FTD-ID(RS)T-1267-81

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FOREIGN TECHNOLOGY DIVISION

EVALUATION OF PARAMETERS OF THE OSCILLATORY MOTION OF DIGITAL SERVO SYSTEM IN A STEADY-STATE MODE

by

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EDITED TRANSLATION

FTD-ID(RS)T-1267-81 4 December 1981

MICROFICHE NR: FTD-81-C-001085

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English pages: 5

Source: Izvestiya Vysshikh Uchebnykh Zavedeniy, Priborostroyeniye, Vol. 14, Nr. 6, 1971, pp. 44-47

Country of origin: USSR
Translated by: Robert D. Hill
Requester: AFWAL/FIGL
Approved for public release; distribution unlimited.

111 600

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

EVALUATION OF PARAMETERS OF THE OSCILLATORY MOTION OF DIGITAL SERVO SYSTEM IN A STEADY-STATE MODE

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Examined is a digital servo system (TsSS) characterized by the quantization of a signal with respect to level and time. The steady-state mode in the system is characterized by the emergence of an oscillating motion. The dependence of parameters of the oscillating motion on parameters of the system is given.

A TsSS with a three-positional relay characteristic of the control element was examined in reference [2]. Shownis the possibility of an analysis of the dynamics of the TsSS by means of the representation of its motion on a phase surface in the presence of quantization of the signal with respect to level and time.

The purpose of this work is to evaluate parameters of the oscillatory motion in the transient and steady-state modes.

The trajectory of the representing point, which characterizes the behavior of the TsSS on the phase plane, will consist of segments each of which can be described by the equation of trajectories:

$$X = X_0 + Y_0 - Y + \Phi(\delta) \ln \frac{Y_0 + \Phi(\delta)}{Y + \Phi(\delta)} \qquad (1)$$

with the appropriate values of $\Phi(\delta)$.* [Translator's note: see footnote on next page.]

Let us examine the case when the representing points M_1 and M_2 during time $T_{\text{ц}}$ occupy the extreme positions H and C, respectively, (Fig. 1).

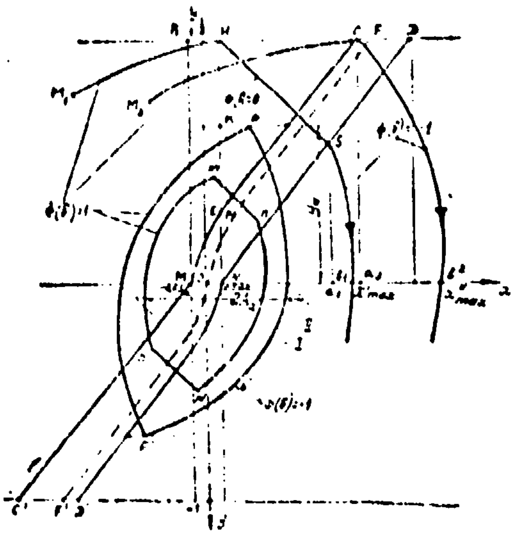

Fig. 1. Determination of parameters of the oscillatory motion in the transient and steady-state modes.

The maximum overswing in the transient mode with respect to the position of equilibrium corresponds to $X'_{\text{макс}}$ for the trajectory of motion of the representing point M_1 and $X''_{\text{макс}}$ for the trajectory of motion of the representing point M_2. $X'_{\text{макс}} = X_s + a_1 b_1$.

Having taken in quation (1)

$$\Phi(\delta) = 0; \quad X_0 = 0{,}5\Delta_x; \quad Y_0 = 1,$$

we get

$$X_s = 0{,}5\Delta_x + 1 + Y.$$

Having substituted into equation of line ND: $Y = \frac{1}{e^{T_{\text{ц}}} - 1} X - \frac{0{,}5\Delta_x}{e^{T_{\text{ц}}} - 1}$ [2] the value X_s, after conversions, we obtain $Y_s = \frac{1}{e^{T_{\text{ц}}}}$, then

$$X_s = \frac{e^{T_{\text{ц}}} - 1}{e^{T_{\text{ц}}}} + 0{,}5\Delta_x.$$

*The change in the velocity and coordinate are described, correspondingly, by these equations:

$$Y = Y_0 e^{-\tau} + \Phi(\delta)(1 - e^{-\tau}), \quad (2) \quad X = X_0 + Y_0(1 - e^{-\tau}) + \Phi(\delta)[\tau - (1 - e^{-\tau})].$$

From equation (1), having assumed that $X_0 = 0;\ Y = 0;\ Y_0 = Y_c;\ X = a_1 b_1$, we get

$$a_1 b_1 = \frac{1}{e^{T_ц}} - \ln\left(\frac{1}{e^{T_ц}} + 1\right),$$

$$X'_{макс} = \frac{e^{T_ц} - 1}{e^{T_ц}} + 0{,}5\Delta_x + \frac{1}{e^{T_ц}} - \ln\frac{(1 + e^{T_ц})}{e^{T_ц}},$$

or

$$\begin{aligned} X'_{макс} &= T_ц + 1 + 0{,}5\Delta_x - \ln\left(1 + e^{T_ц}\right), \\ X''_{макс} &= X_c + a_2 b_2. \end{aligned} \tag{4}$$

The segment BC is numerically equal to the time of the comparison of codes $T_ц$, then $X_c = T_ц - 0{,}5\Delta_x$.

From equation (1), having assumed that $\Phi(\delta) = -1;\ X_0 = 0;\ Y_0 = Y_c = 1;\ X = a_2 b_2;\ Y = 0$, we obtain $a_2 b_2 = 1 - \ln 2$, and then $X''_{макс} = 1 - \ln 2 + T_ц - 0{,}5\Delta_x$ or

$$X''_{макс} = T_ц - 0{,}5\Delta_x + 0{,}3069. \tag{5}$$

From equations (4) and (5) it is seen that the maximal overswing of the system in the transient mode is determined by the quantity $T_ц$. The mutual position of of points $X'_{МАКС}$ and $X''_{МАКС}$ varies from the magnitude $T_ц$.

Having equated equations (4) and (5), after transformations, we obtain the value $T_ц$, for which $X'_{МАКС} = X''_{МАКС}$:

$$T_ц = \ln(1 - 2e^{\Delta_x}). \tag{6}$$

The presence of regions of switching of function $\Phi(\delta)$ lying in the quadrants I and III of plane XY, is the reason for the emergence in the TsSS of the undamped oscillating motion in the steady-state mode. Each trajectory from any initial point with the lapse of a definite time interval enters into a defined region and from this moment no longer emerges from it.

Two kinds of closed trajectories of the representing point are possible in the steady-state mode. Figure 1 gives closed trajectories of types I and II. The trajectory of type I consists of two segments of the integral curve which correspond to the controlling action

$\Phi(\delta) = 1$ and $\Phi(\delta) = -1$. Points p and p' are switching points.

Trajectory II consists of segments of the integral curve with values $\Phi(\delta) = 1$; $\Phi(\delta) = 0$ and $\Phi(\delta) = -1$. Segments mn and m'n' correspond to the absence of the controlling action, i.e., $\Phi(\delta) = 0$. The time of the movement of the representing point from m to n (or from m' to n') is equal to $\Delta t = T_u$.

From Fig. 1 it is evident that the region of switching CHE (closed trajectory of type I) is considerably larger than the switching region MBHE (closed trajectory of type II). To determine parameters of the oscillating motion in the steady-state mode, let us examine the maximal possible closed trajectory, i.e., the trajectory of type I.* The period of the closed trajectory T_0 should consist of a finite even number of time segments T_u (sampling is considered). The time of the half-cycle $T_0/2$, i.e., the time between the switching points p and p' should correspond to

$$\frac{T_0}{2} = mT_u, \tag{7}$$

where m = 1, 2 and 3.

The maximal value of the velocity with a steady-state oscillating motion (at the moment of switching of the controlling action) can be found by assuming that in equation (2) $Y_0 = Y_a$; $Y = -Y_a$; $\Phi(\delta) = -1$; and $\tau = mT_u$. By examining the movement from point p to p', we find

$$Y_a = \frac{1 - e^{-mT_u}}{1 + e^{-mT_u}}. \tag{8}$$

According to equation (8), several limiting closed trajectories can be established in the system. It is necessary to determine the parameters of the maximally possible closed trajectory.

The period of the maximally possible closed trajectory $T_{0\,max} = 2mT_u$ must be less than the period of the closed trajectory, the switching points of which lie on lines AF and A'F'.**

From the condition of point transformation of line AF into line

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*The presence of several closed trajectories of the representing point is possible in the region [1].

**Lines AF and A'F' are limits of regions of switching with the two-position characteristic of the controlling element.

AF', it is possible to find the dependence of Y^* on $T_ц$, where Y^* is the maximum velocity of the closed trajectory, the switching points of which lie on the indicated line AF and A'F'.

The period of the steady-state oscillations in the system of maximal amplitude can be determined from condition $Y_{a\,\text{макс}} < Y_a^*$. The dependence of Y^* on $T_ц$ is given on Fig. 2.

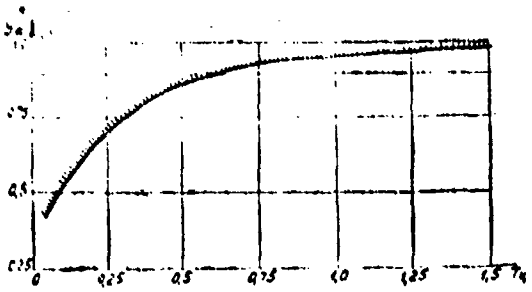

Fig. 2. Limit of the region of the existence of the limiting closed trajectory with the largest parameters.

Having used equation (1), let us define the doubled amplitude of oscillations of the outlet coordinate $2A_x$ in the steady-state mode

$$2A_x = \ln \frac{1}{(1 - Y_{a\,\text{макс}})(1 + Y_{a\,\text{макс}})}. \tag{9}$$

Dependences $Y_{a\,\text{макс}}, T_0$ and $2A_x$ define the basic relations of the oscillating motion from parameters of the system.

Submitted
25 May 1970

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