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PERIODIC SOLUTIONS OF LARGE NORM OF HAMILTONIAN SYSTEMS.(U)
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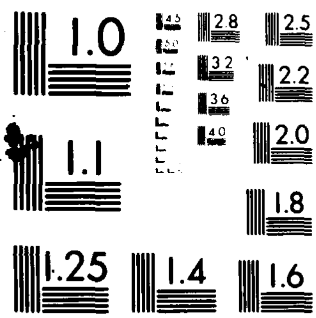

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PERIODIC SOLUTIONS OF LARGE NORM OF HAMILTONIAN SYSTEMS

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ABSTRACT

This paper studies Hamiltonian systems of ordinary differential equations. The only assumption made on the Hamiltonian is appropriately rapid growth at infinity. It is proved that for any given period, there is an unbounded sequence of periodic solutions of the system having the given period.

AMS(MOS) Subject Classification: 34C15, 34C25

Key Words: periodic solution, Hamiltonian system, minimax, variational methods, critical point

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SIGNIFICANCE AND EXPLANATION

Hamiltonian systems of ordinary differential equations model the motion of a discrete mechanical system. This paper considers a class of such systems assuming only suitably rapid growth for the Hamiltonian near infinity. Minimax and comparison arguments from the calculus of variations are then used to show that for any prescribed period, there exist arbitrarily large solutions of the system having the given period.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

PERIODIC SOLUTIONS OF LARGE NORM OF HAMILTONIAN SYSTEMS

Paul H. Rabinowitz

Introduction

This paper concerns the existence of periodic solutions of large norm of the Hamiltonian system

$$\text{(HS)} \qquad \dot{z} = JH_z(z)$$

where $z \in \mathbb{R}^{2n}$, $\dot{z} \equiv \frac{dz}{dt}$, $J = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$, I is the identity matrix on $\mathbb{R}^n$, $H: \mathbb{R}^{2n} \to \mathbb{R}$, and H_z is its gradiant. Let $(a,b)_{\mathbb{R}^j}$ denote the usual inner product in $\mathbb{R}^j$. The following result was presented in [1]:

Theorem 0.1: Let $H \in C^1(\mathbb{R}^{2n}, \mathbb{R})$ and satisfy

(H_0) There is an $r > 0$ and $\mu > 2$ such that

$$0 < \mu\, H(z) \leq (z, H_z(z))_{\mathbb{R}^{2n}}$$

for all $|z| \geq r$.

Then for all T, $R > 0$, (HS) possesses a T periodic solution $z(t)$ with

$$\max_{t \in [0,T]} |z(t)| \geq R.$$

However the proof of Theorem 0.1 given in [1] was not complete. Under the additional assumption of power growth for H, the result was proved in [2]. Our goal here is to show that Theorem 0.1 holds as stated. The proof we give is in the spirit of the argument in [1]. Solutions of (HS) are obtained as critical points of a corresponding functional $I_K(z)$ by minimax arguments. The proof here, however, is more direct avoiding the finite dimensional approximation arguments of [1]. Moreover the choice of sets with

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respect to which we minimax $I_K(z)$ permits a multiplicity theorem for the corresponding critical values of $I_K(z)$ as well as rather sharp lower bounds for critical values of a comparison problem. The latter estimates play a critical role in establishing the unboundedness of the set of solutions of (HS). The lower bounds given for critical values in [1] are probably too weak for the argument given there to succeed without a power growth assumption for H.

§1. The proof of Theorem 0.1.

By rescaling time if necessary we can assume $T = 2\pi$. Let $z(t) = (p(t),q(t))$ with $p,q \in \mathbb{R}^n$ and set

$$A(z) \equiv \int_0^{2\pi} (p(t),\dot{q}(t))_{\mathbb{R}^n} dt,$$

the so-called action integral. The basic idea we use in trying to find periodic solutions of (HS) is to obtain them as critical points of the corresponding functional

$$I(z) = A(z) - \int_0^{2\pi} H(z)dt \tag{1.1}$$

defined on the class of 2π periodic functions under a suitable norm. The form of $A(z)$ suggests working in $E \equiv (W^{1/2,2}(S^1))^{2n}$, the space of $2n$ tuples of 2π periodic functions which possess a square integrable "derivative" of order $1/2$ (See [3]). Unfortunately the H term in I is not necessarily smooth enough for our later purposes nor is I appropriately compact (i.e. I does not satisfy the Palais-Smale condition). Thus following [3] or [4], we truncate H by taking $\chi_K(s) \in C^\infty(\mathbb{R},\mathbb{R})$ such that $\chi_K(s) \equiv 1$ for $s \leq K$; $\equiv 0$ for $s \geq K+1$; and $\chi_K'(s) < 0$ for $s \in (K,K+1)$ and setting

$$H_K(z) = \chi_K(|z|)\, H(z) + (1 - \chi_K(|z|))\, r_K|z|^4 \tag{1.2}$$

where r_K satisfies

$$r_K = \max_{K \leq |z| \leq K+1} \frac{|H(z)|}{|z|^4}.$$

With this choice of r_K, it is easy to verify that H_K satisfies (H_0) with μ replaced by $\bar{\mu} = \min(\mu,4)$. Integrating (H_0) then shows that

$$H_K(z) \geq a_1|z|^{\bar{\mu}} - a_2 \tag{1.3}$$

for all $z \in \mathbb{R}^{2n}$ with a_1,a_2 independent of K.

Let E^+, E^-, E^0 denote respectively the subspaces of E on which $A(z)$ is positive definite, negative definite, and null. A basis for these spaces can be written down explicitly, e.g. if $e_1, \cdots, e_{2n}$ denote the usual orthonormal basis in $\mathbb{R}^{2n}$, set

$$\varphi_{jk} = (\sin jt)e_k - (\cos jt)e_{k+n}$$
$$\psi_{jk} = (\cos jt)e_k + (\sin jt)e_{k+n}$$
$$\theta_{jk} = (\sin jt)e_k + (\cos jt)e_{k+n}$$
$$\zeta_{jk} = (\cos jt)e_k - (\sin jt)e_{k+n}.$$

Then

$$E^+ = \text{span}\{\varphi_{jk}, \psi_{jk} \mid j \in \mathbb{N},\ 1 \leq k \leq n\}$$
$$E^- = \text{span}\{\theta_{jk}, \zeta_{jk} \mid j \in \mathbb{N},\ 1 \leq k \leq n\}$$
$$E^0 = \text{span}\{\varphi_{0k}, \psi_{0k} \mid 1 \leq k \leq n\}$$

and $E = E^+ \oplus E^- \oplus E^0$. Thus for $z \in E$, $z = z^+ + z^- + z^0 \in E^+ \oplus E^- \oplus E^0$ and we will take as norm for E

$$(1.4) \qquad \|z\|^2 \equiv A(z^+) - A(z^-) + |z^0| = \|z^+\|^2 + \|z^-\|^2 + |z^0|^2.$$

It is easy to verify that this norm makes E a Hilbert space and E^+, E^-, E^0 are orthogonal subspaces of E with respect to the inner product associated with (1.4) as well as with the L^2 inner product. Moreover

$$(1.5) \qquad I_K(z) \equiv A(z) - \int_0^{2\pi} H_K(z)\,dt$$

belongs to $C^1(E,\mathbb{R})$. (See [3]).

We will show that $I_K(z)$ possesses an unbounded sequence of critical points which for appropriately chosen K are also critical points of I. This will be done by minimaxing I_K over certain families of sets Γ_j. To show the minimax values $c_j(K)$ produced in this fashion are indeed critical values of I_K requires sufficiently sharp lower bounds for $c_j(K)$. These lower bounds are obtained by minimaxing a comparison functional. Rather than pause now to introduce all of the properties required for the comparison problem, we will simply assume there is an $M \in C^2(\mathbb{R},\mathbb{R})$ such that

(m_1) For all $K > 0$, M has a truncation $M_K \in C^2(\mathbb{R},\mathbb{R})$ such that $M_K(s) = M(s)$ for $s \leq K$,

(m_2) $M_K(|z|) \geq H_K(z)$ for all $z \in \mathbb{R}^{2n}$

and

$$J_K(z) \equiv A(z) - \int_0^{2\pi} M_K(z)dt \tag{1.6}$$

satisfies $J_K \in C^1(E,\mathbb{R})$. We will make further assumptions concerning M and M_K as necessary. Then we will conclude the proof of Theorem 0.1 by constructing M and M_K having the desired properties. Note that (m_1) and (m_2) imply $M(|z|) \geq H(z)$ for all $z \in \mathbb{R}^{2n}$ and (1.6) and (m_2) show $J_k(z) \leq I_K(z)$ for all $z \in E$.

The minimax procedure we will use takes advantage of an S^1 invariance possessed by I_K and I. For $z \in E$ and $\theta \in [0,2\pi] \approx S^1$, set

$$(T_\theta z)(t) = z(t + \theta). \tag{1.7}$$

Then for fixed $z \in E$, $\|T_\theta z\|_{L^2}$, $I_K(T_\theta z)$, and $J_K(T_\theta z)$ remain unchanged as θ varies. We call a subset B of E an <u>invariant set</u> (under $\{T_\theta\}$ or S^1) if for all $z \in B$, $T_\theta z \in B$ for all $\theta \in [0,2\pi]$. If B is an invariant set, we say $h \in C(B,E)$ is an <u>equivariant map</u> if $h(T_\theta z) = T_\theta h(z)$ for all $\theta \in [0,2\pi]$ and $z \in B$. Note that the fixed point set of this group of symmetries,

$$\text{Fix } \{T_\theta\} \equiv \{z \in E \mid T_\theta z = z \text{ for all } \theta \in [0,2\pi]\} = E^0. \tag{1.8}$$

Let $\mathcal{E}$ denote the family of closed invariant subsets of $E\setminus\{0\}$. In [5], an index theory defined on $\mathcal{E}$ was introduced and we shall use it below. The properties we need are summarized in the following result:

Lemma 1.9: There is an index theory, i.e. a mapping $i: E \to \mathbb{N} \cup \{\infty\}$ such that if $B, B_1 \in E$,

1° $\quad i(B) \le i(B_1)$ if there is a $\varphi \in C(B,B_1)$ with φ equivariant

2° $\quad i(B \cup B_1) \le i(B) + i(B_1)$

3° $\quad$ If $B \subset E \backslash E^0$ and B is compact, $i(B) < \infty$ and there is a $\delta > 0$ such that $i(N_\delta(B)) = i(B)$ where $N_\delta(B) = \{x \in E \mid \|x - B\| \le \delta\}$.

4° $\quad$ If $S \subset E \backslash E^0$ is a $2n$ dimensional invariant sphere, $i(S) = n$.

With these preliminaries in hand, several families of sets can be introduced. For $m \in \mathbb{N}$, let

$$(1.10) \qquad V_m = \mathrm{span}\{\varphi_{jk}, \psi_{jk} \mid j \le [m/n],\ k \le m - nj\} \oplus E^- \oplus E^0$$

where $[a]$ denotes the greatest integer in a. Then V_m is an invariant subspace of E. By (1.3) and the Hölder inequality,

$$(1.11) \quad J_K(z) \le I_K(z) \le \|z^+\|^2 - a_3\|z\|^{\bar{\mu}}_{L^2} + 2\pi a_4 \le \|z^+\|^2 - a_3\|z^+\|^{\bar{\mu}}_{L^2} + 2\pi a_4.$$

Since $V_m \cap E^+$ is m dimensional and $\bar{\mu} > 2$, (1.11) shows there is an $R_m > 0$ and independent of K such that

$$(1.12) \qquad I_K(z) \le -2\pi M(0)$$

for all $z \in V_m$ such that $\|z\| \ge R_m$. Let B_R denote the closed ball of radius R in E centered about 0. Set $D_m = B_{R_m} \cap V_m$. Then D_m is an invariant set. Let P^- denote the orthogonal projector of E onto E^-. Let G_m denote the class of mappings $h \in C(D_m, E)$ which satisfy the following properties:

(g_1) $\quad h$ is equivariant

(g_2) $\quad h(z) = z$ for $z \in (\partial B_{R_m} \cap V_m) \cup E^0$

(g_3) $\quad P^-h(z) = \alpha(z)z^- + \psi(z)$ where $\psi(z)$ is compact and $\alpha \in C(D_m, [1,\bar{\alpha}])$, $\bar{\alpha}$ depending on h.

Since $h(z) = z \in G_m$ for all $m \in \mathbb{N}$, $G_m \ne \phi$.

Finally for $j \in \mathbb{N}$, define

$$(1.13) \qquad \Gamma_j = \{\overline{h(D_m \setminus Y)} \mid m \geq j,\ h \in G_m,\ Y \in \mathcal{E}, \text{ and } i(Y) \leq m-j\}.$$

This class of sets resembles somewhat a class used in [5]. We will minimax I_K and J_K over this class. First we briefly study Γ_j:

<u>Lemma 1.14:</u> The classes Γ_j possess the following properties:

1° (<u>Monotonicity</u>): $\Gamma_{j+1} \subset \Gamma_j$

2° (<u>Excision</u>): If $B \in \Gamma_j$ and $Z \in \mathcal{E}$ with $i(Z) \leq s < j$, then
$$\overline{B \setminus Z} \in \Gamma_{j-s}$$

3° (<u>Invariance</u>): If $\varphi \in C(E,E)$ and satisfies (g_1), (g_3) and (g_2) for all $m \geq j$, then $B \in \Gamma_j$ implies $\overline{\varphi(B)} \in \Gamma_j$.

Proof: The definition of Γ_j implies 1°. To prove 2°, let $B = \overline{h(D_m \setminus Y)} \in \Gamma_j$. We claim

$$(1.15) \qquad \overline{B \setminus Z} = \overline{h(D_m \setminus (Y \cup h^{-1}(Z)))}.$$

Assuming this for the moment, since $h \in G_m$, $Y \cup h^{-1}(Z) \in \mathcal{E}$. Hence by 2° and 1° of Lemma 1.9,

$$i(Y \cup h^{-1}(Z)) \leq i(Y) + i(h^{-1}(Z)) \leq i(Y) + i(Z) \leq m - (j-s).$$

Thus $\overline{B \setminus Z} \in \Gamma_{j-s}$. To verify (1.15), note first that $b \in h(D_m \setminus (Y \cup h^{-1}(Z)))$ implies $b \in h(D_m \setminus Y) \setminus Z \subset B \setminus Z$, i.e.

$$(1.16) \qquad h(D_m \setminus (Y \cup h^{-1}(Z))) \subset \overline{B \setminus Z}.$$

Similarly,

$$(1.17) \qquad B \setminus Z \subset \overline{h(D_m \setminus (Y \cup h^{-1}(Z)))}$$

so combining (1.16)-(1.17) yields (1.15). Lastly to get 3°, again let $B = \overline{h(D_m \setminus Y)} \in \Gamma_j$. It is straightforward to show that

$$\varphi(B) \subset \overline{(h(D_m \setminus Y))} \subset \overline{\varphi(B)}.$$

Therefore

$$(1.18) \qquad \overline{\varphi(B)} = \overline{\varphi(h(D_m \setminus Y))} \in \Gamma_j$$

since $\varphi \circ h \in G_m$.

The next result which is based on related intersection theorems in [6] is crucial for our later estimates.

<u>Proposition 1.19:</u> Let $h \in G_m$, $j \leq m$, $\rho < R_m$, and

$$\Theta = \{z \in D_m | h(z) \in \partial B_\rho \cap V_{j-1}^\perp\}$$

Then Θ is compact and $i(\Theta) \leq m-j+1$.

Proof: Due to the way in which it is defined, Θ is closed and invariant. Since $h(E^0) = E^0 \subset V_0$ via (g_2) and $\Theta \cap V_0 = \phi$, $\Theta \cap E^0 = \phi$. To see that Θ is compact, let (z_i) be a sequence in Θ. Since D_m is bounded, by restricting to a subsequence if necessary, we can z_i converges weakly to some $z \in E$, i.e. $z_i \rightharpoonup z$. Since D_m is closed and convex, it is weakly closed so $z \equiv z^+ + z^- + z^0 \in D_m$. Writing $z_i \equiv z_i^+ + z_i^- + z_i^0$, we can assume $z_i^+, z_i^0 \to z^+, z^0$ since E^0 and $V_m \cap E^+$ are finite dimensional subspaces of E. Moreover by (g_3)

$$P^- h(z_i) = \alpha(z_i) z_i^- + \psi(z_i) \tag{1.20}$$

where $1 \leq \alpha(z_i) \leq \bar{\alpha}$, $\bar{\alpha}$ depending on h, and ψ is compact. Thus

$$z_i^- = -\alpha(z_i)^{-1}\psi(z_i)$$

so z_i^- and hence z_i has a strongly convergent subsequence. Consequently Θ is compact and by 3° of Lemma 1.9, $i(\Theta) < \infty$ and there is a $\delta > 0$ such that

$$i(\Theta) = i(N_\delta(\Theta)) \tag{1.21}$$

To estimate $i(\Theta)$, a finite dimensional approximation argument will be used. Let

$$E_k = \text{span}\{\varphi_{\sigma i}, \psi_{\sigma i}, \theta_{\sigma i}, \zeta_{\sigma i} | 0 \leq \sigma \leq k,\ 1 \leq \ell \leq 2n\}$$

and let P_k denote the orthogonal projector of E onto E_k. Thus E_k is an invariant subspace of E, $P_k h \in C(P_k D_m, E_k)$ is equivariant, and for $k > m$, $P_k h(z) = z$ for $z \in E^0 \cup (\partial B_{R_m} \cap V_m \cap E_k)$. Therefore $(P_k h)^{-1}(B_\rho \cap E_k)$ is a closed invariant neighborhood of 0 in $V_m \cap E_k$. Let Ω denote the component of $(P_k h)^{-1}(B_\rho \cap E_k)$ which contains 0. Then Ω is contained in the interior of $B_{R_m} \cap V_m \cap E_k$. Let $\tilde{P}_j$ denote the orthogonal projector of $V_m \cap E_k$ onto $V_{j-1} \cap E_k$. Thus $f \equiv \tilde{P}_j P_k h \in C(\Omega, V_{j-1} \cap E_k)$, is equivariant, and $f(z) = z$ for $z \in E^0 \cap \Omega$. But then f, Ω satisfy the hypotheses of Theorem 2.3 of [6] which guarantees that

$$i(f^{-1}(0) \cap \partial\Omega) \geq m-j+1. \tag{1.22}$$

At zeroes of f on $\partial\Omega$, we have $P_k h(z) \in \partial B_\rho \cap V_{j-1}^\perp$. Thus (1.22) and 1° of Lemma 1.9 imply

$$\Theta_k \equiv \{z \in D_m \mid P_k h \in \partial B_\rho \cap V_{j-1}^\perp\}$$

satisfies

$$i(\Theta_k) \geq m-j+1. \tag{1.23}$$

We claim $\Theta_k \subset N_\delta(\Theta)$ for all large k. The completion of the proof is then immediate via 1° of Lemma 1.9, (1.23), and (1.21). Arguing indirectly, if $\Theta_k \not\subset N_\delta(\Theta)$ for all large k, then there is a sequence of k's $\to \infty$ for which $z_k \in \Theta_k$ but $z_k \notin N_\delta(\Theta)$. Writing $z_k = z_k^+ + z_k^- + z_k^0$, as above we can assume z_k^+, z_k^0 converge and

$$P^- P_k h(z_k) = \alpha(z_k) z_k^- + P_k \psi(z_k) = 0.$$

This implies z_k^- also converges so $z_k \to z \in D_m$. Moreover since

$$\|h(z) - P_k h(z_k)\| \leq \|h(z) - P_k h(z)\| + \|P_k(h(z) - h(z_k))\| \to 0 \tag{1.24}$$

as $k \to \infty$ and $P_k h(z_k) \in \partial B_\rho \cap V_{j-1}^\perp$, it follows that $z \in \Theta$. On the other hand $z \notin N_{\delta/2}(\Theta)$, a contradiction. Thus $\Theta_k \subset N_\delta(\Theta)$ for large k and the proposition is proved.

<u>Corollary 1.25:</u> Under the hypotheses of Proposition 1.19, if $Y \in E$, $i(Y) \leq m-j$ and $W \equiv \overline{\Theta \backslash Y}$, then

$$\overline{h(D_m \backslash Y)} \cap \partial B_\rho \cap V_{j-1}^\perp \supset h(W) \neq \phi. \tag{1.26}$$

Proof: W is compact and

$$h(W) \subset \overline{h(D_m \backslash Y)} \cap \partial B_\rho \cap V_{j-1}^\perp.$$

Hence by 1° and 2° of Lemma 1.9 and Proposition 1.19,

(1.27) $$i(\overline{h(D_m\setminus Y)}) \cap \partial B_\rho \cap V_{j-1}^{\perp} \geq i(h(W)) \geq i(W) \geq i(\mathcal{O}) - i(Y) \geq 1$$

so (1.26) follows.

Having completed the above preliminaries, we can now define a sequence of minimax values for I_K and J_K. Let

(1.28) $$c_j(K) = \inf_{B\in\Gamma_j} \sup_{z\in B} I_K(z),$$

(1.29) $$b_j(K) = \inf_{B\in\Gamma_j} \sup_{z\in B} J_k(z).$$

By (1.11) we have

(1.30) $$c_j(K) \geq b_j(K), \quad j \in \mathbb{N}, \quad K \in \mathbb{R}^+$$

and by 1° of Lemma 1.14, we see that

(1.31) $$c_{j+1}(K) \geq c_j(K); \quad b_{j+1}(K) \geq b_j(K) \geq b_1(K)$$

An estimate for $b_1(K)$ will be needed later. Set

$$\bar{M}(s) = M(s) - M(0); \quad \bar{M}_K(s) = M_K(s) - M(0).$$

We assume that

(m_3) $$\bar{M}(s) = o(s^2) \quad \text{at} \quad s = 0$$

and

(m_4) $M_K(s)$ is strictly monotonically increasing in s and tends to ∞ as $s \to \infty$.

Let

$$\bar{J}_K(z) \equiv A(z) - \int_0^{2\pi} \bar{M}_K(z)dt.$$

Then

(1.32) $$b_j(K) = \inf_{B\in\Gamma_j} \sup_{z\in B} \bar{J}_K(z) - 2\pi M(0) \equiv \bar{b}_j(K) - 2\pi M(0).$$

<u>Lemma 1.33:</u> $\bar{b}_1(K) > 0$.

Proof: Since $\bar{M}(s) = o(s^2)$ at $s = 0$, by Lemma 3.35 of [3],

(1.34) $$\int_0^{2\pi} \bar{M}_k(z)dt = o\,(\|z\|^2) \text{ at } z = 0.$$

Let $B \in \Gamma_1$ so $B = \overline{h(D_m \backslash Y)}$ for some $h \in G_m$, $m > 1$, $Y \in E$ and $i(Y) \leq m-1$. Since $V_0^{\perp} = E^+$, by Corollary 1.25 with $j = 1$, for any $\rho < R_m$, there is a $\hat{z} \in D_m \backslash Y$ such that $h(\hat{z}) \in \partial B_\rho \cap E^+$. Hence

$$\sup_B \bar{J}_K(z) \geq \bar{J}_K(h(\hat{z})) = \|h(\hat{z})\|^2 - \int_0^{2\pi} \bar{M}_k(h(\hat{z}))dt$$

$$= \rho^2 - \int_0^{2\pi} \bar{M}_k(h(\hat{z}))dt.$$

By (1.34), $\rho = \rho(K) \leq R_1$ can be chosen so that

$$\int_0^{2\pi} \bar{M}_K(z)dt \leq \tfrac{1}{2}\|z\|^2$$

for $\|z\| \leq \rho$. Therefore

(1.35) $\quad \sup \bar{J}_k(z) \geq \rho^2 - \tfrac{1}{2}\rho^2 = \tfrac{1}{2}\rho^2.$

Since $B \in \Gamma_1$ was arbitrary, (1.35) shows $\bar{b}_1(K) \geq \tfrac{1}{2}\rho^2 > 0$ where $\rho = \rho(K)$.

Our next goal is to prove that the minimax values $\bar{b}_j(K)$ are critical values of $\bar{J}_k$. This requires a variant of a standard "Deformation Theorem". Let $\Psi \in C^1(\mathbf{R}^{2n},\mathbf{R})$ and for some constants $s, \alpha_1, \alpha_2 > 0$ satisfy

$$|\Psi(z)| \leq \alpha_1|z|^s + \alpha_2$$

for all $z \in \mathbf{R}^{2n}$. Then

$$\int_0^{2\pi}\Psi(z)dt \quad \text{and} \quad \Phi(z) = A(z) - \int_0^{2\pi} \Psi(z)dt$$

belong to $C^1(E,\mathbf{R})$ - see [3]. We say Φ satisfies the Palais-Smale condition (PS) if whenever (i) $\Phi(z_m)$ is uniformly bounded and (ii) $\Phi'(z_m) \to 0$, then (z_m) possesses a convergent subsequence. Let $K_c = \{z \in E \mid \Phi(z) = c$ and $\Phi'(z) = 0\}$ and $A_c = \{z \in E \mid \Phi(z) \leq c\}$.

<u>Lemma 1.36:</u> Let Ψ be as above with $\Phi \in C^1(E,\mathbf{R})$. If Φ also satisfies (PS), then for any $c \in \mathbf{R}$, $\bar{\varepsilon} > 0$, and invariant neighborhood $\mathcal{O}$ of K_c, there is an $\varepsilon \in (0,\bar{\varepsilon})$ and $\eta \in C([0,1] \times E,E)$ such that

1° $\eta(t,\cdot)$ is equivariant for all $t \in [0,1]$

2° $\eta(t,\cdot)$ is a homeomorphism of E onto E for all $t \in [0,1]$

3° $\eta(0,z) = z$

4° $\eta(t,z) = z$ if $\Phi(z) \notin [c-\bar{\varepsilon}, c+\bar{\varepsilon}]$

5° $\eta(1, A_{c+\varepsilon} \vartheta) \subset A_{c-\varepsilon}$

6° If $K_c = \phi$, $\eta(1, A_{c+\varepsilon}) \subset A_{c-\varepsilon}$

7° $P^-\eta(1,z)$ satisfies (g_3).

Proof: The result without assertions 1° and 7° is well known - see e.g. [7] or [8]. Moreover given an equivariant pseudogradiant vector field $V(z)$ for $\Phi'(z)$, 1° also follows via the proof of [8]. The existence of such a $V(z)$ of the form $V(z) = A'(z) + P(z)$ with P compact is given e.g. in [9]. Lastly 7° follows since $P^-\eta(t,z)$ is determined as the solution of the initial value problem for the ordinary differential equation:

$$\frac{dP^-\eta}{dt} = -\beta(\eta)\, P^-(A'(\eta) + P(\eta)) \tag{1.37}$$

$$P^-\eta(0,z) = P^-z = z^-$$

where β is a scalar function with $0 \leq \beta \leq 1$. Since $P^-A'(\eta) = -2P^-\eta$,

$$\begin{aligned} P^-\eta(t,z) &= z^- \exp \int_0^t 2\beta(\eta(s,z))ds \\ &+ \int_0^t (\exp \int_0^t 2\beta(\eta(s,z))ds) P(\eta(\tau,z))d\tau. \end{aligned} \tag{1.38}$$

Hence $P^-\eta(t,z)$ has the form (g_3).

<u>Remark 1.39:</u> Due to the form of the truncation involved, $I_K \in C^1(E,\mathbb{R})$ and as we shall see later, J_K, $\bar{J}_K \in C^1(E,\mathbb{R})$. Moreover this form implies I_K, J_K, $\bar{J}_K$ satisfy (PS) - see [3]. Actually [3] only proves any sequence (z_m) satisfying (i) and (ii) (for I_K, J_K, or $\bar{J}_K$) is bounded. Therefore z_m converges weakly in E and z_m^0 converges strongly in E (along some subsequence). Since $P^{\pm}\Phi'(z) = \pm z^{\pm} + P^{\pm}\tilde{P}(z)$ with $\tilde{P}$ compact - see [3] - (ii) and the weak convergence of $z_m^{\pm}$ imply the strong convergence of $z_m^{\pm}$ and hence (PS).

Now we are in a position to establish that the $\bar{b}_j(K)$'s are critical values of $\bar{J}_K$.

<u>Lemma 1.40:</u>

1° $\bar{b}_{j+1}(K) > \bar{b}_j(K)$

2° $\bar{b}_j(K)$ is a critical value of $\bar{J}_K$

3° Any critical points of $\bar{J}_K$ corresponding to $\bar{b}_K(K)$ lie in $E \setminus E^0$

4° If $\bar{b}_{j+1}(K) = \cdots = \bar{b}_{j+\ell}(K) \equiv b$ and $\mathcal{K} \equiv (\bar{J}_K')^{-1}(0) \cap \bar{J}_K^{-1}(b)$, then $i(\mathcal{K}) \geq \ell$.

Proof: Statement 1° follows from (1.31) and (1.32). To prove 2°, it suffices to prove the stronger multiplicity assertion 4°. Note first that since $\bar{J}_K$ satisfies (PS), $\mathcal{K}$ is compact. For $z \in E^0$, $\bar{J}_K \leq 0$ via (m_4) and the definition of $\bar{M}_K$. Moreover by 1° of this lemma and Lemma 1.33, $\bar{b}_j(K) \geq \bar{b}_1(K) \geq \frac{1}{2}\rho^2(K) > 0$. Hence $\mathcal{K} \cap E^0 = \phi$ and 3° follows. Now by 3° of Lemma 1.9 there is a $\delta > 0$ such that $i(N_\delta(\mathcal{K})) = i(\mathcal{K})$. Suppose $i(\mathcal{K}) \leq \ell-1$. We invoke Lemma 1.36 with $\Phi = \bar{J}_K$, $c = \bar{b}$, $\bar{\varepsilon} = \frac{1}{4}\rho^2(K)$, and $\mathcal{O} = N_\delta(\mathcal{K})$. Thus there is an $\varepsilon \in (0,\bar{\varepsilon})$ and $\eta \in C([0,1] \times E, E)$ satisfying 1°-7° of Lemma 1.36. Choose $B \in \Gamma_{j+\ell}$ such that

$$\sup_B \bar{J}_K \leq b + \varepsilon \tag{1.41}$$

By 2° of Lemma 1.14, $\overline{B \setminus \mathcal{O}} \in \Gamma_{j+1}$. The definition of R_m - see (1.12) - implies that $\bar{J}_K(z) = J_K(z) + 2\pi M(0) \leq 0$ for $z \in \partial B_{R_m} \cap V_m$. As was noted above $\bar{J}_K \leq 0$ on E^0. Thus by 4° of Lemma 1.36, $\eta(1,z) = z$ for $z \in E^0 \cup (\partial B_{R_m} \cap V_m)$ for all $m \in \mathbb{N}$ and $\eta(1,z)$ satisfies (g_2). Moreover 1° and 7° of Lemma 1.36 imply $\eta(1,z)$ satisfies (g_1) and (g_3). Hence $\eta(1,z) \in \underline{G_m}$ for all $m \in \mathbb{N}$. Consequently by 3° of Lemma 1.14, $Q \equiv \eta(1,\overline{B \setminus \theta}) \in \Gamma_{j+1}$. Note that $Q = \eta(1,\overline{B \setminus \theta})$ via 2° of Lemma 1.36. Thus by the definition of $\bar{b}_{j+1}(K)$,

(1.42)
$$\sup_{Q} \bar{J}_K > b$$
while by (1.41) and 5° of Lemma 1.36

(1.43)
$$\sup_{Q} \bar{J}_K < b-\varepsilon,$$
a contradiction. Thus the Lemma is proved.

Next we will make a closer study of the critical values $\bar{b}_j(K)$ of $\bar{J}_K$. Let $z = (p,q)$ be a corresponding critical point. Then - see e.g. [3] - z is a classical solution of

(1.44)
$$\begin{cases} \dot{p} = -\frac{\partial}{\partial q}\bar{M}_K(|z(t)|) = -M_K'(|z(t)|)\frac{q}{|z|} \\ \dot{q} = \frac{\partial}{\partial p}\bar{M}_K(|z(t)|) = M_K'(|z(t)|)\frac{p}{|z|} . \end{cases}$$

Condition (m_3) guarantees that there are no problems with the right hand side of (1.44) if $z(t_0) = 0$. Since (1.44) is a Hamiltonian system, $M_K(|z(t)|)$ is independent of t. Therefore by (m_4), $|z(t)|$ must be constant and nonzero since $\bar{b}_j(k) > \bar{b}_1(K) > 0$. Differentiating (1.44) then yields

$$\ddot{p} = -\frac{\bar{M}_K'(|z(t)|)}{|z|}\dot{q} = -\left(\frac{\bar{M}_K'(|z|)}{|z|}\right)^2 p$$

with q satisfying the same equation. We know exactly what all solutions of (1.45) are and in order for them to be 2π periodic, it must be the case that

(1.46)
$$\frac{\bar{M}_K'(|z|)}{|z|} = k$$

for some $k \in \mathbb{N}$. Then p,q have the form

(1.47)
$$\begin{cases} p(t) = \alpha \cos kt + \beta \sin kt \\ q(t) = \alpha \sin kt - \beta \cos kt \end{cases}$$

where $\alpha,\beta \in \mathbb{R}^n$ and $|z(t)|^2 = \alpha^2 + \beta^2$. Thus for each $k \in \mathbb{N}$, we get a 2n-1 dimensional sphere S_k in E (or L^2) of solutions of (1.44). Since S_k is also an invariant set and lies in $E\backslash E^0$, by 4° of Lemma 1.9, $i(S_k) = n$.

Suppose that

$$(m_5) \qquad M_K(s) \equiv \frac{\bar{M}_K'(s)}{s} \text{ is strictly monotone and tends to infinity as } s \to \infty.$$

Then (1.46) shows that $|z(t)|$ is a monotone increasing function of k which goes to infinity as $k \to \infty$. The critical value of $\bar{J}_K$ corresponding to any $z \in S_k$ is

$$\bar{J}_K(z) = \int_0^{2\pi} (p,\dot{q})_{\mathbb{R}^n} - \bar{M}_K(z))dt \tag{1.48}$$

$$= 2\pi(\tfrac{1}{2}|z|\,\bar{M}_K'(|z|) - \bar{M}_K(|z|)).$$

Thus if M_K satisfies

(m_6) $\quad \tfrac{1}{2} s\,\bar{M}_K'(s) - M_K(s)$ is strictly monotone increasing in s

then on the set of its critical points, $\bar{J}_K$ is a monotone function of $|z|$ and via (m_5) of k.

<u>Lemma 1.49:</u> $\bar{b}_j(K) > J_K|_{S_k}$ where $k = [j/n]$.

Proof: This follows by combining our above observations. By 1° of Lemma 1.40, the critical values $\bar{b}_j(K)$ form a nondecreasing sequence in j and by 4°, a multiple critical value of "multiplicity" ℓ has a corresponding set of critical points of index at least ℓ. All critical points of $\bar{J}_K$ are of the form (1.47) and combine in families S_k of index n. All $z \in S_k$ have $|z(t)| = \text{constant} \equiv \gamma_k$ with γ_k independent of z and by (m_5), γ_k is a monotonically increasing function of k. Moreover by (m_6) and (m_5) $\bar{J}_K|_{S_k} \equiv \sigma_k$ also is a monotonically increasing function of k. Thus the j^{th} minimax value $\bar{b}_j(K)$ must come from family $\bar{k}$ where $\bar{k} > [j/n] \equiv k$.

Corollary 1.50: If $\bar{M}_K$ satisfies

(m_7) $$\tfrac{1}{2} s \bar{M}_K'(s) \geq \theta \bar{M}_K(s) \quad \text{where} \quad \theta > 1,$$

then

$$(1.51) \qquad \bar{J}_K|_{S_n} \geq 2\pi(\theta-1)\bar{M}_K(M_K^{-1}(k)) \to \infty \quad \text{as} \quad k \to \infty.$$

Proof: By (1.45) and (m_7), for $z \in S_k$,

$$\bar{J}_K(z) \geq 2\pi(\theta-1)\bar{M}_K(\gamma_k)$$

so the result follows from (1.46) and (m_5).

Remark 1.52: Note that from (1.51) for any k, by choosing $K(k)$ sufficiently large, we have $\bar{M}_K(M_K^{-1}(k)) = \bar{M}(M^{-1}(k))$ independently of K.

With the aid of the lower bounds established above for $\bar{b}_j(K)$ and therefore $b_j(K)$, we will study the minimax values $c_j(K)$.

Lemma 1.53: If $c_j(K) > 2\pi a_2$,

(i) $c_j(K)$ is a critical value of I_K.

(ii) Any corresponding critical point lies in $E \backslash E^0$.

(iii) If $c_{j+1}(K) = \cdots = c_{j+\ell}(K) \equiv c > 2\pi a_2$, $i(I_K^{-1}(c) \cap (I_K')^{-1}(0)) \geq \ell$.

Proof: Note that

$$\sup_{E^0} I_K = 2\pi \sup_{E^0} (-H_K(z)) \leq 2\pi \sup_{E^0} (a_2 - a_1|z|^{\bar{\mu}})$$

via (1.3). Thus if $c_j(K) > 2\pi a_2$, an argument paralleling that of Lemma 1.40 yields (i)-(iii) above. We will omit the details.

Remark 1.54: Since $c_j(K) \geq b_j(K) \to \infty$ as $j \to \infty$ via Lemma 1.49, (1.51) and the definition of $\bar{b}_j(K)$, the requirement that $c_j(K) > 2\pi a_2$ is satisfied for all large j, say $j \geq j_0(K)$. Moreover Remark 1.52 shows j_0 can be chosen independently of K for K suitably large, say $K \geq K_0$. For what follows we restrict ourselves to $K \geq K_0$.

The next two lemmas provide K independent bounds for $c_j(K)$ and corresponding critical points $z_j(K)$.

<u>Lemma 1.55:</u> For $j > j_0$, there is a constant d_j independent of K such that $c_j(K) \leq d_j$.

Proof: Choosing $h(z) = z$ and $Y = \phi$ in the definition of Γ_j we see $B = D_j \in \Gamma_j$. Hence by (1.28) and our choice of j,

$$(1.56) \qquad 0 < c_j(K) \leq \sup_{D_j} I_K(z).$$

Let $z \in D_j$ such that $I_K(z) > 0$. Since $D_j \subset V_j$,

$$(1.57) \qquad A(z) \leq \|z^+\|^2 \leq j\|z^+\|^2_{L^2}.$$

On the other hand, by (1.56) and (1.3),

$$(1.58) \quad A(z) \geq \int_0^{2\pi} H_K(z)dt \geq a_1 \int_0^{2\pi} |z|^\mu dt - 2\pi a_2 \geq a_3\left(\int_0^{2\pi} |z^+|^2\right)^{\bar{\mu}/2} - 2\pi a_2$$

where a_3 is independent of K and $\bar{\mu} > 2$. Consequently (1.57)-(1.58) successively imply K independent bounds for $\|z^+\|_{L^2}$ and $\|z^+\|$. Hence by (1.3) again,

$$I_K(z) \leq \|z^+\|^2 + 2\pi a_2$$

which is bounded from above by a constant d_j independent of K and any such $z \in D_j$. The lemma now follows from (1.56).

<u>Lemma 1.59:</u> Let $z_j(K)$ be a critical point of I_K with critical value $c_j(K)$. Then there is a constant δ_j independent of K such that $\|z_j(K)\|_{L^\infty} \leq \delta_j$.

Proof: For notational convenience we will drop the K when referring to $z_j(K)$. Since $I'_K(z_j)z_j = 0$, by (H_0) (for H_K),

$$c_j(K) = I_K(z_j) - \tfrac{1}{2} I_K'(z_j)z_j$$

$$(1.60) \qquad = \int_0^{2\pi} [\tfrac{1}{2}(z_j, H_{Kz}(z_j))_{\mathbb{R}^{2n}} - H_K(z_j)]dt$$

$$\geq (2^{-1} - \bar{\mu}^{-1}) \int_0^{2\pi} (z_j, H_{Kz}(z_j))_{\mathbb{R}^{2n}} dt - a_4.$$

where a_4 is a constant independent of K. Then (1.60) and Lemma 1.55 yield a K independent upper bound for

$$\|(z_j, H_{Kz}(z_j))_{\mathbb{R}^{2n}}\|_{L^1}$$

Next observe that by (H_0) again and the fact that z_j is a solution of a Hamiltonian system, we have

$$(1.61) \qquad 2\pi H_K(z_j) = \int_0^{2\pi} H_K(z_j)dt \leq \bar{\mu}^{-1}\|(z_j, H_{Kz}(z_j))_{\mathbb{R}^{2n}}\|_{L^1} + a_5$$

where a_5 is a K-independent constant. Thus $H_K(z_j)$ and therefore by (1.3) z_j are bounded in L^∞ independently of K. Hence the Lemma.

Modulo the construction of M and M_K, we can now complete the:

<u>Proof of Theorem 0.1:</u> It suffices to show that $I(z)$ has an unbounded sequence of critical values c_j. Indeed if z is a critical value of I, as in (1.60) we have

$$(1.62) \qquad I(z) = \int_0^{2\pi} [\tfrac{1}{2}(z, H_z(z))_{\mathbb{R}^{2n}} - H(z)]dt$$

so if the set of critical points of I were bounded in L^∞, the corresponding set of critical values also would be bounded via (1.62).

For each $j \geq j_0$, choose $K_j > \max(\delta_j, M^{-1}(j))$. Let $z_j \equiv z_j(K_j)$ be a critical point of I_{K_j} with critical value $c_j(K_j)$. By Lemma 1.59, $\|z_j\|_{L^\infty} \leq \delta_j$. Hence by our choice of K_j, $H_{K_j}(z_j) = H(z_j)$ and $H_{Kz}(z_j) = H_z(z_j)$. Consequently z_j is a solution of (HS) and a critical point of I with critical value $c_j \equiv c_j(K_j)$ via (1.62). By (1.30), (1.32), (1.51), Remark 1.52, and our choice of K_j,

$$I_{K_j}(z_j) = c_j(K_j) > \bar{b}_j(K_k) - 2\pi M(0)$$

(1.63)

$$> 2\pi(\theta-1)\hat{M}(\mathcal{M}^{-1}(j)) - 2\pi M(0) \to \infty \quad \text{as} \quad j \to \infty.$$

Hence $c_j \to \infty$ as $j \to \infty$.

It remains to construct the functions $M(s)$ and $M_K(s)$ satisfying (m_1)-(m_7). To begin, choose $\varphi(z)$ such that

(a) $\varphi(s) = \alpha_0 + \alpha_1 s^4$ for $s \in [0,1]$ where $\alpha_0 > 2^8 \max_{|z|\leq s} |H(z)|$

(b) $\varphi|s| > 2^5 \max_{|z|\ s+1} |H(z)|$

(c) $\varphi \in C^2$ and $\varphi'(s), \varphi''(s) > 0$ if $s > 0$.

Set $M(s) \equiv e^{\varphi(s)}$. Then with the aid of (a), (b), (c) we have:

(a') $M \in C^2$ and $M'(s), M''(s) > 0$ if $s > 0$

(b') $M(s) > \varphi(s)$

(c') $s\, M''(s) > 3\, M'(s)$ for $s > 0$.

These facts and simple computations imply:

(i) $\bar{M}(s) = M(s) - M(0) = o(s^2)$ at $s = 0$

(ii) $M(|z|) > |H(z)|$ for all $z \in R^{2n}$

(iii) $M(s)$, $\mathcal{M}(s) \equiv \frac{M'(s)}{s}$, $\frac{s}{2}\bar{M}'(s) - \bar{M}(s)$ are strictly monotonically increasing

(iv) $\frac{1}{2} s\, \bar{M}'(s) > 2\bar{M}(s)$ for all $s > 0$.

Define $M_K(s) \equiv M(s)$ for $s \leq K$ and for $s > K$

$$M_K(s) = M(K) + M'(K)(s-K) + \frac{M''(K)}{2}(s-K)^2 + \rho_1(s-K)^4.$$

We can assume $K > 1$. Then $M_K \in C^2$ and satisfies (m_1) and (m_3). Moreover

(1.64) $$s\, M_K''(s) > 3\, M_K'(s)$$

for $s \in [K, K+\varepsilon_K]$ for some $\varepsilon_K > 0$ via (c') above. Therefore by choosing $\rho_1(K)$ sufficiently large, (1.64) holds for all $s > K$. This fact and (iii) - (iv) quickly yield (m_4)-(m_7). Lastly to verify (m_2), i.e. $M_K(|z|) > |H_K(z)|$, note that this is true for $|z| \leq K$ via (ii). For $|z| > K+1$, comparing M_K and H_K shows the desired inequality holds if

$\rho_1(K) > 8r_K(1+K^4)$. Lastly for $K < |z| < K+1$, by the definition of r_K,

$$|H_k(z)| \le |H(z)| + r_K(K+1)^4 \le$$

$$\le \max_{K\le|\zeta|\le K+1} |H(\zeta)| + \max_{K\le|\zeta|\le K+1} \frac{|H(\zeta)|}{|\zeta|^4}(K+1)^4$$

$$\le \left[1 + \left(\frac{K+1}{K}\right)^4\right] \max_{|\zeta|\le K+1} |H(\zeta)| \le M(K) \le M(|z|).$$

The proof of Theorem 0.1 is complete.

References

1. Rabinowitz, P. H., A variational method for finding periodic solutions of differential equations: Nonlinear Evolution Eq., M. G. Crandall, ed.; Academic Press, New York, 1978, 225-251.

2. Rabinowitz, P. H., On large norm periodic solutions of some differential equations, to appear in Ergodic Theory and Dynamical Systems, Proc. Sp. Yr- Maryland 1979-80, A. Katok, ed.

3. Benci, V. and P. H. Rabinowitz, Critical point theorems for indefinite functionals, Inv. Math., 52, (1979), 336-352.

4. Rabinowitz, P. H., Periodic solutions of Hamiltonian systems, Comm. Pure Appl. Math., 31, (1978), 157-184.

5. Fadell, E. R. and P. H. Rabinowiz, Generalized cohomological index theories for Lie group actions with an application to bifurcation questions for Hamiltonian systems, Inv. Math., 45, (1978), 139-174.

6. Fadell, E. R., S. Husseini, and P. H. Rabinowiz, Borsuk-Ulam theorems for arbitrary S^1 actions and applications, Math Research Center Technical Summary Report, University of Wisconsin-Madison, 1981.

7. Clark, D. C., A variant of the Ljusternick-Schnirelman theory, Ind. Univ. Math. J., 22, (1972), 65-74.

8. Rabinowitz, P. H., Variational methods for nonlinear eigenvalue problems, Proc. Sym. on Eigenvalues of Nonlinear Problems, Edizioni Cremonese, Rome, 1974, 141-195.

9. Benci, V., On critical point theory for indefinite functionals in the presence of symmetry, to appear Trans. Amer. Math. Soc.

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This paper studies Hamiltonian systems of ordinary differential equations. The only assumption made on the Hamiltonian is appropriately rapid growth at infinity. It is proved that for any given period, there is an unbounded sequence of periodic solutions of the system having the given period.

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