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Surface Effects Due to Subsurface Processes: A Survey

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The purpose of this document is to provide a survey of the oceanic subsurface dynamic processes which have signatures ~~or effects~~ on the ocean surface. These surface effects can appear in the form of changes in both surface current distributions and local wave structures (or profiles). Based upon their physical scales, these effects are ~~categori~~ classified into phenomena produced by mechanisms associated with weak and strong interactions. Two types of phenomena are distinct among surface wave weak interactions: ~~The first type~~ broad band phenomena, includes wave-wave energy transfer and general wave train instability. ~~The second type~~ narrow band phenomena includes

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20. ABSTRACT (Continued)

following

Benjamin-Feir instability, recurrence, and envelope solitons. All these phenomena share the physical characteristics ~~of the following~~: (1) they may be manifested only by the long dominant waves in a wave field and yet the behavior of short waves is determined by the strong interactions with the long waves rather than by these processes; (2) they are evolutionary phenomena with a time scale of $(ak)^{-2}$ times the wave period where a is the wave amplitude and k is the wave number of the dominant wave. ~~Whereas~~ the strong interactions are those phenomena whose time scale is of the order of the wave period and space scale is of the order of the wavelength. ~~The~~ detailed wave structures (or profiles) are the prime sources of information. Strong interactions include the strong Longuet-Higgins instability, wave-current interactions, long wave-short wave interactions, and the processes of parasitic capillary formation and microscale breaking induced by surface wind drift. Due to the rapid surface wave responses to local oceanic disturbances accounted by the strong interactions, their phenomena can offer at least qualitatively as the indicator of, for example, the existence of internal wave patterns.

Surface effects due to bathymetry, subsurface mesoscale features, internal waves, subsurface Ekman layer and breaking wave, and their relationship to the subsurface processes, then, are discussed within the context set forth for weak and strong interactions. Finally, in order to highlight the dramatic recent scientific and technical advances made for the space age, a new approach to simulate a wide variety of both surface and subsurface dynamic phenomena is briefly mentioned for evaluating the influence of breaking waves on the dynamics of the upper ocean by using a single measurable parameter, significant slope, from a spaceborne altimeter.

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1. INTRODUCTION

The purpose of this document is to provide a survey of the oceanic subsurface dynamic processes which have signatures, or effects, on the ocean surface. These surface effects can appear in the form of changes in both surface current distributions and local wave structures (profiles). Although the microscale breaking of the surface wave disrupts the diffusive surface layer, the thermal surface effects are not included because of the uncertainties in the mechanisms involved in the heat transfer across the air-sea interface. Interested readers should refer to Phillips (1974) on the subject of thermal surface effects or of slicks. Nevertheless, the thermal surface effects are treated here only as the surface temperature boundary conditions in the context of oceanic transport of heat.

Oceanic subsurface dynamic processes, in this case, can be those shown in the internal-wave-modified current and wave field, the mesoscale subsurface eddies and fronts, and the bottom-topography-modified current field. Except for the internal wave induced field, which has an additional direct wavy surface pattern, all the subsurface dynamic processes have surface current effects. These surface current effects present themselves as variations in amplitudes and directions over the underlying current field. The magnitude of these variations can be of the order of cm/sec, i.e., small, but their effects can be manifested through wave-current and wave-wave interactions. Although it is not the intent of this document to identify the remote sensing techniques by which the variations in current patterns and/or the modulation of surface waves can be observed (it should be, however, the subject of the next document), it is easier to observe remotely the modulation of surface waves. Therefore, this document, more or less, concentrates on the response of the wave field to a moving current pattern or stationary current variations generated by subsurface dynamic processes.

2. SURFACE WAVE EFFECTS: THE NATURE OF WEAK AND STRONG INTERACTIONS

It has been known that surface waves of sufficiently long wavelength and sufficiently small slope can propagate for long distances without significant change of their wave form. This phenomenon, basically, asserts the utilization of the principle of superposition, whose validity requires linear relationship without mutual interference, in waves with sufficiently

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small slope. However, these waves have also been observed to be dispersive in amplitude as well as in phase after, say, a time of 100 wave periods or a distance of 100 wavelengths. It is obvious that there are small regular perturbations not only in amplitude but also in phase to the linear solution of the governing equations. The outstanding response of the waves to these small perturbations is slow and small. The effect, which may be cumulatively large, cannot be appreciated significantly in local space and time scales. In fact, locally the waves are almost like linear waves, but as a result of the small regular perturbations, they change very slowly in space or time or both. These changes are classified as the weak interactions which include wave-wave interactions (Phillips, 1977), Benjamin-Feir instabilities (Benjamin and Feir, 1967), the existence of envelope solitons (Yuen and Lake, 1975), etc. It is the nature of the changes which is of the third order and is of interest, not the individual character of each wave. Except for the bottom-bathymetry induced wave-wave interactions (Herterich and Hasselmann, 1980), it should be noted, however, that all the weak interactions are surface wave induced phenomena.

In contrast are the strong interactions which can be interaction of a wave with itself, another wave or a current. The changes are local occurrences but widespread and distributed. The time scale of these changes is comparable with or shorter than the wave period. Therefore, the individual structures (or profiles) of the waves are important quantities to be measured. Strong interactions include the phenomenon such as wave breaking with the formation of whitecaps. Wave breaking certainly is a non-linear phenomenon and cannot be treated mathematically as a small perturbation to the linear solution.

The difference between the strong and the weak interaction phenomena is distinctly fundamental: weak interaction phenomena take many wave periods or many wavelengths to become signficant although their cumulative effects may be large whereas strong interaction phenomena occur rapidly and locally although their net effects may be small. Therefore, wave property changes locally should be accounted for by the strong interactions, and wave property changes in a gradual manner over a long distance or long duration of time should be accounted for by the weak interactions. Phenomena produced by the weak interactions are considered as noise when one is seeking to account for phenomena produced by the strong interactions, and vice versa. Although this document is primarily interested in the wave-current interaction--a strong interaction--all aspects of the relevant weak and strong interactions should be explored.

3. WEAK INTERACTIONS

The essential approximation in the study of the weak interaction is that the wave shape ak is small where a is wave amplitude, $k = 2\pi/L$ is wave number, and L is wavelength. Let ζ be the wave surface displacement at position $\vec{x}$ and time t with respect to the mean sea surface along the vertical axis z, positive upward. The nonlinear boundary conditions, specified at the free surface $z = \zeta(\vec{x}, t)$, can be referred, by means of a Taylor series expansion, to the mean sea level $z = 0$ and the resulting terms can be collected in the order of ak. This is the Stokes' expansion and its important role in the standard procedure for the solution of a set of nonlinear governing equations for non-zero but small ak is well known. Restriction, commented by Phillips (1979), on the small amplitudes of the longest waves imposed by Stokes' expansion about the mean sea level $z = 0$ can be alleviated by expanding on the longest wave surface with the Lagrangian reference frame which moves with this longest wave surface (Liu and Benney, 1980).

Employing the techniques of Stokes' expansion, theoretical studies (Phillips, 1977) indicate that energy exchange and amplitude modulation of weak interactions appear at the third order, not at the first and the second orders, and only among sets of wave components whose wave-number vectors form a closed quadrilateral, which satisfies,

$$\vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{k}_4 \tag{3.1}$$

and whose wave frequencies satisfy the resonant conditions,

$$\sigma_1 + \sigma_2 = \sigma_3 + \sigma_4 \tag{3.2}$$

where

$$\sigma_j^2 = g|\vec{k}_j|, \ j = 1, 2, 3, 4 \tag{3.3}$$

g is the gravitational acceleration and $|\vec{k}_j|$ is the magnitude of the wave-number vectors $\vec{k}_j$. Because theoretical results predict that terms describing energy exchanges and amplitude modulations are of the third order, the time or spatial scales for weak interactions are $(ak)^{-2}$ times the wave period or wavelength, respectively.

The governing equations for weak interactions are found to be

$$\begin{Bmatrix} \sigma_1 \dot{a}_1 \\ \sigma_2 \dot{a}_2 \\ \sigma_3 \dot{a}_3 \\ \sigma_4 \dot{a}_4 \end{Bmatrix} = \begin{Bmatrix} a_1 g_{11} & a_1 g_{12} & a_1 g_{13} & a_1 g_{14} \\ a_2 g_{21} & a_2 g_{22} & a_2 g_{23} & a_2 g_{24} \\ a_3 g_{31} & a_3 g_{32} & a_3 g_{33} & a_3 g_{34} \\ a_4 g_{41} & a_4 g_{42} & a_4 g_{43} & a_4 g_{44} \end{Bmatrix} \begin{Bmatrix} i a_1 a_1^* \\ i a_2 a_2^* \\ i a_3 a_3^* \\ i a_4 a_4^* \end{Bmatrix} + ih \begin{Bmatrix} a_2^* a_3 a_4 \\ a_1^* a_3 a_4 \\ a_1 a_2 a_4^* \\ a_1 a_2 a_3^* \end{Bmatrix} \qquad (3.4)$$

where "•" denotes time derivative ($\partial/\partial t$), "*" designates complex conjugate, i is the imaginary number, a_j's are wave amplitudes, $g_{j\ell}$ and h are real functions of the wave-number vector configuration, represented by Equation (3.1). If wave amplitude varies spatially as well as in time, $\dot{a}_j$ is replaced by $(\partial/\partial t + \vec{c}_g \cdot \vec{\nabla}) a_j$ where $\vec{c}_g$ is the group velocity. The term $g_{k\ell}$ is the square matrix in Equation (3.4) and describes interactions of wave $\vec{k}_j$ to wave $\vec{k}_\ell$ per unit wave energy, and gives changes in phase speeds. These changes of phase speeds of infinitesimal waves associated with finite wave amplitude are called amplitude dispersion effect. The second column matrix at the right hand side specifies the energy exchange.

The phenomena of weak interactions involve the interrelationship of the two effects - amplitude dispersion and energy exchange. Benney (1962) has shown that the total wave energy and momentum densities of each tetrad of waves are conserved. The general solution of Equation (3.4) can be written in terms of elliptic functions (Bretherton, 1964).

3.1 Physical Phenomena of Weak Interactions

3.1.1 Wave-Wave Interaction

Replacing $\vec{k}_4$ and σ_4 in Equations (3.1) and (3.2) by $\vec{k}_3$ and σ_3, respectively, Equations (3.1) and (3.2) reduce to

$$\vec{k}_1 + \vec{k}_2 = 2\vec{k}_3 \qquad (3.5)$$

and

$$\sigma_1 + \sigma_2 = 2\sigma_3 \ . \tag{3.6}$$

The existence of this physical phenomenon of energy transfer among different wave modes was confirmed separately by Longuet-Higgins and Smith (1966) and McGoldrick et al. (1966). Because the experiment performed by the former has considerably higher wave slopes, the amplitude dispersion effects were significant which implies simple shifts in the resonance band to the smaller wave-number vector.

3.1.2 The Instability of a Wave Train

When there is a slight mismatch from resonance such that, from Equation (3.6),

$$\sigma_1 + \sigma_2 - 2\sigma_3 = \xi \ . \tag{3.7}$$

Then, the right hand side of Equation (3.4) contains an additional factor $e^{-i\xi t}$ (Longuet-Higgins and Smith, 1966). For components $\vec{k}_1$ or $\vec{k}_2$ Equation (3.4) would be reduced to

$$\ddot{\alpha}_1 = \frac{1}{4} h^2 \sigma_1 \sigma_2 (A_3 A_3^*)^2 \alpha_1 \tag{3.8}$$

$$\ddot{\alpha}_2 = \frac{1}{4} h^2 \sigma_1 \sigma_2 (A_3 A_3^*)^2 \alpha_2 \tag{3.9}$$

$$\dot{A}_3 = i g_{33} A_3^2 A_3^* \tag{3.10}$$

where

$$\alpha_1 = a_1 \exp(-i g_{13} A_3 A_3^* t) \tag{3.11}$$

and

$$\alpha_2 = a_2 \exp(-i g_{23} A_3 A_3^* t) \tag{3.12}$$

as wave component $\vec{k}_3$, represented by,

$$A_3 = a_3(1 + i)$$

is perturbed slightly by wave component $\vec{k}_1$ or $\vec{k}_2$ under the assumption that the mismatch ξ in the resonant condition, Equation (3.7), is compensated by the influence of amplitude dispersion, i.e.,

$$\xi = -(g_{13} + g_{23})A_3A_3^* \quad . \tag{3.13}$$

Solutions for α_1 and α_2 from Equations (3.8) and (3.9) are unstable. Furthermore, if the mismatch ξ cannot compensate the amplitude dispersion, as shown in Equation (3.13), the stability analysis is valid only for $|a_1|$ and $|a_2| << |A_3|$. The detailed description of the Instability of a Wave Train should be found in Benjamin and Feir (1967) and Yuen (1977).

3.2 Narrow Band Spectrum Approximation

For a wave group or a slowly varying wave train whose surface displacement

$$\zeta(\vec{x},t) = \exp\left[i(\vec{k}_o\cdot\vec{x} - \sigma_o t)\right]\int a(\vec{k})\exp\{i[\vec{k}\cdot\vec{x} - (\sigma(|\vec{k}| - \sigma_o)t]\}d\vec{k} \tag{3.14}$$

where the only significant contribution come from a small range of wavenumbers and frequencies about $\vec{k}_o$ and σ_o, respectively, Equation (3.4) can be considerably simplified and reduced to the nonlinear Schrödinger equation,

$$i\left(A_t + \frac{\sigma_o}{2|\vec{k}_o|}A_x\right) - \frac{\sigma_o}{8|\vec{k}_o|^2}A_{xx} + \frac{\sigma_o}{4|\vec{k}_o|^2}A_{yy} - \frac{1}{2}|\vec{k}_o|^2\sigma_o|A|^2A = 0 \tag{3.15}$$

where A is the local amplitude,

$$A = \int a(\vec{k})\exp\{i[\vec{k}\cdot\vec{x} - (\sigma(|\vec{k}|) - \sigma_o\}t]\}d\vec{k} \tag{3.16}$$

from Equation (3.14) and the subscripts t, xx, and yy represent partial derivatives. This equation was derived by Benney and Newell (1967). The physical phenomena modelled by Equation (3.15) are still fundamentally those of weak interaction, amplitude dispersion and inter-mode energy transfer.

The physical dynamic processes of Benjamin-Feir instability, recurrence, and envelope solitons can all be classified as the narrow band weak interaction phenomena. Benjamin-Feir instability is a special case of the instability of a wave train (Benjamin and Feir, 1967). Recurrence is essentially a phenomenon predicted by numerical integration of Schrödinger's equation, Equation (3.15). Yuen and Ferguson (1978) have shown that the solution to Equation (3.15) with essentially its initial condition, will return after a time interval of order $(a|\vec{k}|)^{-2}\sigma^{-1}$ in the numerical integration scheme. There are some questions raised against the validity of this numerical solution, for example, Huang (1981) stated that, from experimental evidence, the wave pattern may be back to the original form but the wavelength has certainly increased. More research work is required in order to resolve the differences.

Envelope solitons are groups of surface waves whose envelope propagates without change in the wave form. Phase shift may happen if the envelope solitons run through other envelope solitons, but their basic forms still remain the same. Yuen and Lake (1975) among others have given the exact solution to the one-dimensional nonlinear Schrödinger equation for sufficiently small slopes and band-widths. Their solution indicates the involvement of a balance between amplitude dispersion and linear dispersion about the carrier frequency. The problem of stability of an envelope soliton to infinitesimal directional perturbations has been studied by, for example, Saffman and Yuen (1978).

4. STRONG INTERACTIONS

The strong interaction phenomena in waves are those which are produced in a time scale or spatial scale comparable with or shorter than the wave period or the wavelength, respectively. Among these phenomena are local wave breaking (Longuet-Higgins, 1978a and 1978b;

Longuet-Higgins and Cokelet, 1978), the interaction of waves with non-uniform current, and the interaction of waves with another long wave. These phenomena are highly nonlinear in general and are the result of interactions with a finite amplitude disturbance.

Due to the rapid response of the strong interactions in waves, the influence of weak interactions can almost be ignored. Because of the instrumentational difficulties in observing and mathematical difficulties in analyzing the strong interaction phenomena, they are much less explored than the weak interaction phenomena.

4.1 Physical Phenomena of Strong Interactions

4.1.1 Wave-Current Interactions

As the waves consist of a train or group of local wave-number $\vec{k}$ and frequency σ, neglecting the weak interactions, the kinematic conservation equation

$$\frac{\partial \vec{k}}{\partial t} + \vec{\nabla}\,(\sigma + \vec{k} \cdot \vec{U}) = 0 \tag{4.1}$$

should be satisfied where $\vec{U}(\vec{x},t)$ is the current system encountered and $\sigma = (g|\vec{k}|)^{1/2}$. The distribution of energy density E, in this case, should be governed by the conservation of wave action, $\frac{E}{\sigma}$,

$$\frac{\partial}{\partial t}\left(\frac{E}{\sigma}\right) + \vec{\nabla} \cdot \left[(\vec{U} + \vec{c}_g)\,\frac{E}{\sigma}\right] = 0 \tag{4.2}$$

where $\vec{c}_g$ is the group velocity. Equation (4.2) can be rewritten as

$$\frac{d}{dt}\,\ell n\left(\frac{E}{\sigma}\right) + \vec{\nabla} \cdot (\vec{U} + \vec{c}_g) = 0 \tag{4.3}$$

where $\frac{d}{dt}$ represents the rate of change moving with the energy packet which moves with the combined current and group velocities.

One interesting situation is that as the current distribution moves through the ambient fluid field such as when an internal wave pulse or train moves, the phenomenon of blockage (Longuet-Higgins and Stewart,

1961) may result. Let $\vec{C}$ be the velocity of the current pattern and the distribution of current be $\vec{U} = \vec{U}(x - |\vec{C}|t)$, then the modulations in surface wave energy density E is governed by

$$E|\vec{c}|(|\vec{U}| + \frac{1}{2}|\vec{c}| - |\vec{C}|) = \text{const.} \qquad (4.4)$$

where $\vec{c}$, $\vec{U}$, and $\vec{C}$ are all collinear in direction; the local surface wave phase velocity $\vec{c}$ is found from Equation (4.1). For internal waves, in most cases, $|\vec{U}| << |\vec{c}|$ or $|\vec{C}|$. The intensity of the surface wave modulation becomes large as the magnitude of group speed $\frac{1}{2}|\vec{c}|$ approaches that of the propagation speed $|\vec{C}|$ of the pattern. In situtations where surface and internal waves are oblique with each other, the treatise is the same as for the collinear situation except the velocities $\vec{c}$, $\vec{C}$ and $\vec{U}$ are projected in the same direction (Longuet-Higgins and Stewart, 1961).

4.1.2 The Instabilities of Wave Breaking

Great progresses have been made in recent years in the detailed analyses of periodic deep water waves of their amplitudes up to the limiting form, their responses to applied surface pressure pulses, their stability properties to large scale perturbations, the structure and the stability properties of their small scale normal modes, and their evolution towards wave breaking. The whole series of developments on these topics are based upon the ability to calculate the free surface deformation (Longuet-Higgins and Cokelet, 1976) of an irrotational periodic motion.

4.1.2.1 Large Scale Subharmonics

Longuet-Higgins (1978b) considered a set of M number of waves in a spatial interval containing NM cycles of the perturbation waves. He was able to show that the coalescence of the frequencies and the onset of Benjamin-Feir instability are delayed to larger wave slope as the relative change in wave number increases. One interesting result is that once the growth rates of these subharmonic (large scale) waves reach the unstable states, they will restabilize themselves at larger value of wave slopes. Mathematically, the value of wave slope where wave breaking may occur has been suggested. Consequently, Longuet-Higgins and Cokelet (1978) trace the initially small unstable disturbances upon the finite amplitude wave all the way, ultimately, to breaking. Their results indicate that it is necessary to consider that the short-time behavior of waves and the surface configurations of the wave crests before breaking are similar under a wide range of physical conditions. The local theory of wave breaking has not been achieved, however.

4.1.2.2 Small Scale Superharmonics

Longuet-Higgins (1978a) also derived the normal mode perturbations of a nonlinear gravity wave for smaller scales than that of the basic wave. As the steepness of the basic wave increases the frequency of each perturbation shifts to the lower value.

4.1.3 Long Wave-Short Wave Interactions

With the conservative Equations (4.1) and (4.2), encouraged by Longuet-Higgins' (1978a) results on superharmonics, the interactions between relative long and short waves can be considered. Of course, the accuracy of analytical procedures depends asymptotically on the ratio of long to short wavelengths. This ratio is equivalent to the mode number N defined in Section 4.1.2.1.

Considering a train of short waves interacting with the underlining long wave in a Lagrangian reference frame moving with the long wave, the local energy density of the short wave train is simply

$$E = \frac{1}{2}\rho\tilde{g}a^2 \tag{4.5}$$

$$= \frac{1}{2|\vec{k}|}\rho\tilde{\sigma}^2 a^2 \tag{4.6}$$

where, neglecting capillary waves,

$$\sigma = (\tilde{g}|\vec{k}|)^{1/2} \tag{4.7}$$

$$\tilde{g} = g\cos\theta + \frac{|\vec{u}|^2}{R} \quad . \tag{4.8}$$

ρ is the density, a is the wave amplitude, $\tilde{\sigma}$ is the intrinsic wave frequency, $|\vec{k}|$ is the wave number, θ is the angle of wave slope with the horizontal plane, R is the surface curvature, and $|\vec{u}|$ is the tangential speed at the surface in the reference frame. From Bernoulli's Equation,

$$|\vec{u}(s)|^2 = |\vec{c}|^2 - 2g\zeta \tag{4.9}$$

where s is a measure of distance along the surface of long wave, $|\vec{c}|$ is the phase speed of the long wave, and ζ is the free surface elevation.

For both waves travelling in the same direction, Equation (4.1) gives

$$\tilde{\sigma} + |\vec{k}|\ |\vec{u}(s)| = \tilde{\sigma}_m + |\vec{k}_m|\ |\vec{c}| = \tilde{\sigma}_o \tag{4.10}$$

where $\tilde{\sigma}_o$ is a constant, and $\tilde{\sigma}_m$ and $|\vec{k}_m|$ are the intrinsic wave frequency and wave number at the mean water level. From Equation (4.7), $|\vec{k}| = \tilde{\sigma}^2/\tilde{g}$, Equation (4.10) becomes

$$(|\vec{u}(s)|/\tilde{g})\ \tilde{\sigma}^2 + \tilde{\sigma} - \tilde{\sigma}_o = 0$$

or

$$\tilde{\sigma}(s) = (\tilde{g}/2|\vec{u}(s)|)\{-1 + [1 + 4\tilde{\sigma}_o|\vec{u}(s)|/\tilde{g}]^{1/2}\} \tag{4.11}$$

where $\tilde{\sigma}_o$ is a constant, and $\tilde{g}$ and $|\vec{u}(s)|$ are functions of distance s along the long wave free surface. The intrinsic wave number $|\vec{k}|$ can be found, subsequently, from Equation (4.10)

$$|\vec{k}(s)| = \frac{\tilde{\sigma}_o}{|\vec{u}(s)|} - (\tilde{g}/2|\vec{u}(s)|)\{-1 + [1 + 4\tilde{\sigma}_o\ |\vec{u}(s)|/\tilde{g}]^{1/2}\} \tag{4.12}$$

Comparing the values of $|\vec{k}|$ given by Equation (4.12) along s against those calculated by Longuet-Higgins (1978a) for small scale superharmonics, it has been found that Equations (4.11) and (4.12) are indeed valid for the ratio of long wavelength to short wavelength as small as 3 to 1 (Phillips, 1979).

From Equation (4.2), the conservation of wave action flux, the short wave energy density can be specified as

$$(|\vec{u}(s)| + |\vec{c}_g|) \frac{E(s)}{\tilde{\sigma}(s)} = (|\vec{C}| + \frac{1}{2} |\vec{c}_m|) \frac{E_m}{\tilde{\sigma}_m} \tag{4.13}$$

where $|\vec{c}_m|$ and E_m are the phase speed and the short wave energy density at the mean water level.

From Equations (4.11), (4.12), (4.13) and (4.6) for $\tilde{\sigma}(s)/\tilde{\sigma}_m$, $|\vec{k}(s)|/|\vec{k}_m|$, $E(s)/E_m$, and $(a|\vec{k}(s)|)^2/(a|\vec{k}_m|)^2$ it is important to notice that small change in long wave slope can produce large change in the mean square slope of short wave.

4.1.4 Interactions Between Short Waves, with Spectral Band, and Long Wave

The variations of the apparent wave frequency with the phase of the underlining long wave observed in Eulerian reference frames are the result of two effects. One is the variation in intrinsic wave frequency $\tilde{\sigma}$, discussed in Section 4.1.3, and the other is the variation due to Doppler effect associated with the convection of short waves by the orbital velocity of the long underlining wave. The apparent wave frequency σ_a can be obtained as

$$\sigma_a = - \frac{\partial \chi}{\partial t} = \tilde{\sigma} + \frac{|\vec{k}(s)|}{\cos\theta} (- |\vec{C}| + |\vec{u}(s)|\cos\theta)$$

where χ is the phase function of the long wave and the rest of the symbols are defined as before. It should be noted, however, $|\vec{k}(s)|/\cos\theta$ is the magnitude of the horizontal component of $|\vec{k}(s)|$ and $(- |\vec{C}| + |\vec{u}(s)|\cos\theta)$ is the magnitude of the horizontal component of the long wave orbital velocity at the surface. From Equation (4.10)

$$- \frac{\partial \chi}{\partial t} = \tilde{\sigma}_o - \frac{|\vec{k}(s)|}{\cos\theta} |\vec{C}| \tag{4.14}$$

The last term at the right hand side is simply a Doppler shift modification on Equation (4.10) associated with the change of reference frame from that of Lagrangian to Eulerian. The Lagrangian reference frame is moving at the constant phase speed $|\vec{C}|$ of the long wave.

For this short wave train, at a fixed position $\vec{x}$,

$$\zeta(\vec{x},t) = a(\vec{x} - \vec{C}\, t)e^{i\chi} \tag{4.15}$$

at a fixed frequency ω the Fourier component

$$F(\vec{x},\omega) = \frac{1}{T} \int_0^T \zeta(\vec{x},t)e^{i\omega t}dt$$

$$= \frac{1}{T} \int_0^T a(\vec{x} - \vec{C}t)e^{i\phi}dt \tag{4.16}$$

where T is the long wave period and $\phi = \chi + \omega t$. It is quite obvious, from Equation (4.14), that ω should lie at and between the extremes of $-\partial\chi/\partial t$. Phillips (1979), employing the stationary phase method, obtains the spectral density $\Phi(\omega)$ for ω lying between the extremes of $-\partial\chi/\partial t$ and also obtains the spectral densities for ω lying at the extremes of $-\partial\chi/\partial t$. The spectral transformation of a short wave train in the presence of long wave, in general, is distributed over a range of frequencies of width $(\sigma|\vec{k}|)|\vec{C}|$ where $\delta|\vec{k}|$ is the difference in short wave-numbers between at the crest and at the trough of the long wave and $|\vec{C}|$ is the long wave phase speed. The maximum spectral density occurs at the highest wave frequency, and is derived for short waves at the crest of the long wave.

4.1.5 Interactions Between Microscale Surface Waves and Long Wave

At the crest of the long wave, it is indicated, in the previous section, that the slopes of short waves will increase. If these wave slopes (for wavelengths between 5 to 30 cm) become sufficiently large, the short waves may splash down, produce microscale breaking, or may form parasitic capillary waves.

The splashing phenomenon of small gravity waves has been observed near the crests of the long waves and cannot be explained as yet. Microscale wave breaking requires the presence of the wind-driven current (Banner and Phillips, 1974; Phillips and Banner, 1974) and can be enhanced by longer gravity waves somewhat. Microscale wave breaking can also be suppressed by longer gravity waves when the wind stress is sufficiently large enough to regenerate the surface wind-driven layer. The thin thermal conductive layer at the water surface can be disrupted by microscale wave breaking. The formation of parasitic capillary waves ahead of short gravity wave crests has been analyzed theoretically by Longuet-Higgins (1963). The phenomenon is commonly observed but first measured by Chang et al. (1978) in a laboratory. There are qualitative agreements on the distribution of capillary wavelengths and attenuation rate with respect to distance along the long wave profile just ahead of the crest. The amplitudes of the capillary waves were measured to be several orders of magnitude larger than those predicted theoretically by Longuet-Higgins.

In field conditions these three processes may all occur at different proportions with various wind speeds. Nevertheless, their spatial densities are the indicators of local wave energy dissipation at capillary scale and, also, short gravity waves in the wavelengths of 5 to 30 cm.

4.2 Application of Long Wave-Short Wave Interactions to Internal Wave Modulations of Surface Wave Field

In Section 4.1.1 brief mention was made of the phenomenon when a current distribution moves through the ambient fluid such as when an internal wave pulse or packet moves across the ocean surface. If $|\vec{C}|$ is the speed of the internal wave pulse (or packet) pattern, the surface current field associated will have the distribution of $\vec{U} = \vec{U}(x - |\vec{C}|t)$ along the x direction. The modulation in surface wave energy density E is given by Equation (4.4) where $|\vec{c}|$, evaluated from Equation (4.1), is the phase speed of the surface wave and $\vec{c}$, $\vec{C}$, and $\vec{U}$ are all collinear in the direction of x coordinate, say. For internal waves, $|\vec{U}| << |\vec{c}|$ or $|\vec{C}|$. As the group speed $\frac{1}{2}|\vec{c}|$ approaches the propagation speed of internal wave pattern $|\vec{C}|$, the surface wave modulation will increase its intensity. This physical condition was analyzed by Phillips (1974) in which the situation where $\vec{c}$, $\vec{C}$, and $\vec{U}$ are not collinear has also been described. Lewis et al. (1974) demonstrated, in their theoretical and experimental study, the existence of the modulation at $|\vec{C}| = \frac{1}{2}|\vec{c}|$, the so-called "Resonant" or "Blockage" condition, and obtained good agreement quantitatively between their theory and experiments.

The velocity $\vec{U}$ and strain rates $\frac{\partial U_i}{\partial x_j}$ induced at the ocean surface by the internal wave are small. It can induce surface wave modulation significantly only when the group velocity of the surface wave is fairly close or equal to the propagation speed (or phase speed) of the internal wave, according to the resonant (or blockage) condition. The fastest phase speed of the internal wave (lowest mode) is of the order of 50 cm/sec. This condition certainly places a restriction on the wavelengths of surface waves which would have significant modulations at 60 cm or less. Generally, the ocean surface wave spectrum contains much longer wave components than those in the wavelengths of 60 cm or less. Since these longer waves are more energetic, the short waves are in fact strained and distorted by these longer waves in a manner, described in Section 4.1.4, far more rapidly and extensively than they are by the internal wave induced velocity and strain rates. Nevertheless, the surface wave modulations by the internal wave can still be shown observable.

Let the surface displacement of this long surface wave be ζ with the phase function χ such that

$$\zeta(x,t) = a \cos\chi, \quad \chi = |\vec{k}|x - \sigma t \tag{4.17}$$

which is travelling also along the same direction x as the internal wave. The short gravity wave of wave-number $\vec{k}'(s)$ and energy density $E'(s)$ is riding on this long wave in a collinear direction. Both $\vec{k}'(s)$ and $E'(s)$ of this short gravity wave vary, of course, with respect to the phase function χ of the long wave, described in Section 4.1.3, and position x within the internal wave current pattern. In the Lagrangian reference frame moving with the internal wave phase speed $|\vec{C}|$, the surface current induced by the internal wave is $|\vec{U}| - |\vec{C}|$ and the phase function χ is changed to $\chi = |\vec{k}|[x - (|\vec{c}| + |\vec{U}| - |\vec{C}|)t]$ where $\vec{c}$ is the phase velocity of the long wave. According to Phillips (1979), the key to the existence of an analytical solution to this long- short-internal wave interaction problem is that the <u>intrinsic</u> wave frequency $\tilde{\sigma}'(s)$ for the short gravity waves is constant to the order of $a|\vec{k}|$ of the long wave within its phase function cycle. The presence of long wave, therefore, produces no modulation in $\tilde{\sigma}'$ at this order for the short gravity waves. The significant modulation in $\tilde{\sigma}'$ at the order of $a|\vec{k}|$ will only arise from the short wave-current interaction or $\tilde{\sigma}' = \tilde{\sigma}'(x)$. Therefore, the surface wave modulations by internal wave is theoretically observable.

5. SURFACE EFFECTS DUE TO BATHYMETRY, SUBSURFACE MESOSCALE FEATURES, INTERNAL WAVES, SUBSURFACE EKMAN LAYER AND BREAKING WAVES, AND THEIR RELATIONSHIP TO THE SUBSURFACE PROCESSES

In the previous three chapters, the responses of the ocean surface waves to a moving current pattern or stationary current variations (or gradients) generated by subsurface dynamic processes have been discussed briefly. There are two areas, pertinent to this survey, required to be addressed.

The first area, of course, is what kind of accuracy can be expected on the surface current variations derived from the observed surface wave effects. Intuitively, this accuracy is of the order of $a|\vec{k}|$ because the surface current variations would have to be derived through strong interaction phenomena.

The second area of concern is what can be achieved, by knowing the surface wave and the surface current variation fields, in resolving the subsurface features such as bathymetry, mesoscale subsurface eddies and fronts, internal waves, and subsurface shear currents, for example. The validities of the inverse algorithms which would have to be used to infer the subsurface features depend on how complete the mathematical field equations would be for modelling the dynamic physical phenomena. The complete set of mathematical field equations which are briefly summarized by Chen and Noble (1980) govern the three-dimensional current and density fields in the ocean, and the kinematic and the dynamic boundary conditions at the oceanic boundary. These boundary conditions describe the states of heat, mass, and momentum exchanges as functions of position and time. At the ocean surface these surface boundary conditions include strong surface wave interaction phenomena to the order of $a|\vec{k}|$ and weak surface wave interaction phenomena to the order of $(a|\vec{k}|)^2$. Heat exchange surface effects (Phillips, 1974) are weak interaction phenomena while mass exchange surface effects can be strong (e.g., estuarine or coastal run-off) or weak (e.g., evaporation) interaction phenomena.

The steady uniform current field creates no surface wave effects but has dynamic topographic signatures (Hill, 1962) through the geostrophic assumptions (baroclinic and barotropic). Various types of dynamic topography have been observed over currents (Chen and Noble, 1980) and subsurface mesoscale eddies and fronts (Chen and Noble, 1980; Dugan and Chen, 1980). The treatise on dynamic topography for the inference of geostrophic features such as currents and eddies is discussed in detail by Chen and Noble (1980). In a special situation where barotropic instability is assumed to be excited by tidal oscillations, Mollo-Christensen et al. (1981) successfully give estimates of current speed in the Gulf Stream.

5.1 Bathymetry

As ocean surface waves propagate from deep to shallow water with variable depth, significant changes occur in wave spectral characteristics. These changes are the combined effects due to physical dynamic processes which may or may not be governed by the conservation of energy. The physical dynamic processes whose total wave energy densities are conserved include those processes such as wave refraction, wave shoaling, and wave-wave interaction. The physical dynamic processes whose total wave energy densities are not conserved can be those involved in energy dissipation and generation.

5.1.1 Wave Refraction and Shoaling

Based on the conservation of energy flux, Longuet-Higgins (1957) considered the transformation of continuous wave spectrum by analysis of the refraction process for just two components. As an extension, Karlesson* (1969) generalized the transformation for the continuous directional wave spectrum by analysis of its distribution under steady state conditions with finite difference techniques. Using a geometrical-optical approximation, Krasitskiy (1974) derived the explicit analytical solution for the spectral transformation over two-dimensional parallel bottom contours.

The laboratory and field experiments which were designated to observe the phenomena in wave spectral transformation include Wu et al. (1977) and Shemdin et al. (1980) among others. Bear in mind that the data observed contains information on phenomena beyond wave refraction and shoaling. Wu et al. (1977) made some laboratory observations on wave energy dissipation, wave energy generation, and also on nonlinear wave-wave interaction phenomena in the

*Collins (1972) further extended the transformation to include energy dissipation and generation and applied his results to an irregular bottom bathymetry. By tracing wave energy along wave rays, his numerical scheme, however, made the computational procedures quite impractical for the determination of shallow water wave spectrum at predetermined and designated locations.

context of wave spectral transformation. Recently, Shemdin et al. (1980) summarized some of the linear and nonlinear physical dynamic processes which are relevant to the wave spectral transformation in finite-depth water and applied the results against data obtained from the JONSWAP (Hasselmann et al., 1973) site in the North Sea. They concluded that the rate of wave energy dissipation is dominated by yet unidentified physical dynamic processes which are determined by the bottom conditions.

5.1.2 Wave Energy Dissipation*

There are five different physical dynamic processes associated with wave energy dissipation. They are: Percolation, Wave-Induced Bottom Motion, Wave Attentuation, Bottom Friction, and White-caps and Wave Breaking. All but the last two are linear processes (or mechanisms).

5.1.2.1 Percolation

The wave pressure field induces a flow in the sandy bottom (or bed) below. The magnitude of this induced flow is a dependent of the permeability of the sandy bed. The wave energy dissipation can be computed from the cross-correlation between the wave induced pressure above the bed and the vertical velocity just at the top of the bed. Putnam (1949) made the first investigation for an isotropic sand layer. McClain et al. (1977) made corrections to the model regarding the continuity at the bottom interface. Hsiao and Shemdin (1978) improved Putnam's results to accommodate different permeabilities in the horizontal and the vertical directions. The rate of energy dissipation for water depth h is given by

$$\frac{\partial E}{\partial t} = - E|\vec{k}|\sqrt{\kappa_H \kappa_V}\,\frac{\tanh\sqrt{(\kappa_H/\kappa_V)}\,|\vec{k}|d}{\cosh^2|\vec{k}|h} \tag{5.1}$$

where κ_H and κ_V are the horizontal and vertical permeability coefficients (Darcy's numbers), respectively, and d is the thickness of the sand layer.

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*Wave Energy Generation is beyond the scope of this survey.

5.1.2.2 Wave-Induced Bottom Motion

The bottom responds to the wave-induced pressure field with viscoelastic characteristics when the material of bottom sediment is mud or decomposed organic matters. Some field and laboratory observations have been made on this phenomenon by e.g., Tubman and Suhayda (1976), Rosenthal (1978) among others. A solution for a two-layer coupled-flow model was given by Hsiao and Shemdin (1980) for a finite depth mud layer.

5.1.2.3 Wave Attenuation

The direct attenuation by molecular viscosity strongly influences the state of capillary wave. If a slick covers the surface, the amplitude of shorter gravity waves may also be attenuated as well. On the longer gravity waves, the direct effect of molecular viscosity is negligible. Phillips (1959) investigated this physical mechanism extensively.

5.1.2.4 Bottom Friction

Energy dissipation due to bottom friction primarily is the result of two physical mechanisms. One involves the dissipative work done against turbulent shear stresses, the so-called turbulent bottom friction, induced at the bottom bed by water particle motions. The other is responsible for the dissipative work done against the viscous forces induced at the permeable bottom where percolation occurs.

In general, turbulent bottom friction is dominant over other dissipative processes when the sediment is composed of sand with mean diameter in the range of 0.1 - 0.4 mm where low permeability prohibits percolation and granular friction prevents viscous flow behavior. Putnam and Johnson (1949) investigated this problem initially. They used the quadratic friction law to derive the rate of energy dissipation for sinusoidal surface waves. Assuming Gaussian-distributed surface wave field, Hasselmann and Collins (1968) derived the rate of energy dissipation for a random sea. With the concern placed on bottom friction coefficient, Bretschneider and Reid (1954) evaluated the coefficients for the sandy bottoms. Furthermore, relying on laboratory and semi-theoretical studies, Johnson (1965) found that bottom coefficients are functions of Reynolds' number, and also of relative roughness. Hsiao and Shemdin (1978) have investigated this problem in detail.

5.1.2.5 White-Caps and Wave Breaking

It is a well-observed fact that under sustained wind action, waves grow to some maximum amplitudes before the onset of breaking or white-caps would limit their growth. Hasselmann (1974) treated the white-cap interactions in terms of an equivalent ensemble of random pressure pulses. By assuming irrotational motion Longuet-Higgins and Cokelet (1976; 1978) calculated the deformation of the free wave surface up to breaking for horizontally periodic surface waves. Subsequently, Longuet-Higgins (1978a; 1978b) calculated, for the periodic deep water waves, their amplitudes up to limiting forms, their responses to applied surface pressure pulses, their stability properties to subharmonic (large scale) perturbations, the structure and stability of their superharmonic normal modes and their evolution toward breaking. The results provide great progress in this field. The subjects were covered more extensively in Section 4.1.2 in the chapter on strong interactions. Nevertheless, more research work is still required, especially for surface waves over variable depth.

5.1.3 Wave-Wave Energy Transfer in Finite-Depth Water

The nonlinear wave energy transfer due to weak nonlinear wave-wave interactions is derived from Equations (3.1), (3.2) and (3.4) and is given by a Boltzman integral of the form,

$$\frac{\partial F(\vec{k}_4,h)}{\partial t} = \int \pi\sigma_4 \left(\frac{3g^2 D}{2\sigma_1\sigma_2\sigma_3\sigma_4}\right)^2 \left[\sigma_4 F_1 F_2 F_3 + \sigma_3 F_1 F_2 F_4 - \sigma_1 F_2 F_3 F_4 - \sigma_2 F_1 F_3 F_4\right] \cdot \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4)\, \delta(\sigma_1 + \sigma_2 - \sigma_3 - \sigma_4)\, d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 \tag{5.2}$$

where the rate of change of the spectrum at wave-number $\vec{k}_4$ due to interactions with other components $\vec{k}_1$, $\vec{k}_2$, and $\vec{k}_3$ was described in chapter 3. F_i is the spectral density at $\vec{k}_i$, or $F_i = F(\vec{k}_i)$. The depth dependence in Equation (5.2) enters via the coefficient D and the relationship, given by Equation (3.2) and (3.3), particularly in the frequency delta function. Exact computations of this integral were performed by Hasselmann and Hasselmann (1980) in finite-depth water for directional wave spectra.

In the presence of currents the radiative transfer equation for the wave spectrum $F(\vec{x}, \vec{k}, t)$ has the form,

$$\frac{\partial}{\partial t}\left(\frac{F}{\sigma}\right) + \vec{c}_g \cdot \vec{\nabla}\left(\frac{F}{\sigma}\right) + \frac{\partial \vec{k}}{\partial t} \cdot \vec{\nabla}_{\vec{k}}\left(\frac{F}{\sigma}\right) = S \tag{5.3}$$

where $\vec{c}_g$ is the group velocity, σ is the intrinsic wave frequency given in Equation (4.1) with respect to the reference frame moving with the current, and S is the net source function. The source function S is the sum of several components such as

$$S = -S_p - S_b - S_f + S_w - S_a + S_g \tag{5.4}$$

where S_p, S_b, S_f are associated with energy dissipation by percolation, wave-induced bottom motion, and bottom friction, respectively. S_w is associated with the energy transfer by weak nonlinear wave-wave interaction. S_a is associated with wave attenuation, and S_g is associated with energy generation by wind.

The terms F/σ is changed to F in the absence of current systems. In the case of steady wind-driven and thermohaline-driven barotropic velocity or current field in a variable-depth ocean under the mild assumption that the bathymetry does not penetrate up to the depth of appreciable horizontal density gradients, the bottom torque (or vorticity) can be related directly to the stretching term in the barotropic velocity balance (Rattray and Dworski, 1978). Therefore, even this barotropic current field can be modified by the bathymetry at the ocean surface. The plausibility of zero current field description over a coastal region with finite-depth is questionable.

5.2 Subsurface Mesoscale Features

Although oceanic features such as currents, mesoscale eddies, and mesoscale fronts are shown to exhibit measurable temperature gradients at both the surface and subsurface levels (for example, Stommel, 1966; Stommel and Yoshida, 1972; Bruce, 1979; Halliwell and Mooers, 1979; Richardson et al., 1979; Vukovich and Crissman, 1978), in large mid-ocean areas, poor correlation coefficients of the order of 0.2-0.3

between surface temperature patterns and the thermocline structure were found by Dugan (1980) and Dugan and Chen (1980). A majority of these oceanic features do possess measurable dynamic heights and slopes (for example, Stommel, 1966; Stommel and Yoshida, 1972; Vukovich and Crissman, 1978; Dugan, 1980; Dugan and Chen, 1980). These dynamic heights and slopes are the result of different density and current profiles in the horizontal and vertical directions through the currents, the eddies, and to some extent the fronts. These dynamic heights and slopes are the deviations from the local geoid heights and slopes which, theoretically, represent a motionless ocean. The dynamic slopes as well as the variations of dynamic heights can induce horizontal pressure gradients. Under the steady state situation, by neglecting the effects due to friction and removing the effects due to wind set-up and atmosphere pressure, the geostrophic current velocity is related orthogonally to the horizontal pressure gradient by the dynamic method. In the case of eddies, the horizontal pressure gradient is further balanced by the centrifugal force. Therefore, under the geostrophic approximation, the current velocity field can be evaluated if the dynamic heights and slopes over the current, the eddy, or the fronts are known.

Theoretically, a complete set of conditions required for a mathematically and physically unique and bounded current velocity field are well defined. These conditions, derived from the Euler's equation of motion and the equation for continuity of fluids, are the known bottom topography, the known vertical and horizontal density profiles, and the known boundary kinematic and dynamic conditions. In other words, if the current velocity field is known, the unknown three-dimensional density field, which includes the subsurface mesoscale eddies and fronts, can be uniquely determined.

Assuming a linear constitutive law for the surface strain rate tensor, the basic hydrodynamic equations describing oceanic circulation in a general time dependent case are (Hill, 1962) the Euler's equation of motion,

$$\frac{\partial\vec{U}(\vec{x},t)}{\partial t} + [\vec{U}(\vec{x},t)\cdot\nabla]\vec{U}(\vec{x},t) + 2\vec{\Omega}\times\vec{U}(\vec{x},t) + \vec{\omega}(\vec{x},t)\times[\vec{\omega}(\vec{x},t)\times\vec{x}]$$

$$= -\frac{1}{\rho(\vec{x},t)}\vec{\nabla}p(\vec{x},t) + \vec{b}(\vec{x},t) + \nu\nabla^2\vec{U}(\vec{x},t) \quad , \tag{5.5}$$

and the equation of continuity,

$$\frac{\partial\rho(\vec{x},t)}{\partial t} + \vec{\nabla}\cdot[\rho(\vec{x},t)\vec{U}(\vec{x},t)] = 0 \tag{5.6}$$

with the associated kinematic and dynamic boundary conditions. In Equations (5.5) and (5.6), $\vec{U}$ is the current velocity, $\vec{\Omega}$ is the angular velocity of the earth, $\vec{\omega}$ is the angular velocity of the eddy, $\vec{x}$ is the position vector, ρ is the density of sea water, p is the pressure, $\vec{b}$ is the body force, and ν is the kinematic viscosity of sea water. The position vector $\vec{x}$ is a function of time. The third term on the left hand side, $2\vec{\Omega}\times\vec{U}$, of Equation (5.5) is the Coriolis force induced by the fluid motion $\vec{U}$ and the fourth term, $\vec{\omega}\times(\vec{\omega}\times\vec{x})$, is the centrifugal force induced by the eddy motion. In the absence of eddy motion the centrifugal force term always vanishes. For steady state situation, Equations (5.5) and (5.6) become

$$(\vec{U}\cdot\vec{\nabla})\vec{U} + 2\vec{\Omega}\times\vec{U} + (\vec{\omega}\cdot\vec{x})\vec{\omega} - (\vec{\omega}\cdot\vec{\omega})\vec{x} = -\frac{1}{\rho}\vec{\nabla}p + \vec{b} + \nu\nabla^2\vec{U} \tag{5.7}$$

and

$$\vec{U}\cdot\vec{\nabla}\rho + \rho\vec{\nabla}\cdot\vec{U} = 0 \tag{5.8}$$

With the Boussinesq, the hydrostatic, the Rectangular Cartesian Coordinates', and the scale approximations and, also, the eddy term assumption, Equations (5.7) and (5.8) are further reduced to the component form as

$$2\sin\lambda|\vec{\Omega}|U_2(\vec{x}) - \omega_i x_i \omega_1 + |\vec{\omega}|^2 x_1 = \frac{1}{\rho(\vec{x})}\ \frac{\partial p(\vec{x})}{\partial x_1}, \tag{5.9}$$

$$i = 1, 2, 3$$

$$2\sin\lambda|\vec{\Omega}|U_1(\vec{x}) + \omega_i x_i \omega_2 - |\vec{\omega}|^2 x_2 = -\frac{1}{\rho(\vec{x})}\frac{\partial p(\vec{x})}{\partial x_2},$$

$$i = 1, 2, 3 \tag{5.10}$$

$$0 = \frac{1}{\rho(\vec{x})}\frac{\partial p(\vec{x})}{\partial x_3} - g \tag{5.11}$$

and

$$\frac{\partial U_1(\vec{x})}{\partial x_1} + \frac{\partial U_2(\vec{x})}{\partial x_2} = 0, \tag{5.12}$$

respectively, where $\vec{x} = (x_1, x_2, x_3)$, λ is the latitude (positive in northern hemisphere) and g is the gravitational acceleration. The Cartesian Coordinates x_1, x_2, and x_3 are in the local east, north, and vertical downward directions. However, since usually

$$\vec{\omega} = (0,0,\omega_3) \tag{5.13}$$

Equations (5.9) to (5.12) can be simplified as

$$2\sin\lambda|\vec{\Omega}|U_2(\vec{x}) + \omega_3^2 x_1 = \frac{1}{\rho(\vec{x})}\frac{\partial p(\vec{x})}{\partial x_1} \tag{5.14}$$

$$2\sin\lambda|\vec{\Omega}|U_1(\vec{x}) - \omega_3^2 x_2 = -\frac{1}{\rho(\vec{x})}\frac{\partial p(\vec{x})}{\partial x_2} \tag{5.15}$$

$$\frac{\partial p(\vec{x})}{\partial x_3} = \rho(\vec{x})g \tag{5.16}$$

and

$$\frac{\partial U_1(\vec{x})}{\partial x_1} + \frac{\partial U_2(\vec{x})}{\partial x_2} = 0 \tag{5.17}$$

Equation (5.16) is the hydrostatic equation. In the case of eddy motion, Equations (5.14) to (5.17) become

$$2\sin\lambda|\vec{\Omega}|\,|\vec{U}(r)| + \frac{|\vec{U}(r)|^2}{r} = \frac{1}{\rho}\frac{\partial p}{\partial r} \tag{5.18}$$

$$\frac{\partial p}{\partial \theta} = 0 \tag{5.19}$$

$$\frac{1}{\rho} \frac{\partial p}{\partial z} = g \tag{5.20}$$

and

$$\frac{\partial |\vec{U}(r)|}{\partial r} = 0 \tag{5.21}$$

where r, θ, and z are the coordinates for the cylindrical coordinates along the radial, angular, and vertical directions.

The second terms at the left hand sides of Equations (5.14) and (5.15) are high order terms, for current systems other than mesoscale eddies, these terms can be dropped as

$$2\sin\lambda|\vec{\Omega}|U_2(\vec{x}) = \frac{1}{\rho(\vec{x})} \frac{\partial p(\vec{x})}{\partial x_1} \tag{5.22}$$

$$2\sin\lambda|\vec{\Omega}|U_1(\vec{x}) = -\frac{1}{\rho(\vec{x})} \frac{\partial p(\vec{x})}{\partial x_2} . \tag{5.23}$$

Equations (5.22), (5.23), (5.16), and (5.17) are the governing equations for the geostrophic current systems. From Equation (5.16) or (5.20), let Equation (5.16) be

$$p(\vec{x}) - \tilde{p}_a(\vec{x}) = g \int_{\zeta}^{x_3} \rho(\vec{x})dx_3 \tag{5.24}$$

where $\tilde{p}_a(\vec{x})$ is the reference atmospheric pressure at the mean sea surface ζ. The mean sea surface ζ is obtained by referring the dynamic height to an arbitrary datum where the horizontal pressure gradient and current velocity are zero (Chen and Noble, 1980).

Recent results of numerical multi-layer model studies (Piacsek, 1980) indicate that the subsurface mesoscale features such as eddy, a predominant geostrophic baroclinic feature, should exhibit a surface effect (or signature) in the form of deviation (dynamic topography) from the local geoid. SEASAT-1 altimeter has been able to observe this deviation induced by the eddy in the Gulf of Mexico. Through the surface effects due to wave-current interactions (strong interactions) the surface current boundary conditions of these subsurface mesoscale features may further enhance the possibility of resolving their three-dimensional density structures to the first internal wave mode if the local stability (Brunt-Väisälä) frequency history is known.

5.3 Internal Wave with Subsurface Shear Current, Layered Density Anomaly, Bathymetry, and Heat Storage

To the Boussinesq approximation, and neglecting the earth's rotation, the equation describes the motion of a stratified fluid having the form (Phillips, 1969) of

$$\frac{\partial^2}{\partial t^2} (\nabla^2 U_3) + N^2(x_3)\nabla_h^2 U_3 = Q(\vec{x},t) \tag{5.25}$$

where the nonlinear terms,

$$Q(\vec{x},t) = \frac{\partial^3}{\partial x_\alpha \partial x_3 \partial t} \left(U_j \frac{\partial U_\alpha}{\partial x_j}\right) - \nabla_h^2 \left\{U_j \frac{\partial B}{\partial x_j} + \frac{\partial}{\partial t}\left(U_j \frac{U_3}{x_j}\right)\right\},$$

$$\alpha = 1,2; \qquad j = 1,2.3 \tag{5.26}$$

the Brunt-Väisälä frequency,

$$N(x_3) = \left(- \frac{g}{\rho_0} \frac{\partial \bar{\rho}}{\partial x_3}\right)^{1/2}, \tag{5.27}$$

the continuity equation,

$$\frac{\partial U_j}{\partial x_j} = 0, \qquad j = 1,2,3 \tag{5.28}$$

and

$$B = -g \frac{\rho - \rho_0}{\rho_0} \tag{5.29}$$

where ρ_0 is the reference water density at $x_3 = 0$ or at the ocean surface.

For infinitesimal disturbances in the absence of a mean shear, the term $Q(\vec{x},t)$ in Equation (5.25) is negligible and the governing equation is linear; but the thermocline is often a region with appreciable mean vertical shear. Vertical shear has the effect of reducing the frequency and vertical scale of small scale internal wave motions (Phillips, 1969). In the absence of energy dissipation phenomena, energy is eventually lost from the internal waves and is being transferred to the mean shear. The vertical velocity fluctuations are thus suppressed and the motion eventually becomes totally horizontal. Bell (1979) used real background oceanographic data and examined, by numerically simulated internal wave patterns, how the energy envelope (that region of space within which appreciable density perturbations occur) and the phase information (the structure of the density perturbation within the energy envelope) observed in the absense of shear are distorted by the introduction of background shear. The relevant parameter Bell used for the determination of relative importance of the background shear is the bulk Richardson Number $R(x_3)$ as

$$R(x_3) = \frac{N^2(x_3)}{(\partial|\vec{U}_h|/\partial x_3)^2} \tag{5.30}$$

where $\vec{U}_h = (U_1,U_2)$. Because of the influence of the large scale averaged properties of the shear profile on the wave propagation process, Bell found that the net drift velocity of the energy envelope at any depth level does not accurately reflect the background current speed at that level. Shear induced restrictions of the vertical extent of the energy envelope are shown to be consistent with the wave propagation process relating to the critical layer phenomenon. Significant coherent losses relative to unsheared patterns over several Brunt-Väisälä periods were observed on the phase information. Furthermore, the energy envelope hardly reached the ocean surface.

Based upon Bell's numerical simulation, there is no surface effect on internal wave processes due to the presence of the subsurface shear current structures or patterns except that the amplitude and the frequency of the internal wave may be reduced somewhat through the high order mechanism. It would be rather difficult to observe these changes in amplitude and frequency of the internal wave, induced by the presence of subsurface shear current, through the mechanism identified in Section 4.2 above.

An internal wave can be reflected when it incidents upon a layered density anomaly embedded in a stably stratified fluid with constant Brunt-Väisälä frequency. Mied and Dugan (1974) found that the reflection coefficient is a complicated function of the horizontal and vertical wave numbers and the parameters related to the Brunt-Väisälä profile. All wave energy is transmitted through the anomalous layer for waves propagating almost horizontally and a significant amount of wave energy is reflected for vertically propagating ones. Through WKB description, there can exist for some stratification conditions certain other directions of propagation for which the conditions for energy transfer are more favorable than they are for nearly contiguous incident angles of inclination with respect to the horizontal. The existence of these reflected internal waves can be an indicator of the presence of a layered density anomaly.

Internal waves do, of course, reflect from a sloping bottom but the significance of this mechanism to ocean surface phenomena is much less important than those described in Section 5.1. Internal waves can also be reflected (Mollo-Christensen, 1977) by a moving surface layer which can be induced by spatially varying bathymetry, for example.

In the instances where the internal waves are generated by tidal flow over bottom topography one can find the propagation speed of these internal waves from their wave packet spacing (Mollo-Christensen and Mascarenhas, 1979). The propagation speed depends upon the density anomaly and depth of the upper mixed layer. Attributing the density anomaly to temperature rather than to salinity, Mollo-Christensen and Mascarenhas can calculate the heat storage in the upper oceanic layer. In this way, heat storage was calculated from LANDSAT satellite data over the New England continental shelf and was compared with available in-situ observations. They found that the method may have merit and is deserving of further refinement. Furthermore, if one could detect surface signs of internal waves on the main thermocline and then find that such waves are emitted over bottom topography, it should then be possible to follow the integrated density anomaly over large regions of the ocean (Mollo-Christensen, 1981).

5.4 Horizontal Distortion in the Surface Ekman Layer

As pointed out by Mollo-Christensen (1981), the surface wind-driven Ekman layer will show signs of variations of internal vorticity through horizontal divergence, and signs of bottom Ekman layer and free internal shear layer variations. Faller and Kaylor (1966) showed that Ekman layers develop rolls; such rolls may show in the surface Ekman layer as Langmuir circulations. Mollo-Christensen observed, from LANDSAT images and SEASAT-1 Synthetic Aperture Radar (SAR) imagery, that the rolls or the striations seem to be parallel to the velocity field beneath the Ekman layer, rather than at an angle with the geostrophic flow as found by Faller and Kaylor (1966). These striations may be due to a periodic structure, speculated by Mollo-Christensen, in the Ekman layers at the bottom or the Ekman layers at shear interfaces beneath the surface.

Mollo-Christensen does not have an analytical solution for the Ekman layer created by stratified flow parallel to bottom topography, but he feels that there has to be a periodic solution because of the effects of buoyancy that limit vertical excursions of fluid particles. The direction of isobaths at the bottom or of isobaths in the free shear layer that exist below the surface current can be deduced.

5.5 Influence of Breaking Waves on the Dynamics of the Upper Ocean

Using the statistical model proposed by Longuet-Higgins (1969), Huang et al. (1981) found that, based upon the balance of energy between wind and waves, the influence of breaking waves on the dynamics of the upper ocean depends only on one parameter, the significant slope, defined and used by Huang and Long (1980). If the energy released from the breaking waves is treated as the sole source of turbulence energy, various models can be proposed to simulate a wide variety of both surface and subsurface dynamic phenomena. The quantities calculated include wave attenuation rate, equivalent eddy viscosity, wave induced drag coefficient, surface drift, white-capping percentage, and mixing efficiency for the mixing layer models.

The treatment, by Huang et al. (1981), of using breaking waves as the turbulence energy source alone may be perceived only as the first order of approximation of the quantities calculated. Therefore, their results are the lower bound of the real values of these quantities. The quantitative values of all their results are found to agree with the conventional adapted values very well.

The important and revolutionary advantage of their approach is that the inputs to the models are all obtainable from remote sensors. Therefore, the practical implications of their results can be enormous and their approach is deserving of further theoretical, experimental and field investigations.

6. GENERAL SUMMARY

6.1 Weak Interactions

Two types of phenomena, based upon their spectral spreads, can be categorically classified among surface wave weak interactions. The first type, broad band phenomena, includes wave-wave energy transfer and general wave strain instability. The second type, narrow band phenomena, includes Benjamin-Feir instability, recurrence, and envelope solitons. All these phenomena share the physical characteristics of the following: (a) they may be manifested only by the long dominant waves in a wave field and yet the behavior of short waves is determined by the strong interactions with the long waves rather than by these processes; (b) they are evolutionary phenomena with a time scale of $(ak)^{-2}$ times the wave period, where a is the wave amplitude and k is the wave number of the dominant wave. In a typical oceanic condition, this time scale is roughly of the order of at least 100 wave periods of the dominant wave. Equivalently speaking, the space scale is of the order of at least 100 wavelengths of the dominant wave. Therefore, due to their modifications to wave field, which are of the same order of $(ak)^{-2}$, weak interactions cannot be used to account for any local disturbance to the wave field.

6.2 Strong Interactions

The strong interactions are those phenomena whose time scale is of the order of the wave period and space scale is of the order of the wavelength. The detailed wave structures (or profiles) are the prime sources of information. Therefore, the techniques of Fourier Transform representation of wave field, mathematically, is not valid in the interpretation for strong interactions.

Strong interactions include the strong Longuet-Higgins instability, wave-current interactions, long wave-short wave interactions, and the processes of parasitic capillary formation and microscale breaking induced by surface wind drift. Due to the rapid surface wave responses to local oceanic disturbances accounted by the strong interactions, their phenomena can offer at least qualitatively as the indicator of, for example, the existence of internal wave patterns. Additional research work, absolutely necessary for the better quantification of these phenomena, is currently hindered by the lack of mathematical tools. Therefore, quantitative laboratory investigations of these phenomena are at present the only feasible approach toward the understanding of the processes involved.

Modulation of a wave train produced by internal waves or currents is a strong interaction effect which has been observed in the laboratory, and possibly by synthetic aperture radar onboard satellite. More explicitly, a wave train or group can interact with a long wave, and has a spectral signature spreading over a range of frequencies proportional to the long wave speed and approximately proportional to the long wave slope. The effect of internal waves in the form of wavy current distribution can be an integrated part of these long wave-short wave interactions without being smeared, to the surprise of almost everyone.

Microscale surface properties have time spatial densities that respond rapidly to energy exchanges with short gravity waves. Recent theoretical and laboratory studies indicate that these densities will not have a state of saturation. Therefore, a remote sensor estimating the spatial strength of the return signal should be able to measure distributions and variations in these densities.

6.3 Surface Effects due to Bathymetry, Subsurface Mesoscale Features, Internal Waves, Subsurface Ekman Layer and Breaking Waves, and their Relationship to the Subsurface Processes

As ocean surface waves propagate from deep to shallow water with variable depth, significant changes occur in wave spectral characteristics. These changes are the combined effects due to physical dynamic processes which may or may not be governed by the conservation of energy. The physical dynamic processes whose total wave energy densities are conserved include those processes such as wave refraction, wave shoaling, and wave-wave interaction. The physical dynamic processes whose total wave energy densities are not conserved can be those involved in energy dissipation and generation. Wave energy dissipation mechanisms include percolation, wave-induced bottom motion, wave attenuation, bottom friction, whitecaps, and wave breaking.

The physical dynamic process involved in deriving surface effects due to bathymetry is modelled by the radiative transfer equation for the wave spectrum. The source function of this equation includes those terms derived for mechanisms pertinent to the physical process. Considerable research is still required to properly model these source terms.

The results of numerical multi-layer model studies indicate that subsurface mesoscale features such as an eddy, a predominant baroclinic feature, should exhibit a surface effect (or signature) in the form of deviation (dynamic topography) from the local geoid. Satellite-borne altimeter measurements have been able to observe this deviation induced by the Gulf Stream, a predominant geostrophic barotropic feature, and also to confirm the existence of deviations due to eddies. Through the surface effects due to wave-current interaction (strong interaction) the surface current boundary conditions of these subsurface mesoscale features may further enhance the possibility of resolving their three-dimensional density structures to the first internal wave mode if the local stability (Brunt-Väisälä) frequency history is known.

The internal waves and their surface effects, discussed in Section 4.2, can be the sources of the surface signatures due to subsurface shear current, layered density anomaly, bathymetry, and can be used to evaluate heat storage. (in the mixed layer). The propagation speed of the internal wave packets, obtained by knowing the spacing of those internal wave packets or through the internal wave-surface wave interactions, depends upon the density anomaly and depth of the upper mixed layer.

In addition to surface distortion of the wave field, caused by internal waves through strong interactions, the surface wind-driven Ekman layer will show signatures of variations of internal vorticity through horizontal convergence, and may show signs of bottom Ekman layer and free internal shear layer variations through several surface signatures. It has been shown that Ekman layers develop rolls; such rolls may show in the surface Ekman layer as Langmuir circulations. However, the rolls or the striations observed, from LANDSAT images and SEASAT-1 imagery, seem to show their orientations are parallel to the velocity field beneath the Ekman layer, rather than at an angle with the geostrophic flow. These striations may be due to a periodic structure in the Ekman layers at the bottom or the Ekman layers at shear interfaces beneath the surface. The direction of isobaths at the bottom or of isobaths in the free shear layer that exist below the surface current can be deduced.

Of practical importance to the first order approximation, a recent approach to simulate a wide variety of both surface and subsurface dynamic phenomena has been developed for evaluating the influence of breaking waves on the dynamics of the upper ocean. Based upon the balance of energy between wind and waves, the influence was found to be dependent of only one parameter, the significant slope. This parameter is defined as the ratio of the root mean square wave amplitude to the dominant wavelength of the random ocean surface waves. The energy released from the breaking waves is treated as the sole source of turbulence energy. Various models can be proposed to calculate other parameters such as wave attenuation rate, equivalent eddy viscosity, wave induced drag coefficient, surface drift, white-capping percentage, and mixing efficiency for the mixing layer model.

Their treatment of using breaking waves as the turbulence energy source alone may be perceived only as the first order of approximation on these parameters or the lower bound of their real values. The quantitative values of all these parameters agree with the conventional adapted values. Since the significant slope is a measureable quantity by remote sensors, this new approach deserves further investigation.

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