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Research Report CCS 400

PRELIMINARY REPORT 1 ON RAPID RESPONSE ALGORITHMS FOR OPTIMIZING THE UTILIZATION OF HUMAN RESOURCES IN FLIGHT CREWS: SCHEDULING AIRCREWS TO AIRCRAFTS

by

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1. THE PROBLEM

Consider an airlift operation which consists of several routes, each having missions which are subject to given time schedules. The distinction between a mission and a route is that the latter is a purely physical entity, whereas a mission is a route with the associated attributes of aircraft and starting time. Consider the situation in which the scheduling period (cycle) for each mission commences and terminates at the same location which is called "home base." It is further assumed that within the scheduling cycle each aircraft completes exactly one mission.[1]

The aircraft are manned with aircrews that are required to rest for a certain period of time after each leg of a mission. A mission may be continued whenever a rested aircrew is available at the location. Given the number of missions that are needed to be flown on the different routes, and given the schedule timetable that is associated with those missions, we consider the problems:

(1) What is the minimum number of crews that are needed to maintain the operation?

(2) How many aircrews are needed to be staged at each location?

(3) If the number of available aircrews is less than the minimum needed, which legs of what missions may be delayed so that the minimum required number of aircrews is reduced? Cont'd

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[1] Actually, a single aircraft may be scheduled to complete more than one mission, but we shall consider it here as a single, multiple loop route, mission.

We will exclude from the analysis the trivial case where the rest period of the aircrews is always less than the period of time for which the aircrafts are delayed.

If this is the case, then it is clear that the minimum number of aircrews is equal to the number of missions and all the aircrews must be staged initially at the home base.

2. TIME EXPANDED NETWORK

It is assumed that all changes in the system take place at integer multiples of a basic time unit which will be taken as 1 for simplicity. Thus, the scheduling cycle is discretized so that the system is fully described by its states at time 0, 1, 2, ...

Suppose the aircraft operation covers L locations B^{ℓ}, $\ell = 1 \ldots L$ and is scheduled over a cycle of T time periods. Since the number and the schedule of all missions are given, the aircrews' scheduling problem is fully determined by the departure times of the legs of the missions and by the availability of aircrews in the various locations at the different time periods. A time-expanded network that depicts the aircrews' scheduling problem may be constructed as follows:

"Aircrew" network

The following gives a description of the network which depicts the travel of crews throughout the airlift.

Nodes

A node $B^{\ell}(j)$ represents location B^{ℓ}, $\ell = 1 \ldots L$ at time period j, $j = 1 \ldots T$.

Arcs

Two types of arcs are defined in this network; inter-location arcs and intra-location arcs. An inter-location arc has the form $(B^k(i), B^\ell(j))$, $i + r < j$, $k \neq \ell$, where r is the rest period for an aircrew. This arc indicates the existence of at least one aircraft that is scheduled to leave B^k at time peirod i and arrive at B^ℓ at time period $j - r$.

Each aircrew that had flown an aircraft on this leg is available for another flight from location B^ℓ at time period j. (Note that there is a one-to-one correspondence between an "aircrew" arc defined above and an "aircraft" arc that simply defines the departure and arrival times of a leg.) An intra-location arc is of the form $(B^\ell(i), B^\ell(j))$, $i < j$, $\ell = 1...L$.

An arc of this type represents available aircrews that are delayed (after finishing their rest peirod) from time period i to time period j at location B^ℓ. For each inter-location arc, $(B^k(i), B^\ell(j))$, we associate a number $C^{k\ell}(ij)$ which is equal to the number of aircrafts scheduled to depart from B^k at time period i and arrive to B^ℓ at time period $j - r$.

Example

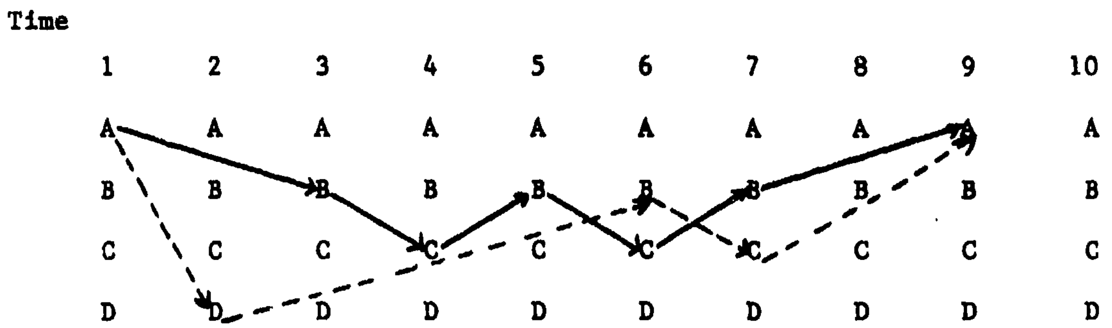
Suppose the airlift operation consists of two routes, four locations, and nine time periods. There is one mission defined for each route. The following table details the two missions along with their time table.

TABLE 1

	FROM		TO	
	Location	Period	Location	Period
Route 1	A	1	B	3
	B	3	C	4
	C	4	B	5
	B	5	C	6
	C	6	B	7
	B	7	A	9
Route 2	A	1	D	2
	D	2	B	6
	B	6	C	7
	C	7	A	9

The "Aircraft" network that depicts the above operation in terms of the missions' flight legs alone is given in Figure 1.

Figure 1

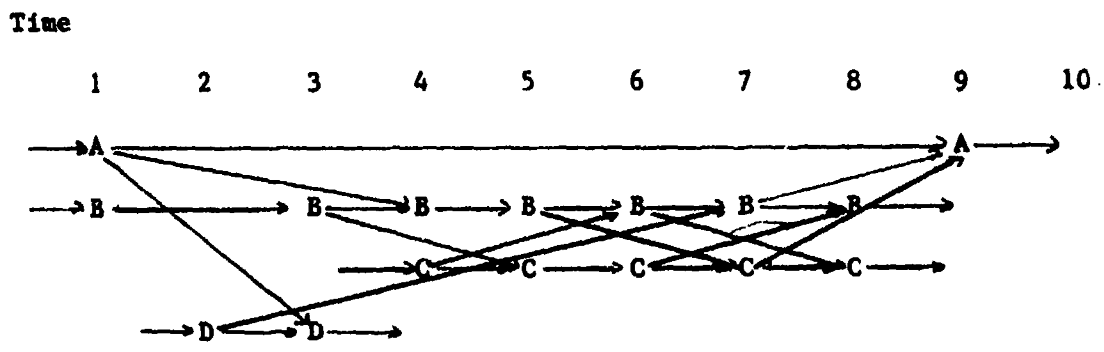


__________ route 1

--------- route 2

Next, we need to adjust the above network to account for crews' rest time. Suppose the rest period after each leg is one time period; then the adjusted "Aircrew" network defined above has the following form:

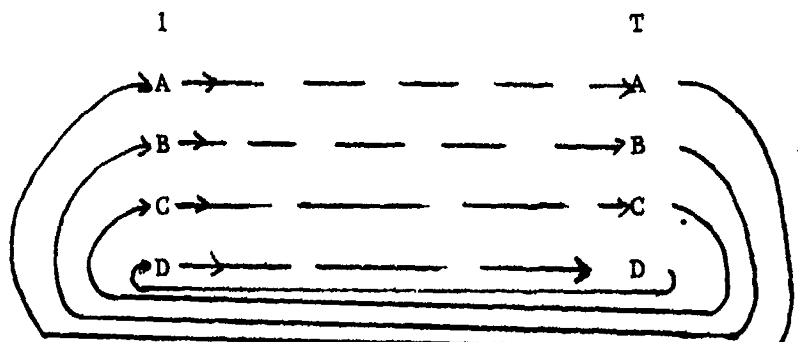
Figure 2



The airlift operation is cyclic in the scheduling horizons; therefore, the number of aircrews left in any location at the end of one scheduling cycle must be equal to the number needed in the same location at the beginning of the next scheduling cycle.

The problem is to find a minimum flow on the circulation network defined by the "Aircrew" network (Figure 2) with the additional arcs $(B^{\ell}(T), B^{\ell}(1))$ $\ell = 1. . .L$, See Figure 3, below.

Figure 3



In linear programming formulation the crew minimization problem may be stated as:

$$\text{Minimize} \sum_{\ell=1}^{L} f(B^{\ell}(T),\ B^{\ell}(1))$$

s.t.

$$\sum_{j=1}^{T} \sum_{\ell=1}^{L} f(B^{i}(i),\ B^{\ell}(j)) - \sum_{j=1}^{T} \sum_{\ell=1}^{L} f(B^{\ell}(j),\ B^{k}(i)) = 0$$

$$i = 1,\ldots,T$$

$$k = 1,\ldots,L$$

(1)

$$c^{k\ell}(ij) \le f(B^{k}(i)\ ,\ B^{\ell}(j)) \le C^{k\ell}(ij)$$

$$f(B^{\ell}(i),\ B^{\ell}(j)) \ge 0 \qquad j > i,\ \ \ell = 1,\ldots,L$$

where $f(B^{k}(i), B^{\ell}(j))$ is the flow on arc $(B^{k}(i), B^{\ell}(j))$

Problem (1) may be solved by an appropriate network algorithm; however, the special structure of the model can be exploited to obtain a simpler and more efficient solution procedure.

In addition, through this procedure one can identify the system's bottlenecks. This will yield insights into where and when to delay an aircraft such that the delay will reduce aircrew requirements and have a minimal effect on the airlift schedule.

The details of this procedure are given in the next section.

3. THE PROCEDURE FOR DETERMINING THE MINIMUM NUMBER OF CREWS AND THEIR ALLOCATION

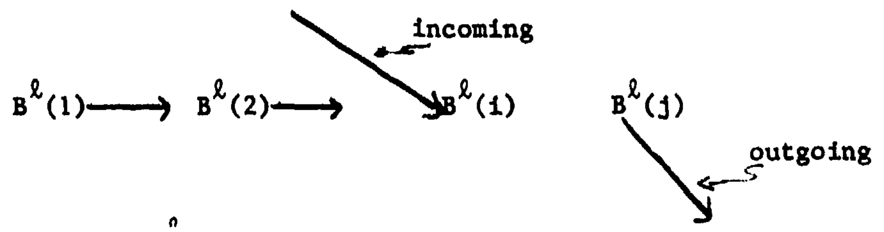
The basic idea of the following procedure is similar to the one proposed by Bartlett [1] in an algorithm for determining the minimum number of transport units to maintain a fixed schedule cyclic in time.

Define

(3.1) $$\overline{B}^{\ell} \equiv \{B^{\ell}(1),\ B^{\ell}(2),\ldots,B^{\ell}(T)\} \quad \ell = 1,\ldots,L$$

to be the set of all nodes in the adjusted "Aircrews" network that corresponds to location B^ℓ. We divide the set of interlocation arcs into two sets; outgoing arcs and incoming arcs. An outgoing arc of $\overline{B}^\ell$ is an arc that is rooted in one of the nodes in $\overline{B}^\ell$ and is connected to a node in $\overline{B}^k$, for $k \neq \ell$, i.e. an arc of the form $(B^\ell(j), B^k(i))$, $i > j+r$. An incoming arc of $\overline{B}^\ell$ is an arc that is rooted in some node in $\overline{B}^k$, $k \neq \ell$, and its end point is in $\overline{B}^\ell$, i.e. an arc of the form $(B^k(i), B^\ell(j))$. (See Figure 4)

<u>Figure 4</u>



For each node $B^\ell(j)$ we associate a number which is equal to the excess of outgoing flow over incoming flow. In other words, the difference between the total number of flights scheduled to depart from B^ℓ at time period j and total number of flights scheduled to arrive to B^ℓ at time period j - r. Namely

(3.2)

$$\delta_j^\ell = \sum_{k\neq\ell} \sum_{i>j+r} C^{\ell k}(ji) - \sum_{k\neq\ell} \sum_{i<j-r} C^{k\ell}(ij)$$

We now define Δ_j^ℓ as the cumulative difference between the outgoing and incoming flow at $B^\ell(j)$.

(3.3) $$\Delta_j^\ell = \sum_{i=1}^{j} \delta_i^\ell$$

and Δ^ℓ as the maxirum cumulative difference at location B^ℓ.

(3.4) $$\Delta^\ell = \underset{j=1...T}{\mathrm{Max}} \Delta_j^\ell$$

Property 1

For all locations B^ℓ

(3.5) $$\sum_{j=1}^{T} \delta_j^\ell = 0 \qquad \ell = 1,\dots,L$$

(3.6) $$\Delta^\ell \geq 0 \qquad \ell = 1,\dots,L$$

Proof

Within a scheduling cycle all routes (missions) start and terminate in the same location (home-base), therefore the number of incoming flights in each location must be equal to the number of outgoing flights. From (3.3) and (3.5) it follows that

(3.7) $$\Delta_T^\ell = 0 \qquad \ell = 1,\dots,L$$

Therefore

(3.8) $$\Delta^\ell = \underset{j=1,\dots,T}{\mathrm{Max}} \Delta_j^\ell \geq 0$$

QED

Property 2

Δ^{ℓ} is equal to the minimum number of aircrews that are needed to be available in location B^{ℓ} at the beginning of the scheduling cycle.

Proof

Suppose $\Delta^{\ell} = \Delta^{\ell}_{j_{\ell}} = m$

That is, the maximum excess of departures over arrivlas is obtained in the j_{ℓ}-th time period and is equal to m. In other words, by time period j_{ℓ}, m flights scheduled to leave B^{ℓ} will not have crews unless m aircrews are staged at location B^{ℓ}.

If we consider the m crews that are staged initially at B^{ℓ} as m arrivals at time period $i = 1$, then from (3.3) and (3.4) it follows that for each time period j the difference between departures and arrivlas is non-positive, which implies that the schedule is flexible.

QED

Property 3

The minimum total number of aircrews needed to maintin the schedule is

$$M = \sum_{\ell=1}^{L} \Delta^{\ell} \qquad (3.9)$$

Proof

Follows directly from Property 2

QED

As we have already seen in Section 2, the cyclic nature of the airlift operation problem dictates additional constraints on the system;

that is, the number of aircrews left at any location at the end of a scheduling cycle should match the number needed at that location at the beginning of the next cycle. The next property shows that this requirement is satisfied by the procedure.

Property 4

Let $B^\ell(k_\ell)$ be the last (in terms of time periods) node of $\tilde{B}^\ell$ such that it is an end point of an interlocation arc (a node of either incoming or outgoing arcs). For a feasible schedule,

$$f\left(B^\ell(k_\ell),\ B^\ell(k_\ell + 1)\right) = \Delta^\ell \qquad (3.10)$$

Proof

CASE 1: $\Delta^\ell = m = 0$

No crews are staged at location B^ℓ at the beginning of the scheduling cycle. But, from Property 1

$$\Delta^\ell_{k_\ell} = 0 \qquad (3.11)$$

Therefore

$$f\left(B^\ell(k_\ell),\ B^\ell(k_\ell + 1)\right) = 0$$

CASE 2: $\Delta^\ell = m > 0$

According to Property 2, at least m aircrews must be staged at B^ℓ at the beginning of the scheduling cycle. Consider an augmented network with m arrivals at time 1. The new Δ'^ℓ_j for the augmented network satisfied

$$\Delta'^\ell_j = \Delta^\ell_j - m \qquad j = 1,\ldots,T \qquad (3.12)$$

In particular

(3.13) $\Delta'^{\ell}_{k_\ell} \quad \Delta^{\ell}_{k_\ell} - m$

But since $\Delta^{\ell}_{k_\ell} = 0$ (Property 1), it follows that

(3.14) $\Delta'^{\ell}_{k_\ell} = -m$

which means that at the last node with either incoming or outgoing arc the cumulative difference between the flow in incoming arcs and outgoing arcs is positive (= m).

To satisfy the network balance equations we must have

(3.15) $f(B^{\ell}(k_\ell), B^{\ell}(k_\ell + 1)) = m$

QED

Corollary 1

The flow on $(B^{\ell}(T), B^{\ell}(1))$ is equal to Δ^{ℓ}.

Aircraft Delays

There are some tradeoffs in this model between the minimum number of aircrews needed and the delay structure of the aircrafts' mission.

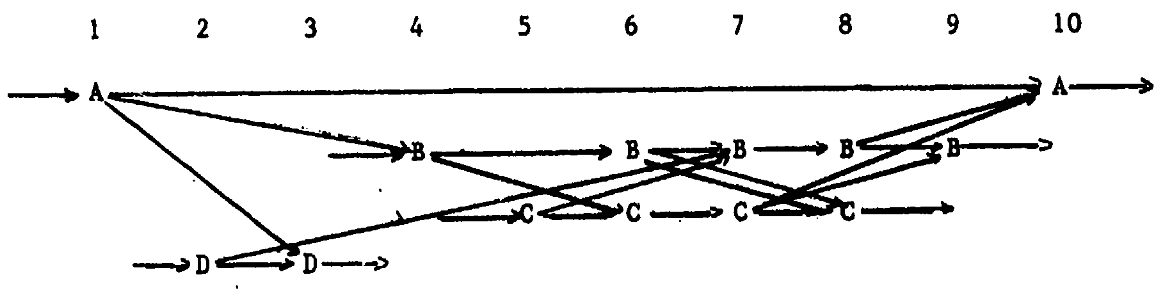
Delaying aircrafts at some locations or stretching the scheduling cycle horizon may yield a reduction in the number of aircrews. It is, however, possible for aircraft delays to increase the number of crews required. For example, consider the airlift operation which corresponds to the aircrew network in Figure 2. The Δ^{ℓ}'s are:

$\Delta^A = 2, \quad \Delta^B = 1, \quad \Delta^C = 1, \quad \Delta^D = 1$

and $M = \Delta^A + \Delta^B + \Delta^C + \Delta^D = 5$

Delaying the aircraft in route 1 from B(3) to B(4) yields the following aircrew network:

Figure 5



Here,

$$\Delta^A = 2,\ \Delta^B = 2,\ \Delta^C = 1, \Delta^D = 1$$

and

$$M = 6$$

Hence, delaying an aircraft resulted in a larger number of aircrews.

Next it is shown that for certain locations and time periods it is impossible to reduce the number of aircrews no matter how long the aircrafts are delayed. The following definitions are needed.

(3.16) $$\hat{j}^{\ell} \equiv \text{Max}\ \{j;\ \Delta_j^{\ell} = \Delta^{\ell}\}$$

and

(3.17) $$\hat{j} = \underset{\ell}{\text{Max}}\ \hat{j}^{\ell}$$

Property 5

No matter how long an aircraft is delayed in $B^{\ell}(j)$, $\ell = 1,\ldots,L$, $j \geq \hat{j} + 1$, no reduction in the minimum number of aircrews can be achieved.

Proof

Delaying one aircraft at location/time period $B^{\ell}(j)$ is equivalent to reducing δ_j^{ℓ} by one unit. But doing this for time periods which are past $\hat{j}^{\ell}$, $\ell = 1,\ldots,L$ means that Δ^{ℓ} is not changed for all $\ell = 1,\ldots,L$, and according to Property 2, no reduction in the number of aircrews needed can occur.

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