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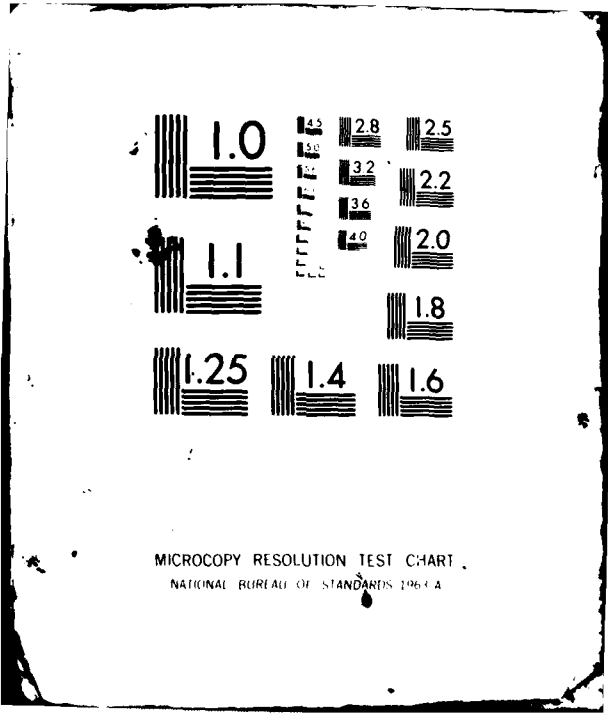
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# Effects of Currents on Waves

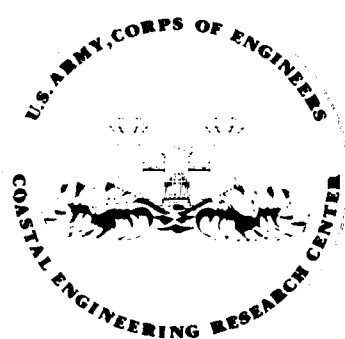
by

Barry E. Herchenroder

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PREFACE

This report presents some ways in which a horizontal current influences surface gravity waves and their measurement. Relatively simple hand-calculation methods are described which provide ways to estimate how much currents affect the wavelength and measured height of waves. These methods are the initial results of a research effort designed to furnish practical guidance on how to account for the effects of currents on waves. Such guidance fills an information gap in the Shore Protection Manual (SPM). The work was carried out under the waves on currents program of the U.S. Army Coastal Engineering Research Center (CERC).

The report was prepared by Dr. Barry E. Herchenroder, Oceanographer, under the general supervision of Dr. C.L. Vincent, Chief, Coastal Oceanography Branch.

Comments on this publication are invited.

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TED E. BISHOP  
Colonel, Corps of Engineers  
Commander and Director

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CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.852	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	$1.0197 \times 10^{-3}$	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6	grams
	0.4536	kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.01745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins <sup>1</sup>

<sup>1</sup>To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula:  $C = (5/9) (F - 32)$ .

To obtain Kelvin (K) readings, use formula:  $K = (5/9) (F - 32) + 273.15$ .



## SYMBOLS AND DEFINITIONS

$\bar{d}_T$	time-averaged water depth
$\bar{d}_{T,L}$	largest time-averaged water depth expected
$\bar{d}_{T,S}$	smallest time-averaged water depth expected
F	effective Froude number (eq. 3)
FM	minimum effective Froude number for which waves with dimensionless frequency $\Omega$ can propagate (Table)
FL	largest F that can be expected (eq. 15)
FS	smallest F that can be expected (eq. 16)
g	acceleration of gravity = 32.2 ft (9.8 m)/s <sup>2</sup>
H	height of a monochromatic wave
H <sub>A</sub>	height of monochromatic wave when current is absent (eq. 10)
H <sub>V</sub>	height of monochromatic surface gravity wave when current is present (eqs. 9 and 12)
<u>K</u>	horizontal wave vector
L	wavelength of monochromatic wave
L <sub>A</sub>	wavelength of monochromatic wave when current is absent (solution of eq. 1)
L <sub>V</sub>	wavelength of monochromatic wave when current is present (eq. 7 and solution of eq. 2)
P	bottom pressure fluctuation magnitude accompanying monochromatic surface gravity wave
R <sub>L</sub>	dimensionless wavelength factor = L <sub>V</sub> /L <sub>A</sub>
R <sub>H</sub>	dimensionless wave height factor = H <sub>V</sub> /H <sub>A</sub> (eq. 11)
R <sub>H,L</sub>	largest value of R <sub>H</sub> expected
R <sub>H,S</sub>	smallest value of R <sub>H</sub> expected
T	period of monochromatic wave
TS	smallest wave period of interest
TS'	smallest wave period for which currents can be neglected when observed bottom pressures are used to estimate wave heights (eq. 17)
V	horizontal current speed
<u>V</u>	horizontal current velocity vector

SYMBOLS AND DEFINITIONS--Continued

VL	largest value of $V$ expected
VS	smallest value of $V$ expected
VST	"stopping velocity" of a wave (eq. 6)
$\pi$	3.14159...
$\theta$	angle between $\underline{V}$ and $\underline{K}$
$\theta_L$	largest value of $\theta$ expected
$\theta_S$	smallest value of $\theta$ expected
$\rho_w$	water density = 2.00 slugs/ft <sup>3</sup> (1,031 kg/m <sup>3</sup> )
$\Omega$	dimensionless frequency (eq. 4)
$\Omega_{A,B,C,D}$	defined in equation (19)
$\Omega_L$	largest value of $\Omega$ that can be expected (eq. 14)
$\Omega_S'$	defined in equation (18)

## EFFECTS OF CURRENTS ON WAVES

by  
Barry E. Herchenroder

### I. INTRODUCTION

A horizontal current can modify surface gravity waves in several ways. One modification is to the wavelength. A current can locally stretch or shrink features in a wave train. This wave-train distortion produces a "Doppler shift" in the wave period. As a result, a stationary observer measures a different wave period than an observer moving with the local current velocity. To a good approximation, the period of a monochromatic wave is fixed in the stationary coordinate system. To an observer moving with the local current velocity, the measured period of this wave changes in time and at each point in response to the changing current.

The wave orthogonal, crest, and ray directions are also modified. An orthogonal is a line perpendicular to the local wave crest direction. A ray is a line parallel to the local group velocity vector, i.e., tangent to the local direction of wave energy flow. In the absence of a current, rays are parallel to orthogonals. When a current is present, orthogonals (by definition) are still perpendicular to the local crest orientation but rays are not parallel to the orthogonals unless the wave is propagating in the same direction as the current.

Another way a current modifies surface waves is to change the wave energy by causing an exchange of energy between wave and current. Energy per unit length of wave crest is no longer approximately conserved between rays, but rather a quantity called "wave action" is conserved. This new quantity, a generalization of wave energy, is given by the energy per unit length of wave crest divided by the "intrinsic angular frequency" of the wave, the latter being the Doppler-shifted angular frequency seen by an observer moving with the local current velocity. Since wave action rather than energy is conserved between rays, enough wave energy is gained from the current or lost to the current to keep the wave action the same as the wave propagates.

A modification also occurs in the pressure field accompanying the wave. This change can be an appreciable source of error in measuring wave characteristics if an existing current is not accounted for. In particular, a significant error can sometimes arise if bottom pressure measurements are used to determine surface wave heights and lengths.

Prediction of current-modified wave energy, heights, directions, and pressures usually involves the use of complex numerical models and computer programs. Such models and programs are under development and will be available over the next few years. However, relatively easy but useful calculations can be done without a computer. This report presents some of these simpler calculations, such as methods to determine (a) the wavelength modification due to a current, (b) whether a current can stop a wave from reaching a particular location, (c) a correction for the presence of a current when bottom pressure measurements are used to determine wave heights, and (d) the range of periods (if any) for which currents can be neglected when these pressure measurements are used to determine wave heights.

## II. WAVELENGTH MODIFICATION AND THE STOPPING OF WAVES

When a current is absent, Peregrine (1976)<sup>1</sup> and the Shore Protection Manual (SPM) (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977)<sup>2</sup> indicate that the wavelength,  $L$ , of a monochromatic wave is given by  $L = L_A$ , where  $L_A$  is computed by solving the "dispersion relation"

$$\frac{2\pi}{T^2 g} = \frac{1}{L_A} \tanh\left(\frac{2\pi \bar{d}_T}{L_A}\right) \quad (1)$$

and

$\tanh$  = the hyperbolic tangent

$T$  = the wave period

$g$  = the acceleration of gravity 32.2 feet (9.8 meters) per second squared

$\bar{d}_T$  = the time-averaged (over enough wave periods to filter out waves) water depth

$\pi$  = the factor 3.14159...

When a current is present, Peregrine (1976)<sup>3</sup> indicates that  $L$  is no longer equal to  $L_A$  but to  $L_V$ , where  $L_V$  is given by solving the equation

$$2\pi \frac{[(1/T) - (V \cos \theta / L_V)]^2}{g} = \frac{1}{L_V} \tanh\left(\frac{2\pi \bar{d}_T}{L_V}\right) \quad (2)$$

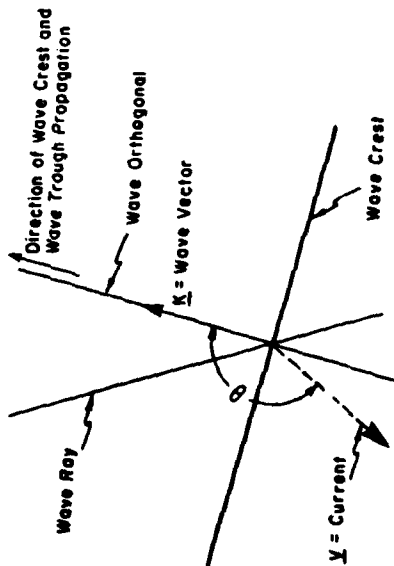
where  $\cos$  is the cosine,  $V$  the horizontal current speed, and  $\theta$  the angle between the horizontal current vector,  $\underline{V}$ , and the horizontal wave vector,  $\underline{K}$ . The angle,  $\theta$ , is taken to be greater than or equal to zero and less than or equal to  $180^\circ$  ( $\pi$  radians). The vector  $\underline{K}$  has a magnitude of  $2\pi/L_V$  and points along the *wave orthogonal* in the direction of wave crest and trough propagation. The relationship between horizontal current velocity,  $\underline{V}$ , horizontal wave vector,  $\underline{K}$ , wave orthogonal, wave crest, wave ray, and angle,  $\theta$ , is shown in Figure 1. The figure shows that when a current is present, wave rays are not parallel to wave orthogonals and hence not perpendicular to wave crests.

Figure 2 shows a set of curves which represents the dimensionless solution of equation (2). The  $F$  axis represents various values of the effective Froude number

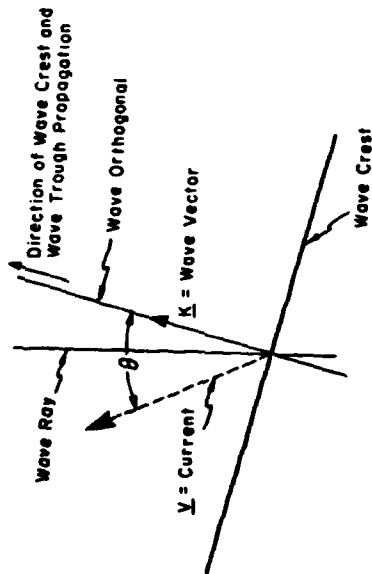
<sup>1</sup>PEREGRINE, D.H., "Interaction of Water Waves and Currents," *Advances in Applied Mechanics*, Vol. 16, Academic Press, New York, 1976, pp. 9-117.

<sup>2</sup>U.S. ARMY, CORPS OF ENGINEERS, COASTAL ENGINEERING RESEARCH CENTER, *Shore Protection Manual*, 3d ed., Vols. I, II, and III, Stock No. 008-022-00113-1, U.S. Government Printing Office, Washington, D.C., 1977, 1,262 pp.

<sup>3</sup>PEREGRINE, D.H., *op. cit.*



a.  $90^\circ \leq \theta \leq 180^\circ$  ( $\pi/2 \text{ Rad} \leq \theta \leq \pi \text{ Rad}$ )



b.  $0^\circ \leq \theta \leq 90^\circ$  ( $0 \text{ Rad} \leq \theta \leq (\pi/2) \text{ Rad}$ )

Figure 1. Relationship between horizontal current velocity,  $\underline{V}$ , horizontal wave vector,  $\underline{K}$ , wave orthogonal, wave crest, wave ray, and angle,  $\theta$ .

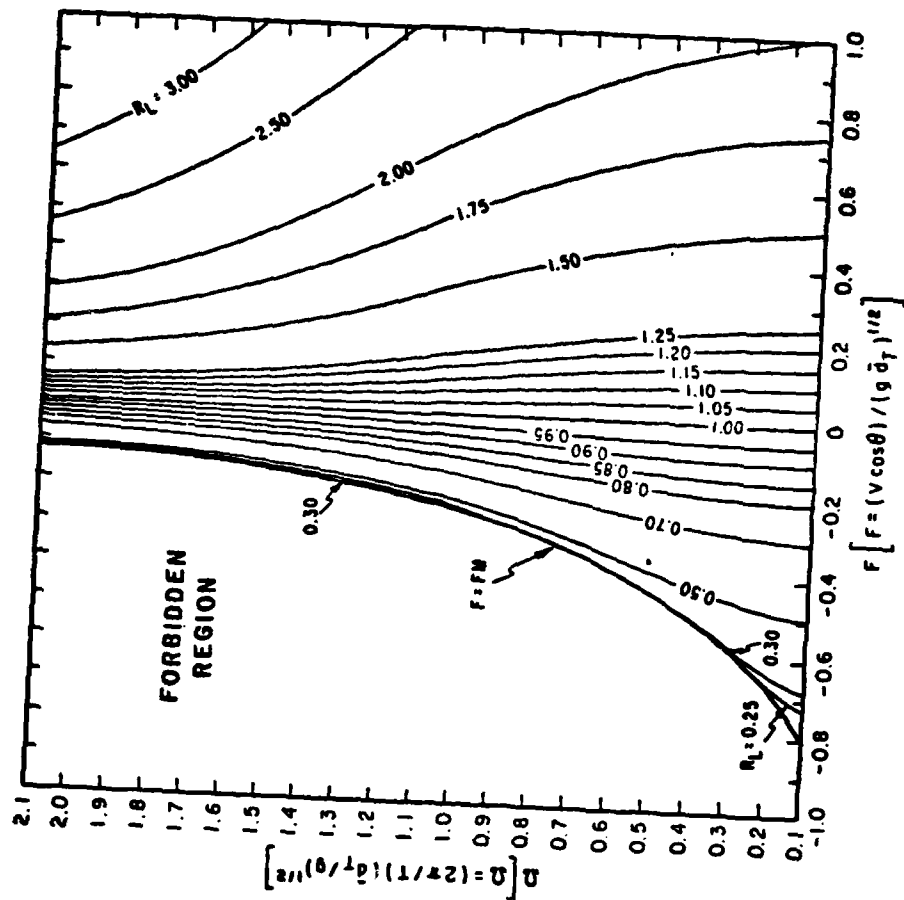


Figure 2. Contours of dimensionless wavelength factor,  $R_L$ , given by equation (7); curves represent the dimensionless solution of equation (2). Waves cannot propagate for values of  $F$  and  $Q$  which lie in the forbidden region (boundary line  $F = FM$ ).

$$F = \frac{(V \cos \theta)}{(g\bar{d}_T)^{1/2}} \quad (3)$$

where the  $\Omega$  axis represents values of dimensionless frequency

$$\Omega = \frac{(2\pi)}{T} \left( \frac{\bar{d}_T}{g} \right)^{1/2} \quad (4)$$

Each curve is a fixed value of wavelength factor,  $R_L$ , where

$$R_L = \left( \frac{L_V}{L_A} \right) = \frac{\text{wavelength with current}}{\text{wavelength without current}} \quad (5)$$

Negative values of  $F$  correspond to waves with a component of phase velocity in the direction opposite to that of the current. The phase velocity is the velocity that a point on the crest or trough is moving. The curve  $R_L = 1.0$  is the case where a current is absent; i.e., where  $L = L_A$ ,  $L_A$  fulfilling the usual dispersion relationship (eq. 1).

The curve  $F = FM$  in Figure 2 represents the limiting effective Froude numbers for which waves can propagate. If  $F$  is less than  $FM$  for a particular  $\Omega$ , then waves cannot propagate against the current for the particular  $\Omega$  and  $F$  values. In other words, *the current acts as a filter under certain conditions and prevents waves from ever reaching a given point.* The limiting value of "effective current velocity,"  $V \cos \theta$ , for which waves of a given period,  $T$ , in a mean depth of water,  $\bar{d}_T$ , can propagate is the "stopping velocity,"  $VST$ , given by

$$VST = (FM) (g\bar{d}_T)^{1/2} \quad (6)$$

$VST$  is negative (since  $FM$  is negative) and depends on dimensionless frequency,  $\Omega$  (since  $FM$ , as shown in Fig. 2, depends on  $\Omega$ ), and mean depth,  $\bar{d}_T$  (through the  $(g\bar{d}_T)^{1/2}$  factor). If a wave is propagating against the current (i.e.,  $\cos \theta$  is less than zero), then the wave cannot reach any area where the current speed  $V$  is greater than the local value of  $VST$ . Such an unreachable area in physical space or on a plot like Figure 2 is called the "forbidden region." Waves traveling with the current (i.e., with a  $\cos \theta$  greater than or equal to zero) have no forbidden region.

The Table gives the minimum effective Froude number,  $FM$ , and the ratio

$$\frac{L_A}{\bar{d}_T} = \frac{\text{wavelength without current}}{\text{average (in time) depth}}$$

for various values of dimensionless frequency,  $\Omega$ . For a particular  $\Omega$  value, this table (used in conjunction with Fig. 2 and linear interpolation) allows the following to be found: (a) whether a wave can propagate against the current (i.e., is  $F$  less than  $FM$ ), and (b) if a wave can propagate, the value of its current modified wavelength,  $L_V$ . Using the value of  $(L_A/\bar{d}_T)$  from the table and  $R_L$  from Figure 2,  $L_V$  is computed from

Table. ( $L_A/\bar{d}_T$ ) and FM for various values of  $\Omega$ .

$\Omega$	$L_A/\bar{d}_T$	FM	$\Omega$	$L_A/\bar{d}_T$	FM
0.10	62.73	-0.787	1.12	4.45	-0.223
0.12	52.23	-0.761	1.14	4.33	-0.219
0.14	44.73	-0.737	1.16	4.22	-0.216
0.16	39.10	-0.714	1.18	4.11	-0.212
0.18	34.72	-0.693	1.20	4.00	-0.208
0.20	31.21	-0.673	1.22	3.90	-0.205
0.22	28.33	-0.653	1.24	3.80	-0.202
0.24	25.93	-0.635	1.26	3.70	-0.198
0.26	23.89	-0.617	1.28	3.61	-0.195
0.28	22.15	-0.600	1.30	3.52	-0.192
0.30	20.63	-0.584	1.32	3.43	-0.189
0.32	19.30	-0.568	1.34	3.34	-0.187
0.34	18.12	-0.553	1.36	3.26	-0.184
0.36	17.08	-0.538	1.38	3.18	-0.181
0.38	16.14	-0.524	1.40	3.10	-0.179
0.40	15.29	-0.510	1.42	3.02	-0.176
0.42	14.52	-0.497	1.44	2.95	-0.174
0.44	13.82	-0.484	1.46	2.87	-0.171
0.46	13.18	-0.472	1.48	2.80	-0.169
0.48	12.59	-0.459	1.50	2.74	-0.167
0.50	12.04	-0.448	1.52	2.67	-0.164
0.52	11.54	-0.436	1.54	2.61	-0.162
0.54	11.07	-0.425	1.56	2.55	-0.160
0.56	10.63	-0.415	1.58	2.49	-0.158
0.58	10.22	-0.404	1.60	2.43	-0.156
0.60	09.84	-0.394	1.62	2.37	-0.154
0.62	09.48	-0.385	1.64	2.32	-0.152
0.64	09.14	-0.375	1.66	2.26	-0.151
0.66	08.83	-0.366	1.68	2.21	-0.149
0.68	08.52	-0.357	1.70	2.16	-0.147
0.70	08.24	-0.349	1.72	2.11	-0.145
0.72	07.97	-0.340	1.74	2.07	-0.144
0.74	07.71	-0.332	1.76	2.02	-0.142
0.76	07.47	-0.324	1.78	1.98	-0.140
0.78	07.24	-0.317	1.80	1.93	-0.139
0.80	07.01	-0.310	1.82	1.89	-0.137
0.82	06.80	-0.303	1.84	1.85	-0.136
0.84	06.60	-0.296	1.86	1.81	-0.134
0.86	06.40	-0.289	1.88	1.77	-0.133
0.88	06.22	-0.283	1.90	1.74	-0.132
0.90	06.04	-0.277	1.92	1.70	-0.130
0.92	05.86	-0.271	1.94	1.67	-0.129
0.94	05.70	-0.266	1.96	1.63	-0.128
0.96	05.54	-0.260	1.98	1.60	-0.126
0.98	05.39	-0.255	2.00	1.57	-0.125
1.00	05.24	-0.250	2.02	1.54	-0.124
1.02	05.09	-0.245	2.04	1.51	-0.123
1.04	04.96	-0.240	2.06	1.48	-0.121
1.06	04.82	-0.236	2.08	1.45	-0.120
1.08	04.69	-0.231	2.10	1.42	-0.119
1.10	04.57	-0.227			

$$L_V = (R_L) \left( \frac{L_A}{\bar{d}_T} \right) (\bar{d}_T) \quad (7)$$

### III. CURRENT-INDUCED CORRECTIONS WHEN BOTTOM PRESSURE MEASUREMENTS ARE USED TO DETERMINE SURFACE WAVE HEIGHTS

Peregrine (1976)<sup>4</sup> and Jonsson, Skovgaard, and Wang (1970)<sup>5</sup> indicate the errors involved in not correcting for the current when using bottom pressure measurements to determine wave heights. They give numerical results which cover some of the conditions encountered in practice. A more extensive set of correction curves is presented in this section. These curves allow an estimate of current corrections to be made for the range of circumstances which usually occur in the field.

Peregrine (1976)<sup>6</sup> indicates that if a monochromatic surface wave height, H, has a bottom pressure fluctuation magnitude, P, then H can be estimated from P by

$$H = H_V \quad (8)$$

where

$$H_V = 2 \left[ \cosh \left( \frac{d\pi\bar{d}_T}{L_V} \right) \right] \frac{P}{\rho_w g} \quad (9)$$

with cosh the hyperbolic cosine,  $\rho_w$  the water density (2.00 slugs per cubic foot or 1,031 kilograms per cubic meter);  $g$ ,  $\bar{d}_T$ , and  $L_V$  are the same as previously defined. Equation (9) takes the current into account since the wavelength  $L_V$  for a nonzero current rather than  $L_A$  for no current is used. Usually the current is not taken into account when estimating H from P. Rather, the equation analogous to (9) but with  $L_A$  instead of  $L_V$  is used to estimate H from P; i.e.,  $H = H_A$  is usually used where

$$H_A = 2 \left[ \cosh \left( \frac{2\pi\bar{d}_T}{L_A} \right) \right] \frac{P}{\rho_w g} \quad (10)$$

The wave height factor,  $R_H$ , where

$$R_H = \frac{H_V}{H_A} = \frac{\cosh(2\pi\bar{d}_T/L_V)}{\cosh(2\pi\bar{d}_T/L_A)} \quad (11)$$

<sup>4</sup>PEREGRINE, D.H., op. cit., p. 10.

<sup>5</sup>JONSSON, I.G., SKOVGAARD, C., and WANG, J.D., "Interaction Between Waves and Currents," *Proceedings of the 12th Conference on Coastal Engineering*, American Society of Civil Engineers, Vol. 1, 1970, pp. 489-509.

<sup>6</sup>PEREGRINE, D.H., op. cit.



allows the "correct" wave height value,  $H$ , to be determined from the uncorrected height,  $H_A$ , by using the simple equation

$$H = (R_H) H_A \quad (12)$$

Contours of  $R_H$  are shown in Figure 3. The  $F$  and  $\Omega$  axes are the same as in Figure 2. The boundary of the forbidden region is again indicated by the curve labeled  $F = FM$ . Figure 3 shows that  $R_H$  is always greater than 1.0 for  $F$  less than zero (for "adverse" currents where waves propagate against the current) and  $R_H$  is less than 1.0 for  $F$  greater than zero (for "following" currents where waves propagate with the current). Figure 3 also shows that for a given value of  $F$ , adverse currents have a larger effect (i.e., give  $R_H$  values further removed from 1.0) than following currents.

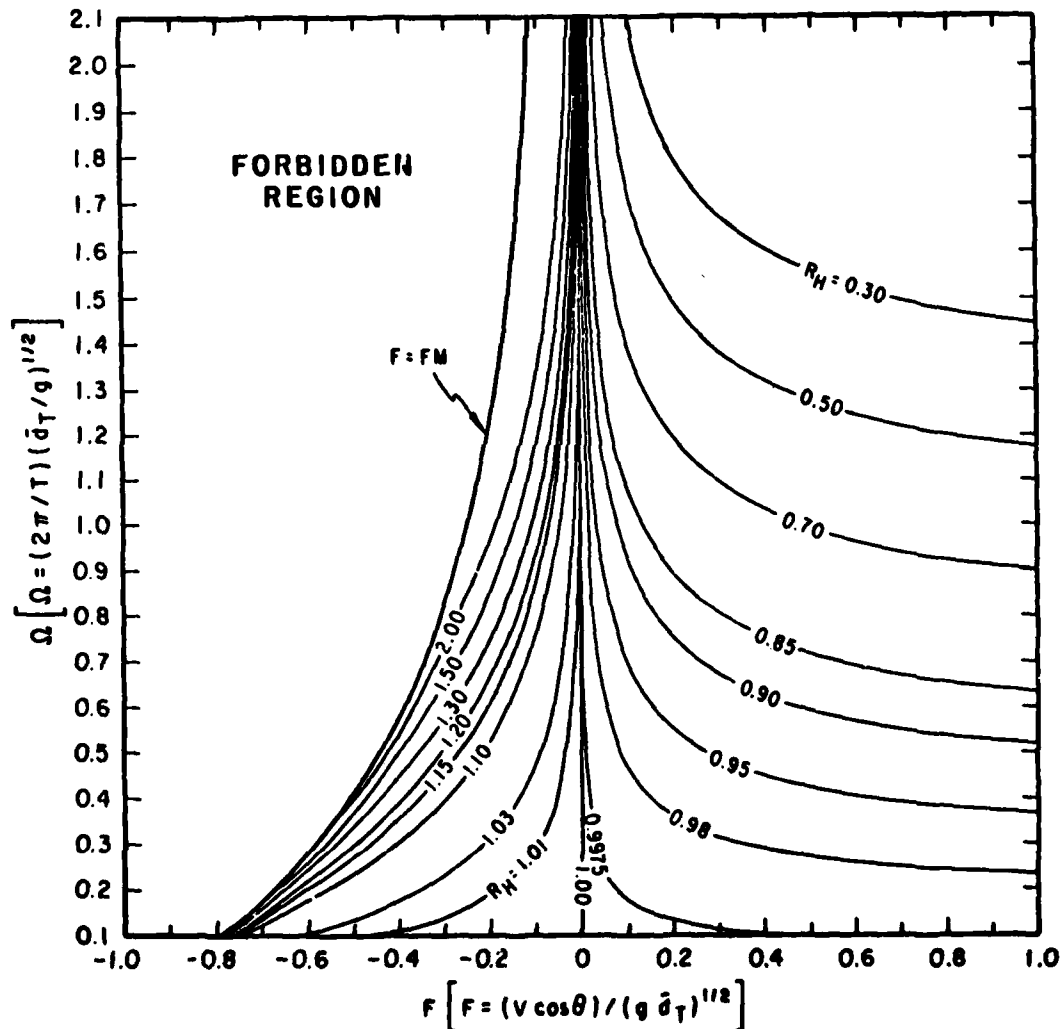


Figure 3. Contours of dimensionless wave height factor,  $R_H$ , given by equation (11). Waves cannot propagate for values of  $F$  and  $\Omega$  which lie in the forbidden region (boundary line  $F = FM$ ).

Knowledge of  $R_H$  can also be used to estimate whether current effects are great enough to warrant being taken into account when using an observed bottom pressure fluctuation magnitude,  $P$ , to get a wave height,  $H$ . For the range of current speeds, average depths, angles between the current vector and wave orthogonal, and wave periods, the largest and smallest values of  $R_H$  (denoted by  $R_{H,L}$  and  $R_{H,S}$ , respectively) are estimated. If both are within the acceptable range, then the current correction can be neglected and the wave height estimated from

$$H = H_A \quad (13)$$

where  $H_A$  is given by equation (10). If either  $R_{H,L}$  or  $R_{H,S}$  or both are out of range, then equation (12) must be used to compute  $H$ . Practical limits on the acceptable range for both  $R_{H,L}$  and  $R_{H,S}$  are 1.15 and 0.85; i.e., *if the wave current interaction produces  $R_H$  values which differ from 1.0 by  $\pm 0.15$  or less, the effect of the current can be neglected.* Figure 3 shows that  $R_{H,L}$  is the  $R_H$  value corresponding to the *smallest*  $F$  and *largest*  $\Omega$  that is expected. Analogously,  $R_{H,S}$  is the  $R_H$  value corresponding to the *largest*  $F$  and *largest*  $\Omega$  that is expected. The largest expected  $\Omega$ ,  $\Omega_L$ , is given approximately by

$$\Omega_L = \frac{2\pi}{TS} \left( \frac{\bar{d}_{T,L}}{g} \right)^{1/2} \quad (14)$$

where  $TS$  is the smallest wave period of interest and  $\bar{d}_{T,L}$  the largest time-averaged water depth expected (i.e., largest value of  $\bar{d}_T$  expected). The largest  $F$ ,  $FL$ , and the smallest  $F$ ,  $FS$ , that can be expected are given approximately by

$$FL = \frac{(VL) \cos(\theta S)}{(g\bar{d}_{T,S})^{1/2}} \quad \text{if } 0^\circ \leq \theta S \leq 90^\circ \quad (0 \text{ rad} \leq \theta S \leq \frac{\pi}{2} \text{ rad}) \quad (15a)$$

$$FL = -\frac{(VS) \cos(\theta S)}{(g\bar{d}_{T,L})^{1/2}} \quad \text{if } 90^\circ < \theta S \leq 180^\circ \quad \left(\frac{\pi}{2} \leq \theta S \leq \pi \text{ rad}\right) \quad (15b)$$

and

$$FS = \frac{-(VL) |\cos(\theta L)|}{(g\bar{d}_{T,S})^{1/2}} \quad \text{if } 90^\circ < \theta L \leq 180^\circ \quad \left(\frac{\pi}{2} \text{ rad} < \theta L \leq \pi \text{ rad}\right) \quad (16a)$$

$$FS = \frac{(VS) \cos(\theta L)}{(g\bar{d}_{T,L})^{1/2}} \quad \text{if } 0^\circ \leq \theta L \leq 90^\circ \quad (0 \text{ rad} \leq \theta L \leq \frac{\pi}{2} \text{ rad}) \quad (16b)$$

where  $(VL, VS)$  are the (largest, smallest) current speeds expected,  $(\theta L, \theta S)$  are the (largest, smallest) values of the angle  $\theta$  expected,  $\bar{d}_{T,S}$  is the smallest time-averaged water depth expected (i.e., smallest value of  $\bar{d}_T$  expected), and  $\bar{d}_{T,L}$  has already been defined. If  $FS$  is less than the  $FM$  value corresponding to  $\Omega = \Omega_L$ , then  $FS$  is reset equal to  $FM$ .

In computing the wave height elevation,  $H$ , from the fluctuating bottom pressure magnitude,  $P$ , there may be a range of wave periods over which a current can be neglected ( $H$  is given approximately by  $H_A$  where  $H_A$  is

defined by eq. 10) and another range over which the current must be accounted for ( $H$  is given by  $(R_H) H_A$  where  $R_H$  is defined by eq. 11). Figure 3 and the values of  $FL$  and  $FS$ , once computed, can be used to determine the smallest wave period beyond which currents can be neglected,  $TS'$ . For all wave periods greater than or equal to  $TS'$ , the current can be neglected when computing  $H$  from  $P$ .  $TS'$  is given by

$$TS' = \frac{2\pi}{\Omega S'} \left( \frac{\bar{d}_{T,L}}{g} \right)^{1/2} \quad \text{for } \Omega S' \neq 0 \quad (17a)$$

$$TS' = \infty \quad (\text{i.e., currents are never important}) \quad \text{for } \Omega S' = 0 \quad (17b)$$

where

$$\Omega S' = \text{MIN}(\Omega A, \Omega B, \Omega C, \Omega D) \quad (18)$$

$\text{MIN}()$  means take the minimum value of the numbers in parentheses, and the factors  $\Omega A$ ,  $\Omega B$ ,  $\Omega C$ , and  $\Omega D$  are estimated from Figure 3 by using the following definitions:

- $\Omega A$  = the value of  $\Omega$  which corresponds to  $R_H = 0.85$  when  $F = FL$   
( $\Omega A = 0$  if  $FL$  is less than or equal to 0)
  - $\Omega B$  = the value of  $\Omega$  which corresponds to  $R_H = 1.15$  when  $F = FL$   
( $\Omega B = 0$  if  $FL$  is greater than or equal to 0)
  - $\Omega C$  = the value of  $\Omega$  which corresponds to  $R_H = 0.85$  when  $F = FS$   
( $\Omega C = 0$  if  $FS$  is less than or equal to 0)
  - $\Omega D$  = the value of  $\Omega$  which corresponds to  $R_H = 1.15$  when  $F = FS$   
( $\Omega D = 0$  if  $FS$  is greater than or equal to 0)
- (19)

Figure 4 gives a schematic representation of how  $\Omega A$  and  $\Omega D$  are related to  $FL$  and  $FS$  for the example problem presented in the next section (in the problem,  $\Omega B = \Omega C = 0$ ).

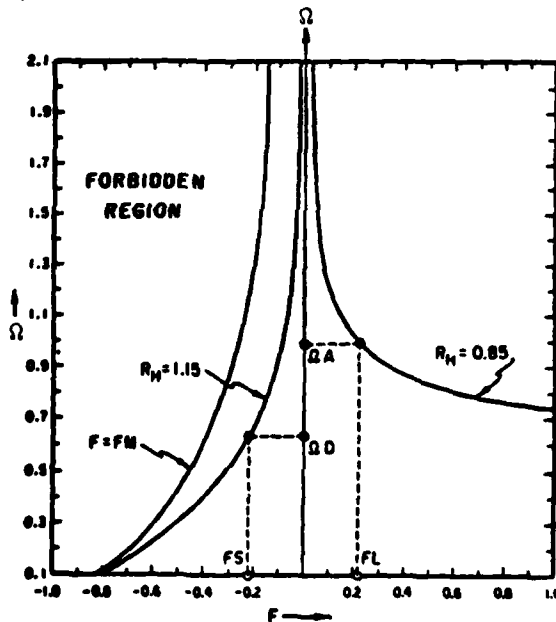


Figure 4. Schematic representation of how  $\Omega A$  and  $\Omega D$  are related to  $FL$  and  $FS$  for the example problem in Section IV. For the problem,  $\Omega B = \Omega C = 0$ .

#### IV. EXAMPLE PROBLEM

This example illustrates the method used to estimate the maximum and minimum wave height factors,  $R_{H,L}$  and  $R_{H,S}$ , whether the current can be neglected completely in computing wave height,  $H$ , from the fluctuating bottom pressure magnitude,  $P$ , and if not what range (if any) the current can be neglected.

GIVEN: A bottom-mounted wave pressure gage is located near an inlet at an elevation of -19.68 feet (-6 meters) referenced to the 1929 National Geodetic Vertical Datum (NGVD). The tide elevation ranges from -3.28 to 3.6 feet (-1 to 1.1 meters) NGVD. Current speeds from 0 to 4.92 feet (1.5 meters) per second are known to occur at the location of the gage. The wave climatology indicates that the dominant waves have periods between 6 and 15 seconds and can travel at any angle with respect to the current.

FIND:

(a) The largest and smallest values of  $R_H$  ( $R_{H,L}$  and  $R_{H,S}$ ) for the conditions expected and the range of wave periods of interest (periods between 6 and 15 seconds).

(b) Whether the currents can be neglected for the entire range of wave periods of interest when bottom pressure observations are used to determine wave heights.

(c) The range of wave periods over which currents can and cannot be neglected if they cannot be neglected over the entire range of interest.

SOLUTION:

Step 1. Set  $\theta_S$  and  $\theta_L$ . Since the angle between the direction of wave travel and the current can take on any value,

$$\theta_S = 0; \theta_L = 180^\circ = \pi \text{ rad}$$

Step 2. Set  $V_L$ ,  $V_S$ , and  $T_S$ . From the given values of the problem,

$$V_L = 4.92 \text{ ft (1.5 m)/s}$$

$$V_S = 0$$

$$T_S = 6 \text{ s}$$

Step 3. Compute  $\bar{d}_{T,S}$  and  $\bar{d}_{T,L}$ . From the tide elevation and bottom elevation information given,

$$\bar{d}_{T,S} = 19.68 - 3.28 = 16.4 \text{ ft (5 m)}$$

$$\bar{d}_{T,L} = 19.68 + 3.60 = 23.28 \text{ ft (7.1 m)}$$

Step 4. Compute  $\Omega_L$  using equation (14).

$$\Omega_L = \frac{2(3.14159)}{6 \text{ s}} \left( \frac{23.2 \text{ ft}}{32.2 \text{ ft/s}^2} \right)^{1/2} = 0.89 \text{ (dimensionless)}$$

Step 5. Compute FL using equation (15) and FS using equation (16). Since  $\theta_S$  is zero, equation (15a) is used for FL, giving

$$FL = \frac{(4.92 \text{ ft/s}) (1.0)}{[(32.2 \text{ ft/s}^2) 16.40 \text{ ft}]^{1/2}} = 0.214 \text{ (dimensionless)}$$

Since  $\theta_L$  is  $180^\circ$  ( $\pi$  rad), equation (16a) is used for FS, giving

$$FS = \frac{-(4.92 \text{ ft/s}) (1.0)}{[(32.2 \text{ ft/s}^2) 16.40 \text{ ft}]^{1/2}} = -0.214 \text{ (dimensionless)}$$

Step 6. In Figure 3, find the value of  $R_H$  corresponding to FL and  $\Omega_L$ . This value of  $R_H$  is  $R_{H,S}$  given approximately by

$$R_{H,S} = 0.77 \text{ (dimensionless)}$$

Step 7. In Figure 3, find the value of  $R_H$  corresponding to FS and  $\Omega_L$ . This value of  $R_H$  is  $R_{H,L}$ , given approximately by

$$R_{H,L} = 1.75 \text{ (dimensionless)}$$

Steps 1 through 7 solve part 1 under "Find."

Step 8. To answer part 2 under "Find," note that  $R_{H,S}$  is less than 0.85 and  $R_{H,L}$  is greater than 1.15. As a result, the current *cannot* be neglected for all wave periods between 6 and 15 seconds.

Step 9. To solve part 3 under "Find," Figure 3 is first used to determine the factors  $\Omega_A$ ,  $\Omega_B$ ,  $\Omega_C$ , and  $\Omega_D$ . Invoking the definitions for these parameters given in equation (19), it is found that approximately (see Fig. 4)

$$\Omega_A = 0.80; \Omega_B = \Omega_C = 0; \Omega_D = 0.59 \text{ (all dimensionless)}$$

Step 10. Compute  $\Omega_S'$  using equation (18). The result is

$$\Omega_S' = \text{MIN}(0.80, 0.0, 0.0, 0.59) = 0.59 \text{ (dimensionless)}$$

Step 11. Compute  $TS'$  using equation (17) and the value of  $\bar{d}_{T,L}$  found in step 3. The result is

$$TS' = \frac{[2(3,14159)]}{0.59} \left( \frac{23.28 \text{ ft}}{32.2 \text{ ft/s}^2} \right)^{1/2} = 9.1 \text{ s}$$

This value of  $TS'$  shows that only for waves between 9.1 and 15 seconds can the current be neglected in computing H from P. Waves between 6 and 9.1 seconds must have the influence of the current taken into account when using bottom pressure measurements to compute wave heights.

## V. SUMMARY

Some of the ways in which a horizontal current influences surface gravity waves and their measurement have been presented. Relatively simple hand-calculation methods have been described which provide a means to estimate (a) the wavelength modification due to a current, (b) whether a current can prevent waves from reaching a particular location, (c) the correction needed to compensate for a current when observed bottom pressure fluctuations are used to estimate surface gravity wave heights, and (d) the range of periods (if any) where the effects of currents can be neglected when surface wave heights are estimated from bottom pressure fluctuations.

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Effects of currents on waves / by Barry E. Herchenroder.--Fort  
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