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A RAY THEORY FOR NONLINEAR SHIP WAVES AND WAVE RESISTANCE

by

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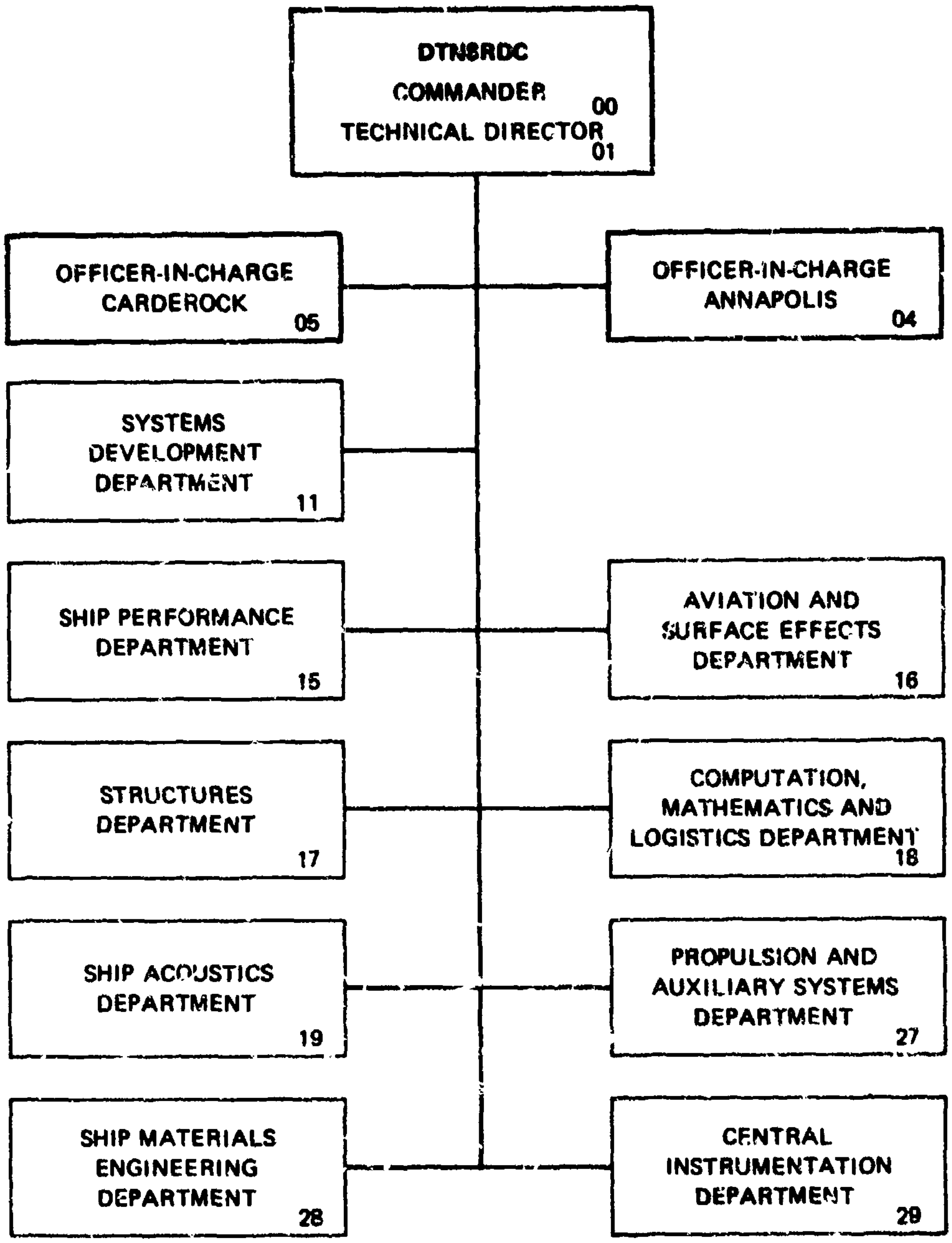

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For the Wigley hull, ray paths, wave phases, and directions of elementary waves are computed by the ray theory and a method of computing wave resistance is developed. The wave phase is compared with that of linear theory as a function of ship-beam length ratio to identify the advancement of the bow wave phase which influences the design of bow bulbs. The wave resistance of the Wigley hull is computed using the amplitude function from Michell's then ship theory and compared with values of Michell's wave resistance. The total wave resistance has the phase of hump and hollow shifted considerably.

TABLE OF CONTENTS

LIST OF FIGURES

A RAY THEORY FOR NONLINEAR SHIP WAVES AND WAVE RESISTANCE

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Abstract

Analytical and numerical methods for application of ray theory in computing ship waves are investigated. The potentially important role of ray theory in analyses of nonlinear waves and wave resistance is demonstrated. The reflection of ship waves from the hull boundary is analyzed here for the first time.

When a wave crest touches the ship surface, the ray exactly follows the ship surface. When the wave crest is nearly perpendicular to the ship surface the ray is reflected many times as it propagates along the ship surface. Many rays of reflected elementary waves intersect each other. The envelope to the first reflected rays forms a line like a shock front which borders the area of large waves or breaking waves near the ship.

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Introduction

Significant developments in the ship wave theory have been made in recent years. These include the application of ray theory[1,2*] and the experimental discovery of a phenomenon called a free surface shock wave.[3]

Because a ship is not thin enough to apply both the thin ship theory and the complicated free-surface effect, theoretical development of an accurate ship wave theory has been slow. The problem should be evaluated differently from the conventional means. Ray theory has been used in geometrical optics or for waves having small wave lengths. Ursell[4] used the theory to consider wave propagation in nonuniform flow, and Shen, Meyer, and Keller[5] used it to investigate water waves in channels and around islands. Recently Keller[2] developed a ray theory for ship waves and pointed out that the theory could supply useful information about the waves of thick ships at relatively slow speeds. However, he had difficulty obtaining the excitation function for wave amplitude and solved only a thin-ship problem. Inui and Kajitani[1] applied the Ursell ray theory[4] to ship waves, using the amplitude function from linear theory.

Because the ray is the path of wave energy, it is not supposed to penetrate the ship surface; and this was emphasized by Keller.[2] However, neither Keller nor Inui and Kajitani consider nonpenetration of the ray seriously. Recently Yim[6] found the existence of rays which emanated from the ship bow and reflected from the ship surface.

In the present paper, further study has revealed many bow rays reflecting from the ship surface, creating an area in which these rays intersect each other. The envelope to the first reflected rays forms a line like a shock front which borders the area of large waves or breaking waves near the ship. This will be referred to as the second caustic. This phenomenon was observed by Inui et al.[7] in the towing tank and called a free surface shock wave (FSSW). Much has been done concerning the theoretical investigation of FSSW by introducing a fictitious depth for a shallow water nondispersive wave.[3]

The ray equation and the ship boundary condition are analyzed further to show the existence of reflecting rays, except in the case of a flat wedgelike ship surface. The ray path is very sensitive to the initial boundary condition near the bow or the stern, and should be identified at downstream infinity not by the initial condition. The conservation law of wave energy in the nonuniform flow is different from that of the uniform flow due to the exchange of energy with the local flow.[8] Therefore, the linear wave amplitude as a function of initial wave angle near the bow or stern is meaningless and cannot be used in the ray theory. It is shown that the amplitude function for the ray theory matched with the

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*A complete listing of references is given on page 15 linear theory far downstream is reasonable and likely to produce a reasonable result as in the two dimensional theory.[9]

The wave phase in the ray theory is obtained, together with the ray path, independent from the amplitude function. Both the phase and the ray path are quite different from the prediction of linear theory near the ship. The rays far downstream are straight as in the linear theory yet have phases different from the linear theory; they are parallel to the linear ray with the same wave angle but do not coincide. The difference in wave phases is the main factor that makes the wave resistance different from the linear theory. Ray paths and phase difference are computed for various parameters of the Wigley ship, with and without a bulbous bow. The ray paths for different drafts and different beam-length ratio of the Wigley hull are slightly different, widening the wave area near the hull for the wider beam and/or larger draft. However, the phase difference is more sensitive to the beam-length ratio and/or draft-length ratio, by always advancing the linear wave phase.

The wave resistance of the Wigley hull is computed and shown to have a considerable shift of phase of hump and hollow.

The most interesting phenomenon of a ship with a bulbous bow is the reduction of slopes of rays and the second caustic, i.e., the larger the bulb, the greater the reduction. This fact was observed in the towing tank.[3] There exists a bulb size which totally eliminates the reflecting rays.[10] However, the phase difference due to the bulb[11] is very small showing that the phase difference, which has been observed in the towing tank, is the effect of nonuniform flow caused by the main hull.

Ray Equations

The concept of ray theory in ship waves is analogous to the concepts used in geometrical optics and in geometric acoustics. A ship is considered advancing with a constant velocity -U which is the direction of the negative x axis of a right handed rectangular coordinate system O-xyz with the origin O at the ship bow on the mean free surface, z=0, z is positive upwards.

First the phase function s(x,y,z) is defined so that the equation (s = constant) rerepresents the wave front where the value of s is the optical distance from the wave source, e.g., the ship bow. When Keller[2] developed his ray theory of ships by expanding boundary conditions and a solution, which should satisfy both the Laplace equation and the boundary conditions, in a series of Froude number squares F^2, he obtained:

$$(\nabla s)^2 = 0$$

$$s_z = -i\,(S_t + \nabla\phi \cdot \nabla s)^2 \text{ at } z = 0 \qquad (1)$$

By eliminating S_z from these two equations he obtained a dispersion relation

$$(s_x{}^2 + s_y{}^2)^{1/2} = (S_t + \phi_x s_x + \phi_y s_y)^2 \text{ at } z = 0 \qquad (2)$$

where ϕ is the double model potential. When steady state motion is assumed with respect to the moving coordinate system, O - xyz,

$$s_t = 0$$

When the angle between the normal $\bar{n}$ to the phase curve s and the x axis is denoted by θ,

$$\bar{n} = \bar{i}\cos\theta + \bar{j}\sin\theta \qquad (3)$$

Then the wave number vector is defined by

$$\bar{k} \equiv k_1\bar{i} + k_2\bar{j} \equiv s_x\bar{i} + s_y\bar{j}$$
$$= \bar{n}k = \bar{i}k\cos\theta + \bar{j}k\sin\theta \text{ at } z = 0 \qquad (4)$$

From equations (2) and (4)

$$k = \left[\frac{1}{u\cos\theta + v\sin\theta}\right]^2 \qquad (5)$$

where

$$-\phi_x = u \text{ and } -\phi_y = v$$

These results are an approximation within the order of F^4, and the phase function and its related equations are all limited to their values at z = 0. Thus, from now on, unless otherwise mentioned, all the physical values are at z = 0. The ray equation of ship waves is obtained from the irrotationality of the wave number vector

$$\frac{\partial k_2}{\partial x} - \frac{\partial k_1}{\partial y} = 0 \qquad (6)$$

From Equations (4) through (6),

$$\{2\sin\theta\,(u\sin\theta - v\cos\theta) + \cos\theta\,(u\cos\theta + v\sin\theta)\}\frac{\partial\theta}{\partial x}$$
$$+ \{2\cos\theta\,(-u\sin\theta + v\cos\theta) + \sin\theta\,(u\cos\theta + v\sin\theta)\}\frac{\partial\theta}{\partial y} \qquad (7)$$
$$= 2\sin\theta\,(\cos\theta\frac{\partial u}{\partial x} + \sin\theta\frac{\partial v}{\partial x})$$
$$- 2\cos\theta\,(\cos\theta\frac{\partial u}{\partial y} + \sin\theta\frac{\partial v}{\partial y})$$

This is the ray equation which can be solved by the method of characteristics, and is equivalent to simultaneous ordinary differential equations.

$$\frac{dy}{dx} = \frac{\{v - \frac{1}{2}\sin\theta\,(u\cos\theta + v\sin\theta)\}}{\{u - \frac{1}{2}\cos\theta\,(u\cos\theta + v\sin\theta)\}} \tag{8}$$

$$\frac{d\theta}{dx} = \{2\sin\theta\,(\cos\theta\,\frac{\partial u}{\partial x} + \sin\theta\,\frac{\partial v}{\partial x})$$

$$- 2\cos\theta\,(\cos\theta\,\frac{\partial u}{\partial y} + \sin\theta\,\frac{\partial v}{\partial y})\}/\{2\sin\theta\,(u\sin\theta \tag{9}$$

$$- v\cos\theta) + \cos\theta\,(u\cos\theta + v\sin\theta)\}$$

Here $u\cos\theta + v\sin\theta$ is the velocity component normal to the phase curve of the flow relative to the ship. Because the wave is stationary relative to the ship, the phase velocity through the water surface should be

$$- u\cos\theta - v\sin\theta$$

The group velocity is one-half of the phase velocity. Thus the ray direction in Equation (8) is along the resultant of the group velocity taken normal to the phase curve and of the velocity of the basic flow as Ursell has shown.[4] Because the wave energy is propagated at the group velocity, the ray path is interpreted as the path of energy relative to the ship. This can be obtained by solving Equations (8) and (9) with the proper initial condition. The phase s can be obtained from equations (4) by

$$s = \int \bar{k}\,d\bar{r}$$

as in potential theory; s(x,y) is a function of (x,y) but is unrelated to the integration path. However,

$$ds = k\cos\theta\,dx + k\sin\theta\,dy \tag{10}$$

can be solved together with the ray equations along the ray path.

Rays of Ship Waves and Linear Theory

To investigate the path of a ray of a ship wave, a ship, represented by a double model source distribution m (x,y) on $y = o$, $h>z>-h$, is considered. Although the linear relation between the source strength m and the ship surface

$$y = \pm f(x,z) \tag{11}$$

is

$$m = \frac{1}{2\pi}\frac{dy}{dx} \tag{12}$$

the actual double model ship body streamline should be obtained by solving

$$\frac{dy}{dx} = \frac{v}{u} \tag{13}$$

through the stagnation point where u and v are the velocity components of the total velocity caused by the double model source distribution m and the uniform flow relative to the ship.

In the linear ship wave theory a smooth source distribution produces two systems of regular ship waves: the bow and the stern waves starting from the bow and stern, respectively, represented[11] as

$$\zeta(x,y) = \int_{-\pi/2}^{\pi/2} A(\theta)\exp\{i\,s_1\,(x,y,\theta)\}\,d\theta \tag{14}$$

where

$$s_1 = k_o\sec^2\theta\,\{(x-x_i)\cos\theta + y\sin\theta\} \tag{15}$$

x_i = the x coordinate of the bow or stern

$k_o = \frac{g}{U^2}$

g = acceleration due to gravity

$A(\theta)$ = amplitude function which is a function of source distribution m

The regular wave ζ is the solution of linear ship wave theory far from the ship.[11] Actually, it is easy to see that the exponential function satisfies both the Laplace equation and the linear free-surface boundary condition for any value of x_i. Havelock interpreted Equation (14) by discussing the integrand as elementary waves,[12,13] i.e., the regular waves are aggregates of elementary waves starting from the bow and stern of a ship. The normal direction of each elementary wave crest is $\bar{n} = \bar{i}\cos\theta + \bar{j}\sin\theta$.

Because the local disturbance of the double model decays rapidly away from the ship, even in nonlinear theory in the far field, the regular wave should be of the form of Equation (14) with possibly different values of x_i and $A(\theta)$. When the integral of ζ is evaluated by the method of the stationary phase[14] by taking roots of

$$\frac{\partial s}{\partial\theta}\,(x,y,\theta) = 0 \tag{16}$$

or

$$2\tan^2\theta + \frac{x}{y}\tan\theta + 1 = 0 \tag{17}$$

in general two values of θ are obtained for a given value of each x and y, satisfying,

$$\frac{x}{|y|} \geq 8^{\frac{1}{2}} \tag{18}$$

Furthermore, when

$$\frac{x}{|y|} = 8^{1/2}$$

$$|\tan\theta| = 8^{-1/2}$$

or

$$\theta = 35 \text{ deg}$$

and the value of the integral of Equation (14) for any x,y in $x/|y| \geq 8^{1/2}$ can be evaluated by the stationary phase method as the sum of two waves: transverse waves (0 deg < $|\theta|$ < 35 deg) and divergent waves (35 deg < $|\theta|$ < 90 deg).

Except for the amplitude function exactly the same result for the relation of x,y and θ as in the linear theory can be obtained from Equation (8) by substituting u = 1 and v = 0. Thus, this means that the energy of each elementary wave propagates on the uniform flow along a straight line ray. The rays are contined in $x/|y| \geq 8^{1/2}$, both of the elementary waves at θ = 0 and θ = 90 deg correspond to the ray y/x = 0, and θ = 35 deg corresponds with the ray $x/|y| = 8^{1/2}$. One ray between these two rays corresponds to two elementary waves where one is transversel and the other is divergent.

When the velocity is not uniform due to the flow perturbation caused by a ship, the ray is not straight but curved near the ship as shown in Figure 1. It is known that wave energy flux divided by the relative frequency with respect to the coordinates for which the fluid velocity is zero, is conserved along the curved ray tube.[8] If the coordinates are fixed in space, then the ship waves are unsteady relative to the fixed coordinates and the frequency is

$$s_t = U\, k \cos\theta \qquad (19)$$

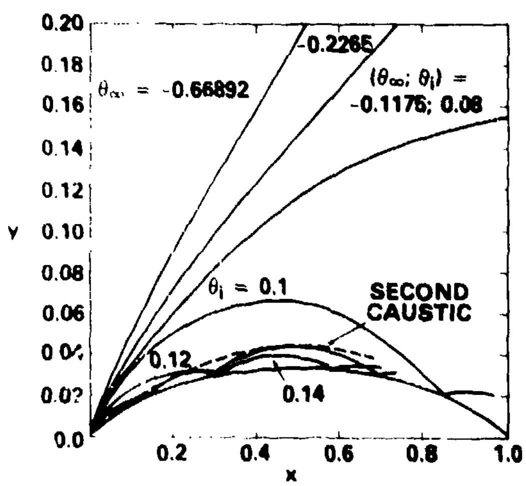

Figure 1 - Ray Paths for a Wigley Hull (b = 0.1, h = 0.0625)

because only the transformation of coordinates (x,y) to (x - Ut, y) is needed to get the unsteady flow. Strictly speaking the relative frequency is

$$s_t = k\,u \cos\theta + k\,v \sin\theta = k\,U \cos\theta$$

but for our discussion, the approximate value serves the purpose. This means that for a uniform flow the wave energy is constant along the ray because for a uniform flow u = U, v = 0, $k = g/(U^2 \cos^2\theta)$, and θ is constant along the straight ray. However, the wave energy is not constant along the curved ray in nonuniform flow, but is dependent upon the local velocity components and θ because the relative frequency is a function of the local velocity and θ as in Equations (5) and (19) and θ changes along the curved ray. The wave energy will change considerably along the curved ray near the bow or stern because u and v change to zero at the wave source, the stagnation point, while in the far field it will be constant along the ray as in linear theory. Near the stagnation point, the wave number increases according to Equation (5) and the wave energy also increases due to the energy conservation law.[8] Thus, the wave may break near the stagnation point.

Because wave amplitude is so difficult to obtain from the ray theory[2], the linear theory has been considered as an approximation.[1] A good result was obtained for a two dimensional flow problem.[9] However, extreme care is needed in the three dimensional theory because θ changes by a large amount along the ray, near the stagnation point, and the wave energy depends upon θ. The matching amplitude function of ray theory with the values from linear theory should be done at the far field where θ and the direction of the ray are, respecitvely, identical for both cases for each elementary wave. In addition, the initial condition has to be taken in the neighborhood, but not exactly at, the stagnational point because, although the stagnation point is the wave source, at the stagnation point the ray equations are indeterminant, u and v being zero. However, because of the large change of u and v near the stagnation point, the ray path is very sensitive to the initial values of x, y and θ; the nearer to the stagnation point, the more sensitive. Therefore, the identification of ray paths should be correlated with the values of θ at infinity. Then all the ray paths can be properly and uniquely identified by θ_∞.

Since perturbations of the ship decay rapidly away from the ship, θ also rapidly approaches θ_∞. The relation between the initial value and the value at infinity of θ has little meaning, although it was misunderstood before[1,6] because θ changes very rapidly near the stagnation point.

Rays of Ship Waves and Ship Boundaries

Ship waves created by a smooth ship hull propagate as regular bow and stern waves from the bow and stern stagnation points to infinity along rays. Because a ray carries the wave energy it cannot penetrate the ship surface. If the linear free-surface condition is considered with the exact hull boundary condition, as has been popular in recent ship wave analysis[11], the straight rays which pass through the hull boundary have to be considered. For a ray theory with the exact hull boundary condition, the ray is not allowed to penetrate the ship. In fact, when the initial wave crest touches the ship boundary it can be proved that the ray of such a wave grazes along the ship boundary without penetrating the ship boundary.

In Equation (8) when the wave crest touches the hull

$$u \cos\theta + v \sin\theta = 0 \tag{20}$$

because θ is the angle between the normal to the crest and to the x axis, and the velocity normal to the ship hull is zero on the hull. Equation (8) then becomes

$$\frac{dy}{dx} = \frac{v}{u}$$

showing that the ray touches the ship hull streamline from Equation (13).

When the wave crest touches the hull, from Equations (11) and (13)

$$\frac{v}{u} = f_x \tag{21}$$

and from Equations (20) and (21)

$$\tan\theta = \frac{-1}{f_x} \tag{22}$$

Differentiating Equation (22) with respect to x along the ship hull

$$\frac{d\theta}{dx} = \frac{f_{xx}}{f_x^{\,2} + 1} \tag{23}$$

However, inserting Equation (20) into Equation (9) yields

$$\frac{d\theta}{dx} = \frac{\cot\theta \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} - \cot^2\theta \frac{\partial u}{\partial y} - \cot\theta \frac{\partial v}{\partial y}}{u(1 - \frac{v}{u}\cot\theta)} \tag{24}$$

Differentiating Equation (21) with respect to x along the hull yields

$$\frac{\partial}{\partial x}\frac{v}{u} = \frac{-\frac{v}{u}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} - \frac{v}{u}\frac{\partial u}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial x}}{u} \tag{25}$$

From Equations (20), (21), (22), and (25) it can be shown that Equations (23) and (24) are equivalent. That is, when the wave crest touches the ship boundary, the ray equations and the ship hull streamline equations are equivalent.

When the wave crest is perpendicular to the ship hull

$$\frac{v}{u} = \tan\theta \tag{26}$$

If Equation (26) is inserted into Equation (8)

$$\frac{dy}{dx} = \tan\theta = \frac{v}{u} \tag{27}$$

The ray also touches the ship initially. However, Equation (9) is not compatible with Equation (26) on the hull. This can be proved in a similar way as follows:

Differentiating Equation (26) with respect to x along the ship hull

$$\frac{d\theta}{dx} = \frac{f_{xx}}{f_x^{\,2} + 1} \tag{28}$$

Inserting Equation (26) into Equation (9) gives

$$\frac{d\theta}{dx} = \frac{2\tan\theta\left(\frac{\partial u}{\partial x} + \tan\theta\frac{\partial v}{\partial y}\right) - 2\left(\frac{\partial u}{\partial y} + \tan\theta\frac{\partial v}{\partial y}\right)}{u(1 + \frac{v}{u}\tan\theta)}$$

From Equations (25), (27), and (29), noting

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

$$\frac{d\theta}{dx} = \frac{-2 f_{xx}}{f_x^{\,2} + 1} \tag{30}$$

Equations (28) and (30) are compatible only when $f_{xx} = 0$, or f_x = constant.

That is, only when the ship is a flat plate does the ray of the wave, whose crest is perpendicular to the ship, follow the ship boundary. This means that when the ship bow is like a wedge, the ray of the bow wave, whose crest is perpendicular to the wedge surface, initially follows the wedge surface.

When the ray equations are solved numerically by the Runge-Kutta method, with initial values near the bow stagnation point, the ray touches and follows the ship boundary at

$$\theta_i = -\frac{\pi}{2} + \alpha \tag{31}$$

where α is the half entrance angle of the ship bow and θ_i denotes the initial value of θ.

When θ_i increases from this value the ray moves gradually away from the ship as shows in Figures 1 through 4. The rays are curved near the ship but at far downstream they are straight and the ray direction becomes exactly the same function of θ as the linear theory. Thus at infinity the rays are inside $|dx/dy| = 8^{1/2}$. However, when θ_∞ approaches zero, the curved ray near the ship gradually approaches the ship, and eventually crosses the ship boundary, as in Figures 1 through 4. Here the wave reflection should be considered to prevent the ray penetration of the ship hull.

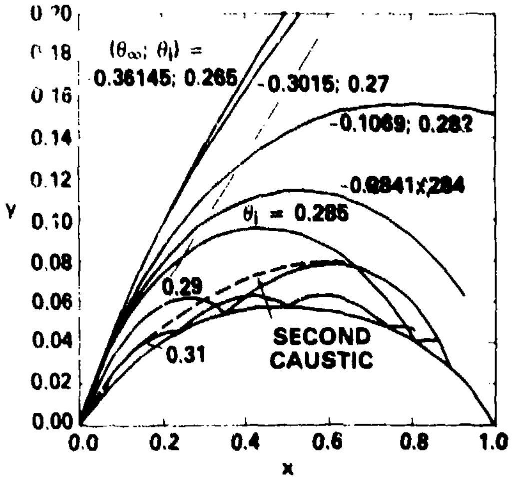

Figure 2 - Ray Paths for a Wigley Hull
(b = 0.2, h = 0.0625)

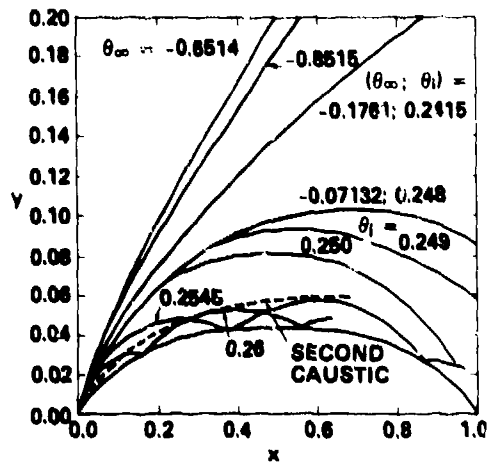

Figure 3 - Ray Paths for a Wigley Hull
(b = 0.2, h = 0.03)

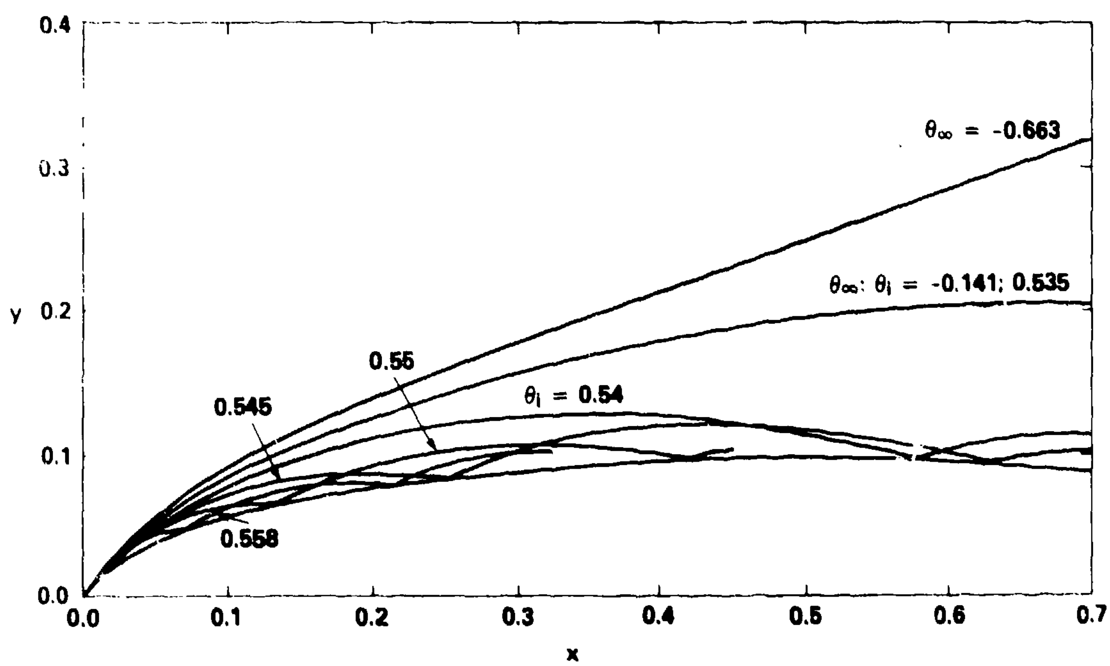

Figure 4 - Ray Paths for a Wigley Hull (b = 0.5, h = 0.0625)

Ray Reflection

There are no waves coming from places other than the ship bow or the ship stern. In the ray theory, the flow field perturbed by the ship deflects the ray path starting from the ship bow toward the ship. Thus, some rays of bow waves impinge on the ship hull. Because the ray theory is for small Froude numbers and the wave phenomenon is considered only on the free surface, it is reasonable to consider only reflected waves, as in geometrical optics, neglecting transmitted waves when the oncoming waves impinge on the ship hull.

Let the ray at the wave angle θ on the ship boundary (x,y) be reflected to θ_r and $f_x = \tan\theta_o$ as in Figure 5.

Then,

$$\theta_r - \theta_o = \theta_o - \theta$$

or

$$\theta_r = 2\theta_o - \theta \qquad (32)$$

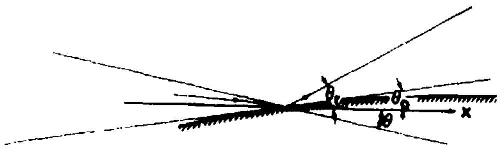

Figure 5 - Angles of Waves Reflecting on the Ship Hull

Whenever the ray intersects the ship boundary at (x,y) with angle θ, then the value of θ at (x,y) will be changed to $\theta_r = 2\theta_o - \theta$, and x,y,θ are continuously calculated by the Runge-Kutta's method. Then the ray will reflect as in Figures 1 through 4. Here if θ_o is zero, it is easily seen in Equation (8) that the impinging ray angle Ⓗ changes to −Ⓗ for the reflected ray angle. This fact can be easily shown to be true even for $\theta_o \neq 0$ by just the rotation of the coordinate system.

Numerical Experiments of Ray Paths and Free Surface Shock Waves

Because the ray equations can be solved only numerically, careful numerical experiments with the ray equations may give valuable information. For simplicity, the Wigley hull source distribution with

$$m = \frac{1}{2\pi}\frac{df_1}{dx}$$

where

$$f_1 = 2\pi b\ (1 - x^2)\ \{1 - (\frac{z}{h})^2\} \qquad (33)$$

is considered for various numbers of b and h which are related to the hull beam and draft, respectively. The actual hull shape corresponding to the source Equation (33), is obtained by plotting the body streamline passing through stagnation points as the solution of

$$\frac{dy}{dx} = \frac{v}{u}$$

where u and v corresponding to Equation (32), are shown in Appendix A together with u_x, u_y, and v_y for the ray equation. Equations (8), (9), and (10), together with the streamline equation, are solved by the Runge-Kutta method with initial conditions (x,y,θ) near the stagnation points with various values of θ.

Many ray paths both reflecting and nonreflecting from the surfaces of various Wigley hulls are shown in Figures 1 through 4. These paths were computed by a high speed Burroughs computer at David Taylor Naval Ship Research and Development Center. The reflection condition is incorporated in the high-speed computation with a routine to find the intersection of the ray and the ship boundary which is precalculated and saved in the memory. The step sizes of integrations and interpolation were determined after many numerical tests, and the shown results are considered to be reasonably accurate. For a given initial condition, the solution is stable and converges well.

In Figures 1 through 4, some common features of rays can be drawn as follows. The rays in $-\pi/2 < \theta_\infty < 0$ far behind the ship behave like rays of linear theory except wave phases are advanced in the ray theory and those near $\theta_\infty = 0$ are rays propagating from the ship bow and reflecting from those of ship hull. The rays near the ship are very different from the linear theory as Inui and Kajitani[1] pointed out. The curved rays from the bow have a far larger slope than those of linear theory and the phase of each ray is considerably advanced. The magnitude of the phase difference is more sensitive to the beam-length ratio and the draft-length ratio than the magnitude of the ray slope.

When $\theta_\infty = 35$ deg, the ray will be the outermost ray and the ray angle at ∞ will be approximately $\tan^{-1} 8^{-1/2}$ as in the linear theory and there is the corresponding initial value of θ or θ_i near the bow, or the origin. However, the θ_i which is corresponding to a single value of θ_∞ is very sensitive to small changes of x and y near the origin. At a fixed point near the origin there exists a unique correspondence between θ_i and θ_∞.

When from the θ_i which is corresponding to $\theta_\infty \doteq 35$ deg, the initial value of θ_i increases, θ_∞ also increases and the ray angle decreases. In general, when $\theta_i = 0$, θ_∞ is still a negative value. When θ_i increases further, θ_∞ approaches zero and the ray pass is very close to the stern. At this point, there exists a certain increment of θ_i which makes the ray barely touch the ship stern, at $\theta_i = \theta_{io}$. When θ_i slightly increases from θ_{io}, the ray reflects from the ship hull near the stern. With the increment of θ_i the reflection point moves toward the bow. When $\theta = \theta_{i1}$ the ray once reflected touches the stern again. When θ_i increases further from θ_{i1}, the ray reflects twice from the ship hull. In this way, further increment of θ_i makes the ray reflect from the ship hull three, four ... times. However, at θ_i near the value of θ_{iu}, the ray tries to penetrate the ship hull at the starting point of the bow. This cannot be allowed because this kind of ray should come from outside of the ship. Let the border point of θ_i be θ_{iu}. This means that rays of initial value of θ between $\theta_{io} < \theta_i < \theta_{iu}$ reflect from the ship hull. As is clear in Figures 1 through 4, in general, all the rays before reflection do not intersect each other. However, reflected rays intersect other rays. The once reflected rays intersect not only with each ray once reflected from ship boundary points close to each other, but also with at least one ray before reflection.

In the stationary phase, each ray has an amplitude. Likewise in the ray theory, each ray carries it's energy. The reflected ray may have approximately the same energy as the ray at $\theta_i \approx \theta_{io}$ or $\theta_\infty \approx 0$ where the amplitude function of the linear theory is in general more significant than amplitudes of the other values of θ_∞. Because the phase must be approximately close to each other for the waves near $\theta_\infty \to 0$, the wave height of the once reflected ray may be close to three times that of the transversal wave for $\theta_\infty = 0$. When the envelope of the once reflected waves is drawn, the domain bounded by the ship surface and the envelope, denoted by Dm, must be distinctly different from other domains because in Dm there are not only once reflected rays but also twic or multi-reflected rays on which more than three reflected rays intersect by an argument similar to that used for the once reflected rays. The envelope of the once reflected rays behaves like the shock front which was observed in Japan.[3]

In general, a line is called a caustic when on one side of the line one can find a continuous distribution of rays but not on the other side, and along the caustic the wave slope is found to be large. The wave near the ray angle θ_∞ = 35 deg is the caustic, and the wave height near the caustic far downstream can be obtained by an application of the Airy function in the linear theory. The envelope of the once reflected rays may be a kind of caustic formed by the refracted bow wave rays due to the nonuniform flow perturbed by the ship. Shen, Meyer, and Keller[5] studied such caustics caused by the sloping beach of channels and around islands. Thus, the additional caustic of ship waves may be called the second caustic of ship waves, and should not be confused with the first caustic which is the known caustic at θ_∞ = 35 deg.

Second Caustic of Ship Waves of Various Ships

For the Wigley hull, several different values of parameters b and h were taken to find their effect on the second caustic. In addition, the effect of a bulbous bow on the second caustic was considered. The most distinguishable physical characteristic of the second caustic is its distance from the ship hull. This is related to the distance of reflected rays from the ship hull. If the number of reflection rays increases, or if θ_i increases from θ_{io}, the maximum distance between the ray and the ship hull decreases. The distance, before or after the ray reflection, approximately behaves like a sine curve. The maximum distance between the ray before the first reflection and the ship hull divided by the x coordinate of the point of the first reflection a/x_1 is plotted in Figure 6 for various ships. The value of a/x_1 for different values of θ_i are approximately the same for a given hull and are related to the area between the second caustic and the ship hull where there may be breaking waves or turbulent waves. Thus, if the area is large, viscous dissipation of energy becomes large. Accordingly, the measured momentum loss behind the ship for the breaking waves becomes large.

The values of a/x_1, increase with increasing beam-length ratio. However, the most interesting part is the effect of the bulbous bow.[10] When the bulb size is increased or the doublet strength is increased the curvature of the ray near the bow becomes less, although the streamline near the bow is such that the entrance angle is slightly large The values of a/x_1 decrease with increasing bulb size, and eventually the ray for $\theta_\infty \approx 0$ cannot propagate without penetrating the ship hull at the beginning. That is, there is no reflecting ray coming out of the bow with a bulb of the proper size, as shown in Figure 7.

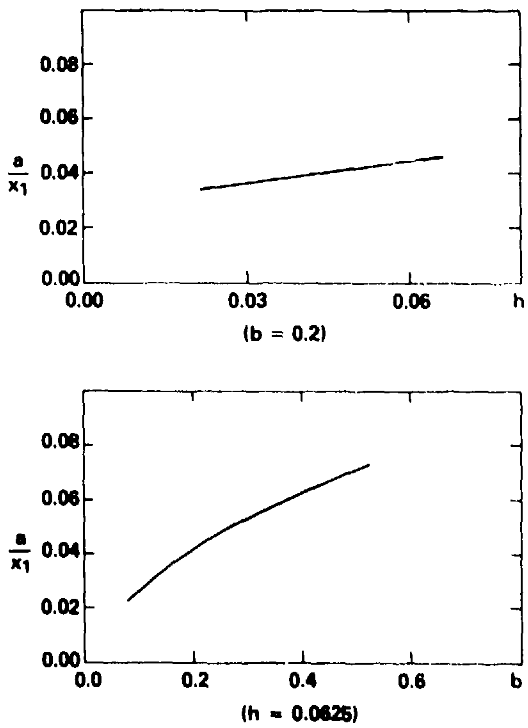

Figure 6 - Width of the Second Caustic a/x_1 for Wigley Hulls

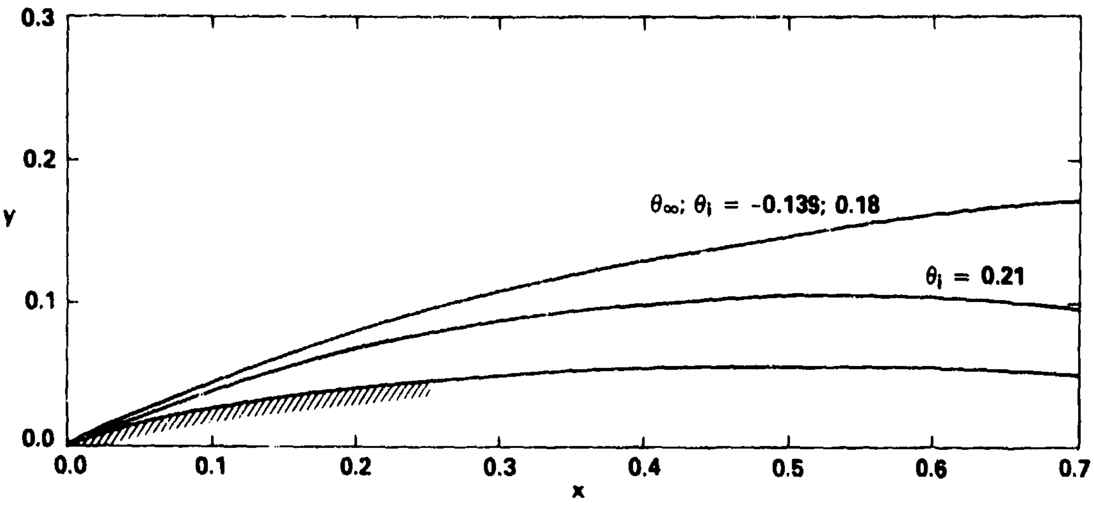

Figure 7 - Rays of the Wigley Ship ($b = 0.2$, $h = 0.0625$) with Bulb (radius, $r_b = 0.0214h$, depth, $z_1 = 0.5h$)

These phenomena associated with the second caustic are exactly the same as those of the "free surface shock wave" which was observed in the towing tank.

Finally it should be pointed out that the ray of the wave whose crest is perpendicular to the hull surface follows exactly along the flat surface as was proven by Equations (28) and (30) and the following paragraphs. This means that rays originating from the vertex of a wedge very likely never reflect on the wedge surface. This is because rays near the ray which follows the ship surface did not intersect each other near $\theta_i = -\pi/2 + \alpha$, or $\theta_\infty = -\pi/2$. Rays near $\theta_\infty \approx 0$ impinge on the ship surface due to the effect of the waterline curvature of the ship. Therefore, if a ship has a wedge bow, the second caustic must be found near or behind the shoulder. Because the first caustic near the bow may be very prominent and both waves near the first caustic and waves near the stagnation point break, careful experimental analysis and more theoretical study of the bow near field may be needed.

Wave Amplitude and Phase

Because the perturbation due to a ship, or both regular waves and local disturbances, decays at far field, the linear theory must hold in the far field. In particular, the wave resistance can be calculated from the energy passing through a vertical plane x= constant far downstream; the linear theory which is properly matched to the near field of the ship will be used for calculating the wave resistance. As explained in Equation (14) and the following paragraphs, the expression for regular waves far downstream is known to be of the form of Equation (14) where the amplitude function

$$A(\theta) = P(\theta) + i\,Q(\theta) \tag{34}$$

may be taken as an approximation from the linear theory but the phase difference x_1 must be obtained from matching with the near field. If the near field is also expressed by the linear theory

$$P + i\,Q = \pi k_o \int_{-h}^{o}\int_{o}^{1} dx\,dz\, f_x\, e^{k_o z \sec^2\theta} \sec^3\theta\, e^{i k_o x \sec\theta} \tag{35}$$

where f_x is the derivative of Equation (11) with respect to x. When the inner integrand of Equation (35) is integrated with respect to x, the value with the limit $x = 1$ will form stern waves and the value with the limit $x = 0$ will form bow waves.[11] Then the bow waves can be represented by

$$\zeta_b = \int_{-\pi/2}^{\pi/2} A_b(\theta) \exp\{i\, s_1(\theta)\}\, d\theta \tag{36}$$

where

$$A_b(\theta) = P_b + i\,Q_b \tag{37}$$

When the phase s is computed from Equation (10) along with the ray path from Equations (8) and (9) considering that $s = 0$ at the bow near the origin, there are two results different from those of linear theory: (1) the ray path is deflected as if the elementary wave of the linear theory started from $(x_1, 0)$ not from the origin, and (2) the phase change denoted by Δs should be considered. That is, the equivalent linear elementary wave may be written as

$$A_b(\theta) \exp\Big[i\,k_o \sec^2\theta \Big\{(x-x_1)\cos\theta + y\sin\theta\Big\} + i\,k_o\,\Delta s\Big] \tag{38}$$

where x_1 is obtained as an intersection of the tangent to the ray at ∞ and the x axis and

$$k_o\Delta s = k_o s - k_o \sec^2\theta \Big\{(x-x_1)\cos\theta + y\sin\theta\Big\} \tag{39}$$

The value of

$$s_2(\theta) = \Delta s - y_1 \sec\theta \tag{40}$$

can be obtained at any point along the ray. In general, x_1 is negative and $s_2(\theta)$ is positive meaning that the bow wave phase in the ray theory is larger or more advanced than the phase of the linear theory. This fact has long been observed in experiments in towing tanks.

The advancement of wave phase is computed for various ships and the values of s_2 and x_1 at $x = 2$ are shown in Figures 8 through 10. When the beam-length ratio increases and/or the draft-length ratio increases, the values of s_2 increase and the values of x_1 decrease for all values of θ_o. As compared with the increment of the slopes of rays near the ship, the increment of the phase angle is more sensitive to the beam- and/or draft-to-length ratios.

The most interesting phenomenon about the phase difference is in regard to the bulbous bow.[10] That is, the phase differences for hulls with and without bulbs are almost the same even with a considerably larger bulb. In the past, because of the observed phase difference of ship waves, the bulb was located far forward to obtain good bow wave cancellation.[11] According to the present analysis, if there is no other reason, the bulb position need not be far forward. Because the nonuniform flow created by the ship is much more significant than that of the bulb, as far as phase change is concerned, both the ship bow waves (in general, positive sine waves) and the bulb waves (negative sine waves) propagate through the same region and cancel each other.

As for the amplitude function, although it was shown by Doctors and Dagan[9] that ray theory produced the best result for a two-dimensional submerged body even though they used a linear amplitude function, the surface piercing three-dimensional case may be quite different. The amplitude function is mainly related to the singularity strength which satisfies the ship hull boundary condition and some improvement might result by considering the sheltering effect. However, in the present study, the linear amplitude function is used to simplify the problem, showing the effect of curved rays.

Wave Resistance

If all the elementary waves are assumed to be propagated without reflection, the amplitude function and the phase difference studied in the previous sections will supply enough information for the calculation of wave resistance. Because at far downstream, the wave height may be considered linear, the Havelock wave resistance formula[15] may be used for the waves represented by Equations (36) through (40), considering that the wave with the changed phase $s_{2b}(\theta)$ has the amplitude

$$(P_b + i\,Q_b)\, e^{ik_o s_{2b}(\theta)}$$

$$\frac{R}{\rho/2\; U^2 L^2} = C_w = \tfrac{1}{4}\int_{-\pi/2}^{\pi/2} \left|(P_b + iQ_b)\, e^{ik\, s_{2b}(\theta)}\right|^2 \cos^3\theta\, d\theta \tag{41}$$

or in Srettensky's formula[16]

$$C_w = \frac{16\pi^2 k_o}{w} \sum_{j=0}^{\infty} \epsilon_j \frac{1 + \left\{1 + \left(\frac{4\pi j}{k_o w}\right)^2\right\}^{\frac{1}{2}}}{\left\{1 + \left(\frac{4\pi j}{k_o w}\right)^2\right\}^{\frac{1}{2}}} \left|A_b(dj)\, e^{ik_1 s_{2b}(dj)}\right|^2 \tag{42}$$

where

$$A_b(dj)\, e^{ik_o s_{2b}(\theta j)} = \int_{-h}^{0}\int_{0}^{1} dx\,dz\; m \exp\left\{k_0 d_j (z d_j + ix) + ik_o s_2\right\}$$

$$m = \frac{1}{2\pi} f_x \tag{43}$$

$$dj = \left[\tfrac{1}{2} + \tfrac{1}{2}\left\{1 + \frac{4\pi j}{k_o w}^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{2}} = \sec\theta_j$$

w = width of the towing tank nondimensionalized by ship length L

$$\epsilon_o = 1$$

$$\epsilon_j = 2 \text{ for } j \geq 1$$

Because

$$\left|(P_b + iQ_b)\, e^{ik_o s_2(\theta)}\right|^2 = \left|P_b + iQ_b\right|^2$$

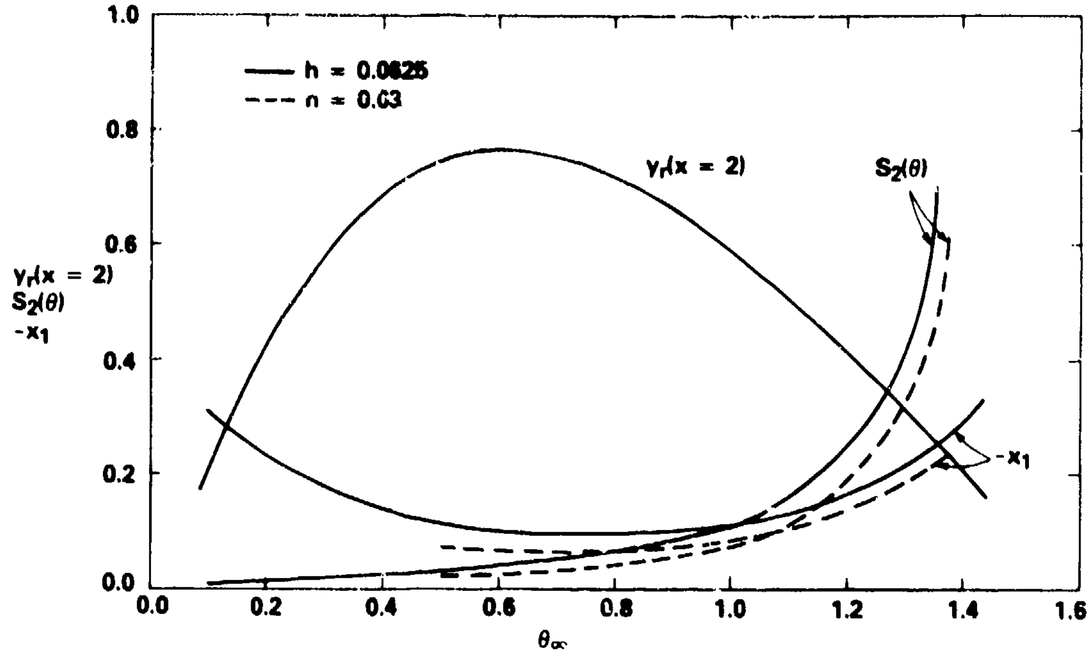

Figure 8 - The y Coordinate of Ray at x = 2, Phase Function $S_2(\theta)$ and the Starting Point of Elementary Wave, x_1 of a Wigley Hull (b = 0.2)

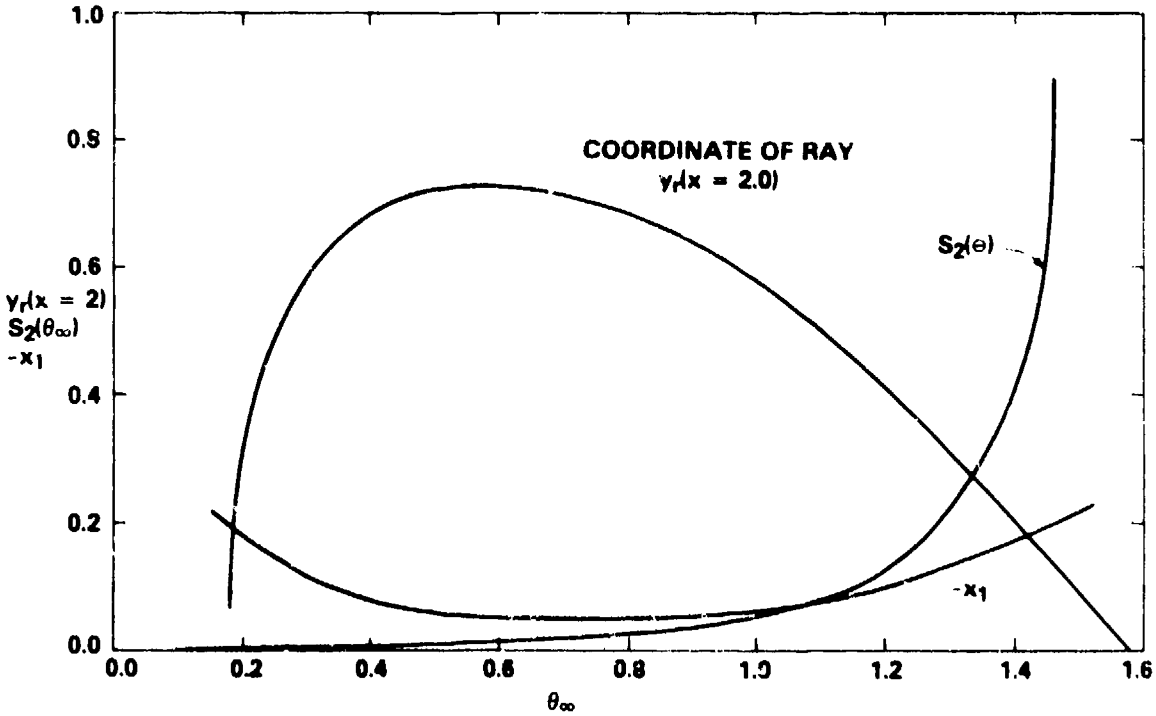

Figure 9 - y (x = 2), $S_2(\theta_\infty)$, and -x of a Wigley Hull (b = 0.1, h = 0.0625)

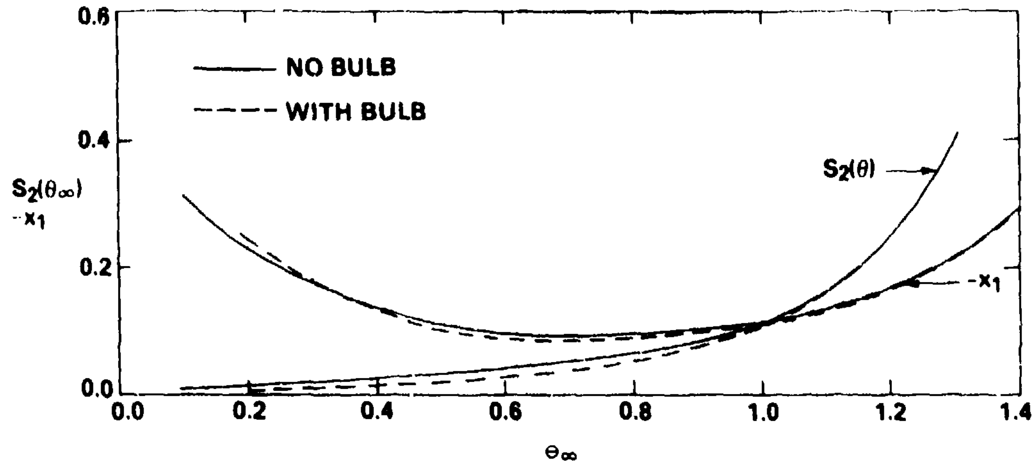

Figure 10 - $S_2(\theta_\infty)$, and $-x_1$ of a Wigley Hull (b = 0.2, h = 0.0625) with and without Bulb (r_b = 0.0285h, z_1 = 0.7h)

the phase difference of each elementary wave does not change bow wave resistance from the linear theory.

The stern wave amplitude is exactly the same as the linear value

$$A_s(\theta) = P_s + iQ_s$$

but the phase difference $s_{2s}(\theta)$ should be computed by the ray theory together with the ray path which is shown in Figure 11. Then it is also obvious that the stern wave resistance is the same as the linear stern wave resistance. The total wave resistance may be obtained similarly by considering that the total wave amplitude is

$$(P_b + iQ_b)\, e^{iks_{2b}(\theta)} + (P_s + iQ_s)\, e^{ik_0(s_{2s}(\theta) - \sec\theta)} \qquad (44)$$

Here, the bow and stern wave interaction appears in the wave resistance. That is, only the interaction term changes due to the phase change caused by the nonuniform flow. This fact is exactly the same as in two-dimensional theory.[9]

The actual computation of wave resistance is performed by the Srettensky formula using the relation

$$\theta_j = \cos^{-1}\left(\frac{1}{d_j}\right) = \tan^{-1}(d_j^2 - 1)^{1/2} \qquad (45)$$

and the corresponding values of s_2 are obtained by interpolation. When a portion of elementary waves near $\theta_\infty = 0$ is reflected from the ship hull, the larger part of the energy in this portion of elementary waves will be dissipated by breaking waves, and the wave resistance will decrease, but the momentum loss due to breaking waves will increase. If such energy was considered to be totally missing in the wave resistance, j in the Srettensky formula for bow waves would start not from zero but from a certain number minimum $J = J_1$ such that

$$|\theta_{j_1}| \geq |\theta_{\infty 1}|$$

where $\theta_{\infty 1}$ is the value of θ_∞ from which bow waves start to reflect. If $\theta_{\infty 1} = \theta_{j_1}$, $\varepsilon_{j_1} = 1$, and $\varepsilon_j = 2$ for $J > J_1$, and if $\theta_{j_1} > \theta_{\infty 1}$, $\varepsilon_j = 2$ for $J \geq J_1$.

The result is shown in Figure 12 where a considerable shift of the phase of hump and hollow of the wave resistance due to bow stern wave interaction is noticeable.

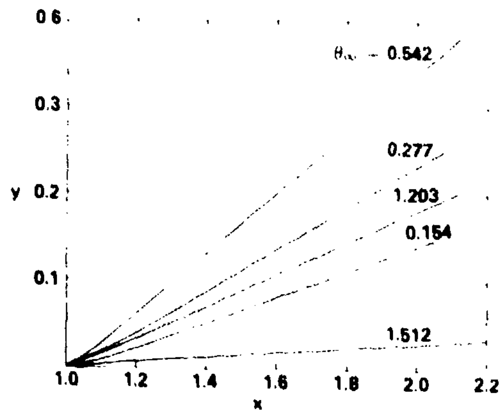

Figure 11 - Ray Paths of Stern Waves of a Wigley Hull (b = 0.2, h = 0.0625)

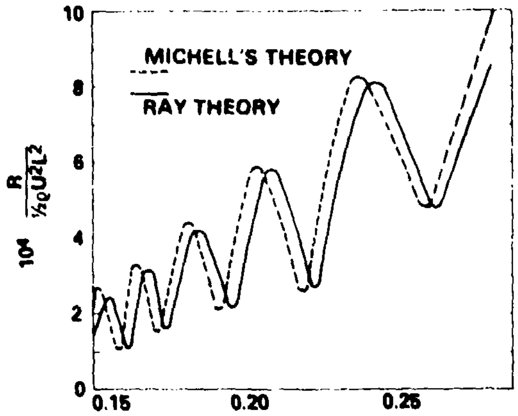

Figure 12 - Wave Resistance of the Wigley Ship ($b = 0.2$, $h = 0.0625$)

Discussion

Recently Eggers[17] obtained a dispersion relation from a low speed free-surface boundary condition which was a slightly modified version of the one Baba[18] used. Eggers showed that there was a small region near the stagnation point where the wave number became negative; since this was not permissible for a wave and it could be interpreted to mean that there was no wave in the region. He suggested that his form might help alleviate the sensitivity of the initial condition to the ray paths. According to the energy conservation law[8] of the wave propagating through nonuniform flow the wave energy flux is proportional to the wave number along the ray tube. When the wave number along a ray is considered, although the Keller dispersion relation has an infinite wave number only at the stagnation point, the Eggers dispersion relation has an infinite wave number in the flow near the bow. Thus, near such a singular point or a singular line, waves may break and the present theory cannot be applied. When Eggers[1] equation was incorporated into the ray equation in the present ray computer program it was found that the ray path was still very sensitive to a small change of initial value (x,y,…) and the curved ray still reflected from the ship hull.

In the experiments[7] conducted at Tokyo University, the second caustic can be noticed, and near the second caustic the flow field is violently different from the linear theory. However, it seems that extreme caution is needed to distinguish the second caustic from the first caustic. The case of experiments[3] with wedges is interesting because, according to the present theory, there cannot be a second caustic on the wedge although the first caustic should be there. However, the wave phases of the wedge should be advanced and quite sensitive to the draft- and beam-length ratios due to the nonuniform flow caused by the wedge. On the other hand, the present theory is still not exact although it takes into account the effect on the propagation of water waves of nonuniform flow due to the double model. Thus, the nonlinear effect of water waves is not totally analyzed here. Nevertheless, the present theory supplies a great deal of hope to an entirely different approach to the ship wave theory - the ray theory.

Although the strictly linear amplitude function is used for simplicity in the present study, a slightly improved amplitude function may be easily incorporated by adding the sheltering effect or other effects. However, it should be noted here that, in any case, the bow wave resistance and the stern wave resistance are the same as the results without the ray theory and the effect of the ray theory would appear as a shift of the phase of hump and hollow in the total wave resistance. Therefore, the computational results would not match the results of towing tank experiments unless the viscous boundary layer effect on the stern wave or some other effect is considered.[19]

Acknowledgements

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Appendix A

For the computation of ray paths of a ship, the flow velocity and its derivatives on (x,y,o), u, v, u_x, v_y, u_y are needed. A Wigley hull has the double model source distribution

$$m = b\,(-2x_1 + 1)\left\{1 - \left(\frac{z_1}{h}\right)^2\right\}$$

in

$$0 < x_1 < 1, \quad y_1 = 0, \quad h > z_1 > -h$$

Thus,

$$-u\,(x,y,o) = 2 \int_o^h \int_o^1 m \frac{\partial}{\partial x}\left(\frac{1}{r}\right) dx_1\, dz_1 - 1$$

$$= -2 \int_o^h \int_o^1 m \frac{\partial}{\partial x}\left(\frac{1}{r}\right) dx_1\, dz_1 - 1$$

$$= 2b \left\{ \int_o^h \frac{1 - \frac{z_1^2}{h^2}}{r\,(x_1 = 1)} dz_1 + \int^h \frac{1 - \frac{z_1^2}{h^2}}{r\,(x_1 = o)} dz_1 \right\} - 1$$

$$+ 4b\left[\frac{2}{3} h \log\left\{x_1 - x + r(z_1 = h)\right\} - \right.$$

$$\int_0^h \frac{(x_1 - x)\, y^2}{(z_1^2 + y^2)\, r}\, dz_1$$

$$+ \int_0^h \frac{(x_1 - x)}{r}\, dz_1$$

$$- \frac{1}{3h^2}\int_0^h \left\{ y^4 \frac{(x_1 - x)}{(z_1^2 + y^2)\, r} - \frac{(x_1 - x)(z_1^2 - y^2)}{r}\right\} dz_1 \Bigg]_{x_1 = 0}^{x_1 = 1}$$

$$-v(x,y,0) = 2 \int_0^h \int_0^1 m \frac{\partial}{\partial y}\left(\frac{1}{r}\right) dx_1\, dz_1$$

$$= -2\, by \int_0^h \Bigg[\frac{2x^2 - 3x + 1}{(z_1^2 + y^2)\, r(x_1 = 1)}\left(1 + \frac{y^2}{h^2}\right) - \frac{2x^2 - 3x + 1}{r(x_1 = 1)\, h^2} - \frac{2z_1^2}{r(x_1 = 1)\, h^2} + \frac{2}{r(x_1 = 1)} - \frac{2x^2 - x + 2z_1^2}{r(x_1 = 0)\, h^2} - \frac{2}{r(x_1 = 0)}\Bigg] dz_1$$

$$-u_x = -4b \log\left|\frac{r(x_1 = 1,\ z_1 = 0)}{r(x_1 = 0,\ z_1 = 0)}\right|$$

$$+ \frac{4}{h^2}\Bigg[-\left(\tfrac{1}{2} z_1\, r(x_1 = 1) - \frac{a_1^2}{2}\log|z_1 - r(x_1 = 1)|\right)\frac{(x-1)}{a_1^2} + 2\left\{\left(\frac{z_1^2}{2} + \frac{a_1^2}{4}\right)\log|z_1 + r(x_1 = 1)| - \frac{z_1}{4}\, r(x_1 = 1)\right\}$$

$$- \frac{x}{a_0^2}\left(\tfrac{1}{2} z_1\, r(x_1 = 0) - \frac{a_0^2}{2}\log|z_1 + r(x_1 = 0)|\right) - 2\left\{\left(\frac{z_1^2}{2} + \frac{a_0^2}{4}\right)\log|z_1 + r(x_1 = 0)| - \frac{z_1}{4}\, r(x_1 = 0)\right\}\Bigg]_{z_1 = 0}^{h}$$

where $a_1^2 = (x-1)^2 + y^2$

$a_0^2 = x^2 + y^2$

$$-u_y(x,y,0) = 2 \int_0^h \int_0^1 m \frac{\partial^2}{\partial y \partial x}\left(\frac{1}{r}\right) dx_1\, dz_1$$

$$= -2by\Bigg[\frac{h}{\{(1-x)^2 + y^2\}\, r(1,h)} + \frac{1}{r(1,h)\, h} + \frac{1}{r(0,h)\, h} - \frac{1}{h^2}\log\frac{(h + r(1,h))\{h + r(0,h)\}}{r(1,0)\cdot r(0,0)} + \frac{h}{(x^2+y^2)\, r(0,h)}\Bigg]$$

$$+4by\Bigg\{\int_0^h \frac{(1-x)\left(1 + \frac{y^2}{h^2}\right)}{(z_1^2 + y^2)\, r(x_1 = 1)}\, dz_1 + \int_0^h \frac{x\left(1 + \frac{y^2}{h^2}\right)}{(z_1^2 + y^2)\, r(x_1 = 0)}\, dz_1 - \frac{1}{h^2}\int_0^h \frac{1-x}{r(x_1 = 1)}\, dz_1 - \frac{1}{h^2}\int_0^h \frac{x}{r(x_1 = 0)}\, dz_1\Bigg\}$$

$$-V_y = b\left(6y^2 + \frac{6y^4}{h^2}\right) \times$$

$$\times \left[\frac{1}{3} (1-2x) \left(\frac{1}{x_1 - x} + \frac{(x_1-x)^2 - y^2}{(x_1-x)\, y^2} \right) \right.$$

$$\times \int_0^h \frac{dz_1}{(z_1^2+y^2)\, r}$$

$$+ \frac{2}{3} \frac{h}{\left\{(x_1-x)^2 + y^2\right\} r(z_1=h)}$$

$$+ \frac{1}{3}(1-2x) \left(\frac{-h}{(x_1-x) \left\{(x-x_1)^2+y^2\right\} r(z_1=h)} \right.$$

$$\left.\left. + \frac{h + (z_1=h)}{(h^2 + y^2)\,(x_1-x)\, y^2} \right) \right]_{x_1=0}^{1}$$

$$- 6\, by^2 \left[\frac{1}{3h^2} \left(\frac{2y^2 + (1-2x)\,(x_1-x)\, h}{(x_1-x)^2 + y^2 \quad r(z_1=h)} \right. \right.$$

$$\left. + 2 \log \frac{h + r(z_1=h)}{r(z_1=0)} - \frac{2h}{r(z_1=h)} \right)$$

$$\left. + \frac{2(x_1-x)\,(1-2x)}{3h^2} \int_0^h \frac{dz_1}{(z_1^2 + y^2)\, r} \right]_{x_1=0}^{1}$$

$$- \frac{v\,(x,y,0)}{y}$$

where $r(a,b) \quad r(x_1=a,\ z_1=b)$

In these expressions the integrals

$$\int^h \frac{z_1}{r}\, dz_1$$

and

$$\int \frac{dz_1}{z_1^2 + y^2\, r}$$

are available in closed forms.[20]

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