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THE DEPARTMENT OF DEFENSE WORLD GEODETIC SYSTEM 1972

Prepared by the
World Geodetic System Committee
and presented by
Thomas O. Seppelin

May 1974

Presented at the International Symposium
on Problems Related to the Redefinition
of North American Geodetic Networks,
Fredericton, New Brunswick, Canada
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ABSTRACT

An improved World Geodetic System has been developed to satisfy the mapping, charting, and geodetic needs of the Department of Defense. The system, designated WGS 72, represents the culmination of approximately five years of data collection within the Department of Defense and the scientific community. Principal agencies involved in the developmental cycle include the USAF, Defense Mapping Agency, US Naval Weapons Laboratory, and Naval Oceanographic Office. The method of solution for an adjustment of this magnitude is characterized by the formation of a large scale matrix by combining normal matrices from each of the major data input sets. The scaling and weighting processes for the final matrix are discussed. The resultant ellipsoid parameters, datum shifts and their related accuracies are presented and compared against similar quantities in other recent geocentric systems.

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INTRODUCTION

The Department of Defense (DOD) in the late 1950s generated a geocentric reference system to which different geodetic networks could be referred and compatibility established between the coordinates of sites of interest. Efforts of the Army, Navy and Air Force were combined leading to the development of the DOD World Geodetic System 1960 (WGS 60). In accomplishing WGS 60, a combination of available surface gravity data, astrogeodetic data and results from HIRAN and Canadian SHORAN surveys were used to obtain a best-fitting ellipsoid for the major datum areas. The sole contribution of satellite data to the development of WGS 60 was the value for the ellipsoid flattening (1/298.3 $\pm$0.1), which was obtained from the nodal motion of Satellite 1958 β. The semimajor axis of the WGS 60 Ellipsoid was determined as 6,378,165 $\pm$50 meters.

In January 1966, a World Geodetic System Committee was charged with the responsibility of developing an improved WGS needed to satisfy mapping, charting and geodetic requirements. Additional surface gravity observations, results from the extension of triangulation and trilateration networks and large amounts of Doppler and optical satellite data had become available since the development of WGS 60. Using the additional data and improved techniques, the WGS Committee produced WGS 66 which has served DOD needs since its implementation in 1967. The defining parameters of the WGS 66 Ellipsoid were the flattening (1/298.25 $\pm$0.02), determined from satellite data and the semimajor axis (6,378,145 $\pm$20m), determined from a combination of Doppler satellite and astrogeodetic data involving a geoid-match technique. A 24th degree and order geopotential coefficient set derived from a harmonic analysis of a worldwide 5°x5° mean free air gravity anomaly field was selected as the WGS 66 Gravitational Model. This geopotential coefficient set was also used in a spherical harmonic expansion to obtain the Worldwide WGS 66 Geoid. Also, a geoid referenced to the WGS 66 Ellipsoid, providing a detailed representation for limited land areas, was derived from available astrogeodetic data. Datum shift constants for the North American Datum 1927 (NAD 27), European Datum (ED) and Tokyo Datum (TD) were obtained for each datum.

The same WGS Committee began work in 1970 to develop a replacement for WGS 66. Since the development of WGS 66, large quantities of additional data had become available from both Doppler and optical satellites, surface gravity surveys, triangulation and trilateration surveys, high precision traverses and astronomic surveys. In addition, greater capabilities had been developed in both computers and computer software. Further, continued research in improved computational procedures and error analyses had produced better methods and a greater facility for handling and combining data.

After an extensive effort extending over a period of approximately three years, the Committee completed the development of the Department of Defense World Geodetic System 1972 (WGS 72). Selected satellite, surface gravity and astrogeodetic data available through 1972 from both DOD and non-DOD sources was used in a Unified WGS Solution (a large scale least squares adjustment). The results of the adjustment consist of corrections to initial station coordinates and geopotential coefficients. This paper presents a brief review of the data and methods used and a summary of the results.

DATA USED IN WGS 72 DEVELOPMENT

The largest collection of data ever used for WGS purposes was assembled, processed and applied in the development of WGS 72. Both optical and electronic satellite data was used. The electronic satellite data consisted, in part, of Doppler data provided by US Navy and cooperating non-DOD satellite tracking stations established in support of the Navy's Navigational Satellite System [1]. Doppler data was also available from the numerous sites established by Geoceivers (geodetic receiver) during the 1971-1972 time period [2]. Additional electronic satellite data was provided by the SECOR (Sequential Collation of Range) Equatorial Network completed by the US Army in 1970 [3]. Optical satellite data from the Worldwide Geometric Satellite Triangulation Program was provided by the BC-4 camera system [4] [5]. The Baker-Nunn camera data of the Smithsonian Astrophysical Observatory (SAO) was also used [6].

Doppler

Geopotential coefficients and some station coordinates have been determined simultaneously in general geodetic solutions using sampled Doppler observations, taken between 1962 and 1970, of satellites at various inclinations. However, an accurate (and most complete) set of station coordinates has been derived by the point positioning method using sampled or integrated Doppler observations of the Navy Navigation Satellites (NAVSATs), whose polar orbits are described by current mathematical models.

Since 1964, locations of sites occupied by mobile Doppler vans have been determined using a minimum of 40 passes of polar NAVSAT data. Although coordinates of many of the sites have been subsequently determined together in a general geodetic solution using observations of several satellites, an independent solution, if determined by current point positioning methods, is considered more accurate because of the sufficiency of data, the high accuracy of polar orbit computations, and the precision of NAVSAT time calibration. With this in mind, the semi-permanent TRANET stations were independently repositioned in 1970 using four sufficient sets of modern data in a way that was consistent with the Bureau International de l'Heure (BIH) pole computations. The resulting station coordinates, defined as the NWL-9D geocentric system, is the basic Doppler system contributing to WGS 72.

Normal equations, representing satellite data from approximately 30,000 Doppler and 500 optical passes, were provided as a matrix and vector of dimension 822 allowing corrections to unnormalized gravitational field coefficients, Love's constant, and the geocentric polar coordinates of 114 observing stations. The gravitational field coefficients include all coefficients through degree and order 19 (except the first degree), zonals through degree 24 and additional resonance terms through order 27. Correlated variations in orbit and bias parameters, implicit in the computations but not appearing in the matrix, are fully accounted for in any solution.

Normal equations for stations and the full set of 479 gravitational field coefficients were developed for about 70 percent of the satellite data processed, including all 330 days of non-polar and 92 days of polar Doppler observations and all 96 days of optical observations. In an effort to include a larger number of Doppler National Geodetic Satellite Program (NGSP) sites in the matrix, the normal equations for the remaining 30 percent of the data consisting of 82 two-day polar arcs, were developed only for station coordinates and three gravitational field coefficients: $C_{0,0}$; $C_{2,0}$; $C_{3,0}$. Use of the NWL-9B gravitational coefficients set insured an RMS orbit error of only three meters for each two-day polar arc, allowing the neglect of additional gravity variations on station determination.

Stations which contributed data fall into the semi-permanent and mobile classifications (Figure 1). Nearly 85 percent of the data was received at 18 stations of the semi-permanent Doppler network. The remaining 15 percent was taken by six similarly equipped mobile vans operating for six to twelve weeks at over 120 worldwide locations. Some 20 of these sites do not appear in the matrix because of size limitations in the processing program. Additionally, a small number of observations were contributed by the twelve original Baker-Nunn stations operated by SAO.

BC-4

In the worldwide network observed with BC-4 cameras, the measurements of approximately 2350 plates were used to establish the relative positions of 49 tracking stations (Figure 2). The BC-4 camera observations were introduced in the Unified WGS 72 Solution in the form of normal equations developed by National Ocean Survey (NOS) for their TRI Solution. The procedure used by NOS was to develop normal equations separately for each event; i.e., a set of two or more simultaneous photographs of the satellite. The unknowns in the normal equations were corrections to the rectangular coordinates of the satellite and the observing stations. Normally, there were seven satellite positions per event and as many as four observing

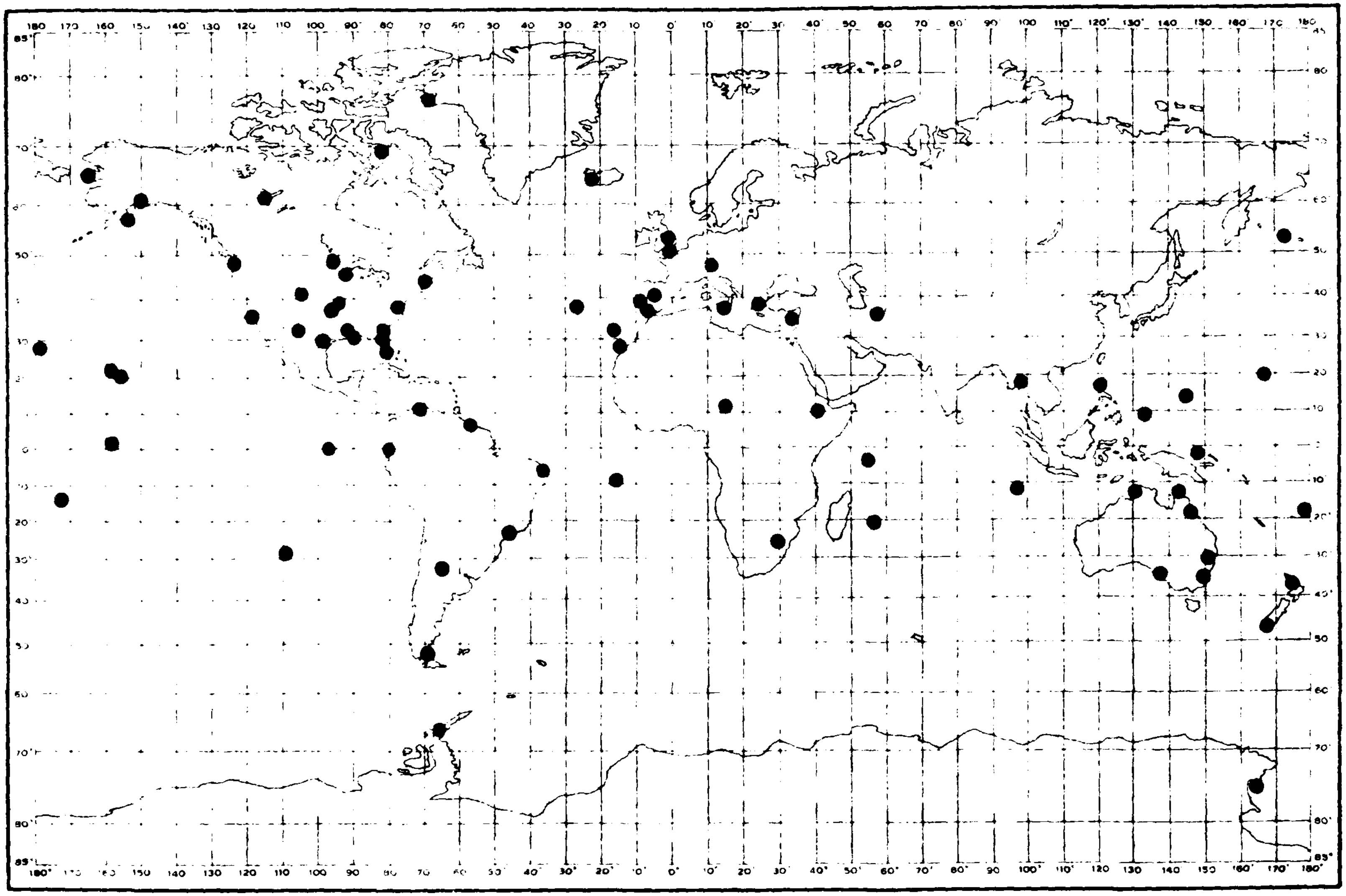

Figure 1. Doppler Satellite Ground Stations Providing Data for WGS 72 Development.

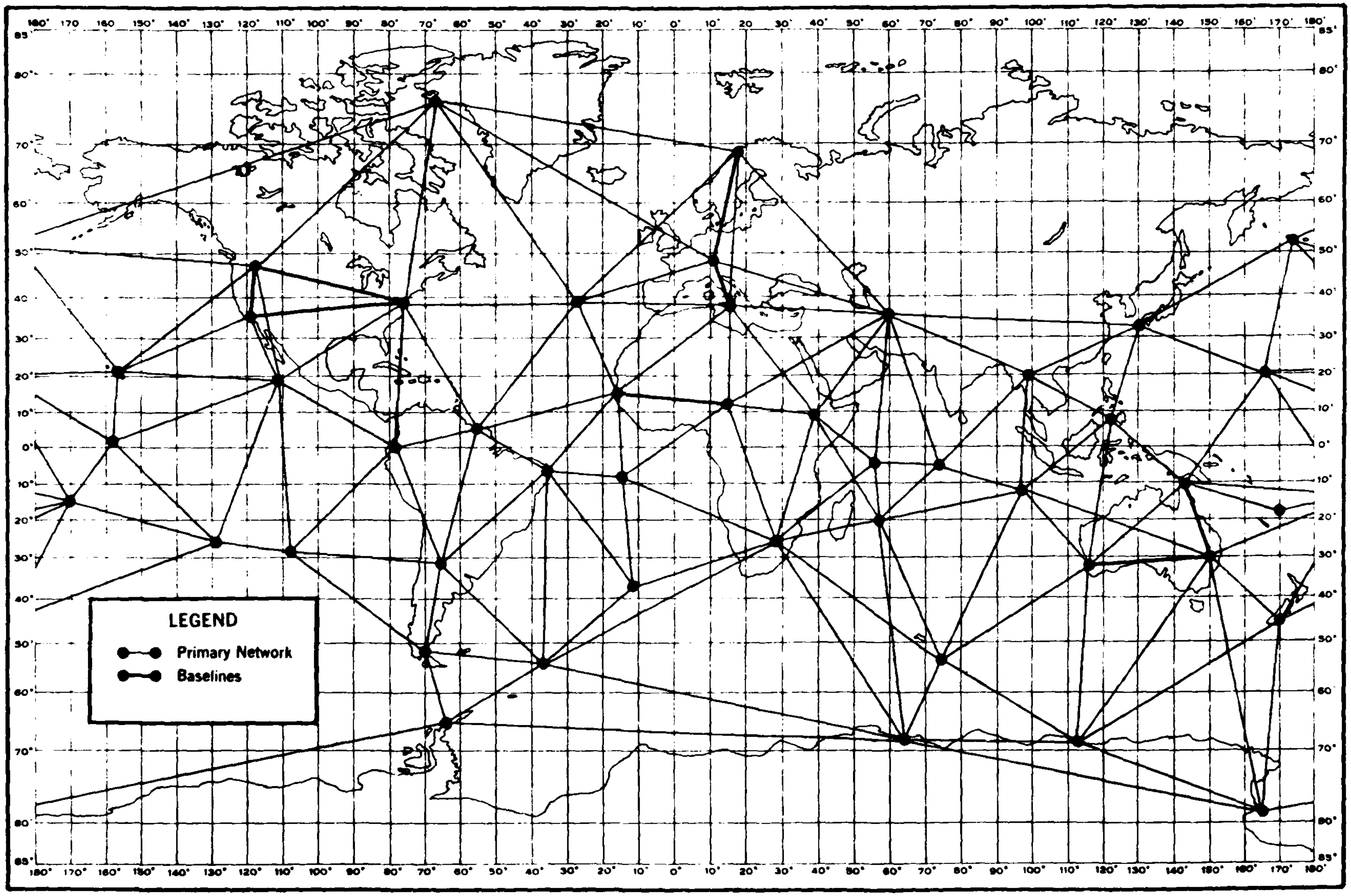

Figure 2. Worldwide Geometric Satellite Triangulation Network, BC-4 Cameras.

stations. The satellite position unknowns were then eliminated leaving a set of event normal equations relating the coordinates of the stations that observed that event. The event normal equations were then summed over all events and the resulting set of equations used in the Unified WGS 72 Solution. A total of 856 two-station, 194 three-station and 14 four-station events were used to relate the 49 observing stations.

Baker-Nunn

Several geodetic solutions have been accomplished by the Smithsonian Astrophysical Observatory using data obtained from the optical observations of satellites by Baker-Nunn cameras. A recent solution, the 1969 Smithsonian Standard Earth II [7], utilized optical data from Baker-Nunn and Mini-track Optical Tracking System (MOTS) cameras (Figure 3). This SAO solution for 117 station coordinates (39 stations) and 296 tesseral harmonic coefficients is based on optical and laser satellite data as well as surface mean free air gravity anomalies. In addition, some observations of deep-space probes are included. The complete solution represents a combination of dynamic and geometric satellite data with surface gravity observations. The normal equations developed by SAO while deriving Standard Earth II were incorporated into the Unified WGS 72 Solution. Standard Earth II is itself the result of a combined solution that incorporated laser and optical satellite tracking data, surface gravity observations and space probe data.

SECOR

The SECOR measurements used in the Unified WGS Solution for WGS 72 were those obtained at stations forming the SECOR Equatorial Network (Figure 4). The results of the SECOR Equatorial Network Project were incorporated into the Unified WGS 72 Solution in the form of normal equations resulting from short arc adjustments.

Surface Gravity

The surface gravity field used in the Unified WGS Solution consisted of a set of 410 10°x10° equal area mean free air gravity anomalies determined solely from terrestrial data. This gravity field includes mean anomaly values compiled directly from observed gravity data wherever the latter was available in sufficient quantity. The values for areas of sparse or no observational data were developed from geophysically compatible gravity approximations using gravity-geophysical correlation techniques. Approximately 45 percent of the 410 mean free air gravity anomaly values were determined directly from observed gravity data. Using the surface gravity data, normal equations were formed for a total of 319 gravitational parameters. These included the zonal harmonic coefficients through degree 20 and the tesseral harmonic coefficients through degree 17 and order 15. Table 1 shows the quantity of gravity data available through November 1972 for WGS development and evaluation purposes.

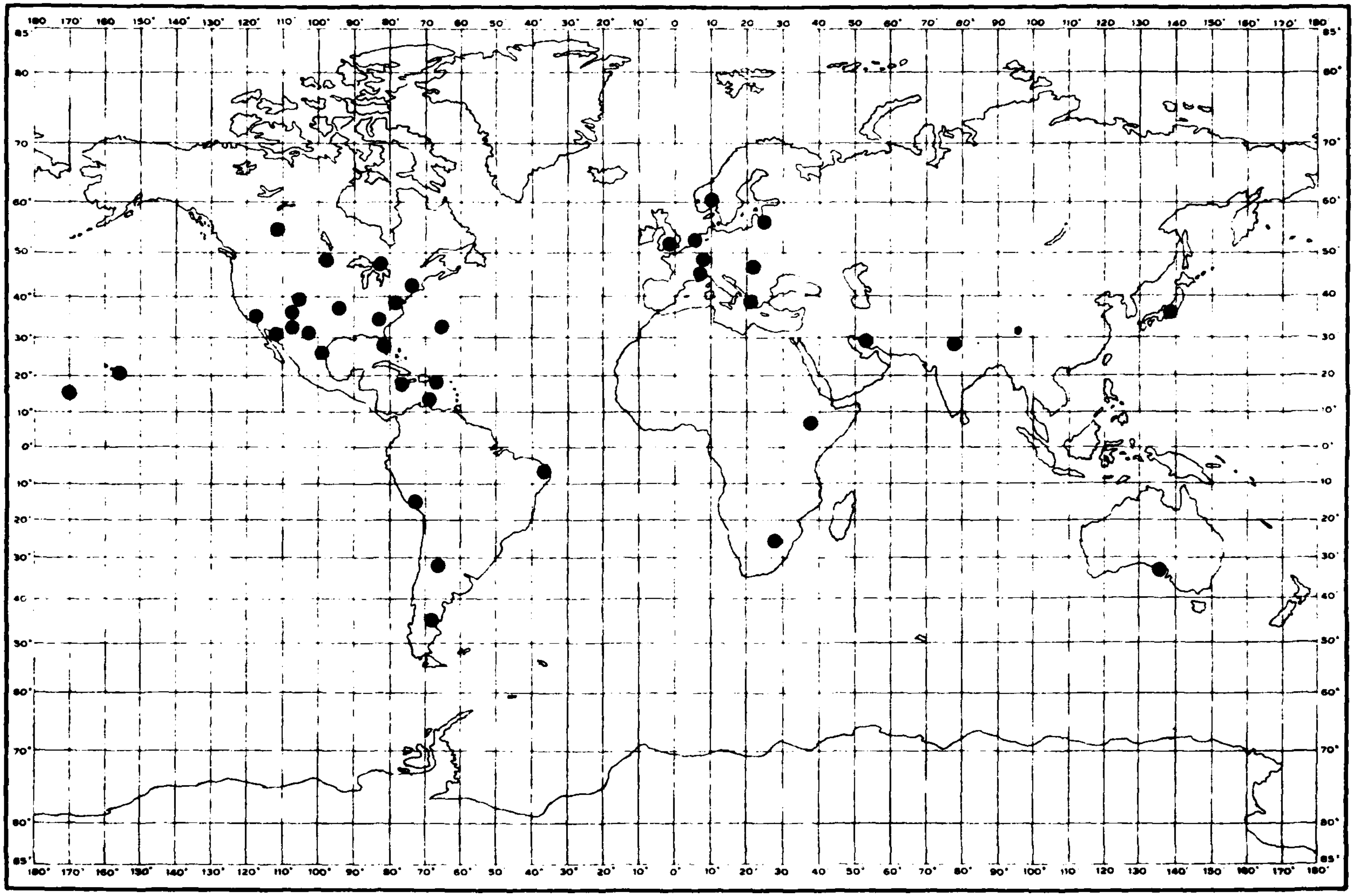

Figure 3. Baker-Nunn and MOTS Stations.

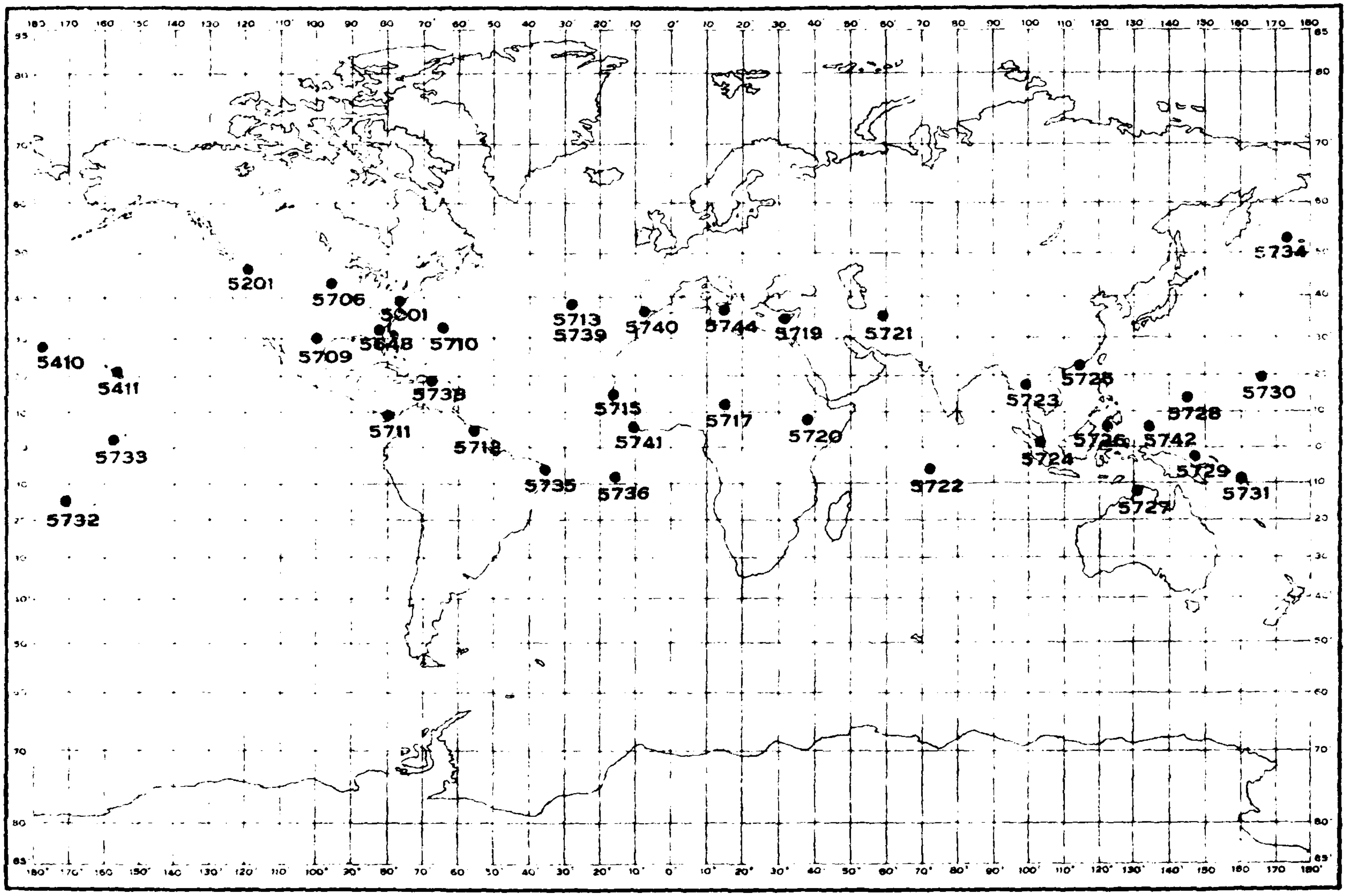

Figure 4. SECOR Equatorial Network.

Table 1

Mean Free Air Gravity Anomaly Information - November 1972 Field

Region	1°x1° $\overline{\Delta g}$ Data			5°x5° $\overline{\Delta g}$ Data		
	Number of Values	RMS Value* (Mgals)	Average Value* (Mgals)	Number of Values	RMS Value* (Mgals)	Average Value* (Mgals)
North America	10,557	±26.9	-1.4	489	±17.9	-1.7
South America	2,830	36.0	-1.4	144	18.9	-0.5
Europe	5.256	32.6	0.0	251	20.2	-1.4
Asia	7,871	34.5	-0.1	353	18.4	-0.7
Eurasia	13,127	33.9	-0.1	604	19.1	-1.0
Australia	1,618	27.4	-4.6	63	20.4	-4.8
Africa	963	21.1	6.2	193	15.4	4.8
Antarctica	296	39.9	-4.7	49	22.1	-8.7

*All $\overline{\Delta g}$ data referenced to the WGS 72 Ellipsoidal Gravity Formula.

Astrogeodetic Geoid Height Data

The astrogeodetic data in its basic form consists of deflection of the vertical components referred to the various national geodetic datums. These deflection values were integrated into astrogeodetic geoid charts referred to these national datums using the methods described in [8]. The increase in the holdings of astrogeodetic data since 1966 is summarized in three reports to the International Union of Geodesy and Geophysics (IUGG) Assemblies in 1967 [8] and 1971 [9] [10]. The astrogeodetic geoid height data was inserted into the Unified WGS Solution in the form of a 430x430 normal equation matrix, the right hand side being a 430x1 column of constants. The astrogeodetic geoid height matrix contributed to the Unified WGS Solution by virtue of providing additional and more detailed data for land areas.

Survey

Normal equations representing conventional ground survey data were included in the Unified WGS Solution to enforce a consistent adjustment of the coordinates of neighboring observation sites of the BC-4, SECOR, Doppler, and Baker-Nunn systems. Also, normal equations representing eight geodimeter long line precise traverses were included for the purpose of controlling the scale of the unified solution.

To relate each set of two to five stations located up to 100 kilometers apart, a normal equation submatrix was produced, treating the survey coordinates as observables with a given standard error (usually 0.5 meter) and allowing as parameters the coordinates of all involved stations and seven transformation constants for the survey data: three rotation, three translation, and one scale. Rotation and scale parameters were given non-zero a priori standard errors (usually 50 ppm) to weaken the tie over long distances. The final tie matrix used was the sum of 54 such sub-matrices after individual elimination of the transformation parameters. A matrix representing the eight geodimeter scalars was formed for coordinates of paired BC-4 stations located at endpoints of the long survey lines.

UNIFIED WGS SOLUTION

The Unified WGS Solution is a solution for geodetic positions and associated parameters based on an optimum combination of available data. Initially, a normal equation matrix was formed based on each of the previously mentioned data sets. Then, the individual normal equation matrices were combined and the resultant matrix solved to obtain the WGS parameters. In order to merge or combine the individual normal equation matrices, the following information is needed or required:

- The normal equations themselves free of extraneous conditions.

- The number of degrees of freedom.
- The weighted sum of the squares of the discrepancies (adjusted if necessary for any eliminated unknowns).
- An estimate of the variance of unit weight.
- Initial estimates of the parameters.

Given this information, a Unified WGS Solution is theoretically very simple. The normal equations for each data set are adjusted so that the unknowns are homogeneous; i.e., they must be corrections to the same initial values and in the same units. Statistically, they are homogenized by dividing each by its variance of unit weight. Then, it is only necessary to sum the various normal equation matrices and solve for the unknowns.

Unfortunately, all of the required information was not available for all the normal equation matrices. In such cases, the only recourse, without going to the expense of reforming the normal equations, was to adjust the weights assigned to each system to bring the results into conformity with reality. That is, various unified solutions were made using different weights and the results were tested against external standards. As one test, for example, the geopotential coefficients from unified solutions were used to determine goodness-of-fit with respect to ground-based satellite tracking data for an actual orbit. Of course, each unified solution was tested for consistency with the individual data sets which it incorporated.

Most of the data sets are complementary to each other, but limited in some way in their ability to uniquely define WGS parameters. For example, due to the lack of high quality worldwide gravity coverage, in particular in the southern hemisphere, existing mean free air gravity anomaly data poorly defines the zonal and lower degree tesseral harmonic coefficients of the geopotential. Similarly, many of the higher degree tesseral harmonic coefficients are not well defined from satellite tracking data due to the small magnitude of the orbital perturbations at geodetic satellite altitudes. In addition, a sufficient number of satellites at distinctly different inclinations is not available to reduce the correlation between the coefficients. Factors such as these provide the logic which justifies a Unified WGS Solution which exploits the advantages and at the same time minimizes the disadvantages of each data set.

Data sets available in the form of normal equations matrices and in addition to ground survey information, were Doppler, surface gravity, astrogeodetic, SECOR, BC-4, and SAO (Standard Earth II). The surface gravity matrix had only gravitational model parameters. The astrogeodetic matrix, in addition to gravitational model parameters, contained unknowns representing datum shift parameters and a change in the semi-major axis. SECOR and BC-4 had only satellite tracking station parameters, and Doppler and Standard Earth II included both satellite tracking station and gravitational model parameters. The ground survey data was used to provide a tie between stations of the different data sets; for example, Doppler and BC-4, Doppler and SECOR, and SECOR and BC-4. However, to minimize the effect of possible survey errors on the solution, the use of ground survey information to form condition equations was restricted to collocated stations. This does not eliminate the use of condition equations for precisely surveyed long baselines to better establish scale for the system.

A starting assumption for the Unified WGS 72 Solution was that the normal equations obtained from local survey data and geometrically processed satellite observational material (BC-4 and SECOR), could be taken at face value. Due to neglect of gravitational and other model errors, normal equations obtained from dynamic satellite methods and geophysical data reduction might require some deweighting. For the surface gravity data, modification of data weights to include a spatially correlated model term with appropriate standard error and decorrelation distance would have given more balance to the worldwide data representation by that matrix. Because the astrogeodetic normal equations were not formed from the measurements themselves, but from equally weighted observations of the finished astrogeodetic geoid at selected locations, important strengths, weaknesses, and singularities of the data have been obscured.

In examining the matrices used, it was noted that only the SAO Standard Earth II and Doppler matrices required joint improvement of both stations and gravitational parameters. (The connection between astrogeodetic datum shifts and station positions was never completed with a datum survey normal matrix.) Experience with Doppler matrices has shown that only limited correlation exists between gravitational coefficient and station coordinate improvements. Further, the Doppler matrix was expected to be a strong contributor to the unified solution. Therefore, the basic procedure adopted was a separate determination of the weights of station-only or gravity-only matrices in preliminary solutions, followed by the full combined solution for all parameters. In the preliminary phase, the SAO Standard Earth II matrix was treated with the other gravity determining matrices.

As weights are changed in combined solutions, each set of normal equations prefers to yield, along its own ill-determined parameter, directions which can make the unit variances insensitive acceptability criteria. As alternative criteria for judging station solutions, the RMS deviations of the trial solution coordinates were expected not to exceed similar deviations of Doppler coordinates (adjusted by survey differences to the non-Doppler positions) from the originators sets. According to this standard, the highest satisfactory trial weight for the BC-4 (and survey) matrix was 10, relative to the Doppler weight. The SECOR equations were entered into this combination at the highest weight relative to BC-4 recommended by DMATC and, as disturbances were minimal, this completed the relative weighting determination for station matrices. Table 2 compares residuals achieved with the goals for each data set.

Table 2

Residuals From Station Solutions

Data Set	Type of Residual	Goal	Achieved
Doppler BC-4 SECOR SAO	RMS Deviation From Originators Solution	2.2 m 6.6 12.0 9.0	2.2 m 5.4 10.0 9.0
Survey	RMS Over 54 Sets Of Tied Stations	0.5 m	0.6 m
Scalar	RMS of 8 Lines	2×10^{-6}	1.5×10^{-6}

The calibration consisted of a determination of biases for scale and longitude for transformation of NWL-9D positions to WGS 72. Originally, the scale correction was based on the comparison of satellite and terrestrial results as previously discussed. Since the equatorial radius corresponding to the dynamic geoid defined by an interim solution (WGSN-44) geopotential coefficients and the mean sea level values of selected Doppler stations is 6378133 meters, a scale correction is necessary for conversion to the adopted WGS 72 semimajor axis (6378135 meters). The longitude correction was defined by enforcing a selected set of Doppler coordinates to agree with gravimetrically derived values in the NAD 27 area.

The merger was effected through an adjustment of scale, longitude, and z-axis origin of WGSN-44, and justified by the low residual difference (2.2 meters, RMS) between the WGS 72 and WGSN-44 (adjusted) station coordinates used in determining the transformation constants. Of these, the scale and z-shift were required to compensate biases inherited from the Doppler normal equation matrix, while the longitude adjustment was necessary to allow for the difference between the gravimetric and satellite standards. Table 3 contains the formulas and parameters for relating NWL-9D coordinates to WGS 72.

Table 3

Formulas and Parameters
To Transform NWL-9D Coordinates
To WGS 72 Coordinates

Formulas	$\Delta\phi'' = \Delta f \sin 2\phi / \sin 1''$ $\Delta\lambda'' = 0\overset{''}{.}260$ $\Delta H = a\ \Delta f \sin^2\phi - \Delta a + \Delta r$
Parameters	$\Delta r = -5.27$ meters $\Delta a = -10.0$ meters $\Delta f = -0.112415 \times 10^{-6}$ $a = 6378135$ meters
Instructions	To obtain WGS 72 coordinates, add the $\Delta\phi$, $\Delta\lambda$, ΔH corrections to the NWL-9D coordinates (ϕ, λ, and H, respectively). Latitude is positive North and Longitude is positive East.

WGS 72 PARAMETERS

Mean Earth Ellipsoid

In determining the WGS 72 Ellipsoid and associated parameters, the Committee decided quite early to closely adhere to the thoughts and approach used by the International Union of Geodesy and Geophysics (IUGG) in establishing the Geodetic Reference System 1967 (GRS 67) [11]. Accordingly, an equipotential ellipsoid of revolution was taken as the form for the WGS 72 Ellipsoid. An equipotential ellipsoid is simply an ellipsoid defined to be an equipotential surface; i.e., a surface on which all values of the potential are equal. Given an ellipsoid of revolution, it can be made an equipotential surface of a certain potential function, the normal gravity potential, U. This normal gravity potential can be uniquely determined, independent of the density distribution within the ellipsoid, by using any system of four independent parameters as the defining constants of the ellipsoid. To determine the normal gravity potential without resorting to the use of a mass distribution model for the ellipsoid, U can be expanded into a series of zonal ellipsoidal harmonics of linear eccentricity in $(a^2-b^2)^{\frac{1}{2}}$. The coefficients in the series are determined by using the condition that the ellipsoid is an equipotential surface (U = constant). Since all the zonal coefficients vanish, except the two of degree zero and two, a closed finite expression is obtained for U. Normal gravity (γ), the gradient of U, is given at the surface of the ellipsoid by the closed formula of Somigliana [12]:

$$\gamma = \frac{a\gamma_e \cos^2\phi + b\gamma_p \sin^2\phi}{(a^2\cos^2\phi + b^2 \sin^2\phi)^{\frac{1}{2}}} \tag{1}$$

where

a = semimajor axis of the ellipsoid

b = semiminor axis of the ellipsoid

γ_e = normal gravity at the equator

γ_p = normal gravity at the poles

ϕ = geodetic latitude

Thus, the equipotential ellipsoid serves not only as the reference surface or geometric figure of the earth but leads to a closed formula for normal gravity at the ellipsoidal surface, a formula easily modified for spatial applications.

Consistent with the IUGG definition of GRS 67, the Committee took the four defining parameters of the WGS 72 Ellipsoid to be the semimajor axis (a), the earth's gravitational constant (GM) and angular velocity (ω), and the second degree zonal harmonic coefficient of the geopotential ($\bar{C}_{2,0}$). Other parameters associated with the ellipsoid, such as the semiminor axis (b) and the flattening (f), including the normal gravity formula, are calculated using the defining parameters. These and other parameters associated with the ellipsoid are given in Table 4.

Semimajor Axis

The value adopted by the Committee for the semimajor axis (a) of the WGS 72 Ellipsoid is

$$a = 6378135 \quad \pm 5 \text{ meters.}$$

The adoption of an a-value 10 meters smaller than that for the WGS 66 Ellipsoid was based on several calculations and indicators. One of the more extensive calculations was made using an approach based on the general combination of satellite and surface gravity data for position and gravitational field determinations [13]. There are two variations of the general combination procedure differing principally in that one does not involve a gravitational field determination. Using that procedure and a related computer program, various sets of satellite derived station coordinates and gravimetric deflection of the vertical and geoid height data were used to determine local-to-geocentric datum shifts, datum rotation parameters, a datum scale parameter and a value for the semimajor axis of the WGS Ellipsoid. Eight solutions were made with the various sets of input data, both from an investigative point of view and also because of the limited number of unknowns which could be solved for in any individual solution due to computer program limitations. Doppler satellite tracking and astro-gravimetric datum orientation stations were used on a selected basis in the various solutions. In these eight solutions, the input values for the semimajor axis and flattening of the ellipsoid were 6378145 ±15m and 1/298.26, respectively. Different combinations of Doppler satellite tracking and astro-gravimetric datum orientation stations provided values for the semimajor axis ranging from 6378134.5 to 6378137.2 meters. Also, eight additional solutions were made in which the only change was in the input value for the ellipsoidal semimajor axis. Using 6378130 ±15m for this input parameter, the solutions for the semimajor axis ranged from 6378133.6 to 6378136.6 meters. Based on these results and those from other related studies accomplished by the Committee, the value a = 6378135 meters was adopted.

Table 4

WGS 72 Ellipsoid

Geodetic and Geophysical Parameters			
Parameters	Notation	Magnitude	Standard Error (68.27%)
Gravitational Constant*	GM	398600.5 km^3/sec^2	±0.4
Second Degree Zonal*	$\bar{C}_{2,0}$	-484.1605×10^{-6}	--
Angular Velocity*	ω	$0.7292115147 \times 10^{-4}$ rad/sec	$\pm 0.1 \times 10^{-13}$
Semimajor Axis*	a	6378135 meters	±5
Flattening	f	1/298.26	$\pm 0.6 \times 10^{-7}$
Equatorial Gravity (Absolute System)	γ_e	978033.26 mgal	±1.8
Gravitational Constant (Mass of Earth's Atmosphere Included)	GM'	398600.8 km^3/sec^2	±0.4

*The defining parameters of the WGS 72 Ellipsoid.

Table 4 (Cont'd)

WGS 72 Ellipsoid

Associated Constants			
Constants	Notation	Formula	Value
Semiminor Axis	b	$b = a(1-f)$	6,356,750.5 m
Major Eccentricity	e	$e = [f(2-f)]^{1/2}$	0.08181881066
Major Eccentricity Squared	e^2	$e^2 = f(2-f)$	0.006694317778
Minor Eccentricity	e'	$e' = e/(1-f)$	0.08209405392
Axis Ratio	b/a	$b/a = 1-f$	0.9966472205
Radius of Sphere with Equal Area		R_A	6,371,005.2 m
Radius of Sphere with Equal Volume		R_V	6,370,998.9 m
Ellipsoid Potential		U_o	6,263,688 kgal m
Mean Value of Normal Gravity		$\overline{G}_{Potsdam}$	979,772.87 mgal
Mean Value of Normal Gravity		$\overline{G}_{Absolute}$	979,758.87 mgal

Gravitational Constant

The earth's gravitational constant (GM), the product of the universal gravitational constant and the earth's mass, can be obtained by different techniques [14]. During recent years, the most reliable values of GM have resulted from the analysis of lunar and planetary probe tracking data. For example, the GM value determined by the Jet Propulsion Laboratory (JPL) from the combination and analysis of radio tracking data from four lunar missions, Rangers VI - IX, was adopted for use with WGS 66. Of the similar more recent determinations, JPL has identified the GM value determined from Mariner 9 radio tracking data as probably a more accurate result to date [15]. This value

$$GM = 398600.8 \pm 0.4 \text{ km}^3/\text{sec}^2$$

which contains the mass of the earth's atmosphere, is one of the two GM values adopted for use with WGS 72. The second WGS 72 GM value does not include the mass of the earth's atmosphere and, along with a, ω and $\bar{C}_{2,0}$, is one of the four defining parameters of the WGS 72 Ellipsoid. It is discussed later.

In determining GM, JPL used the value

$$c = 299792.5 \pm 0.3 \text{ km/sec}$$

for the velocity of light. Since that time, new determinations of the value for c have been reported by the National Bureau of Standards (NBS) [16] [17]. The two values are:

$$c = 299792.4562 \pm 0.0011 \text{ km/sec}$$

$$c = 299792.462 \pm 0.018 \text{ km/sec}$$

In June 1973, the value of c was revised by the Consultative Committee for the Definition of the Meter (under the International Bureau of Weights and Measures). This action occurred as a result of their selection of a recommended value for the wavelength of the methane-stabilized laser, which thereby (in conjunction with NBS laser frequency measurements) established a recommended value for c [18]. The recommended value was adopted by the Committee for use in DOD applications:

$$c = 299792.458 \pm 0.0012 \text{ km/sec}$$

As previously stated, the above JPL determined value for GM contains the mass of the earth's atmosphere. To derive a GM value which does not contain the mass of the earth's atmosphere, values are needed for the universal gravitational constant (G) and the mass of the earth's atmosphere (m_a). The value

$$m_a = (5.136 \pm 0.007) \times 10^{21} \text{ grams}$$

was selected along with

$$G = 66.720 \pm 0.041 \times 10^{-9} \text{ cm}^3/\text{sec}^2$$

for this purpose. The m_a value, above, was selected on the basis of the greater rigor applied in its determination. The product of G and m_a, when subtracted from GM, gives

$$GM = 398600.5 \pm 0.4 \text{ km}^3/\text{sec}^2$$

The result is a GM value which does not contain the mass of the earth's atmosphere and is also adopted for use with WGS 72.

The selection of the proper value depends on the type of application considered. For example, some satellite and space studies require the GM which includes the earth's atmosphere. In geodetic computations involving the normal potential (gravity formula, gravity anomalies, geoid heights, etc.) it is proper to use the GM excluding atmosphere.

Angular Velocity

The angular velocity of the WGS 72 Ellipsoid was taken to be equal to that of the actual earth. The value selected for the WGS 72 Ellipsoid

$$\omega = 7.292115147 \times 10^{-5} \text{ radians/second}$$

is a rounded value of the GRS 67 value which has an extra digit [11]. The GRS 67 value was obtained by two solutions and is consistent with the International Astronomical Union (IAU) System of Astronomical Constants, which were adopted by the IAU in 1964 [19].

Second Degree Zonal Harmonic Coefficients of the Geopotential

The second degree zonal harmonic coefficient of the geopotential is the $\bar{C}_{2,0}$ value in the following series where the geopotential form is restricted to zonal harmonics:

$$U = \frac{GM}{r} \left[1 + \sum_{n=2}^{\infty} \bar{C}_{n,0} \left(\frac{a}{r}\right)^n \bar{P}_{n,0} (\sin\phi') \right] \tag{2}$$

Due to the limitation of available surface gravity data, satellite observational data provides the best means for obtaining coefficients for equation (2). The $\bar{C}_{n,0}$ coefficients are expressed in terms of the rates of change of the orbital elements, the changes in the elements being observed after several revolutions rather than directly. Values for the even degree coefficients are principally influenced by secular or long-periodic variations in the ascending node and perigee, odd degree coefficients are principally influenced by similar-type variations in the eccentricity and inclination.

Using satellite data and an equation such as equation (2), various determinations have been made of a finite number of zonal harmonic coefficients. For example, King-Hele [20] [21] and Kozai [22] made several determinations during the mid and late 1960s. Also, using the more general expression for the geopotential,

$$U = \frac{GM}{r}\left[1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^n \overline{P}_{nm}(\sin\phi')(\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda)\right] \quad (3)$$

the zonal harmonic coefficients are determined along with the tesserals in the standard least squares solution for the gravitational model coefficients ($\overline{C}_{nm}$, $\overline{S}_{nm}$). Selecting a $\overline{C}_{2,0}$ value to serve as one of the four defining parameters of the WGS 72 Ellipsoid is more desirable from a consistency standpoint since it is part of a larger set of correlated coefficients forming the earth's gravitational model. A $\overline{C}_{2,0}$ value selected from a solution involving equation (2) would not necessarily be consistent with the coefficient set from a solution involving equation (3). Therefore, the Committee adopted as a defining parameter of the WGS 72 Ellipsoid the $\overline{C}_{2,0}$ value from the geopotential coefficient set selected as the WGS 72 Earth Gravitational Model:

$$\overline{C}_{2,0} = -484.1605 \times 10^{-6}$$

Flattening (Ellipticity)

The flattening (f) or ellipticity of the ellipsoid is related to the second degree zonal harmonic coefficient of the geopotential by the equation

$$C_{2,0} = -\frac{2f[1-(1/2)f]}{3} + \frac{m[1-(3/2)m-(2/7)f]}{3} + O(f^3) \quad (4)$$

where $m = \omega^2 a/\gamma_e$. Using WGS 72 values for $C_{2,0}$ and each parameter in the equation for m, equation (4) was solved for f by iteration. From this computation, the flattening of the WGS 72 Ellipsoid is:

$f = 1/298.2638$

This value has been rounded to 1/298.26 which is the adopted value.

Normal (Ellipsoidal) Gravity Formula

The WGS 72 Ellipsoidal Gravity Formula is in the form of the Chebychev approximation [11]:

$$\gamma \approx A\,(1 + c_2 \sin^2\phi + c_4 \sin^4\phi) \tag{5}$$

where

$$A = \gamma_e\,(1 + \frac{1}{32}a_6)$$

$$c_2 = a_2 - \frac{9}{16}a_6$$

$$c_4 = a_4 + \frac{3}{2}a_6$$

The coefficients for this formula were obtained by using the WGS 72 parameters, resulting in the WGS 72 Gravity Formula:

$$\gamma = 978.03327\,(1 + 0.005278994 \sin^2\phi + 0.000023461 \sin^4\phi) \text{ gal}$$

where 1 gal = 1 cm/sec^2 and 1 milligal (mgal) = 10^{-3} gal.

The WGS 72 Geoid

In geodetic applications, three different surfaces or earth figures are normally involved. In addition to the earth's natural or physical surface, these include a geometric or mathematical reference surface, the ellipsoid, and an equipotential surface called the geoid. The geoid theoretically coincides with mean sea level over the oceans and extends hypothetically beneath all land surfaces. Normally, the geoid makes its appearance in geodetic positioning applications by the relationship

$$H = N + h \tag{6}$$

where

H = geodetic height

N = geoid height

h = height above mean sea level.

For general purposes, the form of the geoid is usually indicated by a contour chart which shows the location of the geoid above or below a geometric reference surface or figure (the ellipsoid) selected to approximate the earth. When available, an astrogeodetic contour chart is the first practical choice used to obtain geoid height data (Figure 5). Although astrogeodetic charts are very detailed and accurate, there are gaps in the coverage. For practical applications, geoid heights can be computed mathematically from the appropriate formulation rather than scaled from a worldwide geoid contour chart.

THE CONVERSION OF GEODETIC DATUM TO WGS 72

In the development of local-to WGS 72 datum shifts, different geodetic disciplines have been used and the results from each investigated, analyzed and compared. This redundancy of disciplines and data provides assurance that the system accepted as WGS 72 is the best attainable using the methods and data available in 1972. Noting the variations in results obtained from the different disciplines was helpful in assigning accuracy statements to the adopted parameters. Table 5 compares results from gravimetric data, Doppler satellite tracking data, and a combination of the two sources used to orient four principal datums to a geocentric system. (The local-to-WGS 66 datum shifts are shown for comparison purposes.) Corrections to shift the previously existing operational Doppler coordinate set (NWL-9D) to WGS 72 were shown in Table 3. Using these corrections, a large number of Doppler TRANET and Geoceiver station coordinates was available worldwide for deriving local-to-WGS 72 datum shifts. The derivation of the shifts is discussed below.

NAD 27 to WGS 72

Stations distributed over the continental United States and Mexico were selected from the available coordinates. The selection, listed in Table 6, consists principally of Geoceiver stations with some permanent Doppler Tracking Network (TRANET) stations and ITT-5500 stations. Four locations (Howard County, MD; Beltsville, MD; Las Cruces, NM; and Moses Lake, WA) have both TRANET and Geoceiver stations. These stations were used to derive datum shifts for the NAD 27 area, excluding Alaska and Canada. Figures 6, 7 and 8 show the NAD 27-to-WGS 72 datum shifts in terms of contours of the ΔX, ΔY, ΔZ components. The datum shifts contained in Tables 5 and 7 are mean values of the datum shifts for individual stations. Variations of several meters (up to 10 m) exist between the mean NAD 27-to-WGS 72 datum shifts and the shifts observed at each location. The standard deviations from the mean are $\sigma_{\Delta X} = \pm 4.4$ m, $\sigma_{\Delta Y} = \pm 3.5$ m, $\sigma_{\Delta Z} = \pm 3.9$ m. Thus, while the mean NAD 27-to-WGS 72 datum

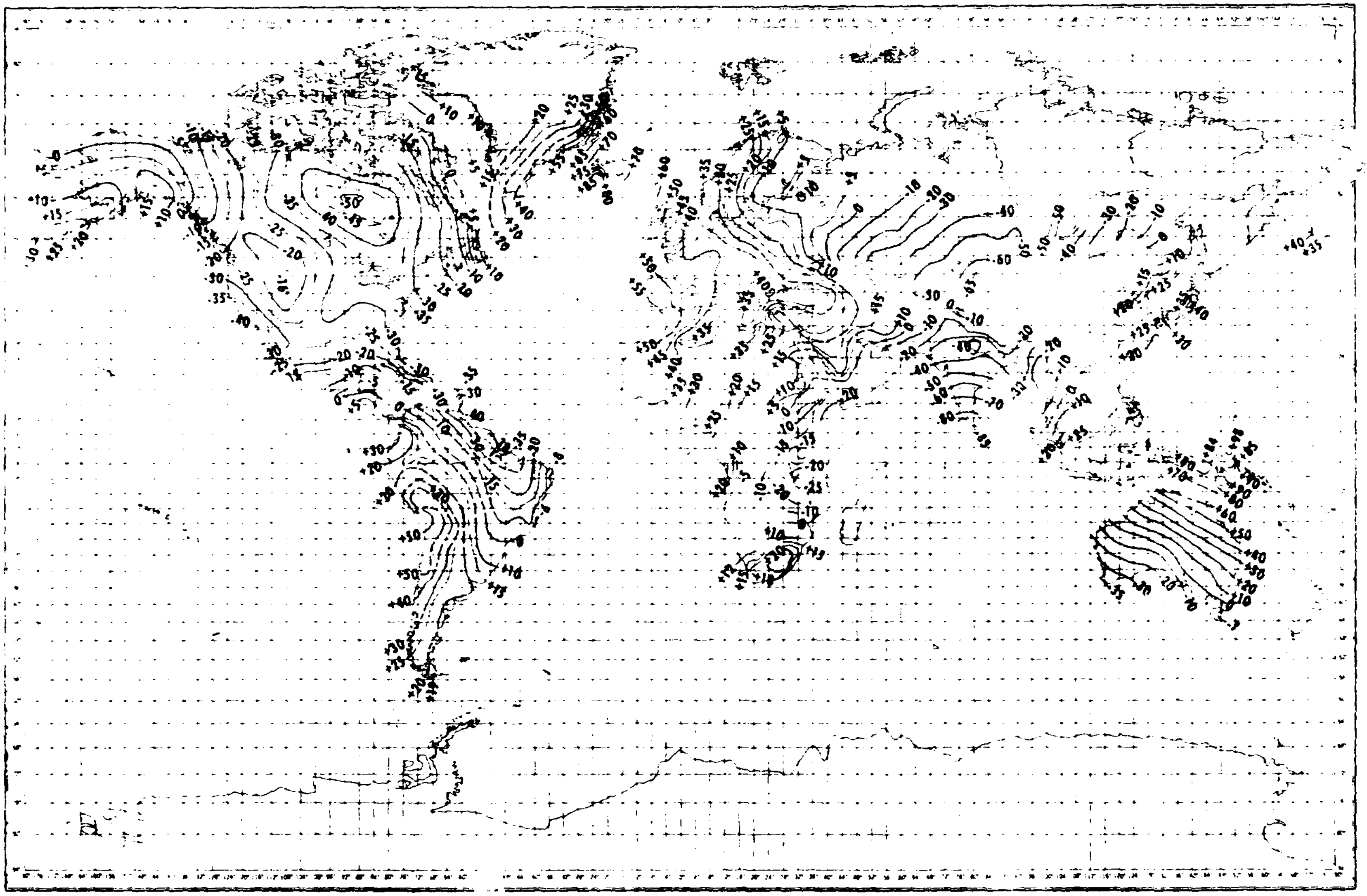

Figure 5. Astrogeodetic WGS 72 Geoid (meters).

Table 5

Datum Shift Constants
Derived From Various Disciplines

Datums	Datum Shift Components	WGS 66	Gravimetric	Doppler *NWL-9D	Trial Unified Solution	WGS 72
NAD 27	$\Delta X(m)$	- 24	- 23	- 24	- 21	- 22
	$\Delta Y(m)$	154	161	154	152	157
	$\Delta Z(m)$	188	179	179	182	176
ED	$\Delta X(m)$	- 73	- 64	- 81	- 84	- 84
	$\Delta Y(m)$	- 82	- 79	-104	-106	-103
	$\Delta Z(m)$	-119	-129	-126	-130	-127
TD	$\Delta X(m)$	-156	-152	-140	-138	-140
	$\Delta Y(m)$	508	543	518	517	516
	$\Delta Z(m)$	674	640	673	678	673
AGD	$\Delta X(m)$	-123	-167	-127	-129	-122
	$\Delta X(m)$	- 33	- 77	- 34	- 31	- 41
	$\Delta Z(m)$	138	154	145	146	146

*Longitude constraint imposed.

Table 6

Stations Used to Derive WGS 72 Datum Shifts

NAD 27 Area, Alaska and Canada Excluded			
Station Number	Location	Station Number	Location
103	Las Cruces, NM	20002	Las Cruces, NM
111	Howard Co., MD	20003	Wrightwood, CA
311	Prospect Harbor, ME	20015	Woodbine, GA
321	Rosemount, MN	20016	Columbia, MS
738	Moses Lake, WA	30025	Bloomfield, OH
742	Beltsville, MD	30027	Greenville, OH
10003	Greenville, MS	30028	Metamora, IL
10006	Tipton, KS	30029	Moses Lake, WA
10018	Jonestown, TX	30030	Green River, UT
10019	Frankton, IN	30031	Seligman, AZ
10020	Marysville, IN	30032	Navajo, AZ
10021	Summit, KY	30033	Miami, AZ
10023	Mathiston, MS	30034	Ajo, AZ
10029	Patrick AFB, FL	30038	Blaine, WA
10031	Goldstone, CA	30078	Westford, MA
10046	Salt, NM	30089	Los Mochis, Mexico
10055	Pillar Point, CA	30098	Orland, CA
20000	Howard Co., MD	30099	Malta, MT
20001	Beltsville, MD	8004	Marshall Point, ME
NAD 27 Area, Alaska and Canada Only			
Station Number	Location	Station Number	Location
14	Anchorage, AK	30053	Barter I., AK
740	Nome, AK	30054	Boundary, AK
807	Hall Beach, Canada	30056	Cold Bay, AK
30050	Kotzebue, AK	30065	Cape Sarichef, AK
30051	Clear, AK	30067	Gilmore Creek, AK
30052	Point Barrow, AK		

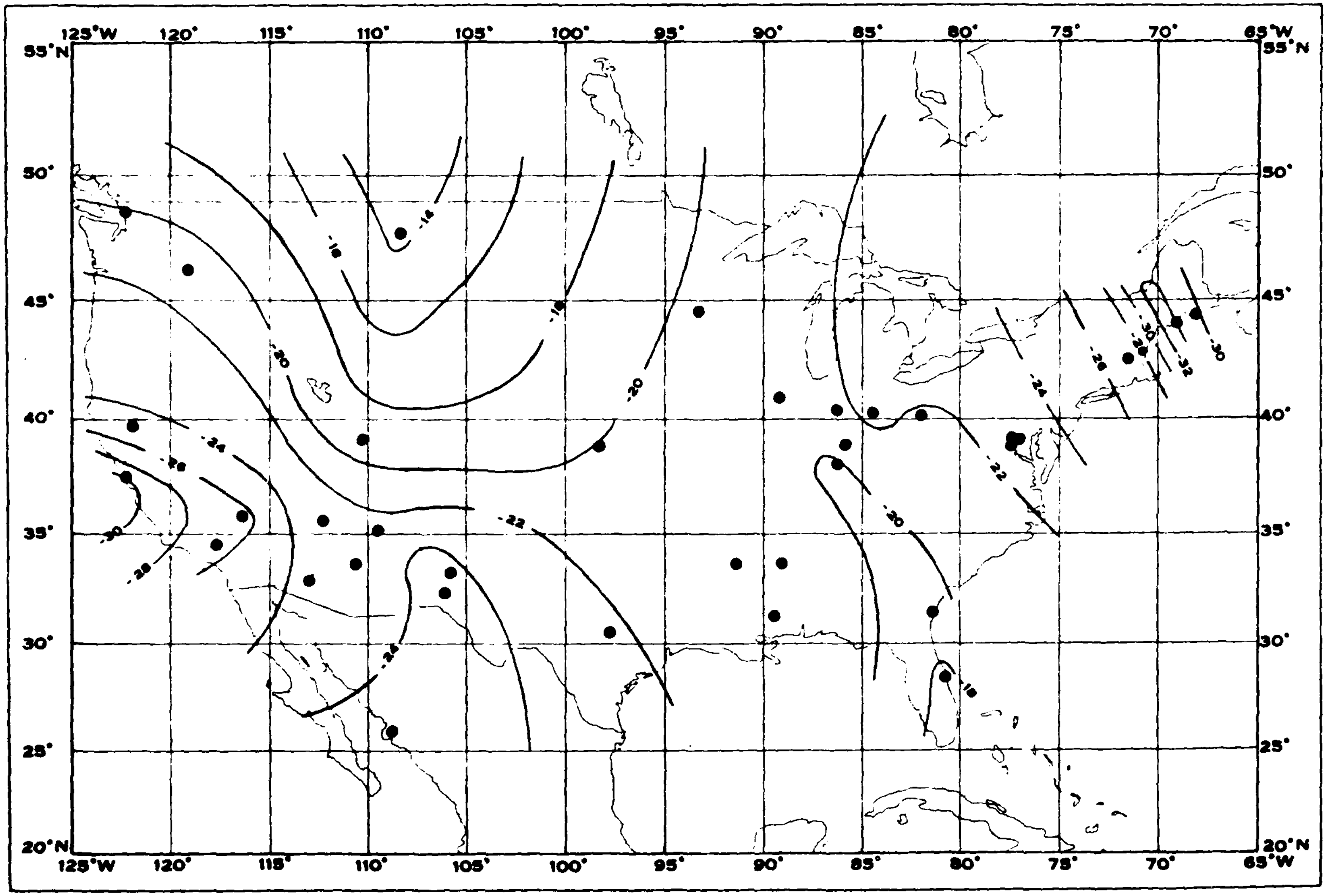

Figure 6. Contour Chart - NAD 27 to WGS 72 Datum Shifts, ΔX Component (Contour Interval = 2 meters).

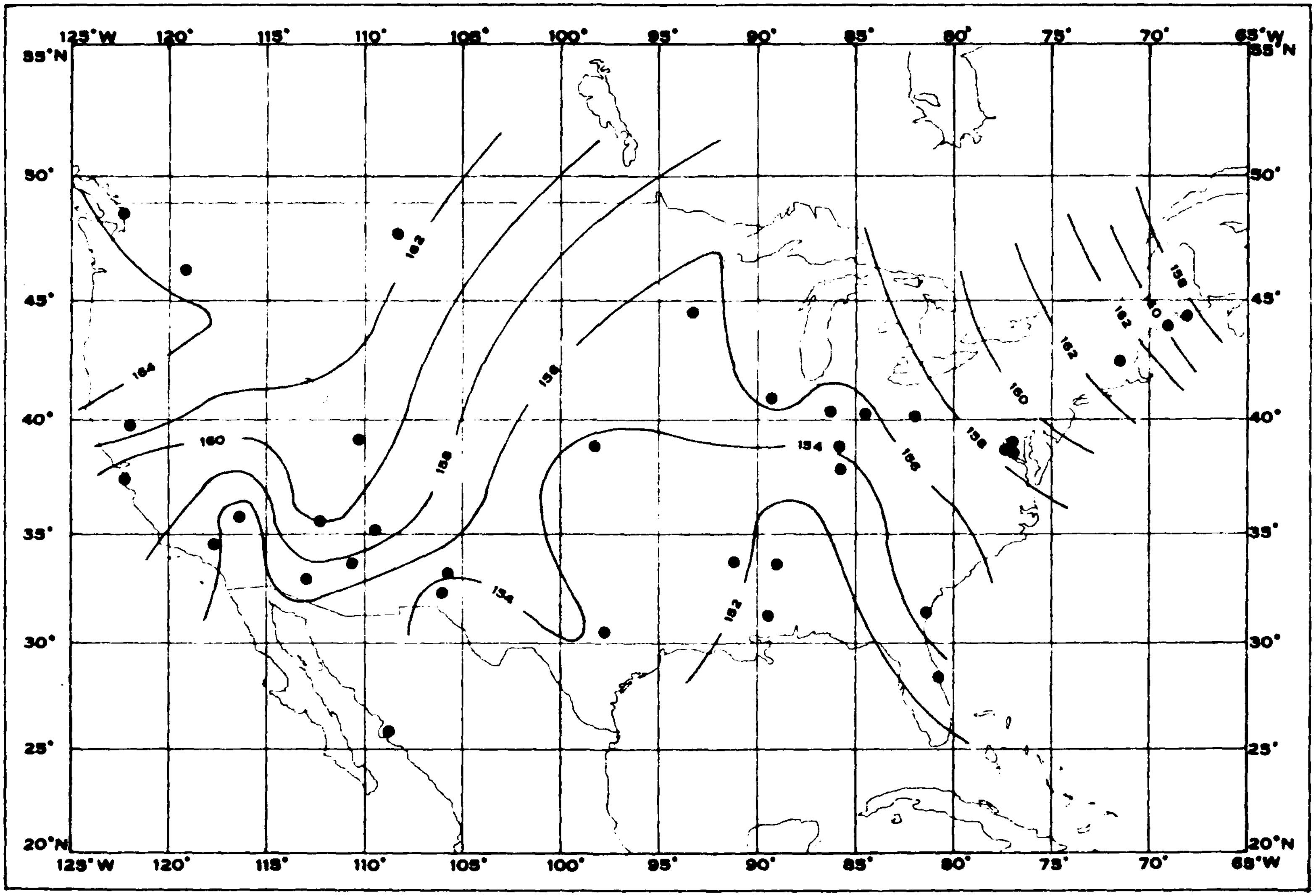

Figure 7. Contour Chart - NAD 27 to WGS 72 Datum Shifts, ΔY Component (Contour Interval = 2 meters).

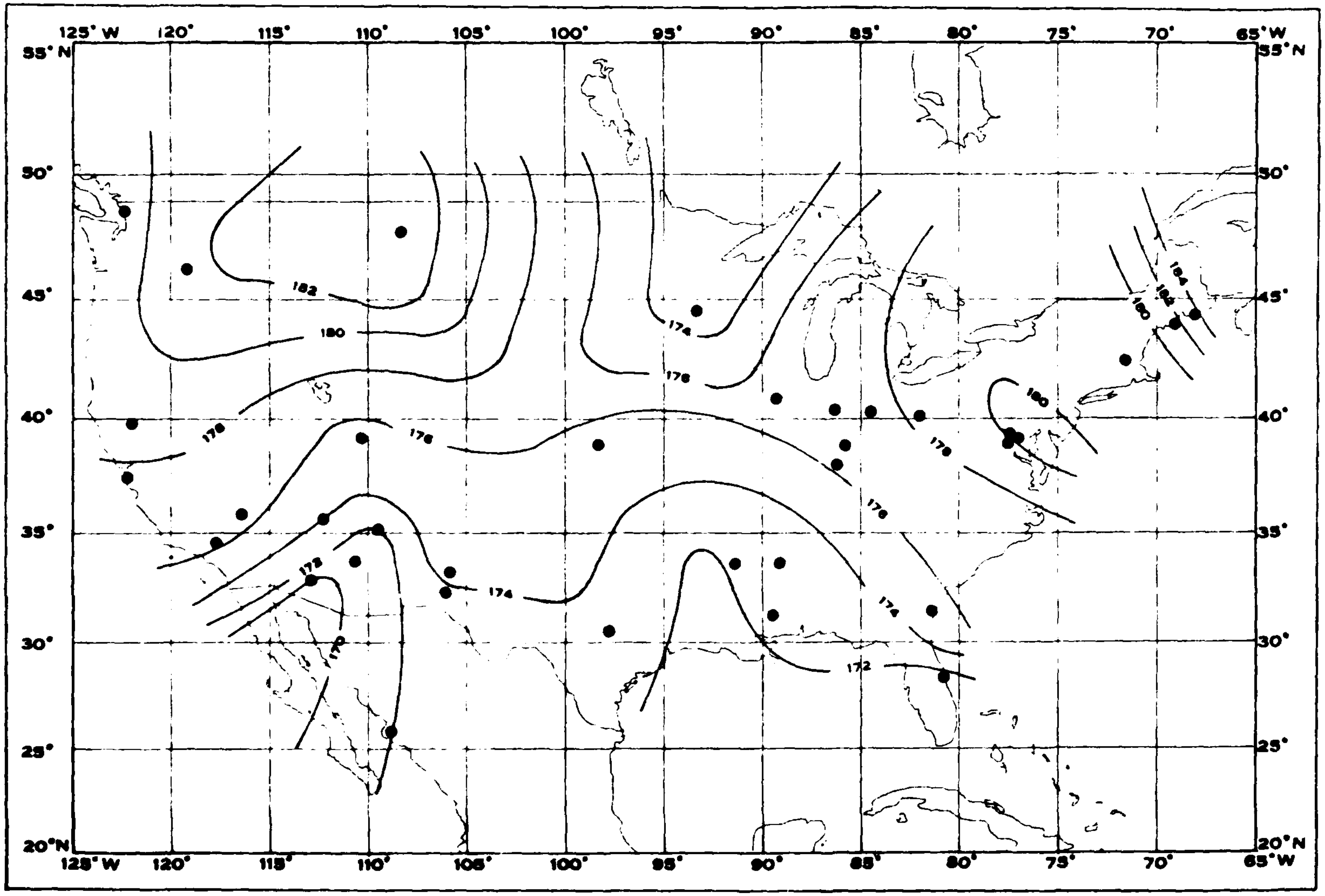

Figure 8. Contour Chart - NAD 27 to WGS 72 Datum Shifts, ΔZ Component (Contour Interval = 2 meters).

Table 7

Datum Shift Constants
(Geodetic Datum to WGS 72)

Geodetic Datums and Reference Ellipsoids	Constants				
	ΔX(m)	ΔY(m)	ΔZ(m)	Δa(m)	$\Delta f X 10^{-4}$
North American 1927 (Clarke 1866)	- 22*	157*	176*	- 71.400	-0.37295850
Alaska and Canada	- 9	139	173	- 71.400	-0.37295850
European (International)	- 84	-103	-127	-253.000	-0.14223913
Tokyo (Bessel)	-140	516	673	737.845	0.10006272
Australian Geodetic (Australian National)	-122	- 41	146	- 25.000	-0.00112415
Ordnance Survey of Great Britain 1936 (Airy)	368	-120	425	571.604	0.11928812
South American 1969 (South American 1969)	- 77	3	- 45	- 25.000	-0.00112415
Old Hawaiian (Clarke 1866)					
Maui	65	-272	-197	- 71.400	-0.37295850
Oahu	56	-268	-187	- 71.400	-0.37295850
Kauai	46	-271	-181	- 71.400	-0.37295850
Johnston Island Astro 1961 (International)	192	- 59	-211	-253.000	-0.14223913
Wake-Eniwetok 1960 (Hough)					
Kwajalein Atoll	112	68	- 44	-135.000	-0.14223913
Wake Island	121	62	- 22	-135.000	-0.14223913
Eniwetok Atoll	144	62	- 38	-135.000	-0.14223913

*Mean value for the NAD 27 area excluding Alaska and Canada; see also Figures 6, 7, and 8.

Table 7 (Cont'd)

Datum Shift Constants
(Geodetic Datum to WGS 72)

Geodetic Datums and Reference Ellipsoids	Constants				
	ΔX(m)	ΔY(m)	ΔZ(m)	Δa(m)	$\Delta f X 10^{-4}$
Wake Island Astro 1952 (International)	283	- 44	141	-253.000	-0.14223913
Canton Island Astro 1966 (International)	294	-288	-382	-253.000	-0.14223913
Guam 1963 (Clarke 1866)	- 89	-235	254	- 71.400	-0.37295850
Ascension Island Astro 1958 (International)	-214	91	48	-253.000	-0.14223913
South Asia (Fischer 1960)	21	- 61	- 15	- 20.000	0.00449585
Nanking 1960 (International)	-131	-347	0	-253.000	-0.14223913
Arc 1950 (Clarke 1880)	-129	-131	-282	-114.145	-0.54781925
Adindan (Clarke 1880)	-152	- 26	212	-114.145	-0.54781925
Mercury 1960 (Fischer 1960)					
NAD 27 Area	- 25	46	- 49	- 31.0	0.00449585
ED Area	- 13	- 88	- 5	- 31.0	0.00449585
TD Area	18	-132	60	- 31.0	0.00449585
Modified Mercury 1968 (Fischer 1968)					
NAD 27 Area	- 4	12	- 7	- 15.0	0.00449585
ED Area	- 3	1	- 6	- 15.0	0.00449585
TD Area	22	34	2	- 15.0	0.00449585

shifts are valid within ±5 m in each component, users may prefer the greater precision attainable by using the datum shifts shown in Figures 6, 7, and 8. Such shifts are more closely related to the geodetic control in a localized area in that survey errors which accumulate over long distances from the datum origin are eliminated. The result is a form of "local" to WGS 72 datum shift. In this sense, individual NAD 27 to WGS 72 datum shifts are also shown in Table 7 for Alaska and Canada.

ED to WGS 72

The number of Doppler stations for ED is, of course, much less than that available in the NAD 27 area and the distribution less desirable. A total of eight stations, four TRANET and four Geoceiver stations, was selected for deriving ED to WGS 72 datum shifts. Tables 5 and 7 contain the mean of the ΔX, ΔY, ΔZ shifts for the eight stations. The mean values define the ED to WGS 72 datum shifts.

TD to WGS 72

Datum shifts for Tokyo Datum were determined in a manner similar to that for the ED area; however, only four stations were used (two TRANET and two Geoceiver). The TD to WGS 72 datum shifts are shown in Tables 5 and 7.

AGD to WGS 72

Four TRANET stations were used to derive WGS 72 datum shifts for the Australian Geodetic Datum (AGD). The datum shifts are contained in Tables 5 and 7.

Other Datum Conversions

In addition to the datum shift constants for the preceding datums, Table 7 also contains constants for converting other significant geodetic datums to WGS 72. The shifts are also based on Doppler derived coordinates in combination with conventional survey data. Included are datum shifts for several island areas. These are good examples of the utilization of Geoceiver surveys to accomplish geocentric positioning in terms of the WGS 72. Several smaller datums exist for which there are no direct ties to WGS 72. Transformations between these local datums and major datums have been derived where possible using terrestrial and satellite information. Datum shifts for this purpose are shown in Table 8. For information purposes, some interdatum relationships for datums in Table 7 are also included.

Table 8

Interdatum Shifts for WGS 72

Geodetic Datums		Constants				
From	To	ΔX(m)	ΔY(m)	ΔZ(m)	Δa(m)	$\Delta f X 10^{-4}$
SAD 69	NAD 27	- 55	-154	-221	46.4	0.3718344
Chua Astro	SAD 69	- 77	239	5	-228.0	-0.1411150
Yacare	SAD 69	- 90	160	78	-228.0	-0.1411150
Campo Inchauspe	SAD 69	- 83	130	120	-228.0	-0.1411150
Hito XVIII	SAD 69	87	198	125	-228.0	-0.1411150
Bogota	SAD 69	354	288	-283	-228.0	-0.1411150
La Canoa	SAD 69	-225	102	-326	-228.0	-0.1411150
Aerodist	SAD 69	-222	108	-317	-228.0	-0.1411150
Adindan	ED	- 68	77	339	138.855	-0.4055801
Arc 1950	Adindan	23	-105	-494	0	0
Arc 1950	ED	- 45	- 28	-155	138.855	-0.4055801
South Asia	ED	105	42	112	233.0	0.1467350
Malayan Revised Triangulation	South Asia	- 33	918	30	850.937	0.2788057

Datum Conversion Formulas

The Molodensky Datum Conversion Formulas [23] previously selected for use with WGS 66 from an analysis of several datum conversion formulas are retained for WGS 72 purposes. To accomplish the conversion, the data in Table 7 and the reference ellipsoid constants (Table 9) are used in the Molodensky Datum Transformation Formulas (Table 10). In the selected rectangular coordinate system, the axes are defined as follows:

X-axis = Intersection of the plane through the Greenwich meridian and the plane of the equator.

Y-axis = Measured 90° east of Greenwich in the plane of the equator.

Z-axis = Coincident with the earth's axis of rotation.

EVALUATION

Accuracy estimates of the parameters defining WGS 72 are included in Table 4. In addition, Figure 9 contains the uncertainties of the datum shift constants for selected datums. These range from ±5 meters, one sigma, in the NAD 27 area up to ±15 meters in the TD and AGD areas. The accuracy values were determined in the WGS 72 development primarily by error analyses involving Doppler and physical geodesy results. A relative evaluation of WGS 72 is provided by a comparison with other contemporary geocentric systems. Table 11 compares parameters and datum shifts between WGS 72 and two recent solutions each of the Smithsonian Astrophysical Observatory (Standard Earth II, 1969 [7], Standard Earth III, 1973 [24]) and the Goddard Space Flight Center (GSFC GEM 4, 1972 [25], GSFC 1973 [26]). Generally, the datum shifts agree well in the NAD 27 area with larger differences in the ED and TD areas. Additional datums are compared in Figure 10 between WGS 72 and GSFC 1973. The good agreement in the NAD 27 area includes the separate shifts for Alaska and Canada. With some exceptions, the differences are generally less than the one sigma accuracy values quoted for WGS 72 and well within the three sigma level.

The WGS 72, adopted by the DOD, represents a state of the art WGS solution using worldwide geodetic/gravimetric surveys and electronic/optical satellite data, and as such, should be considered a candidate system for use in the readjustment of the North American triangulation. Further, the WGS 72 associated with increased worldwide use of the geoceiver, provides a unique combination which, with international cooperation, could rapidly lead to a true world datum.

Table 9

Reference Ellipsoid Constants

Reference Ellipsoids	a(m)	f	$f \times 10^{-2}$	a/b=1/(1-f)	b/a=1-f	e^2	$1-e^2$
Clarke 1866	6378206.4	1/294.9786982	0.339 007 530 4	1.003 401 607	0.996 609 924 7	0.006 768 658 0	0.993 231 342 0
International	6378388	1/297	0.336 700 336 7	1.003 378 378	0.996 632 996 6	0.006 722 670 0	0.993 277 330 0
Bessel	6377397.155	1/299.1528128	0.334 277 318 2	1.003 353 985	0.996 657 226 8	0.006 674 372 2	0.993 325 627 8
Australian National, South American 1969	6378160	1/298.25	0.335 289 186 9	1.003 364 172	0.996 647 108 1	0.006 694 541 9	0.993 305 458 1
Airy	6377563.396	1/299.3249646	0.334 085 064 2	1.003 352 049	0.996 659 149 4	0.006 670 540 0	0.993 329 460 0
Hough	6378270	1/297	0.336 700 336 7	1.003 378 378	0.996 632 996 6	0.006 722 670 0	0.993 277 330 0
Fischer 1960 (South Asia)	6378155	1/298.3	0.335 232 986 9	1.003 363 606	0.996 647 670 1	0.006 693 421 6	0.993 306 578 4
Clarke 1880	6378249.145	1/293.465	0.340 756 137 9	1.003 419 213	0.996 592 438 6	0.006 803 511 3	0.993 196 488 7
Everest	6377276.345	1/300.8017	0.332 444 929 7	1.003 335 538	0.996 675 550 7	0.006 637 846 6	0.993 362 153 4
WGS 66	6378145	1/298.25	0.335 289 186 9	1.003 364 172	0.996 647 108 1	0.006 694 541 9	0.993 305 458 1
Fischer 1960 (Mercury)	6378166	1/298.3	0.335 232 986 9	1.003 363 606	0.996 647 670 1	0.006 693 421 6	0.993 306 578 4
Fischer 1968	6378150	1/298.3	0.335 232 986 9	1.003 363 606	0.996 647 670 1	0.006 693 421 6	0.993 306 578 4

Table 10
COORDINATE TRANSFORMATION FORMULAS
GEODETIC DATUM TO WGS 72

A. The Standard Molodensky Formulas

$$\Delta\phi'' = \{-\Delta X \sin\phi \cos\lambda - \Delta Y \sin\phi \sin\lambda + \Delta Z \cos\phi$$
$$+ \Delta a \, (R_N e^2 \sin\phi \cos\phi)/a$$
$$+ \Delta f \, [R_M (a/b) + R_N (b/a)] \sin\phi \cos\phi\} \cdot [(R_M + H) \sin 1'']^{-1}$$

$$\Delta\lambda'' = [-\Delta X \sin\lambda + \Delta Y \cos\lambda] \cdot [(R_N + H) \cos\phi \sin 1'']^{-1}$$

$$\Delta H = \Delta X \cos\phi \cos\lambda + \Delta Y \cos\phi \sin\lambda + \Delta Z \sin\phi$$
$$- \Delta a \, (a/R_N) + \Delta f \, (b/a) R_N \sin^2\phi$$

B. The Abridged Molodensky Formulas

$$\Delta\phi'' = [-\Delta X \sin\phi \cos\lambda - \Delta Y \sin\phi \sin\lambda + \Delta Z \cos\phi + (a\Delta f + f\Delta a) \sin 2\phi]$$
$$\cdot [R_M \sin 1'']^{-1}$$

$$\Delta\lambda'' = [-\Delta X \sin\lambda + \Delta Y \cos\lambda] \cdot [R_N \cos\phi \sin 1'']^{-1}$$

$$\Delta H = \Delta X \cos\phi \cos\lambda + \Delta Y \cos\phi \sin\lambda + \Delta Z \sin\phi + (a\Delta f + f\Delta a) \sin^2\phi - \Delta a$$

C. Definition of Terms in the Molodensky Formulas

ϕ, λ, H = geodetic coordinates (old ellipsoid)

ϕ = geodetic latitude. The angle between the earth's equatorial plane and the ellipsoidal normal at a point (measured positive north from the equator, negative south).

λ = geodetic longitude. The angle between the plane of the Greenwich meridian and the plane of the geodetic meridian of the point (measured in the plane of the equator, positive east from Greenwich).

H = the distance of a point from the ellipsoid measured along the ellipsoidal normal through the point.

$H = N + {}^*h$

---

*Indicates parameters which do not appear in the Abridged Molodensky Formulas.

Table 10 (Cont'd)

N = geoid-ellipsoid separation. The distance of the geoid above (+N) or below (−N) the ellipsoid.

$^{*}h$ = distance of a point from the geoid (elevation above or below mean sea level).

$\Delta\phi$, $\Delta\lambda$, ΔH = corrections to transform the geodetic coordinates from the old datum to WGS.

ΔX, ΔY, ΔZ = shifts between ellipsoid centers of the old datum and WGS.

a = semimajor axis of the old ellipsoid.

$^{*}b$ = semiminor axis of the old ellipsoid.

$^{*}b/a$ = $1 - f$

f = flattening of the old ellipsoid.

Δa, Δf = differences between the parameters of the old ellipsoid and the WGS ellipsoid (WGS minus old).

e = eccentricity

$e^2 = 2f - f^2$

R_N = radius of curvature in the prime vertical.

$^{**}R_N = a/(1 - e^2 \sin^2 \phi)^{1/2}$

R_M = radius of curvature in the meridian.

$^{**}R_M = a(1 - e^2)/(1 - e^2 \sin^2 \phi)^{3/2}$

NOTE: All Δ-quantities are formed by subtracting old ellipsoid values from WGS ellipsoid values.

---

*Indicates parameters which do not appear in the Abridged Molodensky Formulas.

**For desk calculator computations involving commonly used ellipsoids, these values are given in Latitude Function Tables, i.e., Latitude Functions Clarke 1866 Spheroid, AMS TM No. 68, 1957.

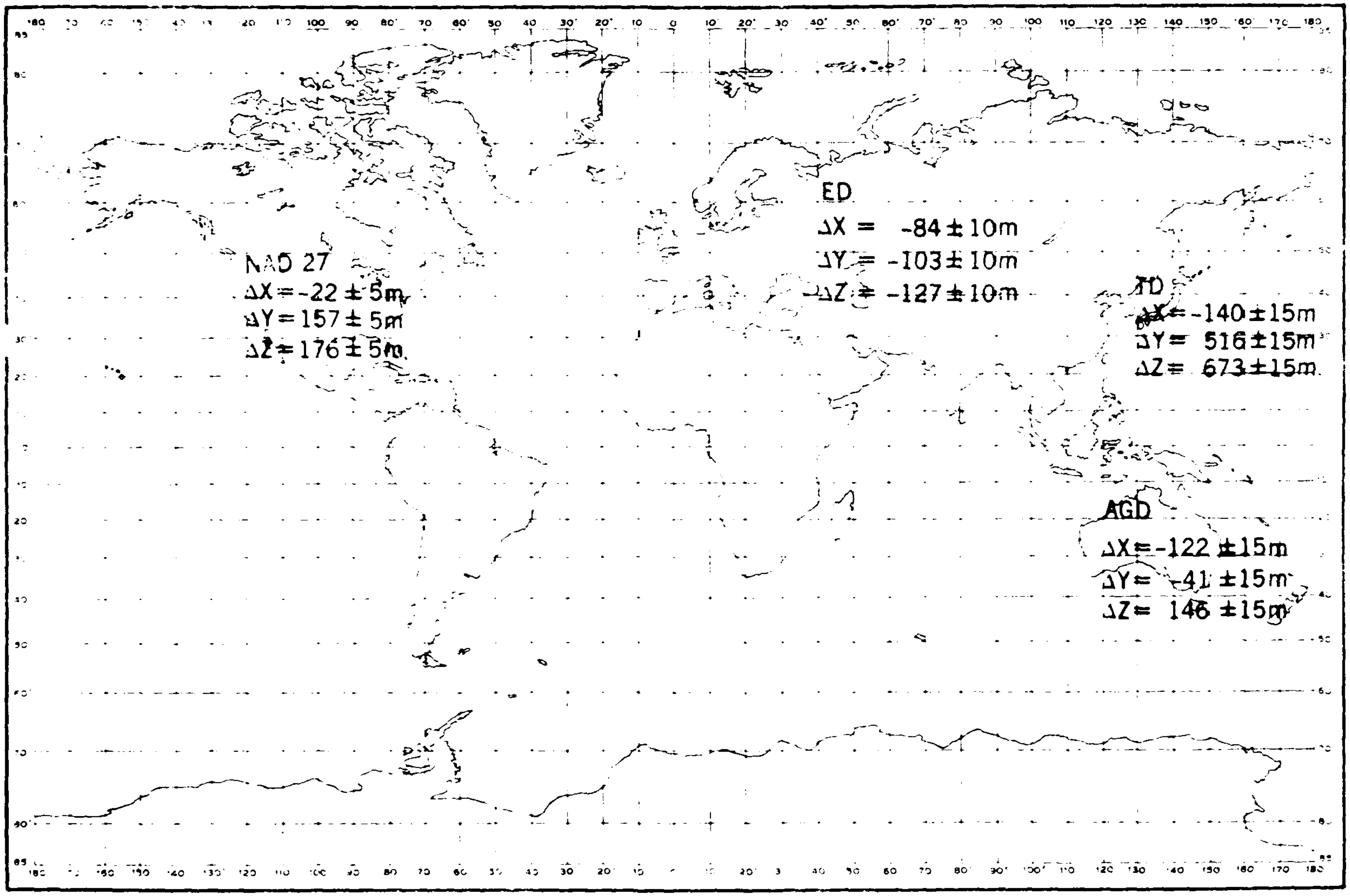

Figure 9. WGS 72 Datum Shifts and Accuracies for Selected Datums.

Table 11

Comparison of WGS 72 and Other
Ellipsoid and Major Datum Transformation Parameters

Parameters	WGS 72	Differences (WGS 72 Minus Other)				Units
		SAO 1969	SAO 1973	GSFC 1972	GSFC 1973	
Ellipsoid						
Semimajor Axis	6378135	-20	-20	-10	-20	m
Flattening	1/298.26	-0.56×10^{-7}	-0.56×10^{-7}	-0.56×10^{-7}	-0.56×10^{-7}	---
GM (With Atmosphere)	398600.8	-0.5	-0.5	-0.5	0	km^3/sec^2
Second Degree Zonal	-484.1605×10^{-6}	0.0055	0.0095	0.0085		---
Equatorial Gravity	978033.3	4.2		1.1		mgals
Datum Transformation						
NAD 27:						
ΔX	- 22	3	8	1	9	m
ΔY	157	3	0	- 6	1	m
ΔZ	176	- 5	4	- 4	- 5	m
ED:						
ΔX	- 84	0	0	10	- 2	m
ΔY	-103	26	23	16	- 4	m
ΔZ	-127	11	22	6	-15	m
TD:						
ΔX	-140	- 3	- 1	- 2	15	m
ΔY	516	- 8	- 9	13	21	m
ΔZ	673	- 6	- 3	-10	5	m

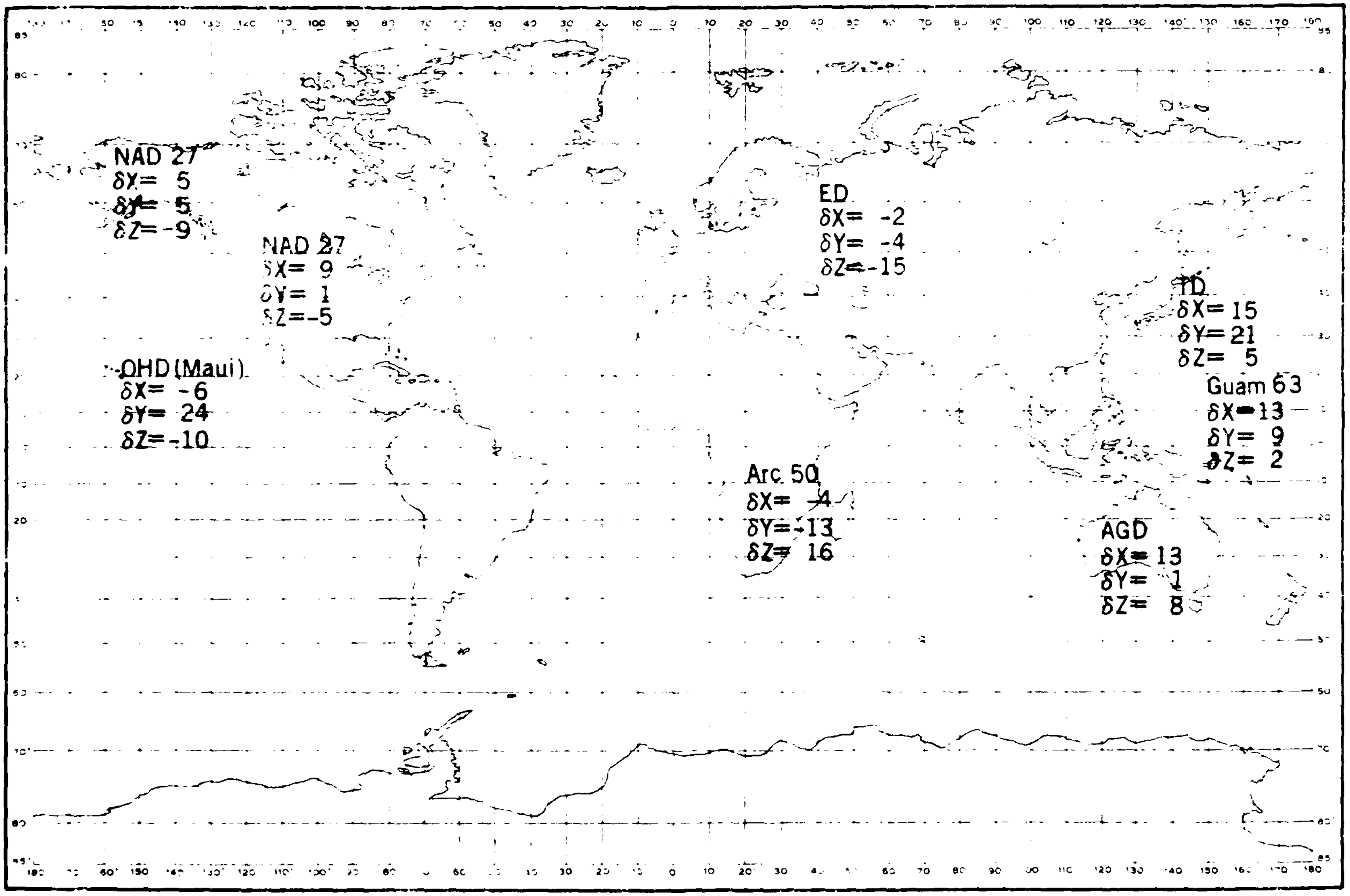

Figure 10. Comparison of Datum Shifts, WGS 72 Minus GSFC 1973 (meters).

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