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ANALYSIS OF THE CURVED JUNCTION EDGE BETWEEN A FLAT PLATE AND A PROLATE SPHEROID

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#### I. INTRODUCTION

The prolate spheroid will be used to simulate a wide class of aircraft fuselage in the future. The far-zone and near-zone radiation patterns of a prolate spheroid mounted antenna have already been studied [1,2]. It is obvious that if one wishes to simulate an aircraft, one must allow the flat plates (model used for wings) to attach to the fuselage which was modeled by the prolate spheroid in our study. Therefore, the object of this study is to study the curved junction edge resulting from attaching the plates to the prolate spheroid.

For the future development of the computer program to simulate the aircraft antenna, it is assumed that all corners of the plate are outside of the prolate spheroid.

Our approach to this problem is first to find the intersection point between a line (i.e., one edge of the plate) and the prolate spheroid. Then one can follow the same idea to find the curved junction edge between a flat plate and the prolate spheroid. The method is described in detail in the following sections.

#### II. INTERSECTION POINT BETWEEN A LINE AND THE PROLATE SPHEROID

Using the geometry as shown in Figure 1, the spheroid surface is defined by  $\vec{R}(v \phi) = a \cos v \cos \phi \hat{x} + a \cos v \sin \phi \hat{y} + b \phi \hat{R}(v \phi)$ 

b sin v z (1)

where

$$v = \tan^{-1} \left( \frac{b \cos \theta}{a \sin \theta} \right)$$
 (2)

The line direction is given by

.

$$\hat{e}(x_{1},y_{1},z_{1}) = \frac{\vec{P}_{1}(x_{1},y_{1},z_{1}) - \vec{P}_{2}(x_{2},y_{2},z_{2})}{|\vec{P}_{1}(x_{1},y_{1},z_{1}) - \vec{P}_{2}(x_{2},y_{2},z_{2})|}$$

$$= e_{x}\hat{x} + e_{y}\hat{y} + e_{z}\hat{z}$$
(3)



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Figure i. Geometry of spheroid.

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where  $\vec{P}_1(x_1, y_1, z_1)$  and  $\vec{P}_2(x_2, y_2, z_2)$  are the position vectors of two points with respect to the origin of the coordinate system. From Figure 1, one observed that the position vector of the intersection point  $\vec{P}_h(x_h, y_h, z_h)$  with respect to the origin of the coordinate system can be easily defined by

$$\vec{P}_{h}(x_{h}, y_{h}, z_{h}) = \vec{P}_{1}(x_{1}, y_{1}, z_{1}) + \hat{\ell}(x_{1}, y_{1}, z_{1})$$
(4)

From Eductions (1), (3) and (4), one obtains the following educations:

$$a \cos v \cos \phi = x_1 + e_v$$
 (5a)

$$a^{CCS} v \sin \phi = y_1 + \ell e_v$$
 (5b)

$$ab \sin v = z_1 + 2e_7 . \tag{5c}$$

From Equations (Ea), (5b), and (5c), one finds

$$\frac{(x_1 + ge_x)^2 + (y_1 + ge_y)^2}{a^2} + \frac{(z_1^2 + ge_z)^2}{b^2} = 1 \quad . \tag{6}$$

Defining

$$A = b^{2}(e_{x}^{2} + e_{y}^{2}) + a^{2}e_{z}^{2}, \qquad (7a)$$

$$B = b^{2}(x_{1}e_{x} + y_{1}e_{y}) + a^{2}z_{1}e_{z}, \qquad (7b)$$

$$C = b^2(x^2 + y^2) + a^2z^2 - a^2b^2$$
, (7c)

and employing Equations (6) and (7), one obtains the distance between the points  $\rm P_1$  and  $\rm P_H$  such that

$$\mathfrak{L}_1 = \frac{-B + \sqrt{B^2 - AC}}{A} , \text{ and}$$
(8a)

$$k_2 = \frac{-B - \sqrt{B^2 - AC}}{A}$$
 (8b)

The smaller one of  $(l_1, l_2)$  will be used in equation (4) to define the position vector  $P_h(x_h, y_h, z_h)$ .

#### III. CURVED JUNCTION EDGE BETWEEN A PLATE AND THE PROLATE SPHEROID

Using the geometry as shown in Figure 2,  $\vec{P}_1$ ,  $\vec{P}_2$ ,  $\vec{P}_3$  and  $\vec{P}_4$  are the position vectors of the corners of the plate, then one can define the unit vectors of the plate edge as follows:

$$\hat{e}_{i}(x_{i},y_{i},z_{i}) = \frac{\vec{p}_{i+1}(x_{i+1},y_{i+1},z_{i+1}) - \vec{p}_{i}(x_{i},y_{i},z_{i})}{|\vec{p}_{i+1}(x_{i+1},y_{i+1},z_{i+1}) - \vec{p}_{i}(x_{i},y_{i},z_{i})|}$$
(9)

with i = 1, 2, 3.

The plate normal unit vector is given by

$$\hat{N}_{p}(x, y, z) = \frac{\hat{e}_{1}(x_{1}, y_{1}, z_{1}) \times \hat{e}_{2}(x_{2}, y_{2}, z_{2})}{|\hat{e}_{1}(x_{1}, y_{1}, z_{1}) \times \hat{e}_{2}(x_{2}, y_{2}, z_{2})|}$$
(10)

Using the procedure described in Section II, one can determine the position vectors of the intersection points  $\vec{P}_{HI}$  and  $\vec{P}_{HF}$  between the prolate spheroid and two plate edges  $\hat{e}_1$  and  $\hat{e}_3$ , respectively. Then one can define the unit vector

$$\hat{e}_{FI}(x,y,z) = \frac{\hat{p}_{HF}(x_{HF},y_{HF},z_{HF}) - \hat{p}_{HI}(x_{HI},y_{HI},z_{HI})}{|\hat{p}_{HF}(x_{HF},y_{HF},z_{HF}) - \hat{p}_{HI}(x_{HI},y_{HI},z_{HI})|}, \quad (11)$$

and the binormal unit vector

$$\hat{t}_{b}(x,y,z) = \hat{e}_{FI}(x,y,z) \times \hat{N}_{p}(x,y,z)$$
 (12)

According to the variation of the spheroid surface, one can divide the line  $\overline{P_{HF}P_{HI}}$  into N-1 unequal length segments,  $\ell_i$ , with  $i=1,2,\ldots,n-1$ . Then one can get N-2 position vectors  $\overline{P}_i(x,y,z)$  along  $\overline{P_{HF}P_{HI}}$  by using the recursive equation,

$$\vec{p}_{i}(x,y,z) = \vec{p}_{i-1}(x,y,z) + \ell'_{i} \hat{e}_{FI}(x,y,z)$$
 (13)

with

$$\vec{P}_2(x,y,z) = \vec{I}_{HI}(x,y,z) + \hat{k}_i e_{FI}(x,y,z) \text{ and } i = 2,3,...,N-1$$





By using the position vectors  $P_i(x,y,z)$  just found, one can get N position vectors along the curved junction edge or

$$\tilde{P}_{CEi}(x,y,z) = \tilde{P}_{i}(x,y,z) + \ell_{i}^{*} t_{b}(x,y,z), i = 2,3,...N-1$$
 (14)  
with

$$\vec{P}_{CE1}(x,y,z) = \vec{P}_{HI}(x,y,z)$$
, and (14a)

$$\vec{P}_{CEN}(x,y,z) = \vec{P}_{HF}(x,y,z)$$
 (14b)

note  $\mathfrak{L}_{i}^{"}$  can be found by using the same idea employed in Section II. Therefore, the curved junction edge was found by connecting all N position vectors  $\vec{P}_{CE\,i}(x,y,z)$ . Note that this approach provides a piecewiselinear approximation for the curved junction edge resulting from the intersection of the flat plate and the prolate spheroid.

## IV. CONCLUSION

The process of attaching the flat plate to the prolate spheroid (aircraft fuselage) and finding the curved junction edge has been studied in this report. The diffraction associated with the curved junction edge will be studied later. A computer sub-routine for the curved junction edge had already been developed and incorporated into the general aircraft program. The complete numerical solution of the aircraft antenna problem will be studied later. The results will be verified by comparing them with the numerous experimental radiation patterns taken at various organization.

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- H. Chung, W.D. Burnside and N. Wang, "The Near Field Radiation Patterns of A Spheroid-Mounted Antenna," Report 712527-2, November 1980, The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering; prepared under Contract N00019-80-C-0050 for Department of the Navy.

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