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MULTIPLE ELECTROMAGNETIC SCATTERING FROM A CLUSTER OF SPHERES

VOLUME II. SYMMETRIZATION

by

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Group theory is used to put the system of equations for the expansion of coefficients of the electromagnetic field scattered by a cluster of spheres in factorized form.

PREFACE

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CONTENTS

MULTIPLE ELECTROMAGNETIC SCATTERING FROM A CLUSTER OF SPHERES. VOLUME II. SYMMETRIZATION

1. INTRODUCTION

In a preceding paper[1] (hereafter referred to as Volume I), we proposed a method to study the scattering properties of a cluster of spheres. Our approach, aimed to improve Rayleigh-Debye theory[2,3] for the scattering of electromagnetic waves from molecules, is quite general as it does not imply any restriction either on the effective index of refraction or on the size and geometry of the cluster. The only approximation required is the truncation of the multipolar expansions used to represent both the incident and the scattered waves as well as the field within the spheres. The expansion coefficients of the scattered wave are the solution of a system of linear nonhomogeneous equations whose order, for large clusters and for convergency reasons, may be rather high. If the cluster possesses symmetry properties, however, as is the case for actual molecules, group theory can be used to get the above system in factorized form.[4] This is easily done by expanding both the incident and the scattered fields in terms of linear combinations of *multipoles transforming* according to the rows of the irreducible representations of the symmetry group. The symmetrization need not be applied to the field within the spheres for, as seen in section 4 of Volume I, the expansion coefficients of the internal field can be eliminated from the final equations.

Although the techniques we are going to use are well known in molecular physics, two fundamental differences should be borne in mind. First, the electric and magnetic multipoles with the same L have opposite parity so that they transform differently under improper rotations.[5] Second, the inhomogeneities of the factorized systems depend on the row index of the multidimensional irreducible representations. As a consequence, we have to solve all the systems arising from the factorization procedure. It will become apparent, however, that these peculiarities of the parent problem do not cancel the undoubted advantages of group theoretical techniques.

2. SYMMETRY ADAPTED MULTIPOLAR EXPANSIONS OF THE FIELDS

The system whose scattering properties to be studied is a cluster of N nonmagnetic spheres whose centers lie at $\underset{\sim}{R}_\alpha$ and whose radii and (possibly complex) refractive indexes are b_α and n_α, respectively. The cluster is referred to a fixed system of axes with its origin at the center of symmetry, and the direction of incidence of the incoming plane wave determined by the direction cosines of its wave vector.

Let us assume that the cluster is left unchanged by the operations of a group G of order g. The effect of any of the operations is to permute, among themselves, the spheres in the cluster, but in general, not all the spheres are linked to each other by a group operation. Therefore, we partition the cluster in sets of spheres which are transformed into each other by the operations of G. We remark that this partitioning does not imply renumbering the spheres, but only that we associate to each site index, α, the appropriate set index, σ. The transformation properties of the multipoles centered at the sites of the σ-th set are then easily found. Indeed, if $f_L(kr)$ denotes a spherical Bessel or Hankel function, then

$$f_L(kr)\underset{\sim}{X}_{LM}(\hat{\underset{\sim}{r}}) = -f_L \underset{\sim}{T}^{M}_{LL} \tag{1a}$$

$$\nabla\times f_L(kr)\underset{\sim}{X}_{LM}(\hat{\underset{\sim}{r}}) = -ik\left[\sqrt{\frac{L+1}{2L+1}}f_{L+1}\underset{\sim}{T}^{M}_{LL+1} - \sqrt{\frac{L+1}{2L+1}}f_{L-1}\ \underset{\sim}{T}^{M}_{LL-1}\right] \tag{1b}$$

where the vector spherical harmonics, $\underset{\sim}{X}_{LM}$, are defined according to Jackson[6] and the irreducible spherical tensors, $\underset{\sim}{T}^{M}_{JL}$, according to Rose[5]. The above equations show that both the magnetic, equation (1a) and the electric, equation (1b), 2^L-poles transform under *"proper"* rotations according to the representation $D^{(L)}$ of the full rotation group.[7] Accordingly, let S be an operation of G such that

$$S\underset{\sim}{R}_\alpha = \underset{\sim}{R}_\beta$$

with α and β in the same set, of course, and let O_s be the associated operator. If S is a *"proper"* rotation, then[8]

$$O_s f_L(kr_\alpha)\underset{\sim}{X}_{LM}(\hat{\underset{\sim}{r}}_\alpha) = f_L(kr_\beta)\sum_{M'} D^{(L)}_{M'M}(S)\underset{\sim}{X}_{LM'}(\hat{\underset{\sim}{r}}_\beta) \tag{2a}$$

$$O_s \nabla\times f_L(kr_\alpha)\underset{\sim}{X}_{LM}(\hat{\underset{\sim}{r}}_\alpha) = \sum_{M'} \nabla\times f_L(kr_\beta) D^{(L)}_{M'M}(S)\underset{\sim}{X}_{LM'}(\hat{\underset{\sim}{r}}_\beta). \tag{2b}$$

If S is an *"improper"* rotation, however, the magnetic and electric 2^L-poles have opposite parity, so that the right-hand side of equation (2a) must be multiplied by $(-)^L$ and that of equation (2b) by $(-)^{L+1}$, while the argument of $D^{(L)}_{M'M}$ is to be understood as the proper, rotational part of S itself. Therefore, if the joint group G also includes improper rotations, to obtain the symmetrized combinations of multipoles belonging to the rows of the ν-th irreducible representation of dimension g_ν we have to apply the projection operators[7,9,10]

$$P^\nu_{pq} = \frac{g_\nu}{g}\sum_s D^{(\nu)*}_{pq}(S) O_s \tag{3}$$

both to the magnetic and the electric 2^L-poles. Accordingly we write

$$\underset{\sim}{H}^{\nu p\sigma}_{NL} = \sum_{\alpha\in\sigma}\sum_{M\in N} a^{\nu p\alpha}_{NLM} h_L(kr_\alpha)\underset{\sim}{X}_{LM}(\hat{\underset{\sim}{r}}_\alpha) \tag{4a}$$

and

$$\underset{\sim}{K}^{\nu p\sigma}_{N'L} = \sum_{\alpha\in\sigma}\sum_{\bar{M}\in N'} b^{\nu p\alpha}_{N'L\bar{M}} \frac{1}{k} \nabla\times h_L(kr_\alpha)\underset{\sim}{X}_{L\bar{M}}(\hat{\underset{\sim}{r}}_\alpha) \tag{4b}$$

for the combinations of magnetic and electric 2^L-poles centered at the sites of the σ-th set. The superscripts ν, p indicate that the combination belongs to the p-th row of the ν-th irreducible representation, and the index N recalls, when appropriate, that one can get more than a set of basis functions for a given L. Of course, if G includes only proper rotations $b^{\nu p\alpha}_{NLM} \equiv a^{\nu p\alpha}_{NLM}$ and $\underset{\sim}{K}^{\nu p\sigma}_{NL} \equiv \frac{1}{k}\nabla\times\underset{\sim}{H}^{\nu p\sigma}_{NL}$. The field scattered by the cluster, equation (3) of Volume I, can be written in symmetrized form as

$$\underset{\sim}{E}^S_\eta = \sum_{\nu p}\sum_{\sigma L}\left[\sum_N A^{\nu p\sigma}_{\eta NL}\underset{\sim}{H}^{\nu p\sigma}_{NL} + \sum_{N'} B^{\nu p\sigma}_{\eta N'L}\underset{\sim}{K}^{\nu p\sigma}_{N'L}\right] \tag{5}$$

the corresponding expression for $i\underline{B}^{s}_{\eta}$ being obtained through the Maxwell equation $i\underline{B} = \frac{1}{k}\nabla\times\underline{E}$; the index $\eta = \pm 1$ according to the polarization. It is useful to notice that since we work with unitary irreducible representations, as shown by the structure of the projection operators, equation (3), the coefficients $a^{\nu p\alpha}_{NLM}$ and $b^{\nu p\alpha}_{NLM}$ have the property

$$\sum_{\alpha\in\sigma}\sum_{M}(a^{\nu p\alpha}_{NLM})^{*}a^{\nu p\alpha}_{N'LM} = \delta_{NN'}\ ,\ \sum_{\alpha\in\sigma}\sum_{M}(b^{\nu p\alpha}_{NLM})^{*}b^{\nu p\alpha}_{N'LM} = \delta_{NN'} \tag{6}$$

Now we have to decompose the incident field, equation (1) of Volume I, into parts belonging to the rows of the irreducible representation of G. This is easily done for the projection operators, equation (3), has the completeness property[7,9]

$$\sum_{\nu,p}\mathcal{P}^{\nu}_{pp} = 1 \tag{7}$$

Therefore, we can write

$$\begin{aligned}\underline{E}^{(i)}_{\eta} &= \sum_{LM} W_{\eta LM}(\hat{\underline{k}})\sum_{\nu p}\mathcal{P}^{\nu}_{pp}\left[j_{L}(kr)\underline{X}_{LM}(\hat{\underline{r}}) + \eta\frac{1}{k}\nabla\times j_{L}(kr)\underline{X}_{LM}(\underline{r})\right]\\ &= \sum_{\nu p}\sum_{LM} W_{\eta LM}(\hat{\underline{k}})\left[\underline{J}^{\nu p}_{LM} + \eta\underline{L}^{\nu p}_{LM}\right],\end{aligned} \tag{8}$$

where $W_{\eta LM}$ is defined by equation (2) of Volume I and

$$\begin{aligned}\underline{J}^{\nu p}_{LM} &= \frac{g_{\nu}}{g}\sum_{S} D^{(\nu)*}_{pp}(S)\sum_{M'} D^{(L)}_{M'M}(S)j_{L}(kr)\underline{X}_{LM'}(\hat{\underline{r}})\\ &= \sum_{M'} C^{\nu p}_{LMM'}\, j_{L}(kr)\underline{X}_{LM'}(\hat{\underline{r}})\end{aligned} \tag{9a}$$

$$\underline{L}^{\nu p}_{LM} = \sum_{M'} d^{\nu p}_{LMM'}\,\frac{1}{k}\nabla\times j_{L}(kr)\underline{X}_{LM'}(\hat{\underline{r}}). \tag{9b}$$

The analogous equation for $i\underline{B}^{(i)}_{\eta}$ is easily obtained through the relation[6]

$$i\underline{B}^{(i)}_{\eta} = \eta\underline{E}^{(i)}_{\eta} \tag{10}$$

Of course, even in this case, if G does not include *"improper"* rotations

$$C^{\nu p}_{LMM'} \equiv d^{\nu p}_{LMM'} \text{ and } \underline{L}^{\nu p}_{LM} = \frac{1}{k}\nabla\times\underline{J}^{\nu p}_{LM}$$

This completes the symmetrization procedure for, as noticed in Volume I, the internal field need not be symmetrized and is therefore given by equation (4) of Volume I, which is shown below for convenience

$$\underset{\sim}{E}_{\eta}^{(t)\alpha} = \sum_{LM} \left[C^{\alpha}_{\eta LM} R^{\alpha}_{L}(r_{\alpha}) \underset{\sim}{X}_{LM}(\hat{\underset{\sim}{r}}_{\alpha}) + \frac{1}{n_{\alpha}^{2}} D^{\alpha}_{\eta LM} \frac{1}{k} \nabla \times S^{\alpha}_{L}(r_{\alpha}) \underset{\sim}{X}_{LM}(\hat{\underset{\sim}{r}}_{\alpha}) \right] \tag{11a}$$

$$i\underset{\sim}{B}_{\eta}^{(t)\alpha} = \sum_{LM} \left[D^{\alpha}_{\eta LM} S^{\alpha}_{L}(r_{\alpha}) \underset{\sim}{X}_{LM}(\hat{\underset{\sim}{r}}_{\alpha}) + C^{\alpha}_{\eta LM} \frac{1}{k} \nabla \times R^{\alpha}_{L}(r_{\alpha}) \underset{\sim}{X}_{LM}(\hat{\underset{\sim}{r}}_{\alpha}) \right] \tag{11b}$$

where R^{α}_{L} and S^{α}_{L} are still the solutions of equation (5) of Volume I.

3. EQUATIONS FOR THE COEFFICIENTS

The equations for the coefficients $A^{\nu p \sigma}_{\eta NL}$ and $B^{\nu p \sigma}_{\eta NL}$ of the scattered wave can now be easily found through the same procedure as used in Volume I. First of all, we rewrite equations (5) and (8) in terms of multipoles centered at the single site $\underset{\sim}{R}_{\alpha}$, by means of the appropriate addition theorem[11]

$$\begin{aligned}
\underset{\sim}{E}_{\eta}^{(s)} = \sum_{\nu p \sigma} \sum_{L} \Bigg\{ & \sum_{N} A^{\nu p \sigma}_{\eta NL} \sum_{M \in N} a^{\nu p \alpha}_{NLM} h_{L}(kr_{\alpha}) \underset{\sim}{X}_{LM}(\hat{\underset{\sim}{r}}_{\alpha}) + \sum_{N'} B^{\nu p \sigma}_{\eta N'L} \sum_{\bar{M} \in N'} b^{\nu p \sigma}_{N'L\bar{M}} \frac{1}{k} \nabla \times h_{L}(kr_{\alpha}) \underset{\sim}{X}_{L\bar{M}}(\hat{r}_{\alpha}) \\
& + \sum_{N} \sum_{\tau} A^{\nu p \tau}_{\eta NL} \sum_{\beta \in \tau} \sum_{M \in N} a^{\nu p \beta}_{NLM} \sum_{L'M'} \left[H^{\alpha\beta}_{L'M'LM} j_{L'}(kr_{\alpha}) \underset{\sim}{X}_{L'M'}(\hat{\underset{\sim}{r}}_{\alpha}) + K^{\alpha\beta}_{L'M'LM} \frac{1}{k} \nabla \times j_{L'}(kr_{\alpha}) \underset{\sim}{X}_{L'M'}(\hat{\underset{\sim}{r}}_{\alpha}) \right] \\
& + \sum_{\tau} \sum_{N'} B^{\nu p \tau}_{\eta N'L} \sum_{\beta \in \tau} \sum_{\bar{M} \in N'} b^{\nu p \beta}_{N'L\bar{M}} \left[K^{\alpha\beta}_{L'M'LM} j_{L'}(kr_{\alpha}) \underset{\sim}{X}_{L'M'}(\hat{\underset{\sim}{r}}_{\alpha}) + H^{\alpha\beta}_{L'M'L\bar{M}} \frac{1}{k} \nabla \times j_{L'}(kr_{\alpha}) \underset{\sim}{X}_{L'M'}(\hat{\underset{\sim}{r}}_{\alpha}) \right] \Bigg\}
\end{aligned} \tag{12}$$

$$\begin{aligned}
\underset{\sim}{E}_{\eta}^{(i)} = \sum_{LM} W_{\eta LM}(\hat{\underset{\sim}{k}}) \Bigg\{ & \sum_{\nu p} \sum_{M'} c^{\nu p}_{LMM'} \sum_{L''M''} \left[J^{\alpha}_{L''M''LM'} j_{L''}(kr_{\alpha}) \underset{\sim}{X}_{L''M''}(\hat{\underset{\sim}{r}}_{\alpha}) + L^{\alpha}_{L''M''LM'} \frac{1}{k} \nabla \times j_{L''}(kr_{\alpha}) \underset{\sim}{X}_{L''M''}(\hat{\underset{\sim}{r}}_{\alpha}) \right] \\
& + \sum_{\nu p} \sum_{M'} \eta d^{\nu p}_{LMM'} \sum_{L''M''} \left[L^{\alpha}_{L''M''LM'} j_{L''}(kr_{\alpha}) \underset{\sim}{X}_{L''M''}(\hat{\underset{\sim}{r}}_{\alpha}) + J^{\alpha}_{L''M''LM'} \frac{1}{k} \nabla \times j_{L''}(kr_{\alpha}) \underset{\sim}{X}_{L''M''}(\hat{\underset{\sim}{r}}_{\alpha}) \right] \Bigg\}
\end{aligned} \tag{13}$$

Analogous equations hold for $iB^{(s)}_{\eta}$ and $iB^{(i)}_{\eta}$ while the matrix elements of the dyadic Green's function $H^{\alpha\beta}_{L'M'LM}$, $K^{\alpha\beta}_{L'M'LM}$, $J^{\alpha}_{L'M'LM}$ and $L^{\alpha}_{L'M'LM}$ are defined by equation (8) of Volume I. Moreover, we recall that equations (6) and (7) are valid only for $r_{\alpha} \le R_{\alpha\beta} = \left| \underset{\sim}{R}_{\beta} - \underset{\sim}{R}_{\alpha} \right|$; i.e., near the surface of the α-th sphere. Now we proceed as in Volume I; i.e., we take the dot product of equations (11), (12), and (13) in turn with $\hat{\underset{\sim}{r}}_\alpha Y^*_{\ell m}(\hat{\underset{\sim}{r}}_\alpha)$, $\underset{\sim}{X}^*_{\ell m}(\hat{\underset{\sim}{r}}_\alpha)$ and $\hat{\underset{\sim}{r}}_\alpha \times \underset{\sim}{X}^*_{\ell m}(\hat{\underset{\sim}{r}}_\alpha)$ and get the radial and tangential components of the fields. Imposition of the boundary conditions and integration over the angles then yields, for each ν, p, α, six equations from which the coefficients of the internal fields, $C^\alpha_{\eta\ell m}$ and $D^\alpha_{\eta\ell m}$, can easily be eliminated. This circumstance clarifies the inessentiality of the symmetrization of the internal fields, and yields, for each ν, p, α, two equations involving only the A's and B's as unknowns:

$$\sum_\tau \sum_{\beta\in\tau} \sum_L \left\{ \sum_N \sum_{M\in N} \left(\delta_{L\ell}\, \delta_{Mm}\, \delta_{\sigma\tau} \left[R^\tau_L \right]^{-1} + H^{\alpha\beta}_{\ell m LM} \right) a^{\nu p\beta}_{NLM} A^{\nu p\tau}_{\eta NL} + \sum_{N'} \sum_{\bar{M}\in N'} K^{\alpha\beta}_{\ell m L\bar{M}}\, b^{\nu p\beta}_{N'L\bar{M}} B^{\nu p\tau}_{\eta N'L} \right\}$$

$$= -\sum_{LM} \sum_{M'} W_{\eta LM}(\hat{\underset{\sim}{k}}) \left[\eta c^{\nu p}_{LMM'} L^\alpha_{\ell m LM'} + d^{\mu p}_{LMM'} J^\alpha_{\ell m LM'} \right] \tag{14a}$$

$$\sum_\tau \sum_{\beta\in\tau} \sum_L \left\{ \sum_{N'} \sum_{\bar{M}\in N'} \left(\delta_{L\ell}\, \delta_{Mm}\, \delta_{\sigma\tau} \left[S^\tau_L \right]^{-1} + H^{\alpha\beta}_{\ell m L\bar{M}} \right) b^{\nu p\beta}_{N'L\bar{M}} B^{\nu p\tau}_{\eta N'L} + \sum_N \sum_{M\in N} K^{\alpha\beta}_{\ell m LM}\, a^{\nu p\beta}_{NLM} A^{\nu p\tau}_{\eta NL} \right\}$$

$$= -\sum_{LM} \sum_{M'} W_{\eta LM}(\hat{\underset{\sim}{k}}) \left[c^{\nu p}_{LMM'} L^\alpha_{\ell m LM'} + \eta d^{\mu p}_{LMM'} J^\alpha_{\ell m LM'} \right] \tag{14b}$$

where the quantities R^α_ℓ and S^α_ℓ are defined by equation (10) of Volume I. Now, since the spheres within a set are identical to each other, the quantities R^α_ℓ and S^α_ℓ are actually independent of the site index, α, but dependent on the set index, σ. This circumstance, together with the orthogonality relations, equation (6), allows putting equation (14) in a more symmetrical form. Indeed, multiplication of equation (14a) by $(a^{\nu p\alpha}_{\eta\ell m})^*$ and of equation (14b) by $(b^{\nu p\alpha}_{\eta\ell m})^*$ and summation over the α's belonging to and over m, yield, for each ν, p, τ, the equations

$$\sum_s \left(\delta_{rs} \left[R_s \right]^{-1} + H^\nu_{rs}(m) \right) A^{\nu p}_{\eta s} + \sum_{s'} K^\nu_{rs'}(m,e)\, B^{\nu p}_{\eta s'} = -P^{\nu p}_{\eta r} \tag{15a}$$

$$\sum_{s'} \left(\delta_{r's'} \left[S_{s'} \right]^{-1} + H^\nu_{r's'}(e) \right) B^{\nu p}_{\eta s'} + \sum_s K^\nu_{r's}(e,m)\, A^{\nu p}_{\eta s} = -Q^{\nu p}_{\eta r'} \tag{15b}$$

In equation (15) we put, for the sake of simplicity, $r \equiv (\sigma,\ell,n)$, $s \equiv (\tau,L,N)$ $r' \equiv (\sigma, \ell, n')$, $s' \equiv (\tau, L, N')$ and define

$$H^{\nu}_{rs}(m) = \sum_{\alpha m}\sum_{\beta M}(a^{\nu p\alpha}_{n\ell m})^* H^{\alpha\beta}_{\ell mLM}\, a^{\nu p\beta}_{NLM} \tag{16}$$

$$K^{\nu}_{rs}(m,e) = \sum_{\alpha m}\sum_{\beta M}(a^{\nu p\alpha}_{n\ell m})^* K^{\alpha\beta}_{\ell mLM}\, b^{\nu p\beta}_{NLM} \tag{17}$$

with an obvious meaning of the parameters e,m. The quantities $H^{\nu}_{rs}(e)$ and $K^{\nu}_{rs}(e,m)$ are identical to $H^{\nu}_{rs}(m)$ and $K^{\nu}_{rs}(m,e)$, respectively, but for the mutual exchange of the a's with the b's. Moreover

$$P^{\nu p}_{\eta r} \sum_{LM}\sum_{\alpha m}\sum_{M'} W_{\eta LM}(\hat{k}) \left[\eta c^{\nu p}_{LMM'}\, L^{\alpha}_{\ell mLM'} + d^{\nu p}_{LMM'}\, J^{\alpha}_{\ell mLM'}\right]\left(a^{\nu p\alpha}_{n\ell m}\right)^* \tag{18a}$$

$$Q^{\nu p}_{\eta s'} = \sum_{LM}\sum_{\alpha m}\sum_{M'} W_{\eta LM}(\hat{k}) \left[c^{\nu p}_{LMM'}\, L^{\alpha}_{\ell mLM'} + \eta d^{\nu p}_{LMM'}\, J^{\alpha}_{\ell mLM'}\right]\left(b^{\nu p\alpha}_{n'\ell m}\right)^* \tag{18b}$$

We remark that on the left-hand side of equations (16) and (17) the superscripts p on H^{ν}_{rs} and K^{ν}_{rs} is missing. As will be shown later, these quantities are actually independent of the row index, p, which has accordingly been dropped. Anyway, the systems composed of the equation (15) with a given ν, p for all values of σ solve completely our scattering problem. In fact, it was shown, in section 4 of Volume I how to compute the relevant properties of the cluster from the knowledge of $A^{\alpha}_{\eta LM}$ and $B^{\alpha}_{\eta LM}$, the coefficients of the scattered field. Now, direct comparison of equation (5) with equation (3) of Volume I yields

$$A^{\alpha}_{\eta LM} = \sum_{\nu p}\sum_{N} A^{\nu p\sigma}_{\eta NL}\, a^{\nu p\alpha}_{NLM}\ ,\quad B^{\alpha}_{\eta LM} = \sum_{\nu p}\sum_{N} B^{\nu p\sigma}_{\eta NL}\, b^{\nu p\alpha}_{NLM} \tag{19}$$

Therefore, all the properties of the cluster can also be expressed in terms of $A^{\nu p}_{\eta NL}$ and $B^{\nu p}_{\eta NL}$, the coefficients of the symmetrized expansion.

4. DISCUSSION

The consequences of the factorization effected by group theory can be best discussed by writing the system with a given ν, p, in matrix form:

$$\begin{vmatrix} \underset{\sim}{R}^{-1} + \underset{\sim}{H}^{\nu}(m) & \underset{\sim}{K}^{\nu}(m,e) \\ \underset{\sim}{K}^{\nu}(e,m) & \underset{\sim}{S}^{-1} + \underset{\sim}{H}^{\nu}(e) \end{vmatrix} \begin{vmatrix} A^{\nu p} \\ B^{\nu p} \end{vmatrix} = \begin{vmatrix} P^{\nu p} \\ Q^{\nu p} \end{vmatrix} \tag{20}$$

Since equation (20) has the same overall structure of equation (15) of Volume I, the matrix on its left-hand side is easily recognized as the inverse of the electromagnetic T-matrix for the whole cluster.[12] The lack of spherical symmetry of the cluster does not permit getting a diagonal T-matrix, but equation (20) shows that group theory effects the decomposition of the whole scattering process into "modes of scattering" belonging to the irreducible representations of the symmetry group, G. Of course, the dependence on the row index of the inhomogeneity of equation (20) forces us to solve *all* the systems arising from the factorization procedure, while in the case of secular determinants one needs to solve only one system for irreducible representation.[9] However, as explicitly indicated by the omission of the superscript p, the T-matrix for the ν-th mode of scattering does not depend on p, as we shall show presently. The p-independence of $\underset{\sim}{R}$ and $\underset{\sim}{S}$ follows from the very definition of their matrix elements, equation (10) of Volume I. These diagonal matrices occur even in the theory of scattering from a single sphere and are the electromagnetic analog of the transition matrix of quantum scattering theory.[13] The elements $H^{\nu}_{\gamma_s}(m)$, $H^{\nu}_{\gamma_s}(e)$, $K^{\nu}_{\gamma_s}(m,e)$ and $K^{\nu}_{\gamma_s}(e,m)$ of the matrices appearing in equation (20), are in turn the symmetrized counterparts of $H^{\alpha\beta}_{\ell mLM}$ and $K^{\alpha\beta}_{\ell mLM}$, which, as shown in the appendix of Volume I, are the matrix elements, in the site and angular momentum representation, of $\underset{\sim}{G}$, the dyadic Green's function for free space propagation of spherical vector waves. The p-independence of $\underset{\sim}{H}^{\nu}(m)$, $\underset{\sim}{H}^{\nu}(e)$, $\underset{\sim}{K}^{\nu}(m,e)$ and $\underset{\sim}{K}^{\nu}(e,m)$ is then a direct consequence of the invariance of $\underset{\sim}{G}$ under the symmetry operations. Therefore the whole T-matrix for the ν-th model of scattering turns out to be independent of the row index, p. This circumstance greatly reduces the computational work in the case of multidimensional irreducible representations. Of course the size of the factorized systems depends not only on the number of spheres in the cluster and on the number of L-values included in the multipolar expansion of the scattered wave, but also on the structure of the symmetry group. Therefore the effect of the factorization can be illustrated only through examples. Table 1 shows the order of the systems to be solved for a cluster of 5 spheres with point group T_d the CH_4 molecule and the SO_4^{++} ion have just this structure). Although table 1 considers the case in which terms up to $L = 4$ are included in the multipolar expansions, from the discussion of section 6 of Volume I, we expect that terms, up to and including $L = 3$ will be quite sufficient to get fairly converged values for the scattered field, even if the n_α's are not close to unity.[14,15] The usefulness of group theory requires no further comment.

Table 1.

L_M	1	2	3	4
A_1	1	2	6	10
A_2	1	2	6	10
E	2	8	12	20
F_1	4	10	19	30
F_2	4	10	19	30
U	30	80	150	240

Dimension of the symmetrized and unsymmetrized systems for L_M up to 4 for a cluster of 5 spheres with joint group T_d. The entry U means "unsymmetrized" while the other entries indicate irreducible representations.[7]

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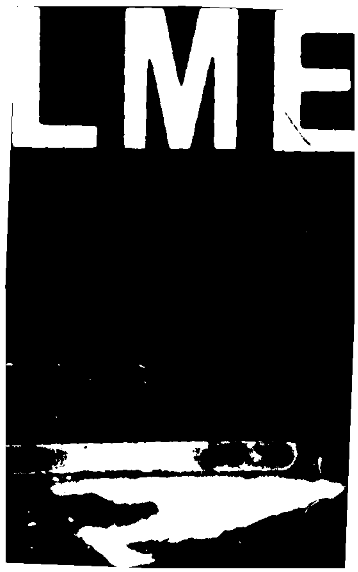
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