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ON DIFFERENTIATING HYPEROSCULATORY ERROR TERMS. (U)

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ON DIFFERENTIATING HYPEROSCILLATORY
ERROR TERMS.

by

J. Barzilai

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ABSTRACT

We generalize Ralston's result on differentiating error terms to the hyperoscillatory and nonpolynomial cases.

KEY WORDS

Interpolation errors, Hyperoscillatory interpolation, Nonpolynomial interpolation

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We generalize Ralston's result [5] on differentiating error terms to the hyperosculatory and nonpolynomial cases. This result is used implicitly in [6,7] and explicitly in [1,2,3].

Theorem 1. Let $f: R \rightarrow R$ have continuous derivatives in an interval J . Let $x_j \in J$ $j=0,1,\dots,n$. Let $P(x)$ be the unique hyperosculatory interpolation polynomial of degree $< r = \sum_{j=0}^n \gamma_j$ satisfying

$$(1) \quad \left. \begin{aligned} P^{(k_j)}(x_j) &= f^{(k_j)}(x_j) \\ k_j &= 0, 1, \dots, \gamma_j - 1, \gamma_j \geq 1 \end{aligned} \right\} \quad j = 0, 1, \dots, n,$$

with $x_k \neq x_\ell$ for $k \neq \ell$. Then for $x \in J$, $x \neq x_j$ $j=0,1,\dots,n$, we have

$$(2) \quad f'(x) - P'(x) = \frac{f^{(r)}(\xi)}{r!} W'(x) + \frac{f^{(r+1)}(\eta)}{(r+1)!} W(x),$$

where $W(x) = \prod_{j=0}^n (x-x_j)^{\gamma_j}$, and ξ and η are in the interval spanned by

$x, x_0, x_1, \dots, x_n$.

Proof. The error term in the hyperosculatory interpolation (1) is given by

$$(3) \quad f(x) - P(x) = \frac{f^{(r)}(\xi)}{r!} W(x)$$

with ξ and $W(x)$ as above (see e.g. [4]). To simplify notation we will prove the theorem for the case $\gamma_j = s$, $j=0,\dots,n$.

Let $\psi_{ij}(x)$ be the unique interpolation polynomial of degree $< r$ satisfying

$$\psi_{ij}^{(k)}(x_\ell) = \delta_{ik} \delta_{j\ell}$$

for $j, l = 0, \dots, n$; $i, k = 0, \dots, s-1$. Then

$$P(x) = \sum_{i=0}^{s-1} \sum_{j=0}^n f^{(i)}(x_j) \psi_{ij}(x),$$

and

$$(4) \quad \frac{d}{dx} \frac{f(x)}{W(x)} = \sum_{i=0}^{s-1} \sum_{j=0}^n f^{(i)}(x_j) \frac{d}{dx} \frac{\psi_{ij}(x)}{W(x)} + \frac{d}{dx} \frac{f^{(r)}(\xi)}{r!}.$$

Let $x_{n+1} \neq x_j$ $j = 0, \dots, n$ and define interpolation polynomials $\bar{\psi}_{ij}(x)$ by

$$\bar{\psi}_{ij}^{(k)}(x_l) = \psi_{ij}^{(k)}(x_l) = \delta_{ij} \delta_{jl},$$

$$\bar{\psi}_{ij}(x_{n+1}) = 0,$$

$$\bar{\psi}_{0, n+1}^{(k)}(x_l) = 0,$$

$$\bar{\psi}_{0, n+1}(x_{n+1}) = 1,$$

for $j, l = 0, \dots, n$ and $i, k = 0, \dots, s-1$.

The polynomial $\bar{P}(x) = \sum_{i=0}^{s-1} \sum_{j=0}^n f^{(i)}(x_j) \bar{\psi}_{ij}(x) + f(x_{n+1}) \bar{\psi}_{0, n+1}(x)$ of degree $\leq r$

satisfies (1) and $P(x_{n+1}) = f(x_{n+1})$. Therefore we have

$$f(x) - \bar{P}(x) = W(x)(x - x_{n+1}) \frac{f^{(r+1)}(\eta)}{(r+1)!} \text{ with } \eta \text{ in the interval spanned by } x, x_0, \dots, x_n.$$

The uniqueness of the interpolation polynomials implies

$$(5) \quad \left\{ \begin{array}{l} \bar{\psi}_{ij}(x) = \psi_{ij}(x) - \frac{\psi_{ij}(x_{n+1})}{W(x_{n+1})} W(x) \text{ for } j = 0, \dots, n; i = 0, \dots, s-1, \\ \bar{\psi}_{0, n+1}(x) = \frac{W(x)}{W(x_{n+1})}. \end{array} \right.$$

Hence

$$\frac{f(x)}{W(x)} = \sum_{i=0}^{s-1} \sum_{j=0}^n f^{(i)}(x_j) \frac{\bar{\psi}_{ij}(x)}{W(x)} + \frac{f(x_{n+1})}{W(x_{n+1})} + \frac{f^{(r+1)}(\eta)}{(r+1)!} (x - x_{n+1})$$

and

$$(6) \quad \frac{1}{x - x_{n+1}} \left(\frac{f(x)}{W(x)} - \frac{f(x_{n+1})}{W(x_{n+1})} \right) = \sum_{i=0}^{s-1} \sum_{j=0}^n f^{(i)}(x_j) \frac{1}{x - x_{n+1}} \cdot \frac{\bar{\psi}_{ij}(x)}{W(x)} + \frac{f^{(r+1)}(\eta)}{(r+1)!}.$$

For $x \neq x_j$ $j = 0, \dots, n$ we now let $x_{n+1} \rightarrow x$. Since $W(x_{n+1}) \neq 0$ and $\bar{\psi}_{ij}(x_{n+1}) = 0$ we have

$$(7) \quad \lim_{x_{n+1} \rightarrow x} \frac{1}{x - x_{n+1}} \cdot \frac{\bar{\psi}_{ij}(x)}{W(x)} = \lim_{x_{n+1} \rightarrow x} \frac{1}{x - x_{n+1}} \left(\frac{\bar{\psi}_{ij}(x)}{W(x)} - \frac{\bar{\psi}_{ij}(x_{n+1})}{W(x_{n+1})} \right) \\ = \frac{d}{dx} \frac{\bar{\psi}_{ij}(x)}{W(x)} = \frac{d}{dx} \frac{\psi_{ij}(x)}{W(x)}$$

where the last equality holds by (5).

From (6) and (7) we have

$$(8) \quad \frac{d}{dx} \frac{f(x)}{W(x)} = \sum_{i=0}^{s-1} \sum_{j=0}^k f^{(i)}(x_j) \frac{d}{dx} \frac{\psi_{ij}(x)}{W(x)} + \frac{f^{(r+1)}(\eta)}{(r+1)!}.$$

Comparing (4) and (8) we conclude

$$(9) \quad \frac{d}{dx} \frac{f^{(r)}(\xi)}{r!} = \frac{f^{(r+1)}(\eta)}{(r+1)!}.$$

Differentiating (3) using (9) finally yields (2). $\square$

Theorem 2. If $T \in C^{(r+1)}(J)$ is a hyperoscillatory interpolating function for f satisfying (1) (i.e. (1) holds with P replaced by T), then Theorem 1 holds with (2) replaced by

$$(2') \quad f'(x) - T'(x) = \frac{f^{(r)}(\xi) - T^{(r)}(\xi)}{r!} W'(x) + \frac{f^{(r+1)}(\eta) - T^{(r+1)}(\eta)}{(r+1)!} W(x) .$$

Proof. Replace f in Theorem 1 by $h = f - T$, and note that the unique interpolation polynomial satisfying (1) for h is $P = 0$. $\square$

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