steel mounting structure which changes dimensions as the ambient temperature changes. The values of the baseline length B which were determined from the data ranged from 5.4791 m to 5.4796 m over the 10 nights of observations. This variation was generally consistent with the variation in air temperature, assuming an expansion rate of 0.06 millimeters K^{-1}. Thermally induced changes in the baseline orientation should be much smaller since they require a distortion of the mounting structure instead of equal expansion. No changes were seen in the value of the baseline hour angle to a precision of about 0.1 millimeters in the relative positions of the two telescopes. Assuming that the amount of distortion is only about 2% of the thermal expansion, a relative displacement of the two telescopes by 0.01 millimeters would be expected, consistent with the above limit. Any such distortion causing a displacement of one telescope out of the equatorial plane would change the declination of the baseline. Since the baseline declination directly affects the phase difference between sources at different declinations and since in this analysis it is assumed to be constant, such a distortion would introduce a systematic error in the results. If the above estimate is realistic, this error would amount to an uncertainty of ±0.03 arcsec in the difference in right ascension between α Orionis and o Ceti.

An additional thermally induced phase error can result from changes in temperature, and hence baseline length, which occur during the course of a night's observations. Typical nightly changes in the temperature of the heliostat mounting structure were about 1 K in 24 hours, as determined by the baseline lengths measured on successive nights. Assuming the same

rate of change for the shorter three-hour interval separating observations of α Orionis and o Ceti, an error of 0.06 arcsec would be introduced in the measured relative positions of these sources. It is likely that a combination of this effect together with the variation in baseline declination, causing a total positional error of about 0.07 arcsec, is the limiting source of error in the measurements described here.

Other systematic errors can arise from mechanical instabilities in the telescopes. For example, bearing runout and bearing surface irregularities can produce displacements of the heliostat mirrors amounting to several microns and hence produce correspondingly large phase shifts. However, to the extent that such irregularities are reproducible with hour angle these phase shifts should be largely cancelled by comparing different sources at the same hour angle. Similarly, flexure of the telescope structure can cause systematic errors. Although the McMath telescopes have relatively sturdy and compact heliostat mountings, the differential flexure between the two telescopes can produce phase shifts of several cycles at extreme hour angles. However, this effect should be much smaller for observations near the meridian and also should be fairly reproducible.

The precision of the positional measurements described here is limited by the telescopes being used, which do not have the desired sub-micron tolerances. Although the errors introduced can only be estimated, they seem to be on the order of 0.07 arcsec. These errors are not fundamental, however, since it is possible to construct telescopes which are either sufficiently stable or in which the systematic phase shifts are continuously monitored. Ultimately, the accuracy of positional measurements with an

infrared interferometer is limited by the stability of the earth's atmosphere, which should allow measurements to be made to a precision several times greater than described here.

III. Experimental Results

A. The Data

The data were obtained on ten nights during the two week period from September 22, 1980 to October 6, 1980. This period was characterized by unseasonably hot weather in Arizona and by exceptionally good and stable observing conditions. The sky was generally entirely free of clouds, the surface wind typically less than or equal to 5 meters per second, and the seeing approximately 1-2 seconds of arc. Of the four nights during this period not included in the present analysis, two nights were lost completely, one due to equipment failure and one due to the passage of a weak weather disturbance. The remaining two nights were excluded since the data obtained showed substantial periods of poorer phase stability, associated with somewhat worse seeing conditions.

The objects observed were the bright infrared sources o Ceti, α Orionis, and R Leonis. In addition to their brightness, these sources were selected because of their similar declinations and their roughly equal spacings of about 4 hours in right ascension. A procedure was adopted whereby the first source, o Ceti, was observed starting as soon as it rose to a convenient hour angle, usually about 4 hours east of the meridian. The phase of o Ceti was then tracked continuously until it approached the meridian, at which point the telescopes were switched to α Orionis. The observations of α Orionis were continued either until sunrise or until R Leonis rose to

4 hours east of the meridian. In the latter case, a relatively brief (approximately 1 hour) integration was obtained on R Leonis, terminating at sunrise. These long continuous observations served two main purposes. First, they provided the best picture of the systematic motions of the telescopes due to effects such as bearing irregularities and flexure of the telescope mounts. Also, the absence of any lengthy gaps in the data eliminated possible ambiguities of 2π which otherwise could enter in the determination of the phase. By comparing the phases observed on successive stars at the same hour angles, the systematic errors in telescope position are largely eliminated and the observed phase difference becomes a good measure of the difference in right ascension between the objects.

One thirty-four minute stretch of data on o Ceti is plotted in Figure 1. Each data point represents the difference between the observed fringe phase and that calculated from equation (1), averaged over 20 seconds. The short-term scatter in phase, as determined by the RMS deviations of the points from a third-order polynomial fit to the data, is 32° for each 20 second measurement. This is equivalent to a positional precision of 0.04 arcsec for a source on the meridian. This short-term scatter is due to both noise intrinsic to the heterodyne receiver and random phase fluctuations in the atmosphere. In this stretch of data the receiver noise contributes about 15° RMS to the phase fluctuations in the 20 second averages, indicating that the bulk of the short-term phase scatter is due to atmospheric irregularities.

The accuracy of positional determinations is limited not by these short-term fluctuations, but by the long-term wandering of the average phase. In Figure 1 this wandering can be seen in the initial increase

of phase through almost 180° followed by a leveling off of the phase. Slow phase drifts of this sort are due to longer-period structures in the earth's atmosphere as well as changes in the geometry of the telescopes due to flexure, irregular bearings, and thermal drifts. In the present data, the instrumental effects are probably dominant, although the ultimate accuracy of positional measurements is limited by the amount of large-scale structure in the atmosphere.

The data from one of the nights during which all three sources were observed are plotted in Figure 2. This figure shows several features of the long-term phase drift which are characteristic of all of the nights studied. First, in the region within 2 hours of the meridian, the phase progression is fairly flat. What irregularities there are reproduce well between o Ceti and α Orionis, leaving the difference between them almost constant. At larger hour angles, the tracks for the different sources begin to diverge. This presumably is due to differential flexure between the heliostat mounts, which can cause rather complicated declination-dependent phase offsets which change with hour angle. The shapes of these diverging tracks reproduce from night-to-night to better than one wavelength for hour angles as large as 4 hours east. However, because of the complex nature of these flexure effects and since it is not expected that they should be entirely reproducible, data from these large hour angles are not used in the following analysis, except in the case of R Leonis where no data are available from the more favorable region near the meridian.

B. Comparison of o Ceti and α Orionis

Listed in Table 1 are the results of measuring the positional difference between o Ceti and α Orionis on the ten nights during which

sufficient data were obtained on both sources. For each night the data taken within two hours of the meridian were averaged into samples of 512 seconds each. These samples were then used to calculate an overall positional difference for the night as well as the standard deviation of the samples. For the night of October 3-4 there was an ambiguity of one cycle present in about half of the data, therefore there were two possible values of the average result differing by one-half cycle (0.2 arcsec). Finally, the ten different nights were compared, yielding an overall average and a standard deviation for each night. The overall average of 0.12 arcsec is nominally the difference in the right ascension errors between o Ceti and α Orionis. However, as discussed in Section II, an unknown offset is produced in this quantity due to the uncertainty in the declination of the baseline. Thus, the exact value of this overall average is not significant, although the night-to-night deviations provide a good measure of the precision of astrometric measurement using this technique.

The standard deviation for the 512 second samples, averaged over the ten nights, is 0.09 arcsec. This is considerably larger than the 0.04 arcsec error derived from Figure 1 for 20 second averages, even after accounting for the factor of $\sqrt{2}$ increase in error due to measuring a difference between two stars and neglecting any improvement due to the longer averaging time. This is due to the fact that the short-term phase scatter was measured in terms of deviations from a smooth but slowly wandering curve, thus eliminating any error contributed by this long-term wandering of the phase. These longer period variations are, in fact, the dominant source of phase error. One consequence is that only very slow improvement in precision will occur with increased averaging time. This is illustrated by the 0.08 arcsec standard

deviation of the nightly averages, which provides little further reduction in error beyond that available with the 512 second samples.

C. Comparison of α Orionis and R Leonis

The results of comparing the positions of α Orionis and R Leonis are listed in Table 2 for the five nights during which R Leonis was observed. The data were limited to fairly large hour angles due to the fact that the sun rose while R Leonis was still about 3 hours east of the meridian. At such hour angles the phase difference between the two sources was not constant but varied due to flexure of the telescopes. To roughly correct for this effect, a phase varying linearly with hour angle between 180° at 4 hours and 900° at 3 hours was subtracted from each night's data before averaging. Despite this added complication, the positional difference was repeatable from night-to-night and the errors were comparable to those obtained using the difference between o Ceti and α Orionis.

D. Experimental Summary

The nightly precision of the differential positional measurements described here is approximately 0.08 arcsec. This precision is limited in part by systematic errors due to mechanical instabilities in the instrument and in part by atmospheric disturbances. The sizes of the systematic errors were estimated in Section II to be on the order of 0.07 arcsec, indicating that they may be responsible for the bulk of the observed error. Thus the atmospheric disturbances, which limit the ultimate precision of such positional measurements, may produce deflections much smaller than this. The other significant feature of the experimental errors is that they do not decrease significantly for longer averaging times. This behavior is

consistent with the presence of long-term systematic effects such as thermal distortions of the instrument. It is also consistent with atmospheric phase fluctuations, which are thought to be dominated by very low frequency atmospheric disturbances.

IV. Theoretical Interpretation

A. Theory of Random Turbulence

The astrometric potential of an infrared heterodyne interferometer, as well as that of any earth-based astrometric system, is ultimately limited by irregularities in the earth's atmosphere. The theory of random turbulence (cf. Tatarskii, 1971; Hufnagel, 1978; Fried, 1979) provides a fairly comprehensive description of atmospheric refractive index fluctuations over a limited range of length scales. The small scale limit of this range is determined by the rate of turbulent energy dissipation and the viscosity of air. This lower bound is known as the inner scale of turbulence and is on the order of several millimeters under normal conditions. The outer scale is determined by the physical extent of the turbulent field and therefore is probably on the order of 10-100 meters since most atmospheric turbulence occurs within the approximately 100 meter thick turbulent boundary layer just above the earth's surface. The region between these limits, known as the inertial subrange, includes those spatial scales of turbulence which are relevant to the short-term imaging properties of all optical telescope apertures although not the largest scales which might be relevant for conceivable long-baseline visible and infrared interferometric systems. In addition, the properties of turbulence in the inertial subrange are not adequate to describe the large-scale, long-period variations which affect all wide-field astrometric systems. Nevertheless, the theory of

random turbulence is useful in studying such large-scale variations since these variations are most easily understood in terms of their similarities to or departures from the behavior expected for a randomly turbulent field.

Under the assumption of random turbulence the atmospheric temperature fluctuations can be described by the structure function

$$D_T(r) = \overline{[T(\vec{r}_o+\vec{r})-T(\vec{r}_o)]^2} = C_T^2\, r^{2/3} \qquad (5)$$

for values of r in the inertial subrange. The constant C_T has been measured by Coulman (1969) to be on the order of 10^{-2} K $cm^{-1/3}$ in the lower regions of the atmosphere under typical evening conditions. The density changes associated with these temperature fluctuations are generally thought to be the main source of refractive index fluctuations in the atmosphere and hence the ultimate cause of seeing. These index fluctuations are also described by a structure function of the form

$$D_n(r) = \overline{[n(\vec{r}_o+\vec{r})-n(\vec{r}_o)]^2} = C_n^2\, r^{2/3} \qquad (6)$$

where $C_n = (\partial n/\partial T)C_T$ and $\partial n/\partial T$ has a value of about 10^{-6} K^{-1} for visible and infrared radiation. Thus the RMS fluctuations in refractive index described by the constant C_n are on the order of 10^{-8} $cm^{-1/3}$.

Differences in water vapor content can, in principle, also contribute to variations in the refractive index. Fluctuations in water vapor obey a similar structure function relationship and contribute a term of the form $(\partial n/\partial e)_{P,T}\, C_e$ to the value of C_n, where e is the partial pressure of water vapor. Since $(\partial n/\partial e)_{P,T}$ is equal to 6×10^{-8} $torr^{-1}$ for visible light and typical values of C_e are about 10^{-2} torr $cm^{-1/3}$, the contribution of water vapor to the fluctuations in refractive index will generally be an order of magnitude less than that due to temperature fluctuations. However,

under some circumstances these effects may be more nearly comparable and water vapor may then be important (Friehe et al., 1975).

B. Propagation of Light Through Random Turbulence

The problem of interest for astrometry and astronomical imaging is that of a plane electromagnetic wavefront incident on a randomly turbulent medium. After traversing a region with refractive index fluctuations described by equation (6), the initially planar wavefront acquires phase distortions. The mean-squared phase difference between two points separated by a distance ρ along the wavefront can be described, under the approximation of geometrical optics, by the structure function

$$D_S(\rho) = 2.91\left(\frac{2\pi}{\lambda}\right)^2 \rho^{5/3} \int_0^L C_n^2(z)\, dz \qquad (7)$$

where λ is the wavelength of the radiation and ρ is restricted to values in the inertial subrange. The constant C_n has been given an explicit dependence on the altitude z within an atmosphere of overall height L. The use of geometrical optics in deriving this equation is not unduly restrictive since a more exact solution only slightly changes the numerical constant and the weighting of $C_n^2(z)$ in the integral over height. The $\rho^{5/3}$ dependence of the mean-squared phase fluctuations may be used to predict a $\lambda^{6/5}$ wavelength dependence for the maximum diffraction-limited aperture of a telescope and a $\lambda^{-1/5}$ behavior for the angular size of a seeing disc.

In a plane parallel to the initial unperturbed wavefront the two-dimensional spectral density of the phase fluctuations, derived from equation (7), is given by

$$F_S(\kappa) = 0.21 \left(\frac{2\pi}{\lambda}\right)^2 \kappa^{-11/3} \int_0^L C_n^2(z)\, dz \qquad (8)$$

where $\vec{\kappa}$ is the wavenumber of the fluctuations. The strong divergence in this equation for $F_S(\kappa)$ as $\kappa \to 0$ illustrates that the atmospheric fluctuations are dominated by the low frequency, large scale disturbances. The actual fluctuations do not diverge at $\kappa=0$ since the approximations used in deriving this result break down as $2\pi/\kappa$ approaches the outer scale of turbulence.

The strength of the turbulence in these formulae is represented by the quantity $\int_0^L C_n^2(z)\, dz$ which is typically on the order of 3×10^{-12} cm$^{1/3}$ for good seeing conditions. Another common measure of turbulence strength is the coherence length r_0 (Fried, 1966, 1979) which is related to $\int_0^L C_n^2(z)\, dz$ by the formula

$$r_0 = \left[0.42 \left(\frac{2\pi}{\lambda}\right)^2 \int_0^L C_n^2(z)\, dz\right]^{-3/5} \qquad (9)$$

The significance of r_0 is related to the fact that the RMS phase distortion across an aperture of diameter r_0 is nearly 1 radian. Hence it corresponds, roughly speaking, to the maximum diameter aperture which will be diffraction limited. The turbulence strength adopted here, given by $\int_0^L C_n^2(z)\, dz \sim 3 \times 10^{-12}$ cm$^{1/3}$, corresponds to a value of $\sim$10 cm for r_0 at 5000Å. The value of r_0 at 11 μm, the operating wavelength of the interferometer, is then about 410 cm, indicating that the 81 cm diameter telescopes used in these measurements are easily diffraction-limited at this wavelength. Such telescopes are not, of course, diffraction-limited at visible wavelengths.

These formulae may be used to estimate the size of the interferometer phase fluctuations which are due to random atmospheric turbulence. The phase of the interference signal is the difference in the phases of the

wavefront at the two telescopes. Using equation (7) with ρ set to equal 5.5 meters, the length of the interferometer baseline, and with the value of the turbulence strength given above, the mean-squared fluctuations in this phase difference are predicted to be $D_s(5.5m) \approx 10.5\ rad^2$.

Thus the phase of the interference signal is expected to fluctuate typically through most of an entire cycle. The observations reported here, such as those displayed in Figures 1 and 2, show considerably smaller atmospheric phase variations, especially considering that much of the observed phase variations are due to instrumental effects. This is largely due to the fact that the observations were obtained during exceptionally good seeing conditions when the strength of the atmospheric turbulence was less than the more typical value assumed here. Also equation (7) tends to overestimate the fluctuations since it ignores the fact that $D_S(\rho)$ should begin to saturate as ρ approaches the outer scale of turbulence.

The spectral distribution of the interferometer's phase fluctuations is considerably different than that given by equation (8) since the baseline vector preferentially selects fluctuations with certain length scales and orientations. For an interferometer with baseline length B, the one-dimensional spectral density of the phase difference between the two ends of the baseline is

$$V_{\Delta S}(\kappa') = 2.79 \left(\frac{2\pi}{\lambda}\right)^2 (\kappa')^{-8/3} \sin^2\left(\frac{\kappa' B}{2}\right) \int_0^L C_n^2(z)\, dz \qquad (10)$$

where κ' is the projection of $\vec{\kappa}$ onto the baseline (Greenwood and Fried, 1976). The term $\sin^2(\kappa' B/2)$ shows the sensitivity of the interferometer to phase variations with projected wavenumber κ'. It shows, for example,

that the interferometer does not respond to phase fluctuations when the baseline is an integer multiple of their wavelength. In addition, the interferometer is relatively insensitive to fluctuations with wavelengths much larger than the baseline. As a result, in the limit $\kappa' B/2 \ll 1$, $V_{\Delta S}(\kappa')$ diverges only as $(\kappa')^{-2/3}$, a much weaker divergence than that of equation (8). The bulk of the phase fluctuations, nevertheless, comes from atmospheric variations ranging in size from the baseline length B up to the point where this theory breaks down, the outer scale of turbulence. The $(\kappa')^{-2/3}$ dependence of the spectral density in this region indicates that increased averaging times will be ineffective at reducing the phase errors of the interferometer. For integration times T up to a time given by the outer scale of turbulence divided by the wind velocity, the RMS phase error will be diminished only as $T^{-1/6}$.

C. Large Scale Atmospheric Structure

The theory of random turbulence probably provides a good description of atmospheric irregularities on scales of up to several meters. Beyond this, in the region from about 10 to 100 meters, the theory is increasingly inadequate. In this region the magnitude of the fluctuations should begin to saturate as the outer scale of turbulence is approached. In addition, under conditions of thermal inversion the fluctuations can saturate at even smaller distances due to the action of buoyancy forces (Obukhov, 1959; Bolgiano, 1959). In this region the turbulence becomes more and more anisotropic as the scale size approaches the height of the turbulent region above the ground or the thickness of an inversion layer. At the same time the turbulent flow becomes less random and more dependent on local surface features and terrain. The very largest spatial scales are best discussed

in meteorological terms. Disturbances on these scales are typically major weather fronts separated by hundreds or thousands of kilometers with smaller but significant variations of atmospheric pressure or water vapor content down to kilometer scales. All of these can have an important effect on the errors in astrometric measurements. When two stars are measured at times separated by several hours their inferred relative positions will depend on the changes in the structure of the atmosphere which have occurred during that interval.

The intermediate length scales of tens to hundreds of meters are the least well understood since they are too large to be described by random turbulence, yet too small to be measured by standard meteorological data. Most evidence suggests that the refractive index fluctuations continue to grow throughout this region but at a rate somewhat less than that extrapolated from smaller scale turbulence (Gossard, 1960; Bouricius and Clifford, 1970). A fluctuation with a wavelength of 100 meters has a corresponding time scale of 10 seconds, assuming a wind speed of 10 meters per second. Thus fluctuations of these sizes will be fairly well averaged out in a typical 100 second astrometric measurement and will not contribute as significantly as the large scale disturbances to the residual errors of such measurements. However, the atmospheric behavior in this region is important in determining the maximum practical interferometer baseline length.

Abundant data are available on the behavior of the atmosphere on the very largest spatial scales from measurements of atmospheric pressure. In the static case atmospheric surface pressure is a direct measure of the total mass of the vertical column of air, which in turn determines the

phase delay of light from a source at the zenith. Pressure gradients of 0.2 mbar/km can occur within a few tens of kilometers of strong frontal disturbances. Such gradients are sufficient to produce 0.1 arcsec deflections in the apparent position of stellar images. However, gradients of this sort are short-lived phenomena which furthermore occur during bad weather when observations would not be made. More typical long-standing gradients are less than 0.02 mbar/km which can be responsible for astrometric errors of only 0.01 arcsec.

Dynamical effects in the atmosphere can also produce pressure variations and hence variations in apparent stellar position. Gossard and Munk (1954) report measurements of relatively long-lived wave phenomena in the atmosphere which they refer to as gravity waves. These waves typically have amplitudes of several tenths of millibars and wavelengths of about 5 km. The very largest of these are sufficient to produce errors of 0.1 arcsec in a single astrometric measurement. However, strong gravity waves seem to be quite infrequent since they require, among other things, a strong temperature inversion. Also since these fluctuations are periodic with periods of about 10 minutes, they are unable to account for the very long term (night-to-night) variations in stellar positions.

In this context it is interesting to consider the reported fluctuations in the apparent solar diameter (Brown et al., 1978) and the attempts to explain these measurements using variations in the earth's atmosphere (KenKnight et al., 1977; Fossat et al., 1981). The observed fluctuations have amplitudes of about 0.005 arcsec in each of several modes with periods of ten minutes and longer as well as in modes corresponding to the well known five-minute solar oscillations. Brown et al. claim that atmospheric variations in the mHz frequency range are insufficient to

account for the observed power in these modes. Using measurements of fluctuations in the relative positions of stars separated by one solar diameter, KenKnight et al. concluded that the atmospheric variations are similar in magnitude to those assumed by Brown et al. On the other hand, Fossat et al. claim that the atmospheric noise level is significantly higher and that therefore the existence of solar oscillations with periods longer than five minutes is not well established. They conclude that the total contribution of atmospheric noise, integrated over the frequency range of from 0.15 to 3.2 mHz, is sifficient to produce an RMS fluctuation of the solar diameter equal to 0.06 arcsec. The corresponding value for the models of KenKnight et al. is an RMS fluctuation of 0.04 arcsec. Although these are errors in differential positional measurements they are closely related to the errors in absolute positional measurements, which in these cases would be approximately 0.1 arcsec for a 1000 second integration. Interestingly enough, Lindgren (1980) points out that these values are comparable in magnitude to the errors calculated using a straightforward extrapolation of the power law dependence derived for random turbulence in the inertial subrange. This result is somewhat surprising since the scale sizes of the mHz fluctuations are many kilometers, which is at least an order of magnitude larger than the scales at which the theory of random turbulence can reasonably be expected to apply. Assuming this extrapolation to be valid, atmospheric variations in the mHz frequency range would be able to contribute errors of about 0.1 arcsec to individual 1000 second long astrometric measurements. However, these variations are still short term relative to the night-to-night differences in stellar positions. Night-to-night changes cannot be explained by such variations but only by persistent atmospheric gradients.

Long standing deflections in apparent stellar positions can be produced by persistent gradients in either atmospheric pressure or water vapor content. Since the gradients in atmospheric pressure are typically less than 0.02 mbar/km, the resulting deflections of 0.01 arcsec are insufficient to explain long-term astrometric errors. Similarly it can be inferred from the measurements of Guiraud et al. (1979) that gradients of water vapor content which persist for many kilometers are generally less than 0.1 mm of precipitable water per kilometer under dry conditions. Such gradients would produce positional errors of only 0.002 arcsec at visible wavelengths. As discussed by Townes and Sutton (1981) the refractive effect of water vapor gradients is much smaller at a wavelength of 10 μm than in the visible. Thus water vapor gradients are unlikely to be a dominant source of error at visible wavelengths and are almost certainly negligible in this part of the infrared.

V. Conclusions

Although ground-based astrometric measurements are ultimately limited by the stability of the earth's atmosphere, a significant improvement in precision seems possible. Short term variations in the atmosphere introduce positional errors of about 0.13 arcsec for integration times of about 100 seconds. Both zenith-tube measurements and the interferometric measurements reported here are able to approach this precision. Over the longer term it is more difficult to avoid being dominated by instrumental errors. For night-to-night variations, zenith tubes do as well as about 0.10 arcsec. The interferometric measurements do somewhat better but still only about 0.08 arcsec. Both techniques are probably limited on these time scales by instrumental uncertainties since meteorological data indicate

that the long term atmospheric variations should produce errors of only 0.01 arcsec. Thus a considerable improvement in astrometric precision seems possible if sufficient attention is devoted to instrumental design.

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Table 1. Differences in right ascension between o Ceti and α Orionis

Date	Average Positional Difference (arbitrary zero point)	Standard Deviation of Samples	Number of Samples
Sept 22-23	0".03	0".11 (in 512 sec)	10 (512 sec each)
Sept 24-25	0".25	0".05	4
Sept 26-27	0".13	0".18	12
Sept 27-28	0".18	0".10	9
Sept 28-29	0".06	0".10	12
Oct 1-2	0".06	0".05	8
Oct 2-3	0".05	0".10	6
Oct 3-4	0".24 (0".04)	0".08 (0".16)	8
Oct 4-5	0".07	0".04	4
Oct 5-6	0".16	0".04	8
Total	0".12	0".08 (night to night)	10 nights

Table 2. Differences in right ascension between α Orionis and R Leonis

Date	Average Positional Difference (arbitrary zero point)	Standard Deviation of Samples	Number of Samples
Oct 1-2	0".11	0".04 (in 512 sec)	7 (512 sec each)
Oct 2-3	-0".07	0".06	4
Oct 3-4	0".10	0".09	6
Oct 4-5	-0".02	0".04	4
Oct 5-6	-0".06	0".06	7
Total	0".01	0".09 (night to night)	5 nights

Figure Captions

Figure 1. Observations of the phase of the interference fringe for o Ceti. The phase plotted is the difference between the observed fringe phase and that calculated from equation (1). Each data point represents 20 seconds of integration.

Figure 2. Phase measurements on o Ceti, α Orionis, and R Leonis for a single night. Each point represents 100 seconds of integration. The deviations at large hour angles are probably due to systematic distortions of the telescopes.

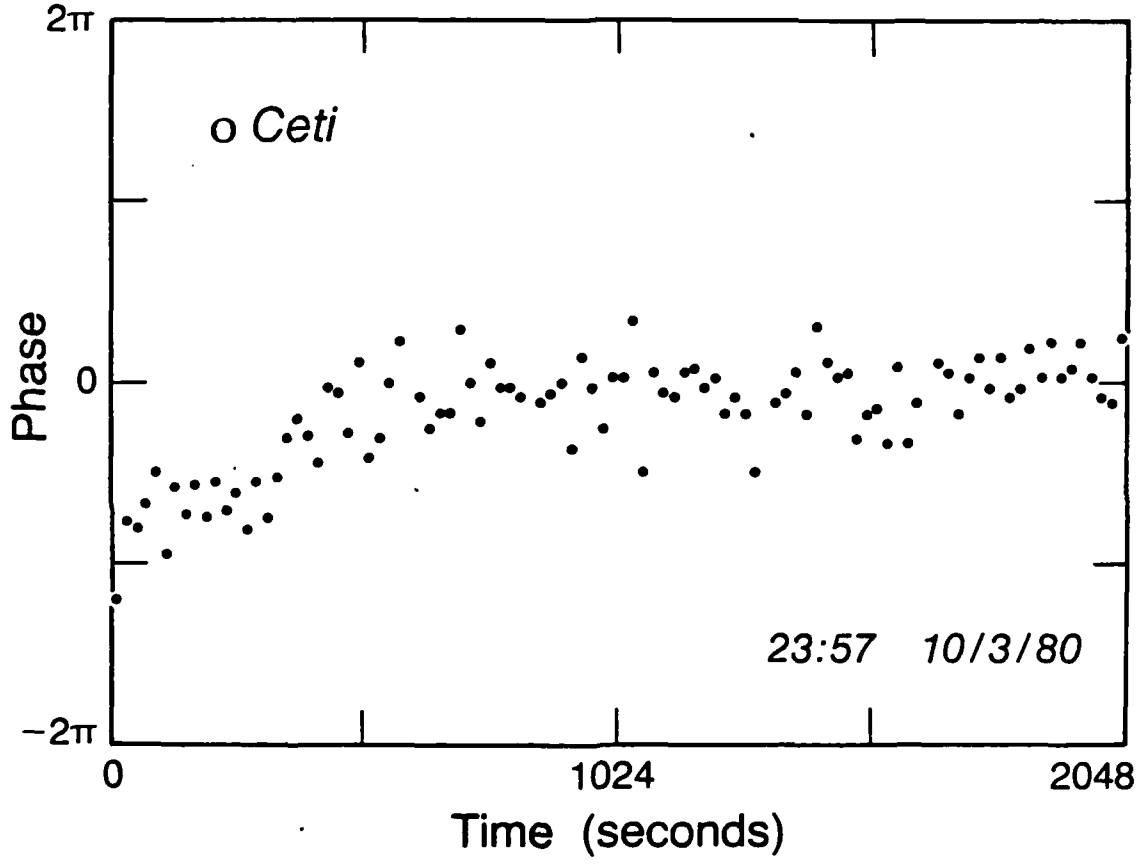

Fig. 1

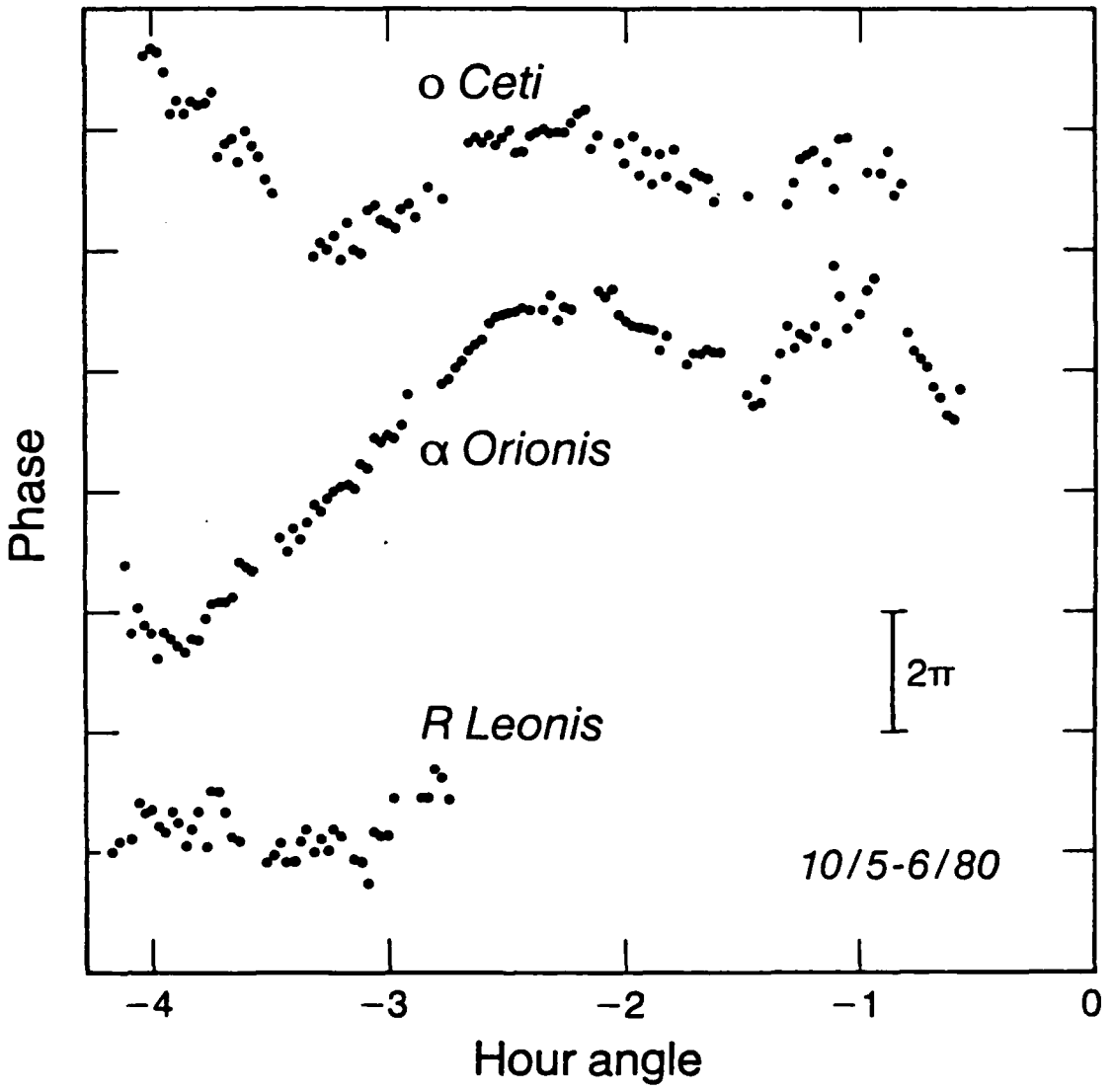

Fig. 2

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