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Error statistics for astrogeodetic positions for an RGSS test course

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In the experimental mobile inertial survey system, geodetic parameters are measured relative to an origin point. Certain errors build up along the survey course. Typically, these errors are adjusted and distributed both during and after completing each mission. During testing, the adjusted derived data are compared with "known" data that are determined by conventional "classical" surveys. In its research, ETL requires a more accurate adjustment of errors obtained from conventional methods. → cont.

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The approach to the problem involved evaluating and analyzing the various error sources in the conventional data recalculation of certain data by new methods and developing a new statistical error treatment that applies not only to the objectives of this in-house research, but to the general application of inertial geodetic systems.

In this report, a new statistical error procedure has been developed to improve astro-geodetic positions that are useful in ETL's core program of RGSS tests and in any future tests of inertial systems.

PREFACE

This study was conducted under DA Project 4A161101A91D, Task 01, Work Unit 76, "Error Statistic for Astrogeodetic Positions for RGSS Test Course."

The study was done during 1979 under the supervision of Mr. Melvin Crowell, Jr., Director, Research Institute.

COL Daniel L. Lycan, CE was Commander and Director and Mr. Robert P. Macchia was Technical Director of the Engineer Topographic Laboratories during the study period.

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ERROR STATISTICS FOR ASTROGEODETIC POSITIONS FOR AN RGSS TEST COURSE

INTRODUCTION

The objective of this research is to provide error statistics for the series of astrogeodetic positions in a test course required to evaluate the U.S. Army Engineer Topographic Laboratories (ETL) Rapid Geodetic Survey System (RGSS).

The National Geodetic Survey (NGS) of National Oceanic and Atmospheric Administration (NOAA), U.S. Department of Commerce, observed and computed the astronomic latitudes and longitudes for the ETL test course. The latitudes and longitudes were obtained at each station of the test course from observations taken on two nights with a Wild T–4 Universal theodolite and Datametrics model S–P 300 digital timing system.

Latitude determinations were made using a modified version of the Sterneck method. Longitude determinations were made using the meridian transit method. Time synchronization for the Datametrics time system was maintained from radio signals transmitted by the National Bureau of Standard Time Service Station (WWV) located at Fort Collins, Colorado. Stellar positions taken from the Fourth Fundamental Catalogue (FK4) were used for computing latitudes and longitudes. For latitude determination, the modified Sterneck method involves two zenith distance measurements of both occular positions that are symmetrically east and west of the meridian. The time of the stellar bisection is recorded. The latitude determination depends on observations of 32 stars with occurrences divided north and south of the zenith. Stellar positions taken from the Fourth Fundamental Catalogue were used for reducing astronomic latitudes and longitudes.

ANALYSIS OF ERRORS IN LATITUDE DETERMINATION BY THE STERNECK METHOD

ANALYSIS OF ERRORS IN LATITUDE • Erroneous latitude determinations result from observing zenith star angle errors that are due to personal equation of bisection, index error, refraction uncertainties, and scale reading errors from inaccurate graduation line coincidence.

The index and bisection errors are cancelled when two observations are made in the direct and reverse theodolite position. In the modified Sterneck latitude method, each star is observed symmetrically east and west of the meridian plane. Corrections are applied to the observed zenith distances and reduced as the zenith distances are observed in the meridian plane. Thus, the latitude can be determined from the general formula:

$$\phi = \delta \pm Z \tag{1}$$

where δ is the star's declination, and Z is the mean value of observed zenith distances after applying all the corrections. Latitude is determined from observations of 32 stars with occurrences distributed north and south of the zenith.

Evaluation of Personal Error. After reducing the star zenith distances from direct and reverse instrument position to a common plane of references, one can compute the index error. Let Z_1 be the observed zenith distance in the direct position measured 0° ➡ to 90° and Z_2 be the zenith distance on the reverse instrument position measured 0° ➡ to 270°. Let ϵ be the instrumental index error that results from observing a star. The personal error is small and can be treated as an index error because the observer sees the star before or after the star crosses the horizontal reticle line. The total effect of the instrument index error and personal error is considered as $(\epsilon + p)$.

A fixed object with direct and reverse zenith distance observations gives the following equation of condition:

$$Z_1 + Z_2 + 2(\epsilon + p) = \pi \tag{2}$$

where Z_1 refers to the direct observation and Z_2 to the reverse instrument position. The value of p differs when the observation is made with respec. to a moving object, and $(\epsilon + p)$ must be referred to as star observations. Owing to the star motion within the zenith distance observations, corrections are to be introduced to the values Z_1 and Z_2 to satisfy equation (2). This can be made by reducing Z_1 and Z_2 to a common instant of time, T_0. Let T_1 and T_2 equal the references times that correspond to the observation of Z_1 and Z_2 respectively. Applying corrections ΔZ_1 and ΔZ_2, reduce Z_1 and Z_2 to the time T_0. But to fit equation (2), one must include the level and refraction correction also. Let R_1 and R_2 equal the refraction corrections to Z_1 and Z_2, respectively. Therefore, the level corrections are $\rho/2\ b_1$ and $\rho/2\ b_2$. After adding the corrections, equatio: (2) changes to

$$Z_1 + Z_2 + R_1 - R_2 + \frac{\rho}{2}(b_1 - b_2) + \Delta Z_1 - \Delta Z_2 + 2(\epsilon + p) = 2\pi \qquad (3)$$

P = Level value in seconds of arc per division of the vial

East star provides an equation where ΔZ_1 and ΔZ_2 are known through computation and the personal equation is the only unknown to be considered. If n is the number of observed stars, the personal equation is derived from

$$p = \pi - \frac{1}{2n}[\Sigma(Z_1 + Z_2) + \Sigma(R_1 - R_2) + \frac{\rho}{2}(\Sigma(b_1 - b_2) + (\Sigma\Delta Z_1 - \Sigma\Delta Z_2)] - \epsilon \quad (4)$$

EVALUATION OF ΔZ_1 and ΔZ_2 • Let Z_0 equal the zenith distance that corresponds to a time T_0, and let

$$T_1 = T_1 - T_0$$

$$T_2 = T_2 - T_0$$

According to Taylor's theorem,

$$Z_1 = Z_0 + \left(\frac{dZ}{dt}\right)_{T_0} T_1 + \frac{1}{2}\left(\frac{d^2Z}{dt^2}\right)_{T_0} T_1^2 + \frac{1}{6}\left(\frac{d^3Z}{dt^3}\right)_{T_0} T_1^3 + \ldots$$

$$Z_2 = Z_0 + \left(\frac{dZ}{dt}\right)_{T_0} T_2 + \frac{1}{2}\left(\frac{d^2Z}{dt^2}\right)_{T_0} T_2^2 + \frac{1}{6}\left(\frac{d^3Z}{dt^3}\right)_{T_0} T_2^3 + \ldots \qquad (5)$$

Select T_0 as the time of the star crossing the meridian plane, so that

$$\frac{dZ}{dt} = \frac{d^3Z}{dt^3} = \frac{d^5Z}{dt^5} = 0 \tag{6}$$

Then ΔZ_1, ΔZ_2 can be computed from the general equation that gives ΔZ in seconds of arc

$$\Delta Z'' = C_1 \left.\frac{d^2Z}{dt^2}\right|_{T_0} (T^\circ)^2 + C_2 \left.\frac{d^4Z}{dt^4}\right|_{T_0} (T^\circ)^4 + \ldots \tag{7}$$

where T° is expressed in degrees and fraction of it, and the constants C_1 and C_2 are

$$C_1 = \frac{1}{2} \frac{\pi}{180} 3600'' = 31''.4159 \tag{8}$$

$$C_2 = \frac{1}{24} \left(\frac{\pi}{180}\right)^3 3600'' = 0''.7979 \times 10^{-3} \tag{9}$$

The corrections ΔZ_1 and ΔZ_2 are subtracted always from the observed zenith distances.

For the Taylor's coefficients, the following expressions exist:

$$\left(\frac{d^2Z}{dt^2}\right)_{A=0,\pi} = \sin A \frac{dA}{dt} \left(\frac{dA}{dt} \cos Z - \sin\delta\right)$$

$$\left(\frac{d^4Z}{dt^4}\right)_{A=0,\pi} = -\frac{d^2Z}{dt^2} \left(1 + 3 \cot Z \frac{d^2Z}{dt^2}\right) \tag{10}$$

From the condition

$$\left(\frac{dZ}{dt}\right)^2 + \left(\sin Z \frac{dA}{dt}\right)^2 = \cos^2\delta \tag{11}$$

and from equation (6) is obtained

$$\sin Z \frac{dA}{dt} = \cos \delta \qquad (12)$$

in which Z represents the zenith distance on the meridian plane. If a star has an azimuth A = 0, then dA/dt has a positive value. If the azimuth is $A = \pi$, dA/dt has a negative value.

Analysis of the observed field data shows first that no index error was evaluated and second that the star zenith distances were not measured with respect to the central intersection of the vertical and horizontal lines of the reticle. The theodolite position was fixed with respect to the meridian plane. The stars were collimated outside of the vertical center line. Specifically, the analyses shows that the index error disappears when averaging the direct and reverse zenith distances. Equation (5) cannot be applied to obtain the zenith distances corrections because the equations were derived for observations made for the intersection of the vertical and horizontal center vertical lines. To apply equation (5), a meridian projection, PZ, is used on the horizontal plane (figure 1).

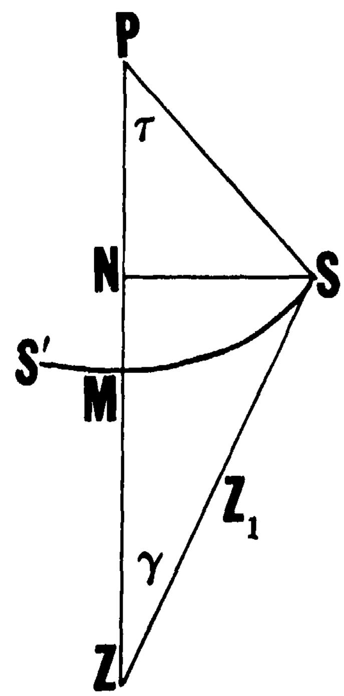

FIGURE 1. Observing a Northern Star.

Where SMS′ is the parallel of the north star's declination. Let τ equal the hour angle of the star when the star is collimated at S. Since the theodolite is clamped with respect to the meridian, the horizontal line of the collimation is SN. The zenith distance ZN is measured instead of the zenith distance ZS. With meridian zenith distance as ZM, the correction is

$$MN = PM - PN = PS - PN \tag{13}$$

The value of PN is obtained from the spherical triangle PNS as a function of the hour angle τ, and the polar distance $PS = 90° - \delta$. Setting $PN = 90° - \delta'$, we obtain

$$\tan \delta' = \tan \delta / \cos \tau$$

δ' is very close to δ and allowing

$$\cos \tau = 1 - 2 \sin^2 \frac{1}{2} \tau$$

and

$$\delta Z = \delta - \delta = \frac{\sin 2\delta \, \sin^2 \frac{1}{2} \tau}{\sin 1''} \tag{14}$$

therefore,

$$MN = \delta' - \delta = \delta Z_n$$

The meridian zenith distance of the north star is

$$Z_m = ZM - \delta Z_n \tag{15}$$

Where δZ_n is always subtracted from the observed zenith distance of a north star. The correction to be applied to a southern star is shown in figure 2, where ZN is the observed zenith distance and the meridian distance is ZM.

Thus,

$$ZM = ZN + NM$$

For a south star, the observed zenith distance is less than the meridian zenith distance. A correction $\delta Z = \delta' - \delta$ is added to the observed zenith distance.

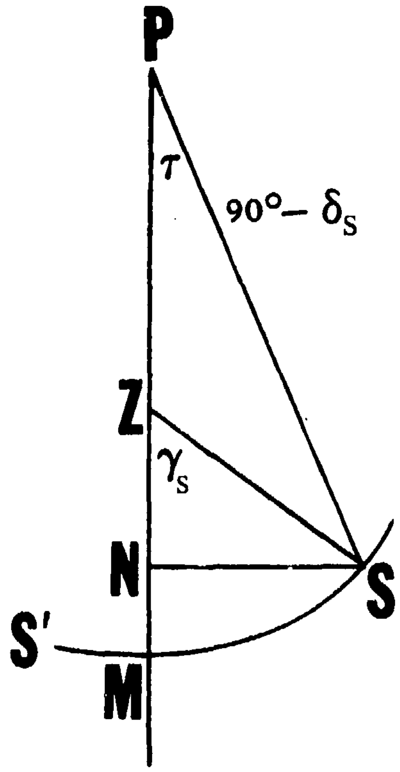

FIGURE 2. Observing a Southern Star.

OBSERVATION ERROR • The Sterneck method yields the latitude from the generalized equation

$$\phi = \delta \pm Z \tag{16}$$

in which Z represents the meridian zenith distance. The positive sign is for the south star and the negative sign for the north star.

In the NGS modified Sterneck method, the index error is eliminated and the need for the zenith point correction is avoided. The mean zenith distance of a star is corrected for refraction and inclination errors, and is reduced to the meridian to obtain the Z value that appears in equation (16). The probable error of an observation may be found by comparing the values of the index error and the personal equation that results from the direct and reverse star's zenith distances. For each star, the observed values are reduced to a common instant of time after applying all the corrections. According to the theory of least squares, where e denotes the error of an observation, V_1 denotes the residuals obtained by comparing the mean $(\epsilon + p)$ of the first star with n_1 observations

$$
\begin{array}{c}
V_2 \cdots\cdots n_2 \\
\cdots\cdots\cdots \\
\cdots\cdots\cdots \\
\cdots\cdots\cdots \\
\cdots\cdots\cdots \\
V_n \cdots\cdots n_m
\end{array}
$$

then,

$$(n_1 - 1)\ ee = p^2 [V_1\ V_1]$$

$$(n_2 - 1)\ ee = p^2 [V_2\ V_2]$$

$$(n_m - 1)\ ee = p^2 [V_m\ V_m]$$

where $[V_1\ V_1]$, $[V_2\ V_2]$, etc. denote the sum of the squares.

The sum of this equation yields

$$(n - m)\ ee = p^2 [V\ V]$$

where n denotes the whole number of an individual star's zenith distance and results in

$$n = n_1 + n_2 + \ldots\ldots + n_m$$

$$[V\ V] = [V_1\ V_1] + [V_2\ V_2] + \ldots\ldots + [V_m\ V_m]$$

Hence, we have

$$e = \pm p \sqrt{\frac{[V\ V]}{n - m}}$$

The NGS modified Sterneck formulation involves two symmetrical circummeridian zenith distances in direct and reverse instrument positions;

$$n_1 = n_2 = n_3 = n_m = 2$$

and

$$m = n$$

Hence, we have

$$e = \pm p \sqrt{\frac{[V V]}{n}}$$

PROCEDURE FOR OBTAINING OBSERVATION ERROR • As defined in the previous section, the values of V come from the star's zenith distances on the meridian plane at the instant of time T_0. Each star is observed from the meridian at an hour angle τ defined by

$$\tau = \theta_0 + T_0(1 + C) - (\lambda + \alpha) \tag{17}$$

where

T_0 = Greenwich Universal Time
θ_0 = Greenwich hour angle of vernal equinox
C = Constant
λ = longitude
α = star right ascension

The τ values given by equation (17) are used to obtain δZ_i. Let Z_1 represent the direct observed zenith distance at the instant of time T_1; R_1 the error due to refraction; δZ_1 the correction to reduce the observed zenith distance to the meridian; $(\epsilon + p)$ the index and personal error of observation; and Z the true value of the star's zenith distance. For direct instrument position, the following equation must be satisfied:

$$Z = Z_1 + R_1 + (\epsilon + p) + \gamma \cdot \delta Z_1 + V_1 \tag{18}$$

where γ symbolizes the sign to be applied because the correction δZ_1 and V_1 is the error of observation. A similar equation is obtained for the reverse instrument position, which is indicated by the subindex 2.

$$Z = 2\pi - Z_2 - R_2 - (\epsilon + p) - \gamma \cdot \delta Z_2 - V_2 \tag{19}$$

Let

$$Z_d = Z_1 + R_1 + \frac{\delta}{2} b_1 - \gamma \cdot \delta Z_1$$

$$Z_r = 2\pi - Z_2 - R_2 - \gamma \cdot \delta Z_2 + \frac{\delta}{2} b_2 \qquad (20)$$

Introducing the values in (20) and subtracting (19) from (18) results in

$$0 = Z_d - Z_r + 2(\epsilon + p) + V_2 + V_1 \qquad (21)$$

Each star furnishes an equation, such as (21), with constant value $(\epsilon + p)$. With n number of stars there are m equations, which when added give

$$\Sigma(Z_d - Z_r) + 2n(\epsilon + p) + \Sigma(V_1 + V_2) = 0 \qquad (22)$$

Solving for the unknown $(\epsilon + p)$ yields

$$\epsilon + p = -\frac{\Sigma(Z_d - Z_r)}{2n} - \frac{\Sigma(V_1 + V_2)}{2n} \qquad (23)$$

The V's are random, and when n is large, then

$$\frac{\Sigma(V_1 + V_2)}{2n} = 0$$

and

$$\epsilon + p = -\frac{\Sigma(Z_d - Z_r)}{2n} \qquad (24)$$

Knowing $(\epsilon + p)$, the V's can be defined from equations (18) and (19).

As an example of computation, we took the observation of Station Aero USAETL 1978, shown in table 1. Table 2 is self explanatory. The hour angles were computed using equation (17). The δZ's from equation (15) and the refraction correction from the author's formula results in[1]

[1] A. A. Baldini, "Formulas for Computing Atmospheric Refraction for Objects Inside or Outside the Atmosphere." *GIMRADA Research Note 8.* 1963. AD – 419 915.

TABLE 1. Modified Sterneck Latitude Computation

1 STATION AERO USAETL 1978 MD INST NO. T–4, 86994 CHRON NO. D-253
2 OBS BY COHEN REC BY JCH ACC NO. ECC 0.000M AZI ODEG 0.0MIN
3 LATITUDE 39 19 53.00 LONGITUDE 77 11 35.00 LEVEL VALUE 1.217
4 78 7 6 78 7 6 WWV 4 1 28.000 4 1 28.000
4 78 7 6 78 7 6 WWV 6 15 49.000 6 15 49.000

CHRONOMETER OFFSET 0 0 0.000
CHRONOMETER RATE 0.00000 SECONDS/MINUTE

1	676	NE	12	9	54.6	14.8	14.6	4	8	41.355	15.90C	.2	29.4
1	676	NW	347	50	39.0	14.0	15.2	4	10	41.126	29.471	–1.2	29.2
2	684	NW	357	10	36.8	14.3	15.0	4	27	38.286	– 0.00	– .7	29.3
2	684	NE	2	49	23.4	14.8	14.6	4	29	28.846	– 0.00	.2	29.4
3	695	NE	33	23	7.8	14.9	14.4	4	33	55.076	15.60C	.5	29.3
3	695	NW	326	36	50.0	14.0	15.3	4	36	8.696	29.471	–1.3	29.3
4	1483	NW	353	7	41.1	14.8	14.6	4	45	50.728	– 0.00	.2	29.4
4	1483	NE	6	52	19.5	14.9	14.5	4	47	36.161	– 0.00	.4	29.4
5	1488	SE	347	18	47.2	15.2	14.2	4	57	56.713	– 0.00	1.0	29.4
5	1488	SW	12	41	10.6	14.0	15.5	4	59	28.983	– 0.00	– 1.5	29.5
6	705	SW	5	59	21.2	15.7	13.8	5	1	56.975	15.30C	1.9	29.5
6	705	SE	354	0	37.6	15.0	14.4	5	3	40.786	29.471	.6	29.4
7	709	SE	324	51	26.6	14.5	15.0	5	7	56.375	– 0.00	– .5	29.5
7	709	SW	35	8	32.0	14.7	14.9	5	9	24.263	– 0.00	– .2	29.6
8	719	SW	3	15	43.1	14.7	15.0	5	19	7.406	– 0.00	– .3	29.7
8	719	SE	356	44	18.0	14.0	15.8	5	20	48.604	– 0.00	– 1.8	29.8
9	723	NE	28	17	9.4	14.9	14.7	5	25	6.539	– 0.00	.2	29.6
9	723	NW	331	42	48.0	14.2	15.2	5	26	52.965	– 0.00	– 1.0	29.4
10	729	NW	326	1	25.6	14.9	14.8	5	28	29.914	0.00	.1	29.7
10	729	NE	33	58	33.7	14.7	15.0	5	30	19.911	– 0.00	– .3	29.7
11	1506	NE	5	33	24.7	15.2	14.4	5	37	43.384	15.00C	.8	29.6
11	1506	NW	354	26	33.5	14.7	15.0	5	39	38.108	29.461	– .3	29.7
12	1510	SW	4	55	15.5	14.8	15.0	5	43	30.488	– 0.00	– .2	29.8
12	1510	SE	355	4	43.4	15.5	14.2	5	45	10.503	– 0.00	1.3	29.7
13	738	NE	10	50	17.0	15.0	14.6	5	48	26.230	– 0.00	.4	29.6
13	738	NW	349	9	41.0	13.6	16.3	5	50	1.141	– 0.00	– 2.7	29.9
14	741	SW	28	45	37.8	14.6	15.2	5	57	52.125	– 0.00	– .6	29.8
14	741	SE	331	14	21.2	16.2	13.3	5	59	15.669	– 0.00	2.9	29.5
15	749	SE	327	1	59.3	14.9	14.8	6	6	52.920	– 0.00	.1	29.7
15	749	SW	32	57	58.2	14.0	15.7	6	8	14.614	– 0.00	– 1.7	29.7
16	1523	SW	11	37	59.4	14.1	15.6	6	12	43.687	15.00C	– 1.5	29.7
16	1523	SE	348	22	2.6	14.3	15.6	6	14	16.649	29.461	– 1.3	29.9

SOURCE: NOAA

TABLE 2. Reduction of Zenith Distance to the Meridian

Star	Obs. Zenith Distances	Hour Angle	Refr.	$\frac{\rho b}{2}$	δZ	Red. Zenith Dist.
676	12° 9′ 54″.6	−63ˢ.37	−12″.00	″.12	+1″.07	12° 09′ 57″.88
	347 50 39 .0	56 .73	12 .09	− .73	− .86	347 50 36 .12
684	357 10 36 .8	−55 .35	− 2 .80	− .43	″.84	357 10 34 .41
	2 49 23 .4	55 .51	2 .80	.12	− .83	2 49 25 .43
695	33 23 07 .8	−68 .36	36 .98	.30	.72	33 23 44 .36
	326 36 50 .0	65 .63	−36 .98	− .79	.67	326 36 12 .90
1483	353 7 41 .10	−52 .69	6 .76	.12	.77	353 7 35 .23
	6 52 19 .50	53 .03	6 .76	.24	.76	6. 52 25 .74
1488	347 18 47 .2	−45 .54	−12 .62	.61	−.48	347 18 34 .81
	12 41 10 .6	46 .98	12 .62	−.91	.45	12 41 22 .76
705	5 59 21 .2	−49 .42	5 .89	1.16	.61	5 59 28 .86
	354 00 37 .6	54 .68	− 5 .89	.37	− .75	354 00 31 .33
709	324 51 26 .6	−40 .62	−39 .48	− .30	− .09	324 50 46 .73
	35 8 32 .0	47 .51	39 .48	− .12	.07	35 9 11 .43
719	3 15 43 .1	−51 .17	3 .20	− .18	.68	3 15 46 .80
	356 44 18 .0	50 .31	− 3 .20	−1.10	− .68	356 44 13 .02
723	28 17 9 .4	−54 .33	30 .18	.12	− .57	28 17 9 .40
	331 42 48 .0	52 .38	30 .18	− .61	.63	331 42 48 .00
729	326 1 25 .6	−56 .90	−37 .80	.06	.49	326 00 48 .35
	33 58 33 .7	53 .40	37 .80	− .18	− .43	33 58 10 .89
1506	5 33 24 .7	57 .63	5 .46	.49	− .91	5 33 29 .74
	354 25 33 .5	−57 .41	− 5 .46	− .18	.90	354 26 28 .75
1510	4 55 15 .50	−50 .36	4 .83	− .12	.64	4 55 20 .85
	355 4 43 .30	49 .93	− 4 .83	.79	− .63	355 4 38 .75
738	10 50 17 .0	−47 .92	10 .74	.24	− .62	10 50 27 .56
	349 9 41 .0	47 .25	−10 .74	−1.64	.60	349 9 29 .22
741	28 45 37 .8	−41 .89	30 .78	− .37	.17	28 46 08 .38
	331 14 21 .2	41 .88	−30 .78	1.76	− .17	331 13 52 .01
749	327 1 59 .3	−40 .75	−36 .37	.06	− .09	327 1 22 .89
	32 57 58 .2	41 .17	36 .37	-1.03	.09	32 57 33 .63
1523	11 37 59 .4	−46 .77	11 .35	− .91	.38	11 38 10 .22
	348 22 2 .6	46 .45	−11.35	− .79	− .38	348 21 50 .08

$$R = A_0(n_0 - 1)\ \tan Z + A_1(n_0 - 1)\ \tan^3 Z$$

The meanings of columns in table 1 are

Column 1	=	Sequence of observations
Column 2	=	Star's number
Column 3	=	Observed zenith distances
Column 4	=	Level readings, in the order left-right
Column 5	=	Universal time (WWV)
Column 6	=	Temperature and barometer readings
Column 7	=	Difference between level readings
Column 8	=	Sum of level readings

$$\Sigma Z_d = 245^\circ\ 32'\ 33''.83$$
$$\Sigma Z_r = 245^\circ\ 32'\ 15''.36$$

From equation (24)

$$\epsilon + p = -\frac{1}{2n}\left[\Sigma Z_d - \Sigma Z_r\right]$$

$$\epsilon + p = -\frac{18''47}{30} = -0''.66$$

Adding the value $(\epsilon + p)$ to the Z_d and subtracting it from Z_r, we get the values $(Z_d + \epsilon + p)$ and $Z_r - (\epsilon + p)$ and are shown in column 4 of table 3. By subtracting these values (column 4) from each mean (column 5), the V values are obtained. The probable error of an observation is obtained by squaring and adding the V values and applying equation (17).

$$e = p\sqrt{\frac{[V\,V]}{n}}$$

The sum of the V values squared gives

$$[V\,V] = 26.7781$$

and

$$n = 15$$

TABLE 3. Determination of Observation Error

Star	Zenith Dist Z_d/Z_r	$2(\epsilon+\rho)+V_1+V_2$	$Z_d+\epsilon+\rho$ $Z_r-(\epsilon+\rho)$	Means	V
684	2° 49 ′25 ″.49	−0.10	2° .49 ′ 24 ″.83	25.54	+ .71
	2 49 25 .59		26 .25		− .71
695	33 23 47 .10		33 23 46 .44		− .71
	33 23 44 .36	+2.74	45 .02	45.73	+ .71
1483	6 52 25 .74	+0.97	6 52 25 .08	25.26	+ .18
	6 52 24 .77		25 .43		− .17
1488	12 41 22 .76	−4.43	12 41 22 .10	23.98	+1.88
	12 41 25 .19		25 .85		−1.87
705	5 59 28 .86	+0.09	5 59 28 .20	28.76	+ .56
	5 59 28 .67		29 .33		− .57
709	35 09 11 .43	−1.84	35 09 10 .77	12.35	+1.58
	35 09 13 .27		13 .93		−1.58
719	3 15 46 .80	−0.18	3 15 46 .14	46.89	+ .75
	3 15 46 .90		47 .64		− .75
723	28 17 39 .13	−3.03	28 17 38 .47	40.64	−2.17
	28 17 42 .16		42 .82		+2.17
729	33 58 10 .89	−0 76	33 58 10 .23	11.27	1.04
	33 58 11 .65		12 .31		−1.04
1506	5 33 29 .74	−1.50	5 33 29 .08	30.49	1.41
	5 33 31 .24		31 .90		−1.41
1510	4 55 20 .85	−0.42	4 55 20 .19	21.06	0.87
	4 55 21. 27		21 .93		−0.87
738	10 50 27 .36	−3.42	10 50 26 .70	29.07	2.03
	10 50 30 .78		31 .44		−2.03
741	28 46 08 .38	+0.39	28 46 7 .72	8.18	0.46
	28 46 07 .99		8 .65		−0.46
749	32 58 33 .63	−3.48	32 58 32 .97	35.37	2.40
	32 58 37 .11		37 .77		−2.40
1523	11 38 10 .22	+0.30	11 38 9 .60	10.09	0.49
	11 38 09 .92		10 .58		−0.49

The probable error of an observation is

$$e = 0.6745 \sqrt{26.7781/15} = \pm 1''.20$$

Further investigations using random stations show large discrepancies owing to changes in the TALCOTT level position from one star to the next. However, these discrepancies do not affect the mean value of the individual star zenith distances that result from the direct and reverse instrument position because of the cancellation in the mean value. As a result, the probable error of an observation cannot be used by this method. Thus, another procedure must be used. One such procedure is discussed next.

Probable Error of One Observation as Function of Zenith Distances. The probable error can be computed by computing the zenith distances of any northern star with all southern stars, and any southern star with all of the northern stars. The zenith distances are reduced to the meridian plane as shown previously. The condition becomes

$$Z_{1n} = \delta_{1n} - \delta_{is} - Z_{is}$$

for the northern star, and

$$Z_{1s} = \delta_{1s} - \delta_{in} - Z_{in}$$

for the southern star.

Note that only two stars can be chosen. The V values are formed with respect to the means of each star (see table 4).

TABLE 4. Error Of An Observation

Star	Zenith Dist.		Means	V	VV
	12°09′	48.″75		− .15	.0225
		48. 95		− .35	.1225
676 N		49. 79		−1.19	1.4161
		48. 20	48 .60	+ .40	.1600
		48. 66		− .06	.0036
		47. 94		+ .66	.4356
		48. 87		− .27	.0729
		47. 62		+ .98	.9604
	11°38′	8.″68		+ .29	.0841
		8. 35		+ .62	.3844
		9. 11		− .14	.0196
1523 S		8. 72	8″.97	+ .25	.0625
		9. 10		− .15	.0225
		10. 04		−1.07	1.1449
		9. 08		− .11	.0121
		8. 67		+ .30	.0900

$n - m = 14$ $\qquad$ $[V\,V] = 5.0137$

Error of one observation

$$e_o = \sqrt{\frac{[V\,V]}{m-n}} = 0''.60$$

and the probable error

$$e_p = 0.67\, e_o = 0''.40$$

INFLUENCE OF REFRACTION IN MEAN LATITUDE VALUE BY THE STERNECK METHOD

In the measurements of astronomic coordinates, a degree of uncertainties is due to a lack of knowledge of the atmospheric refraction during the acquisition of stellar positions. Atmospheric refraction comprises both astronomic and terrestrial refraction; however, only the effect of astronomic refraction is considered. Astronomic refraction is defined as the bending of light from objects at distances above the earth's mean radius. From a star, the light moves as it passes through strata of different densities. Turbulence in the earth's atmosphere causes changes in the brightness of a star. Small angular displacement can be expected during optical observations.

Since the determination of the actual air density profile is difficult, it is generally approximated by using a quasi-standard model. The solution of the refraction integral may then be represented in the form

$$R = A \tan Z + B \tan^3 Z + C \tan^5 Z + Em \qquad (25)$$

where the coefficients A, B, and C depend on the quasi-standard model. There remains, however, an error owing to the replacement of the actual air density profile by a model profile. Using the former requires practically the numerical evaluation of the refraction integral.

In geodetic astronomy, the first two terms are considered that give satisfactory results to zenith distances up to $Z = 75°$.

The stars used for latitude determination are chosen in a way with no zenith angle greater than 30°. For the mean latitude value that results from a night of observation, the latitude is

$$\varphi = \frac{1}{n}(\Sigma\delta n + \Sigma\delta s) + \frac{1}{n}(\Sigma Z_n - \Sigma Z_s) + d\varphi \qquad (26)$$

where

$$d\varphi = \frac{A}{n}(\Sigma \tan Zn - \Sigma \tan Zs) + \frac{B}{n}(\Sigma \tan^3 Zn - \Sigma \tan^3 Zs) \qquad (27)$$

and n represents the number of observed stars. A program for latitude comprises 16 stars, which are evenly divided to north and south of the zenith. The coefficient B is small, less than 0.07". Since no zenith distance is greater than 30°, the second term of equation (27) vanishes for practical purposes.

ERROR LEVEL CORRECTION • A TALCOTT level is used by NOAA for zenith distances observations. The graduation readings increase outwardly from each side from the center zero mark on the vial-level scale.

Figure 3 shows the position of the instrument with an inclination i.

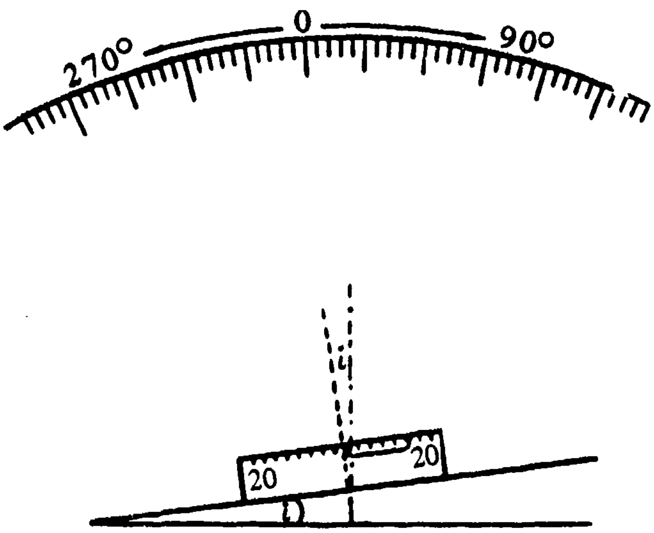

FIGURE 3. The Inclination i.

The bubble reads higher to the right and lower to the left. Let R and L equal the right and left readings and the mean reading is denoted by L. Let ρ be the level sensibility.

The inclination i is

$$i = \rho(R - L_0) = \rho(L - L_0) \tag{28}$$

The direct zenith distance for 0° to 90° is greater by i, and the reverse Z_r for 0° to 270° is lesser by i. For Z_d, the correction for inclination is negative. Then, R is greater than L_0, therefore

$$i = \rho(L_0 - R) = \frac{\rho}{2}(L - R)$$

where the direct zenith distance is

$$i = \frac{\rho}{2} (L - R)$$

and the reverse zenith distance is

$$i = -\frac{\rho}{2} (L - R)$$

When the observer maintains a small difference $(L - R)$, an error in the value of ρ has little influence. For an error $\delta\rho = 0''.1$ and $(L - R) = 3$, the error in the inclination is $d_i = 0''.15$.

CONDITIONAL EQUATION • When summing equations (18) and (19), the mean value is free of the index error and personal equation, as shown in equation (20).

$$Z = \frac{1}{2}(Z_d + Z_r) + \frac{1}{2}(V_1 - V_2)$$

Let index n refer to the north star and index s refer to the southern star. The sum of all of the star's zenith distances have to satisfy the equation

$$\Sigma Z_n + \Sigma Z_s + \Sigma V_1 - \Sigma V_2 = \Sigma\delta_n - \Sigma\delta_s \qquad (29)$$

Station values used to determine the observational error are applied to equation (29) as follows:

$\Sigma\delta_n$ -------	448° 35 '22 ".53	ΣZ_n -------	133° 56 '17. "02
$\Sigma\delta_s$ -------	179 15 02 .09	ΣZ_s -------	135 24 06. 79
$\Sigma\delta_n - \Sigma\delta_s$ =	269° 20 '20 ".44	$\Sigma Z_n + \Sigma Z_s$ =	269° 20 '23. "81

Then

$$\Sigma V_1 - \Sigma V_2 = -3''.37$$

The sum of V_1 and V_2 is inseparable.

LEAST SQUARES SOLUTION • The discrepancy shown in equation (26) can be minimized if a solution by a least-square method is followed. Two sets of equations of condition are established that are based on previously derived general equations having all stars observed, or reduced, to a constant hour angle. The equation is

$$\sin \varphi \, \sin(\delta_n - \delta_s) + \cos \delta_n \, \cos Z_s = \cos \delta_s \, \cos Z_n \tag{30}$$

The index n and s refer to the north and south star, respectively. Since the zenith angles are small, the formula fulfills the accuracy requirement. Assuming small errors in Z_s and Z_n and differentiating relative to ϕ and Z_s, Z_n yields

$$\cos \varphi \, d\varphi = \frac{\cos \delta_n \, \sin Z_s}{\sin(\delta_n - \delta_s)} \, dZ_s - \frac{\cos \delta_s \, \sin Z_n}{\sin(\delta_n - \delta_s)} \, dZ_n \tag{31}$$

Since the sin Z's are always small, the errors in the zenith distances decreases with the factor sin Z.dZ.

The least-square method continues as follows:

A star with large declination is selected from the north set. The zenith distance for the star is unknown. The star is used with each of the stars from the south to obtain a number of free equations of the form

$$- X + a_i Y_1 = \ell_i \tag{32}$$

where the unknowns X and Y_1 are

$$\begin{aligned} X &= \sin \varphi \\ Y_1 &= \cos Z_n \end{aligned} \tag{33}$$

and the coefficients a_i, ℓ_i, are

$$a_i = \frac{\cos \delta_{is}}{\sin(\delta_n - \delta_s)} \tag{34}$$

$$\ell_i = \frac{\cos \delta_n \, \cos Z_{is}}{\sin(\delta_n - \delta_s)} \tag{35}$$

$$i = 1, 2, \ldots\ldots, n$$

Similarly, by holding a fixed southern star with each one from the north set, one can obtain m observation equations such as

$$X + b_j Y_2 = \ell_j \tag{36}$$

where the unknown Y_2 is

$$Y_2 = \cos Z_s \tag{37}$$

and the coefficients b's and ℓ's are obtained from

$$b_j = \frac{\cos \delta_{in}}{\sin(\delta_{in} - \delta_s)}$$

$$\ell_j = \frac{\cos \delta_s \cos Z_{in}}{\sin(\delta_{in} - \delta_s)} \tag{38}$$

$$j = 1, 2 \ldots\ldots, m$$

Solving equations (32) and (36) simultaneously by the method of least squares and eliminating the unknown X, one obtains two normal equations as

$$\begin{aligned} AY_1 + BY_2 &= L_1 \\ BY_1 + CY_2 &= L_2 \end{aligned} \tag{39}$$

To these normal equations is added

$$Z_1 + Z_2 = \delta_n - \delta_s \tag{40}$$

Let Z_1 and Z_2 be the observed values and dZ_1, dZ_2 be the corrections needed to satisfy equation (40), and let

$$W_{12} = \delta_n - \delta_s - (Z_1 + Z_2) \tag{41}$$

then

$$dZ_1 + dZ_2 = W_{12} \tag{42}$$

To solve equation (40) with equation (42), proceed as follows.

By replacing the unknowns Y_1, Y_2 by

$$Y_1 = \cos \bar{Z}_1 - dZ_1 \sin \bar{Z}_1$$
$$Y_2 = \cos \bar{Z}_2 - dZ_2 \sin \bar{Z}_2$$

equation (40) becomes

$$\begin{aligned} A \sin \bar{Z}_1\, dZ_1 + B \sin \bar{Z}_2\, dZ_2 &= W_1 \\ B \sin \bar{Z}_1\, dZ_1 + C \sin Z_2\, dZ_2 &= W_2 \end{aligned} \qquad (43)$$

where

$$\begin{aligned} W_1 &= (A \cos \bar{Z}_1 + B \cos \bar{Z}_2 - L_1)/\sin 1'' \\ W_2 &= (B \cos \bar{Z}_1 + C \cos \bar{Z}_2 - L_2)/\sin 1'' \end{aligned} \qquad (44)$$

The normal equation (44) is to be solved by using equation (43).

It is solved by introducing the unknown K as follows:

$$\begin{aligned} A \sin \bar{Z}_1\, dZ_1 + B \sin \bar{Z}_2\, dZ_2 + K &= W_1 \\ B \sin \bar{Z}_1\, dZ_1 + C \sin \bar{Z}_2\, dZ_2 + K &= W_2 \\ dZ_1 + d\bar{Z}_2 &= W_{12} \end{aligned} \qquad (45)$$

Solving for dZ_1 and dZ_2 and using

$$Y_1 = \cos(\bar{Z}_1 + dZ_1) \qquad Y_2 = \cos(\bar{Z}_2 + dZ_2)$$

The unknown X is computed from the first normal equation, and the latitude is computed from

$$\varphi = \arcsin X$$

The test conducted refers to the Station Aero USAETL 1978. Star number 676 is chosen for the north star and star 1488, for the southern set. The data used is shown in table 5.

TABLE 5. Final Meridian Zenith Distances

	STAR	LST	RA	DEC	ZD
1	676	17 56 5.11	17 56 8.43	51 29 42.20	12 9 49.30
2	684	18 15 .54	18 15 .46	42 9 18.06	2 49 25.49
3	695	18 21 29.92	18 21 31.29	72 43 38.94	33 23 45.61
4	1483	18 33 13.40	18 33 13.23	46 12 18.18	6 52 25.24
5	1488	18 45 14.77	18 45 14.05	26 38 29.45	12 41 24.00
6	705	18 49 21.48	18 49 18.85	33 20 24.48	5 59 28.75
7	709	18 55 13.88	18 55 10.43	4 10 40.14	35 9 12.27
8	719	19 6 33.42	19 6 33.85	36 4 7.13	3 15 46.87
9	723	19 12 36.16	19 12 37.13	67 37 33.96	28 17 40.64
10	729	19 16 1.88	19 16 3.63	73 19 5.46	33 59 11.20
11	1506	19 25 19.24	19 25 19.35	44 53 23.78	5 33 30.48
12	1510	19 30 59.92	19 31 .13	34 24 32.48	4 55 21.06
13	738	19 35 53.91	19 35 54.24	50 10 21.95	10 50 29.06
14	741	19 45 15.66	19 45 15.66	10 33 46.19	28 46 8.15
15	749	19 54 17.00	19 54 16.79	6 21 19.00	32 58 35.33
16	1523	20 0 14.38	20 0 14.54	27 41 44.22	11 38 10.36

The following equations of observation are

$-X$	$+2.12664562Y_1$		$= 1.44509638$
	$+2.68118197$		$= 1.98718370$
	$+1.35671000$		$= .69244814$
	$+3.03876241$		$= 2.33673594$
	$+2.80804611$		$= 2.11119945$
	$+1.50046665$		$= .83297305$
	$+1.40210030$		$= .73681725$
$-X$	$+2.19417462Y_1$		$= 1.51110765$
X		$+1.48127975Y_2$	$= 2.07890158$
		$+2.77169144$	$= 3.33779276$
		$+.41216626$	$= 1.03590425$
		$+2.06682246$	$= 2.65014159$
		$+.53038532$	$= 1.20001419$
		$+.39458299$	$= 1.01875357$
		$+2.26246721$	$= 2.84100790$
X		$+1.60419822Y_2$	$= 2.19881757$

Normal Equations

$$16X - 17.10808768Y_1 + 11.57359364Y_2 = 4.70777185$$
$$39.70290799Y_1 = 27.96836505$$
$$22.50285187Y_2 = 29.28858325$$

Eliminating the unknown X yields

$$21.40999149Y_1 + 12.37512842Y_2 = 33.00217590$$
$$12.37512842Y_1 + 14.13109751Y_2 = 25.88321835$$

From these equations and from the data in table 5, the result is

$$W_{12} = 0''.55$$
$$W_1 = 1\ .36$$
$$W_2 = -0\ .14$$

From equation (45), the result is

$$4.511\,dZ_1 + 2.719\,dZ_2 + K = -1.36$$
$$2.608\,dZ_1 + 3.104\,dZ_2 + K = -0.14$$
$$dZ_1 + dZ_2 = -0.55$$

Then, solving for dZ_1 and dZ_2,

$$dZ_1 = -0''.687 \qquad Z_1 = Z_1 + dZ_1 = 12^\circ\ 09'\ 48''.61$$
$$dZ_2 = +0\ .137 \qquad Z_2 = Z_2 + dZ_2 = 12\ \ 41\ \ 24\ .14$$

With these values of Z_1 and Z_2, the values of Y_1 and Y_2 are obtained. From the first normal equation is obtained

$$X = \sin\varphi = 0.633800660$$
$$\varphi = 39^\circ\ 19'\ 53''.50$$

The mean latitude from Sterneck's method for 16 stars from table 6 is

$$\varphi = 39° \ 19' \ 53''.40$$

More tests must be done using different star pairs. One example, which follows, is observing several zenith distances in a direct and reverse theodolite position.

A star whose declination is defined by the equation

$$\tan \delta \ \overline{>} \ \sqrt{2} \cdot \frac{\sin (45 + \phi)}{\cos \phi}$$

for a star north of the zenith

$$\tan \delta \ \overline{<} \ \sqrt{2} \cdot \frac{\sin (45 - \phi)}{\cos \phi}$$

for a star south of the zenith, has a very small variation in zenith distance with respect to the minimum zenith distance when it crosses the upper meridian. Hence, if we observe it continuously and record the zenith distance with the corresponding times for approximately 10 to 15 minutes before and after it crosses the meridian and continue to observe it for about the same time after it has crossed the meridian, the star's image will always be in the field of view of the telescope. Thus, many observations can be made in a direct and reverse theodolite position by using the tangent screw of the vertical circle or by measuring the small angles with the eyepiece micrometer. These zenith distances can be reduced to the meridian by using equation (2.1) to (2.6). The average value is then free from index error and personal equation error. The refraction corrections are practically the same for all observations since the changes in zenith distances reach, at most, 3 to 5 minutes.

TABLE 6. Results of Latitude Determined by the Sterneck Method

1 STATION AERO USAETL 1978 MD INST NO. T-4, 86994
2 OBS BY COHEN REC BY JCH ACC NO. ECC 0.00
3 LATITUDE 39 19 53.00 LONGITUDE 77 11 35.00 LEVEL VALUE 1.
4 78 7 6 78 7 6 WWV 4 1 28.000 4 1 28.000
5 78 7 6 78 7 6 WWV 6 15 49.000 6 15 49.000

1	676 N	39°	19′	52″.90	.50
2	684 N	39	19	52 .57	.83
3	695 N	39	19	53 .34	.06
4	1483 N	39	19	52 .94	.46
5	1488 S	39	19	53 .45	- .05
6	705 S	39	19	53 .23	.17
7	709 S	39	19	52 .41	.99
8	719 S	39	19	54 .00	- .60
9	723 N	39	19	53 .32	.08
10	729 N	39	19	54 .27	- .87
11	1506 N	39	19	53 .30	.10
12	1510 S	39	19	53 .54	- .14
13	738 N	39	19	52 .89	.51
14	741 S	39	19	54 .34	- .94
15	749 S	39	19	53 .34	.06
16	1523 S	39	19	54 .58	-1.18

MEAN LATITUDE IS 39 19 53.400
STD ERR SINGLE OBS IS .6240 SECS
STD ERR OF THE MEAN IS .1560 SECS
ECC LAT IS 0.00 SECS
ECC LON IS 0.00 SECS

USING 16 ACCEPTABLE OBSERVATIONS FROM THE 16 OBSERVATIONS TAKEN AT THE STATION AND APPLYING THE ECCENTRIC LATITUDE REDUCTION.

THE ADJUSTED LATITUDE IS 39° 19′ 53″.400

EVALUATING LATITUDE FROM A SET OF NORTH STARS • A test was conducted for Aero Station using the author's formula

$$\sin \varphi = \frac{\cos Z_1 \cos \delta_2 - \cos Z_2 \cos \delta_1}{\sin (\delta_2 - \delta_1)} \qquad (46)$$

Star number 1 is held fixed with respect to the remainder of the same set. For star number 2, the large declination is used. For the north set, star number 676 and for the south set, star number 719 are selected. Results are shown in table 7.

TABLE 7. Evaluating Latitude By Set

	North Set			South Set		
	39°	19′	53″.43	39°	19′	53″.23
			53 .36			52 .24
			53 .37			52 .18
			53 .35			52 .58
			53 .33			54 .40
	39	19	53 .41			53 .23
				39	19	54 .84
mean	39	19	53 .38	39	19	53 .24

The results of these two sets show that better quality of observation was achieved from the north stars. The differences may be caused by sky illumination and refraction anomalies. More observations are recommended for testing the different methods.

STATION LATITUDE ERROR • The generalized form of Gauss's propagation error law is

$$m_{\varphi}^2 = \left(\frac{\partial f}{\partial Z} m_z\right)^2 + \left(\frac{\partial f}{\partial \delta} m_\delta\right)^2 + 2 \frac{\partial f}{\partial Z} \frac{\partial f}{\partial \delta} m_{z\,\delta} \qquad (47)$$

where $m_z\delta$, the covariance of Z and δ, is given by

$$m_{Z\delta} = \rho \cdot m_Z \cdot m_\delta$$

Because m_z and m_δ are independent,

$$m_{Z\delta} = 0$$

In the Sterneck latitude method,

$$\frac{\partial f}{\partial Z} = \frac{\partial f}{\partial \delta} = 1$$

therefore

$$m_{\varphi}^2 = m_e^2 + m_\delta^2 \qquad (48)$$

The standard error of the FK4 system of the FK4 for declination is 1,925; so the standard error in declination is calculated from formula

$$\left(m_\delta\right)_T^2 = \left(m_\delta^2\right)_{T_o} + \left(\frac{T - 1925}{100}\right)^2 m_\mu^2$$

For Aero Station T = 1,958, so we have

$$\left(m_\delta^2\right)_T = \left(m_\delta^2\right)_{T_o} + 0.281\ m_\mu^2$$

For the mean value of the latitude derived from 16 stars,

$$\left(m_\delta^2\right)_{T_o} = 0''.016$$

$$\left(m_\mu\right)_{T_o}^2 = 0\ .06 \qquad\qquad \left(m_\delta^2\right)_T = 0.04$$

The standard error of an observation is $m_Z = 0.60$, with $n = 16$, the result is $m_{\bar{Z}} = 0.15$.

The standard error of the mean latitude is

$$m_\varphi = \sqrt{m_\delta^2 + m_Z^2} = \pm 0''.16$$

For two nights at each station, the average value for the entire test course was between 0.18 to 0.20. The most frequent value, 0.20, is used as the mean error of latitude for any given station.

IMPROVING ACCURACY IN LATITUDE DETERMINATION ● Improving the accuracy in latitude can be achieved by reducing all stars to a common hour angle, i.e. $t = 60^s$. The east and west reduced zenith angles, having the mean value free of the index error, are the same. With respect to the same hour angle, one star from the north and two from the south are selected with indices 1, 2, and 3. Let

$$\delta_1 > \delta_2 > \delta_3 >$$

An equation of condition that is free of the latitude and is related to the zenith angles and declinations is derived as follows:

$$\cos Z_1 \sin(\delta_2 - \delta_3) + \cos Z_3 \sin(\delta_1 - \delta_2) = \cos Z_2 \sin(\delta_1 - \delta_3) \qquad (49)$$

with

$$\begin{aligned} Z_1 + Z_3 &= \delta_1 - \delta_3 + \delta Z_1 + \delta Z_3 \\ Z_1 + Z_2 &= \delta_1 - \delta_2 + \delta Z_1 + \delta Z_2 \end{aligned} \qquad (50)$$

Similarly, selecting two stars from the south and one from the north with the condition

$$\delta_1 > \delta_2 > \delta_3 >$$

and with the equation (49) the condition is

$$\begin{aligned} Z_2 + Z_3 &= \delta_2 - \delta_3 + \delta Z_2 + \delta Z_3 \\ Z_1 + Z_3 &= \delta_1 - \delta_3 + \delta Z_1 + \delta Z_3 \end{aligned} \qquad (51)$$

The zenith distances in equations (30) to (33) are obtained as follows:

With the original hour angles, the observed zenith distances are corrected by applying equations (14) and (15)

In figure 4, the configuration is depicted for a star north of the zenith $\delta > \phi$, where the circle SM is the star trajectory and S is the star position.

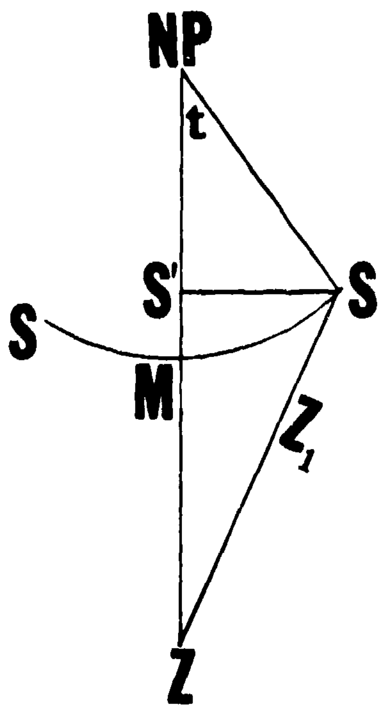

FIGURE 4. The Configuration for a Circumpolar Star.

Let t be the selected common hour angle.

$$ZM = Z_m, \text{ Meridian Zenith Distance}$$
$$ZS = Z_1$$
$$SP = MP = 90° - \delta_x$$

then

$$S'P = 90° - \delta_x$$

from

$$\tan \delta_x = \tan \delta \sec t \qquad (52)$$

resulting in

$$\cos Z_1 = \cos ZS' \frac{\sin \delta}{\sin \delta_x} \tag{53}$$

where

$$ZS' = Z_m + \delta_x - \delta$$

LONGITUDE DETERMINATION BY TRACKING STARS CROSSING THE MERIDIAN PLANE. • Longitudes were determined using a Wild T – 4 'eo-dolite. The observer tracks the star halfway toward the center of the field of ew with the eye piece micrometer. The instrument is then reversed 180°, the telescope reset for the same star, and the observer resumes tracking the star. This procedure of tracking the star before and after crossing the meridian must be discussed.

The procedure of reversing a transit instrument by lifting it from its bearings to track the star before and after crossing the meridian plane is different from the procedure used with the Wild T – 4 in which the theodolite is rotated along a vertical axis 180° with respect to the first position through the horizontal scale readings. This procedure results in errors which influence the longitude or time computations. Errors requiring analysis are

1. Error of a transit observation.
2. Instrumental errors.
3. Rotation of the pier, if any.
4. Equation of bright.

1. Error of a Transit Observation. The error in the observed time of star transit is independent of the personal equation and other constant errors. The final result is affected by the error when the observations of the two observers are not combined. The error is determined by comparing the values of the thread intervals or micrometer position from the observations. Let τ equal the equational interval between two closely ruled vertical lines on glass, and let T_1, T_2 equal the observed transit times. Then

$$\tau = (T_2 - T_1) \cos \delta \tag{54}$$

Each observation furnishes a value of τ. When considering a great number of values, the error of a single determination is obtained from

$$r = \sqrt{\frac{[V V]}{(m-1)}} \tag{55}$$

in which the values of V are the residuals computed by subtracting the known value of τ from each value in the M number of observations. The error ϵ of the observed time transit over each thread is

$$2\epsilon^2 = r^2$$

and

$$\epsilon = \sqrt{\frac{[V V]}{2(m-1)}} \tag{56}$$

If n indicates the number of thread with the mean for a star having declination $\delta = 0$, the error is

$$\epsilon = \sqrt{\frac{[V V]}{2(m-1)n}} \tag{57}$$

For a star with nonzero declination, the error is

$$\epsilon = \sqrt{\frac{[V V]}{2(m-1)n}} \sec \delta \tag{58}$$

2. Instrumental Errors are from Defects in the Instrument. Also included are errors made in setting up the instrument (collimation, inclination, and azimuth of the horizontal axis) and imperfections in the pivots. A good observational procedure can reduce the instrumental errors.

3. Rotation of the Pier. The instrumental errors are controlled by placing two ground marks north and so[illegible] of the meridian plane. Any changes will affect the instrument azimuth orientation and influence the longitude or the time determination.

4. Equation of Bright. The eye perceives the light of a dim star later than that of a bright star. To overcome the difference of perception, the observer tracks the star with a micrometer that has two parallel threads in place of one. The star is centered in the middle of the two threads and then tracked.

Analysis of Data. The micrometer-contact times from the longitude observations are not available for these analyses. Instead NGS (National Geodetic Survey, NOAA) provided a computed mean time for a star's meridian transit. The transit time error is estimated in this report by methods independent of instrument orientation errors.

The star is tracked before and after crossing the meridian by revolving the theodolite through the vertical axis. The observation is set to a different vertical plane because a special attachment is not provided to the theodolite.

Before the star meridian transit, the time refers to an azimuth a_1 and to an azimuth a_2 after the star crosses the meridian. A small error exists in the meridian time crossing.

The error in the star transit time is given by

$$\delta T = \frac{a_1 - a_2}{2} \cdot \frac{\sin(\varphi - \delta)}{\cos \delta} \qquad (59)$$

To avoid the error, stars are selected between declinations that are close to the station latitude. The effect of the error on the observed transit time of a star set is examined. The longitude is determined from the same star set.

Inherent Error in the Transit Time. To set up the Wild T – 4 theodolite position without an angular deviation when reversing to observe the star transit time is practically impossible. The first longitude determination of Aero Station is considered. The deviation of azimuth coefficient results from a fixed north star related to stars from the south as follows:

$$\beta_i = -\delta\lambda + a \cdot A_i$$
$$\beta_i = -UT_i + \theta_o + \Delta\theta_i + b\,B_i + k_i - (\alpha_i - \lambda_o) \qquad (60)$$

where

$\delta\lambda$	=	Correction to the assumed longitude λ_o
UT_i	=	WWV observation time
θ_o	=	Greenwich Vernal Equinox hour angle
A	=	$\sin(\varphi - \delta) \sec \delta$
B	=	$\cos(\varphi - \delta) \sec \delta$
b	=	Level error
k	=	Correction for diurnal aberration
a	=	Azimuth error expressed in time

Table 8 shows the data used in this investigation and table 9 shows the evaluation of mean azimuth orientation.

Equations to be used are

$$\beta_i = -\lambda\delta + a A_i$$

$$\beta_i = U.T_i + \theta_o + \Delta\theta_2 + b B_i + k_i - (\alpha_i + \lambda_o)$$

TABLE 8. Evaluation of B_i

	U.T	α_i	δ	bB+K	β_i
1	$23^h40^m43^s.663$	$21^h43^m\ 8^s.847$	9° 46 '.9	+.053	$2^s.834$
2	43 35 .064	46 1 .294	49 13 .1	+.194	3 .270
3	49 39 .466	21 52 6 .212	25 49 .8	-.007	2 .989
4	23 57 35 .429	22 0 3 .650	13 1 .3	+.235	2 .919
5	0 3 32 .706	6 1 .874	25 14 .8	+.133	2 .990
6	7 37 .684	10 8 .405	58 6 .2	+.451	3 .554
7	20 12 .117	22 44 .682	52 7.7	+.398	3 .386

$$\theta_o = 3^h 07^m 18^s .368$$
$$\lambda = 5 \quad 8 \quad 49 \ .467$$
$$\Delta\theta = (1 + C_o) \ U.T$$
$$1 + C_o = 1.0027379051$$

The coefficients A are

Star	A
1	.500
2	−.263
3	.259
4	.455
5	.269
6	−.609
7	−.361

TABLE 9. Evaluation of Azimuth

South \ North	2	6	7
1	−.571	−.643	−.640
3	−.538	−.651	−.640
4	−.489	−.591	−.572
5	−.526	−.642	−.629

Table 10 shows that star numbers 2 and 4 have errors in the time crossing because of misorientation. A graphic representation of equation (47) shows a deviation of the stars 2 and 4 with the other set (figure 5).

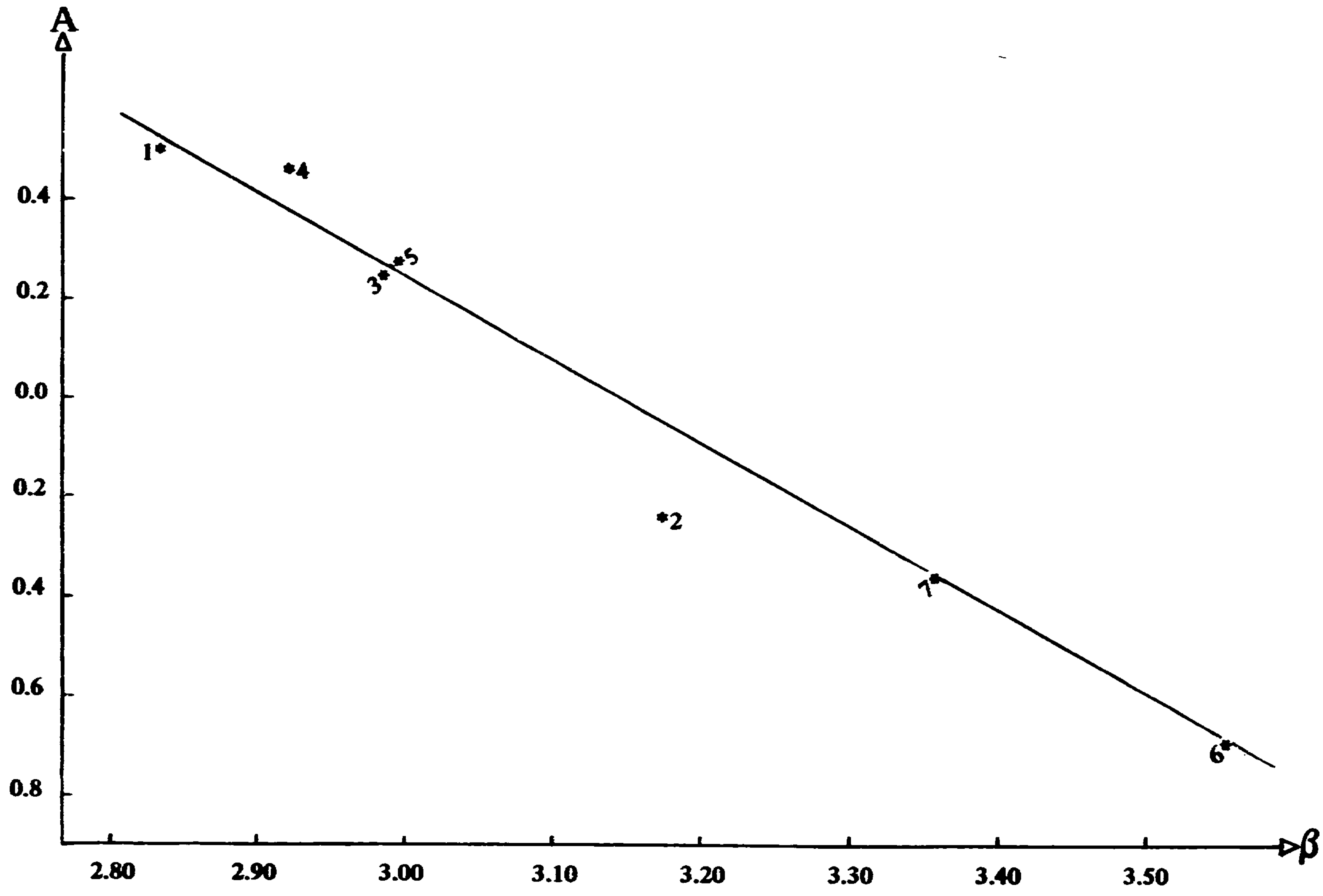

FIGURE 5. The Deviation of Observations 2 and 4.

The resulting deviation in the correction $\lambda\delta$ from the author's equation is

$$\Delta T = -\Delta\lambda = \frac{\beta_2 \cdot M_1 - \beta_1 / \cdot M_1}{M_1 - M_2} \qquad (61)$$

where

$$\begin{aligned} M_1 &= \tan\varphi - \tan\delta_1 \\ M_2 &= \tan\varphi - \tan\delta_2 \end{aligned} \qquad (62)$$

Solving by pairs of stars, one can see in table 10 that the star numbers 2 and 4 have an erroneous time of crossing, which is caused mainly by the difference in azimuth orientation.

TABLE 10. Evaluation Accuracy of $\Delta\lambda$

Star	1	2	3	4	5	6	7
1		3.120	3.156		3.171	3.159	3.155
2	3.120		3.129		3.132		
3	3.156			3.081		3.158	3.155
4		3.141	3.081		3.093	3.191	3.179
5	3.171	3.132		3.093		3.163	3.159
6	3.159		3.159	3.191			
7	3.155		3.155	3.179			

The author recommends that future astro teams observe pairs of stars with instrument position clamp east (or west) and pairs of stars clamp west (or east).[2] The author's formulas are suggested since the formulas are independent of instrumental azimuth orientation and reduce the number of unknowns by 50 percent.

[2] *National Ocean Survey*, formerly the *Coast and Geodetic Survey*, "Manual of Geodetic Astronomy," 1947, Special Publication No. 237, p. 54.

TABLE 11. Results From The Baldini Method

		N	S	S	N	N	S	N
STAR		**ν Andr.**	**α Triang.**	**α Arietis**	**ϕ Persei**	**δ Andr.**	**γ Triang.**	**τ Persei**
ν Andromedae	N		−0.729	−0.720			−0.736	
α Trianguli	S	−0.736			−0.738	−0.743		−0.736
α Arietis	S	−0.720			−0.725	−0.737		−0.721
ϕ Persei	N		−0.738	−0.725			−0.743	
δ^1 Andromedae	N		−0.744	−0.737			−0.745	
γ Trianguli	S	−0.736			−0.743	−0.745		−0.742
τ Persei	N		−0.736	−0.721			−0.742	
MEAN		−0.731	−0.737	−0.726	−0.735	−0.742	−0.741	−0.733

$$\Delta T = -9^s.00 + \delta T$$

$$\Delta T = -9^s.735$$

SOURCE: National Ocean Survey, formerly the Coast and Geodetic Survey, "Manual of Geodetic Astronomy, 1947, Special Publication No. 237, pp. 44 and 45.

NOTE: As the star is observed before and after crossing the meridian plane, the effect of collimation is cancelled out, so Baldini's method solves for only one unknown: ΔT. Each star can be combined with another star, forming a pair. The results for any pair can be seen at intersecting a column and a line on a table. The values are the decimal part of seconds in longitude. The final value is:

$$\Delta T = -\Delta\lambda = -9^s + \delta t = \Delta T = -9^s.735$$

DEFLECTIONS OF THE VERTICAL • The ξ and η components of the vertical deflection in the meridian and in the prime vertical are related to the astronomic latitude and longitude φ, λ, with respect to the geodetic B and L, by the equations

$$\xi = \varphi - B$$
$$\eta = (\lambda - L) \cos \varphi \tag{63}$$

Table 12 gives the results of the final astronomic positions and the mean errors in its positions. Table 13 gives the errors in computing ξ and η.

TABLE 12. Final Astronomic Positions

STATION	ϕ (N)	σ_ϕ	Λ(W)	σ_Λ
AERO USAETL 1978	39° 19′ 52″.82	±0″.17	77° 11′ 31″.08	±0″.38
BRINK 1969 AZ. MK.	39 12 27 .98	±0 .19	77 14 32 .76	±0 .38
CEDAR HEIGHTS 1969	39 15 14 .73	±0 .20	77 13 45 .65	±0 .38
CHEVY USAETL 1978	39 04 54 .05	±0 .20	76 56 45 .35	±0 .38
DALE 1943 RM 1	39 17 21 .22	±0 .20	77 15 49 .02	±0 .38
FIRE USAETL 1978	39 17 43 .40	±0 .20	77 19 00 .77	±0 .38
FREEWAY USAETL 1978	39 11 29 .16	±0 .20	77 15 04 .02	±0 .38
HORSE USAETL 1978	39 05 02 .22	±0 .20	77 18 27 .58	±0 .38
LAYTON 1943	39 12 45 .52	±0 .18	77 08 32 .89	±0 .38
MAYNE USAETL 1978	39 11 13 .90	±0 .20	77 07 04 .71	±0 .38
MILL 1958	39 20 11 .29	±0 .20	77 22 49 .22	±0 .38
RIFFLE USAETL 1978	39 08 52 .88	±0 .20	77 17 29 .15	±0 .38
SHERWOOD USAETL 1978	39 08 54 .81	±0 .20	77 00 56 .85	±0 .38
SPENCER 1969	39 07 14 .06	±0 .20	76 59 04 .65	±0 .38
TABOR 1969	39 15 06 .70	±0 .20	77 08 37 .35	±0 .38
TOWER USAETL 1978	39 03 40 .08	±0 .20	76 55 10 .99	±0 .38
VADER USAETL 2978	39 10 11 .11	±0 .17	77 16 36 .92	±0 .38
WELFARE 1970	39 03 15 .95	±0 .17	76 51 23 .40	±0 .38

TABLE 13. Deflections of the Vertical

STATION	ξ	σ_ξ	n	σ_n
AERO USAETL 1978	+4".25	±0".17	+2".9	±0".29
BRINK 1969 AZ. MK.	+1 .56	±0 .19	+1 .8	±0 .29
CEDAR HEIGHTS 1969	+2 .49	±0 .20	+2 .4	±0 .29
CHEVY USAETL 1978	-1 .35	±0 .20	-5 .1	±0 .29
DALE 1943 RM 1	+4 .49	±0 .20	+5 .0	±0 .29
FIRE USAETL 1978	+5 .15	±0 .20	+6 .4	±0 .29
FREEWAY USAETL 1978	+2 .12	±0 .20	+1 .8	±0 .29
HORSE USAETL 1978	+1 .63	±0 .20	+3 .0	±0 .29
LAYTON 1943	+1 .50	±0 .18	-0 .9	±0 .29
MAYNE USAETL 1978	+0 .61	±0 .20	-1 .7	±0 .29
MILL 1958	+5 .23	±0 .20	+6 .2	±0 .29
RIFFLE USAETL 1978	+0 .64	±0 .20	+2 .5	±0 .29
SHERWOOD USAETL 1978	-1 .39	±0 .20	-5 .5	±0 .29
SPENCER 1969	-0 .31	±0 .20	-4 .8	±0 .29
TABOR 1969	+1 .42	±0 .20	-0 .5	±0 .29
TOWER USAETL 1978	-1 .28	±0 .20	-5 .2	±0 .29
VADER USAETL 1978	+2 .39	±0 .17	+2 .1	±0 .29
WELFARE 1970	-1 .58	±0 .17	-7 .3	±0 .29

Expected Accuracy Between Two Stations. The expected accuracy between stations can be derived by applying the law of error propagation. The meridian deflections of the vertical components ξ_1 and ξ_2 of the stations 1 and 2 have standard errors σ_1 and σ_2 respectively.

The standard error of the difference

$$d = \xi_2 - \xi_1 \quad (64)$$

is obtained by the special law of error propagation:

$$\sigma_d = \sqrt{\left(\sigma_1 \frac{\partial d}{\partial \xi_1}\right)^2 + \left(\sigma_2 \frac{\partial d}{\partial \xi_2}\right)^2 - 2\left(\frac{\partial d}{\partial \xi_1} \frac{\partial d}{\partial \xi_2}\right)\sigma_{12}} \tag{65}$$

ξ_1, and ξ_2 are quite independent of each other, the covariance $\sigma_{12} = 0$. Therefore,

$$\sigma_d = \sqrt{\sigma_1^2 + \sigma_2^2} \tag{66}$$

From the values shown in table 13, the following is obtained:

$$(\sigma_d)_\xi = \pm 0''.20\sqrt{2} = \pm 0''.29 \tag{67}$$

In a similar way, the standard error of the difference of the deflections of the vertical component in the prime vertical is found

$$(\sigma_d)_n = \pm 0''.41 \tag{68}$$

COMMENTS ON LATITUDE DETERMINATION • The latitude determination by the NGS modified Sterneck method involves two symmetrical circum-meridian zenith distances made in the direct and reverse theodolite positions east and west of the meridian. The index error is eliminated and the need for the zenith point correction is avoided. An additional salient feature of the index error is the elimination of the personal error of star bisection.

CONCLUSIONS

1. The Sterneck Method of determining astronomic latitude entails measurements of zenith distances, which are subjected to circle and reading errors, as well as systematic error due to atmospheric refraction. These errors, determined by a relationship between zenith distances and stars' declinations, are excessive for the needs of baseline research.

2. Such errors are difficult to control, measure or eliminate by data processing, but their effects can be minimized by a special least square treatment introduced by the author.

3. A new observing procedure, associated with the Wild T – 4 universal theodolite and observations of transit times of pairs of stars over a fixed vertical plane, significantly reduces the number of unknowns and thereby provides improved precision and reliability.

An essential condition of the NGS observational procedure is that the instrument remains clamped in the meridian and the star is observed at a certain distance from the middle vertical thread, with time being recorded. TALCOTT level vial sensitivity values are highly correlated with the calibration process by displacing the level division intervals with respect to the angular change of the vertical circle.

Absolute zenith distances are to be used for the latitude computation, which is hard to obtain with a minimum source of errors. Anomalous variations in the refraction affect differently the north and south stars and the error in level calibration, which increases both accidental and systematic errors of observations.

The observer probable error of one observation is 0".40. This small probable error is a criterion of the skill of the observer. The stations latitude mean error is less than 0".22.

SYMBOLS

λ	Longitude
ϕ	Latitude
δ	Declination
α	Right assension
t	Hour angle
Z	Zenith distance
P	Personal equation
ρ	Level sensitivity
ε	Index error
τ	Interval of time
A	Azimuth
R	Refraction
i	Inclination