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PREDICTING WHEELED VEHICLE MOTION RESISTANCE IN SHALLOW SNOW

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A vehicle traveling through snow is required to expend a greater amount of energy than is necessary when traveling on a rigid surface. Visually, this energy difference can be explained by the formation of a rut. Various attempts have been made in the past to equate the energy of compaction to vehicle motion resistance. However, many of the previous models use information gathered through the application of a vertical force (with a plate-sinkage device) to predict the horizontal motion resisting force. In an attempt to more accurately quantify the relationship between snow compaction and vehicle motion resistance, a vec-

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torial analysis of compaction by a wheel is performed. A method for separating the compaction due to vehicle weight and forward thrust (horizontal propulsion) is suggested. Two methods of using this compaction force breakdown with field-generated data are proposed for the calculation of vehicle motion resistance in shallow snow.

PREFACE

This report was prepared by George L. Blaisdell, Research Civil Engineer, of the Applied Research Branch, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory. Funding was provided by DA Project 4A762730AT42, Design, Construction, and Operations Technology for Cold Regions; Technical Area A, Combat Operations Support, Work Unit 10, Battlefield Mobility in Cold Regions.

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George Blaisdell

INTRODUCTION

A vehicle traveling through snow is required to expend a greater amount of energy than it would on an undeformable surface. Since this energy difference is visually accounted for by the formation of a rut, many researchers have postulated that the energy required to compact a snow layer should be equal to the increase in motion resistance of a vehicle moving in that snow layer. Provided the energy density of a snow layer can be determined, the compaction energy, and hence the motion resistance, can be calculated for a wheel or track moving through snow.

PAST WORK

The first attempt to use an energetics approach for the prediction of vehicle mobility over snow was made by Landauer and Royse (1956), who generated force vs sinkage curves for a natural snow with a hydraulically driven plate. By integrating the curves using planimetry, they established an energy per unit area term, w. When compared with motion resistance values measured in the same area with a towed tire, the compaction energy determined by their method yielded lower values. The authors thus concluded that the cause of increased motion resistance had little to do with the work of compaction.

Bekker (1956) and Nuttall (1957) continued the idea of equating motion resistance to the work of compaction. Each established a mathematical relationship among pressure, density and sinkage. Upon evaluation of the Bernstein (1913) equation

$$R = b \int_{o}^{z_o} p \, dz \qquad (1)$$

where R = motion resistance

b = contact width

z_o = sinkage

p = pressure,

Bekker (1956) and Nuttall (1957), respectively, express motion resistance as

$$R = \frac{bkz_o^{n+1}}{n + 1} \qquad (2)$$

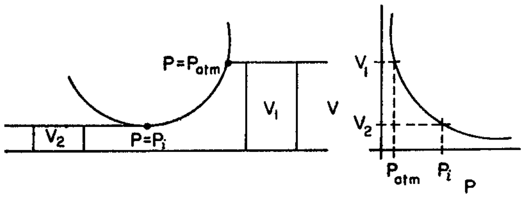


Figure 1. Hyperbolic pressure distribution along the wheel/snow interface (Liston 1974).

where k is a coefficient of proportionality and n a function of snow properties, and

$$R = F \frac{h_1}{\ell} \frac{P_z}{P_o} (1 - \frac{\gamma_i}{\gamma_f}) \tag{3}$$

where F = applied vertical force

h_1 = initial snow depth

ℓ = contact length

P_z = pressure at sinkage z

P_o = average ground pressure

γ_i = initial snow density

γ_f = final snow density.

In a slightly different approach, Liston (1974) defined the work E expended in compacting snow from its initial volume V_1 to a final volume V_2 as

$$E = \int_{V_1}^{V_2} p \, dV, \tag{4}$$

and assumed that the pressure vs volume relationship varies hyperbolically (Fig. 1). This relationship is based on an analysis of the wheel/snow interface and assumes that 1) the only dimensional change that occurs during compaction from V_1 to V_2 is in the snow depth and 2) the pressure along the wheel/snow interface equals the atmospheric pressure at the point of contact between the snow surface and the wheel and equals the inflation pressure at the lowest point of contact. The pressure-volume relationship then becomes

$$p = p_i \frac{V_2}{V} \tag{5}$$

where p_i is tire inflation pressure. Introducing eq 5 into eq 4 and completing the integration yields

$$E = p_i V_2 \ln \frac{V_2}{V_1} . \qquad (6)$$

By defining the force resisting motion as R, its relation to compaction is then

$$E = R\ell . \qquad (7)$$

Noting that the volume V is the product of contact length ℓ, contact width b and the snow depth h, and combining eq 6 and 7, the resistance equation becomes

$$R = p_i b h_2 \ln \frac{h_2}{h_1} \qquad (8)$$

where h_1 is initial snow depth and h_2 is final snow depth after vehicle passage.

The Bekker (1956), Nuttall (1957) and Liston (1974) approaches are all similar in that they equate the work of compaction of snow to the product of distance traveled and force resisting motion. Likewise, each attempts to develop one mathematical expression that describes the force vs sinkage relationship for all snows. Realizing that the effects of temperature, density, grain size, stratification, etc. combine to produce different force vs sinkage curves in different snows, Harrison (1975) suggests obtaining the energy of compaction from an actual force vs sinkage record. All previous approaches require information obtained in the field anyway, so this does not pose any new inconvenience. He then suggests that an energy per unit volume term be used to characterize the snow to calculate motion resistance.

Harrison notes that the deformation z nears a maximum value z_e as maximum compaction is approached during a compression test with snow confined on top and bottom only. This is considered a shallow snow case because the pressure bulb extends to the bearing surface under the snow. Further loading of the snow once critical density γ_{cr} is reached (occurring at sinkage z_e) causes plastic flow (mass movement of the snow rather than further compaction) to take place. It has been found that a critical density does exist for snow and that the majority of vehicles traveling through shallow snow do compact the snow to this critical density (Bennett 1973, Hinic 1974, Kinoshita and Akitaya 1970).

The area under the force vs sinkage curve between the limits of zero sinkage and z_e is, then, equal to the total work E expended in compacting the snow to its critical density. To obtain an energy per unit volume term ω the total work E is divided by the total volume prior to loading that became involved in compaction:

$$\omega = \frac{E}{V} \quad . \tag{9}$$

This equation assumes that all the snow involved in compaction has been brought to its critical density and that this energy term is independent of the volume used to obtain it. Variations from these assumptions and their possible effects will be discussed later.

Harrison (1975, 1981) has suggested the use of a circular plate-sinkage device to obtain values of ω. Because of its simple geometry and the nature of the pressure bulb developed beneath the plate, calculation of the volume of material undergoing compaction is easily accomplished. Figure 2 shows both thin-section and snow pit views of the pressure bulb beneath a plate. It is readily apparent that the volume of snow disturbed is confined to a cylinder with the diameter of the plate used. Since we are concerned with a shallow snow, the cylinder's height is the snow depth. Thus, the expression for energy of compaction for a plate-sinkage device is

$$\omega = \frac{E}{A_p \; h_1} \tag{10}$$

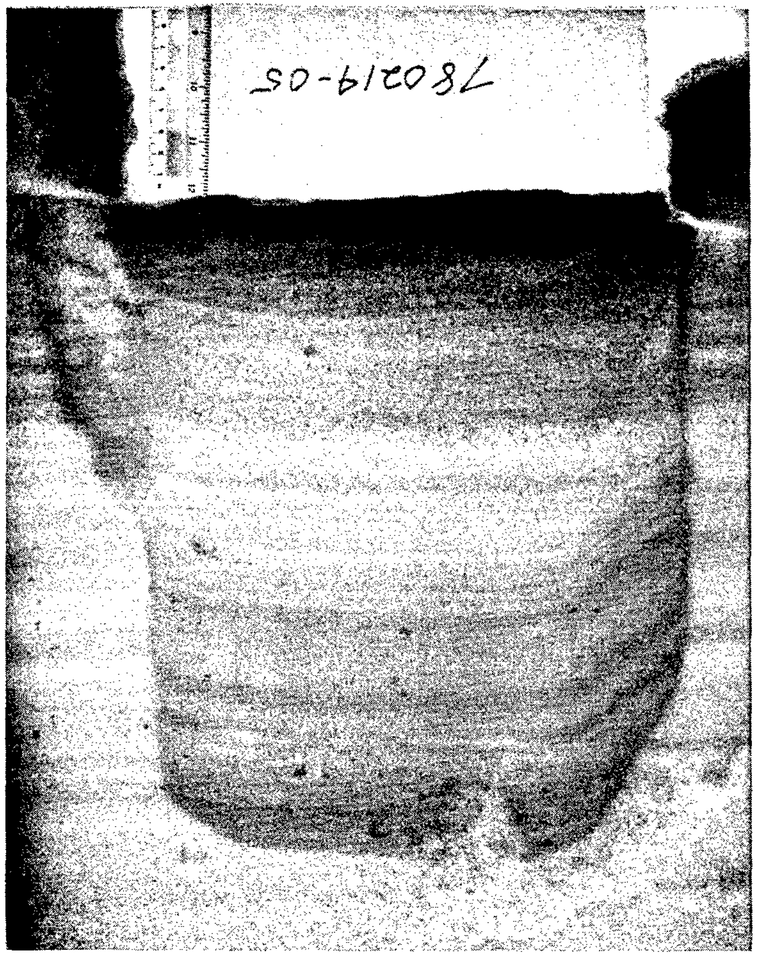
where A_p is the area of the plate and h_1 is initial snow depth.

The E term in eq 9 and 10 is found from a plot of force vs sinkage obtained during the plate test (Fig. 3). Using planimetry, the area under the curve is found between the limits of $z = 0$ and $z = z_e$. The quantity z_e may be found from the force-sinkage curve where the curve becomes asymptotic or by calculation from (Harrison 1973)

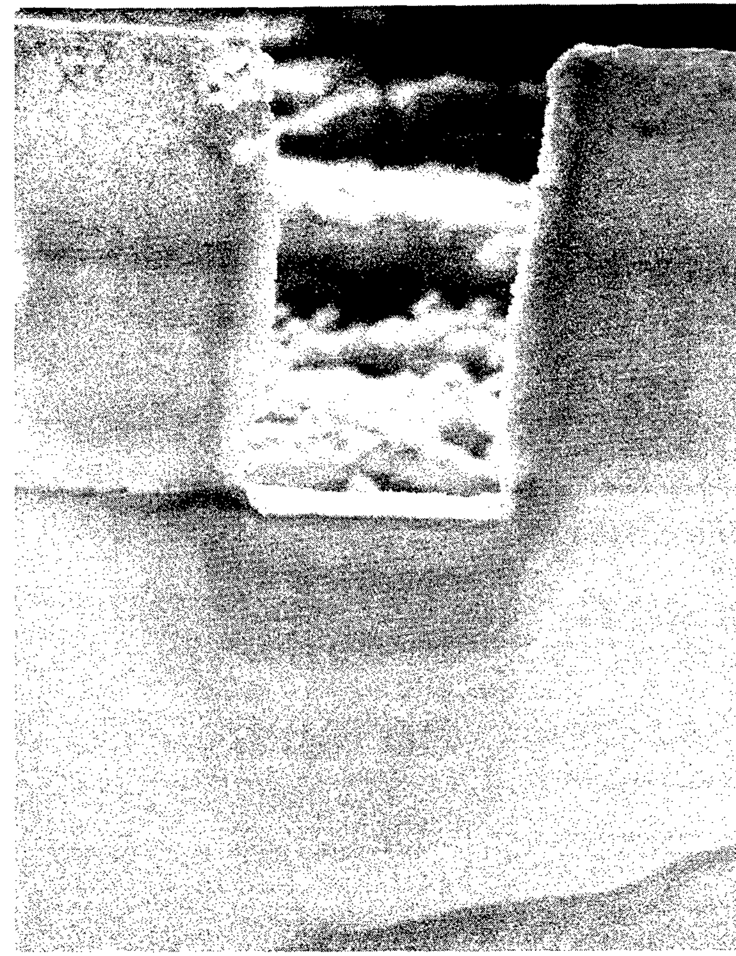
$$z_e = h_1 \; (1 - \frac{\gamma_i}{\gamma_{cr}}) \tag{11}$$

where γ_i is initial snow density and γ_{cr} = critical density.

It should be cautioned that the pressure bulb must intersect the ground surface in order to use h_1 in eq 10 as the pre-compaction height of



a. Thin section.



b. Pit view.

Figure 2. Pressure bulb beneath a plate.

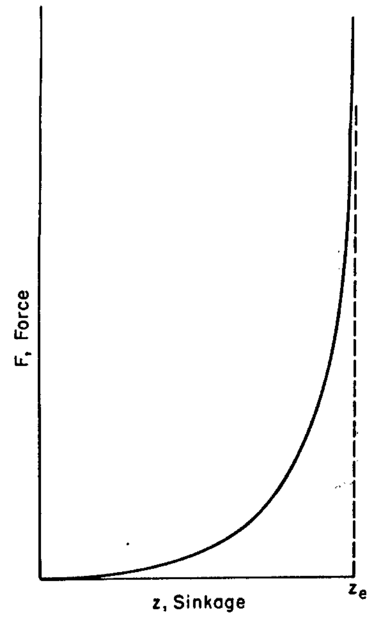


Figure 3. Idealized force vs sinkage curve generated by a plate-sinkage device in shallow snow.

material involved in densification. Harrison (1981) also recommends a minimum plate diameter of 20 cm be used to minimize size effects.

Calculation of ω using other devices is not precluded but may introduce complications in the determination of both E and V. Unlike the plate test, other tests may not compact all of the material disturbed by the loading device to its critical density. The volume disturbed by other pressure-sinkage devices may be of an irregular shape and thus difficult to compute. In addition, one must decide the percentage of the volume not at γ_{cr} to add to that at γ_{cr} for the volume term in eq 9. In determining E, one must choose a point along the load vs deformation curve (generated by the device) that corresponds with having reached critical density. This point should also represent the onset of plastic flow in the snow.

To obtain the resistance to motion R from snow compaction for one tire of a vehicle traveling through shallow snow, Harrison (1975) suggests that the energy per unit volume term be multiplied by the volume of snow which the vehicle compacts, or

$$R = bh_1\omega \qquad (12)$$

where b is the track width or effective tire width and ω the energy required per unit volume to compact snow to its critical density.

An approach similar to Harrison's was used by Selig (1971) for evaluating soil compaction equipment. For a towed rigid wheel roller, the compactive effort e is defined as

$$e = F_t LP \qquad (13)$$

where F_t = average towing force (equivalent to motion resistance in the previous approaches)

L = distance towed

P = number of passes over distance L.

This relationship indicates that the compactive effort is provided by the towing unit through the drawbar. Selig suggests, however, that the towing force can be related to the roller weight by

$$F_t = fW \qquad (14)$$

where f is the coefficient of rolling resistance and W the total roller weight.

The volume of soil V_c considered to be absorbing the compactive effort is defined as

$$V_c = LBt \qquad (15)$$

where B is the roller width and t the layer thickness after compaction.

Selig uses the compacted thickness rather than loose thickness to be consistent with the convention used in the standard laboratory impact compaction test.

Defining a compactive effort per unit volume term E_c by combining eq 13-15 yields

$$E_c = \frac{fWP}{Bt} \quad . \qquad (16)$$

For pneumatic tire rollers this relationship basically remains the same except that B is taken as the sum of all the tire widths (provided the tire separation is greater than the tire width).

Rearranging eq 16 to predict average towing force F_t we see that

$$F_t = fW = E_c \; Bt \qquad (17)$$

for one pass of the vehicle (P = 1). This relationship closely resembles Harrison's expression for motion resistance (eq 12). The primary difference between eq 17 and 12 lies in the height of material used in the volume calculation, one using pre-compaction height (Harrison) and the other using the post-compaction height (Selig).

E_c and ω are clearly similar energy terms and are found in a like manner. As stated previously, ω is found from a plate-sinkage test where a flat circular plate is forced into the snow along a straight path roughly perpendicular to the snow and ground surfaces. The value of E_c is based on the relationship of the measured soil densities in the field to densities produced by the standard and modified laboratory compaction tests (Wu 1976). Both E_c and ω are, then, determined from the application of a force perpendicular to the ground surface. Both approaches, though, use this energy term to predict a horizontal motion resisting force.

Selig attempts through f in eq 14 to relate the force required to advance the wheel with the vertical load acting on that wheel. However, f is an experimentally determined value and thus requires running tests in which the motion resistance F_t is measured. From these tests, tables of f values are generated for various types of soil and compaction equipment. Therefore, f is an empirically derived correction factor which relates motion resistance to soil and vehicle characteristics. Attempts to predict motion resistance using eq 17 require relying on the completeness and applicability of tabulated values of f.

Using Harrison's approach for making predictions of motion resistance, a plate-sinkage test may be performed in the snow of interest from which an ω value is obtained. The ω value can then be used with eq 12 to transfer the information gained from a vertical application of force to a horizontal motion resisting force.

THE MECHANICS OF A WHEEL TRAVELING THROUGH SNOW

Each of the six previous approaches to predicting motion resistance involves calculations based on a vertically applied force. Yet, motion resistance, as a vector quantity, is directed horizontally. Both Harrison (1981) and Selig (1971) recognize this fact and attempt to account for it through an empirical approach. It should be possible, however, to mathematically describe the magnitude and direction of the forces at the tire/material interface.

The wheel can be assumed to be constructed of an infinite number of flat plates arranged tangentially to the circumference. As the wheel moves forward each plate is pushed into the snow along a path described by the motion of a point on the tire surface (Fig. 4a). A pressure bulb develops beneath each flat section of the tire which grows as distance along the sinkage path increases from the initial tire/snow contact point (Fig. 4b).

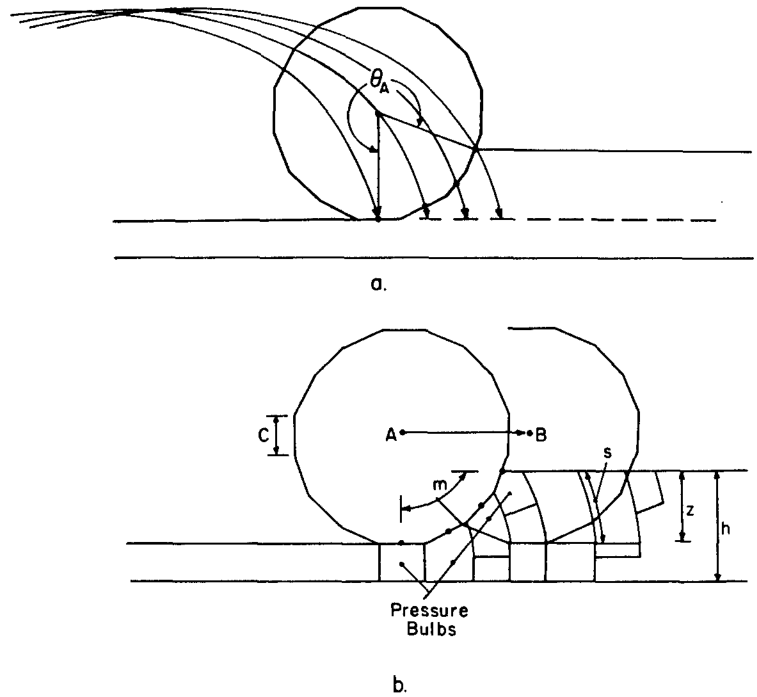


Figure 4. Sinkage path (a) and pressure bulb development (b) beneath a wheel made of plates.

The pressure bulb continues to grow until the influence of the ground or bearing surface is felt, at which point the snow becomes compressed to its critical density and no further sinkage takes place. The volume of snow compacted by a wheel in moving from position A to position B (Fig. 4b) can be found from

$$V = m\ b\ [s + (h - z)] \tag{18}$$

where m is the arc length along the tire/snow contact surface and s the length of the sinkage path.

The energy required to move the wheel from position A to B is the product of the force necessary to advance the compacting tire and the distance traveled. This force has two components. One component is developed parallel to the snow/vehicle contact surface by the driven wheel. The other component is the portion of the vehicle weight acting on the wheel of interest and directed vertically.

The forces resisting forward motion can be found by going back to the wheel made up of plates. If one assumes that the force and deformation vectors coincide as the snow is compacted, each plate feels a resistance to sinkage F_{Ti} which acts tangentially to the sinkage path at the plate/snow contact point (Fig. 5). The magnitude of this resistance is a function of

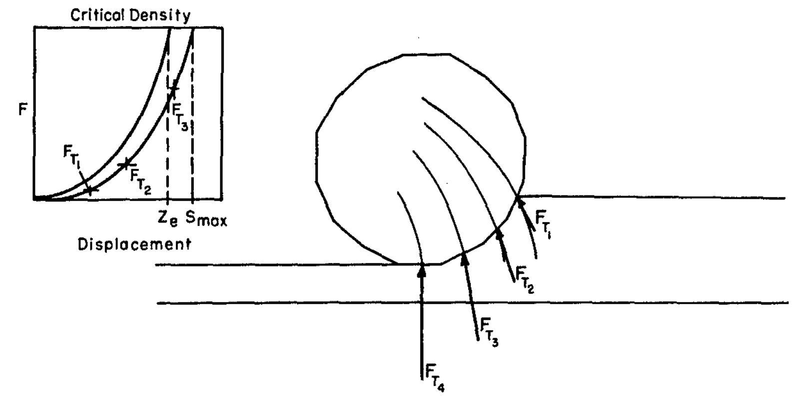


Figure 5. Force vectors resisting forward wheel motion in snow.

the plate's sinkage along the curved path, just as in the case of a simple plate-sinkage test. The vector sum of these forces (acting on all the plates) represents the total resistance to compaction F_T offered by the snow. The horizontal component F_{T_h} of this resultant force represents the true resistance to forward motion that is opposing the energy input by the driving wheels. If one ignores traction, it is only this horizontal component of force that makes travel over deformable surfaces consume more energy than movement over non-deforming materials.

CALCULATION OF THE FORCE NECESSARY TO COMPACT SNOW WITH A WHEEL

To determine the magnitude and direction of the resultant force F_T, the force vs sinkage relationship along a curved sinkage path and the length of the sinkage path is needed. This curved path, called a trochoid, is generated by describing the motion of a point on the circumference of the wheel as it rotates. The general form of the equation for a trochoid describing wheel motion is

$$x = a\theta - r\sin\theta$$
$$y = r(1 - \cos\theta) \qquad (19)$$

where r = wheel radius

a = radius of a hypothetical wheel which would cover the same distance as the real wheel after moving through angle θ, but without any slippage (a = r for a wheel with no slippage)

θ = angle (in radians) between a vertical radius and the radius which ends at the point under consideration.

The angle θ_A for the point of initial tire contact with the snow surface (Fig. 4a) can be found from sinkage by

$$\theta_A = 2\pi - \arccos \frac{r'-z}{r} \tag{20}$$

where r' is the effective radius (r' = r − δ where δ is tire deflection). If we begin by assuming a non-driven wheel with no slip, a = r in eq 19. The length of the sinkage path can then be found from

$$s_{max} = \int_{\theta_A}^{2\pi} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta \tag{21}$$

which yields

$$s_{max} = -4r \cos \frac{\theta}{2} \Big|_{\theta_A}^{2\pi} . \tag{22}$$

It is along this distance s that the force required to compact snow varies. If the snow mass is homogeneous, the force vs sinkage relationship along the curved sinkage path should be very similar to that produced by sinkage along a vertical or straight line path. Thus, the force to compact snow along the curved path will vary with distance s approximately as shown in Figure 3. The point labeled z_e now correlates with s_{max} and still indicates the point at which critical density is reached and mass movement begins. Force at each point along the s axis (previously z axis) not only varies in magnitude but has a different direction as well. In order to treat this situation mathematically it is required that an expression for force vs sinkage be available.

Polynomial curve fitting techniques were performed using existing force vs sinkage records generated with a circular plate-sinkage device. It was found that a fourth-order polynomial expression can be used to adequately describe the force vs sinkage relationship. The force vs sinkage expression can then be written as

$$F = a_1 s^4 + a_2 s^3 + a_3 s^2 + a_4 s + a_5 \tag{23}$$

where F is the force on plate and a_1, a_2 ••• a_5 are constants determined from a polynomial curve fitting routine.

Substituting s_{max} from eq 22 into eq 23 to get F in terms of θ yields

$$F = a_1(-4r \cos \frac{\theta}{2} + C)^4 + a_2 (-4r \cos \frac{\theta}{2} + C)^3$$
$$+ a_3(-4r \cos \frac{\theta}{2} + C)^2 + a_4(-4r \cos \frac{\theta}{2} + C) + a_5 \quad (24)$$

where $C = 4r \cos \theta_A/2$.

This expression describes the magnitude of the force against a point on the tire after the tire has rotated θ radians since the point initially contacted the snow.

The direction of the compaction force can be found by differentiating eq 19 to find the slope of the sinkage path at a given θ:

$$\frac{dy}{dx} = \frac{\sin\theta}{1-\cos\theta} \quad (25)$$

The direction of the unit tangent vector **t** at any point on the sinkage path can be expressed as

$$\mathbf{t} = \frac{\mathbf{i} + \frac{\sin\theta}{1-\cos\theta}\ \mathbf{j}}{\left[1 + \frac{\sin^2\theta}{(1-\cos^2\theta)}\right]^{1/2}} \quad (26)$$

where **i** is the unit vector in x direction (parallel to the ground surface) and **j** the unit vector in y direction (perpendicular to **i**).

The force required for compaction can now be expressed as a vector ($\overline{F}$) for any point along the trochoid describing the sinkage path from

$$\overline{F} = F\ \mathbf{t}\ . \quad (27)$$

Observing the tire/snow contact surface, it is apparent that each point on the tire surface is at a different location along its sinkage path. Points near the rut bottom have traveled through most of their sinkage path. The points on the tire surface that are still near the top of the snow have only begun travel along their sinkage path. Thus, each point on the tire is experiencing a different resisting force. The sum of all the points along the tire surface makes up one complete sinkage path. If the compaction forces are summed along one complete sinkage path, the result will be the total force resisting motion of the tire. This can be accomplished by integrating force $\overline{F}$ over the distance from $\theta = \theta_A$ (entry angle) to $\theta = 2\pi$, or along a complete sinkage path. Mathematically, the total force resisting wheel motion $\overline{F}_T$ is

$$\bar{F}_T = \int_{\theta_A}^{2\pi} \bar{F}\, ds \tag{28}$$

where ds = 2r sin θ/2 dθ (from eq 22).

Combining eq 24, 26 and 27 into eq 28 and simplifying yields

$$\bar{F}_T = \int_{\theta_A}^{2\pi} (a_1K^4 + a_2K^3 + a_3K^2 + a_4K + a_5)\ 2r \sin^2 \frac{\theta}{2}\ d\theta\ \mathbf{i}$$
$$+ \int_{\theta_A}^{2\pi} (a_1K^4 + a_2K^3 + a_3K^2 + a_4K + a_5)\ 2r \sin \frac{\theta}{2} \cos \frac{\theta}{2}\ d\theta\ \mathbf{j} \tag{29}$$

where K = −4r cos θ/2 + C.

Completing the integration, the i and j components become

$$\bar{F}_{T\,i} = \Bigg[\frac{X_1}{3} \cos^3 \frac{\theta}{2} \sin^3 \frac{\theta}{2} + \frac{2X_2}{5} \cos^2 \frac{\theta}{2} \sin^3 \frac{\theta}{2} + \left(\frac{X_3}{2} + \frac{X_1}{4}\right) \cos \frac{\theta}{2} \sin^3 \frac{\theta}{2} + \left(\frac{4X_2}{15} + \frac{2X_4}{3}\right) \sin^3 \frac{\theta}{2}$$
$$- \left(\frac{X_1}{16} + \frac{X_3}{8} + \frac{X_5}{2}\right) \sin \theta + \left(\frac{X_1}{16} + \frac{X_3}{8} + \frac{X_5}{2}\right)\theta \Bigg]_{\theta_A}^{2\pi} \tag{30}$$

$$\bar{F}_{T\,j} = -\Bigg[\frac{X_1}{3} \cos^6 \frac{\theta}{2} + \frac{2X_2}{5} \cos^5 \frac{\theta}{2} + \frac{X_3}{2} \cos^4 \frac{\theta}{2} + \frac{2X_4}{3} \cos^3 \frac{\theta}{2} + X_5 \cos^2 \frac{\theta}{2}\Bigg]_{\theta_A}^{2\pi} \tag{31}$$

where
$X_1 = a_1 A^4 B$
$X_2 = (4a_1A^3C + a_2A^3)B$
$X_3 = (6a_1A^2C^2 + 3a_2A^2C + a_3A^2)B$
$X_4 = (4a_1AC^3 + 3a_2AC^2 + 2a_3AC + a_4A)B$
$X_5 = (a_1C^4 + a_2C^3 + a_3C^2 + a_4C + a_5)B$
$A = -4r$
$B = 2r$

The i component of this force (parallel to the snow-ground surface) can be related to the amount of resistance that must be overcome by the driving wheels in order to advance. The horizontal force obtained from eq 30 is acting over the area of the plate used to generate the force vs sinkage curve from which the polynomial was acquired. By dividing this force by the area of the plate it becomes a pressure term that is independent of area. The area of the tire which is of interest for motion resistance calculations is the curved portion of the tire/snow contact surface. Multiplication of the horizontal pressure component by this contact area then yields motion resistance R (for one wheel), or

$$R = \bar{F}_{T\,i} \frac{br(2\pi - \theta_A)}{A_p} . \qquad (32)$$

MODIFIED PLATE TEST

The common plate-sinkage test compacts a volume of snow equal to the product of the plate area and the snow depth. Following compaction, the final volume of snow is still of the same diameter as the plate but the height has been reduced by z, the sinkage. In comparison, a tire operating in the same snow and at the same sinkage (same final volume of snow), produces compaction along a longer sinkage path than the plate. The initial height of snow compacted by the tire is s + (h - z).

If an additional layer of snow of thickness (s - z) were placed on top of the original snow surface (Fig. 6) and a plate test performed to a sinkage of s (measured from the new snow surface), the resulting force vs sinkage curve would be identical to that produced by any point on the tire surface if coaxiality of force and deformation vectors is assumed. This curve can be used to obtain the coefficients needed in eq 23. The energy found from this modified plate test (area under the force vs sinkage curve) must equal the total amount of energy required to advance the tire. This can also be called the total energy of compaction and includes both the vertical and horizontal sinkage components.

During compaction with a tire, the snow is vertically translated for a distance equal to z, the compaction which takes place during a standard

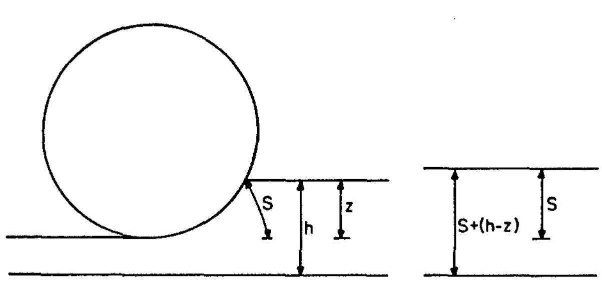


Figure 6. Comparison of sinkage paths during compaction by a plate and a wheel.

plate-sinkage test. The energy found from this test can be equated to the vertical component of compaction.

If the total energy of compaction along the curved sinkage path (from the modified plate test) and the vertical component (from a standard plate test) are known, the horizontal component (the energy required to translate the snow horizontally) can be computed. This energy can also be considered as the sum of the horizontal force components acting on the tire/snow contact surface. Thus, vehicle motion resistance for one wheel (per unit of distance traveled) can be found from plate tests in the same snow from

$$R = \frac{br(2\pi - \theta_A)}{A_p} (E_m^2 - E_s^2)^{\frac{1}{2}} \tag{33}$$

where E_m is the energy (or sum of forces) determined from a modified plate test, and E_s the energy (or sum of forces) determined from a standard plate test.

Although this discussion has chosen to represent the force vs sinkage relationship for snow by a fourth-order polynomial, the same process of breaking down snow compaction into vertical and horizontal components can be accomplished with other mathematical relationships. In some instances, logarithmic, hyperbolic or exponential curves may best fit the data produced by a modified plate test. By redefining F (as a function of sinkage s) in eq 23, vertical and horizontal force components can be computed from eq 27 and 28.

Conditions of positive and negative slip can also be taken into account by not equating a and r in eq 19. This leads to the following expressions for distance along the curved sinkage path and the unit tangent vector:

$$s = \int_{\theta_A}^{2\pi} (a^2 + r^2 - 2\ ar \cos\theta)^{1/2}\, d\theta \tag{34}$$

$$\mathbf{t} = \frac{\mathbf{i} + \frac{r \sin\theta}{a - r\cos\theta}\mathbf{j}}{\left[1 + \left(\frac{r \sin\theta}{a - r\cos\theta}\right)^2\right]^{1/2}}. \tag{35}$$

Substituting these expressions into eq 23 and 27, and completing the integration in eq 28, yields values for the vertical and horizontal components of compaction.

APPLICATION

A simple test was performed to get an idea of the shape of the deformation area ahead of a tire traveling in snow. Several closely spaced vertical holes were punched in a virgin snow underlain by concrete. The holes were each dusted with colored chalk powder to coat the sides of the hole. Approximately half of the holes were then overrun by a tire. A vertical section sliced through the holes revealed the deformed area (Fig. 7). It is readily apparent that a great deal of horizontal displacement has taken place. It can also be seen that the basic outline of the deformation bulb is as depicted in Figure 4b.

The opportunity to validate the concept of the assumed coaxiality of stress and deformation vectors has not yet presented itself. It is concluded, from theoretical concepts (Richmond and Blaisdell 1980) and observed tests (both plates and tires in snow), that this concept is sound. Field and laboratory tests must be performed, however, to assess the accuracy of dividing the energy of compaction into vertical and horizontal components as shown in eq 30 and 31. These tests must also be used to determine the accuracy of eq 33 and the modified plate test concept.

In its present form, vehicle motion resistance can be predicted for a given snow by performing the modified plate test described above in the snow and determining the coefficients of the best fit fourth-order

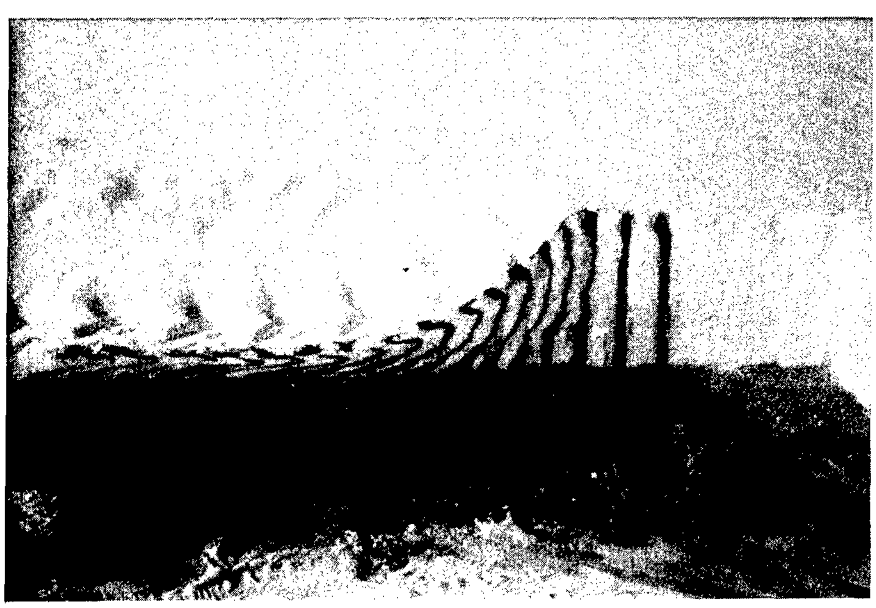


Figure 7. Chalk dust holes depicting deformation area and compaction.

polynomial. These coefficients and the vehicle sinkage can then be used with eq 30 and 32 to predict R.

An alternate approach (which should yield approximately the same value) is to run both a modified and standard plate test in snow and use their force vs sinkage curves with eq 33 to predict R.

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