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HEAT TRANSFER FROM IMPINGING JETS  
A LITERATURE REVIEW

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## FOREWORD

This report represents the results of work done by Prof. Peter Hrycak of the New Jersey Institute of Technology, 323 High Street, Newark, New Jersey 07102, on CERT/POD Application Analysis. Contract Number was F33615-79-C-3417, and Task Number was 240204. This contract was conducted under the sponsorship of the Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Dayton, Ohio 45433. Dr. Alan H. Burkhard (AFWAL/FIEE) was the project engineer. The work reported herein was conducted between September 1979 and November 1980. This report was submitted by the author in March 1981. Mr. Amir J. Daibes a graduate student at NJIT, was of help in carrying out a portion of library research.

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## LIST OF SYMBOLS

<u>SYMBOL</u>		<u>UNITS</u>
$a^*$	dimensionless velocity gradient at stagnation point	
$b$	width of jet	mm
$B_0$	width of slot jet nozzle	mm
$c$	speed of sound	m/sec
$C_p$	specific heat at constant pressure	J/kg K
$D$	diameter of nozzle, circular jet; diameter of sphere, Equation (12)	mm
$D_p$	target diameter	
$f$	solution of momentum equation	-
$g$	solution of energy equation	-
$h$	heat transfer coefficient	$W/m^2 K$
$J$	$z$ - component of momentum (of the jet)	N
$k$	thermal conductivity	W/m K
$M$	Mach Number, $v/c$	-
$Nu$	Nusselt Number, $h D/k$	
$p$	pressure	$N/m^2$
$Pr$	Prandtl Number, $C_p/k$	-
$r$	radial distance, generalized distance away from stagnation point	mm
$R$	radius of target = $D_p/2$	mm
$Re$	Reynolds Number, $vD_p/\mu$	-
$St$	Stanton Number, $Nu/(RePr)$	-
$T$	Temperature	K

## LIST OF SYMBOLS (Concluded)

<u>SYMBOL</u>		<u>UNITS</u>
u	velocity in the negative z direction	m/sec
v	velocity in the direction away from stagnation point along target plate	m/sec
x	direction away from the nozzle to the target	mm
z	direction away from the target	mm
$z_n$	normal distance between nozzle and target plate	mm
$\delta$	boundary layer thickness	mm
$\delta_{.5}$	boundary layer thickness at a point where velocity reduces to 50% of maximum velocity ( $u_{max}$ or $v_{max}$ )	
$\psi$	similarity variable, momentum equation	
$\mu$	dynamic viscosity	m/kg sec
$\nu$	kinematic viscosity	m <sup>2</sup> /sec
$\rho$	density	kg/m <sup>3</sup>

### Subscripts and other Symbols

subscript "o" means conditions at the nozzle exit ( $B_o, r_o, u_o$ )  
and at the stagnation point ( $Nu_o$ ).

A bar over the Nusselt Number designates a mean value.

subscript "p" refers to target plate

subscript "c" refers to calorimeter:  $D_c$  = calorimeter diameter  
or for velocity, to conditions at center of symmetry

Additional symbols explained in text.



## SECTION I

### INTRODUCTION AND SUMMARY

#### 1.1 Introduction

Heat transfer from impinging jets is a relatively new technological development that has already attracted the interest of heat transfer engineers and designers, involved with the design of modern jet power plants and related machinery. This group of people is mainly interested in heat transfer from impinging jets because of its promise to succeed where applications of convective heat transfer often fail - in situations where the rates of heat flux are high and the space is restricted - such as at the leading edge of a turbine blade. This sudden spurt of interest of engineers primarily interested in applications had, as a counterpart, an equivalent response among the more research oriented circles.

Investigation of the topics connected with the heat transfer from the impinging jets can be traced back to about 1951 (Freidman and Mueller, Reference 1) for an experimental approach and to 1956 (Kezios, Reference 2) for a combination of analysis and experimentation. Since that time, many investigators have appeared who tried both analytical and practical approaches, mostly with one ultimate purpose in mind: to furnish to the designer formulas, charts and graphs, that would enable him to handle various heat transfer problems related to the impinging jets in a routine fashion.

In general, the geometry of even a single impinging jet is quite complicated, and the heat transfer aspects depend strongly on the corresponding fluid flow structure. Therefore, in the following literature review, fluid flow aspects of the impinging jets will be reviewed first, and then our attention

will be turned to the items more directly related to heat transfer from the impinging jets.

Among the other hydrodynamic parameters, one of the most critical parameters governing heat transfer from impinging jets appears to be the distance between the nozzle exit and the target. In particular, the heat transfer from the impinging jets to flat targets, appears at first to increase directly with the distance between the nozzle and the target, to reach a flat maximum at six or seven diameters away in the case of circular jets. This distance is for the slot jets slightly higher, and for both kinds of jets, the region of maximum intensity in heat transfer coincides roughly with the tip of the "potential core" - the region where the original jet velocity at the nozzle exit is still preserved at the center of the jet.

Beyond the potential core region, the intensity of heat transfer decreases *pari passu* with the further dissipation of the original jet velocity. This zone of jet development has originally attracted a substantial number of investigators who were concerned primarily with the free jets, unencumbered by the presence of the targets. Here, similarity profiles techniques have been used with success.

After the jet hits the target, eventually a wall jet develops, which exhibits definite boundary layer characteristics. Consequently, within the potential core, heat transfer intensity will depend on the original jet velocity, whereas in the region of the developing jets, heat transfer intensity is a function of the actual velocity of the jet when it hits the target the so called "velocity of arrival".

## 1.2 Summary

In the report, the available literature on the subject of flow patterns and heat transfer resulting from impinging jets has been reviewed. The report shows that despite some accomplishments, there still exists necessity for both analytical and experimental work, before the heat transfer aspects of the impinging jets are properly understood. The state of the art with respect to fluid flow patterns is in a relatively better position, but there, too, appear considerable gaps in knowledge, primarily concerning the effects of turbulence intensity and due to a difficult-to-handle geometry.

The flow from free jets becomes turbulent in the Reynolds number range from 1000 to 3000. Impinging jets behave still like free jets with the target only slightly over one nozzle diameter away from the jet nozzle.

The nominal potential core region, for the non-turbulized jets only, extends six to seven diameters away from the nozzle for the circular jets; for slot jets, this distance seems to be somewhat longer.

The flow near the stagnation point is laminar, but the effects of the jet stream turbulence on the heat transfer after impingement are considerably higher than for the flow over a flat plate. This makes it sometimes difficult to determine how far away from the stagnation point the laminar regime still extends. Outside boundary layer, potential flow solutions are valid near the stagnation point.

Because in the real jets, of a finite thickness, the flow regime depends considerably on the distance away from the nozzle, it seems, at the present time, that with a few exceptions, analytical calculations are only possible when the target is relatively close to its nozzle so that flow may still be considered infinite, in the sense as originally treated by Hiemenz and Homann.

However, there appears to be a real promise in semi-empirical calculations of the hydrodynamic and heat transfer phenomena in the wall jet region,

farther away from the stagnation point, where the flow is already definitively turbulent. For arrays of jets, the results so far have been obtained only experimentally, and there still exist gaps in our knowledge of the subject. It seems that, at the present state of the art, the calculations of heat transfer from arrays of jets will have to rely heavily on experimental techniques, or on analogue computation for some time to come.

It is interesting to note the effects of the turbulence intensity and the turbulence scale on local heat transfer near the stagnation point, for both the two- and the three-dimensional jets, impinging on flat targets. These effects, observed independently from each other by several investigators, manifest themselves in a substantial increase of heat transfer intensity in the neighborhood of the stagnation point, and in an explicit dependence of heat transfer intensity there on the nozzle diameter.

## SECTION II

### FLUID FLOW ASPECTS OF IMPINGING JETS

#### 2.1 General Features of Jets, Potential Core Length

As in virtually all forced convection problems, fluid flow aspects of impinging jets cannot be really separated from the heat transfer aspects (and vice versa), since the energy equation is dependent upon the momentum equation. A reasonable degree of understanding of fluid flow patterns associated with the impinging jets, is the prerequisite for a satisfactory analysis of all the associated heat transfer effects.

Of fundamental importance is here the distinction between the laminar and the turbulent flow regimes. Here, according to Vickers, (Reference 3), it seems that the critical Reynolds number,  $Re$ , based on the nozzle diameter, is most likely 1000. This finding is supported by McNaughton and Sinclair (Reference 4), who report having observed four characteristic jet patterns:

1. What they call a "dissipated laminar jet",  $Re < 300$ .
2. Fully laminar jet,  $300 < Re < 1000$ .
3. Semi-turbulent jet, that starts as a laminar jet, eventually becoming turbulent,  $1000 < Re < 3000$ .
4. Fully turbulent jet,  $Re > 3000$ .

The above observations were made on liquid into liquid jets, in vessels of various shapes, using as a tracer methylene-blue dye. Working with air jets, Hrycak et al. (Reference 5) have obtained very similar results.

The above results are in line with the observations made by Cederwall, (Reference 6) who found that jets with  $Re > 3000$  are already turbulent. Moreover the heat transfer results of Gardon and Cobonpue, (Reference 7) and those by Chamberlain, (Reference 8) seem to indicate that, for impingement in the potential core region, the Nusselt number ceases to be, (for the

given fluid and flow geometry) a function of the Reynolds number alone for  $Re > 7000$ . This means that the Nusselt number must become then also functionally dependent on turbulence patterns prevailing in the jet, as the simplest explanation of this phenomenon.

The above remarks apply directly only to jets issuing from circular openings, but are also roughly applicable to slot jets. Thus, in, another paper dealing with slot jets exclusively, Gardon and Akfirat, (Reference 9) consider jets above  $Re = 2000$  as turbulent.

The above discussion applies, strictly speaking, only to free jets. It seems, however, that similar reasoning can be applied also reasonably well to the impinging jets, in the space not affected by the presence of the target plate. Theoretical calculations show that the effect of the impinging surface is felt by the jet only in the immediate vicinity of that surface (Levey, Reference 10). Even when the impinging surface is only one slot width away from the origin of the jet, it has no effect on either the momentum flux or the mass flux through the jet in question. For axisymmetric jets this result is essentially confirmed by Tani and Komatsu's experimental observations, (Reference 11) and Hrycak et al. (Reference 5). It should be noted that the above observed critical Reynolds number values are one order of magnitude higher than the results obtained from the flow stability calculations, as outlined in Reference 12, for example.

Another important characteristic of a jet is the length of the potential core (Figure 1) - the distance to which the original nozzle exit velocity of the jet persists along the center line of the jet.

Here, some useful theory is available. According to Schlichting, (Reference 12, p. 732), we have the width of the jet,  $b$ , proportional to the distance from the nozzle exit,  $x$ .

$$b = \text{const. } x \quad (1)$$

The relationship between the maximum velocity,  $u_m$ , at the jet centerline, and  $x$ , can be obtained from the conservation of momentum equation, where the  $x$ -component of total jet momentum,  $J$ , must be independent of  $x$ :

$$J = \int_A \rho u^2 dA = \text{const.} \quad (2)$$

In the case of two-dimensional jet, we have  $J' = \text{const.} \times \rho u_m^2 b$  where  $J'$  = momentum per unit length. Equations (1) and (2) may be used to get expressions for  $u_m$ , valid in the region where  $u_m/u_{oc} < 1$  applies. In the equations for  $u_m$  below, the constants  $C_1$  and  $C_2$  must be determined empirically for the best results. Stated in dimensionless form, we have then for  $u_m/u_{oc}$  the relations

$$u_m/u_{oc} = C_1 / \sqrt{x/B_0 + x_0/B_0} \quad (3)$$

for the slot jet, and

$$u_m/u_{oc} = C_2 / (x/D + x_0/D) \quad (4)$$

for the circular jet. Equations (3) and (4) are valid only outside of the potential core region:  $x_0$  term is the virtual origin correction, since a real jet does not start from a point source, but at the nozzle exit with a finite diameter and finite velocity  $u_{oc}$  at the center of jet.

The above results can be used to determine the length of the potential core region, when the ratio  $u_m/u_{oc}$  is plotted against the dimensionless distance from the jet nozzle exit on log-log paper. The slope of the resulting curves is indeed very nearly  $-1/2$  for the slot jets, and  $-1$  for the circular jets (Reference 5). The values of the constants  $C_1$  and  $C_2$  determine the length of the nominal potential core distance. The actual values of these constants depend on the skill of the particular investigator, the specific method used to get  $C_1$  and  $C_2$ , as well on the nozzle construction details. For example, the value of 2.28 for  $C_1$  has been recommended by Albertson et al. (Reference 13) which makes the end of the potential core 5.2 slot widths

away from the slot itself, whereas Schauer and Eustis (Reference 14) used the value of  $C_1=2.35$  according to the results observed by Van der Hegge Zijnen (Reference 15).

For  $C_2$ , Albertson et al., give the value of 6.2. In discussion on Reference 13, Citrini recommends an empirical relationship

$$u_m/u_0 = 6.6 D/x - 0.49 \quad (5)$$

while Baines claims that there is a lengthening of the potential core with increasing Reynolds number from 5.0 at  $Re = 1.4 \times 10^4$ , to 7.0 at  $Re = 1.0 \times 10^5$  (Abramovich (Reference 16), on the other hand, asserts that  $C_2$  does not vary with the Reynolds number directly, but does so with the velocity profile. Since the velocity profile in turbulent flow is a weak function of the Reynolds number, it means that  $C_2$  varies (although to a very limited extent) also with the Reynolds number. After all, the recorded variation of  $C_2$  is from 5.2 (adopted from Tani and Komatsu Reference 11) to 7.7 (Poreh and Cermak, Reference 17); all these figures are based on the average velocity at the jet nozzle exit. The mean value of  $C_2$  may be taken as 7 (cf. Gauntner et al., Reference 18). A comprehensive study of the magnitude of  $C_2$  has been carried out by Hrycak et al. (Reference 5).

Another aspect of the potential core length is its variation with the Mach number as discussed, for example, by Snedeker and Donaldson (Reference 19) who find an apparent potential core length of 7.5 diameters for  $M = 0.52$ . Downstream of  $x/D \approx 11$ , the velocity decay follows the  $1/x$  dependence quite closely. Here Snedeker and Donaldson cite the experiments of Warren (Reference 20), who for  $M = 0.69$  found a core length of 7.2 diameters with  $x/D = 10$  for the start of the fully developed region.

Actually, the exact "measured" length of the potential core depends on the method of its determination. Since there is no sudden, sharp transition at the center between the potential core and the region of the developed



flow, this determination must be, by necessity, a somewhat arbitrary procedure. Thus, for example, Pai (Reference 21) suggests for the constant in Equation (4) the value  $C_2 = 6.5$ .

Pai reports also the existence of the intermittancy in the jet flow, first observed by Corrsin: in a fully developed axi-symmetric jet, a completely turbulent flow exists only in the core region, out to a radius at which the velocity is about one half of the maximum velocity of the given cross-section. Outside of this region, there is a transition region, followed by a region of laminar flow.

Information about the exact length of the potential core is quite important in regard to maximum cooling efficiency; heat transfer results, obtained with single jets, show that the maximum intensity of heat transfer takes place at the tip of the nominal potential core.

The above results on the form of attenuation of the jet centerline velocity with the distance can also be calculated through assumption of a particular similar velocity profile, with a specific velocity distribution, for flow regions where similar velocity profiles can be expected to have already established themselves (Albertson et al., Reference 13), and then working backwards, towards  $u/u_{oc} = 1.0$ . (This is of course only possible when the potential core has already disappeared, and should be applicable, in principle, to both the laminar and the turbulent flow regimes).

There exist essentially two methods of determining the potential core length experimentally. The real potential core,  $L^*$ , ends where the jet centerline velocity starts to deviate for the first time from  $u_{oc}$ . The "nominal" potential core length is obtained from an extrapolation of the developed jet centerline velocities plot on log-log paper towards lower  $x/B_0$  or  $x/D$  values. Equation (3) and (4) here refer to the nominal potential

core length; the nominal potential core length is a useful concept in heat transfer calculations, however. It is about 2.5 nozzle diameters longer than the true potential core,  $L^*$ .

In general, in the case of an impinging jet we can distinguish four characteristic regions of flow (Figure 1). They are (Gauntner et al., Reference 18):

1. a transition zone of flow establishment
2. a zone of established flow in the original direction of the jet
3. a deflection zone
4. a zone of established flow in the radial direction.

In Zone 1, the most interesting fact is the existence of the potential core region with the undisturbed original velocity of the jet, already discussed above. Ordinarily, Zone 1 extends up to nine nozzle diameters downstream from the jet orifice. In Zone 2, the velocity distribution may be considered similar to that of a free jet diffusing into an infinite medium. In the neighborhood of the stagnation point, (Zone 3), flow patterns appear to be close to those computed for the irrotational flow.

Of great importance is the zone of established flow in the radial direction, Zone 4. The jet in this region is called the wall jet, and, for axisymmetric jets, exhibits characteristics somewhat similar to those of a free radial jet - a jet issued from a toroidal slit in the radial direction. The presence of the impinging plate is responsible for the boundary-layer development.

In many practical applications, Zone 2 of jet flow has never a chance to form if  $Z_n/D < 9$ ,  $Z_n$  being the normal distance between the target plate and the nozzle. Zone 1 has been already discussed to some extent. In the following, the implications of existence of Zones 3 and 4 will be covered.

In Zones 3 and 4, the boundary-layer theory can be applied with some far-reaching consequences. It is particularly convenient that near the

stagnation point, the available exact analytical solutions of the complete Navier - Stokes equations, applicable for the laminar flow impinging on a wedge, can be utilized (Reference 12). Brady and Ludwig (Reference 22) prepared a chart to indicate the extent of the laminar region near the stagnation point. A summary of fluid flow characteristics of a round turbulent jet impinging on a flat plate has been made by Gauntner et al., Reference 18.

## 2.2 Analytical Treatment of Impinging Jets

The simplest approach to the analytical handling of impinging jets is based around the assumption of inviscid flow. The essential difficulty here lies in the fact that despite the well defined boundary conditions in the form of the free streamlines in such flows, the location of the free streamlines is not known in advance, and must be determined as a part of the solution (Reference 23).

Exact analytical solutions for two-dimensional impinging flow are available for ideal fluids (e.g. Levey, Reference 10). The available theoretical treatments of inviscid three-dimensional impinging jets have been obtained through analog solutions or approximate analytical and numerical methods (e.g., Strand, Schach, LeClerc, and Daibes, References 24-27). The results of the researchers above are sound, because they check quite well with the available experimental data. The approach based on the inviscid flow can be expected to give solutions which are reasonably close to an actual situation where the dynamic effects predominate over the viscous effects, which occurs at sufficiently high Reynolds numbers. Here, Schach used an integral equation method according to Trefftz, which resulted in numerical solutions by successive approximations, for the location of the free jet boundary. LeClerc used an electrolytic analog to locate the free jet boundary and to determine the shape of the streamlines.

In his approach, once the free surface of the jet had been obtained, LeClerc employed relaxation techniques in the interior of the jet to establish the potential lines and streamlines. Daibes obtained an approximate solution for an inviscid, impinging jet using cylindrical coordinates to determine the free surface shape from consideration of the exact boundary conditions. The governing differential equations were then solved by relaxation methods. Daibes' numerical approach essentially confirmed Schach's and LeClerc's analyses (Figure 2).

The velocity distribution (Figure 3) is connected by Equation (6)

$$v/u_0 = \sqrt{1 - (p-p_0)/\frac{1}{2} \rho u_0^2} \quad (6)$$

to the pressure distribution. At the stagnation point, potential flow solutions result in  $v/u_{0c} = a^* r/D$ , with  $a^* = 1.02$  by Daibes, and  $a^* = 1.03$  by LeClerc. The dimensionless velocity gradient,  $a^*$ , is needed for heat transfer calculations at the stagnation point, and will be the subject of additional discussion below.

In the region where the laminar flow prevails, exact solutions of the Navier - Stokes equations are possible (Reference 12). It also happens to be the case here that the boundary layer approach gives the same results as the corresponding solutions of the complete Navier - Stokes equations. The flow within the boundary is dependent upon the main flow, in the sense that the flow configuration determines the pressure distribution at the surface under consideration (References 2, 12).

For systems of axial symmetry, momentum, energy and continuity equations are then,

$$\rho(v \frac{\partial v}{\partial r} + u \frac{\partial v}{\partial z}) = - \frac{dp}{dr} + \frac{\partial}{\partial z} (\mu \frac{\partial v}{\partial z}) \quad (\text{momentum}) \quad (7)$$

where for  $z=0$ :  $u=0$ ,  $v=0$ ; for  $z=\infty$  :  $v=v_s$

$$C_p \rho (v \frac{\partial T}{\partial r} + u \frac{\partial T}{\partial z}) = \frac{\partial}{\partial z} (k \frac{\partial T}{\partial z}) \quad (\text{energy}) \quad (8)$$

where for  $z=0$ ,  $T=T_w$ ; for  $z=\infty$ ,  $T=T_\infty$

$$\frac{\partial}{\partial r} (\rho rv) + \frac{\partial}{\partial z} (\rho ru) = 0 \text{ (continuity)} \quad (9)$$

When the free stream velocity is proportional to some power of the distance from the stagnation point, this leads to the notion that the heat transfer coefficient at any point along the median profile of a body of arbitrary cross-section is the same as that on a wedge, which has the same velocity and pressure gradient just outside the boundary layer at the same distance from the stagnation point as the real profile. In addition, for heat transfer applications the local temperature ratio has to be the same (Reference 2).

Thus, according to this procedure, the two-dimensional surface is replaced by a series of wedges, each having a uniquely enclosed angle that depends on the Euler number,  $m$ , in question. The three-dimensional stagnation flow represents a special case of wedge flow.

In the stagnation region (Zone 3 in Figure 1) the velocity at stagnation point obeys the relation  $v_s = ar$ ; when  $a^* = aD/u_{oc}$  is introduced, one has outside of the boundary layer also the relation  $v_s = u_{oc} a^* r/D$ ; the system of Equations (7) - (9) can be solved (Reference 12) by reducing it to the total differential equation in  $f(\eta)$ , by using  $\eta = \sqrt{a/v_s} z$ ,  $\psi = r^2 \sqrt{av_s} f$ ,  $vr = \frac{\partial \psi}{\partial z}$ , and  $ur = -\frac{\partial \psi}{\partial r}$ . Letting  $f' = df/d\eta$ , etc., there results

$$f''' + 2ff'' + 1 - f'^2 = 0 \quad (10)$$

where  $f = 0$ ,  $f' = 0$  at  $\eta = 0$ ,  $f' = 1$  at  $\eta = \infty$ , and  $v/v_s = f'$ . This kind of solution was first obtained by Homann (see Reference 12, p. 100). Similarly, one can define a dimensionless temperature function,  $(T-T_\infty)/(T_w-T_\infty) = g(\eta)$ ; through substituting for  $T$ ,  $u$  and  $v$  in terms of  $f$  and  $g$  and their derivatives, one gets from the energy equation the dimensionless expression

$$g'' + 2fPr g' = 0$$

where  $g = 1$  at  $n = 0$ , and  $g = 0$  at  $n = \infty$ . Solution of Equation (11) has been obtained by Sibulkin (Reference 28)

$$Nu_D / Re_D^{1/2} = 0.763 Pr^{0.4} (a^*)^{1/2} \quad (12)$$

where  $a^*$  is a free constant that depends on a particular system's geometry: for example,  $a^* = 3$  for a sphere, and  $a^* \rightarrow 1.0$  as  $Z_n/D \rightarrow 1$ , in the limiting cases of impinging jet flow.

From looking on Equations (9)-(12) it is seen that effects of fluid flow are intimately interrelated with the heat transfer aspects of the impinging jets. Thus, the Nusselt number from Equation (12) is a function of the Reynolds number that is linked to the flow regime and the flow geometry, and to the Prandtl number, which brings in the thermal properties that determine the physical aspects of heat transfer. Finally, there is the  $a^*$  term, the dimensionless velocity gradient at the stagnation point;  $a^*$  is the dimensionless ratio that relates the fluid flow regime at the stagnation point to the conditions at the nozzle exit and to the nozzle-to-target distance.

In practical cases,  $a^*$  is a function of the jet nozzle velocity profile at the exit (which is, in general, also related to the nozzle geometry) and the distance between the target plate and the nozzle exit, in the case of real fluids. Turbulent mixing at the boundary of the potential core (for  $Re_D > 1000$ ) modifies the original velocity profile as a function of the distance from the nozzle; consequently,  $a^*$ -magnitude generally decreases with the distance between the nozzle and the target plate, in real life situations (cf. Reference 5). Additional discussion of  $a^*$  will be found in Section IV, dealing with experimental impinging jet heat transfer.

Stagnation flow changes eventually into wall-jet flow (Zone 4 in Figure 1) after the direction of flow has been turned through a  $90^\circ$  angle. In the wall-jet flow of real fluids, we may distinguish the outer region,  $z > \delta$ , where the

effects of friction between the fluid in the jet and the quiescent surrounding fluid are predominant, and the inner region,  $z < \delta$ , where the presence of the target plate makes itself felt. Because each region has distinct physical characteristics, the flow is only approximately similar in Zone 4.

The flow in Zone 4 has been analyzed by Glauert (Reference 29). Although some of the assumptions used in that analysis proved to be only remotely related to reality (for example, the existence of wall friction coefficient of the Blasius type), Glauert's solution describes reasonably well flow situation in the wall-jet zone, and is potentially useful for the analysis of heat transfer there, with certain limitations. Also, heat transfer in Zone 4 may be still analyzed by semi-empirical methods. One such method is based on the Colburn analogy, applicable to fluids whose Prandtl number is sufficiently close to unity:

$$St Pr^{2/3} = C_f/2 \quad (13)$$

where the Stanton number is defined as  $St = Nu/Re Pr = h/(vC_p)$ . If the friction coefficient  $C_f$  is known, as a function of the local Reynolds number, and other pertinent wall jet parameters are available, the heat transfer coefficients (local and average) can then be calculated. For a turbulent wall jet, the use of Equation (13) appears to be the most convenient way of calculating the wall-jet heat transfer.

Stagnation point mass transfer follows governing equations similar to those for heat transfer. For identical boundary conditions,  $Nu/(Pr)^n = Sh/(Sc)^n$ , where  $Sh$  = Sherwood number, and  $Sc$  = Schmidt number, are the Nusselt number, and Prandtl number, respectively, equivalents for mass transfer.

Many other deserving authors, who have written on the subject of fluid flow patterns of free or impinging jets, have not been mentioned here directly. This was done so because of the time constraints imposed upon the present study, supposed to serve mainly as an introduction to the bibliography, rather than as an exhaustive survey article, concerning the impinging jets.

However, it is hoped that the discussion above has elucidated some of the facts that make analytical calculations of fluid flow parameters and, consequently, of heat transfer from impinging jets, so difficult. This is so because of considerably more involved geometry, and more highly pronounced frictional effects, in comparison with the corresponding analytical and semi-empirical solutions available for a flat plate in the parallel flow, or in the generalized stagnation flow (not restricted to a relatively thin jet).



SECTION III  
ANALYTICAL HEAT TRANSFER

3.1 Results Obtained with Standard Analytical Approaches

The region of the jet impingement, Zone 3, of Figure 1 was investigated by numerous individuals who employed similarity techniques adapted to stagnation flow and the wedge - flow solutions of the boundary layer equations. After the jet hits the target, it is deflected and eventually a wall jet develops (Zone 4, of Figure 1). Since the wall-jet region exhibits boundary-layer characteristics, it may be analyzed by boundary-layer theory.

Initial attempts to analytically predict heat-transfer from an impinging jet to a plane surface were limited to the neighborhood of the stagnation point, where even for turbulent jets, laminar flow predominates in the boundary layer. Kezios, (Reference 2) reports the following expression for the stagnation-point heat transfer:

$$Nu_0 = 0.475 \sqrt{Re_{D/2}} \quad (14)$$

where the nozzle radius  $D/2$  was used as the characteristic dimension. Equation (14) becomes, when  $Re_D$  is used and the term  $Pr^{0.4}$  is introduced, identical with Sibulkin's solution. Kezios found also analytically, at  $r/D = 0.65$ , a maximum for the local Nusselt number, away from the stagnation point. Walz (Reference 30) using the results originally derived by Eckert, obtained the expression

$$Nu = 0.44 Pr^{0.36} Re_r^{0.5} \quad (15)$$

for the local values in the axisymmetrical flow, with the average value of the Nusselt number in the axisymmetrical flow being equal to twice the local value.

For slot jets, using Eckert's wedge flow solutions, Metzger (Reference 31) obtained the equation

$$Nu = 0.57 Pr^{0.37} Re_r^{0.5} \quad (16)$$

for local values of the Nusselt number based on distance from the stagnation point, and found the average value of the Nusselt number to be twice the local value. The inability of such formulas to show explicitly the effect of the nozzle-to-plate separation distance may be considered a weakness.

Schauer and Eustis (Reference 14) present the solution for the hydrodynamics of a two-dimensional wall jet, the basis for the analysis in Reference 32. In Reference 32, the thermal boundary-layer equations for incompressible turbulent flow were solved by applying integral techniques. A modified Reynolds analogy (relating the diffusivity for heat to the diffusivity for momentum) was used to link the thermodynamic to the hydrodynamic solution.

In the impingement region, of a two-dimensional jet, Cadek (Reference 33) used an approach that, although based on the integral techniques, was reducible to Eckert's wedge flow solutions. In the wall jet region, Cadek's efforts resulted in expressions which could be reduced to the well-known flat-plate solution for the corresponding limiting case.

Cadek recognized the fact that, in the wedge flow class of solutions, an infinite uniform stream impinges on a surface, whereas in the case of impinging jets we deal with a nonuniform jet of finite width striking a surface. In the first case, the Euler number  $m$  is constant; in the latter case, it is a variable function of distance from the stagnation point.

Eckert and Livingood developed methods of calculating heat transfer with a variable  $m$  in Reference 34, applicable to stagnation flow. According to Cadek, his method gives results on heat-transfer rates that were essentially similar to those obtainable by Eckert and Livingood's approach, while at the same time yielding more information on the boundary layer itself.

Tomich calculated heat transfer from impinging, turbulent jets (Reference 35) by solving the governing equations by means of finite difference techniques that were extended into the compressible flow region. It was found that the jet Mach number, and jet temperature ratio, were the only two initial jet properties necessary to characterize the dimensionless velocity and temperature variations in the free jet.

In Reference 36, Brdlik and Savin used the integrated energy equation adapted for situations with a radial symmetry and obtained, for  $Z_n/D < 6.7$ , with  $Re$  based on the plate radius  $D_p/2$

$$Nu = 1.09 Pr^{1/3} Re_{D_p/2}^{i/2} \quad (17)$$

which shows reasonably good agreement with the test data of several investigators for  $0.3 < Pr < 1.7$ .

Sparrow and Wong derived, (Reference 37) analytical expressions for heat transfer from a slot jet with a parabolic velocity profile that shows a 26% increase for  $Z_n/B_0=1$ , a 68% increase for  $Z_n/B_0=2$ , etc., in comparison with heat transfer from a slot jet with a flat initial velocity profile at the nozzle.

Miyazaki and Silberman, (Reference 38) obtained the analytical solution for a laminar slot jet heat transfer, and for a finite plate-to-nozzle distance. The final results are given graphically, as apparently no closed form solution was obtainable. The Nusselt number for  $Z_n/B_0=2$  showed a small maximum away from the stagnation point, a phenomenon similar to that found by Kezios before for round impinging jets.

Hrycak, (Reference 39) using results from cold-jet hydrodynamic measurements, showed that a solution for the average heat transfer in the wall-jet region could be obtained through the so-called Colburn analogy,  $StPr^{2/3} = Cf/2$ ; that particular solution included also the stagnation region:

$$Nu = 1.95 Pr^{1/3} Re_D^{0.7} (D/D_p)^{1.23} \quad (18)$$

and is independent of the  $Z_n/D$  term for  $Z_n/D < 7$ . The above expression applies to a single round jet of nozzle diameter  $D$  impinging normally at the center of a round plate of diameter  $D_p$ . Other authors used this approach before, but Equation (18) is based on a  $C_f$  specifically derived for wall jets, and also uses other parameters, characteristic for axisymmetric wall jets (cf. also Reference 5).

From looking at the available analytical solutions, it is seen that they all have certain common features. The general form of the equation for the Nusselt number is

$$Nu = C Pr^m Re^n \quad (19)$$

with  $m$  ranging from  $1/3$  to  $0.4$  and  $n$  ranging from  $0.5$  at the stagnation point to  $0.7$  and more in the wall-jet region.

The value of  $C$  varies from  $0.763$  (Reference 28) to  $1.09$  (Reference 36) for circular jets, and is  $1.14$  for slot jets (Reference 31), according to Eckert's method. For the wall-jet region, a reasonably good agreement between theory and experiment has been found. For the cases of even greater complexity, dimensional analysis appears to be very useful.

### 3.2 Dimensional Analysis

In many physical situations involving impinging jets, the geometry is so complicated that the governing differential equations cannot be solved at all

analytically. Dimensional analysis is very helpful in such cases; additionally, it is very handy in correlating experimental data, by reducing the total number of the variables. Dimensional analysis teaches us (Reference 12) that heat transfer from impinging jets can be correlated in the form

$$Nu = f(Re, Pr, M, r/D, Z_n/D, D_p/D, D_c/D, C_n/D, L_1/D, L_2/D \dots L_n/D) \quad (20)$$

which results can also be represented for restricted ranges of the parameters Re, Pr, and M by the relation

$$Nu = C Re^a Pr^b M^c (r/D)^d (Z_n/D)^e (D_p/D)^f (D_c/D)^g (C_n/D)^h (L_1/D)^i \dots (L_n/D)^u \quad (21)$$

where C is a constant. The parameters Re, Pr and M represent the physical side of the problem. For  $M < 0.5$ , the influence of Mach number is small, and can be safely neglected (here, formally by setting  $c = 0$ ). For boundary layer type laminar flow,  $a = 0.5$  and  $b = 0.4$  (Reference 28). For turbulent flow in the wall-jet region,  $a \approx 0.7$  and higher applies, and  $b = 0.33$  is commonly used. The other dimensionless ratios are characteristic lengths, normalized with the nozzle diameter D for the sake of convenience. Local values of Nu at stagnation point ( $r/D < 1$ ) may be expected to depend on  $Z_n/D$  and  $D_c/D$ , or on the normalized nozzle-to-target plate distance and the calorimeter (or heat flux gauge) diameter, but would not depend on the target plate diameter,  $D_p$ .

Occasionally, other characteristic lengths are useful, like the thickness of the wall jet boundary layer, or the jet spreading pattern parameter b. Perusal of the pertinent literature teaches that, in many cases, information on the significance of certain parameters pertinent to the problem at hand may be lacking, thus making the value of information available greatly reduced for various practical applications. In particular, for the average heat transfer results, the effect of the ratio  $D/D_p$  is often not emphasized sufficiently well. On

the other hand, for strictly local heat transfer results, one can expect some dependence of the local Nusselt number on the relative size of the heat flux gauge or calorimeter, in the form of  $(D_c/D)^g$ . Likewise, for a row of jets, there will be Nusselt number dependence on the nozzle-to-nozzle spacing, or  $(C_n/D)^h$ .

Dimensional analysis is indispensable for a better understanding of the literature on the subject of impinging jet heat transfer. Reports and papers concerned with experimental results obtained by various investigators often appear to be hard to understand and contradictory to the reader. Dimensional analysis, properly applied, is then of great help. In a new field like impinging jet heat transfer, a certain lack of generality in the description of its associated experimental activities is inevitable, and may be considered as a healthy sign of growth. As the interest in impinging jet heat transfer grows, one can also expect that additional terms may be considered to be of significance in equations 21 and 22. For example,  $(L_1/D)^i$  term may be considered as representative of the roughness of the target plate, etc.

SECTION IV  
EXPERIMENTAL HEAT TRANSFER

#### 4.1 Single Circular Jets

Experimental results obtained from the study of heated jets striking cool flat surfaces are reported in References 40 to 44. Perry (Reference 40) considered air heated to 873 K (1112°F) discharging through nozzles 16.5 and 21.6 mm (0.65 and 0.85 in.) in diameter at velocities up to 91 meters per second (300 ft/sec). The nozzle-to-plate spacing was at least 8 nozzle diameters. The effect of the angle of jet impingement was studied. The heat flow was determined by means of a 16.5 mm (0.65-in.) diameter calorimeter; this dimension,  $D_c$ , was used as the characteristic dimension in both the Nusselt and Reynolds numbers.

The test results are shown in Figure 4. They were correlated for a normal impingement by the expression

$$Nu = 0.181 Re^{0.7} Pr^{0.33} \quad (22)$$

Heat transfer was tested also at other angles of impingement. Perry found a twofold increase in the plate heat transfer, as the jet axis of symmetry was turned from 15° to 90° with respect to the plate.

The influence of nozzle-to-plate spacing on the heat transfer of an impinging hot air jet on a flat plate was investigated by Thurlow (Reference 41). Values of  $Z_n/D > 10$  and  $Re_D$  up to 60 000 were considered. Nozzles 2.54 and 12.7 mm (1 and 1/2 in.) in diameter were used to impinge hot air on a copper plate with linear dimensions of 610 by 152 mm (24 by 6 in.). The data were correlated by

$$Nu = C Re_D^{1/3} \exp(-0.037 Z_n/D) \quad (23)$$

where the coefficient  $C$  was found to be equal to 1.06 for the 25.4 mm diameter nozzle, and equal to 0.33 for the 12.7 mm diameter nozzle, or  $C \sim D^{3/2}$ .

The experimental results of Huang (Reference 42) on turbulent hot air jets with temperature ranging from 300 to 350°F, indicate a local value (near the

(stagnation point) of

$$Nu_0 = 0.023 Pr^{1/3} Re_D^{0.87} \quad (24)$$

and the average value

$$Nu = 0.0180 Pr^{1/3} Re_D^{0.87} \quad (25)$$

for  $1 < Z_n/D < 10$  and for Reynolds numbers going up to  $10^4$ . No variation of heat transfer with  $Z_n/D$  was detected for  $Z_n/D < 6$  due most likely to the local heat flow detector sensitive area being  $1 \text{ in.}^2$ , with nozzles ranging from  $1/4$  in. to  $1/8$  in. in diameter, so that only average heat flux was in fact measured. The experimental results showed that the maximum heat transfer was achieved with a 4.76 mm (3/16-in.) diameter nozzle and an  $r/D$  ratio less than 3. It should be pointed out that, in this investigation, no change was found in the heat-transfer coefficient for  $Z_n/D < 6$ ; in contrast, major changes were found in Reference 7 (to be discussed below).

Popiel et al., in Reference 43, show local heat transfer from a normal impinging hot gas jet to a plane isothermal surface, at low Reynolds numbers. Reasonably good agreement is found with Sibulkin's solution (Figure 5), for  $Z_n/D < 5$ . Radial heat transfer distributions were found to be qualitatively consistent with other investigations of the impinging cold jets at low Reynolds numbers. Comfort et al. (Reference 44) also investigated hot jets.

Room-temperature jets, impinging on heated flat surfaces, are reported in References 7,8 and 45-46. Reference 7 presents results of tests of cool air impinging from a circular jet onto an isothermal hot surface by Gardon and Cobonpue. Nozzles with diameters from 2.3 to 9.0 mm (0.089 to 0.354 in.) and nozzle exit velocities up to sonic were used in conjunction with a 152.4 mm (6-in.) square aluminum plate. Heat-flow measuring gauges 0.9 mm (0.035 in.) in diameter were mounted flush in the aluminum plate to measure heat-transfer rates. In a later publication, (Reference 9) Gardon stated that calibration of his heat flow gauges could have been as much as 40 percent too high. Provisions



were made to investigate various nozzle-to-plate spacings (0.25 to 40 diameters).

Stagnation point heat transfer presented in Reference 7 had the Nusselt number based on the nozzle diameter and the Reynolds number on the nozzle diameter and nozzle exit velocity. The results are shown in Figures 5 and 6 for various nozzle diameters and nozzle-to-plate spacings. The figures show for peak heat-transfer rates a tendency to occur at progressively lower nozzle-to-plate spacings, the peaks increasing with increasing nozzle diameter. Figure 6 shows also clearly the residual effects of nozzle diameter on heat transfer for all Reynolds numbers; this effect is most pronounced for  $Z_n/D < 10$ , however. For values of  $Z_n/D > 20$  and nozzle exit Reynolds number bigger than 14 000, the following correlation was obtained:

$$Nu_0 = 13 \sqrt{Re_D} D/Z_n \quad (26)$$

The magnitude of heat transfer intensity reported by Gardon and Cobonpue was substantially less than that found by Comfort et al. (Reference 44) for stagnation point heat transfer, relatively small  $Z_n/D$ , and  $M=0.762$ .

The cooling of flat circular plates by air jets, impinging normally on the plates, was investigated in Reference 30 by Walz. The transient technique was used to determine average heat-transfer coefficients for nozzle Reynolds numbers up to 31 000, nozzle-to-plate spacings from 1 to 8 nozzle diameters, nozzle diameters from 3.2 to 7.6 mm (0.125 to 0.3 in), and target diameters from 12.7 to 25.4 mm (0.5 to 1 in.). Results similar to the original results of Reference 7 were obtained (before announcement of the possibility of inaccurate calibration of the heat flux gauges used in Reference 7). The effect of the ratio of target diameter to nozzle diameter is also discussed.

Experimental heat-transfer coefficients were presented by Brdlik and Savin in Reference 36, for a jet impinging on a heated plate. The data showed that for  $Z_n/D < 6.2$ , the heat transfer coefficient was practically independent of

the distance from the jet exit, and for  $Z_n/D > 6.2$ , the distance  $Z_n$  had an appreciable effect on the heat transfer. Good agreement was obtained when the experimental and analytical results were compared.

Room-temperature turbulent air jets impinging on a segmented flat Invar steam-heated plate perpendicular to the jet axis were studied by Chamberlain (Reference 8). Two nozzle sizes, 6.35 and 9.52 mm (0.250 and 0.375 in.) in diameter; nozzle exit Reynolds numbers up to 67 000; and nozzle-to-plate spacings up to 50 nozzle diameters were investigated. When the value of nozzle-to-plate spacings  $Z_n/D$  was 7 or less (the plate was located within the potential-core region), local stagnation-point Nusselt numbers  $Nu_0$  were correlated by

$$Nu_0 = 1.16 Re^{0.447} Pr^{0.333} \quad (27)$$

When the plate was located outside the potential-core region, the data were correlated by

$$Nu_0 = 0.384 Re_a^{0.569} Pr^{0.333} \text{ for } Z_n/D > 7 \quad (28)$$

where  $Re_a$  is the Reynolds number based on the arrival velocity (as proposed in Reference 7 when multiple-jet data were correlated). Results of thermal conductivity test conducted by the National Bureau of Standards on a sample of invar used in the test plate showed that it was 40 percent higher than that available from the literature previously. The coefficients in the two preceding equations have been modified accordingly.

For locations along the plate away from the stagnation point, the correlation

$$\frac{h_r}{h_0} = \exp. (-1.56 (r/Z_n)^{0.75}) \quad (29)$$

was suggested; here  $h_r$  is the local coefficient at a distance  $r$  from the stagnation point.

Heat-transfer coefficients for a jet impinging on a plate are reported in Reference 45, for jet exit velocities of 61 to 213 meters per second (200 to

700 ft/sec). Calorimeters were used to obtain the heat-transfer distributions on the plate. Turbulence characteristics of the applicable free jets were used as a basis for correction factors to be applied to computed laminar stagnation-point heating.

In particular, near the stagnation point, the heat transfer may be computed from the laminar heat transfer that would take place on a surface having the same pressure distribution as that on the impingement plate and using a correction factor, while farther away from the stagnation point, the heat transfer may be computed using the technique of Reference 46, and very far from the stagnation point, the relation

$$Nu = 0.12 Re_{1/2}^{0.8} \quad (30)$$

should be used, where the Reynolds number is based on the distance at which the velocity is one-half the maximum free-jet velocity in the plane of impingement.

Interesting measurements of heat transfer, from naturally occurring and turbulized jets, have been reported by Dyban and Mazur in Reference 47. They show clearly the enhancing effects of turbulence on stagnation point heat transfer in Figure 7. Heat transfer boosts of the order of 100 percent could be produced by turbulizing the impinging jet, that is by increasing artificially the fluctuating velocity components in the jet stream. Additionally, Dyban et al. in Reference 48 show the effects of turbulizing on the term  $a^*$ , which is needed in analytical heat transfer calculations of impinging jets (Figure 8). In Figure 8, comparisons with results from Reference 5, 11 and 49 are made. It is seen that  $a^*$  may be approximated by

$$a^* = (Z_n/D)^{0.16}, 1 < Z_n/D < 6.2 \quad (31)$$

and

$$a^* = 32.6 (Z_n/D)^{-1.75}, Z_n/D > 6.2 \quad (32)$$

Comparing the results of Popiel, Gardon and Cobcnpue, and those of Dyban et al.

in Figures 5-8, it is seen that turbulence effects enhance heat transfer by nearly 100 percent at the stagnation point, in case of impinging round jets, in proportion to  $(Z_n/D)^{0.16}$  for  $Z_n/D$  less than 6 plus. The dimensionless velocity gradient at the stagnation point is also, at first, increasing with  $Z_n/D$ , until  $Z_n/D = 6.2$  is reached; then a gradual decrease in  $a^*$  is observed. The effects of turbulence influence apparently the rate of fluid mixing in the "mixing cone" of the jet, and also the length of the potential core. These facts influence pressure gradients at the target plate in the direction parallel to the plate, which explains the changes in  $a^*$  shown in Figure 8. In Figure 6, it is seen that many nozzles achieve a high degree of turbulence in the jets they discharge, without any apparent use of turbulizers. This may be concluded by comparing the Figure 6 heat-transfer data with those in Figure 7, where turbulizers were purposely used.

#### 4.2 Arrays of Circular Jets

Reference 1 presents results of an experimental investigation of heat transfer by Freidman and Mueller, for an array of circular air jets impinging on a heated plate. Effects of hole size and spacing, distance between the array of nozzles and the heated plate, and air velocity on the heat-transfer rates were studied. Hole sizes from 6.34 to 19.05 mm (0.250 to 0.750 in.) and spacings between the nozzles and the heated plate from 57.2 to 158.8 mm (2.25 to 6.25 in.) were considered. Results indicated that spacings between holes of from 4 to 6 hole diameters (giving about 5 to 2 percent free-flow area) yielded the best heat transfer results.

In addition to the study of single-jet impingement discussed above, Gardon and Cobonpue also considered multiple arrays of jets; the results are reported in Reference 7, where heat-transfer effects of nozzles with diameters from 1.59 to 12.7 mm (1/16 to 1/2 in.), arranged in square arrays, were studied.

A 5x5 nozzle array on 50.8 mm (2-in.) centers and a 7x7 nozzle array on 30.5 mm (1.2-in.) centers were used. Airflow rates from 50 to 800 lb/(hr ft<sup>2</sup>) were considered. Nozzle exit velocities were kept low enough so that compressibility effects could be neglected.

When attempts to correlate the data based on nozzle exit velocity failed, Gardon and Cobonpue (Reference 7) decided to correlate by using the velocity of arrival. Space-averaged heat-transfer data were successfully correlated when the center-to-center hole spacing was used as the characteristic dimension in the Nusselt and Reynolds numbers, and the velocity of arrival was used in the Reynolds number. The velocity of arrival  $u_a$  for nozzle-to-plate spacings greater than 8 was evaluated as

$$u_a = 6.63 u_{oc} D/Z_n \text{ for } Z_n/D > 8. \quad (33)$$

The average Nusselt number was found to be

$$Nu = 0.286 Re_a^{0.625} \quad (34)$$

The equation for Nu above holds down to  $Z_n/D > 1$ .

The effect of nozzle-to-plate separation distance on the local Nusselt number was also investigated. It was found that the shapes of the curves, notably different, depend on  $Re_a$  as well as on  $Z_n/C_n$  and, when  $Z_n/D < 8$ , on  $Z_n/D$  also. In Reference 42, the maximum heat transfer for a single jet was found for a 4.76 mm (3/16-in.) diameter nozzle and an  $r/D < 3$ ; Huang's investigation of multiple jets was limited to the 4.76 mm (3/16-in.) diameter nozzle, spaced at  $C_n = 15.88$  mm (5/8 in.) center-to-center in a single row and the row-to-row distances of 38.1, 76.2 and 152.4 mm (1.5, 3, and 6 in.).

Huang proposed correlations that came under a heavy criticism during the discussion of his paper, since average heat transfer coefficients from his

arrays of round jets seemed to be higher than those of the equivalent number of single jets. During discussion Huang stated that his heat transfer results from arrays of jets were carefully determined. Comparison of his data is shown in Figure 10.

A triangular array of circular jets, impinging on a flat surface, was investigated by Ott in Reference 50. Only a limited range of Reynolds numbers was considered and only  $Z_n/D$  and  $Re$  were variable. An average value of the heat-transfer coefficient was therefore correlated with only  $Re$  and  $Z_n/D$  as parameters. No variation in free-flow area was included in this study. Ott's results are shown in Figure 10.

Hilgeroth (Reference 51), discovered that jet interaction and spent air effects had a substantial effect on heat transfer. He found also that, for  $C_n/D$ ,  $Z_n/D$ , and  $u$  held constant, the heat-transfer coefficients increased as the hole diameter increased. A 25 percent decrease in heat-transfer coefficient between  $Z_n/D$  of 2 and 6 was observed. Maximum values of  $h$  were obtained for  $Z_n/D$  of 6.3, for a free-flow area of 1.5 percent. The Reynolds number exponent was found to be a function of hole-spacing-to-hole-diameter ratio, increasing, as  $C_n/D$  increased from 3.5 to 12.5. A square array of jets was found to be superior to an equilateral triangular array from the same jet velocity and open-area ratio. Comparison of Hilgeroth's results with those of others is also shown in Figure 10.

Reference 52 reported the investigation of a series of configurations by Kercher. Figure 10 compares Kercher's data and the results of Gardon and Cobonpue (Reference 7). The differences between the results may be attributed to the influence on boundary-layer phenomenon of the effects of jet impingement from perforated plates and from long throat nozzles. Data indicated that the average surface heat-transfer coefficients increased as  $Z_n/D$  was increased from 1 to about 5.

The test results are summarized in Reference 52 as follows: Among the highlights we find that heat-transfer coefficients increase with increasing open area, heat transfer is dominated by the hole-diameter Reynolds number  $Re_D$  and the hole-spacing-to-hole-diameter ratio; within the range tested, increasing  $Z_n/D$  increases heat transfer without crossflow, but decreases heat transfer with cross flow; increasing cross flow decreases heat transfer performance; and decreasing hole diameter with increasing number of holes, everything else being equal, improves heat-transfer performance.

An experimental study of the effects of cross flow on the heat-transfer characteristics of single rows of air jets impinging on plane surfaces is reported by Metzger and Korstad in Reference 53. Rows of 10 circular jets, each 2.54 mm (0.1 in.) in diameter, with center-to-center spacing ratios  $C_n/D$  from 2.5 to 5, were considered. Nozzle-to-plate spacings  $Z_n/D$  from 2 to 6.7 and ratios of cross flow jet flow  $M^*$  from 1 to 3 were studied. The average Stanton number was defined as

$$St = St(Re_{es}, C_n/D, Z_n/D, l/D) \quad (35)$$

where  $St$  is  $h/(GC_p)$  evaluated over the cooled surface of half-length  $l$  and  $Re_{es}$  is based on the width of a slot whose area is equal to that of the 10 holes used, and where  $G = V\rho$ , the main flow rate per unit area, and where  $B_{es} =$  equivalent slot width, or

$$Re_{es} = 2G B_{es}/\mu \quad (36)$$

An extra parameter,  $M^* = m_c/m_j$ , should correlate data with cross flow. As  $M^*$  increases, the general trend shows a decrease in  $St/St_{M^*=0}$  in the region upstream from the nozzle exit plane and an increase in the downstream region. For the ranges of variables covered in this investigation, the cross flow heat-transfer results were correlated by

$$St = 0.822 M^{*-0.049} Re_D^{-0.338} \quad (37)$$

The significant length on which Nu and Re are based may be the nozzle diameter or the radius or the target plate diameter or radius. For arrays of jets, the nozzle-to-nozzle spacing  $C_n$  is often chosen. Most impinging jet results are stated in terms of  $Re_D$ . Equation (34) (for  $Z_n/D < 8$ ) may be converted into expressions involving  $Re_D$  by multiplying each side by  $D/C_n$ , or

$$(D/C_n) hC_n/k = 0.296 (U_{oc} C_n \rho/\mu)^{0.625} (D/C_n)$$

with the result

$$hD/k = Nu = 0.286 (C_n/D)^{-0.375} Re_D^{0.625} \quad (38)$$

Equation (38) allows us to introduce here explicitly the effects of nozzle-to-nozzle spacing and of the diameter size in the form of the ratio  $(C_n/D)$  to some negative power. It indicates that, as  $C_n$  increases, there are fewer nozzles per unit area, and average heat transfer is reduced. Likewise, with an increasing nozzle diameter,  $D$ , average heat transfer improves as the nozzle area increases in proportion to the total area exposed to impinging jets.

The influence of the size of the nozzle array,  $D_{eq}$ , can be also introduced in Equation (38) by multiplying each side with  $(D/D_{eq})^n$ ,  $n$  being obtained experimentally, where  $D_{eq} \equiv 2 (\text{area of nozzle array}/\pi)^{1/2}$ .

In this way, for example, Kercher's result

$$Nu_{C_n} = 0.324 Re_{C_n}^{0.658} \quad (39)$$

can be transformed into

$$Nu_D = 0.324 (C_n/D)^{-0.342} Re^{0.658} \quad (40)$$

to be compared with the results of other investigators based on the nozzle diameter  $D$ , in Figures 9 and 10. Equations (38) and (40) allow us to introduce the effect of  $C_n$  and the nozzle diameter  $D$  explicitly into the correlating equation. The effect of  $(D/D_{eq})^n$ , not shown specifically in above examples, is useful in comparison of nozzle arrays of various sizes. Its function is similar to that of the term  $(D/D_p)^{1.23}$  in Equation (18).



It is sometimes very convenient to use the analogy between heat and mass transfer to overcome some of the difficulties, inherent in getting highly accurate measurements of local heat fluxes due to multiple jet impingement. Thus, Koopman and Sparrow (Reference 54) have applied the naphthalene sublimation technique in conjunction with the semi-automated data acquisition, to determine the local and average mass transfer coefficients resulting from impingement of a row of jets onto a flat surface. The local Sherwood numbers thus obtained could then be used to identify the locations where the impinging jets would provide the most effective heating or cooling. It has been observed, in particular, that because of the collisions of the wall jets originating from the adjacent impinging jets, "the transfer coefficients at positions mid-way between the jets may take on relatively large values". The effect of such collisions is enhanced at high Reynolds numbers, small jet orifice spacings, and small orifice-to-target distances. A two-dimensional wall jet is produced at sufficiently large downstream distances from the row of jets.

In contrast to single jets, where the results depend only on the radial distance to the stagnation point, for multiple jets transfer coefficients depend differently on the spanwise and streamwise relative distances. Under certain circumstances, the multiple jet transfer coefficients were found to exceed those for the single jet.

On the other hand, when the single jet results from Reference 54 were compared with those of Gardon and Akfirat (Reference 9), the agreement was found to be satisfactory for conditions corresponding to common values of the characteristics parameters.

In spite of proven high precision of mass transfer results at lower Reynolds numbers, it remains doubtful if such methods may be considered entirely satisfactory for really high Reynolds numbers and for determination of the maximum possible heat fluxes in cases of complex geometry existing in some applications of impinging jets. Such applications are described in some detail below.

### 4.3 Single Slot Jets

An experimental study of the heat-transfer characteristics of slot jets impinging normally on a flat target was performed by Metzger (Reference 31). Nozzles with lengths of 19.0 mm (0.75 in.) and widths of 0.25, 0.51, 1.02, 1.53, and 2.04 mm (0.01, 0.02, 0.04, 0.06, and 0.08 in.) were studied as they impinged on two copper targets of half-lengths 12.7 and 6.35 mm (0.50 and 0.25 in.) mounted in blocks of balsa wood. Reynolds numbers up to 10 000 (based on nozzle exit velocity and nozzle hydraulic diameter) were considered. Initial tests, with all variables except nozzle-to-target spacing held constant, showed that maximum average heat transfer occurred at a value of  $Z_n/B_0 = 8$ . Since no significant variation was noted as  $Z_n/B_0$  was varied from 7 to 10, it was decided that all other tests would be made for  $Z_n/B_0 = 8$ . This value of 8 is comparable with the value of  $Z_n/D$  of between 6 and 7 for a single circular jet.

The correlation equation for the average heat transfer was found to be

$$St (Re_{D_h})^{0.434} Pr^{0.63} = 0.74 (D_p/B_0)^{-0.434} \quad (41)$$

for  $7 < Z_n/B_0 < 10$  and  $3 < D_p/B_0 < 50$ .

Experimental wall jet heat-transfer data that were obtained by Myers et al. using a 12.7 mm (0.5-in.) wide slot and slot Reynolds numbers from 16 600 to 38 100 are reported in Reference 32. Data were taken for values of  $r/B_0$  from 20 to 186 for five different unheated starting lengths  $L$  ( $r$  is the distance along the wall). The final results are represented by the formula good for correlation of the data for  $r/B_0 > 45$ , namely.

$$St Re_{B_0}^{0.2} (10^2) \left[ 1 - \left( \frac{L}{r} \right)^{9/20} \right]^{1/16} = 12 \left( \frac{r}{B_0} \right)^{-0.5658} \quad (42)$$

Heat-transfer characteristics of air jets issuing from slots 1.59, 3.17, and 6.35 mm (1/16, 1/8, and 1/4 in.) wide and 152.4 mm (6 in.) long and impinging normally on an aluminum 152.4 mm (6 in.) square plate are reported by Gardon and Akfirat, Reference 9. Both average and local heat-transfer rates were obtained, with the help of a 0.9-millimeter (0.035 in.) diameter heat transfer gauge. Maximum stagnation heat transfer was found to be a function of the nozzle exit Reynolds number and occurred between values of  $Z_n/B_0$  from 7 to 10. For turbulent jets ( $Re > 2000$ ) and nozzle-to-plate spacings greater than 14 slot widths, the stagnation-point heat-transfer coefficients were correlated

$$Nu_0 = 1.2 Re_{B_0}^{0.58} (Z_n/B_0)^{-0.62} \quad (43)$$

for  $Re_{B_0}$  up to 50 000 and  $Z_n/B_0$  up to 60. Nozzle size was found to influence the turbulence levels of the jets; for  $Re_{B_0} = 11\ 000$ , the initial turbulence levels were found to be 0.6, 2.5, and 7.5 percent for the 1.59-, 3.17-, and 6.35-mm (1/16-, 1/8-, and 1/4-in.) wide slots, respectively. The observed variation of the local heat-transfer coefficients along the plate has been dependent on  $Z_n/B_0$ . Thus, for nozzle-to-plate spacings  $Z_n/B_0$  greater than 14 slot widths, the variation of heat-transfer coefficients has a bell shape, while for  $14 < Z_n/B_0 < 8$ , the bell shape is modified slightly by an abrupt change in slope in the vicinity of  $r/B_0 = 4$ . On the other hand, as  $Z_n/B_0$  is reduced below 8, two "humps" begin to form at about  $r/B_0 = 7$ ; and for  $Z_n/B_0 < 6$ , they become well-defined secondary peaks in the heat-transfer rate. These secondary peaks are tentatively explained as due to a transition from laminar to turbulent boundary layers, and as  $Z_n/B_0$  is reduced below 1/2, the impinging jet becomes essentially a "wall jet". Heat-transfer coefficients rise sharply with increasing velocities in the gap between the nozzle exit and the plate; the local heat-transfer coefficients show two peaks away from the stagnation line.

Average heat-transfer coefficients for the three slot widths considered are correlated against the Reynolds number based on the arrival velocity by an expression of the form.

$$Nu = 0.36 Re_a^{0.62} \quad (44)$$

for  $Re_a > 10^4$ , and  $Z_n/B_0 > 8$ ,

The heat-transfer characteristics for a slot jet impinging on a flat copper surface for slot widths of 6.35, 12.7 and 19.0 mm (1/4, 1/2, and 3/4 in.), nozzle-to-plate spacings from 2 to 16 slot Reynolds numbers from 4600 to 102 000 were studied by Cadek (Reference 33). Local heat-transfer rates were measured by use of a miniature circular foil heat-flow sensor with a 0.91 mm (0.036-in.) diameter sensing area. Values of  $r/B_0$  from 0 to 36 were considered. Maximum heat transfer was found for  $Z_n/B_0 = 8$ , in agreement with Gardon and Akfirat's data. Cadek concluded from his study that for  $2 < Z_n/B_0 < 4$ , good agreement between theory and experiment was obtained for both the stagnation region and the wall jet region.

Reference 55 presents experimental heat-transfer data obtained by Cartwright and Russel, from an impinging jet for nozzle-to-plate spacings from 8 to 47 slot widths, nozzle exit Reynolds numbers from 25 000 to 110 000, and distances along the wall jet from 0 to 132 slot widths. At low slot Reynolds numbers, the maximum value of the heat-transfer coefficient occurred at the stagnation point, and then decreased monotonically with increasing distance from the stagnation point, while at larger Reynolds numbers, the maximum heat-transfer coefficient occurred some distance from the stagnation point, in a way similar to axisymmetric impinging jet heat-transfer results.

Analytical attempts to predict the heat transfer appeared to have been successful only in the wall jet region beyond  $r/B_0 = 36$ ; failure of predictions in the impingement region was explained by the high level of turbulence existing in the free jet before impingement.

An analytical method for predicting the local Nusselt number in the stagnation region of a slot jet was developed by Andreyev et al. in Reference 56. The prediction was eventually modified, by use of some experimental data so that an empirical correlation resulted, where  $e$  is the degree of turbulence intensity for  $1 < Z_n/B_o < 6.5$ :

$$Nu = 0.48 \frac{Re_{B_o}^{0.5}}{(Z_n/B_o)^{0.1}} \left[ 1 - 0.116 \frac{(r/B_o)^2}{(Z_n/B_o)^2} \right] (1 + 0.015e) \quad (45)$$

The expressions for  $6.5 < Z_n/B_o < 12$  and  $Z_n/B_o > 12$  are of the same form, with some changes in the values of the numerical constants occurring in Equation 45.

#### 4.4 Arrays of Slot Jets

The tests reported in Reference 1 by Freidman and Mueller included information on plates with a multitude of slots. It was found that the slotted plates were generally comparable in performance to the round nozzle equipped plates, with some exceptions.

Gardon and Akfirat in Reference 9 also report the results of an investigation of impingement from an array of slot jets. Data were obtained for two combinations of slot jets, namely, three slots at 50.8 mm (2-in.) centers, and two slots at 101.6 mm (4-in.) centers. It was found that, at the smallest slot-to-plate spacing, the identity of each jet was preserved and the peak heat-transfer rates of the jets differed only slightly from those corresponding to single jets. Interaction between the jets produced secondary peaks midway between the points of impingement. As the length of the jet was increased, interaction occurred before impingement. The peaks did not necessarily lie below the centers of the jets. At the largest spacing, the jets behaved almost like a large single jet.

The data were correlated on the basis of a Reynolds number based on the velocity of arrival defined here in Equation 3 with the average Nusselt number defined as

$$\overline{Nu} = \bar{h} C_n / k \quad (46)$$

in the form

$$\overline{Nu} = 0.36 Re_a^{0.62} \quad (47)$$

which expression should not be extrapolated to  $C_n/B_o < 16$ .

Schuh and Pettersson report heat-transfer data in Reference 57 for impingement of air through arrays of slots. Slot widths of 1 and 5 mm (0.0394 and 0.197 in.), jet spacings of 25, 40, 60 and 100 mm (0.984, 1.57, 2.36, and 3.94 in.), and slot-to-plate spacings of 2, 4, 6 and 8 slot widths were investigated. Slot exit velocities from 4 to 100 meters per second (13.12 to 328 ft/sec) were considered. For  $12\,000 \leq Re \leq 100\,000$ ,  $5 \leq C_n/B_o \leq 100$ , and  $Z_n/B_o = 4$ , it was found that the data could be correlated by

$$St = 0.461 \left( \frac{C_n}{B_o} \right)^{-0.327} Re_{B_o}^{-0.402} \quad (48)$$

within 10 percent. Also, in Reference 57, it was found for rows of jets that lower heat-transfer was achieved for the round jets than for the arrays of slot jets with wide row spacings. Little difference was found for arrays with narrow row spacings.

It was also found that a superimposed wall-parallel flow, of up to 60 percent of the jet flow, could be imposed with no reduction in heat transfer. For similar conditions, but with rows of round jets instead of arrays of slot jets, some reduction in heat-transfer accompanied the 60 percent superimposed wall parallel flow.

In conclusion of discussion related to the slot jet experiments, substantial experimental difficulties in obtaining reproducible slot jets should be pointed out. While actual slot jets are all, by necessity, of finite length, great care must be taken to make sure that the end effects are eliminated. For that reason, the comparison of individual slot jet data must be made with some caution.

Heat transfer from impinging jets literature review, carried out in 1968-72, has been prepared by Livingood and Hrycak (Reference 58). This information, in an updated version, has been used in Sections III and IV of this report. Another, more recent literature related review has been made by Martin (Reference 59). It focusses its attention to a large extent on the results of German workers in the present area of interest, and deals with heat transfer optimization problems of arrays of jets impinging on flat plates. Since heat transfer from arrays of jets has not yet been described in a definitive way, additional work is indicated before heat transfer from impinging jets can be optimized in a physically meaningful fashion. Still, such efforts are very timely. Figure 10 has been adapted from Reference 60. In Reference 60, dimensions of characteristic parameters have been kept down to what may be expected in impingement cooling of a typical gas turbine blade. The results may be quantitatively different from other cooling applications, with substantially larger diameter nozzles, as is positively shown in Figure 10.

#### 4.5 Cooling of Areas with a Curvature

Cooling of areas with a curvature is an important application of rows and arrays of impinging jets, primarily to cool gas turbine blades internally. Such areas may be approximated by concave semicylindrical surfaces. Among the investigators here, we may distinguish Metzger et al., Chupp et al., Jenkins and Metzger, Dyban and Mazur, Tabakoff and Clevenger, and Burggraf

(References 61-66 respectively). Very helpful is here also the monograph by Shvets and Dyban, with an extensive bibliography (Reference 67).

In cooling of areas with a curvature, several unique effects must be considered. The most significant effect is, perhaps, that of the severely constricted space between the nozzle exits and the impingement plate with a curvature. It becomes very significant when the nozzles are formed by a row of round holes, in a tightly fitting plenum chamber. Because of a restricted space in the case of low  $Z_n/D$  values, a true wall-jet cannot develop in all cases, and the stagnation area (where  $Re^{0.5}$  term is still applicable) may be somewhat extended. The restricted free space, and the fact that the ends of the heat transfer surface must be blocked off, to make flow still two-dimensional, imply for low  $Z_n/d$  values the possibility of formation of a channel-like flow, as suggested in Reference 64.

For heat transfer at the stagnation point, Hrycak (Reference 39) developed an expression

$$Nu_o = 1.6 Re_D^{0.5} Pr^{0.39} (D_c/D)^{-0.42} (Z_n/D)^{-0.22} (C_n/D)^{-0.16} \quad (49)$$

that shows explicitly the effect of the nozzle diameter,  $D$ , of  $Z_n/D$  and nozzle-to-nozzle spacing,  $C_n$ .

The expression is similar to a corresponding flat plate formula that is also discussed in Reference 39. For the average heat transfer, Reference 39 reports the expression

$$Nu = 0.72 Pr^{0.33} Re_D^{0.63} (C_n/D)^{-0.16} (D/D_p)^{0.402} \quad (50)$$

Equation (46) is compared in Figures 9 and 10 with arrays-of-nozzles results by Kercher, and Gardon and Cobonpue. It seems to fall almost exactly into the middle between the results of these two investigators. Equation (50) describes heat transfer on a semicylindrical plate 10 in. long and 5 in. in diameter; comparison with the results of other investigators working with a similar



geometry is shown in Figure 11, while local distribution of heat transfer coefficients is shown in Figure 12. In both cases, 10 by 5 in. semicylindrical plate results compare favorably with the results of others. Also, heat transfer on that 10 by 5 in. semicylindrical plate mentioned above has been found similar to heat transfer on a plate scaled down ten times. Performance of that small-scale configuration plate has been compared with the results of several curvilinear geometries in Figure 13. Again the comparison is quite favorable. This figure has been adapted from an illustration prepared by Livingood and Gauntner in Reference 68, while Figure 12 was adopted from Reference 69. In Figures 11-13 there occur references that have not been listed directly in the present report. These additional references may be found in References 68 and 69.

Impinging jet heat transfer from jet rows, and jet arrays, to both flat and curved surfaces depends ultimately on the performance of the individual jets, forming the most essential part of the sometimes very complex heat exchanging system. Fundamental here is still the information on a single, impinging, axisymmetric jet. A lot of important information on behavior of a single impinging jet (applied to mass transfer), where the effects of surface roughness have been studied systematically for the first time, for example, has been collected by Schrader. The behavior of a single jet in cross-flow has been studied by Bouchez and Goldstein. A systematic study of the impinging jet behavior has been carried out in great detail by Popiel (References 70-72). Additional insights into behavior of rows of jets impinging on semicylindrical surfaces may be seen in a recent publication by Hrycak (Reference 73), where also some other fundamental aspects of impinging jet heat transfer are covered.

More information on the interaction between individual jets in an array appears to be very desirable. Details of how a centrally located jet interacts with its eight neighbors in a three-by-three square jet array would be probably the logical first step towards this kind of investigation.

SECTION V  
CONCLUSIONS AND RECOMMENDATIONS

5.1 General Considerations, Individual Circular and Slot Jets

Experimental investigation of heat transfer from impinging jets has a much shorter history than the experimental investigation of free jet flow patterns. Consequently, agreement among the results of various investigators on a number of important aspects of heat transfer from impinging jets is still not as good as could be desired.

Here we can distinguish three areas where experimentation has been carried out: on impinging round jets, impinging slot jets, and arrays of jets. So far, the theory has been able to calculate mainly heat transfer at the stagnation point under some simplifying assumptions (cf. References 2,30,38, 73) and several reasonably successful attempts have been made to do likewise for the wall jet region, (References 14,33). Also, measured heat-transfer coefficients in the stagnation region exceed those determined by use of the several analytical procedures discussed herein. The analyses are affected by a lack of knowledge of the jet turbulence characteristics. Further investigations leading to a more precise evaluation of the jet turbulence characteristics appear to be required for more satisfactory calculations. Additional investigations of single, turbulent, room-temperature air jets, impinging on heated flat surfaces, would be useful.

The main contributions of theory at the present time seem to be the following: for efficient heat transfer, for both circular and slot jets, the impinging surface has to be at the end of the potential cone, where the turbulence pattern helps to maximize heat transfer. This is the idea one gets from perusal, for example, of the papers by Gardon and associates (References 7,9), Chamberlain, Metzger and Walz, (References 8,30,31) where heat flow patterns are studied

at the stagnation point with the Reynolds number as a parameter and  $Z_n/D$  as abscissa (Figure 6). This phenomenon is somewhat obscured when what actually has been measured is not strictly the local heat transfer, but happens to be only the average heat flow. This occurs naturally with the heat transfer-sensitive element larger than the effective jet diameter; it can be seen in Figure 6 of the paper by Huang (Reference 42) and Figure 9 of that by Walz (Reference 30). Into this category seem to fall also observations of Perry (Reference 40).

In general, the experiments of each particular investigator have been carried out over only a relatively limited range of parameters, thus making a general comparison difficult. In addition, some variables apparently could not always be kept completely under control, which led to what appears to be contradictory results at times (Huang, Reference 42, as pointed out by Walz, Reference 30).

Four more general methods for measuring of heat transfer have been used so far: heat transfer gauges (Gardon and associates, References 7,9,Cadek, Reference 33), temperature change of a piece of metal of high conductivity in the nonsteady state (Metzger, Huang, Schauer and Eustis and Walz, References 31,42,14,30), metal rod calorimeters (Chamberlain, Tomich, Hrycak and Comfort et al., References 8,35,39,44) and actual steam calorimeters (Kezios, Reference 2). Additional measuring techniques are possible with mass transfer analogy (cf. Koopman and Sparrow, and Schrader, References 54,70).

While most investigators considered heat transfer from impinging jets to flat surfaces, Walz (Reference 30) concerned himself also with effects of axially symmetric jets impinging on spheres externally, and two-dimensional jets impinging externally on cylinders, with the long axis of the jet impinging parallel to the cylinder axis.

Some investigators considered jet impingement effects on heat transfer for angles other than normal (Perry, Schauer and Eustis, References 40 and

14), and several investigators pursued the effect of cross-flow on heat transfer from the impinging jets, extremely important for many applications. Heat transfer resulting from a single jet in cross-flow has been investigated by Bouchez and Goldstein (Reference 71).

Wall jet heat-transfer predictions, all being of semi-empirical kind, have been found generally to agree with experimental results. The theoretical approaches employ the consideration of two flow layers: an inner layer along the wall and an outer layer assumed to behave as a free jet. Both integral and finite difference solutions have been obtained by various investigators (cf. References 14,33,39,73).

## 5.2 Arrays of Jets

5.2.1 Arrays of round jets represent a configuration which is perhaps the most applicable for cooling of large areas. Care must be taken to properly space the holes so that jet interference is minimized, with spacings of from 4 to 6 hole diameters (about 5 to 2 percent of free-flow area). Results of tests reported in Reference 7 showed that the best correlation could be obtained by use of the velocity of arrival, and the center-to-center hole spacing as the characteristic values, as a single jet yielded better heat-transfer results than one jet in an array of jets. Relatively good agreement was found by comparing the results of several different investigations of arrays of round jets. The effect of crossflow on heat transfer may be accounted for by means of an experimentally obtained correction parameter; generally, a decrease in heat transfer resulted from increasing the crossflow to the jet. Another way the crossflow effects could be accounted for in determining the average Stanton number, by including in the correlation a power of the ratio of the crossflow to the jet flow.

5.2.2 Arrays of Slot Jets - Data for two distinct arrays of slot jets showed that the identity of each jet was preserved at small slot-to-plate spacings, but jet interaction before impingement occurred at larger spacings. The average data obtained for multiple slots have been successfully correlated. It appears that slot jets may be more immune than round jets to the effects of crossflow. When a crossflow, equal to as much as 60 percent of the air flow, was superimposed on an array of slot jets, no reduction in heat transfer was found.

### 5.3 Recommendations

It appears that a great amount of generally reliable information exists already on the use of the individual jets (of the round and slot type) for cooling, or heating of small areas with high intensity heat fluxes (spot cooling or heating). For cooling of larger flat areas, arrays of jets appear to be appropriate and their performance under laboratory conditions can be described with a reasonable degree of accuracy. The point is made here, however, that in actual applications, where arraying of a large number of jets becomes mandatory, the effects of crossflow may substantially reduce the effect of impingement cooling. Also proper evaluation of the effect of spent air becomes here a problem of utmost importance.

While there is generally satisfactory information on impingement cooling/heating of flat and concave surfaces, virtually no information exists on cooling or heating of large convex surfaces. Experimental work should be recommended, therefore, in this area, as well as additional theoretical studies. It is believed that, in the near future, there will be a substantial number of new contributions in the area of impinging jet heat transfer, of theoretical and practical significance. New technological development will be generated by proper organization and channelling of research related to impinging jets in general, and to heat transfer from impinging jets in particular.

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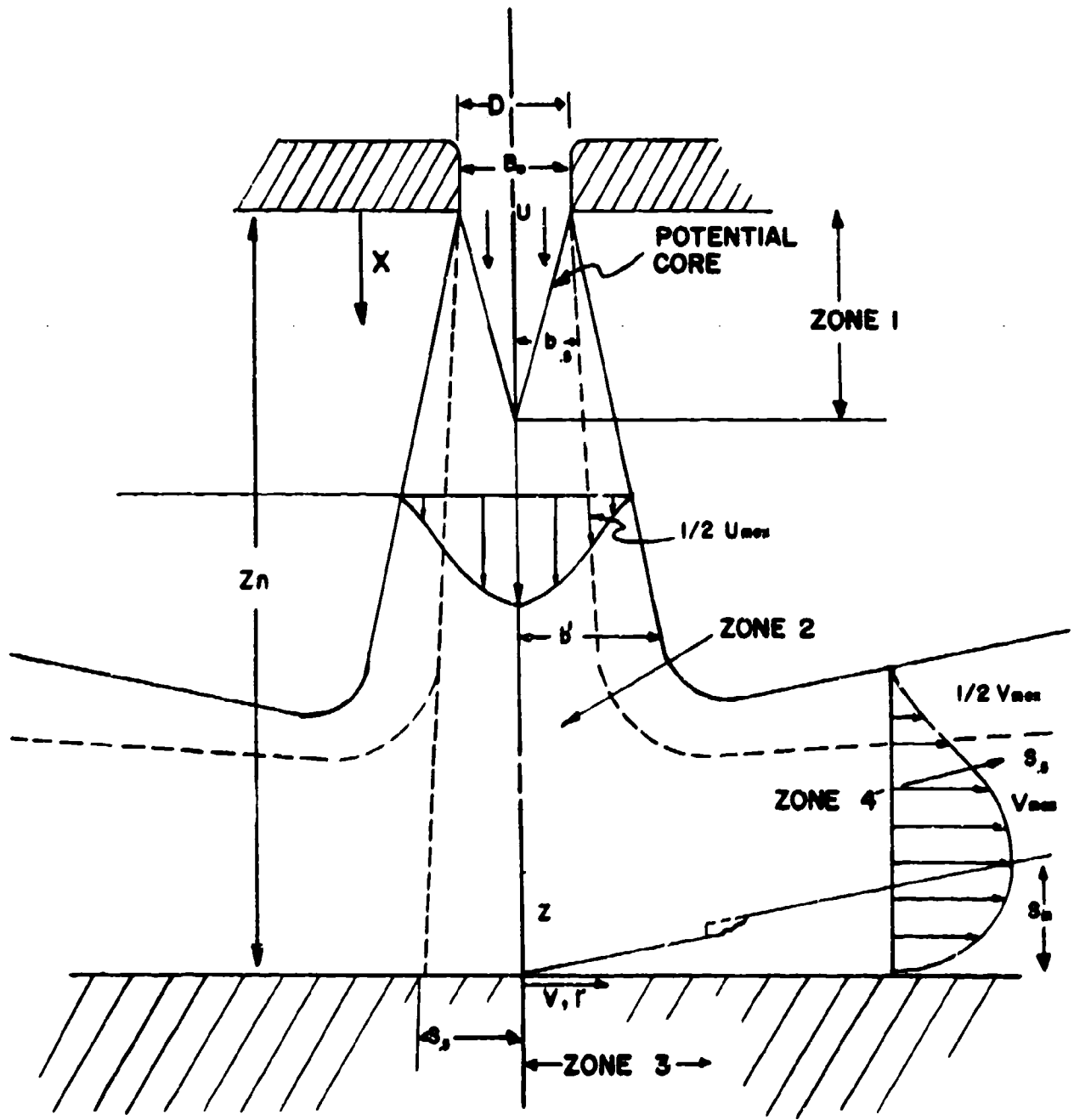


FIGURE 1 Flow Geometry of Impinging Jet.

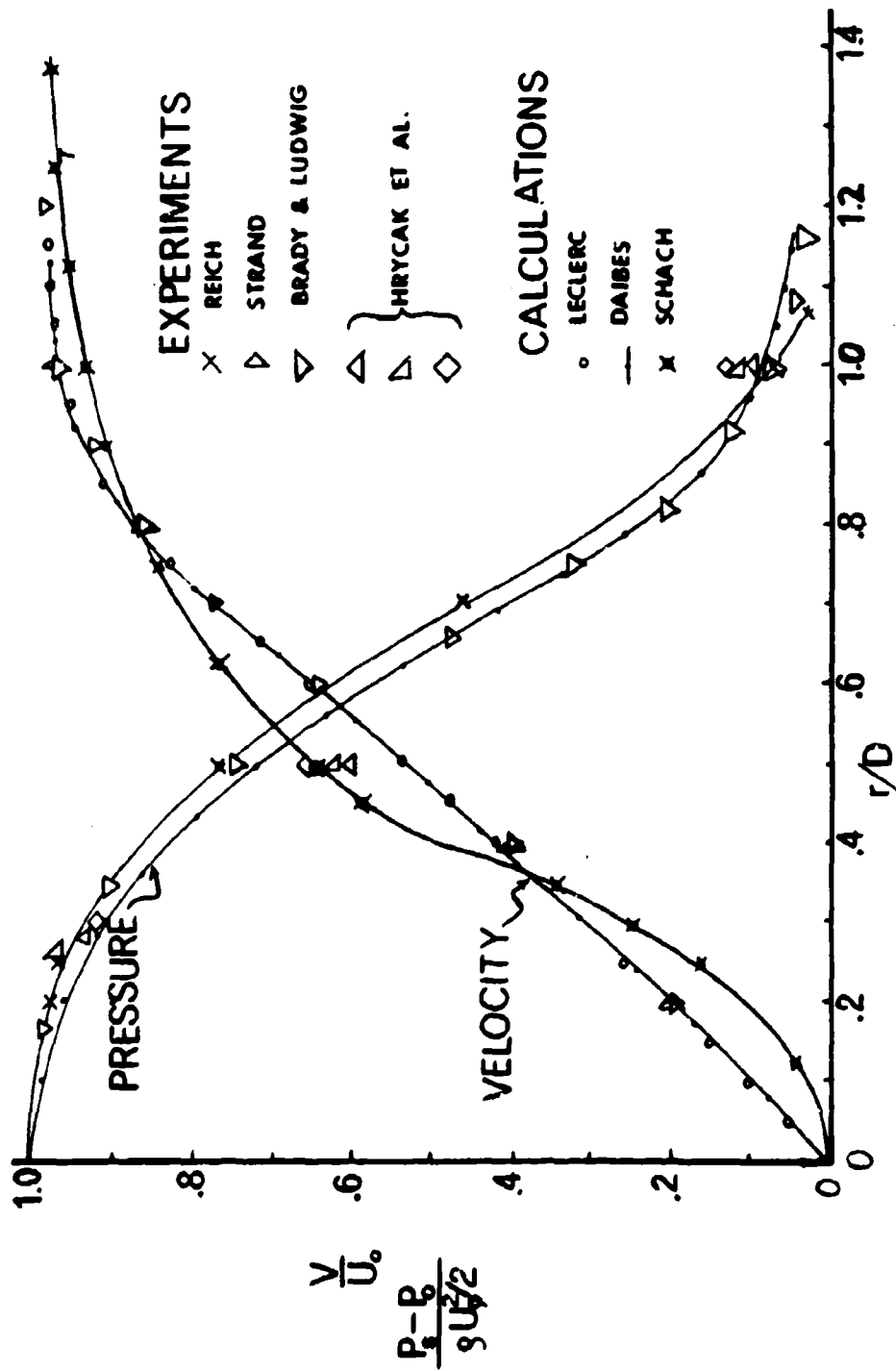


FIGURE 2 Potential Flow Solutions at Stagnation Point; Comparison with Experimental Results.

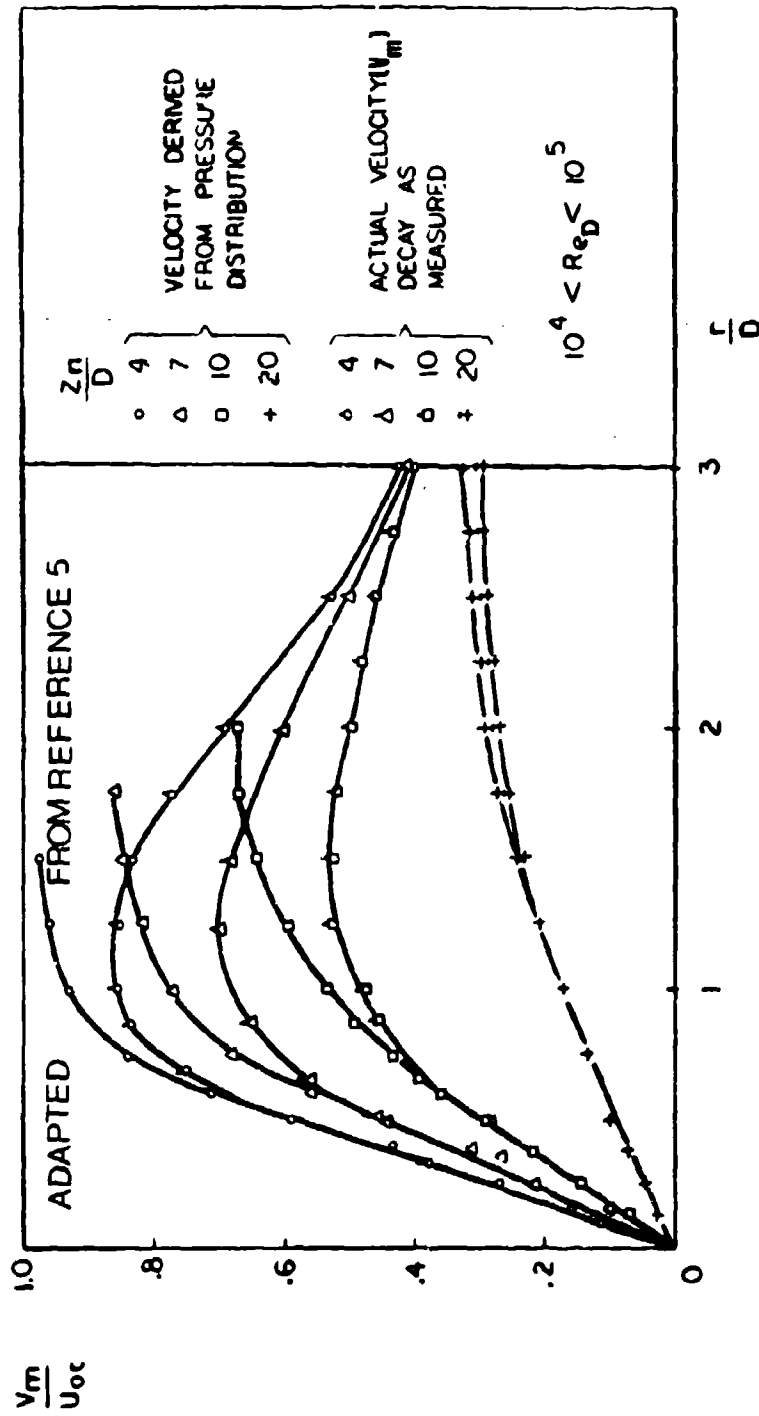


FIGURE 3 Distribution of Velocity Near Stagnation Point.

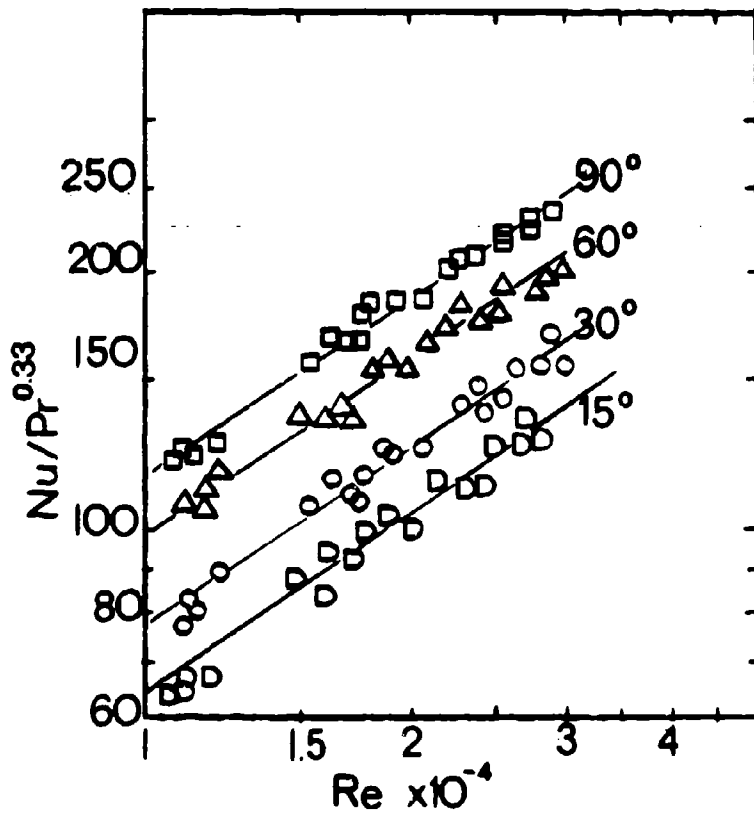


FIGURE 4 Effect of Impingement Angle on Heat Transfer (After Perry, Reference 40).

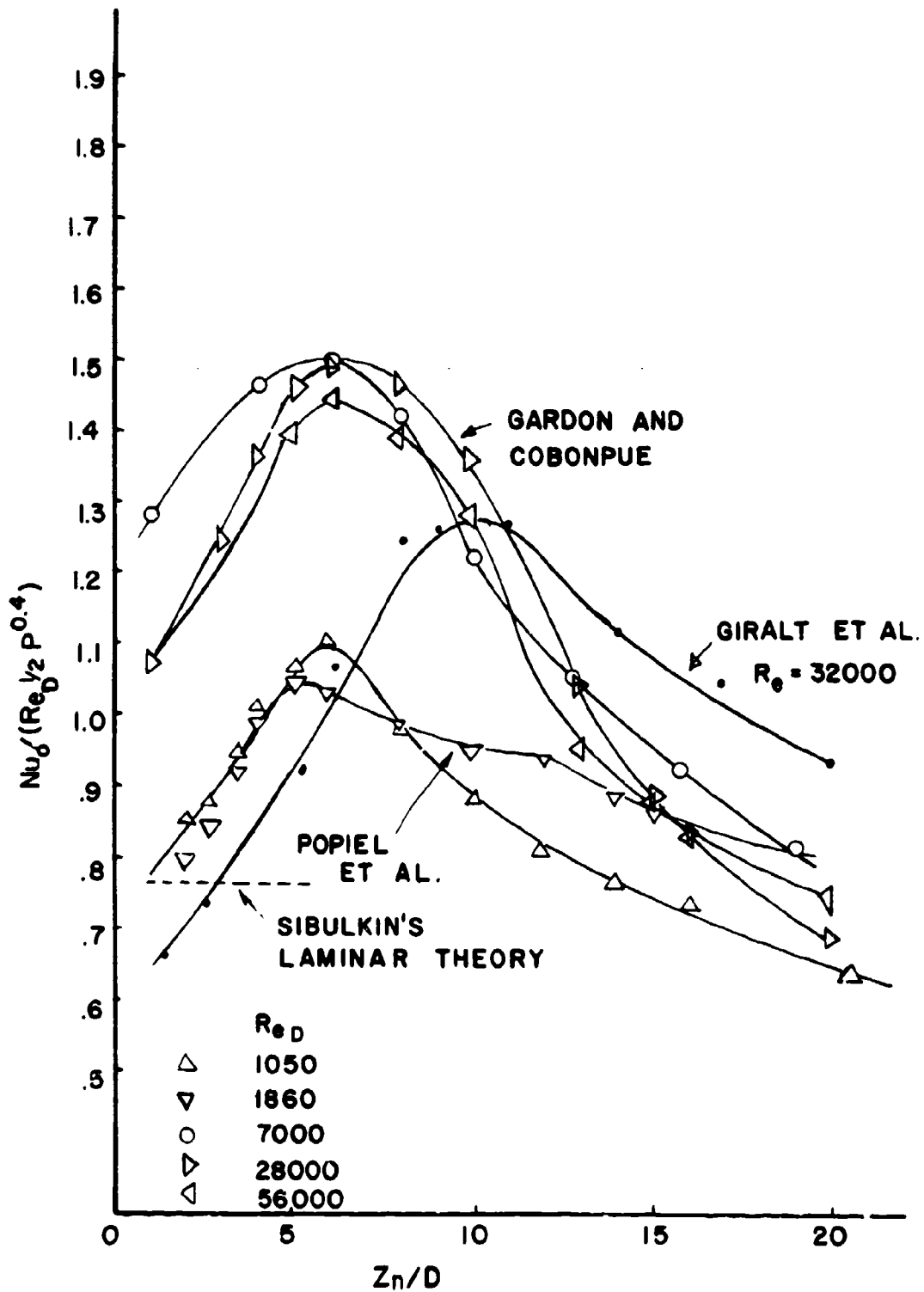


FIGURE 5 Comparison of Popiel's Results with Those of Other Investigators.

Fig.

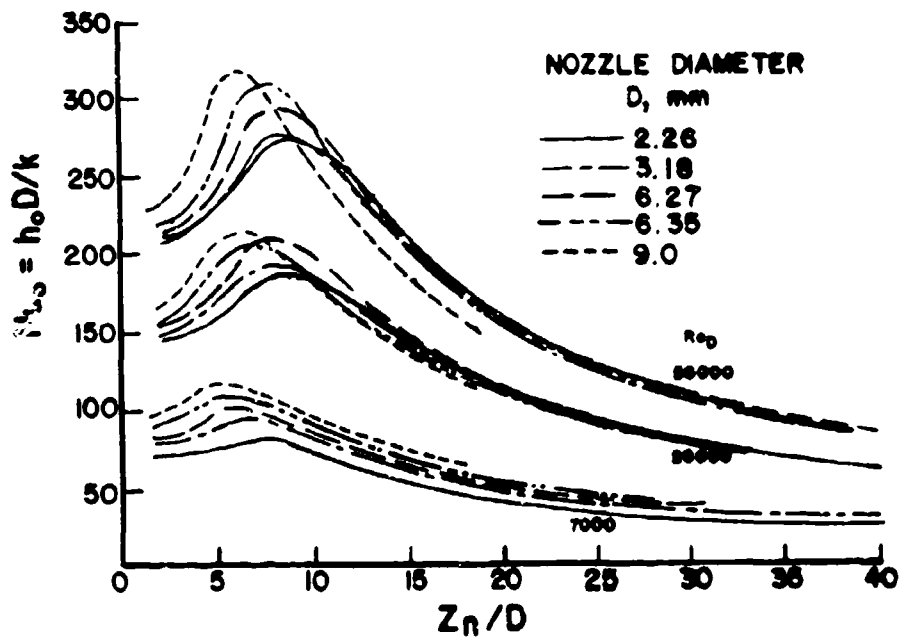


FIGURE 6 Gardon and Cobonpue, Stagnation Point Heat Transfer Results.



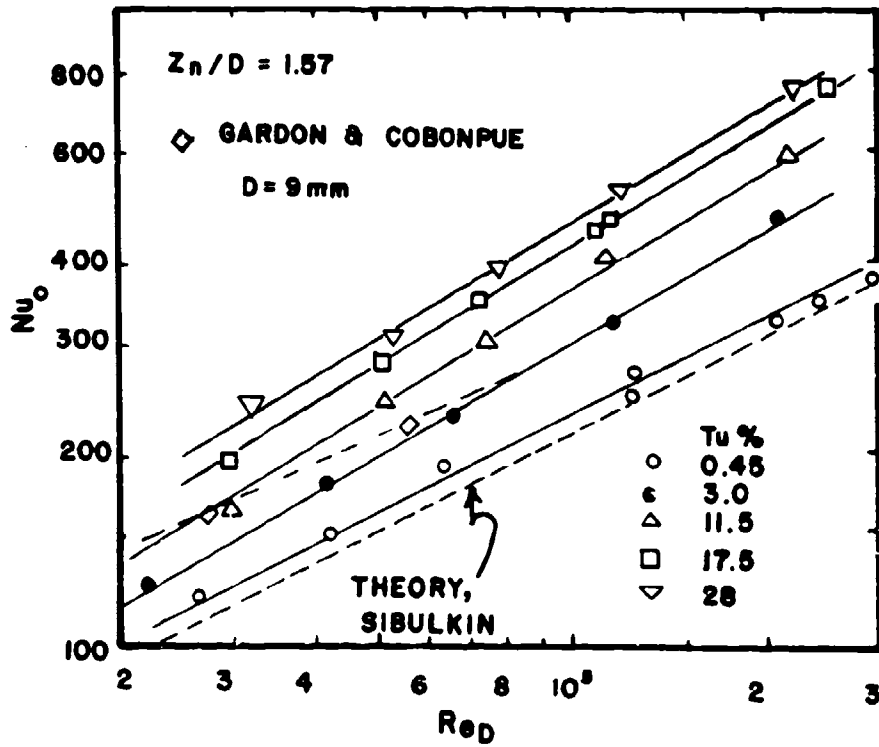


FIGURE 7 Dyban and Mazur's Stagnation Point Heat Transfer Measurements Carried Out on Turbulized Jets.

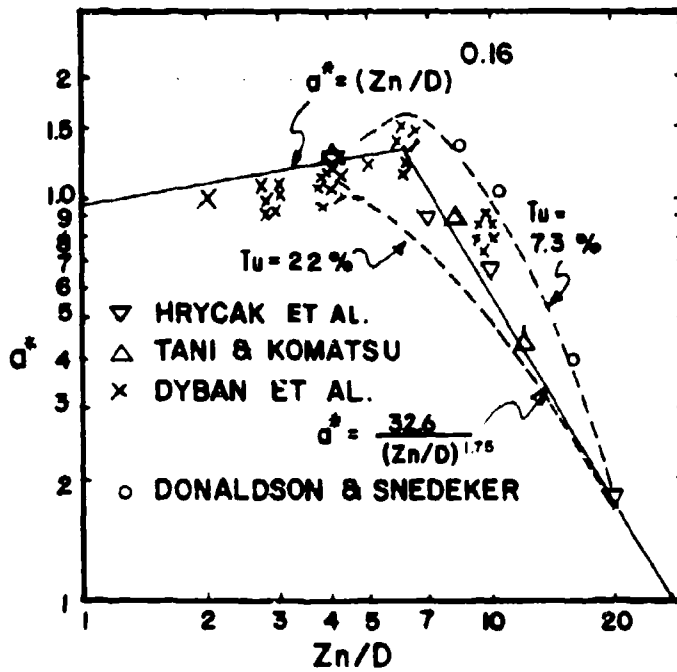


FIGURE 8 Distribution of Dimensionless Velocity Gradient at Stagnation Point; Comparison of Results of Various Investigators.

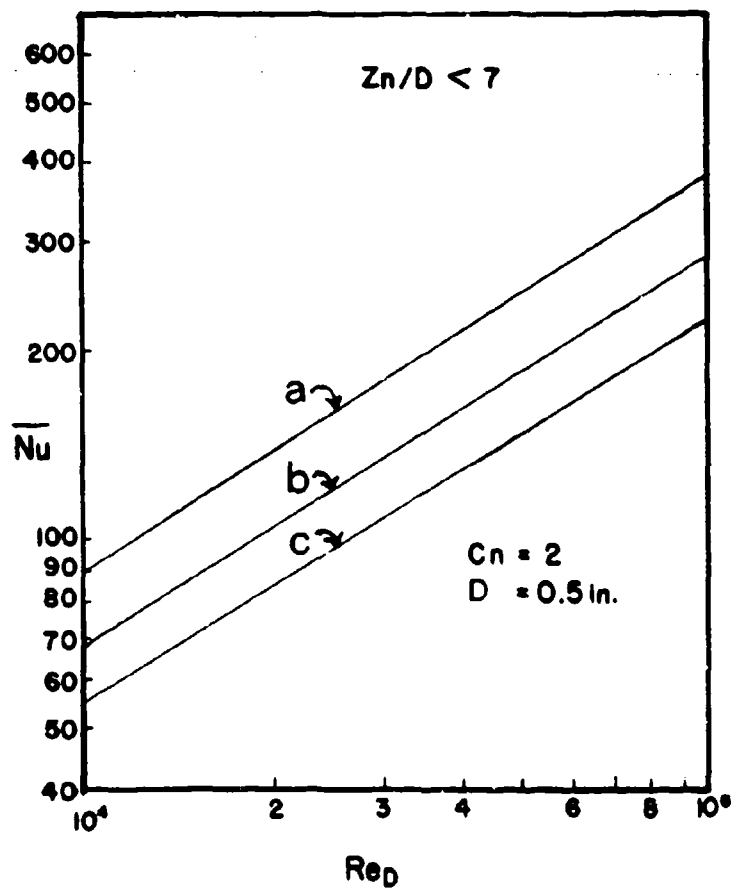


FIGURE 9 Comparison of Heat Transfer Results for Rows of Round Jets and Arrays of Round Jets:  
 (a) Kercher and Tabakoff, (b) Hrycak, (c) Gardon and Cobonpue

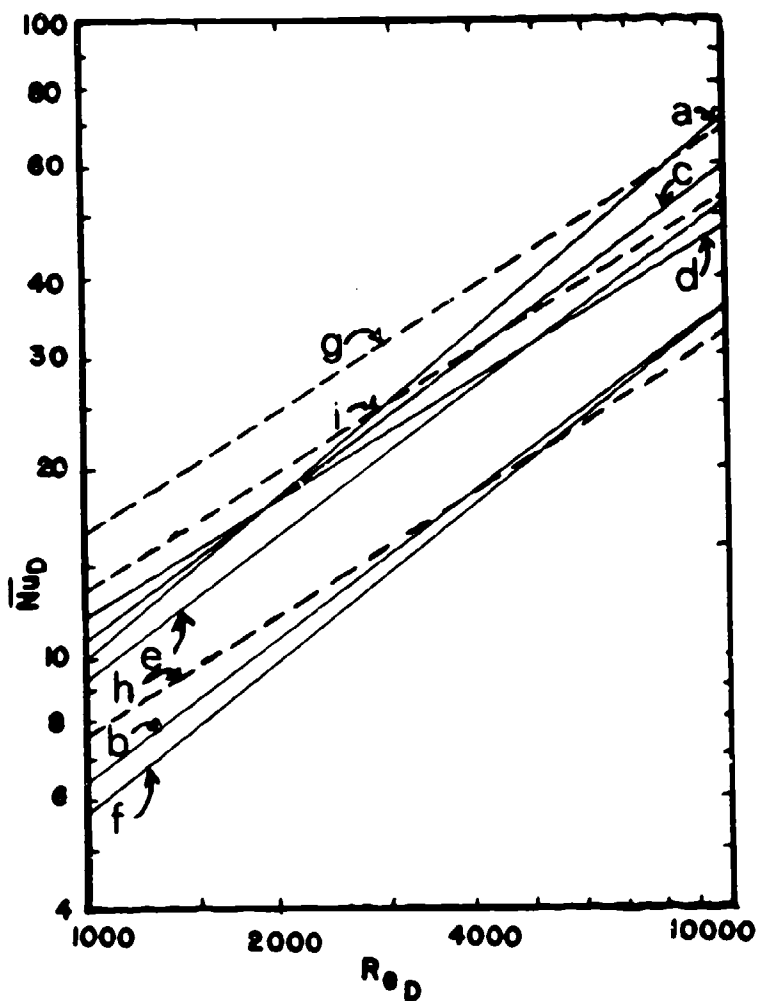


FIGURE 10 Comparison of Average Heat Transfer Results for Rows and Arrays of Round Jets: — Results adapted from Reference 60 (small diameters throughout).

- |  |   |
|--|---|
| (a) Huang, Reference 42                | --- Hrycak, Reference 39                            |
| (b) Gardon and Cobonpue, Reference 7   | (g) $D = 12.7 \text{ mm}$ , $C_n = 50.8 \text{ mm}$ |
| (c) Freidman and Mueller, Reference 1  | (h) $D = 3.18 \text{ mm}$ , $C_n = 50.8 \text{ mm}$ |
| (d) Ott, Reference 50                  | --- Gardon and Cobonpue, Reference 7                |
| (e) Hilgeroth, Reference 51            | (i) $D = 12.7 \text{ mm}$ , $C_n = 50.8 \text{ mm}$ |
| (f) Kercher and Tabakoff, Reference 52 |   |

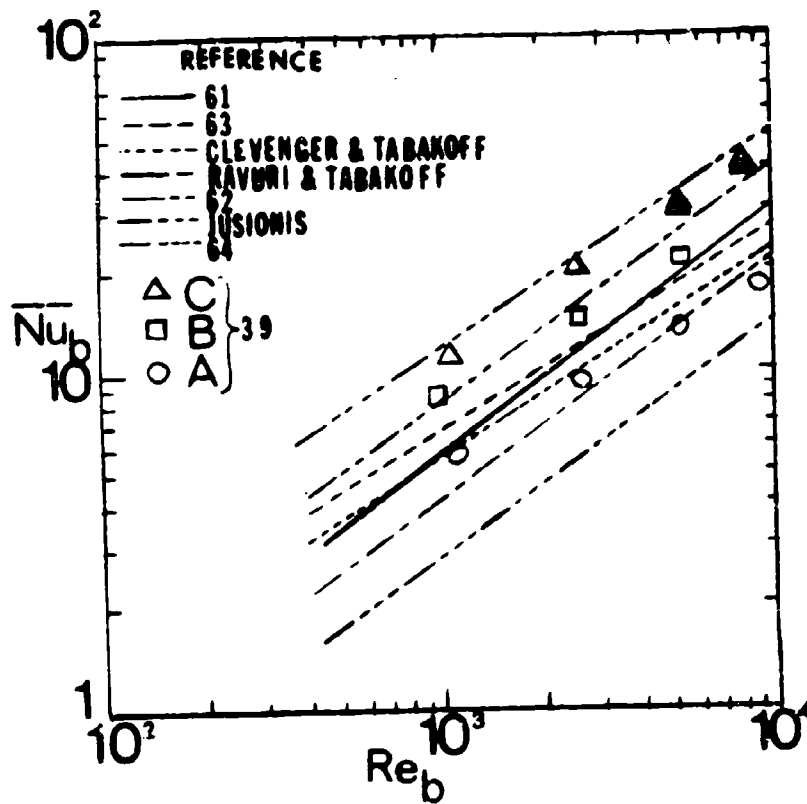


FIGURE 11 Additional Comparison of 10 by 5 in. Semicylindrical Plate Results with Results of Other Investigators.

A = 3.18 mm, B = 6.35 mm, C = 9.52 mm

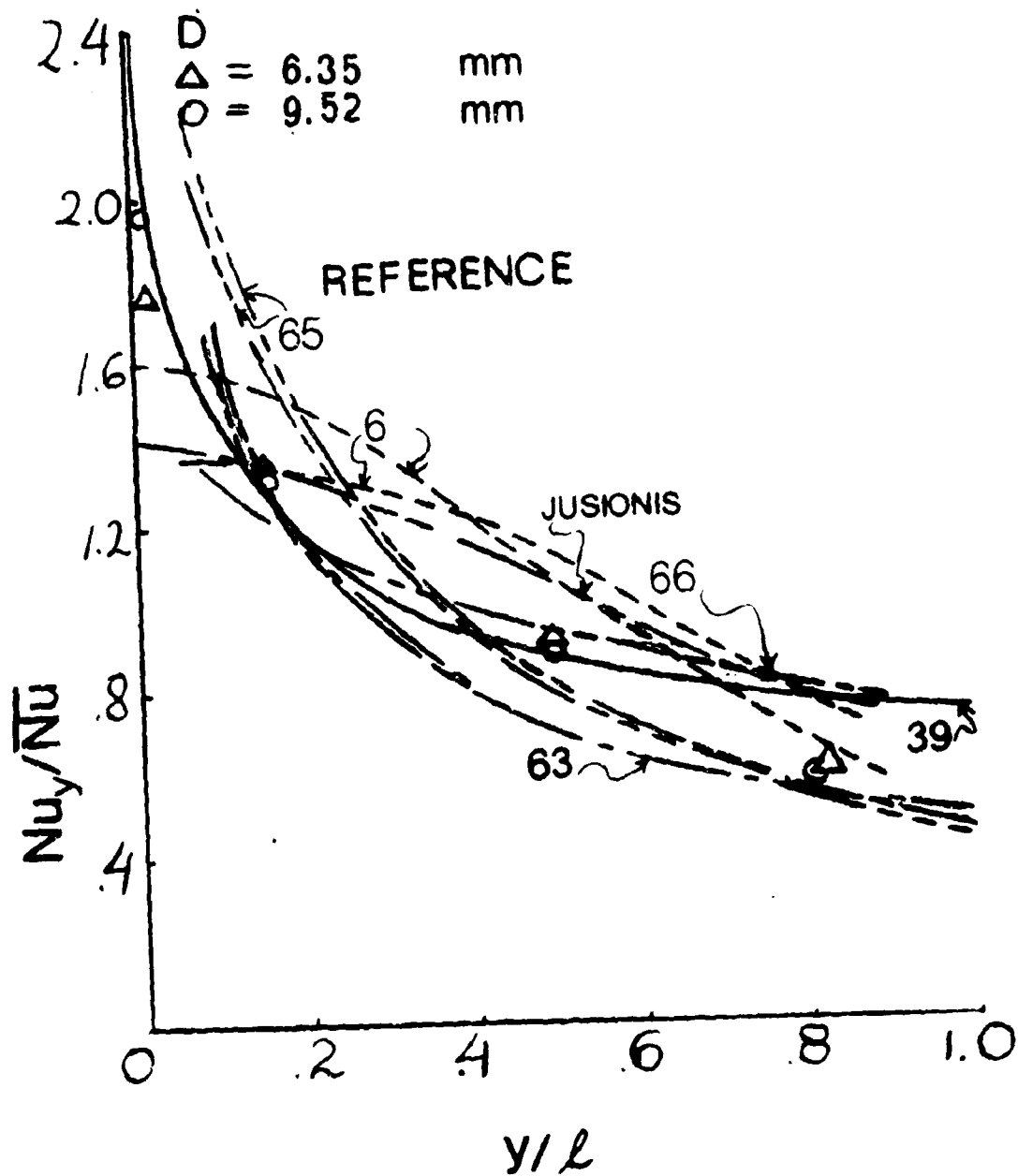


FIGURE 12 Local Distribution of Heat Transfer Coefficients, 10 by 5 in. Semicylindrical Plate (Reference 39); Comparison with Results of Other Investigators.

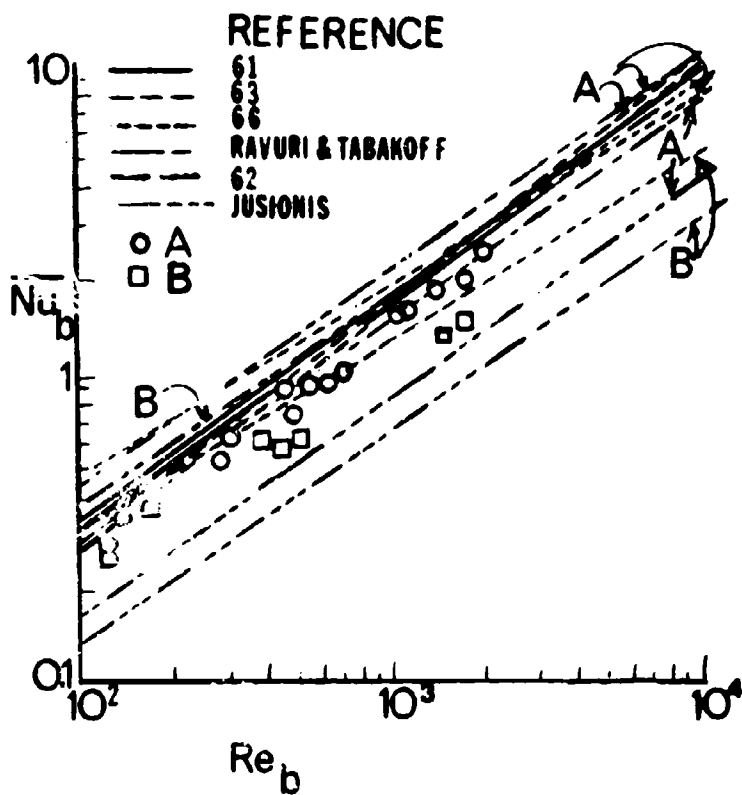


FIGURE 13 Results Obtained with 1.0 by 0.5 in. Semicylindrical Plate (Reference 39). Comparison with Results of Other Investigators.

A:  $Z_n = 12.7$  mm, B:  $Z_n = 25.4$  mm