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CHARACTERISTICS OF IMAGES DERIVED FROM FACTORABLE POLYNOMIALS

Scripps Institute of Oceanography

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Air Force Systems Command
Griffiss Air Force Base, New York 13441

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Image reconstruction techniques exist which use as the input the autocorrelation function of the image. The uniqueness of the reconstructed image has been questioned. This report considers a class of images which are ambiguous in that many different images can be generated which have the same autocorrelation function. It is shown that two dimensional images generated from factorable polynomials which have unity power factors are basically uninteresting images. However, by using (Cont'd)

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non-unity power factors, images with structure results. This structure varies significantly among images with the same autocorrelation function. Finally, it is pointed out that this factoring process is equivalent to convolution of the image with a pair of delta functions, leading to the ability of generating ambiguous images from functions that are not completely factorable. Examples are shown. The conclusion of the work is that caution must be exercised in interpretation of images reconstructed from autocorrelation functions.

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CHARACTERISTICS OF IMAGES DERIVED FROM FACTORABLE POLYNOMIALS

INTRODUCTION

In recent years image processing techniques have been developed which use as an input the autocorrelation function of an image or, equivalently, the modulus square of the Fourier transform of the image. Fienup's[1] technique is a good example of this approach. In his technique an iteration procedure is used which alternates between spatial and frequency domains, enforcing known constraints in each domain. In the spatial domain the constraints are non-negativity of the image brightness values and limited spatial extent. In the frequency domain the constraint is that the processed image Fourier transform must have a modulus equal to the original modulus. This processing has had a promising degree of success on the limited number of images processed thus far.

In the absence of the non-negativity constraint there are an infinite number of objects which can produce the same autocorrelation function since the phase component of the Fourier transform can be varied at will without affecting the image autocorrelation function while affecting the image significantly, usually creating negative image values. The enforcement of non-negativity of the image, however, limits the number of images which have the same autocorrelation function.

An important question is how many non-negative images can produce the same autocorrelation function? There seems to be no general theory for two dimensional images. There has been some work done on specialized two dimensional images. Fried[2], following the work of Bruck and Sodin[3], has developed a method for developing specialized two dimensional images which have a high degree of ambiguity in the sense that a large number of images can produce the same autocorrelation function and satisfy the non-negative constraint. These are images derived from completely factorable polynomial functions. Left unanswered in Fried's work was the question of whether such specialized images are of any interest. Are they similar to real world images of interest and are the ambiguous images different from each other in an interesting way? This report addresses these questions.

REVIEW OF THEORY

Let A_{pq} represent the brightness value of an image in p, q coordinates. We define a polynomial function f(u,v) as

$$f(u,v) = \sum_{pq} A_{pq} u^p v^q. \qquad (1)$$

If the polynomial has the characteristics of being completely factorable, then f(u,v) can also be written as

$$f(u,v) = \prod_{r,s} (u + a_r)(v + b_s). \qquad (2)$$

In Fried's and Bruck and Sodin's work the form used was $(u - a_r)$. The change of sign here is for bookkeeping convenience in computer programs.

The general image $A_{p,q}$ will not have the characteristic of being completely factorable. We are dealing here with special cases. Assuming the image is completely factorable then Equations (1) and (2) can be equated.

$$\sum_{pq} A_{pq} u^p v^q = \prod_{r,s} (u + a_r)(v + b_s). \qquad (3)$$

Given the set of (a_{r}, b_{s}) the image values A_{pq} are found by multiplying the factors on the right hand side of Eq.3, and equating the coefficients of like powers of u and v. For example let the factorable polynomials be

$$f(u,v) = (u + 1)(u + 3)(v + 2)(v + 4)$$

$$= 24 + 32u + 8u^2$$
$$+ 18v + 24uv + 6u^2v$$
$$+ 3v^2 + 4uv^2 + v^2v^2. \qquad (4)$$

The resulting brightness values of the image A_{pq} are:

p→

q↓		u^0	u^1	u^2
	v^0	24	32	8
	v^1	18	24	6
	v^2	3	4	1

To ensure non-negativity of the image the roots a_r, b_s must be positive reals, or, if complex, occur in complex conjugate pairs with positive real components.

Fried has shown that the autocorrelation function of an image which is completely factorable is not unique, i.e., there are many non-negative images, all completely factorable, which produce the same autocorrelation function. These images are found by replacing any combination of the roots by their reciprocals, a_r^{-1}, b_s^{-1}. For example, for the factors $(u + a_1)(v + b_1)$ the complete set of images would be generated by

$$(u + a_1)(v + b_1)$$
$$(u + a_1^{-1})(v + b_1)$$
$$(u + a_1)(v + b_1^{-1})$$
$$(u + a_1^{-1})(v + b_1^{-1}). \qquad (5)$$

For a polynomial of R u factors and S v factors an image of dimensions (R + 1) by (S + 1) results. By flipping the roots 2^{S+T} images can be generated, all having the same autocorrelation function. One of these is the original, the other its 180° rotation so that $2^{S+T} - 2 = 2^{S+T-1}$ ambiguous images can be formed.

It should be noted that the image developed from this method are very constrained in terms of degrees of freedom. The images are really the product of two one dimensional images, each extended in a direction orthogonal to its variable. Let f(p) be the one dimensional image obtained from the expansion of the u roots and f(q) the one dimensional image obtained from the expansion of the v roots. the two dimensional image f(p,q) is then

$$f(p,q) = f(p)\ f(q) . \qquad (6)$$

PRELIMINARY IMAGES

A program was written by B. Fahy of the visibility laboratory which allows specification of the u and v factors or roots, which roots are to be flipped, and computes the resulting image, normalized so that the maximum value is 10,000. Results are shown in Tables 1 to 25 for a variety of root distributions. In all cases the image brightness values decrease monotonically from the maximum value. As the roots increase in value from unity the images become smaller, that is, fall off more rapidly. They also shift towards an edge and become more asymmetrical. Flipping half of the roots changes the location and the symmetry. In general the images produced are uninteresting.

FACTORS AS POINT SPREAD FUNCTIONS

The process of multiplying a polynomial, whose coefficients represent an image, by a factor $(u + a_n)$ is equivalent to convolving that image with a pair of delta functions. For example the polynomial

$$f_{n-1}(u) = A_0 + A_1u + A_2u^2 + \cdots A_{n-1}u^{n-1} \qquad (7)$$

when multiplied by $(u + a_n)$ will consist of two components. The first will be the original polynomial with each coefficient multiplied by a_n. The second component will be the original polynomial with the power of each variable increased by one, which corresponds to a right shift of the image associated with the polynomial coefficients. In terms of the image we have convolved the original image with pair of delta functions, one at the origin with a weighting of a_n and the other shifted to the right by one pixel, with a weighting of unity.

If the polynomial is multiplied by $(u^{\delta} + a_n)$ where δ is an integer, the effect on the one-dimensional image associated with the polynomial is to convolve it with a pair of delta functions, now separated by δ pixels. This allows more interesting images to be constructed, that is, images with more structure. In addition, the ambiguous images can be significantly different from one another. This is illustrated in

Fig.1 which is a graph of two one-dimensional funcitons. The change in going from one image to the other was caused by the flipping of one root.

The convolution concept leads to a less constrained method of generating ambiguous images. It will now be shown that any arbitrary one-dimensional function can be used to produce a pair of ambiguous images by convolving the image with a pair of delta functions. The initial function does not need to be a factorable function.

Let f(p) be a general one dimensional real function. Convolving f(p) with the delta function pair produces a new function g(p), which is the original function weighted by (a) plus the original function shifted by δ.

$$g(p) = a\, f(p) + f(p-\delta) \tag{8}$$

The autocorrelation function of g(p), C(i), is

$$C(i) = \frac{\sum_{p} g(p)\; g(p-i)}{\sum g(p)^2} \tag{9}$$

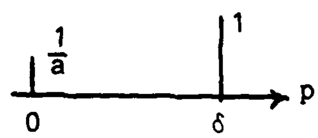
We substitute Eq.(8) into Eq.(9) and use the relationship of

$$\sum_{p} f(p) f(p-i) = \sum f(p-\delta) f(p-\delta-i) \tag{10}$$

to obtain

$$C(i) = \frac{(1+a^2)\sum_p f(p)f(p-i) + a\sum[f(p-\delta)f(p-i) + f(p)f(p-\delta-i)]}{(1+a^2)\sum f^2(p) + 2a\sum f(p)f(p-\delta)} \qquad (11)$$

If we multiply top and bottom of Eq(11) by $1/a^2$, which leaves C(i) unchanged, the resulting equation is identical to that obtained by replacing a by 1/a. In other words convolving the function f(p) with the two point spread functions



produces images with the same autocorrelation function. Again f(p) can be any function, it need not be derived from a factorable polynomial.

Figure 2 shows a more complex set of ambiguous images. The starting image was an array of seven dots, arranged in the pattern found on a domino. The factor, *i.e.*, the horizontal shifts and weightings used to generate the ambiguous images are shown on the figure. The four images have identical autocorrelation functions. This was verified computationally. The four images have the same basic structure in terms of numbers and locations of dots but the relative brightness of the dots are quite different. The convolving process could be done in the vertical direction to produce even more complex ambiguous images.

DISCUSSION AND CONCLUSIONS

The work shown here has demonstrated that: a) Factorable polynomials using unity power factors, such as $(u + a_n)$, result in smooth images which change little apart from spatial shifts when ambiguous images are generated; b) When greater than unity power factors are used, such as $(u^{\gamma(n)} + a_n)$, structured images can result and the ambiguous images can be quite different from one another; c) The concept of multiplication of a polynomial by a factor is equivalent to convolution of the image represented by the coefficients of the polynomial, with a pair of delta functions. This concept leads to the result that the starting function for generating ambiguous images need not be a factorable polynomial but can be any non negative function. That is, convolution of any starting function with appropriate pairs of delta functions can lead to different images which have the same autocorrelation function. An example of four ambiguous images using the technique, shown in Fig. 2, shows that while basic shape and structure was the same for all images, considerable variation in the internal brightness values occurred between the images.

Images which are ambiguous are very specialized, at least as generated by this technique. Such images may not occur frequently in practice. However, if image reconstruction is made from the autocorrelation function, the possibility of the reconstructed image being ambiguous should be considered.

At the present there apparently is no theory available which allows determination of whether a given autocorrelation function is unambiguous, that is, whether the autocorrelation function could have been produced by only one non negative image. Instead, the theory deals with the reverse of the problem: methods of generating images which have either ambiguous or unambiguous autocorrelation functions.

Perhaps the approach for now is to rely on the reconstruction techniques themselves for answer to the question of uniqueness. For a wide class of images, with and without noise, do the techniques provide reconstruction of the image within limits set by resolution of the optical system and sensor noise? Comparisons should be made to other image processing approaches.

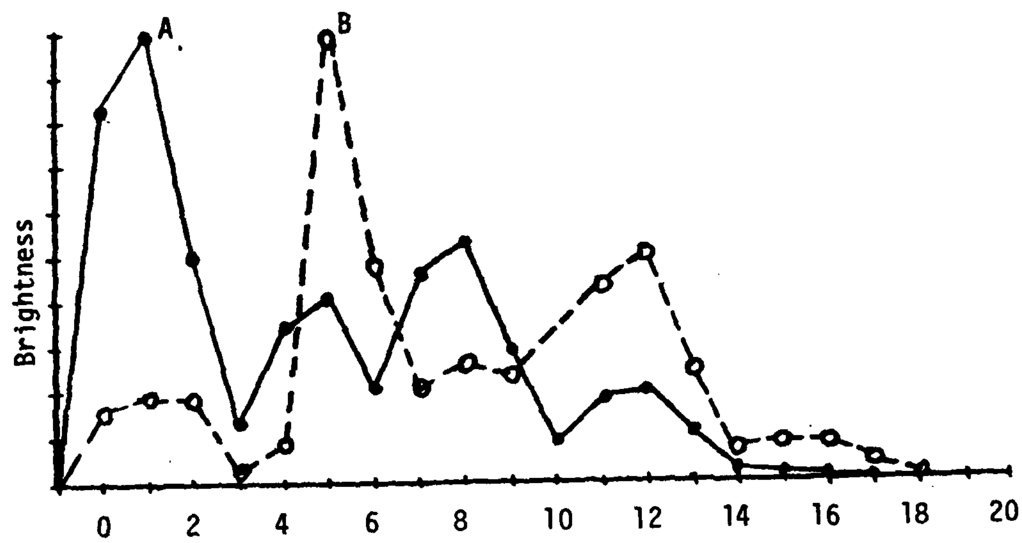


Figure 1. Two factorable functions having the same autocorrelation function.

A factors: $(u+1)^5\ (u^4+5)(u^4+5)(u^7+2)$

B factors: $(u+1)^5\ (u^4+5)(u^4+.2)(u^7+2)$

flipped

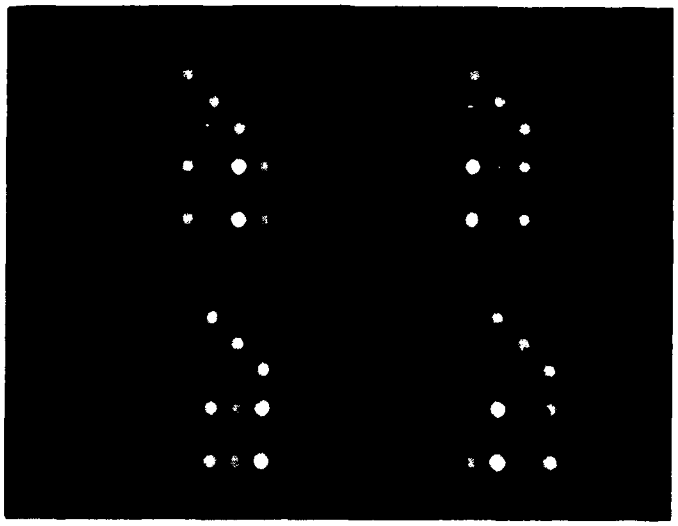
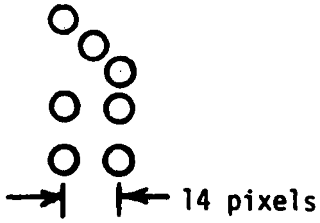


Figure 2. Ambiguous images having the same autocorrelation function. Starting function was a pattern of seven dots arranged as on a domino. Let $f(u,v)$ be the polynomial (not factorable) representing the starting dot pattern. The images are:

Top Left : $f(u,v)(u^7+2)(u^{14}+2)$
Top Right : $f(u,v)(u^7+2)(u^{14}+.5)$
Bottom Left : $f(u,v)(u^7+.5)(u^{14}+2)$
Bottom Right: $f(u,v)(u^7+.5)(u^{14}+.5)$.

Original pattern:



TABLES 1 to 25

These tables represent the brightness values of two dimensional images made from completely factorable polynomials whose factors are of unity power.

TABLE 1.

All roots = 1.

ABOUT TO READ IMAGE# 1

0	1	6	18	32	39	32	18	6	1	0
1	15	70	188	330	396	330	188	70	15	1
6	70	318	850	1488	1785	1488	850	318	70	6
18	188	850	2267	3968	4762	3968	2267	850	188	18
32	330	1488	3968	6945	8334	6945	3968	1488	330	32
39	396	1785	4762	8334	10000	8334	4762	1785	396	39
32	330	1488	3968	6945	8334	6945	3968	1488	330	32
18	188	850	2267	3968	4762	3968	2267	850	188	18
6	70	318	850	1488	1785	1488	850	318	70	6
1	15	70	188	330	396	330	188	70	15	1
0	1	6	18	32	39	32	18	6	1	0

UROOTS =
1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000

VROOTS =
1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000

---------------------- END IMAGE 1 -------------------

TABLE 2.

All roots = 2.

ABOUT TO READ IMAGE# 1

44	222	500	666	583	350	145	41	7	0	0
222	1111	2500	3333	2916	1750	729	208	39	4	0
500	2500	5625	7500	6563	3937	1640	468	87	9	0
666	3333	7500	10000	8750	5250	2187	625	117	13	0
583	2916	6563	8750	7657	4594	1914	546	102	11	0
350	1750	3937	5250	4594	2756	1148	328	61	6	0
145	729	1640	2187	1914	1148	478	136	25	2	0
41	208	468	625	546	328	136	39	7	0	0
7	39	87	117	102	61	25	7	1	0	0
0	4	9	13	11	6	2	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

UROOTS =
2.0000 2.0000 2.0000 2.0000 2.0000 2.0000 2.0000 2.0000 2.0000 2.0000

VROOTS =
2.0000 2.0000 2.0000 2.0000 2.0000 2.0000 2.0000 2.0000 2.0000 2.0000

---------------------- END IMAGE 1 --------------------

TABLE 3.

1/2 of roots = 2.

1/2 of roots = 1/2.

Compare to TABLE 2.

ABOUT TO READ IMAGE# 1

0	0	2	8	16	20	16	8	2	0	0
0	6	35	109	208	257	208	109	35	6	0
2	35	193	591	1126	1391	1126	591	193	35	2
8	109	591	1807	3442	4251	3442	1807	591	109	8
16	208	1126	3442	6556	8097	6556	3442	1126	208	16
20	257	1391	4251	8097	10000	8097	4251	1391	257	20
16	208	1126	3442	6556	8097	6556	3442	1126	208	16
8	109	591	1807	3442	4251	3442	1807	591	109	8
2	35	193	591	1126	1391	1126	591	193	35	2
0	6	35	109	208	257	208	109	35	6	0
0	0	2	8	16	20	16	8	2	0	0

UROOTS =
0.5000 0.5000 0.5000 0.5000 0.5000 2.0000 2.0000 2.0000 2.0000 2.0000

VROOTS =
0.5000 0.5000 0.5000 0.5000 0.5000 2.0000 2.0000 2.0000 2.0000 2.0000

---------------------- END IMAGE 1 --------------------

TABLE 4.

All roots = 4.

ABOUT TO READ IMAGE# 1

1264	3160	3555	2370	1037	311	64	9	0	0	0
3160	7902	8889	5926	2592	777	162	23	2	0	0
3555	8889	10000	6667	2916	875	182	26	2	0	0
2370	5926	6667	4444	1944	583	121	17	1	0	0
1037	2592	2916	1944	850	255	53	7	0	0	0
311	777	875	583	255	76	15	2	0	0	0
64	162	182	121	53	15	3	0	0	0	0
9	23	26	17	7	2	0	0	0	0	0
0	2	2	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

UROOTS =
4.0000 4.0000 4.0000 4.0000 4.0000 4.0000 4.0000 4.0000 4.0000 4.0000

VROOTS =
4.0000 4.0000 4.0000 4.0000 4.0000 4.0000 4.0000 4.0000 4.0000 4.0000

---------------------- END IMAGE 1 -------------------

TABLE 5.

1/2 of roots = 4,
1/2 of roots = 1/4.
Compare to TABLE 4.

ABOUT TO READ IMAGE# 1

0	0	0	0	2	3	2	0	0	0	0
0	0	4	19	49	69	49	19	4	0	0
0	4	37	170	435	608	435	170	37	4	0
0	19	170	781	2002	2796	2002	781	170	19	0
2	49	435	2002	5126	7159	5126	2002	435	49	2
3	69	608	2796	7159	10000	7159	2796	608	69	3
2	49	435	2002	5126	7159	5126	2002	435	49	2
0	19	170	781	2002	2796	2002	781	170	19	0
0	4	37	170	435	608	435	170	37	4	0
0	0	4	19	49	69	49	19	4	0	0
0	0	0	0	2	3	2	0	0	0	0

UROOTS =
0.2500 0.2500 0.2500 0.2500 0.2500 4.0000 4.0000 4.0000 4.0000 4.0000

VROOTS =
0.2500 0.2500 0.2500 0.2500 0.2500 4.0000 4.0000 4.0000 4.0000 4.0000

---------------------- END IMAGE 1 --------------------

TABLE 6.

All roots = 8.

```
ABOUT TO READ IMAGE#           1

 6400  8000  4500  1500   328    49     5     0     0     0     0
 8000 10000  5625  1875   410    61     6     0     0     0     0
 4500  5625  3164  1054   230    34     3     0     0     0     0
 1500  1875  1054   351    76    11     1     0     0     0     0
  328   410   230    76    16     2     0     0     0     0     0
   49    61    34    11     2     0     0     0     0     0     0
    5     6     3     1     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0

UROOTS =
8.0000  8.0000  8.0000  8.0000  8.0000  8.0000  8.0000  8.0000  8.0000  8.0000
VROOTS =
8.0000  8.0000  8.0000  8.0000  8.0000  8.0000  8.0000  8.0000  8.0000  8.0000

----------------------- END IMAGE           1 --------------------
```

TABLE 7.

1/2 of roots = 8,

1/2 of roots = 1/8.

Compare to TABLE 6.

ABOUT TO READ IMAGE# 1

0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	4	8	4	1	0	0	0
0	0	2	17	73	143	73	17	2	0	0
0	1	17	141	610	1191	610	141	17	1	0
0	4	73	610	2629	5127	2629	610	73	4	0
0	8	143	1191	5127	10000	5127	1191	143	8	0
0	4	73	610	2629	5127	2629	610	73	4	0
0	1	17	141	610	1191	610	141	17	1	0
0	0	2	17	73	143	73	17	2	0	0
0	0	0	1	4	8	4	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0

UROOTS =
0.1250 0.1250 0.1250 0.1250 0.1250 8.0000 8.0000 8.0000 8.0000 8.0000

VROOTS =
0.1250 0.1250 0.1250 0.1250 0.1250 8.0000 8.0000 8.0000 8.0000 8.0000

---------------------- END IMAGE 1 ------------------

TABLE 8.

All roots = 16.

```
ABOUT TO READ IMAGE#            1

 10000  6250  1757   292    32     2     0     0     0     0     0
  6250  3906  1098   183    20     1     0     0     0     0     0
  1757  1098   309    51     5     0     0     0     0     0     0
   292   183    51     8     0     0     0     0     0     0     0
    32    20     5     0     0     0     0     0     0     0     0
     2     1     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0

UROOTS =
16.0000 16.0000 16.0000 16.0000 16.0000 16.0000 16.0000 16.0000 16.0000 16.0000
VROOTS =
16.0000 16.0000 16.0000 16.0000 16.0000 16.0000 16.0000 16.0000 16.0000 16.0000

---------------------- END IMAGE             1 -------------------
```

TABLE 9.

1/2 of roots = 16,
1/2 of roots = 1/16.
Compare to TABLE 8.

ABOUT TO READ IMAGE# 1

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	6	22	6	0	0	0	0
0	0	0	13	107	362	107	13	0	0	0
0	0	6	107	873	2955	873	107	6	0	0
0	0	22	362	2955	10000	2955	362	22	0	0
0	0	6	107	873	2955	873	107	6	0	0
0	0	0	13	107	362	107	13	0	0	0
0	0	0	0	6	22	6	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

URDOTS =
0.0625 0.0625 0.0625 0.0625 0.0625 16.0000 16.0000 16.0000 16.0000 16.0000

VRDOTS =
0.0625 0.0625 0.0625 0.0625 0.0625 16.0000 16.0000 16.0000 16.0000 16.0000

---------------------- END IMAGE 1 --------------------

TABLE 10.

All roots = 32.

```
ABOUT TO READ IMAGE#          1
10000  3125   439    36     2     0     0     0     0     0     0
 3125   976   137    11     0     0     0     0     0     0     0
  439   137    19     1     0     0     0     0     0     0     0
   36    11     1     0     0     0     0     0     0     0     0
    2     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
UROOTS =
32.0000 32.0000 32.0000 32.0000 32.0000 32.0000 32.0000 32.0000 32.0000 32.0000 32.0000
VROOTS =
32.0000 32.0000 32.0000 32.0000 32.0000 32.0000 32.0000 32.0000 32.0000 32.0000 32.0000
---------------------- END IMAGE          1 --------------------
```

TABLE 11.

1/2 of roots = 32.

1/2 of roots = 1/32.

Compare to TABLE 10.

```
ABOUT TO READ IMAGE#          1
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     2     0     0     0     0     0
    0     0     0     0    14    95    14     0     0     0     0
    0     0     0    14   237  1540   237    14     0     0     0
    0     0     2    95  1540 10000  1540    95     2     0     0
    0     0     0    14   237  1540   237    14     0     0     0
    0     0     0     0    14    95    14     0     0     0     0
    0     0     0     0     0     2     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
UROOTS =
0.0313  0.0313  0.0313  0.0313  0.0313 32.0000 32.0000 32.0000 32.0000 32.0000
VROOTS =
0.0313  0.0313  0.0313  0.0313  0.0313 32.0000 32.0000 32.0000 32.0000 32.0000
---------------------- END IMAGE           1 -------------------
```

TABLE 12.

All roots = 64.

ABOUT TO READ IMAGE# 1

10000	1562	109	4	0	0	0	0	0	0	0
1562	244	17	0	0	0	0	0	0	0	0
109	17	1	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

UROOTS =
64.0000 64.0000 64.0000 64.0000 64.0000 64.0000 64.0000 64.0000 64.0000 64.0000

VROOTS =
64.0000 64.0000 64.0000 64.0000 64.0000 64.0000 64.0000 64.0000 64.0000 64.0000

---------------------- END IMAGE 1 -------------------

TABLE 13.

1/2 of roots = 64.

1/2 of roots = 1/64.

Compare to TABLE 12.

ABOUT TO READ IMAGE# 1

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	24	1	0	0	0	0
0	0	0	1	60	778	60	1	0	0	0
0	0	0	24	778	10000	778	24	0	0	0
0	0	0	1	60	778	60	1	0	0	0
0	0	0	0	1	24	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

UROOTS =
0.0156 0.0156 0.0156 0.0156 0.0156 64.0000 64.0000 64.0000 64.0000 64.0000

VROOTS =
0.0156 0.0156 0.0156 0.0156 0.0156 64.0000 64.0000 64.0000 64.0000 64.0000

-------------------- END IMAGE 1 ------------------

TABLE 14.

Linearly increasing roots, 1 to 10.

ABOUT TO READ IMAGE# 1

809	2371	2845	1876	762	201	35	4	0	0	0
2371	6946	8334	5495	2233	589	103	11	0	0	0
2845	8334	10000	6594	2679	707	123	14	1	0	0
1876	5495	6594	4348	1766	466	81	9	0	0	0
762	2233	2679	1766	717	189	33	3	0	0	0
201	589	707	466	189	50	8	1	0	0	0
35	103	123	81	33	8	1	0	0	0	0
4	11	14	9	3	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

UROOTS =
1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000

VROOTS =
1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000

---------------------- END IMAGE 1 -------------------

TABLE 15.

Linearly increasing roots, 1 to 10,
first half flipped. Compare to TABLE 14.

ABOUT TO READ IMAGE# 1

0	0	5	15	23	17	6	1	0	0	0
0	13	78	234	360	279	106	21	2	0	0
5	78	478	1424	2187	1694	643	130	14	0	0
15	234	1424	4240	6512	5044	1916	388	43	2	0
23	360	2187	6512	10000	7747	2943	597	66	3	0
17	279	1694	5044	7747	6001	2279	462	51	2	0
6	106	643	1916	2943	2279	866	175	19	1	0
1	21	130	388	597	462	175	35	3	0	0
0	2	14	43	66	51	19	3	0	0	0
0	0	0	2	3	2	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

UROOTS =
1.0000 0.5000 0.3333 0.2500 0.2000 6.0000 7.0000 8.0000 9.0000 10.0000

VROOTS =
1.0000 0.5000 0.3333 0.2500 0.2000 6.0000 7.0000 8.0000 9.0000 10.0000

--------------------- END IMAGE 1 --------------------

TABLE 16.

Linearly increasing roots, 1 to 10,
second half flipped. Compare to TABLE 14.

ABOUT TO READ IMAGE# 1

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	2	3	2	0	0	0
0	0	0	3	19	51	66	43	14	2	0
0	0	3	35	175	462	597	388	130	21	1
0	1	19	175	866	2279	2943	1916	643	106	6
0	2	51	462	2279	6001	7747	5044	1694	279	17
0	3	66	597	2943	7747	10000	6512	2187	360	23
0	2	43	388	1916	5044	6512	4240	1424	234	15
0	0	14	130	643	1694	2187	1424	478	78	5
0	0	2	21	106	279	360	234	78	13	0
0	0	0	1	6	17	23	15	5	0	0

URDOTS =
1.0000 2.0000 3.0000 4.0000 5.0000 0.1667 0.1429 0.1250 0.1111 0.1000

VROOTS =
1.0000 2.0000 3.0000 4.0000 5.0000 0.1667 0.1429 0.1250 0.1111 0.1000

---------------------- END IMAGE 1 --------------------

TABLE 17.

Linearly increasing roots, 10 to 100.

```
ABOUT TO READ IMAGE#             1

10000  2929   351    23     0     0     0     0     0     0     0
 2929   857   102     6     0     0     0     0     0     0     0
  351   102    12     0     0     0     0     0     0     0     0
   23     6     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     0     0     0

 VROOTS =
10.0000 20.0000 30.0000 40.0000 50.0000 60.0000 70.0000 80.0000 90.0000 $100.00

 VROOTS =
10.0000 20.0000 30.0000 40.0000 50.0000 60.0000 70.0000 80.0000 90.0000 $100.00

 ---------------------- END IMAGE          1 --------------------
```

TABLE 18.

Linearly increasing roots, 10 to 100,
first half flipped. Compare to TABLE 17.

ABOUT TO READ IMAGE# 1

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	6	0	0	0	0	0
0	0	0	3	41	185	11	0	0	0	0
0	0	1	41	511	2262	144	3	0	0	0
0	0	6	185	2262	10000	640	16	0	0	0
0	0	0	11	144	640	40	1	0	0	0
0	0	0	0	3	16	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

~~VROOTS =~~
0.1000 0.0500 0.0333 0.0250 0.0200 60.0000 70.0000 80.0000 90.0000 $100.00

VROOTS =
~~0.1000~~ 0.0500 ~~0.0333~~ ~~0.0250~~ 0.0200 60.0000 70.0000 80.0000 ~~90.0000~~ ~~$100.00~~

-------------------- END IMAGE 1 ------------------

TABLE 19.

Linearly increasing roots, 10 to 100,
second half flipped. Compare to TABLE 17.

ABOUT TO READ IMAGE# 1

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	7	1	0	0	0	0
0	0	0	0	20	452	102	8	0	0	0
0	0	0	7	452	10000	2268	185	7	0	0
0	0	0	1	102	2268	514	42	1	0	0
0	0	0	0	8	185	42	3	0	0	0
0	0	0	0	0	7	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

URDOTS =
10.0000 20.0000 30.0000 40.0000 50.0000 0.0167 0.0143 0.0125 0.0011 0.0010

VROOTS =
10.0000 20.0000 30.0000 40.0000 50.0000 0.0167 0.0143 0.0125 0.0011 0.0010

---------------------- END IMAGE 1 --------------------

TABLE 20.

Exponentially increasing roots, $\sqrt{2}$ steps.

ABOUT TO READ IMAGE# 1

500	1654	2236	1622	697	185	30	3	0	0	0
1654	5473	7398	5364	2305	612	101	10	0	0	0
2236	7398	10000	7251	3116	828	137	14	0	0	0
1622	5364	7251	5258	2259	600	99	10	0	0	0
697	2305	3116	2259	971	258	42	4	0	0	0
185	612	828	600	258	68	11	1	0	0	0
30	101	137	99	42	11	1	0	0	0	0
3	10	14	10	4	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

UROOTS =
1.0000 1.4142 2.0000 2.8284 4.0000 5.6568 8.0000 11.3137 16.0000 22.6274

VROOTS =
1.0000 1.4142 2.0000 2.8284 4.0000 5.6568 8.0000 11.3137 16.0000 22.6274

---------------------- END IMAGE 1 ------------------

TABLE 21.

Exponentially increasing roots, $\sqrt{2}$ steps, first half flipped. Compare to TABLE 20.

ABOUT TO READ IMAGE# 1

0	5	26	59	70	42	12	1	0	0	0
5	68	311	700	827	501	146	21	1	0	0
26	311	1415	3186	3762	2280	665	99	7	0	0
59	700	3186	7172	8469	5132	1497	224	17	0	0
70	827	3762	8469	10000	6060	1767	264	20	0	0
42	501	2280	5132	6060	3672	1071	160	12	0	0
12	146	665	1497	1767	1071	312	46	3	0	0
1	21	99	224	264	160	46	7	0	0	0
0	1	7	17	20	12	3	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

UROOTS =
1.0000 0.7071 0.5000 0.3535 0.2500 5.6568 8.0000 11.3137 16.0000 22.6274

VROOTS =
1.0000 0.7071 0.5000 0.3535 0.2500 5.6568 8.0000 11.3137 16.0000 22.6274

--------------------- END IMAGE 1 ------------------

TABLE 22.

Exponentially increasing roots, $\sqrt{2}$ steps, second half flipped. Compare to TABLE 20.

ABOUT TO READ IMAGE# 1

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	3	12	20	17	7	1	0
0	0	0	7	46	160	264	224	99	21	1
0	0	3	46	312	1071	1767	1497	665	146	12
0	0	12	160	1071	3672	6060	5132	2280	501	42
0	0	20	264	1767	6060	10000	8469	3762	827	70
0	0	17	224	1497	5132	8469	7172	3186	700	59
0	0	7	99	665	2280	3762	3186	1415	311	26
0	0	1	21	146	501	827	700	311	68	5
0	0	0	1	12	42	70	59	26	5	0

VROOTS =
1.0000 1.4142 2.0000 2.8284 4.0000 0.1768 0.1250 0.0884 0.0625 0.0442

VROOTS =
1.0000 1.4142 2.0000 2.8284 4.0000 0.1768 0.1250 0.0884 0.0625 0.0442

---------------------- END IMAGE 1 --------------------

TABLE 23.

Exponentially increasing roots, x2 steps.

ABOUT TO READ IMAGE# 1

2505	5005	3330	947	125	7	0	0	0	0	0
5005	10000	6654	1893	250	15	0	0	0	0	0
3330	6654	4427	1260	166	10	0	0	0	0	0
947	1893	1260	358	47	3	0	0	0	0	0
125	250	166	47	6	0	0	0	0	0	0
7	15	10	3	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

UROOTS =
1.0000 2.0000 4.0000 8.0000 16.0000 32.0000 64.0000 $128.00 $256.00 $512.00

VROOTS =
1.0000 2.0000 4.0000 8.0000 16.0000 32.0000 64.0000 $128.00 $256.00 $512.00

------------------------ END IMAGE 1 --------------------

TABLE 24.

Exponentially increasing roots, x2 steps.
First half flipped. *Compare to TABLE 23.*

ABOUT TO READ IMAGE# 1

0	0	0	2	4	2	0	0	0	0	0
0	2	22	92	150	83	4	0	0	0	0
0	22	229	925	1514	842	47	0	0	0	0
2	92	925	3736	6112	3400	191	3	0	0	0
4	150	1514	6112	10000	5563	312	5	0	0	0
2	83	842	3400	5563	3094	173	3	0	0	0
0	4	47	191	312	173	9	0	0	0	0
0	0	0	3	5	3	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

UROOTS =
1.0000 0.5000 0.2500 0.1250 0.0625 32.0000 64.0000 $128.00 $256.00 $512.00

VROOTS =
1.0000 0.5000 0.2500 0.1250 0.0625 32.0000 64.0000 $128.00 $256.00 $512.00

---------------------- END IMAGE 1 -------------------

TABLE 25.

Exponentially increasing roots, x2 steps.
second half flipped. Compare to TABLE 23.

ABOUT TO READ IMAGE# 1

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	3	5	3	0	0	0
0	0	0	0	9	173	312	191	47	4	0
0	0	0	3	173	3094	5563	3400	842	83	2
0	0	0	5	312	5563	10000	6112	1514	150	4
0	0	0	3	191	3400	6112	3736	925	92	2
0	0	0	0	47	842	1514	925	229	22	0
0	0	0	0	4	83	150	92	22	2	0
0	0	0	0	0	2	4	2	0	0	0

UROOTS =
1.0000 2.0000 4.0000 8.0000 16.0000 0.0313 0.0156 0.0078 0.0039 0.0019

VROOTS =
1.0000 2.0000 4.0000 8.0000 16.0000 0.0313 0.0156 0.0078 0.0039 0.0019

---------------------- END IMAGE 1 ------------------

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