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**STRESS INTENSITY AND FATIGUE CRACK GROWTH  
IN MULTIPLY-CRACKED, PRESSURIZED,  
PARTIALLY AUTOFRETTAGED THICK CYLINDERS**

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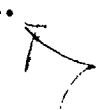
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**ABSTRACT**

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Stress intensity factors are determined for internally and externally cracked, pressurized thick cylinders with partial autofrettage (less than 100% overstrain). The solutions are based on a superposition of existing solutions which does not involve any loss of accuracy.

Implications of the stress intensity factor results for the safe-life design of gun tubes are discussed. Various suggestions for future work are presented.



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## INTRODUCTION

Fatigue crack growth arising from the cyclic pressurization of thick cylinders can produce a regular array of up to 50 equal-length radial cracks emanating from the bore.<sup>1</sup> A knowledge of the crack tip stress intensity factor  $K$  is necessary to predict the fatigue growth rate and critical length of such cracks.

Several solutions for the case of a cracked, pressurized-thick cylinder are available.<sup>1-6</sup> It is likely that the most accurate of these solutions are those derived by use of the modified mapping collocation (MMC) method. These include the solution in Reference 5 for up to four internal or external radial cracks, and that in Reference 6 for up to forty internal radial cracks. The errors associated with the MMC technique are generally estimated as being less than 1%.

To inhibit fatigue growth of internal cracks it is common practice to produce a more advantageous stress distribution involving residual compressive hoop stresses near the bore, by autofrettage treatment of the cylinder prior to use.<sup>7</sup>

$K$  solutions exist for a multiply-cracked, fully autofrettaged (100% overstrain)\* tube.<sup>6,8</sup> Reference 6 is an MMC solution.

Note: It has been demonstrated that the stress distribution arising from full autofrettage is essentially equivalent to that arising from steady-state thermal loading of the tube,<sup>9</sup> and that  $K$  values for one of these loadings may be obtained from the other by a simple scaling operation.<sup>8</sup>

However, the optimum autofrettage condition may not be 100% overstrain<sup>7</sup> since fatigue cracks may develop at the outside radius as a result of the relatively high tensile residual stress. Clearly, the choice of the optimum overstrain condition will involve a consideration of the rates at which external cracks will grow radially inward, and the rates at which internal cracks will grow outward. In each case, prediction of crack growth rate, critical crack length, and residual strength will depend on a knowledge of the crack-tip stress intensity factor.

\*Overstrain is the proportion of the cylinder wall thickness that is subjected to plastic strain during the autofrettage process.

1. GOLDTHROPE, B. D. *Fatigue and Fracture of Thick-Walled Cylinders and Gun Barrels* in Case Studies in Fracture Mechanics, Army Materials and Mechanics Research Center, AMMRC MS 77-5, June 1977, p. 3.8.1-3.8.15.
2. PU, S. L., and HUSSAIN, M. A. *Stress Intensity Factors for a Circular Ring with Uniform Array of Radial Cracks Using Cubic Isoparametric Singular Elements*. Trans. 24th Conference of Army Mathematicians, Army Research Office, Report 79-1, 1979.
3. TWEED, J., and ROOKE, D. P. *The Stress Intensity Factor for a Crack in a Symmetric Array Originating from a Circular Hole in an Infinite Solid*. International Journal of Engineering Science, v. 13, 1975, p. 653-662.
4. BARATTA, F. I. *Stress Intensity Factors for Internal Multiple Cracks in Thick-Walled Cylinders Stressed by Internal Pressure Using Load Relief Factors*. Engineering Fracture Mechanics, v. 10, 1978, p. 691-697; also Army Materials and Mechanics Research Center, AMMRC TN 77-3, July 1977.
5. TRACY, P. G. *Elastic Analysis of Radial Cracks Emanating from the Outer and Inner Surfaces of a Circular Ring*. Engineering Fracture Mechanics, v. 11, 1979, p. 291-300.
6. PARKER, A. P., and ANDRASIC, C. P. *Stress Intensity Prediction for a Multiply-Cracked, Pressurized Gun Tube with Residual and Thermal Stresses* in Army Symposium on Solid Mechanics, 1980 - Designing for Extremes: Environment, Loading, and Structural Behavior, Army Materials and Mechanics Research Center, AMMRC MS 80-5, 1980, p. 35-39.
7. KAPP, J. A., and EISENSTADT, R. *Crack Growth in Externally Flawed, Autofrettaged Thick-Walled Cylinders and Rings* in Fracture Mechanics - A Symposium, ASTM STP 677, C. W. Smith, ed., 1979, p. 746-756.
8. PARKER, A. P., and FARROW, J. R. *Stress Intensity Factors for Multiple Radial Cracks Emanating from the Bore of an Autofrettaged or Thermally Stressed Thick Cylinder*. Engineering Fracture Mechanics, v. 14, 1981, p. 237-241.
9. PARKER, A. P., and FARROW, J. R. *On the Equivalence of Axi-Symmetric Bending, Thermal and Autofrettage Residual Stress Fields*. Journal of Strain Analysis, v. 15, no. 1, 1980, p. 51-52.

The designer requires accurate stress intensity factors for both internally cracked and externally cracked tubes with internal pressure, and any amount of autofrettage from zero to 100% overstrain (full autofrettage). In this report it is demonstrated that many solutions for the apparently more complicated problems of internally and externally cracked, partially autofrettaged tubes may be obtained, without any loss of accuracy, by a straightforward superposition of existing solutions.

### BASIC EQUATIONS FOR PRESSURIZED, AUTOFRETTAGED TUBES

Consider a tube, internal radius  $a$ , external radius  $b$ , which is subjected to an internal pressure  $p$ , Figure 1. The distribution of hoop ( $\sigma_\theta$ ) stress in this case is given by Lamé's equation as:

$$\sigma_\theta^B = \frac{a^2 p}{b^2 - a^2} \left[ 1 + \frac{b^2}{r^2} \right] \quad (1)$$

where  $r$  is the radius at which the stress is defined, and the superscript B indicates pressure in the bore.

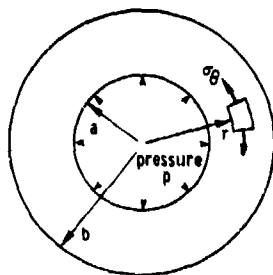


Figure 1. Pressurized, elastic thick cylinder.

Assuming elastic-perfectly plastic material properties and plane strain conditions, and we employ Tresca's yield criterion, but omit the analysis, the pressure  $p^*$  to cause yielding of the tube out to a radius  $r = c$  (Figure 2) is given by:

$$p^* = Y \ln(c/a) + \frac{Y}{2b^2} (b^2 - c^2) \quad (2)$$

where  $Y$  is the uniaxial yield stress for the material. This will give directly the pressure for initial yielding at the bore:

$$p_i^* = \frac{Y}{2b^2} (b^2 - a^2) \quad (3)$$

and the pressure for complete yielding of the tube:

$$p_y^* = Y \ln(b/a) \quad (4)$$

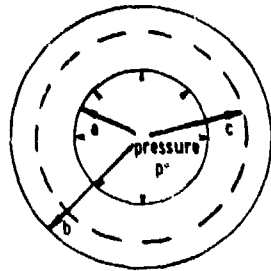


Figure 2. Pressurized, partially plastic thick cylinder.

If the cylinder is subjected to a pressure  $p^*$ , [ $p_i^* < p^* < p_y^*$ ], there will be partial yielding of the tube out to a radius  $c$ , Figure 2. The hoop stresses produced by this pressurization are:

$$\sigma_{\theta}^* = -p^* + Y[1 + \ln(r/a)] , \quad a < r < c \quad (5)$$

$$\sigma_{\theta}^* = \frac{Yc^2}{2b^2} \left[ 1 + \frac{b^2}{r^2} \right] , \quad c < r < b \quad (6)$$

If the pressure  $p^*$  is subsequently removed completely, assuming that the unloading is entirely elastic, with no reversed yielding (valid provided  $b/a < 2.22$ ), the residual hoop stress distribution  $\sigma_{\theta}^R$  is given by:<sup>10</sup>

$$\sigma_{\theta}^R = -p^* + Y[1 + \ln(r/a)] - p^* a^2 / (b^2 - a^2) [1 + (b^2/r^2)] , \quad a < r < c \quad (7)$$

$$\sigma_{\theta}^R = [(Yc^2/2b^2) - (p^* a^2 / (b^2 - a^2))] [1 + (b^2/r^2)] , \quad c < r < b \quad (8)$$

Clearly, a repressurization of the tube to a pressure  $p < p^*$  will produce a stress distribution which may be calculated by the addition of (7) and (1) for  $r \leq c$ , and (8) and (1) for  $r > c$ .

#### SUPERPOSITION METHOD

The superposition principle applies to any linearly elastic body subjected to a statically determinate loading system. By use of this principle it is a straightforward procedure to demonstrate that the stress intensity factor for a crack in a body subjected to external stress is identical to that caused by stresses acting on the surface of the crack equal but opposite to those which would be present in the uncracked body under external load. The implications of this result for the particular cases to be considered are presented.

##### A. Internal Cracking, Pressure in Bore and Cracks, Autofrettage Stresses (Figure 3)

The appropriate crack-line loading will comprise:

- (i) A normal loading on the crack surface, equal and opposite to that predicted by Lamé's Equation 1
- (ii) A constant pressure  $p$  which has infiltrated the cracks from the bore.

10. HILL, R. *The Mathematical Theory of Plasticity*. Oxford University Press, Oxford, England, 1967.

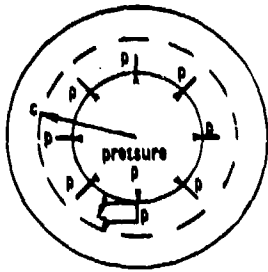


Figure 3. Internally cracked, autofrettaged thick cylinder - pressure in bore and cracks.

[The total effect of (i) and (ii) above is an equal and opposite stress given by:

$$\sigma_{\theta}^{BC} = p \left( 1 + \frac{a^2}{b^2 - a^2} \left[ 1 + \frac{b^2}{r^2} \right] \right) \quad (9)$$

where the superscript BC indicates pressure in bore and cracks.]

(iii) A residual distribution equal and opposite to that predicted by (7) and (8).

#### B. External Cracking, Pressure in Bore, Residual Autofrettage Stresses (Figure 4)

In this case the necessary crack-line loading will comprise:

- (i) A normal loading equal and opposite to that predicted by (1).
- (ii) A residual distribution equal and opposite to that predicted by (7) and (8).

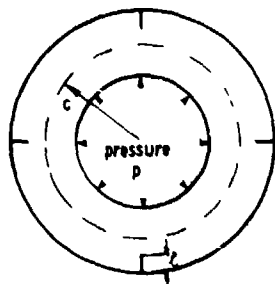


Figure 4. Externally cracked, autofrettaged thick cylinder - pressure in bore.

#### EXTERNALLY CRACKED, PARTIALLY AUTOFRETTAGED TUBE

If a partially autofrettaged, externally cracked tube is repressurized to a pressure  $p < p^*$ , then the total stress distribution  $\sigma_{\theta}^T$  is given by the addition of (1) and (7), and (1) and (8), thus:

$$\sigma_{\theta}^T = \sigma_{\theta}^B + \sigma_{\theta}^R \quad (10)$$

or:

$$\sigma_{\theta}^T = \left[ \frac{a^2 p}{b^2 - a^2} + \frac{\gamma c^2}{2b^2} - \frac{p^* a^2}{b^2 - a^2} \right] \left[ 1 + \frac{b^2}{r^2} \right], \quad c < r < b \quad (11)$$



Apart from a multiplying constant, the above expression is identical to Equation 1. Thus, if stress intensity factors have been derived for a cylinder with radial edge crack(s) of length  $\ell$ , [ $\ell < (b-c)$ ], it is only necessary to scale these results by S, where:

$$S = \left[ \frac{a^2 p}{b^2 - a^2} + \frac{Yc^2}{2b^2} - \frac{p^* a^2}{b^2 - a^2} \right] \left[ \frac{b^2 - a^2}{a^2 p} \right] \quad (12)$$

in order to predict K for the externally cracked, pressurized, partially autofrettaged tube.

#### Example I

A thick cylinder has an external radius twice that of its internal radius. It is subjected to 50% overstrain in the autofrettage process. The cylinder is subsequently repressurized to  $p = Y/N$  ( $< p^*$ ), where N is any suitable number. What is the scaling factor S to be applied to K results for the externally cracked, nonautofrettaged, pressurized tube in order to solve for the partially autofrettaged case? [The solution is to be limited to  $\ell < (b-c)$ .]

#### Solution

From (2), the autofrettage pressure  $p^*$  is given by

$$p^* = Y \ln(c/a) + \frac{Y}{2b^2} (b^2 - c^2) \quad (13)$$

in the example  $c = 1.5a$ ,  $b = 2a$ ; hence:

$$p^* = 0.624 Y \quad (14)$$

Substitute from (14) into (12), noting that  $Y = Np$ , to obtain:

$$S = 1 + 0.2196 N. \quad (15)$$

Typical value would be  $N = 3$ , giving  $S = 1.659$ , hence:

$$K_{50\% \text{ overstrain} + \text{pressure}} = 1.659 K_{\text{zero overstrain} + \text{pressure}}$$

Note: The solution is valid only provided  $\ell < (b-c)$ .

A set of results for external cracks with internal pressure and 50% overstrain, based on the results of Reference 5 and the superposition outlined in this section, is presented graphically in Figure 5.

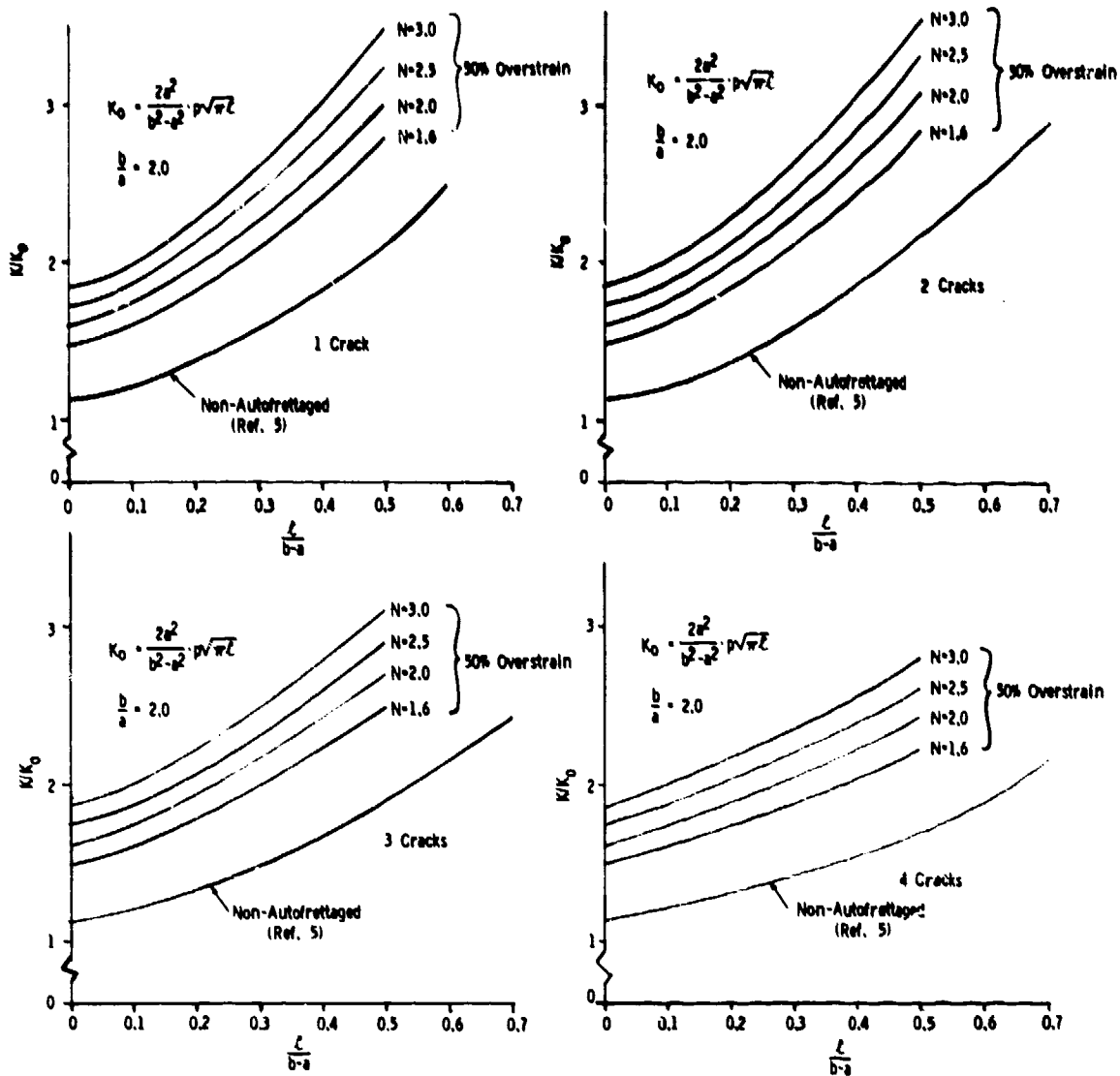


Figure 5. Stress intensity factors for externally cracked, partially autofretted thick cylinder.

### INTERNALLY CRACKED, PARTIALLY AUTOFRETTAGED TUBE

First consider the case of full autofrettage (100% overstrain). The autofrettage pressure required to achieve this is given by Equation 4. Substitute from (4) into (7) to obtain the residual stress field with full autofrettage, namely:

$$\sigma_{\theta}^R \text{ full autofrettage} = -Y \ln(b/a) \left( 1 + \frac{a^2}{b^2 - a^2} \left[ 1 + \frac{b^2}{r^2} \right] \right) + Y [1 + \ln(r/a)] \quad (16)$$

Now, returning to the case of partial autofrettage, the required crack-line stress system is given by the superposition of (9) and (7), thus:

$$\sigma_{\theta}^T = \sigma_{\theta}^R + \sigma_{\theta}^{BC} \quad (17)$$

or:

$$\sigma_{\theta}^T = -p^* + Y[1 + \ln(r/a)] - \frac{p^* a^2}{b^2 - a^2} \left[ 1 + \frac{b^2}{r^2} \right] \quad (18)$$

$$+ p \left( 1 + \frac{a^2}{b^2 - a^2} \left[ 1 + \frac{b^2}{r^2} \right] \right)$$

Substituting into (18) from (2) we obtain:

$$\sigma_{\theta}^T = [p - Y \ln(c/a) - \frac{Y}{2b^2} (b^2 - c^2) + Y \ln(b/a)] \left( 1 + \frac{a^2}{b^2 - a^2} \left[ 1 + \frac{b^2}{r^2} \right] \right) \quad (19)$$

$$- Y \ln(b/a) \left( 1 + \frac{a^2}{b^2 - a^2} \left[ 1 + \frac{b^2}{r^2} \right] \right) + Y[1 + \ln(r/a)]$$

On inspecting the above equation we note that the first term is merely a scaling of the stress field for pressure in bore and cracks, Equation 9, while the second and third terms represent the stress field arising from full autofrettage, Equation 16. Making the substitution  $Y = Np$  and writing in terms of superposition of  $K$  solutions, we obtain the expression:

$$\begin{aligned} K_{\text{partial autofrettage}} + \text{pressure} &= [1 + N \ln(b/c) - \frac{N}{2b^2} (b^2 - c^2)] K_{\text{pressure}} \quad (20) \\ &+ K_{\text{full autofrettage}} \end{aligned}$$

#### Example II

The tube described in Example I has internal cracking. What is the stress intensity factor in terms of that for full autofrettage without internal pressure, and that for internal pressure without autofrettage? [The solution is to be limited to  $L < (c-a)$ .]

#### Solution

Substituting  $c = 1.5a$ ,  $b = 2a$  into (19) gives:

$$K_{\text{partial autofrettage}} + \text{pressure} = (1 + 0.0689 N) K_{\text{pressure}} + K_{\text{full autofrettage}} \quad (21)$$

Thus if  $N = 3$ :  $K_{\text{partial autofrettage}} + \text{pressure} = (1.2067) K_{\text{pressure}} + K_{\text{full autofrettage}}$ .

Note: The solution is valid only provided  $l < (c-a)$ .

A set of results for internal cracks with internal pressure and 50% overstrain, based on the results of Reference 6 and the superposition described in this section, is presented graphically in Figure 6.

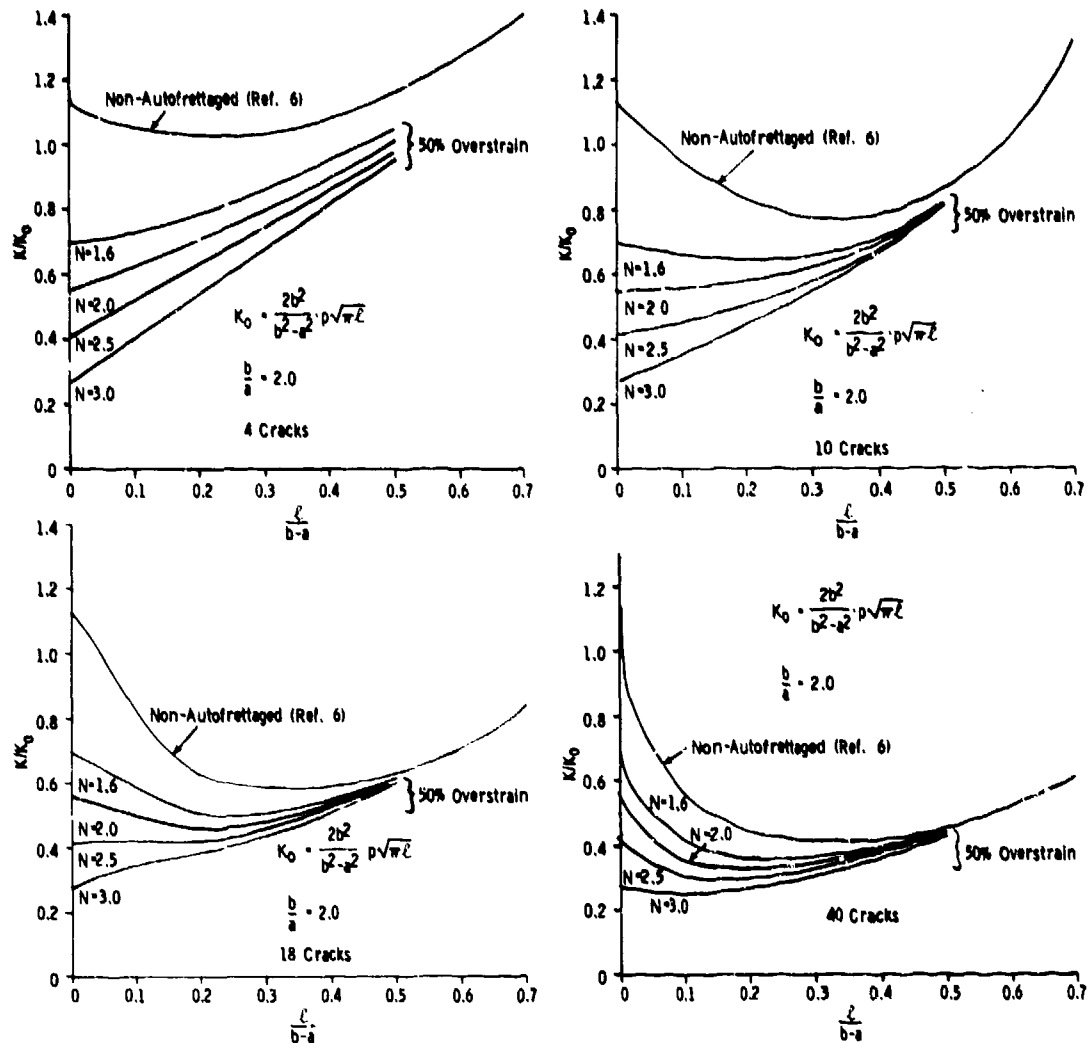


Figure 6. Stress intensity factors for internally cracked, partially autofretted thick cylinder.

### IMPLICATIONS FOR FATIGUE LIFE PREDICTION OF GUN TUBES

The fatigue growth rate of cracks subjected to cyclic loading may be expressed in terms of Paris' Law,<sup>11</sup> namely:

$$\frac{dl}{dN} = C(\Delta K)^m \quad (22)$$

11. PARIS, P. C., and ERDOGAN, F. *A Critical Analysis of Crack Propagation Laws*. Trans. ASME, Journal of Basic Engineering, v. 85, 1963, p. 528-534.

where  $d\ell/dN$  is the fatigue crack growth per loading cycle,  $C$  and  $m$  are empirical constants, and  $\Delta K$  is the range of stress intensity defined by:

$$\Delta K = K_{\max} - K_{\min} \quad (K_{\min} \geq 0)$$

$$\Delta K = K_{\max} \quad (K_{\min} < 0)$$

and  $K_{\max}$  and  $K_{\min}$  are the maximum and minimum values of stress intensity during the loading cycle. (Note, the possibility of "overlapping" of the crack surfaces at some point on the crack line remote from the crack tip is not considered in this report.)

During the lifetime of a particular cracked tube the crack will propagate from some initial length  $\ell_i$  to some final, critical length  $\ell_c$  at which  $K_{\max}$  approaches the fracture toughness of the material. In order to predict the fatigue life, (21) is rearranged to give:

$$\int_{\ell_i}^{\ell_c} \frac{d\ell}{C(\Delta K)^m} = N_c - N_i \quad (23)$$

where  $(N_c - N_i)$  is the number of cycles to propagate from initial to critical crack length.

The implications of the results presented herein for the safe-life design of gun tubes may be summarized as follows.

#### A. Externally Cracked Tubes

All loading contributions in the case of external cracking tend to produce positive contributions to  $K$ , thus crack closure is not a possibility, and the only contribution to the stress intensity range ( $\Delta K$ ) is the cyclic pressurization term. The stress intensity arising from residual stressing will simply serve to increase  $K_{\max}$ , while not affecting  $\Delta K$ . In the case of steels this generally causes a relatively small increase in crack growth rate<sup>12</sup>. In addition, the increase in  $K_{\max}$  will cause the critical crack length  $\ell_c$  to be reduced in comparison with the nonautofrettaged case, hence total lifetime may also be reduced somewhat. This reduction in lifetime may not be significant, since relatively little of the component's lifetime is expended at longer crack lengths.<sup>13</sup>

#### B. Internally Cracked Tubes

In this case autofrettage will produce a negative  $K$  contribution. Positive  $K$  values, and the possibility of crack growth, cannot occur until the bore pressure has been raised to produce a positive  $K$  contribution which exceeds the autofrettage effect. Hence autofrettage tends to reduce the value of  $\Delta K$  for internal cracks, and thus also reduces the crack growth rate in comparison with the nonautofrettaged tube. Furthermore, since  $K_{\max}$  is also reduced, the critical crack length will increase and thus tend to increase component lifetime.

12. POOK, L. P. *Analysis and Application of Fatigue Crack Growth Data*. Journal of Strain Analysis, v. 10, 1975, p. 242-250.

13. DAVIDSON, T. E., and THROOP, J. F. *Practical Fracture Mechanics Applications to the Design of High Pressure Vessels in Application of Fracture Mechanics to Design*, J. J. Burke and V. Weiss, ed., Plenum Press, New York, 1979.

### C. Stability of Crack Growth Patterns

Stable arrays of 40 to 50 near-equal length, internal radial cracks have been observed in rifled, nonautofrettaged gun tubes. Life estimates based on this number of cracks appear to be borne out in practice. Goldthorpe<sup>1</sup> has suggested that the case of multiple (say 40) crack growth is the stable configuration for the nonautofrettaged cylinder. This stability derives from the strong negative slope of the  $K$  versus  $\ell$  curve at short crack lengths. However, this feature is absent in the case of autofrettaged tubes. Initial results from other work<sup>14</sup> indicate that life estimates for internally-cracked, fully autofrettaged tubes should be based on the assumption of six cracks propagating.

In view of the significant effect on  $K$  arising from the assumption of a particular number of cracks, it is clearly important that future work should fully investigate the stability of crack growth patterns in nonautofrettaged, partially autofrettaged, and fully autofrettaged tubes. This work should also address the question of the effect of rifling on crack pattern stability. Only then will it be possible to establish a proper fracture mechanics design procedure.

### D. Characterization of Autofrettage Stresses

There is some evidence to suggest that the residual stress field arising from the autofrettage process may not attain the magnitudes predicted by an elastic/perfectly plastic analysis which ignores strain hardening and Bauschinger effects.<sup>10,13</sup> The effect of this reduction in residual stress has been included as a simple multiplying factor in the determination of stress intensity factors.<sup>15</sup> Nevertheless, in order to have confidence in the fracture mechanics design procedure, it will be necessary to investigate the true nature of the autofrettage stress field, and the effect of this field on stress intensity calculations.

### E. Effects of Crack Shape

Another important feature of the crack growth pattern and rates in gun tubes is crack shape, and the change in shape (and hence growth rate and relative interaction) of thumbnail or semielliptical cracks during the fatigue life.

An understanding of crack-pattern stability, residual stresses, three-dimensional effects and (possibly) crack closure will also be of importance in the fracture mechanics design of other types of structural elements in general use, such as welded components.

## SUMMARY OF AVAILABLE EXPERIMENTAL WORK

Experimental work relating to the fracture mechanics of autofrettaged tubes is scarce. Work has commenced in the United Kingdom\* and in Australia† on the determination

\*AUSTIN, B. A., Private discussion, 1979.

†DeMORTON, M. E., Private discussion, 1979.

14. TABONE, M. V., BURNS, I. W., and GIBSON, A. F. *A Review of Fatigue Life Prediction for In-Service Ordnance*. Army Staff Course, Division II Project Study, Royal Military College of Science, Shrivenham, Swindon, England, 1980.

15. UNDERWOOD, J. H., and THROOP, J. F. *Residual Stress Effects on Fatigue Cracking of Pressurized Cylinders and Notched Bending Specimens*. Presented at SESA Spring Meeting, Boston, Massachusetts, May 1980.

of  $K$  calibrations. Other work is not currently accessible.\* However, some open literature crack-growth rate measurement is available.<sup>7,15</sup> In Reference 7 the fatigue growth rate of a single external crack was determined, using the test specimen configuration illustrated in Figure 7. The ring specimens tested had zero or 50% or 100% overstrain. Crack growth rates for 50% and 100% overstrain are generally within 25% of one another, this difference being easily explained on the basis of scatter and the relative values of  $K_{max}$ . However, the nonautofrettaged ring produced surprisingly low crack growth rates, bearing in mind that it was subjected to the same stress intensity range.

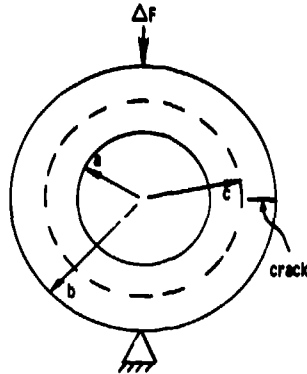


Figure 7. Configuration used in crack growth rate determination for autofrettaged ring (Ref. 7).

Reference 15 contains experimentally determined fatigue crack growth rate data for a single, internal, elliptical crack propagating in an autofrettaged barrel with varying degrees of autofrettage from 0% to 60%. The cyclic loading in this case was produced by internal pressure. The crack growth rates show general agreement with those predicted from a simple, two-dimensional, linearly-varying stress analysis modified by appropriate correction factors.

\*JOHANSSON, S., Private discussion, 1980.

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IN MULTIPLY-CRACKED, PRESSURIZED, PARTIALLY  
AUTOFRETTAGED THICK CYLINDERS -  
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Technical Report AMMRC TR 81-37, August 1981,  
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Stress intensity factors are determined for internally and externally cracked, pressurized thick cylinders with partial autofrettage (less than 100% overstrain). The solutions are based on a superposition of existing solutions which does not involve any loss of accuracy. Implications of the stress intensity factor results for the safe-life design of gun tubes are discussed. Various suggestions for future work are presented.

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