







# Application of Adaptive Techniques to Problems in Control and Communication

Annual Report for period October 1, 1980 ~ September 30, 1981

K. S. Narendra

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### Application of Adaptive Techniques to Problems in Control and Communication

Annual Report for period Oct. 1, 1980 - Sept. 30, 1981 of work done under Navy Contract N00014-76-C-0017

Principal Investigator: Professor K. S. Narendra

#### Yale University

I. <u>Introduction</u>: The global stability of adaptive control schemes for deterministic single-input single-output systems, which had remained an open problem for two decades, was resolved in 1979 by several groups working independently in different parts of the world. Soon after that it was realized that the various schemes, as well as the assumptions that have to be made in each case about the plant, are all equivalent. These assumptions include knowledge of

- (i) the exact relative degree n\* of the plant transfer function
- (ii) an upper bound n on the order of the plant
- and (iii) the sign of the high frequency gain k, of the plant transfer function, whose zeros lie in the open left half plane.

(1)

(i) and (iii) are found to be particularly hard to establish in practice. Hence it was decided at Yale in 1980 that the control problem should be reformulated with more realistic objectives, allowing less restrictive assumptions to be made regarding the plant. For example, in many situations, where the error between plant output and model output cannot be made to tend to zero asymptotically, a reasonable objective would be to assure the boundedness of the output error as well as all the signals in the adaptive loop. Such considerations eventually led to the study of bounded error adaptive control problems. Plants with nonlinearities, output disturbances and timevarying parameters fall into this category.

The work that has been in progress during the past eighteen months at Yale is concerned with the development of procedures for stable identification and control of systems with various degrees of uncertainty. Direct and indirect control techniques for systems with many control inputs and parameters are being investigated

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and methods of combining different techniques to obtain improved performance in practical applications are being examined.

II. Adaptive Control: Theory and Practice:

Cont

On May 27-29 the Second Workshop on Applications of Adaptive Systems Theory was held at Yale and was attended by leading researchers and practicing engineers in this country and abroad. From the discussions at the Workshop it became clear that adaptive techniques are finding application in a wide variety of problems in industry where adequate prior information regarding the plant is not available. It is also evident that many of the adaptive schemes which perform satisfactorily in practice (with bounded error) do not satisfy all the theoretical conditions stated earlier for global stability. Hence modifications of existing schemes will be needed before truly viable design procedures based on adaptive control theory emerge.

The successful practical implementation of adaptive algorithms requires further investigation of many theoretical areas. These include

- (i) the effect of observation noise or output disturbance on the adaptive system
- (11) speed of convergence of the schemes
- (iii) the extension of known results to multivariable systems
- (iv) the effect of nonlinearities and time-varying parameters in the plant
- and (v) the effect of reduced order models in identification and control.

Most of the work done during the period 1980-1981 was concerned with these topics and significant new results were obtained in (i) and (iii). Some of the salient aspects of these adaptive control problems as well as others currently under investigation are outlined below:

(a) Reduced Order Models:

In our opinion, the single most important problem in the area of adaptive control at this time is the problem of the reduced order model of the plant. Its ramifications

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are felt in almost all the major open questions in the field and its resolution will have a significant impact on both theory and applications. In many practical systems it is found that the overall performance is critically dependent only on a few parameters. Identification of these parameters followed by control often leads to acceptable performance. However at present no general theory exists which explains the observed behavior. Work at Yale during the coming years will continue to emphasize this important, yet unresolved problem.

### (b) Speed of Convergence:

The speed of convergence of adaptive algorithms is of practical as well as theoretical interest. In many applications, feasibility of adaptive control is found to depend critically on this factor. It is well known that in the disturbance free case the output error tends to zero asymptotically while the parameter errors tend to zero only when the input is sufficiently rich. "Arbitrarily fast" convergence of identification parameters by increasing the adaptive gain has also been reported in the literature. However extensive theoretical and simulation studies have revealed that the speed of convergence of adaptive observers is far from being completely resolved. In this context work is currently underway comparing well known least-squares methods to the rapidly converging algorithms of Westphal and Kim and Gourishankar.

# (c) "Sufficiently Rich" Inputs:

It is now clear that the assumption of "richness" of the control input will assure the robustness of most of the adaptive algorithms for identification and control, since it is precisely such a condition that assures the exponential (or uniform asymptotic) stability when no disturbance is present. Unfortunately such desirable input properties cannot be assured in many practical cases. Moreover, even when the reference input is sufficiently rich it is not easy to show that the input to the plant satisfies the necessary conditions for richness. This is one of the problems currently being considered.

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#### III. Work Accomplished:

During the period 1980-1981 significant advances were made at Yale in the following four different aspects of the adaptive control problem. The results were presented at various national and international conferences and as invited lectures in several universities in this country and abroad.

- (a) Generation of new error models
- (b) Adaptive control in the presence of bounded disturbance
- (c) Extension of known results to multivariable systems

and (d) Analysis of quadratic differential equations which arise in adaptive control

#### (a) Error Models with External Disturbance:

During the past several years considerable emphasis in our work has been placed on error models. The following three models have been extensively studied.

(i) 
$$\phi^{T} u = e_{1}$$
 (ii)  $W_{\underline{M1}}(s) \phi^{T} u = e$  (iii)  $W_{\underline{M}}(s) \phi^{T} u = e_{1}$ 

where u is an mxl input vector,  $\phi$  is an mxl parameter error vector, e is an nxl state error vector,  $e_1$  is the output error,  $W_{M1}(s)$  is a vector of stable transfer functions and  $W_M(s)$  is a strictly positive real transfer function. The behavior of these models for bounded and unbounded inputs  $u(\cdot)$  is known for both continuous and discrete systems. Hence, most identification and control problems can be considered to be solved once the error equations are expressed in one of these standard forms.

When the plant is nonlinear, has time-varying parameters, or when observation noise is present, it is no longer possible to control the output of the plant to approach the output of the model asymptotically. The same problem arises when the order of the model used to identify the plant is lower than that of the plant. In all these cases the only recourse is to settle for a bounded output error as well as bounded signals within the adaptive loop. The design of efficient and practically

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feasible adaptive controllers for these cases represents perhaps the most important aspect of our research effort.

In this context a new error model has been developed and analyzed extensively. In its simplest form it can be described by the equation

$$\phi^{T} u + v = e_{1}$$

where v is a disturbance at the output. The presence of v may be attributed to any one of the four causes discussed earlier. In the control problem, v may be unbounded in general and it is this fact that substantially complicates the analysis. Recent efforts at Yale have treated the problem when v is uniformly bounded. If the input u is sufficiently rich, the homogeneous equation is uniformly asymptotically stable and hence a bounded disturbance produces a bounded error. However, it has been demonstrated that if u is not sufficiently rich the disturbance may result in the magnitude of the parameter vector increasing without bound. Therefore, when an external disturbance is present at the outputs, special efforts have to be made to assure the boundedness of the state vector e as well as the parameter error vector  $\phi$ .

These results are discussed in Yale S&IS Report No. 8005, Dec. 1980 and were presented at the 1980 CDC.

# (b) Nonlinear Adaptive Law for System with Bounded Disturbance:

Recently two nonlinear adaptive laws for the MRAC problem in the presence of bounded disturbances were developed and are briefly described below:

(1) Use of Dead-Zone in Adaptive Law:

A principal difficulty in the adaptive control problem arises from the fact that the adjustment of the parameter error vector (given by  $\dot{\phi}(t)$ ) is known to be in the "right" direction only when the output e(t) is large. Hence, it is felt that if a bound on the disturbance is known and  $\phi$  is adjusted only when the output error exceeds a computed threshold, the signals and the parameters of the system would be bounded. Such an adjustment corresponds to an adaptive law with a dead-zone

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and in [3] it is shown that the foregoing reasoning is indeed correct. These results were presented as an invited paper at the Johns Hopkins Conference on Information Sciences and Systems and will appear in the IEEE Transactions on Automatic Control in 1982.

(2) Bounds on Parameters:

In [2] an alternate approach is presented in which the structure of the controller used is identical to that used in the disturbance free case. This assures the existence of a constant control parameter vector  $\theta^*$  such that when  $\theta(t) \equiv \theta^*$ the transfer function of the plant together with the controller matches exactly that of the reference model. As part of the prior information, it is assumed that  $\theta^*$  has a norm less than a specified value  $\|\theta\|_{max}$  and that the disturbance is bounded. Since  $\|\theta\|_{max}$  is known, the search for  $\theta^*$  is confined to a sphere S of radius  $\|\theta\|_{max}$  in parameter space. When  $\theta(t)$  lies in the interior of S the adaptive law is identical to that in the disturbance free case. The law is modified in a nonlinear fashion only when  $\theta$  lies either on the boundary of S or outside it. In [2] it is demonstrated that such a scheme results in the integer of all signals in the adaptive system. These results will appear in the IEEE Transactions on Automatic Control.

A brief comparison of the two schemes described above is of interest. The prior information assumed as well as the adaptive laws used are different in the two cases. While the adaptive law in the first case is modified when the output error is small the adaptive law in the second case is modified when the norm of the control parameter exceeds a prespecified value. In the first scheme the parameter tends to a constant value while in the second case the controller is time-varying. However the latter retains the potential of obtaining zero output error in the limit when no external disturbance is present.

(c) Extension to Multivariable Systems:

Considerable work has been done to extend the results available for single variable systems to the multivariable case. In particular, the multivariable analogs

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of conditions (i)-(iv) for single variable systems have been derived. Condition (i) is equivalent to knowing the Hermite form and condition (ii) to knowing the observability index of the plant transfer matrix; condition (iii) reduces to a knowledge of the positive definiteness of the symmetric part of a gain matrix and condition (iv) assures that the "zeros" of the plant transfer matrix (suitably defined) lie in the open left half of the complex plane. It is generally found that assumptions (i) and (iii) are very rarely satisfied in practice and some authors have contended that the practical control of general multivariable systems may be impossible to achieve.

The position taken by our research group is that in general, considerable prior information regarding the plant transfer matrix is available, hence, adaptive control of multivariable systems is practically realizable. Recent work in this area deals with the prior information needed to decouple the plant transfer matrix using pre-filters. This, in turn, allows the straightforward extension of singlevariable methods to multivariable problems.

The above results were presented at the 1981 CDC and will be contained in a forthcoming report.

#### (d) Quadratic Differential Equations which Arise in Adaptive Control:

Quadratic differential equations arise naturally in adaptive control where control parameters become state variables. Further, the special class of bilinear systems

# x = Ax + uDx + bu

which has received considerable attention in the control literature in recent years, becomes a quadratic differential equation when linear state feedback is used. The stability of quadratic differential equations is therefore an important consideration in adaptive control. Work has been in progress in this area for over two years and in 1981 theorems giving necessary and sufficient conditions for

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stability and asymptotic stability in the large of second order quadratic differential equations was developed. This was published as S&IS Report No. 8004 in December 1980 [4] and will appear in the IEEE Transactions on Automatic Control in August 1982.

The work reported in the above paper was extended to the stabilizability of bilinear systems and these results are contained in S&IS Report No. 8109, November 1981 [5].

IV. Publications: [October 1, 1980 - Sept. 30, 1981]

 Benjamin B. Peterson and Kumpati S. Narendra "Bounded Error Adaptive Control, Part I" S&IS Report No. 8005, December 1980.

- Gerhard Kreisselmeier and Kumpati S. Narendra
   "Stable Model Reference Adaptive Control in the Presence of Bounded Disturbances"
   S&IS Report No. 8103, March 1981 (to appear in IEEE Transactions on Automatic Control)
- 3. Kumpati S. Narendra and Benjamin B. Peterson "Bounded Error Adaptive Control, Part II"

S&IS Report No. 8106, April 1981 (to appear in IEEE Transactions on Automatic Control)

4. Daniel E. Koditschek and Kumpati S. Narendra

"The Stability of Second Order Quadratic Differential Equations, Part III" S&IS Report No. 8004, Dec. 1980 (to appear in IEEE Transactions on Automatic Control).

5. Daniel E. Koditschek and Kumpati S. Narendra

"Stabilizability of Second Order Bilinear Systems"

S&IS Report No. 8109, Nov. 1981 (sent to IEEE Transactions for publication).

V. Related Activities of Principal Investigator:

Invited papers were presented at the 1980 CDC, The Johns Hopkins Conference on

Information Sciences and Systems, and the 1981 CDC.

Invited lectures on adaptive control were given at the University of Massachusetts,

Stall vity of New York at Buffalo, Japan, China, Singapore and Aberdeen,

Scotland. Nanking Institute of Technology invited the Principal Investigator to give a series of ten lectures on adaptive control which was attended by approximately one hundred faculty members and students.

#### VI. Personnel:

Principal InvestigatorProfessor K. S. NarendraSecretaryMrs. Jean Gemmell (1/3 time)Graduate StudentsLt. Cmdr. B. B. PetersonA. AnnaswamyR. P. SinghI. H. Khalifa(Only A. Annaswamy and R. P. Singh were supported on the Navy grant)Visiting FacultyDr. G. Kreisselmeieron leave from DFVLR<br/>Oberpfaffenhofen, West Germany

January 1 - June 30, 1981

#### VII. Second Workshop on Applications of Adaptive Systems Theory:

The first Workshop on Applications of Adaptive Systems Theory was held at Yale University on August 1979 with the primary objective of bringing together researchers and practicing engineers. In view of the growing interest in the field and the success of the first Workshop, a second one was organized in May 1981. The Workshop stimulated vigorous exchange of ideas between theorists and practicing engineers and presentations reflected a broad range of interests in adaptive systems. Eight invited papers provided an overview of recent theoretical developments while twenty-two treated specific applications in signal processing, ship dynamics, process control and aircraft and spacecraft.

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We are encouraged by the very favorable responses from the adaptive control community to the two workshops. We feel that this reception together with the high level of current activity in the field forms the basis for hosting future workshops. We are tentatively planning to hold such workshops every other year as long as the interest and activity in the field continue.

Publication: Proceedings of the Workshop on Applications of Adaptive Systems Theory.

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