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## SOURCE RANGING BY A SINGLE ARRAY IN A MULTIPATH OCEAN

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## SUMMARY

### PROBLEM

The objective of this work was to develop a possible way of source ranging by a single array in a multipath ocean and to indicate its limitations.

### RESULTS

It was shown that mean-square sound-level fluctuations for a single sensor in a random multipath environment are directly dependent on the source range. When this was applied to an array of sensors, the result showed that a single horizontal line array might track a moving source in a multipath ocean environment. This result would facilitate the inter-array communication problem for coherent multi-array processing (CMAP) tracking of submarines. In Section IV of Ref 1 this single-array source tracking method was tested with promising results for the Bearing Stake data.

### RECOMMENDATIONS

It is recommended that this tracking method be further tested with even more suitable data.

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## NOTATION

$A$	amplitude of acoustic signal $F$
$\phi$	phase of acoustic signal $F$
$\chi$	sound-level of Eq (15)
$F_{mj}$	acoustic signal on path $m$ at sensor $j$
$\Phi_0$	source bearing
$k_0$	wave number
$\overline{\mu^2}$	the mean-square fluctuation of the refractive index
$L_n$	the integral scale for the correlation function of the refractive index
$A_0$	amplitude for a deterministic medium
$\phi_0$	phase for a deterministic medium
$\overline{(\chi_j')^2}$	mean-square sound-level fluctuation for sensor $j$
$\overline{(\chi')^2}$	mean-square sound-level fluctuation for the array
$N_0$	statistically independent noise power

## I. INTRODUCTION

References 2, 3, and 4 derived and verified useful relations for the mean-square sound-level fluctuation and the mean-square phase fluctuation for single paths to a single sensor in the ocean. In this report, these relations are extended to give the mean-square sound-level fluctuation for a single sensor in a random multipath environment. This relation could permit source ranging by a single sensor, and when applied to an array of sensors, the result shows that a single horizontal line array may track a moving source in a multipath ocean environment.

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2. V. I. Tatarski, *Wave Propagation in a Turbulent Medium*, McGraw-Hill, 1951.
  3. R. F. Shvachko, "Sound Fluctuations in the Upper Layer of the Ocean and Their Relation to the Random Inhomogeneities of the Medium," *Sov. Phys.-Acoust.* 2, 280-282 (1964).
  4. R. F. Shvachko, "Sound Fluctuations and Random Inhomogeneities in the Ocean," *Sov. Phys.-Acoust.* 13, 93-97 (1967).



## II. STATISTICAL CHARACTERISTICS OF MULTIPATH SIGNALS

The multipath signals received by a horizontal line array can be represented in a multipath ocean environment as follows. First assume that an ocean acoustic signal  $F$  received at sensor point  $\underline{x}$  and time  $t$  can be represented by

$$F = F(\underline{x}, t) = A e^{i\phi}, \quad (1)$$

where  $A$  is the amplitude and  $\phi$  is the phase. Next, assume that  $F$  is composed of a linear superposition of multipath signals  $F_m$ ,  $m = 1, 2, 3, \dots, M$  ( $M$  is the total number of multipath signals received) and that each multipath signal received can be represented by

$$F_m = A_m e^{i\phi_m}. \quad (2)$$

The  $j^{\text{th}}$  sensor of a horizontal line array with sensors at  $\underline{x}_j$ ,  $j = 1, 2, 3, \dots, J$  ( $J$  is the total number of array sensors) receives a signal represented by

$$F_j = A_j e^{i\phi_j}. \quad (3)$$

$$= \sum_{m=1}^M F_{mj} = \sum_{m=1}^M A_{mj} e^{i\phi_{mj}}. \quad (4)$$

Note that Eq (4) represents an irreversible linear superposition at the array in the sense that the multipath signal components  $F_{mj}$ , not to mention their individual amplitudes  $A_{mj}$  and phases  $\phi_{mj}$ , cannot be resolved by any processing techniques on the array signal  $F_j$ . For example, take

$$|F_j|^2 = A_j^2 = \sum_{m=1}^M \sum_{n=1}^M A_{mj} A_{nj} \cos(\phi_{mj} - \phi_{nj}). \quad (5)$$

and note that not only are the component amplitudes mixed together, but also that the component phases remain in a manner determined by the algorithm used for determining  $A_j$ . Likewise,

$$\cot \phi_j = \frac{\text{Re } F_j}{\text{Im } F_j} = \frac{\sum_{m=1}^M A_{mj} \cos \phi_{mj}}{\sum_{m=1}^M A_{mj} \sin \phi_{mj}}. \quad (6)$$

illustrates that the component amplitudes remain in  $\phi_j$  in a manner that is determined by the algorithm employed for determining  $\phi_j$ .

For later convenience, define the source bearing  $\Phi_0$  as the appropriate peak  $B(\Phi_0)$  of the phase-only beamformer

$$B(\Phi) = \frac{1}{J^2} \sum_{j=1}^J \sum_{\ell=1}^J \langle \cos [(\phi_j - \phi_\ell) - k_0 (d_j - d_\ell) \cos \Phi] \rangle, \quad (7)$$

where  $k_0$  is the wave number,  $d_j$  is the distance from the first sensor to the  $j^{\text{th}}$  sensor ( $d_1 = 0$ ) and the operator  $\langle \cdot \rangle$  represents a time average. The source bearing  $\Phi_0$  is defined as zero for a forward endfire arrival.

Now consider the statistical characteristics of multipath signals received by horizontal line arrays in the ocean. Assume the length of the array  $d_J$  is much less than the range  $R$  to the array, ie,  $d_J \ll R$ . Since the array is horizontal, the  $m^{\text{th}}$  paths to sensors  $j$  and  $\ell$ ,  $j \neq \ell$ , are approximately parallel in the vicinity of the array (since the  $m^{\text{th}}$  paths are of the same propagation type) so that  $F_{mj}$  and  $F_{m\ell}$ ,  $j \neq \ell$ , are statistically dependent in the sense that

$$A_{mj} \text{ is statistically dependent on } A_{m\ell}, j \neq \ell \quad (8)$$

$$\phi_{mj} \text{ is statistically dependent on } \phi_{m\ell}, j \neq \ell \quad (9)$$

Define the residual phase  $\theta_{mj}$  so that

$$\phi_{mj} \equiv \theta_{mj} + k_0 d_j \cos \Phi_0. \quad (10)$$

Now assume that

$$\theta_{mj} \text{ and } \theta_{nj} \text{ are statistically independent, } m \neq n \quad (11)$$

$$A_{mj} \text{ and } \theta_{n\ell} \text{ are statistically independent, } m \neq n \quad (12)$$

$$A_{mj} \text{ and } A_{nj} \text{ are statistically independent, } m \neq n. \quad (13)$$

### III. TATARSKI'S RELATIONS FOR SOUND-LEVEL AND PHASE FLUCTUATIONS

Modify the development starting on page 124 of Ref 2 as follows. Represent the pressure wave p solution of the stochastic Helmholtz equation as

$$p = F_{mj} = A_{mj} e^{i\phi_{mj}} = e^{\chi_{mj} + i\phi_{mj}}, \quad (14)$$

where the sound-level is defined as

$$\chi = \ell n A \quad (15)$$

and use the accurate and practical approximation for the time average of  $\chi_{mj}$

$$\overline{\chi_{mj}} = \overline{\ell n A_{mj}} \simeq A_0, \quad (16)$$

where  $A_0$  represents the amplitude for a deterministic medium (with the subscripts m and j suppressed). Define the sound-level fluctuation by

$$\chi'_{mj} \equiv \chi_{mj} - \overline{\chi_{mj}} \simeq \ell n (A_{mj}/A_0). \quad (17)$$

From Ref 1 through 3,

$$\overline{(\chi'_{mj})^2} \simeq \overline{[\ell n (A_{mj}/A_0)]^2} = \overline{\mu^2} L_n k_0^2 R, \quad \sqrt{\lambda R} \gg L_0, \quad (18)$$

where  $\overline{\mu^2}$  is the mean-square fluctuation of the refractive index,  $L_n$  is the integral scale for the correlation function of the refractive index fluctuation,  $\lambda$  is the wavelength, and  $L_0$  is the longest fluctuation length scale of interest. Reference 3 found

$$\overline{\mu^2} L_n \sim 4.2 \cdot 10^{-9} \text{ m} \quad (19)$$

in the Atlantic Ocean. An accurate and practical approximation for the time average of  $\phi_{mj}$  is

$$\Phi_{mj} \sim \phi_0 \sim k_0 d_j \cos \Phi_0, \quad (20)$$

where  $\phi_0$  represents the phase for a deterministic medium (with the subscripts m and j suppressed) and  $k_0 d_j \cos \Phi_0$  is the phase delay from the first to the  $j^{\text{th}}$  sensor. Equation (20) is an appropriate relation for array studies. Define the phase fluctuation as

$$\phi'_{mj} \equiv \phi_{mj} - \overline{\phi_{mj}} \quad (21)$$

$$\simeq \phi_{mj} - k_0 d_j \cos \Phi_0 \equiv \theta_{mj}. \quad (22)$$

Equation (9.35) of Ref 2 gives

$$\overline{\theta_{mj}^2} \approx \overline{(\phi_{mj} - \phi_0)^2} = \overline{\mu^2} L_n k_0^2 R, \quad \sqrt{\lambda R} \gg L_0. \quad (23)$$

#### IV. DEVELOPMENT OF THE MEAN-SQUARE SOUND-LEVEL FLUCTUATION FOR A MULTIPATH MEDIUM

The mean-square sound fluctuation relation,  $\overline{(x'_j)^2}$ , for a single array sensor in a random multipath medium can be developed in a straightforward manner by using the expression

$$\begin{aligned} 2x_j &= \ell n A_j \\ &= 2 \sum_{m=1}^M \ell n A_{mj} + 2 \sum_{n=1}^M \ell n A_{nj} + \sum_{m=1}^M \sum_{n=1}^M \ell n \cos(\phi_{mj} - \phi_{nj}), \end{aligned} \quad (24)$$

which comes from Eq (5) and (15). Therefore,

$$\begin{aligned} \overline{(x'_j)^2} &= \overline{(x_j - \bar{x}_j)^2} \\ &= \overline{(\ell n A_j)^2} - (\ell n A_j)^2 \\ &= 4 \sum_{m=1}^M \sum_{n=1}^M \left( \overline{\ell n A_{mj} \ell n A_{nj}} - \overline{\ell n A_{mj}} \overline{\ell n A_{nj}} \right) \\ &\quad + 2 \sum_{m'=1}^M \sum_{m=1}^M \sum_{n=1}^M \left[ \overline{\ell n A_{m'j} \ell n \cos(\phi_{mj} - \phi_{nj})} - \overline{\ell n A_{m'j}} \overline{\ell n \cos(\phi_{mj} - \phi_{nj})} \right] \\ &\quad + \frac{1}{4} \sum_{m=1}^M \sum_{n=1}^M \sum_{m'=1}^M \sum_{n'=1}^M \left( \overline{\ell n [\cos(\phi_{mj} - \phi_{nj})]} \overline{\ell n [\cos(\phi_{m'j} - \phi_{n'j})]} \right. \\ &\quad \left. - \overline{\ell n [\cos(\phi_{mj} - \phi_{nj})]} \overline{\ell n [\cos(\phi_{m'j} - \phi_{n'j})]} \right). \end{aligned} \quad (25)$$

The last term in Eq (25) is approached as follows. Applying

$$\phi_{mj} - \phi_{nj} = \theta_{mj} - \theta_{nj}, \quad (26)$$

from Eq (10), and

$$\ell n \cos x \approx -\frac{x^2}{2}, \quad |x| \ll 1, \quad (27)$$

gives

$$\begin{aligned} \overline{\ln \cos (\phi_{mj} - \phi_{nj})} &= \overline{\ln \cos (\theta_{nj} - \theta_{nj})} \\ &= \begin{cases} \approx -\frac{1}{2}(\overline{\theta_{mj}^2} + \overline{\theta_{nj}^2}), & m \neq n, \\ = 0 & , m = n, \end{cases} \end{aligned} \quad (28)$$

and

$$\begin{aligned} \overline{\ln [\cos (\phi_{mj} - \phi_{nj})] \ln [\cos (\phi_{m'j} - \phi_{n'j})]} &= \overline{\ln [\cos (\theta_{mj} - \theta_{nj})] \ln [\cos (\theta_{m'j} - \theta_{n'j})]} \\ &= \begin{cases} \approx \frac{1}{4} (\overline{\theta_{mj}^2} \overline{\theta_{m'j}^2} + \overline{\theta_{mj}^2} \overline{\theta_{n'j}^2} + \overline{\theta_{nj}^2} \overline{\theta_{m'j}^2} + \overline{\theta_{nj}^2} \overline{\theta_{n'j}^2}), & m \neq n, m' \neq n', \\ = 0 & , m = n \text{ or } m' = n', \end{cases} \end{aligned} \quad (29)$$

via Eq (11).

Now consider the last term in Eq (25), as follows:

$$\begin{aligned} &\sum_{m=1}^M \sum_{n=1}^M \sum_{m'=1}^M \sum_{n'=1}^M \left( \overline{\ln [\cos (\phi_{mj} - \phi_{nj})] \ln [\cos (\phi_{m'j} - \phi_{n'j})]} \right. \\ &\quad \left. - \overline{\ln [\cos (\phi_{mj} - \phi_{nj})]} \overline{\ln [\cos (\phi_{m'j} - \phi_{n'j})]} \right) \\ &\approx \frac{5}{2} \sum_{m=1}^M \left[ \overline{\theta_{mj}^4} - (\overline{\theta_{mj}^2})^2 \right]. \end{aligned} \quad (30)$$

$$= \frac{5}{2} M \left[ 3(\overline{\theta_{mj}^2})^2 - (\overline{\theta_{mj}^2})^2 \right], \quad (31)$$

$$= 5M (\overline{\theta_{mj}^2})^2. \quad (32)$$

$$\approx 5M (\mu^2 L_n k_0^2 R)^2, \quad \sqrt{\lambda R} \gg L_0. \quad (33)$$

Equation (30) follows from Eq (11), (28), and (29). The central limit theorem yields Eq (31), and Eq (23) gives Eq (33).

Equations (12) and (26) give

$$\begin{aligned} \overline{\ln A_{mj} \ln [\cos (\phi_{mj} - \phi_{nj})]} &= \overline{\ln A_{m'j} \ln [\cos (\theta_{mj} - \theta_{nj})]} \\ &= \overline{\ln A_{m'j} \ln [\cos (\theta_{mj} - \theta_{n'j})]} \end{aligned} \quad (34)$$

so that the second term in Eq (35) goes to zero. Applying Eq (13) yields

$$\overline{\ln A_{mj} \ln A_{nj}} = \overline{\ln A_{mj}} \overline{\ln A_{nj}} \quad (35)$$

which simplifies the first term in Eq (29).

Therefore, applying Eq (15), (17), (18), (23), (30), (34), and (35) to Eq (25) produces the mean-square sound-level fluctuation for a single array sensor in a random multipath medium:

$$\overline{(x'_j)^2} \simeq 4 \sum_{n=1}^M \left[ \overline{(\ln A_{mj})^2} - \overline{(\ln A_{mj})}^2 \right] + \frac{5}{2} \sum_{m=1}^M \left[ \overline{\theta_{nj}^4} - \overline{(\theta_{mj}^2)}^2 \right] \quad (36)$$

$$= \sum_{m=1}^M \left[ 4 \overline{(x'_{mj})^2} + 5 \overline{(\theta_{mj}^2)^2} \right] \quad (37)$$

$$\simeq M \left[ \overline{\mu^2} L_n k_o^2 R (4 + 5 \overline{\mu^2} L_n k_o^2 R) \right], \quad \sqrt{\lambda R} \gg L_o, \quad (38)$$

$$\simeq 4M \overline{\mu^2} L_n k_o^2 R, \quad \sqrt{\lambda R} \gg L_o, \quad (39)$$

when

$$1 \gg \overline{\mu^2} L_n k_o^2 R. \quad (40)$$

References 1, 2, and 3 give other relations that are valid when the inequality  $\sqrt{\lambda R} \gg L_o$  is not satisfied. In these cases, the analysis that led to Eq (39) can be extended. From Eq (19) and (38), it is apparent that there is a large range of application for Eq (39) for low frequencies. Shvachko (Ref 3 and 4) has shown that stochastic parameters like  $\overline{\mu^2} L_n$  are stable in the ocean (see also Ref 5). Therefore, when the sound propagation conditions are well understood (so that  $M$  and  $\overline{\mu^2} L_n$  are known), relations like Eq (39) can be used to determine the source range  $R$  from measurements of  $\overline{(x'_j)^2}$ . Thus, passive ranging by a single sensor is possible in a random multipath medium. Equations (38) and (39) apply when there exists only one dominant source, since they depend on only one range.

5. J. A. Neubert, "Experimental Agreement of Stochastic Ray-Theory Relations," *J. Acoust. Soc. Am.* 62, 326-334 (1977).

## V. CONCLUSIONS

Equation (39) can be applied to a horizontal line array so that the source bearing  $\Phi_0$  is available as well as the range R. This could permit source tracking by a single horizontal line array in a multipath ocean environment. The total array gives

$$\overline{(x')^2} = \sum_{j=1}^J \overline{(x'_j)^2} + N_0 \quad (41)$$

$$\approx 4JM \mu^2 L_n k_0^2 R + N_0, \sqrt{\lambda R} \gg L_0 \quad (42)$$

for Eq (39). In Eq (41) and (42)  $N_0$  has been included to show that statistically independent noise  $N_0$  only results in a range-independent offset to the basic source-ranging relation when one dominant source is embedded in a random noise field. Equation (42) shows that the mean-square sound-level fluctuation level increases by the number J of sensors when their outputs are summed. Hence a horizontal line array may determine the source range R even when the fluctuation level of a single sensor is insufficient.

Although Section IV of Ref 1 showed promising results for this array source tracking method when Bearing Stake data were used, there are some definite limitations that must be considered.

1. Although the lower limit on range should present no practical difficulties, upper range limits such as Eq (40) will. In fact, the fluctuations will always have a saturation limit as the range increases.
2. The method is restricted to a relatively simple multipath environment. Bottom loss and refractive effects can invalidate relations like Eq (42) since these latter cannot treat a variable number M of arrivals. By a different method, Ref 6 treats these and related problems.
3. The theory behind this method (Sections III and IV) can only treat the first term in Eq (41) and then only for the presence of a single dominant source (since this method cannot beamform). References 1 and 6 show that  $N_0$  of Eq (41) plays a dominant role. Although  $N_0$  can be measured, it cannot be predicted, so unmeasured changes in  $N_0$  militate against the use of this source-ranging method.
4. As seen in Ref 1 and 6, this source-ranging scheme requires considerable statistical stability and, therefore, may not be feasible unless suitable processing techniques are carefully applied.

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6. Reference available to qualified requestors.



## REFERENCES

1. Reference available to qualified requestors.
2. V. I. Tatarski. *Wave Propagation in a Turbulent Medium*, McGraw-Hill, 1961.
3. R. F. Shvachko, "Sound Fluctuations in the Upper Layer of the Ocean and Their Relation to the Random Inhomogeneities of the Medium." *Sov. Phys.-Acoust.* 9, 280-282 (1964).
4. R. F. Shvachko, "Sound Fluctuations and Random Inhomogeneities in the Ocean," *Sov. Phys.-Acoust.* 13, 93-97 (1967).
5. J. A. Neubert, "Experimental Agreement of Stochastic Ray-Theory Relations," *J. Acoust. Soc. Am.* 62, 326-334 (1977).
6. Reference available to qualified requestors.