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by

D. G. King-Hele
Doreen M. C. Walker

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Procurement Executive, Ministry of Defence
Farnborough, Hants

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SUMMARY

The Earth's gravitational potential is usually expressed as an infinite series of tesseral harmonics, and it is possible to evaluate 'lumped harmonics' of a particular order m by analyses of resonant satellite orbits - orbits with tracks over the Earth that repeat after m revolutions. In this paper we review results on 30th-order harmonics from analyses of 15th-order resonance, and results on 29th- and 31st-order harmonics from 29:2 and 31:2 resonance.

The values available for 30th-order lumped harmonics of even degree are numerous enough to allow a solution for individual coefficients of degree up to 40. The best-determined coefficients are those of degree 30, namely

$$10^9\bar{C}_{30,30} = -1.2 \pm 1.1 \qquad 10^9\bar{S}_{30,30} = 9.6 \pm 1.3$$

The standard deviations here are equivalent to 1 cm in geoid height.

For the 29th- and 31st-order harmonics, and for the 30th-order harmonics of odd degree, there are not enough values to determine individual coefficients, but the lumped values from particular satellites can be used for 'resonance testing' of gravity field models, particularly the Goddard Earth Model 10B (up to degree 36) and 10C (for degree greater than 36). The results of applying these tests are mixed. GEM 10B/C emerges well for order 30, with sd about 3×10^{-9}; for order 31, the GEM 10B values are probably good but the GEM 10C values are probably not; for order 29, the test is indecisive.

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1 INTRODUCTION

A satellite orbit experiences $\beta:\alpha$ resonance if the satellite completes β revolutions about the Earth while the Earth spins α times relative to the satellite's orbital plane. The satellite's track over the Earth then repeats after β revolutions in approximately α days. In such a resonance the orbital perturbations caused by harmonics of order β in the geopotential have the same effect on each revolution, so that they build up and can be analysed to determine accurate values of 'lumped harmonics' of order β. The lumped harmonics are linear sums of individual harmonic coefficients, and these individual coefficients can therefore be determined if values of lumped harmonics can be obtained from a number of satellites at different inclinations to the equator.

The 15:1 resonance, or 15th-order resonance, when the track repeats after 15 revolutions in one day, has proved the most useful in practice, because the height (500-600 km) is such that air drag brings the orbit through resonance quite slowly - but not so slowly that a 50-year wait is needed. An orbit that stays near resonance for about a year is ideal, and analysis of such orbits has given values of 15th-order lumped harmonics accurate to 1½% at best[1,2]. Recently, 23 satellites which experienced 15th-order resonance were used[3] to determine individual harmonics of order 15 and degree 15, 16, 17, ..., 35. Also values of individual 14th-order coefficients have been obtained from analysis of 11 resonant orbits[4].

The 2-day resonances, such as 29:2 and 31:2, are much weaker than the 14:1 or 15:1 resonances, but the effects are often detectable. The first such analysis, of 29:2 resonance by Walker[5] in 1976, has been followed by several studies of 29:2 and 31:2 resonance[5-9], though the accuracies achieved have not been better than about 15%.

Any 15th-order resonant orbit which stays near resonance for a year or more is also appreciably affected by harmonics of order 30, and the analysis of the variations yields values of 30th-order lumped harmonics as a by-product. Again their accuracy has not usually been better than about 20%, except for one satellite, where 7% was achieved.

The aim of this paper is to gather together existing results on lumped harmonics of order 29, 30 and 31 and to see whether it is possible to produce tentative solutions for individual coefficients. The 30th-order harmonics prove to be the most satisfactory and are discussed first in section 3, after a brief résumé of the theory in section 2. Sections 4 and 5 are devoted to the 29th and 31st-order coefficients.

2 THEORY

The longitude-dependent part of the geopotential at an exterior point (r,θ,λ) can be written in normalized form[10] as

$$\frac{\mu}{r}\sum_{\ell=2}^{\infty}\sum_{m=1}^{\ell}\left(\frac{R}{r}\right)^{\ell} P_{\ell}^{m}(\cos\theta)\left\{\bar{C}_{\ell m}\cos m\lambda + \bar{S}_{\ell m}\sin m\lambda\right\}N_{\ell m}\ , \qquad (1)$$

where r is the distance from the Earth's centre, θ is co-latitude, λ is longitude (positive to the east), μ is the gravitational constant for the Earth (398600 km^3/s^2)

and R is the Earth's equatorial radius (6378.1 km). The $P_\ell^m(\cos\theta)$ are the associated Legendre functions of order m and degree ℓ, and $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ are the normalized tesseral harmonic coefficients. The normalizing factor $N_{\ell m}$ is given by[10]

$$N_{\ell m}^2 = \frac{2(2\ell+1)(\ell-m)!}{(\ell+m)!} \quad . \tag{2}$$

A satellite experiences $\beta:\alpha$ resonance when $\dot{\Phi} = 0$, where Φ is the resonance angle defined by

$$\Phi = \alpha(\omega + M) + \beta(\Omega - \nu) \quad .$$

Here ω is the argument of perigee, M the mean anomaly, Ω the right ascension of the node, and ν the sidereal angle.

Near resonance the main changes in inclination i due to the relevant $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ take the form of terms in the sine and cosine of Φ, 2Φ, ..., if the orbital eccentricity e is small[11]. The multiplying coefficients of these terms, which we seek to evaluate by analysis of the resonance, are the lumped harmonics, written as $\bar{C}_m^{0,k}$ and $\bar{S}_m^{0,k}$, which are linear sums of individual coefficients $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ having values of ℓ that increase i steps of 2 with $\ell \geqslant m$ and $(\ell - k)$ even. The index k is equal to $\gamma\alpha$ where $\gamma = 1, 2, 3, \ldots$, and the terms with $\gamma = 1$ are usually dominant.

The variation in eccentricity e is controlled mainly by terms in the sine and cosine of $(\Phi - q\omega)$, the terms with $q = \pm 1$ being dominant. Analysis of the variations in eccentricity yields values of lumped harmonics $\bar{C}_m^{q,k}$ and $\bar{S}_m^{q,k}$, where $k = \gamma\alpha - q$: these lumped harmonics are also linear sums of individual coefficients $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$.

The full theory has been given previously and will not be repeated here. Explicit forms for the variations in i and e at 29:2 and 31:2 resonance can be found in Refs 12 and 6. Explicit forms for the 30th-order terms at 15th-order resonance appear in Ref 2.

3 HARMONICS OF ORDER 30

3.1 Form of the even-degree lumped harmonics

The variation of inclination at 15th-order resonance depends primarily on the 15th-order harmonics (the terms with $\gamma = 1$ as defined in section 2), but there is also a contribution from 30th-order terms ($\gamma = 2$), and the main 30th-order term in di/dt is[2]

$$\frac{2n}{\sin i}\left(\frac{R}{a}\right)^{30}(15 - \cos i)\bar{F}_{30,30,14}\left\{\bar{C}_{30}^{0,2}\sin 2\Phi_{15} - \bar{S}_{30}^{0,2}\cos 2\Phi_{15}\right\} \tag{3}$$

where n $(= \dot{M})$ is the mean motion, a is the semi major axis, $\Phi_{15} = \omega + M + 15(\Omega - \nu)$ is the 15th-order resonance angle, and $\bar{F}_{30,30,14}$ is Allan's normalized inclination function[11]. The lumped coefficients are given by

$$\bar{C}_{30}^{0,2} = \bar{C}_{30,30} + Q_{32}^{0,2}\bar{C}_{32,30} + Q_{34}^{0,2}\bar{C}_{34,30} + \ldots \tag{4}$$

with the same equation for S on replacing C by S throughout. The Q-factors are products of inclination functions $\bar{F}$ and powers of (R/a), the rth Q-factor in equation (4) being given by

$$Q^{0,2}_{30+2r} = \left(-\frac{R^2}{a^2}\right)^r \bar{F}_{30+2r,30,14+r}/\bar{F}_{30,30,14} . \qquad (5)$$

In each analysis of a 15th-order resonant satellite there is a possibility of extracting determinate values for these even-degree lumped 30th-order coefficients, though in practice only the best resonances yield such values. If values of lumped coefficients can be obtained at enough different inclinations, it should be possible to solve equations (4) for the individual harmonics $\bar{C}_{\ell,30}$ and $\bar{S}_{\ell,30}$ (ℓ even).

3.2 Observational values of even-degree lumped harmonics

In practice we have found only seven analyses of 15th-order resonance giving values of 30th-order coefficients that we regard as reliable. They are listed in Table 1 and briefly described below.

The results for *1974-34A*, Intercosmos 11, are from the analysis of 129 weekly US Navy orbits at dates between June 1975 and November 1977 (Ref 2). The analysis yielded excellent values of 15th-order harmonics, but the two 30th-order coefficients are small and scarcely greater than their standard deviation. As the expression (3) shows, however, the effect of the harmonics is best indicated by multiplying by $\bar{F}_{30,30,14}$, which varies greatly with inclination. Table 1 also shows the values after such multiplication and, judged in this light, the sd for 1974-34A emerges as the second best among the seven satellites; the values are of the same order as the sd only because the values happen to be small.

The results for *1963-24B*, Tiros 7 rocket, are from analysis of 112 weekly US Navy orbits at dates between November 1975 and December 1977 (Ref 2). Again the values of the 15th-order coefficients were excellent, but the value for $\bar{C}^{0,2}_{30}$ happens to be small, as is seen from the entry in Table 1 for $\bar{F}_{30,30,14}\bar{C}^{0,2}_{30}$.

The third and fourth satellites, *1970-111A* and *1971-13B*, were in almost identical orbits but exhibited variations in inclination of quite different form[13,14] because resonance occurred at different values of Φ_{15}. The excellent agreement between the values of $\bar{C}^{0,2}_{30}$ and $\bar{S}^{0,2}_{30}$ obtained for the two satellites gives confidence in their reliability.

The fifth satellite is *1967-42A*, Ariel 3, which has served as the standard satellite in Gooding's development of the computer programs THROE and SIMRES used in resonance analysis. The values in Table 1 are from a SIMRES fitting of inclination and eccentricity on 281 orbits at dates between May 1967 and August 1969, with ten coefficients[15].

The sixth satellite is *1971-54A*, for which 269 weekly US Navy orbits over five years were analysed[1]. The results were excellent and give the most accurate values of lumped 30th-order coefficients yet obtained.

The final orbit, that of *1964-52B*, Nimbus 1 rocket, was determined by Hiller[16] at 25 epochs between March and September 1970, from 2000 observations including many Hewitt camera plates. Although the individual orbits are more accurate than those of the other six satellites, only 41 orbits were used in the resonance analysis, so the reliability of the values of the lumped coefficients cannot be guaranteed.

Table 1

Values of even-degree lumped harmonics $(\bar{C},\bar{S})^{0,2}_{30}$ for the seven satellites

No.	Satellite	i (deg)	$10^9\bar{C}^{0,2}_{30}$	$10^9\bar{S}^{0,2}_{30}$	$\bar{F}_{30,30,14}$	$10^9\bar{F}_{30,30,14}\bar{C}^{0,2}_{30}$	$10^9\bar{F}_{30,30,14}\bar{S}^{0,2}_{30}$
1	1974-34A	50.64	597 ± 558	679 ± 651	0.000952	0.57 ± 0.53	0.65 ± 0.62
2	1963-24B	58.20	46 ± 106	253 ± 88	0.01176	0.54 ± 1.25	2.98 ± 1.03
3	1970-111A	74.00	19.2 ± 4.9	4.1 ± 4.4	0.2579	4.95 ± 1.26	1.06 ± 1.13
4	1971-13B	74.05	27.1 ± 5.5	6.0 ± 3.3	0.2594	7.03 ± 1.43	1.56 ± 0.86
5	1967-42A	80.17	-9.1 ± 4.6	-5.0 ± 5.5*	0.4340	-3.95 ± 2.00	-2.17 ± 2.39*
6	1971-54A	90.21	-8.2 ± 0.6	11.1 ± 0.8	0.4755	-3.90 ± 0.29	5.28 ± 0.38
7	1964-52B	98.68	22.8 ± 7.9	38.0 ± 10.3*	0.2502	5.70 ± 1.98	9.51 ± 2.58*

* These standard deviations were doubled in the solutions

3.3 The equations to be solved

We have seven equations of the form (4) for C, and seven for S. The values of the Q coefficients for the seven satellites, evaluated with Gooding's computer program PROF, are given in Table 2 on page 7.

In addition, following the procedure that has proved successful in the past, we add constraint equations of the form

$$\left.\begin{aligned} \bar{C}_{\ell,30} &= 0 \pm 10^{-5}/\ell^2 \\ \bar{S}_{\ell,30} &= 0 \pm 10^{-5}/\ell^2 \end{aligned}\right\} \qquad (6)$$

where $\ell = 30, 32, 34, \ldots$ These equations express the expectation[10] that the order of magnitude of the individual coefficients of degree ℓ is $10^{-5}/\ell^2$ for $30 \leqslant \ell \leqslant 50$, as is confirmed in a general way by the Goddard Earth Model 10C (Ref 17).

Thus we solve $7 + N$ equations by least squares for N harmonics, the optimum value of N being selected empirically. As in our recent solution for 15th-order harmonics[3], we are prepared to consider a relaxation of some standard deviations, if necessary.

3.4 The solutions for individual coefficients of even degree

When the equations were solved, there were no problems with the C-values, but two of the S-values, for 1967-42A and 1964-52B, had large weighted residuals, for all values of N up to 6. The standard deviations for these two satellites were therefore doubled, as indicated in Table 1.

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Table 2

Values of $Q_{32}^{0,2}$, $Q_{34}^{0,2}$, ..., $Q_{42}^{0,2}$ for the seven satellites

Satellite	$Q_{32}^{0,2}$	$Q_{34}^{0,2}$	$Q_{36}^{0,2}$	$Q_{38}^{0,2}$	$Q_{40}^{0,2}$	$Q_{42}^{0,2}$
1974-34A	-11.9	52.9	-131.8	200.3	-171.5	34.5
1963-24B	-7.61	19.01	-20.11	2.71	11.18	-2.92
1970-111A	-1.121	-0.806	-0.128	0.319	0.433	0.314
1971-13B	-1.107	-0.807	-0.140	0.308	0.429	0.317
1967-42A	0.155	-0.161	-0.281	-0.295	-0.252	-0.184
1971-54A	0.430	0.213	0.100	0.036	-0.000	-0.020
1964-52B	-1.113	-0.847	-0.231	0.127	0.313	0.333

When the 7 + N equations were solved for N coefficients, for $2 \leqslant N \leqslant 6$, the values of the measure of fit ε were as follows:

N	2	3	4	5	6
C equations	1.08	1.05	1.04	0.92	0.89
S equations	1.50	0.96	0.95	0.94	0.92

As usual, ε^2 is the sum of squares of the weighted residuals, divided by the number of degrees of freedom; and the weighted residual is the residual for each lumped coefficient divided by the standard deviation for that coefficient, as given in Table 1.

Thus it appears that any value of N between 3 and 6 will give a 'satisfactory' solution with ε near 1. However, Table 2 shows that 1974-34A, the second most accurate of the satellites, has its largest Q coefficients for $\ell = 38$ and $\ell = 40$. So the 6-coefficient solution (ℓ = 30, 32, ..., 40) is required to ensure that 1974-34A plays its proper role.

Table 3 gives the 6- and 4-coefficient solutions, together with the values from the Goddard Earth Model 10B (Ref 18), which only goes to degree 36 and so should be compared with the 4-coefficient solution, and also values for $\ell = 38$ and $\ell = 40$ from GEM 10C (in brackets)

Table 3

Values of $\bar{C}_{\ell,30}$ and $\bar{S}_{\ell,30}$ given by our 6- and 4-coefficient solutions and by GEM 10B (and GEM 10C)

ℓ	$10^9\bar{C}_{\ell,30}$			$10^9\bar{S}_{\ell,30}$		
	6-coefficient	4-coefficient	GEM 10B (GEM 10C)	6-coefficient	4-coefficient	GEM 10B (GEM 10C)
30	-1.2 ± 1.1	-0.4 ± 1.0	-5.2	9.6 ± 1.3	10.1 ± 0.9	11.1
32	-14.5 ± 4.3	-16.2 ± 4.1	-0.6	0.1 ± 4.4	-1.4 ± 3.6	-0.2
34	-4.7 ± 4.4	-3.6 ± 4.8	-11.9	7.2 ± 4.6	7.9 ± 4.4	1.2
36	0.7 ± 4.4	-1.4 ± 4.2	-3.9	-2.4 ± 5.1	-2.3 ± 4.3	-0.9
38	5.7 ± 3.6		(0.6)	2.0 ± 4.5		(2.6)
40	2.5 ± 3.7		(1.5)	2.4 ± 4.1		(-3.0)

Table 3 shows that the solutions are stable: going from the 4- to the 6-coefficient solution does not change any value by as much as its sd. This is largely because 1974-34A dominates for $\ell = 38$ and 40, and, since it has small lumped coefficients, 1974-34A tends to produce small 38th- and 40th-degree coefficients.

The 15th-order coefficients in GEM 10B were judged[3] to be probably accurate to about 3×10^{-9}. If the same figure is used for the 30th-order coefficients of GEM 10B, Table 3 shows that the mean difference between a GEM 10B value and the corresponding value in our 4-coefficient solution is $0.8 \times$ (the sum of the sd). Since the GEM 10B values and ours are believed to be completely independent, this agreement suggests that a standard deviation near 3×10^{-9} may be appropriate for the 30th-order GEM 10B coefficients. The four GEM 10C values given in Table 3 are also consistent with an assumed accuracy of 3×10^{-9}, though a comparison of only 4 out of 32000 coefficients can scarcely be regarded as a useful test.

However, the main conclusion from Table 3 must be that the values in our solution are poorly determined, except for $\ell = 30$. This poor accuracy is to be expected when using only seven satellites, of which three (1970-111A, 1971-13B and 1964-52B) have very similar values for their Q-factors (see Table 2).

The weighted residuals in the 6-coefficient solution, for the seven satellite equations and the six constraint equations, are given in Table 4. It will be seen that the largest weighted residual is 1.60.

Table 4

Weighted residuals in the 13 equations used in the 6-coefficient solution

Satellite equations			Constraint equations		
Satellite	$\bar{C}_{30}^{0,2}$	$\bar{S}_{30}^{0,2}$	Degree ℓ	$\bar{C}_{\ell,30}$	$\bar{S}_{\ell,30}$
1974-34A	0.10	0.01	30	0.11	-0.87
1963-24B	-0.02	0.30	32	1.48	-0.01
1970-111A	-0.50	-0.36	34	0.54	-0.83
1971-13B	1.04	0.09	36	-0.10	0.31
1967-42A	-0.84	-1.18	38	-0.83	-0.28
1971-54A	0.03	0.13	40	-0.40	-0.40
1964-52B	0.33	1.60			

3.5 The variation of even-degree lumped harmonics with inclination

Our basic data are the seven values of lumped harmonics, and the most useful comparisons can be made by considering their variation with inclination after multiplication by $\bar{F}_{30,30,14}$. Fig 1 shows the values of the lumped harmonics with their sd, together with the variations given by the 6-coefficient solution (unbroken line), the 4-coefficient (dot-dash line) and GEM 10B (broken line). The values of $\bar{S}_{30}^{0,2}$ for 1967-42A and 1964-52B are shown in Fig 1 with their standard deviations doubled, and it is clear that the relaxation was justified.

It is worth noting that the curves of Fig 1 are nearly symmetrical about $i = 86^{o}$, so that the 98^{o} satellite (1964-52B) competes with the two at 74^{o}, while the 80^{o} satellite (1967-42A) competes with that at 90^{o}.

Fig 1 shows that our 6-coefficient solution is only slightly better than the 4-coefficient solution, and, if this were the only criterion, the latter would be recommended. However, as already explained, six coefficients are required to take account of 1974-34A, and, since they do not cause instability, the 6-coefficient solution is to be preferred.

Another notable feature of Fig 1 is the good performance of GEM 10B: although there are some discrepancies, GEM 10B generally gives a variation that can fairly be called quite realistic.

3.6 Review of values of $\bar{C}_{30,30}$ and $\bar{S}_{30,30}$

Comparisons with previous evaluations of individual coefficients are not likely to be very illuminating, because of the general inaccuracy. But comparisons *are* worthwhile for $\bar{C}_{30,30}$ and $\bar{S}_{30,30}$, which are well determined here.

Table 5 lists values of these coefficients from various sources. The first, from the European Gravity Field Model GRIM 2 (Ref 19), would not be expected to be accurate, because it is the final coefficient in a 30×30 array and is likely to carry a burden of error from the undetermined coefficients of higher order and degree. The second entry is from Rapp's analysis of $5^{o} \times 5^{o}$ terrestrial gravi _ measurements[20], which goes to order and degree 52 and should therefore avoid this source of error. The same applies to the next entry, GEM 10B, which is complete to order and degree 36. (GEM 10C is not listed because it is the same as GEM 10B to order and degree 36. The gravity field derived[22] by Gaposchkin (1980) goes to degree and order 30, but has no (30,30) coefficient.) All these values are independent of ours.

The values obtained by Kostelecký and Klokočník[21], however, utilize some of our resonances and are therefore not independent of our values. Kostelecký and Klokočník took pairs of lumped coefficients from ten satellites, of which eight pairs were obtained by us. Of these eight, there are three for which we no longer regard the 30th-order harmonics as being reliably separated from the 15th-order, as explained in Ref 3. These three are Saturn SA5, 1964-05A ($i = 31^{o}$), which was omitted even from our 15th-order analysis; Ariel 1, 1962-15A ($i = 53.8^{o}$), and Cosmos 72, 1965-53B ($i = 56.0^{o}$). Of the other five satellites, we use three here with the same values as Kostelecký and Klokočník; and we use two, 1967-42A and 1971-54A, with new values. Kostelecký and Klokočník also took two values from their own analyses. We have not used these because they are both at inclinations where we have more accurate values (50.6^{o} and 74.0^{o}). To summarize, Kostelecký and Klokočník use the same values as we do for three satellites, together with three of our values we have now discarded, two of our values we have now revised, and two of their own.

Table 5

Values of $\bar{C}_{30,30}$ and $\bar{S}_{30,30}$

Date	Source, and type of data	$10^9\bar{C}_{30,30}$	$10^9\bar{S}_{30,30}$
1976	GRIM 2 (Ref 19), comprehensive	14.2	5.8
1977	Rapp (Ref 20), terrestrial gravity	1.4 ± 5.2	12.6 ± 4.9
1978	GEM 10B (Ref 18), comprehensive	-5.2 ± 3?	11.1 ± 3?
1979	Kostelecký and Klokočník, Ref 21 resonance, solution 5	-6.0 ± 0.9	11.0 ± 1.8
	Kostelecký and Klokočník, Ref 21 resonance, solution 7	-8.2 ± 5.7	8.4 ± 5.8
1981	Our solution, resonance, 4-coefficient	-0.4 ± 1.0	10.1 ± 0.9
	Our solution, resonance, 6-coefficient	-1.2 ± 1.1	9.6 ± 1.3

How do the values in Table 5 stand up to comparison? The answer is "surprisingly well", when the earliest (GRIM 2) is ignored. If the values from our solutions are accepted as correct, averaged, and given a 'safety factor' of 2 on the sd, we can say that the C and S values are -1 ± 2 and 10 ± 2 respectively. These are consistent with nearly all the previous values and there is remarkable unanimity on the value of $10^9\bar{S}_{30,30}$, all six values being between 8.4 and 12.6.

3.7 Another look at the two rejected satellites

We rejected Ariel 1 and Cosmos 72 in our analysis, because we preferred[3] a fitting of the 15th-order resonance which had no 30th-order terms. In other words, we judged that the 30th-order terms did not contribute usefully to the fitting. However, this does not necessarily mean that they are wrong, and it is of interest to see how they fit.

Ariel 1 gives[23] $10^9\bar{\bar{FC}} = -4.0 \pm 3.4$ and $10^9\bar{\bar{FS}} = 2.2 \pm 3.8$ at $i = 53.8^\circ$, where the values from the unbroken curve of Fig 1 are 0.8 and 1.6 respectively. The 'residuals' are therefore -1.4σ and 0.2σ respectively. Cosmos 72 gives[24] $10^9\bar{\bar{FC}} = -4.1 \pm 2.1$ and $10^9\bar{\bar{FS}} = -1.7 \pm 2.2$ at $i = 56.0^\circ$, where the values from the unbroken curve of Fig 1 are 0.7 and 2.2 respectively. The 'residuals' are therefore -2.3σ and -1.8σ respectively.

Thus our decision to exclude these two satellites seems reasonable, even with the benefit of hindsight. It would be possible to strengthen the S-solutions by including the value from Ariel 1; but this would be capricious, since we had previously[3] rejected the solution in which it occurs.

3.8 Lumped harmonics of odd degree

So far we have only discussed the coefficients of even degree (30, 32, 34, ...), obtained from analysis of inclination. Lumped coefficients of odd degree (31, 33, 35, ...) can in principle be obtained from analysis of eccentricity at 15th-order resonance (explicit forms are given in Table 4 of Ref 2). But in practice the effects of these 30th-order harmonics are small, and only one satellite, 1971-54A, has yielded determinate values of lumped coefficients of odd degree[1].

Fig 2 shows the values of the lumped coefficients $(\bar{C},\bar{S})_{30}^{1,1}$ and $(\bar{C},\bar{S})_{30}^{-1,3}$ determined from 1971-54A and also the variations of these coefficients with inclination given by GEM 10B and GEM 10C. Since coefficients of degree higher than 36 contribute significantly to the lumped coefficients at most inclinations, and GEM 10B does not go beyond degree 36, there are considerable differences between the GEM 10B and GEM 10C curves. Both curves agree fairly well with the S-coefficients from 1971-54A, but the C-coefficients of GEM appear to be too small.

4 HARMONICS OF ORDER 29

4.1 Form of the even-degree lumped harmonics

At the 29:2 resonance the variation of i is given by[12]

$$\frac{di}{dt} = \frac{n}{\sin i}\left(\frac{R}{a}\right)^{30}(29 - 2\cos i)\bar{F}_{30,29,14}\left\{\bar{S}_{29}^{0,2}\sin\Phi_{29} + \bar{C}_{29}^{0,2}\cos\Phi_{29}\right\} + \text{terms in } e, e^2, \ldots, \tag{7}$$

where

$$\Phi_{29} = 2(\omega + M) + 29(\Omega - \nu) \tag{8}$$

and the lumped coefficients are given by

$$\bar{C}_{29}^{0,2} = \bar{C}_{30,29} - \frac{\bar{F}_{32,29,15}}{\bar{F}_{30,29,14}}\left(\frac{R}{a}\right)^2\bar{C}_{32,29} + \frac{\bar{F}_{34,29,16}}{\bar{F}_{30,29,14}}\left(\frac{R}{a}\right)^4\bar{C}_{34,29} + \cdots \tag{9}$$

with the same equation for S on replacing C by S throughout. Equation (9) shows that the variation of inclination depends on the harmonics of even degree.

4.2 Observational values of even-degree lumped harmonics

There are four satellites for which the orbital inclination has been analysed during passage through 29:2 resonance. These are discussed in turn below, in order of increasing inclination.

The phenomenon of 29:2 resonance was first recognised and analysed for Ariel 1, *1962-15A*, by Walker[5]. The analysis showed the reality of the phenomenon, and the possibility of evaluating lumped harmonics; but the data were not of high accuracy and the values of lumped coefficients obtained, given in Table 6, were inevitably rather inaccurate.

The second satellite is *1976-62E*, Cosmos 837 rocket, recently analysed by Hiller[9]. This is the best example of 29:2 resonance so far studied. But it is still not ideal because the eccentricity was appreciable (0.04) and the e terms in equation (7) should strictly have been included. This was not practicable because the inclination was close to 63.4° where $\dot{\omega} = 0$, and consequently the e terms, which depend on $\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}(\Phi_{29} \pm \omega)$, were strongly correlated with the main terms, in $\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\Phi_{29}$.

The third and fourth satellites in Table 6, *1971-106A* and *1971-18B*, were both of high drag. The changes in inclination at resonance were therefore very small and the values obtained for the lumped coefficients have large standard deviations[12,23].

If the early terms were dominant in equation (9), it might be possible to use the values of the lumped coefficients from these four satellites to solve for two or three individual coefficients; but at inclinations between 50° and 65° the higher-degree terms are important, and often dominate, so a solution is not practicable. The best that can be done is to compare the values with those from GEM 10C.

Table 6

Values of lumped harmonics $(\bar{C},\bar{S})^{0,2}_{29}$ from the four satellites

Satellite	i (deg)	$10^9\bar{C}^{0,2}_{29}$	$10^9\bar{S}^{0,2}_{29}$	$\bar{F}_{30,29,14}$	$10^9\bar{F}_{30,29,14}\bar{C}^{0,2}_{29}$	$10^9\bar{F}_{30,29,14}\bar{S}^{0,2}_{29}$
1962-15A	53.8	-1000 ± 500	1100 ± 500	0.01526	-15.3 ± 7.6	16.8 ± 7.6
1976-62E	62.7	-10 ± 15	-76 ± 12	0.1286	-1.3 ± 1.9	-9.8 ± 1.5
1971-106A	65.7	-90 ± 74	-127 ± 39	0.2089	-18.8 ± 15.5	-26.5 ± 8.1
1971-18B	69.9	50 ± 20	-160 ± 70	0.3372	16.9 ± 6.7	-54.0 ± 23.6

4.3 The variation of even-degree lumped harmonics with inclination

Fig 3 shows the four pairs of values of $\bar{F}_{30,29,14}(\bar{C},\bar{S})^{0,2}_{29}$ from Table 6, and the curve of variation with inclination as given by GEM 10C, with coefficients up to degree 44 included. The comparison is inconclusive. It could be argued that most of the observed values are numerically too large. On the other hand, it may be that the coefficients of order 29 in GEM 10C are much less accurate than those of order 30: certainly the S-coefficients in GEM 10C appear to be too small, and GEM 10C fails to fit the well determined $\bar{F}\bar{S}$ value of -9.8 ± 1.5 at inclination 62.7°. Also Rapp's S-values[20] are much larger and give a variation quite different from the GEM 10C curve.

Further observational results are needed before any conclusions can be drawn.

4.4 Lumped harmonics of odd degree

Lumped coefficients of odd degree can, in principle, be determined from the variations in eccentricity at 29:2 resonance, but in practice the effects are small, and the values so far obtained, from 1962-15A and 1976-62E, cannot be regarded as well-established. It is not worth attempting any comparisons until better observational values are available: it is hoped that analysis of the orbit of 1968-40B, which is currently passing through 29:2 resonance, will provide good values.

5 HARMONICS OF ORDER 31

5.1 Form of the lumped harmonics

At the 31:2 resonance the variation of i is given by[7]

$$\frac{di}{dt} = \frac{n}{\sin i}\left(\frac{R}{a}\right)^{32}(31 - 2\cos i)\bar{F}_{32,31,15}\left\{\bar{S}_{31}^{0,2}\sin\Phi_{31} + \bar{C}_{31}^{0,2}\cos\Phi_{31}\right\}$$

$$+ \text{ terms in } e, e^2, \ldots, \qquad (10)$$

where

$$\Phi_{31} = 2(\omega + M) + 31(\Omega - \nu) \qquad (11)$$

and the lumped coefficients are given by

$$\bar{C}_{31}^{0,2} = \bar{C}_{32,31} - \frac{\bar{F}_{34,31,16}}{\bar{F}_{32,31,15}}\left(\frac{R}{a}\right)^2\bar{C}_{34,31} + \frac{\bar{F}_{36,31,17}}{\bar{F}_{32,31,15}}\left(\frac{R}{a}\right)^4\bar{C}_{36,31} - \ldots \qquad (12)$$

with the same equation for S on replacing C by S throughout. Equation (12) shows that changes in inclination again yield values of harmonics of *even* degree.

However it has also been possible to analyse the variation of eccentricity at 31:2 resonance for three satellites, to obtain values of lumped harmonics of *odd* degree. The variation of e is given by[7]

$$\frac{de}{dt} = n\left(\frac{R}{a}\right)^{31}\Bigg[-17\bar{F}_{31,31,15}\left\{\bar{C}_{31}^{1,1}\sin(\Phi_{31} - \omega) - \bar{S}_{31}^{1,1}\cos(\Phi_{31} - \omega)\right\}$$

$$+ 13\bar{F}_{31,31,14}\left\{\bar{C}_{31}^{-1,3}\sin(\Phi_{31} + \omega) - \bar{S}_{31}^{-1,3}\cos(\Phi_{31} + \omega)\right\}$$

$$+ \text{ terms in } e, e^2, \ldots\Bigg] . \qquad (13)$$

The lumped harmonic coefficients here are given by

$$\left.\begin{aligned} \bar{C}_{31}^{1,1} &= \bar{C}_{31,31} - \frac{18\bar{F}_{33,31,16}}{17\bar{F}_{31,31,15}}\left(\frac{R}{a}\right)^2\bar{C}_{33,31} + \frac{19\bar{F}_{35,31,17}}{17\bar{F}_{31,31,15}}\left(\frac{R}{a}\right)^4\bar{C}_{35,31} - \ldots \\ \bar{C}_{31}^{-1,3} &= \bar{C}_{31,31} - \frac{14\bar{F}_{33,31,15}}{13\bar{F}_{31,31,14}}\left(\frac{R}{a}\right)^2\bar{C}_{33,31} + \frac{15\bar{F}_{35,31,16}}{13\bar{F}_{31,31,14}}\left(\frac{R}{a}\right)^4\bar{C}_{35,31} - \ldots \end{aligned}\right\} \qquad (14)$$

and similarly for S on replacing C by S throughout.

5.2 Observational values

So far, four orbits have been analysed at 31:2 resonance. They are discussed overleaf in order of increasing inclination.

The variations in inclination and eccentricity for Skylab 1 rocket, *1973-27B*, at 31:2 resonance were analysed by King-Hele[7], and quite well-determined values were obtained for all six of the lumped coefficients. These values are given in Tables 7 and 8.

The orbit of Proton 4, *1968-103A*, was analysed by Hiller and King-Hele[6]. Since the orbit was appreciably eccentric, it was found necessary to include the e terms in equation (12), and the 'main' coefficients ($\bar{C}_{31}^{0,2}$ and $\bar{S}_{31}^{0,2}$) turned out to be very small (they are given in square brackets in Table 7). A simultaneous analysis of inclination and eccentricity gave the values for the other four coefficients listed in Table 8.

The third satellite, China 2 rocket, *1971-18B*, suffered high drag and the values obtained from analysis of inclination are barely determinate[25]. They are included in Table 7.

The fourth and best analysis is of Samos 2, *1961 α1*, by Walker[8]. A simultaneous fitting of inclination and eccentricity was successful in giving well-defined values of all six lumped coefficients, as recorded in Tables 7 and 8.

It is not possible to solve for individual coefficients, because the numerical factors multiplying the high-degree terms are extremely large for 1973-27B and 1968-103A. For example[7], with 1973-27B, $\bar{C}_{31}^{1,1}$ has a maximum numerical factor of 979.6 for $\bar{C}_{41,31}$, and $\bar{C}_{57,31}$ has a factor 363.1, so that coefficients would need to be evaluated up to degree 57, and this is impossible with only six equations.

5.3 The variation of the lumped harmonics with inclination, and implications for GEM 10B/C

Fig 4 shows the four pairs of values of $\bar{F}_{32,31,15}(\bar{C},\bar{S})_{31}^{0,2}$ from Table 7 and also the variation of these lumped harmonics with inclination as given by GEM 10C up to degree 46.

Fig 5 shows the values of the four lumped harmonics $(\bar{C},\bar{S})_{31}^{1,1}$ and $(\bar{C},\bar{S})_{31}^{-1,3}$, multiplied by the appropriate $\bar{F}$, for each of the three satellites of Table 8, and also their variation with inclination as given by GEM 10C. Since the multiplying factors in equation (14) remain large up to a higher degree than those in equation (12), terms up to degree 53 in GEM 10C are included.

Table 7

Values of even-degree lumped harmonics $(\bar{C},\bar{S})_{31}^{0,2}$ from four satellites

Satellite	i (deg)	$10^9\bar{C}_{31}^{0,2}$	$10^9\bar{S}_{31}^{0,2}$	$\bar{F}_{32,31,15}$	$10^9\bar{F}_{32,31,15}\bar{C}_{31}^{0,2}$	$10^9\bar{F}_{32,31,15}\bar{S}_{31}^{0,2}$
1973-27B	50.0	860 ± 320	-1210 ± 200	0.002649	2.3 ± 0.8	-3.2 ± 0.5
1968-103A	51.5	[100 ± 1800]	[30 ± 800]	0.004634	0.5 ± 8.3	0.1 ± 3.7
1971-18B	69.8	24 ± 18	30 ± 23	0.3139	7.5 ± 5.7	9.4 ± 7.2
1961 α1	97.2	-2.9 ± 1.2	9.0 ± 2.2	-0.4323	1.3 ± 0.5	-3.9 ± 1.0

Table 8

Values of odd-degree lumped harmonics $(\bar{C},\bar{S})_{31}^{1,1}$ and $(\bar{C},\bar{S})_{31}^{-1,3}$ from three satellites

Satellite	1973-27B	1968-103A	1961 α1
i (deg)	50.0	51.5	97.2
$10^9\bar{C}_{31}^{1,1}$	20800 ± 10100	-49000 ± 14000	-47.3 ± 7.2
$10^9\bar{S}_{31}^{1,1}$	24700 ± 11400	11000 ± 7000	-8.7 ± 4.2
$\bar{F}_{31,31,15}$	0.0002811	0.0005269	0.3430
$10^9\bar{F}_{31,31,15}\bar{C}_{31}^{1,1}$	5.8 ± 2.8	-25.8 ± 7.4	-16.2 ± 2.5
$10^9\bar{F}_{31,31,15}\bar{S}_{31}^{1,1}$	6.9 ± 3.2	5.8 ± 3.7	-3.0 ± 1.4
$10^9\bar{C}_{31}^{-1,3}$	36100 ± 4300	10000 ± 3000	63.9 ± 15.3
$10^9\bar{S}_{31}^{-1,3}$	17800 ± 2800	2000 ± 3000	42.0 ± 15.2
$\bar{F}_{31,31,14}$	0.001139	0.001995	0.2349
$10^9\bar{F}_{31,31,14}\bar{C}_{31}^{-1,3}$	41.1 ± 4.9	19.95 ± 6.0	15.0 ± 3.6
$10^9\bar{F}_{31,31,14}\bar{S}_{31}^{-1,3}$	20.3 ± 3.2	4.0 ± 6.0	9.9 ± 3.6

It is not possible to make an immediate judgment on the accuracy of GEM 10C from Figs 4 and 5: an indication of the range of error on the GEM curves is required. In Tables 9 and 10, therefore, the values of the lumped coefficients from the two best satellites, Samos 2 and 1973-27B, are compared with the values given by GEM 10C at the appropriate inclination (97.24° or 50.04°), on the assumption that the individual coefficients in GEM 10C have sd of 5×10^{-9}.

Table 9

Comparison of lumped harmonics from Samos 2 and GEM 10C

Lumped coefficient	GEM 10C for $i = 97.24^{\circ}$	Samos 2	Lumped coefficient	GEM 10C for $i = 97.24^{\circ}$	Samos 2
$10^9\bar{C}_{31}^{0,2}$	-4.5 ± 6 (-4.4 ± 6)	-2.9 ± 1.2	$10^9\bar{S}_{31}^{0,2}$	-1.7 ± 6 (-1.7 ± 6)	9.0 ± 2.2
$10^9\bar{C}_{31}^{1,1}$	-18 ± 8 (-18 ± 8)	-47 ± 7	$10^9\bar{S}_{31}^{1,1}$	-4 ± 8 (-3 ± 8)	-9 ± 4
$10^9\bar{C}_{31}^{-1,3}$	-25 ± 12 (-23 ± 12)	64 ± 15	$10^9\bar{S}_{31}^{-1,3}$	-11 ± 12 (-11 ± 12)	42 ± 15

The numbers in brackets in Table 9 are the values that would be obtained by using coefficients of degree up to that recommended in Ref 8, namely 44, 55 and 61, rather than the maximum degree used here, namely 46, 53 and 53 respectively. The comparison with GEM 10C made in Ref 8 utilized a preliminary set of GEM 10C coefficients, now superseded: so the values in brackets provide a revised version of the table in Ref 8 - and also show that altering the maximum degree does not have much effect.

In Table 9 the GEM 10C values of $\bar{C}_{31}^{0,2}$ and $\bar{S}_{31}^{0,2}$ differ from the rather accurate observational values obtained from Samos 2 by 0.3 and 1.8 times the GEM sd. This suggests that the chosen sd for GEM is nearly correct. For the odd $(\bar{C},\bar{S})$ coefficients, however, the quoted standard deviations from the two sources are nearly equal and their sum is a better yardstick: the differences between the GEM 10C values of $\bar{C}_{31}^{1,1}$, $\bar{S}_{31}^{1,1}$, $\bar{C}_{31}^{-1,3}$ and $\bar{S}_{31}^{-1,3}$ and the corresponding Samos 2 values are 1.9, 0.4, 3.3 and 2.0 × (the sum of the sd). This suggests a larger sd for GEM 10C. In the odd-degree lumped harmonics, the higher-degree terms are much more important: so the indication is that, although GEM 10B (degree ≤36) may be accurate to 5×10^{-9}, GEM 10C is probably not. For Samos 2, GEM 10B gives $10^9\bar{C}_{31}^{0,2} = -3.2$ and $10^9\bar{S}_{31}^{0,2} = -0.3$, and these are both closer than GEM 10C to the observed values.

Table 10

Comparison of lumped harmonics from 1973-27B and GEM 10C

Lumped coefficient	GEM 10C for $i = 50.04°$	1973-27B	Lumped coefficient	GEM 10C for $i = 50.04°$	1973-27B
$10^6\bar{C}_{31}^{0,2}$	-0.7 ± 0.7	0.86 ± 0.32	$10^6\bar{S}_{31}^{0,2}$	0.1 ± 0.7	-1.21 ± 0.20
$10^6\bar{C}_{31}^{1,1}$	-7 ± 8	21 ± 10	$10^6\bar{S}_{31}^{1,1}$	-8 ± 8	25 ± 11
$10^6\bar{C}_{31}^{-1,3}$	-1 ± 2	-36 ± 4	$10^6\bar{S}_{31}^{-1,3}$	-2 ± 2	18 ± 3

A similar comparison for Table 10 shows that, for $\bar{C}_{31}^{0,2}$ and $\bar{S}_{31}^{0,2}$, the GEM 10C values differ from 1973-27B by 2.2 and 1.9 standard deviations. (The inclusion of terms up to degree 60 in GEM 10C gives no significant improvement: $10^6\bar{C}_{31}^{0,2}$ changes to -0.4 and $10^6\bar{S}_{31}^{0,2}$ to 0.6.) For $(\bar{C},\bar{S})_{31}^{1,1}$ the differences are 1.6 and 1.7 × (the sum of the sd); and for $(\bar{C},\bar{S})_{31}^{-1,3}$ the differences are 5.8 and 4.0 × (the sum of the sd). This can only be called serious disagreement: however, since all the lumped harmonics involve high-degree coefficients, the previous conclusion about GEM 10B (degree ≤36) remains unaffected.

Thus, although there remains room for doubt, our provisional conclusion for 31st-order coefficients is that GEM 10C cannot be recommended, but the GEM 10B coefficients (degree ≤36) are probably within the 5×10^{-9} accuracy.

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6 SUMMARY OF RESULTS

6.1 Harmonics of order 30

The values of even-degree 30th-order lumped harmonics obtained from analyses of 15th-order resonance are numerous enough to allow a solution for individual coefficients (Table 3). The best-determined coefficients are those of degree 30, namely:

$$10^9\bar{C}_{30,30} = -1.2 \pm 1.1 \qquad 10^9\bar{S}_{30,30} = 9.6 \pm 1.3 \ .$$

These standard deviations correspond to an accuracy of about 1 cm in geoid height.

The variation of the even-degree lumped harmonics with inclination given by the Goddard Earth Model 10B is generally similar to the variation given by the resonance solution (see Fig 1), and it appears that a standard deviation as low as 3×10^{-9} may be appropriate for the 30th-order coefficients in GEM 10B. For the (30,30) coefficients, the values from GEM 10B (with sd 3×10^{-9}), and from Rapp's 1977 solution (sd 5×10^{-9}), are consistent with those quoted above.

At the single inclination where a test is possible, the odd-degree 30th-order S-coefficients from GEM agree well but the C-coefficients seem to be too small (see Fig 2).

All in all, progress in evaluating 30th-order coefficients is encouraging, though more analyses of long-enduring 15th-order resonances are required.

6.2 Harmonics of orders 29 and 31

The picture is different for these harmonics. There are not enough analyses of 29:2 or 31:2 resonances to allow the determination of individual coefficients, and all that can be done is to compute the variation of lumped harmonics with inclination as given by GEM 10C (GEM 10B only goes to degree 36, and higher coefficients are usually needed), and to compare lumped harmonics from GEM 10C with the small number of accurate values found from resonance analyses. Figs 3 to 5 show that there is no consistent agreement. For 31st order, there is an indication that the GEM 10B values (degree 31 to 36) may be accurate to 5×10^{-9}, but the GEM 10C values (degree >36) disagree. Since GEM 10C was derived as a model of the ocean surface to degree and order 180, the use of a small group of its coefficients, of degree 37 to 50, say, may be inappropriate. For 29th order, the resonance results are fewer and less accurate, so the disagreement is inconclusive.

Further analyses of 29:2 and 31:2 resonance, particularly the former, are needed if future comprehensive models of the gravity field are to be tested by the resonance method.

6.3 Conclusions

Resonance-testing of comprehensive gravitational field models has previously been applied (*eg* Refs 3 and 26) for various orders up to 15. Here the test is made for orders 29, 30 and 31, with mixed results: GEM 10B/C emerges well for order 30; for order 31, the GEM 10B values (degree ≤36) are probably good, but the GEM 10C values (degree >36) are probably not; for order 29 the test is indecisive.

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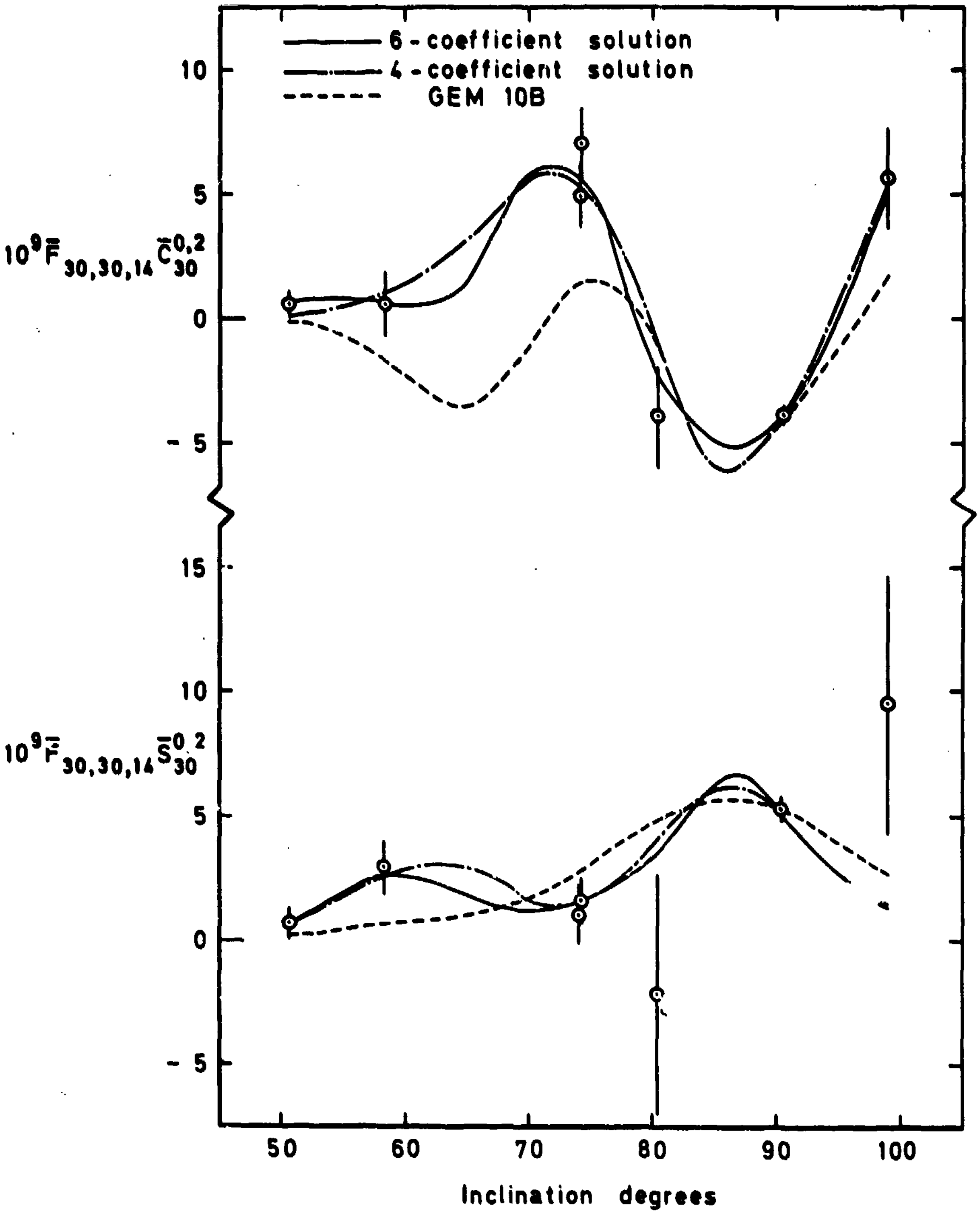

Fig 1 Values of 30th-order lumped harmonics of even degree from Table 1 and the curves given by the 4- and 6-coefficient solutions and GEM 10B

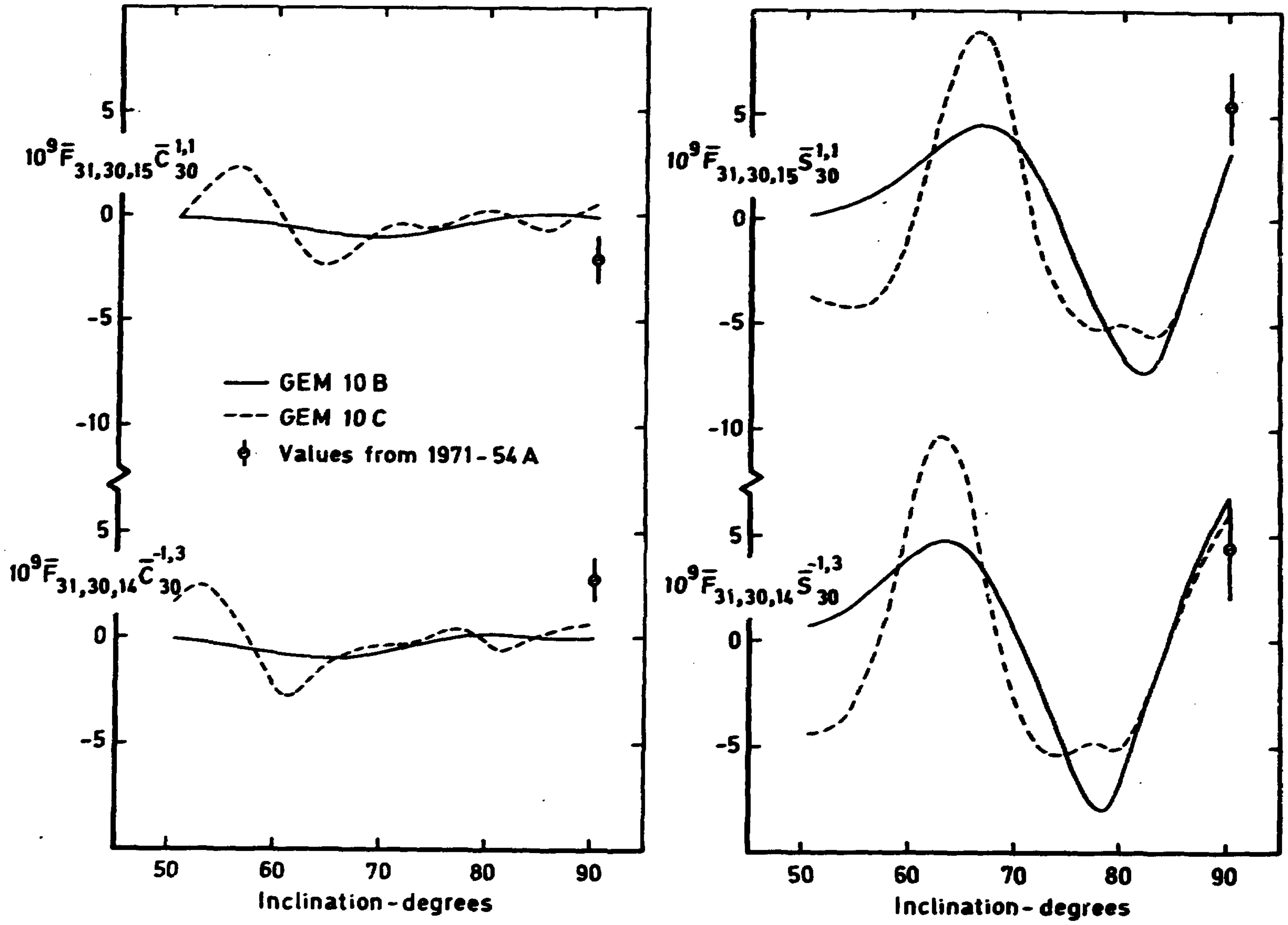

Fig 2 Variation of 30th-order lumped harmonics of odd degree given by GEM 10B and C

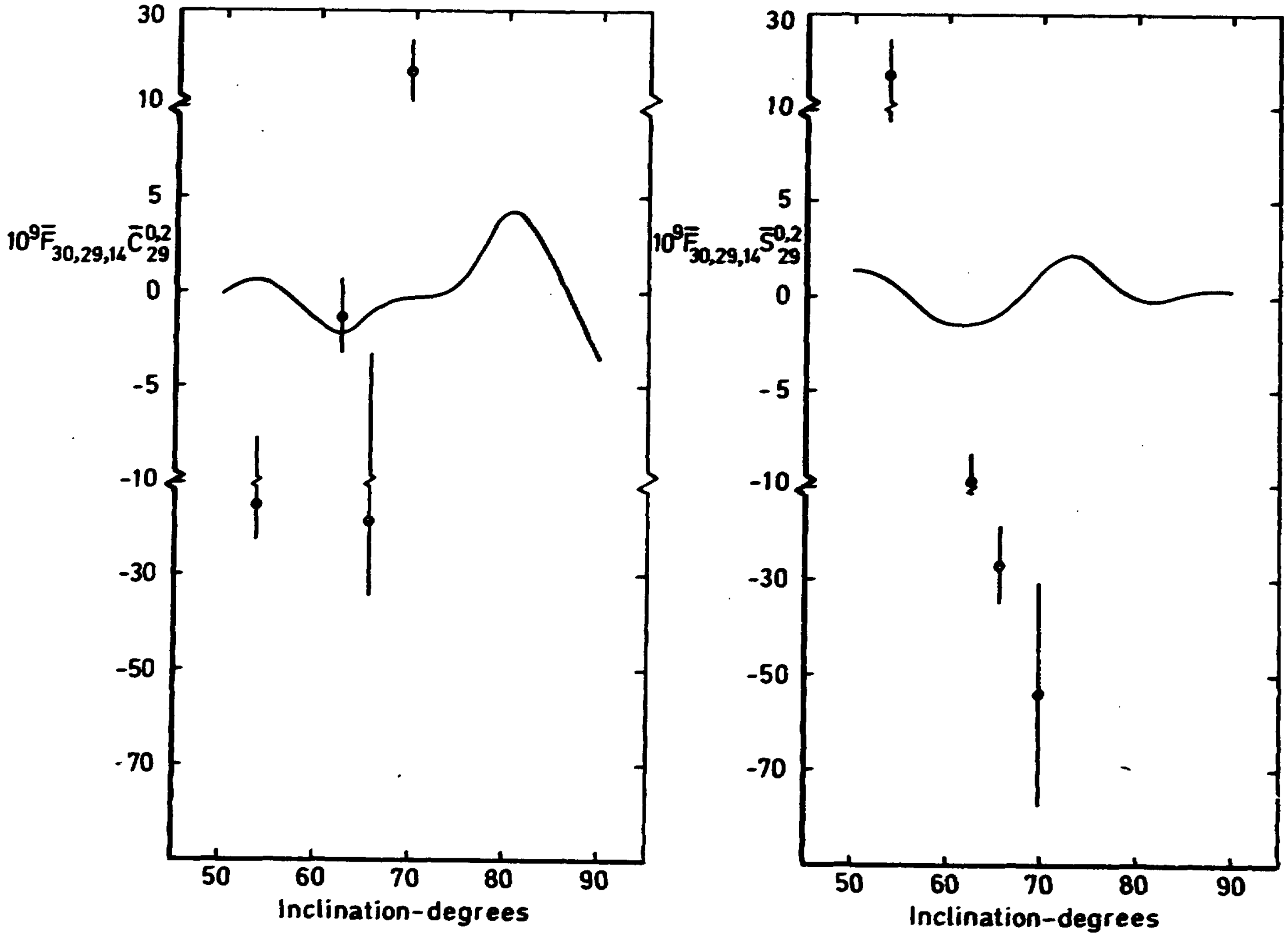

Fig 3 Values of 29th-order lumped harmonics of even degree from Table 6 and the curves given by GEM 10C

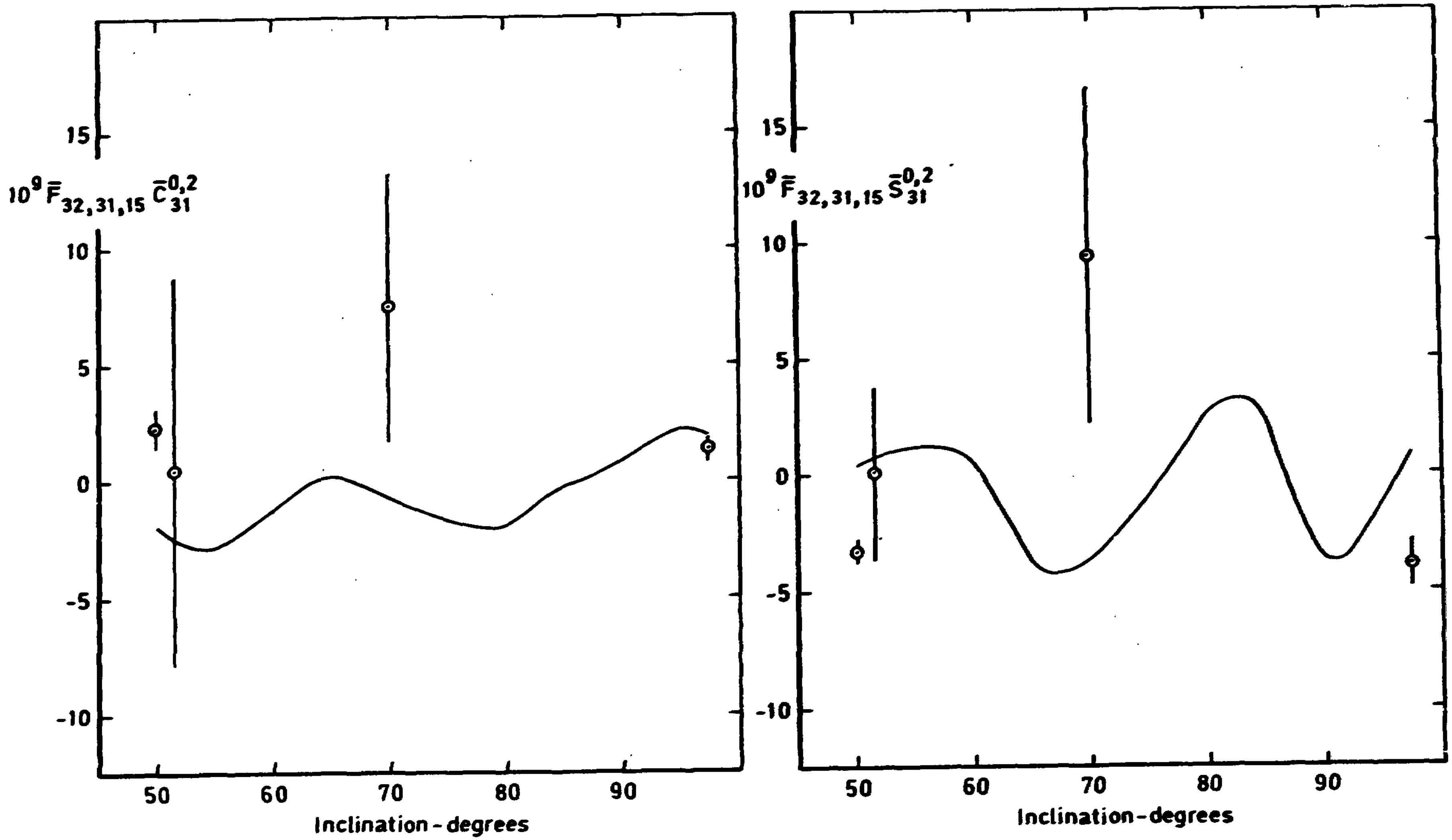

Fig 4 Values of 31st-order lumped harmonics of even degree from Table 7 and the curves given by GEM 10C

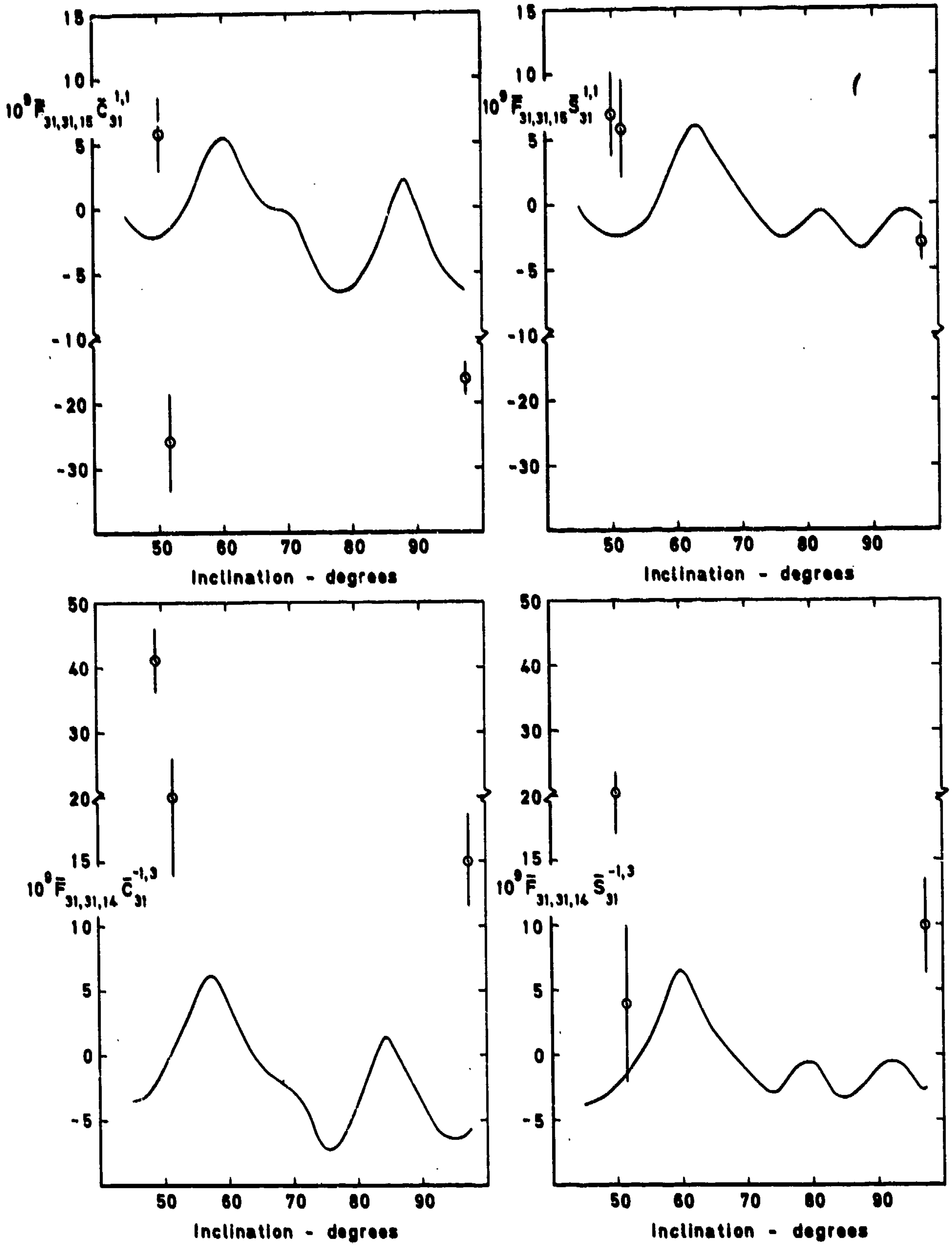

Fig 5 Values of 31st-order lumped harmonics of odd degree from Table 8 and the curves given by GEM 10C

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17. Abstract

The Earth's gravitational potential is usually expressed as an infinite series of tesseral harmonics, and it is possible to evaluate 'lumped harmonics' of a particular order m by analyses of resonant satellite orbits - orbits with tracks over the Earth that repeat after m revolutions. In this paper we review results on 30th-order harmonics from analyses of 15th-order resonance, and results on 29th- and 31st-order harmonics from 29:2 and 31:2 resonance.

The values available for 30th-order lumped harmonics of even degree are numerous enough to allow a solution for individual coefficients of degree up to 40. The best-determined coefficients are those of degree 30, namely

$$10^9\bar{C}_{30,30} = -1.2 \pm 1.1 \qquad 10^9\bar{S}_{30,30} = 9.6 \pm 1.3 \ .$$

The standard deviations here are equivalent to 1 cm in geoid height.

For the 29th- and 31st-order harmonics, and for the 30th-order harmonics of odd degree, there are not enough values to determine individual coefficients, but the lumped values from particular satellites can be used for 'resonance testing' of gravity field models, particularly the Goddard Earth Model 10B.

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