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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

**ye* initially, after vowels, and after ъ, ь; *e* elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

THE DESIGN OF PHASED ANNULAR ANTENNA ARRAYS OF INTERACTING RADIATORS, EXCITED BY A BUTLER CIRCUIT

N. P. Polishchuk and D. M. Sazonov

The article discusses an annular antenna array of arbitrary interacting radiators, excited by means of a passive reactive Butler pattern-forming circuit.

The procedure is given for calculating the phase distribution at the inputs of the pattern-forming circuit, assuring the best rms approximation to the direction pattern with maximum efficiency in the given direction. The amplitude distribution is supposed to be uniform. Several numerical examples are given and discussed.

INTRODUCTION

Annular antenna arrays are interesting because they can realize a rotation of the direction pattern (DN) within limits of 360° (in the plane of the array). However the resulting amplitude-phase distribution of the currents, necessary for the formation of the beam in the given direction, is sharply nonuniform, which complicates the realization of the rotation of this distribution.

The utilization of matrix schemes to energize the antenna array enables the transformation of the amplitude-phase distribution of currents on the radiators so that the control of the direction pattern of the array can be implemented solely by changing the phases in the regulated phase shifters at the inputs of the matrix circuit with constant amplitude excitations [1,2]. In this connection, there occurs the problem of synthesis of the DN with maximum coefficient of directional activity in the given direction, using annular antenna arrays with phase scanning. The synthesis is understood to refer to the discovery of the distribution of phase shifts of the exciting signals at the inputs of the matrix circuit, which assure the best rms approximation to the DN with maximum coefficient of directional activity in the given direction. This paper is devoted to the above problem. It considers a procedure of designing annular antenna arrays of arbitrary radiators, allowing for their interaction.

FORMULATION OF THE PROBLEM. THE CALCULATION METHOD.

Let us consider an antenna array of N identical radiators, arranged at equal distances along a ring of radius a in the plane $\theta=\pi/2$ (Fig. 1). Let us assume that the DN of the radiators are known. We shall designate these by $\vec{f}_n(\theta,\phi)$, where n=1,2...N. We may then consider a symmetrical matrix of mutual impedances to be also known:

$$[z]=[r]+i[x]=\begin{bmatrix} 1 & z_{12} & \dots & z_{1N} \\ z_{12} & 1 & \dots & z_{2N} \\ \vdots & \vdots & & \vdots \\ z_{1N} & z_{2N} & & 1 \end{bmatrix}, \qquad (1)$$

the elements of which may be calculated, e.g., by the method proposed in [3].

We shall consider that the antenna array is excited by means of a passive, reactive, Butler pattern-forming circuit, characterized by the scattering matrix:

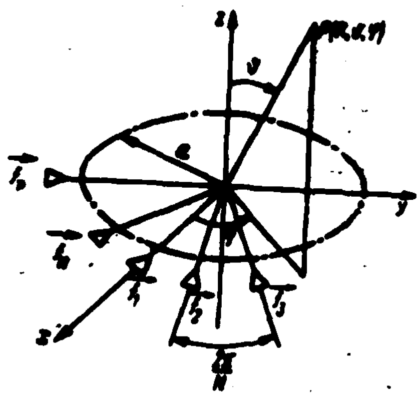

Fig. 1.

$$[S] = \begin{array}{c|cc} & 1 \ldots N & 1^{\circ} \ldots N^{\circ} \\ \hline \begin{matrix} 1 \\ \vdots \\ N \\ 1^{\circ} \\ \vdots \\ N^{\circ} \end{matrix} & \left[\begin{matrix} [0] \\ [T] \end{matrix}\right. & \left.\begin{matrix} [T] \\ [0] \end{matrix}\right] \end{array}, \tag{2}$$

where the transformation block

$$[T] = \frac{1}{\sqrt{N}}\left[e^{i\frac{2\pi}{N}(m-1)(n-1)}\right] \tag{3}$$

coincides with a matrix comprised of the eigenvectors of matrix [z].

The problem consists in finding the amplitude-phase distribution of the exciting signals at the inputs of the pattern-forming circuit, to assure a maximum coefficient of directional activity of the antenna array in the given direction. It is convenient to solve this problem in two stages.

1) Synthesis of the given DN with maximum coefficient of directional activity in the given direction, i.e. finding the optimal amplitude-phase distribution of the exciting currents on the radiators of the antenna array.

2) Realization of an approximation to the optimal distribution of currents in the specific feeder circuit, i.e. rescaling the discovered optimal distribution for the inputs of the pattern-forming circuit.

As was shown in [4], the synthesis of a DN with maximum coefficient of directivity is a partial case of a more general problem of synthesis of a DN of arbitrary shape if the synthesized DN is given by the function:

$$\vec{g}(\theta, \varphi) = \vec{\varepsilon}\delta(\theta - \theta_0)\,\delta(\varphi - \varphi_0), \tag{4}$$

where $\vec{\varepsilon}$ is the vector of radiation, parallel to the vector $\vec{E}$ in the far zone; (θ_0, ϕ_0) is the direction of the maximum of the coefficient of directivity.

The first part of our problem corresponds to the problem of synthesis of an annular antenna array with no limitations imposed by the pattern-forming circuit, and its solution, i.e. the optimal distribution of currents on the array elements, can be written in the form [5]:

$$i\rangle = [r]^{-1}\gamma\rangle, \tag{5}$$ [1]

where

$$\gamma\rangle = \int_0^{2\pi}\int_0^{\pi} \vec{g}(\theta, \varphi)\vec{f}^*(\theta, \varphi)\rangle \sin\theta \, d\theta \, d\varphi. \tag{6}$$

To the discovered currents there corresponds the DN of an antenna array, which approximates the given (in a rms approximation) with the best accuracy:

$$\vec{G}(\theta, \varphi) = \langle\vec{f}(\theta, \varphi) i\rangle. \tag{7}$$

In order to solve the second part of the problem, let us consider the operation of the pattern-forming circuit in the conditions of its loading on the antenna array (Fig. 2).

The normalized incident and reflected waves at the outputs of the matrix circuit can be expressed by the currents in the elements of the antenna array and by the matrix of mutual resistances [z]:

$$u^{*}_{пад}\rangle = \frac{1}{2}[z-E]i\rangle, \tag{8}$$

$$u^{*}_{отр}\rangle = \frac{1}{2}[z+E]i\rangle, \tag{9}$$

[1]The designation a> refers to the column matrix, <a to the row matrix, and {a} to the diagonal matrix.

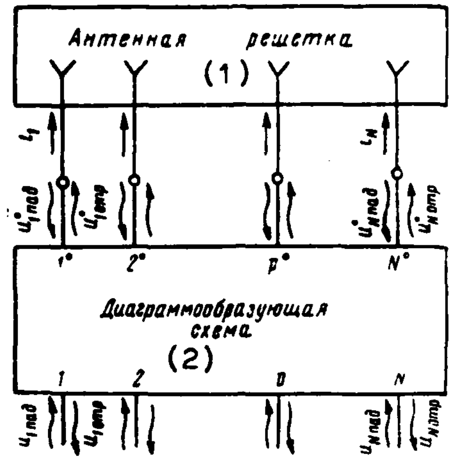

Fig. 2.
Key: (1) Antenna array;
(2) Pattern-forming circuit.

where E is a unit matrix of order N.

On the other hand, it follows from definition (2) of the scattering matrix of the pattern-forming circuit that:

$$u_{\text{отр}}\rangle = [T]\, u^{\circ}_{\text{пад}}\rangle, \tag{10}$$

$$u^{\circ}_{\text{отр}}\rangle = [T]\, u_{\text{пад}}\rangle. \tag{11}$$

From (8)-(11) we find that:

$$u_{\text{пад}}\rangle = \frac{1}{2}[T]^{-1}[z+E]\, i\rangle, \tag{12}$$

$$u_{\text{отр}}\rangle = [T][z-E][z+E]^{-1}[T]\, u_{\text{пад}}\rangle. \tag{13}$$

The column $u_{\text{пад}}\rangle$ gives the sought optimal distribution of the excitation at the inputs of the pattern-forming circuit, while $u_{otp}\rangle$ characterizes the losses due to reflection and the mutual coupling between the channels.

From the discovered column $u_{\text{пад}}\rangle$ let us construct a new

$u'_{\text{пад}}>$, all the amplitudes in which are equal to unity[1] while the phases have not been changed. In this case, the corresponding DN $\vec{G}'(\theta,\phi)$ will differ from the optimal; however, this discrepancy will be a minimum for the described construction of the column $u'_{\text{пад}}>$. In fact, the partial DN, corresponding to each excited input of the pattern-forming circuit, are orthogonal by virtue of the circular symmetry of the antenna array and the choice of matrix [T] in the form (3). These DN have the appearance

$$<\vec{e}(\theta,\ \varphi)=2<\vec{f}(\theta,\ \varphi)[T]\{\zeta+1\}^{-1},$$

where $\{\zeta+1\}$ is a diagonal matrix formed from the eigenvalues of the matrix $[z+E]$.

Then the DN of an antenna array can be represented in the form:

$$\vec{G}(\theta,\ \varphi)=<\vec{e}(\theta,\ \varphi)\,u_{\text{пад}}>,\quad \vec{G}'(\theta,\ \varphi)=<\vec{e}(\theta,\ \varphi)\,u'_{\text{пад}}>.$$

Assuming that $u_{\text{пад}}>=a\,e^{i\psi}>$ and $u'_{\text{пад}}>=e^{i\psi'}>$, we find that the squared norm of the difference between the two diagrams $\vec{G}(\theta,\phi)$ and $\vec{G}(\theta,\phi)$, equalling

$$\|\vec{G}(\theta,\ \varphi)-\vec{G}'(\theta,\ \varphi)\|^2=<\vec{e}(\theta,\ \varphi)\{1+a^2-2a\cos(\psi-\psi')\}\vec{e}^*(\theta,\ \varphi)>,$$

will be a minimum when $\psi_k=\psi'_k$.

[1]The uniform distribution is a good approximation of the optimal distribution, averaged over all the scanning angles.

With the discovery of the column $u'_{\text{выд}}>$, the particular synthesis problem may be regarded as solved. By assigning different beam directions (θ_0, ϕ_0) in (4), we obtain sets of phase shifts, the realization of which by means of phase shifters enables the control of the DN of the antenna array.

Since the synthesis of the DN is approximate, a calculation is necessary to verify the realized DN, using familiar distributions of $u'_{\text{пад}}>$:

$$\vec{G}'(\theta, \varphi) = 2 < \vec{f}(\theta, \varphi)\,[z+E]^{-1}\,[T]\,u'_{\text{пад}}> . \tag{14}$$

CALCULATION EXAMPLES

As an example of the use of the above procedure, several calculations were made for a 32-element annular antenna array. The DN of the individual radiators were given in the form:

$$f_n(\theta, \varphi) = \begin{cases} \cos q\left(\frac{\pi}{2} - \theta\right) \cos p\left[\varphi - (n-1)\frac{2\pi}{N}\right] e^{i\kappa a \cos\left[\varphi - (n-1)\frac{2\pi}{N}\right]\sin\theta} \\ \quad \text{when} \quad \theta \in [\theta_1 \theta_2] \wedge \varphi \in [\varphi_1 \varphi_2], \\ 0 \quad \text{when} \quad \theta \notin [\theta_1 \theta_2] \vee \varphi \notin [\varphi_1 \varphi_2], \end{cases} \tag{15}$$

where $\kappa = 2\pi/\lambda$,

$$[\theta_1 \theta_2] = \begin{cases} \left[\frac{\pi}{2} - \frac{\pi}{2q}\,;\ \ \frac{\pi}{2} + \frac{\pi}{2q}\right] & \text{for} \quad q > 1, \\ [0,\ \pi] & \text{for} \quad q \leqslant 1, \end{cases}$$

$$[\varphi_1\, \varphi_2] = \begin{cases} \left[-\frac{\pi}{2p} + (n-1)\frac{2\pi}{N}\,;\ \ \frac{\pi}{2p} - (n-1)\frac{2\pi}{N}\right]; & p > \frac{1}{2}, \\ [0,\ 2\pi]; & p \leqslant \frac{1}{2}. \end{cases}$$

Under an appropriate choice of the parameters p and q, this form of the equation can provide approximate DN of actual radiators (vibrators, slits, etc.), arranged near an ideally conducting cylinder.

Table 1 shows the values of the active and reactive components of the matrix of mutual resistances, calculated from the DN (15) by the method of [3].

Table 1.

i	$\kappa a = 22$		$\kappa a = 18$	
	r_{1i}	x_{1i}	r_{1i}	x_{1i}
1	1	0	1	0
2	+0,099	−0,164	0,274	−0,134
3	−0,009	0,040	−0,069	0,000
4	0,001	−0,014	0,022	0,015
5	−0,004	0,006	−0,001	−0,015
6	0,004	−0,001	−0,007	0,006

Parameters p and q were chosen to obtain an approximate DN of a transverse half-wave slit on the surface of an ideally conducting circular cylinder: p=1.137; q=1.2. The calculations were performed for two values of the electrical radius of the cylinder: ka=22 and ka=18, i.e. for a weak and strong intercoupling between the radiators.

Figures 3-7 show the amplitude-phase distributions at the inputs of the pattern-forming circuit (a) and the radiators of an antenna array[1] (b), as well as the DN which correspond to these (c) for various phasing directions. Further, the optimal distributions and DN are shown on the figures for comparison.

It can be seen from the graphs that the amplitude distribution at the inputs of the pattern-forming circuit is close to the uniform only for small numbers of harmonics and is oscillatory in nature for the larger numbers. Thus the conversion to an equal-amplitude distribution naturally produces a change in the amplitude-phase distribution of the currents on the radiators and, ultimately, reduces the directivity coefficient of the array. However, as can be seen from the graphs and Tables 2 and 3, this reduction is slight (<6%).

The graphs of Figs. 3-7 also show the distributions of the coefficients of reflection at the inputs of the pattern-forming circuit. These are determined solely by the matrix of mutual resistances [z] and do not depend on the scanning angle. The total reflected power is $\sim$2.0% when ka=22 and $\sim$7.3% when ka=18.

Tables 2 and 3 show the values of the directivity coefficient of an annular antenna array in the case of a scanning over ϕ and Θ for an optimal and an equal-amplitude excitation of the inputs of the pattern-forming circuit when ka=22 and ka=18, respectively. The calculations were performed with and without an allowance for the interaction between the radiators (i.e. when [z]=E).

[1]On the graphs the index n designates the numbers of the radiators n=1,2,...,N, while the index m=0, ±1...±N/2 -1, N/2 designates the numbers of the inputs of the pattern-forming circuit, corresponding to the excited current harmonics at the outputs.

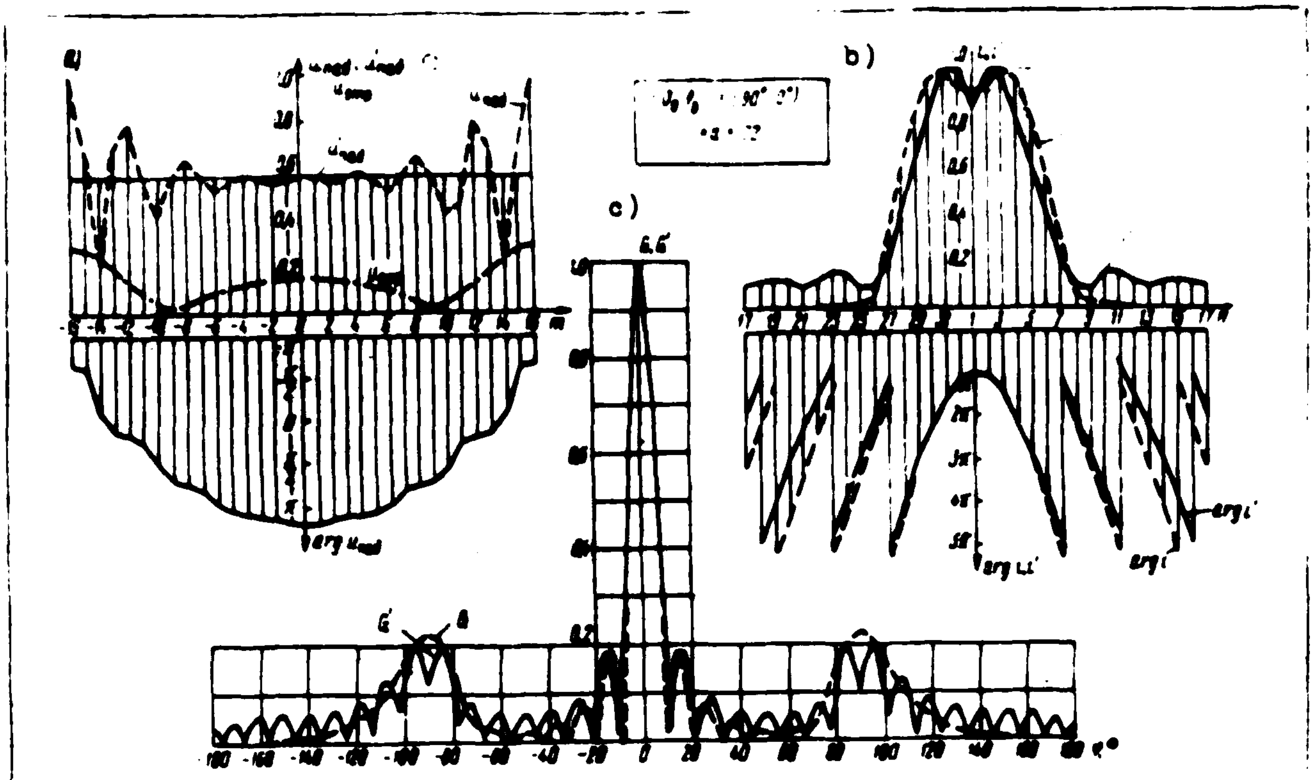

Fig. 3.

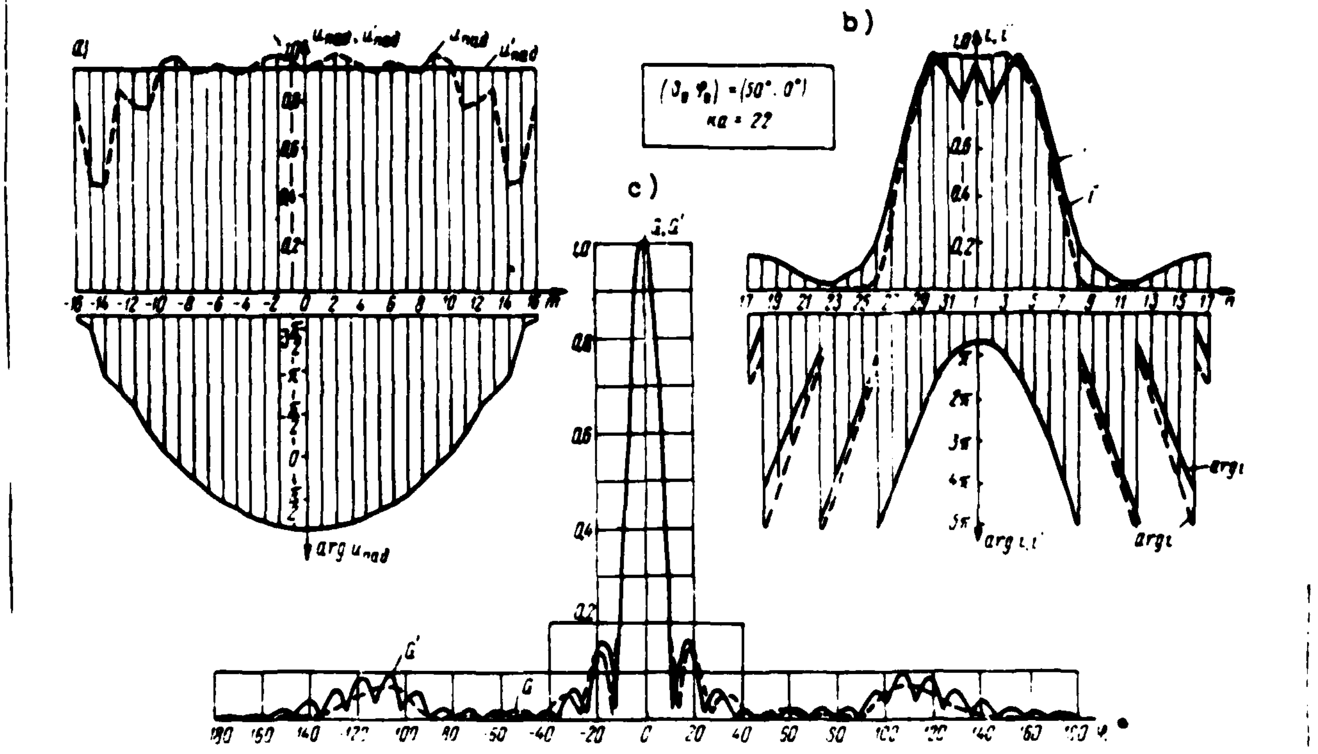

Fig. 4.

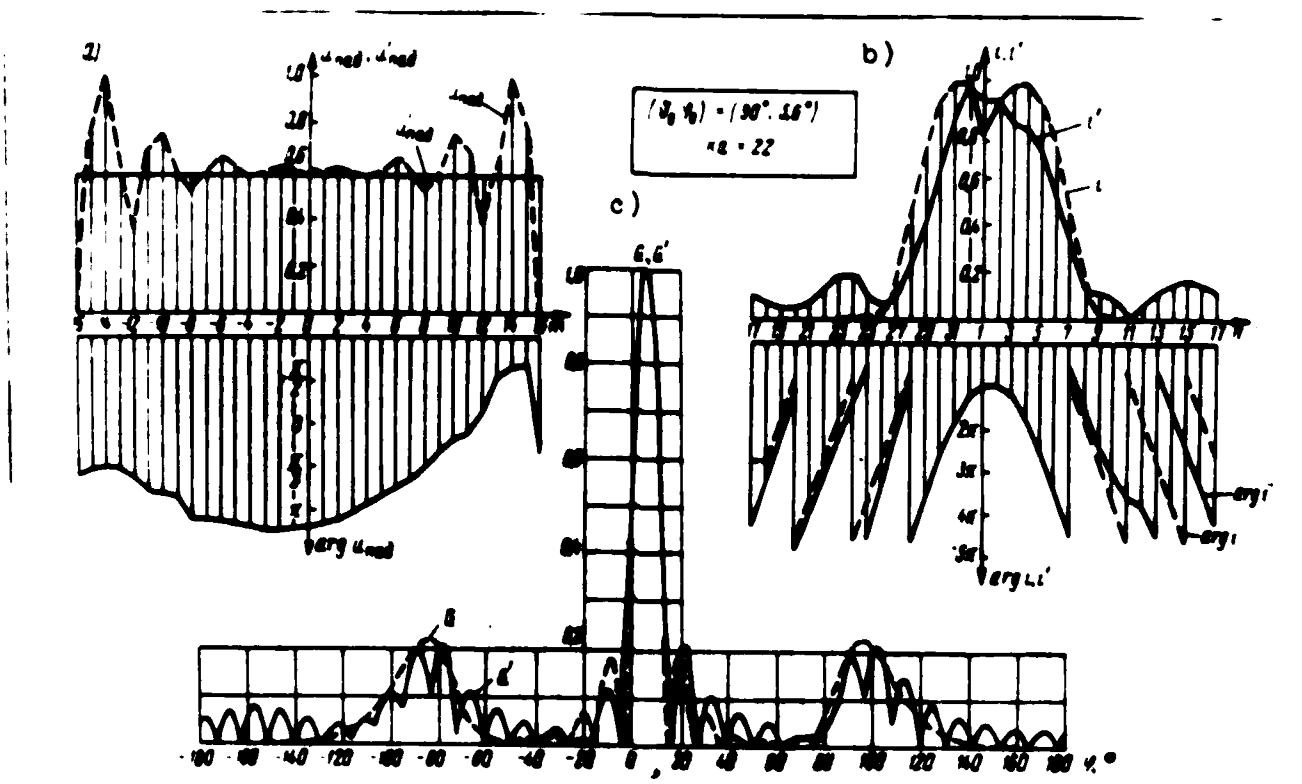

Fig.5.

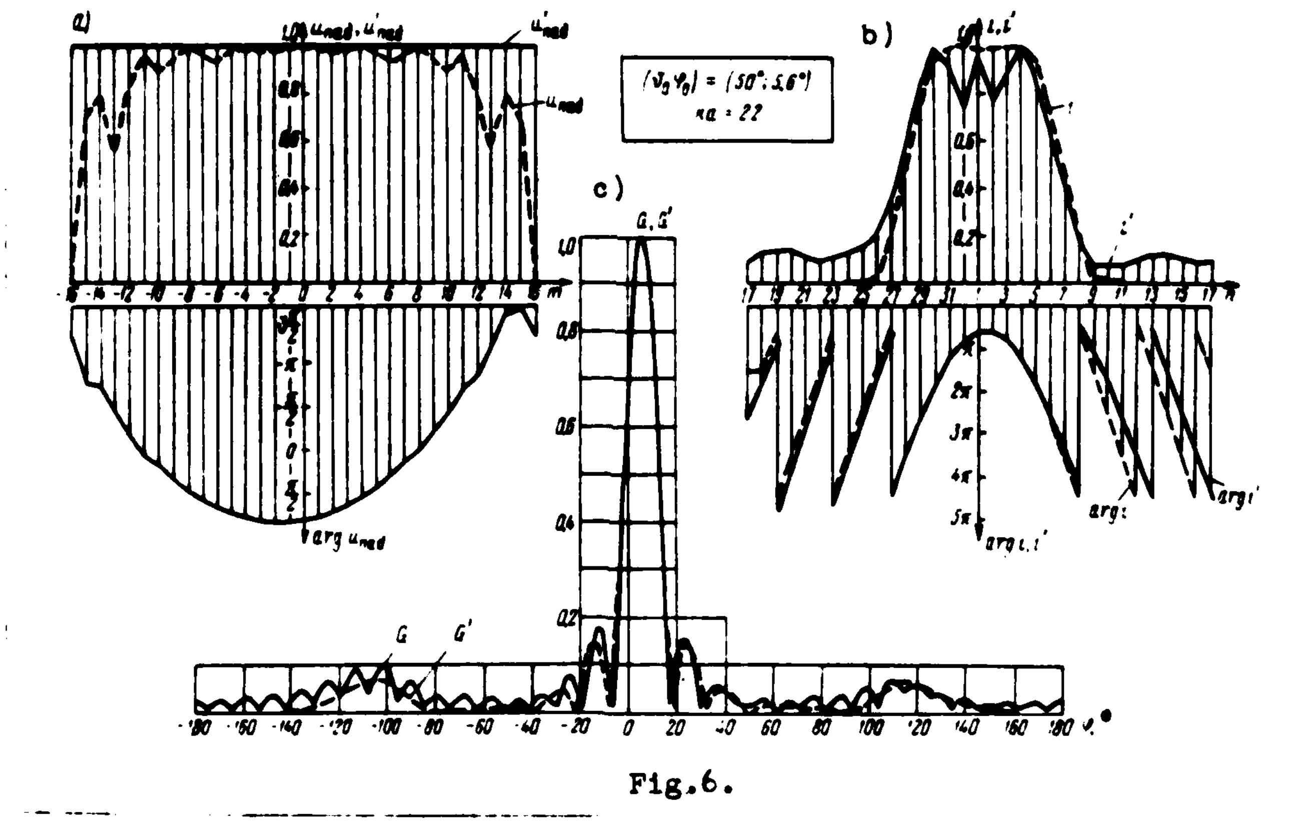

Fig.6.

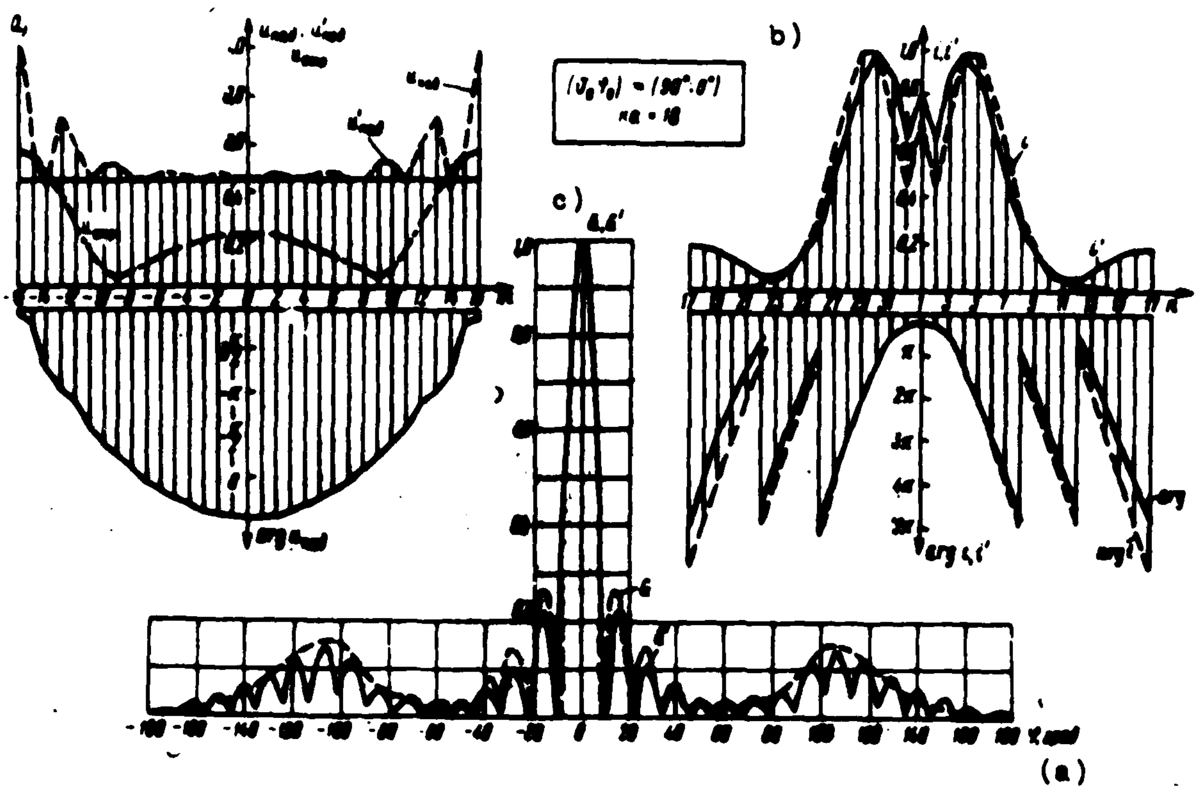

Fig. 7.
Key: (a) Degrees.

When ka=22, the intercoupling is small and the results of the calculations are practically in agreement. When ka=18, a disregarding of the interaction between the radiators will lead to an error in the directivity coefficient of $\lesssim 8\%$, the maximum errors occurring for a beam scanning in the equatorial plane ($\theta_0 = \pi/2$).

Figure 8 shows graphs for the change of the phases of the regulated phase shifters at the inputs of the pattern-forming circuit, which is necessary for a smooth scanning in the antenna array in the equatorial plane when ka=22. The graphs were constructed only for positive harmonics, as it is easy to demonstrate

Table 2.

	θ_0		(1) φ_0: 0°	1,4°	2,8°	4,2°	5,6°
$2N = 32$	90°	Opt	55,8	55,8	55,8	55,8	55,8
		$\lvert z \rvert \neq E$	52,6	53,1	52,4	53,1	51,9
		$\lvert z \rvert = E$	52,1	52,5	52,0	52,6	51,5
	70°	Opt	46,1	46,1	46,1	46,1	46,1
		$\lvert z \rvert \neq E$	44,4	44,4	44,2	44,5	43,6
		$\lvert z \rvert = E$	44,1	44,2	44,0	44,2	43,5
	60°	Opt	35,5	35,5	35,5	35,5	35,5
		$\lvert z \rvert \neq E$	34,7	34,6	34,6	34,7	34,1
		$\lvert z \rvert = E$	34,7	34,4	34,6	34,7	34,1
	50°	Opt	23,7	23,7	23,7	23,7	23,7
		$\lvert z \rvert \neq E$	22,9	22,8	22,8	22,9	22,6
		$\lvert z \rvert = E$	23,2	23,1	23,1	23,2	22,9
	40°	Opt	12,8	12,8	12,8	12,8	12,8
		$\lvert z \rvert \neq E$	11,5	11,5	11,8	11,5	11,4
		$\lvert z \rvert = E$	11,9	11,9	11,9	11,9	11,8

Key: (1) ϕ_0 values.

that, for a symmetrical DN, the phase radiators corresponding to positive and negative harmonics are related by:

$$\arg u_{-\kappa}(\varphi_0) = \arg u_{\kappa}\left(\frac{2\pi}{N} - \varphi_0\right) + \frac{2\pi}{N}\kappa. \tag{16}$$

It can be seen from the graphs that, for inputs which correspond to small harmonic numbers (m<8), the phase change is nearly linear, which is characteristic for a continuous circular antenna. For larger harmonic numbers, the phase distribution becomes sharply nonuniform.

Table 3.

ka	θ_0		(1) Значение φ				
			0°	1,4°	2,8°	4,2°	5,6°
$ka = 18$	90°	Opt	59,4	59,4	59,5	59,6	59,6
		$\|z\| \neq E$	57,0	56,7	56,8	57,0	55,8
		$\|z\| = E$	52,7	52,5	52,5	52,7	51,9
	70°	Opt	46,5	46,5	46,6	46,6	46,6
		$\|z\| \neq E$	45,6	45,4	45,5	45,6	44,8
		$\|z\| = E$	43,2	43,1	43,1	43,3	42,8
	60°	Opt	33,8	33,8	33,9	33,9	33,9
		$\|z\| \neq E$	33,4	33,3	33,4	33,5	33,0
		$\|z\| = E$	32,8	32,8	32,8	32,9	32,6
	50°	Opt	21,2	21,2	21,2	21,2	21,2
		$\|z\| \neq E$	20,3	20,3	20,3	20,3	20,1
		$\|z\| = E$	21,0	21,0	21,0	21,0	20,9
	40°	Opt	10,9	10,9	10,9	10,9	10,9
		$\|z\| \neq E$	9,5	9,4	9,4	9,4	9,4
		$\|z\| = E$	10,5	10,4	10,5	10,4	10,4

Key: (1) ϕ values.

As was pointed out above, the amplitude distribution at the inputs of the pattern-forming circuit differs considerably from the uniform for large harmonic numbers. The question therefore arises as to the feasibility of exciting these harmonics.

Figures 9 and 10 show the DN of an antenna array with the inputs, corresponding to the seven upper harmonics (m=±13, ±14, ±15, 16), closed on matched loads for ka=22 and ka=18, respectively. It can be seen that, although the elimination of the upper harmonics will reduce the directivity coefficient as a result of the beam expansion, the shape of the DN is slightly

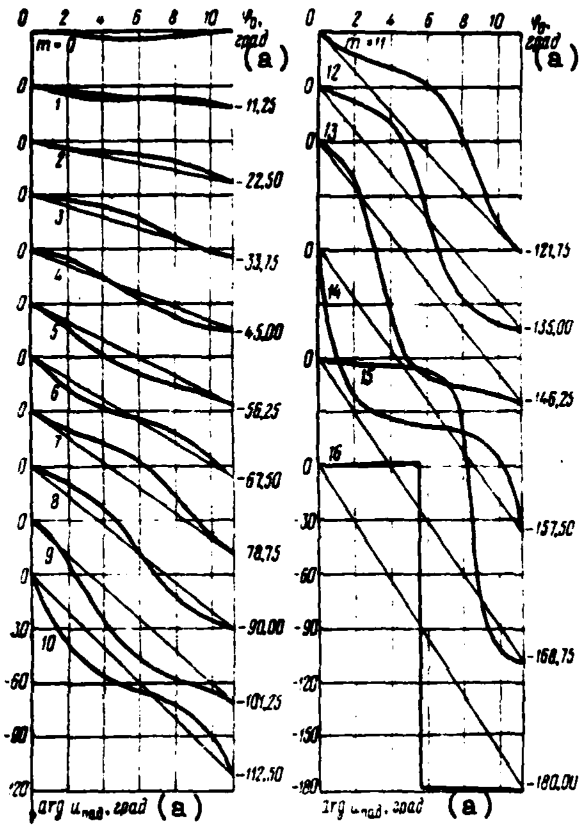

Fig. 8.
Key: (a) Degrees.

improved, since the level of side lobes is reduced. This can be utilized in the selection of the number of elements in the antenna array. The same figures show for comparison the DN obtained for corresponding 25-element arrays when all the harmonics are excited in them. It can be seen that the increase in the number of excited radiators in the case of an identical number of considered harmonics will increase the directivity coefficient of the array as a result of reducing the level of side lobes.

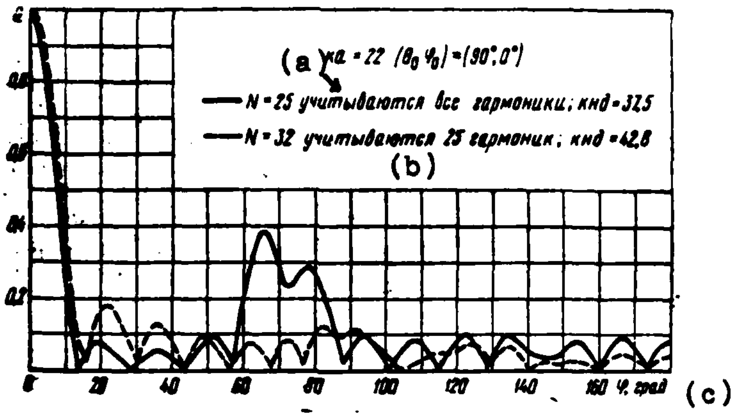

Fig. 9.
Key: (a) All the harmonics taken into account, directivity coefficient 37.5; (b) 25 harmonics taken into account, directivity coefficient 42.8; (c) Degrees.

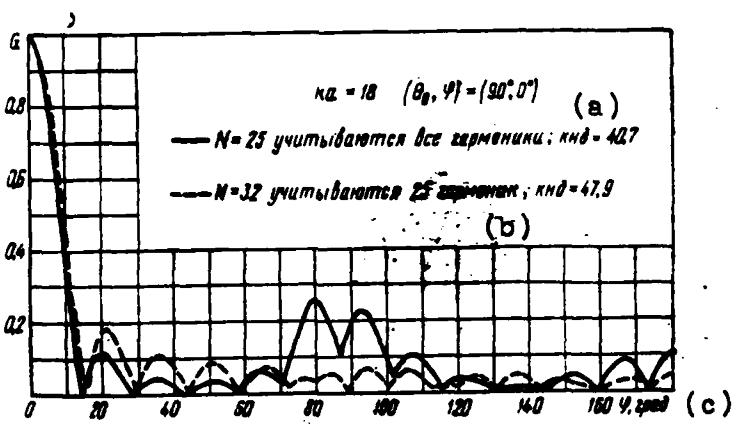

Fig. 10.
Key: (a) All harmonics taken into account, directivity coefficient 40.7; (b) 25 harmonics taken into account, directivity coefficient 47.9; (c) Degrees.

CONCLUSION

This paper, contrary to [1,2], proposes a procedure for calculating annular antenna arrays with phase scanning which takes into account the interaction between the radiators of the array.[1] Furthermore, the direction of the maximum directivity may change in both the equatorial and meridional planes.

These calculations demonstrate that the intercoupling between the radiators should be taken into account only for a scanning close to the equatorial plane. When the beam deviates from the equator ($\theta_0 \leqslant 70°$), the influence of the intercoupling can be disregarded.

This procedure is convenient for calculations by digital computer. The number of computations can be significantly reduced by exploiting the property of symmetry of the annular array, thanks to which in the matrix [z] it is sufficient to calculate merely (N/2 +1) elements. Moreover, the DN of a radiator may be calculated for a single element, the other DN being found by shifting the first through the appropriate angle. All the calculations can be done by hand in the case of small N.

[1]A calculation is given in [6] for partial DN of an annular array of interacting radiators in the form of infinite slits in an ideally conducting cylinder. However, questions of phase scanning were not considered.

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THE DESIGN OF THE CRITICAL WAVE NUMBER AND WAVE RESISTANCE OF A COAXIAL LINE WITH INTERIOR CONDUCTOR OF CROSS-SHAPED SECTION

G. P. Koshelev, Yu. B. Korchemkin and S. I. Shamayev

The paper reports the results of a theoretical investigation of a new waveguide with complicated profile of its cross section. Formulas are derived to calculate the critical wave number and wave resistance for various dimensions of the cross section.

The results of a numerical calculation for a broad range of cross sectional parameters are given in the form of graphs, convenient for practical use.

INTRODUCTION

Recently the theory and technique of waveguide devices is resorting more and more to waveguides with a complex shape of cross section, e.g. U, H, and E-shaped waveguides [1,2,3,4]. This interest can be accounted for by several reasons. In the first place, the use of such waveguides generally assures a gain in the dimensions and weight, and also produces a broader band for the fundamental oscillations, as compared with waveguides of simple shape (rectangular, circular, etc.). In the second place, as a rule, the basic characteristics of various devices (commutators, phase shifters, resonators, switches, etc.) can be significantly improved by employing waveguides with a complex cross section.

Theoretical investigations of any type of waveguide, since it is a transmission line, basically involve the determination of the critical wave numbers, wave resistances, and the dependence of these on the geometrical dimensions. The paper solves a similar problem for a homogeneous cylindrical waveguide, formed by an external circular tube 1 and an internal rod 2 with lengthwise ribs 3, the general appearance of which is shown in Fig. 1. The walls of the waveguide are considered to be ideally conducting. The symmetrical arrangement of the ribs relative to the orthogonal axes AA' and BB' creates conditions for the propagation of waves of circular or any given linear polarization while preserving the advantages of the U or H-shaped waveguides.

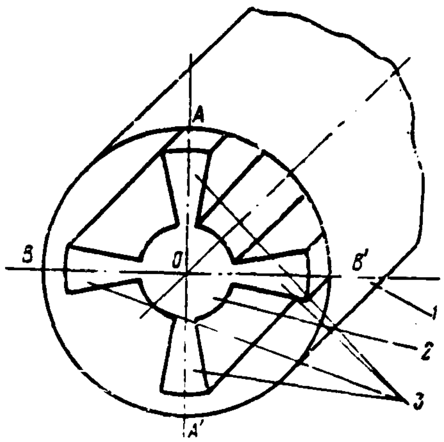

Fig. 1

STATEMENT OF THE PROBLEM. OBTAINING THE CHARACTERISTIC EQUATION

Let us first note that the cross section of the investigated waveguide represents a doubly connected region and, consequently, there may exist here TEM, TE, and TM types of waves. We shall confine ourselves to a consideration of a magnetic wave, similar to a wave of the type H_{11} in a coaxial waveguide.

As is known from the theory of cylindrical waveguides [5], the fields in these are entirely determined by the eigenfunctions ψ and eigenvalues κ of the Helmholtz equation:

$$\Delta_{\perp}\Psi - \varkappa^2\Psi = 0, \tag{1}$$

where $\Delta_{\perp}$ is the Laplacian for the transverse coordinates with boundary conditions on the contour L of the cross section (for a magnetic wave):

$$\left.\frac{\partial\Psi}{\partial n}\right|_{L} = 0. \tag{2}$$

Here, $\frac{\partial}{\partial n}$ is the normal derivative.

Additional conditions which emerge from the features of our chosen magnetic wave (by analogy with the wave H_{11} in a coaxial line) are:

$$\left.\frac{\partial\Psi}{\partial n}\right|_{BB'} = 0, \quad \Psi|_{AA'} = 0. \tag{3}$$

It is presumed in this case that the vector of the intensity of the electrical exciting field coincides in direction with the line AA' (Fig. 1).

The conditions (3) permit a narrowing of the considered region to the quadrant AOB (Fig. 2). Thus, the problem (1), (2) is reduced to finding the eigenfunctions and eigenvalues of equation (1) under the following boundary conditions:

$\Psi=0$ (on the line AC), (4)

$\frac{\partial\Psi}{\partial n}=0$ (on the remaining portion of the complex contour in quadrant AOB, including BD).

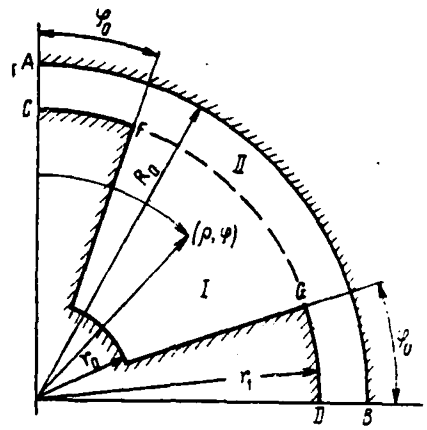

Fig. 2

In view of the fact that the boundary L coincides with the coordinate lines in a polar system (ρ,ϕ), it is most convenient to seek a solution by the method of partial regions. Therefore, let us break down the selected segment AOB into two simple regions I and II (Fig. 2), in which we shall designate the eigenfunctions as ψ_I and ψ_{II}. In order for these to represent the solutions in the entire region AOB, it is necessary to satisfy the conditions of continuity on the line FG[1]:

$$\Psi_I=\Psi_{II}\big|_{\rho=r_1}, \qquad (5)$$

$$\frac{\partial\Psi_I}{\partial\rho}=\frac{\partial\Psi_{II}}{\partial\rho}\bigg|_{\rho=r_1}. \qquad (5')$$

[1]Here and below all the designations conform to Fig. 2.

We shall represent the general solutions of equation (1) for each of the partial regions, satisfying boundary conditions (4), in the form:

$$\Psi_I = \sum_0^\infty A_m \left[\frac{J_{\alpha_m}(\varkappa\rho)}{J'_{\alpha_m}(\varkappa r_0)} - \frac{N_{\alpha_m}(\varkappa\rho)}{N'_{\alpha_m}(\varkappa r_0)} \right] \cos\alpha_m(\varphi-\varphi_0), \tag{6}$$

where $\alpha_m = \frac{\pi}{\pi/2 - 2\varphi_0}$ when m=0,1,2...

$$\Psi_{II} = \sum_0^\infty B_n \left[\frac{J_{\beta_n}(\varkappa\rho)}{J'_{\beta_n}(\varkappa R_0)} - \frac{N_{\beta_n}(\varkappa\rho)}{N'_{\beta_n}(\varkappa R_0)} \right] \sin\beta_n\varphi, \tag{6'}$$

where β_n=1+2n when n=0,1,2...

Let us introduce the auxiliary function:

$$f(\varphi) = \frac{\partial\psi}{\partial\rho}\bigg|_{\rho=r_1}. \tag{7}$$

Using (6) and (6') to calculate the partial derivatives in (5') and treating the obtained result as the expansion of f(ϕ) in a Fourier series of cos $\alpha_m(\phi-\phi_0)$ on the interval $[\phi_0, \frac{\pi}{2}-\phi_0]$ and a Fourier series of sin $\beta_n\phi$ on the interval $[0,\frac{\pi}{2}]$, we shall express the coefficients A_m and B_n by f(ϕ):

$$\left.\begin{aligned} A_m &= \frac{2}{\varkappa\psi_0}\,\frac{\displaystyle\int\limits_{\varphi_0}^{\pi/2-\varphi_0} f(\varphi)\cos\alpha_m(\varphi-\varphi_0)\,d\varphi}{\dfrac{J'_{\alpha_m}(\varkappa r_1)}{J'_{\alpha_m}(\varkappa r_0)}-\dfrac{N'_{\alpha_m}(\varkappa r_1)}{N'_{\alpha_m}(\varkappa r_0)}} \\ m &= 1,\ 2,\ 3,\ \ldots \\ A_0 &= \frac{1}{\varkappa\psi_0}\,\frac{\displaystyle\int\limits_{\varphi_0}^{\pi/2-\varphi_0} f(\varphi)\,d\varphi}{\dfrac{J'_0(\varkappa r_1)}{J'_0(\varkappa r_0)}-\dfrac{N'_0(\varkappa r_1)}{N'_0(\varkappa r_0)}} \end{aligned}\right\} \tag{8}$$

$$B_n = \frac{4}{\varkappa\pi}\,\frac{\displaystyle\int\limits_{\varphi_0}^{\pi/2-\varphi_0} f(\varphi)\sin\beta_n\varphi\,d\varphi}{\dfrac{J'_{\beta_n}(\varkappa r_1)}{J'_{\beta_n}(\varkappa R_0)}-\dfrac{N'_{\beta_n}(\varkappa r_1)}{N'_{\beta_n}(\varkappa R_0)}}, \tag{8'}$$

where $\psi_0=\pi/2-2\phi_0$, $n=0,1,2,3\ldots$ In the last integral, the limits $(0,\ \pi/2)$ have been replaced by $(\phi_0,\ \pi/2-\phi_0)$, since $f(\phi)=0$ on CF and GD in view of (4) and (7).

Inserting (8) and (8') in (6) and (6') and using condition (5), we obtain an integral equation for the auxiliary function $f(\phi)$:

$$\int\limits_{\varphi_0}^{\pi/2-\varphi_0} f(\xi)\,[P(\xi,\varphi)-P_0]\,d\xi=0, \tag{9}$$

where

$$
\left.\begin{aligned}
P(\xi,\ \varphi) &= \frac{4}{\pi}\sum_{0}^{\infty}\mathrm{Ct}_{\beta_n}(\varkappa r_1,\ \varkappa R_0)\sin\beta_n\varphi\sin\beta_n\xi - \\
&- \sum_{1}^{\infty}\frac{2}{\varphi_0}\mathrm{Ct}_{\alpha_m}(\varkappa r_1,\ \varkappa r_0)\cos\alpha_m(\varphi-\varphi_0)\cos\alpha_m(\xi-\varphi_0) \\
P_0 &= \frac{1}{\varphi_0}\mathrm{Ct}_0(\varkappa r_1,\ \varkappa r_0)
\end{aligned}\right\} \quad (10)
$$

The function

$$\mathrm{Ct}_\nu(x,\ y) = \frac{\dfrac{J_\nu(x)}{J'_\nu(y)} - \dfrac{N_\nu(x)}{N'_\nu(y)}}{\dfrac{J'_\nu(x)}{J'_\nu(y)} - \dfrac{N'_\nu(x)}{N'_\nu(y)}}$$

in accordance with [7], is known as the major radial cotangent of the index ν.

We shall solve the integral equation (9) approximately by the method of Galerkin-Bubnov, for which we shall represent the function $f(\phi)$ in the form:

$$f = \sum_{1}^{n} C_i f_i, \quad (11)$$

where $f_i (i=1,2,3\ldots)$ form a complete system of functions on the interval $[\phi_0\ \pi/2-\phi_0]$ and satisfy the same boundary conditions as $f(\phi)$, namely:

$$\frac{\partial f_i}{\partial\varphi} = 0 \text{ (at } \varphi=\varphi_0;\ \ \varphi=\pi/2-\varphi_0). \quad (12)$$

Inserting (11) in (9) and altering the sequence of integration and summation, we obtain a system of equations in terms of the coefficients C_i:

$$\sum_{i}^{n}(M_{ij}-P_0N_{ij})C_i=0, \tag{13}$$

where

$$M_{ij}=\int_{\varphi_0}^{\pi/2-\varphi_0}\int_{\varphi_0}^{\pi/2-\varphi_0} f_i(\xi)P(\xi_1, \varphi)f_j(\varphi)\,d\xi\,d\varphi,$$

$$N_{ij}=\int_{\varphi_0}^{\pi/2-\varphi_0}\int_{\varphi_0}^{\pi/2-\varphi_0} f_i(\xi)f_j(\varphi)\,d\xi\,d\varphi.$$

A condition for the existence of a nontrivial solution of the system of equations (13) is its determinant equaling zero:

$$\det\| M_{ij}-P_0N_{ij}\| = 0. \tag{14}$$

Equation (14) is characteristic and contains, as unknown, the number κ, i.e. the critical wave number. The least value of this number corresponds, evidently, to the investigated type of magnetic wave.

SOLUTION OF THE CHARACTERISTIC EQUATION. DISCUSSION OF THE RESULTS

As a complete system of functions f_i which satisfy condition (12), let us select a sequence of the type

$$f_i=\cos\left[(i-1)\frac{\pi}{\varphi_0}(\varphi-\varphi_0)\right. \tag{15}$$

Such a selection greatly simplifies the ultimate form of the characteristic equation, since $N_{ij}=0$ when $i\neq 1$ and $j\neq 1$. Using (15) to calculate M_{ij} and N_{ij} and inserting the values of these coefficients in (14), we obtain:

1) for the first approximation [n=1 in expansion (11)]

$$\sum_{n=0}^{\infty} Ct_{\beta_n}(\varkappa_I r_1, \varkappa_I R_0) \frac{\sin^2 \beta_n \frac{\psi_0}{2}}{\beta_n^2} - \frac{\pi\psi_0}{8} Ct_0(\varkappa_I r_1, \varkappa_I r_0) = 0, \tag{16}$$

2) for the second approximation [n=2 in expansion (11)]

$$\sum_{0}^{\infty} Ct_{\beta_n}(\varkappa_{II} r_1, \varkappa_{II} R_0) \frac{\sin^2 \beta_n \frac{\psi_0}{2}}{\beta_n^2} - \frac{\pi\psi_0}{8} Ct_0(\varkappa_{II} r_1, \varkappa_{II} r_0) - \frac{\left[\sum_{0}^{\infty} Ct_{\beta_n}(\varkappa_{II} r_1, \varkappa_{II} R_0) \frac{\sin \beta_n \psi_0 \sin \beta_n \frac{\pi}{2}}{\beta_n^2 - (\pi/\psi_0)^2}\right]^2}{4\sum_{0}^{\infty} Ct_{\beta_n}(\varkappa_{II} r_1, \varkappa_{II} R_0) \frac{\beta_n^2 \cos^2 \beta_n \frac{\psi_0}{2}}{\left[\beta_n^2 - \left(\frac{\pi}{\psi_0}\right)^2\right]^2} - \frac{\pi\psi_0}{4} Ct_{\frac{\pi}{\psi_0}}(\varkappa_{II} r_1, \varkappa_{II} r_0)} = 0. \tag{17}$$

The equation obtained for the third approximation κ_{III} is too unwieldy and not given in the paper.

Let us note that, in the case of $\phi_0=\pi/4$, the investigated waveguide becomes coaxial with a radius R_0 (external) and r_1 (internal). In this case, equations (16) and (17) are reduced, by simple transformations and a passage to the limit, to the characteristic equation in terms of the critical wave number for a wave of type H_{11} in a coaxial waveguide.

The solutions of the transcendental equations (16) and (17) are respectively the first κ_I and second κ_{II} approximations of the sought critical wave number κ. In order to estimate their convergence, several trial computations were made, revealing that, when $r_1 \leq 0.9R_0$ and $\phi_0=0$, the error between κ_I and κ_{II} does not exceed 30%, while between κ_{II} and κ_{III} is less than 5%, the error diminishing with increasing ϕ_0. Thus, a third approximation only slightly improves the results of the second; this fact was the basis of a numerical calculation by digital computer.

We found κ in accordance with (17) in a broad region of variation of the cross section parameters of the waveguide, specifically:

$$0 \leqslant \varphi \leqslant \frac{\pi}{2},$$
$$0{,}5R_0 \leqslant r_1 < 0{,}95R_0,$$
$$0{,}1R_0 \leqslant r_0 \leqslant 0{,}7R_0.$$

The calculation results are represented in the form of graphs, indicated by solid lines in Figs. 3-6, where the nondimensional quantity κR_0 is laid off along the vertical axis (at the left). The radial parameters have also been replaced by the nondimensional relations r_0/R_0 and r_1/R_0, allowing the results to be used for any given values of the external radius R_0. An analysis of these graphs reveals that the behavior of the critical wave number κ is characterized by the following features:

1) a weak dependence on the internal radius r_0;

2) a strong dependence on r_1, especially when $r_1 \to R_0$;

3) the presence of an optimum depending on ϕ_0 near the value $\pi/8$.

The latter two circumstances suggest a qualitative agreement with the behavior of the critical wave number of a type H_{10} wave in a U and H-shaped waveguide [8].

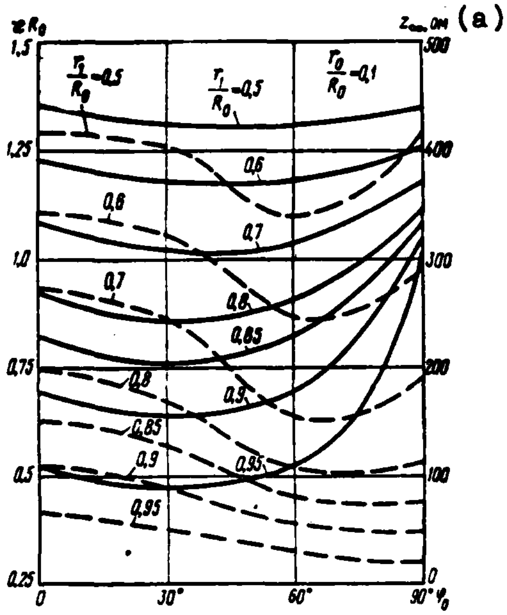

Fig. 3.
Key: (a) Ohms.

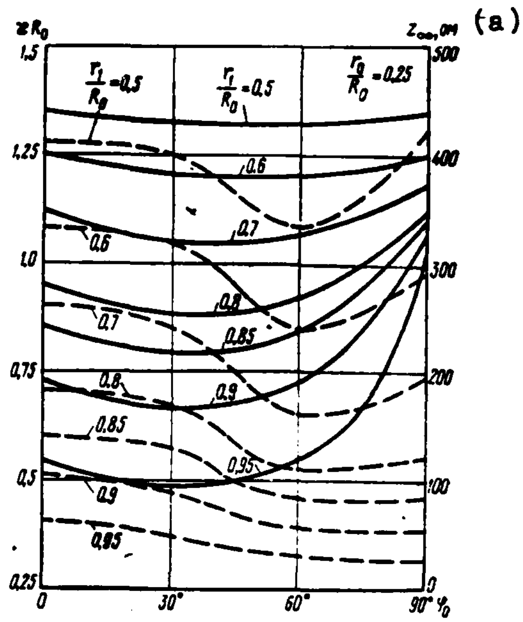

Fig. 4.
Key: (a) Ohms.

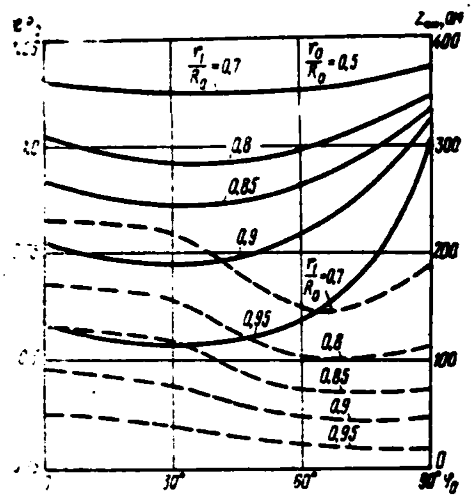

Fig. 5.
Key: (a) Ohms.

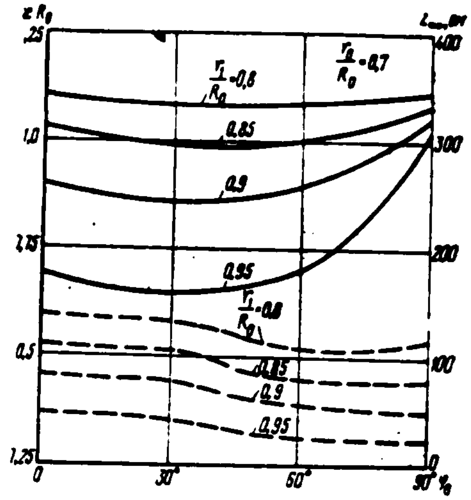

Fig. 6.
Key: (a) Ohms.

CALCULATION OF THE WAVE RESISTANCE

In order to determine the wave resistance of the investigated waveguide, we shall employ the familiar formula

$$Z = \frac{U_m^2}{2P}, \tag{18}$$

where U_m is the amplitude of the "potential" of the traveling wave; P is the power which passes through the cross section S_1.

Presuming that

$$U_m = \int_{AA'} E_\rho \, d\rho ,$$

$$P = \frac{1}{2} \int_{S_l} [\vec{E}\vec{H}] \, d\vec{S}$$

and representing the components of the fields $\vec{E}$ and $\vec{H}$ by the eigenfunctions of the partial regions, we obtain:

$$Z = \frac{Z_0}{\varkappa^2} \frac{\int\limits_{r_1}^{R_0} \frac{\partial \psi_{II}(\rho, 0)}{\partial \varphi} \frac{d\rho}{\rho}}{\int\limits_{r_0}^{r_1} \int\limits_{\varphi_0}^{\pi/2-\varphi_0} \psi_I^2(\rho, \varphi)\,\rho\, d\rho\, d\varphi + \int\limits_{r_1}^{R_0} \int\limits_{0}^{\pi/2} \psi_{II}^2(\rho, \varphi)\,\rho\, d\rho\, d\varphi}, \tag{19}$$

where

$$Z_0 = \frac{\varkappa}{\sqrt{\kappa^2 - \varkappa^2}} \sqrt{\frac{\mu_0}{\varepsilon_0}},$$

κ is the wave number in free space; ε_0 and μ_0 are respectively the electrical and magnetic permeability.

The quantities ψ_I and ψ_{II}, necessary to calculate the integrals in (19), are expressed by inserting (8) and (8') in (6) and (6') in terms of the function $f(\phi)$ which, in accordance with (11), is determined by means of the coefficients C_i, which are a solution of the system of equations (13) when conditions (14) are satisfied. Confining ourselves to a second approximation, as in the case of the calculation of the critical wave number, we obtain the following formula as a result:

$$Z = \frac{64}{\pi^2} \frac{Z_0}{\varkappa^2} \times$$

$$\times \frac{\left\{ \sum\limits_0^\infty D_n \left[\frac{1}{J_{\beta_n}(\varkappa R_0)} \int\limits_{r_1}^{R_0} J_{\beta_n}(\varkappa\rho) \frac{d\rho}{\rho} - \frac{1}{N_{\beta_n}(\varkappa R_0)} \int\limits_{r_1}^{R_0} N_{\beta_n}(\varkappa\rho) \frac{d\rho}{\rho} \right] \right\}^2}{\psi_0 [Q_0(r_1) - Q_0(r_0)] + \frac{\psi_0}{2} F^2 [Q_{\pi/\psi_0}(r_1) - Q_{\pi/\psi_0}(r_0)] + \frac{16}{\pi} \sum\limits_0^\infty D_n^2 [T_n(R_0) - T_n(r_1)]}, \tag{20}$$

where

$$D_n = \frac{\sin\beta_n \frac{\pi}{4} \sin\beta_n \frac{\psi_0}{2} + a_n \cos\beta_n \frac{\pi}{4} \cos\beta_n \frac{\psi_0}{2}}{\frac{J'_{\beta_n}(\varkappa r_1)}{J'_{\beta_n}(\varkappa R_0)} - \frac{N'_{\beta_n}(\varkappa r_1)}{N'_{\beta_n}(\varkappa R_0)}},$$

$$a_n = \frac{F}{1-\left(\frac{\pi}{\psi_0}\right)^2 \frac{1}{\beta_n^2}},$$

$$F = \frac{\sum_0^\infty \mathrm{Ct}_{\beta_n}(\varkappa r_1, \varkappa R_0) \frac{\sin^2\beta_n \frac{\psi_0}{2}}{\beta_n^2} - \frac{\psi_0\pi}{8} \mathrm{Ct}_0(\varkappa r_1, \varkappa r_0)}{\frac{1}{2}\sum_0^\infty \mathrm{Ct}_{\beta_n}(\varkappa r_1, \varkappa R_0) \frac{\sin\beta_n\psi_0 \sin\beta_n \frac{\pi}{2}}{\beta_n^2 - \left(\frac{\pi}{\psi_0}\right)^2}},$$

$$Q_p(\xi) = \frac{\xi^2}{2} \frac{\left[\frac{J_p(\varkappa\xi)}{J'_p(\varkappa r_0)} - \frac{N_p(\varkappa\xi)}{N'_p(\varkappa r_0)}\right]^2 - \left[\frac{J_{p-1}(\varkappa\xi)}{J'_p(\varkappa r_0)} - \frac{N_{p-1}(\varkappa\xi)}{N'_p(\varkappa r_0)}\right] \times \left[\frac{J_{p+1}(\varkappa\xi)}{J'_p(\varkappa r_0)} - \frac{N_{p+1}(\varkappa\xi)}{N'_p(\varkappa r_0)}\right]}{\left[\frac{J'_p(\varkappa r_1)}{J'_p(\varkappa r_0)} - \frac{N'_p(\varkappa r_1)}{N'_p(\varkappa r_0)}\right]^2},$$

$$T_n(\xi) = \frac{1}{\beta_n^2}\frac{\xi^2}{2}\left\{\left[\frac{J_{\beta_n}(\varkappa\xi)}{J'_{\beta_n}(\varkappa r_0)} - \frac{N_{\beta_n}(\varkappa\xi)}{N'_{\beta_n}(\varkappa r_0)}\right]^2 - \left[\frac{J_{\beta_n-1}(\varkappa\xi)}{J'_{\beta_n}(\varkappa r_0)} - \frac{N_{\beta_n-1}(\varkappa\xi)}{N'_{\beta_n}(\varkappa r_0)}\right]\left[\frac{J_{\beta_n+1}(\varkappa\xi)}{J'_{\beta_n}(\varkappa r_0)} - \frac{N_{\beta_n+1}(\varkappa\xi)}{N'_{\beta_n}(\varkappa r_0)}\right]\right\};$$

$$\beta_n = 1+2n \quad n = 0, 1, 2, \ldots$$

A calculation by formula (20), including the computation of the integrals in the numerator, was carried out by digital computer, using a numerical method. The results of the calculation for various dimensions of the cross section are represented in the form of graphs, shown by dotted lines in Figs. 3-6, where the values of the resistances Z_∞ for infinite frequency are laid off along the vertical axis (at the right), so that:

$$Z = \frac{Z_\infty}{\sqrt{1-\left(\frac{\varkappa}{\kappa}\right)^2}}.$$

In view of the fact that the representation of the eigenfunctions ψ_I and ψ_{II} in the second approximation is unsatisfactory for values of ϕ_0 close to zero and r_1 close to R_0, the accuracy of the calculation by formula (20) is reduced when $\phi_0 \to 0$ and $r_1 \to R_0$.

An analysis of the obtained results shows that the nature of the dependence of Z on ϕ_0 and r_1, except for the above-mentioned region, qualitatively agrees with the behavior of the wave resistance of a type H_{10} wave in a U or H-shaped waveguide [8] under a corresponding replacement of the dimensions of the cross section.

EXPERIMENTAL CONFIRMATION

In order to confirm the theoretical calculations of the critical wave number κ_{Teop} by formula (17), an experimental investigation of the waveguide (Fig. 1) was carried out. At various values of the cross section parameters, the wavelength was measured in the waveguide and used to calculate the quantity $\kappa_{\text{экс}}$. The results of the confirmation are shown in the table below. Also shown here are the error $\sigma = \left|\frac{\varkappa_{\text{экс}} - \varkappa_{\text{теор}}}{\varkappa_{\text{теор}}}\right|$ % and the measurement error $\sigma_{\text{изм}}$. As can be seen from the table, $\sigma \leqslant \sigma_{\text{изм}}$.

Table

$2\varphi_0$	r_1/R_0	r_0/R_0	$\chi_{экс} R_0$	$\chi_{теор} R_0$	$\sigma_{изм}$	σ
45°	0,9	0,1875	0,679	0,670	3,6%	1,4%
45°	0,75	0,1875	0,974	0,956	2,5%	1,9%
45°	0,625	0,1875	1,142	1,142	2,0%	0,5%
18°	0,9	0,1875	0,687	0,668	3,4%	2,8%

Thus, the results of the calculation of the critical wave number, shown in Figs. 3-6, have an accuracy which is sufficient for practice.

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THE SYNTHESIS OF DIFFERENCE DIRECTION PATTERNS OF SPHERICAL ANTENNAS

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The paper considers the problem of minimization of the lateral space radiation of difference direction patterns of spherical non-superdirectional antennas in the form of an idealized continuous system of diffraction-type radiators, arranged on an ideally conducting spherical surface.

INTRODUCTION

In radar systems designed for precise measurement of the angular coordinates of objects there arises the problem of constructing difference direction patterns with a small level of side radiation [1-3].

It is known from [2,3] that the problem of Dolf-Chebyshev for difference direction patterns is formulated in the following manner: under a given level of side radiation (R) the slope of the normalized difference characteristic curve in the direction of the bearing (γ) should be a maximum, while the width of both major lobes should be minimal at the zeroes ($\Delta\theta_0$) and, on the contrary, at a given value of γ the level of side radiation and $\Delta\theta_0$ should be minimal. The designations R, γ, and $\Delta\theta_0$ are explained in Fig. 1.

A large number of Soviet and foreign works, e.g. [1-3], have been devoted to the problem of the formulation of difference direction patterns which are optimal or close to optimal, in the

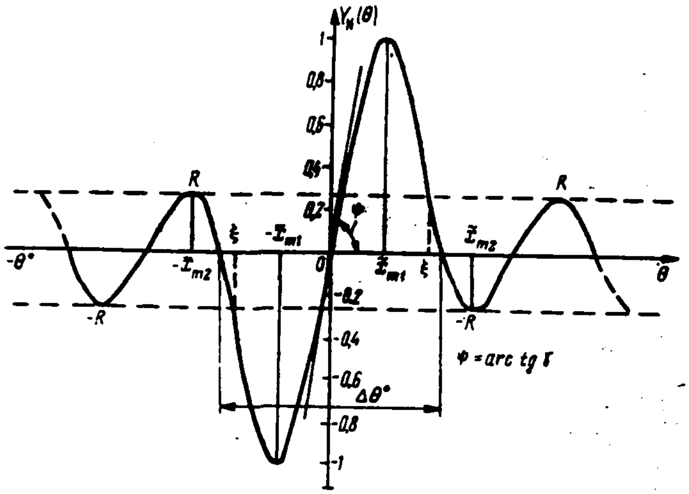


Fig. 1.

above-mentioned sense, by means of planar arrays, as well as linear, rectangular, and circular apertures with continuous distributions of the exciting currents.

This paper considers a solution of an analogous problem for spherical antennas in the form of rather closely spaced diffraction-type radiators, cut out in an ideally conducting spherical surface, so that the distribution of the exciting current may be regarded as continuous. The need to use convex (especially spherical) pencil-beam antenna arrays occurs, for example, when implementing a wide-angle electrical scanning with the shape of the direction pattern changing little or not at all.

The problem is solved in the most general vector form by the method of eigenfunctions, enabling an allowance for the diffraction effects at the surface of the spherical antennas.

THE FUNDAMENTAL STARTING RELATIONSHIPS

We shall show that the problem of minimization of the side radiation in spatial difference direction patterns of spherical antennas can be reduced, under certain conditions, to the solution of two unidimensional problems.

For this purpose, we shall represent the vectorial spatial direction pattern $\bar{F}(\theta,\phi)$ (θ and ϕ are the spherical coordinates) of a spherical antenna in the form of a series in the vectorial eigenfunctions $\bar{E}^{\mu}_{mn}$ and $\bar{E}^{e}_{mn}$ [4]:

$$\bar{F}(\theta, \varphi) = \sum_{m} \sum_{n} \left(C^{e}_{mn} \bar{E}^{e}_{mn} + C^{\mu}_{mn} \bar{E}^{\mu}_{mn}\right), \tag{1}$$

where C^{e}_{mn} and C^{μ}_{mn} are the unknown excitation coefficients.

As the plane for formulation of the difference direction pattern (bearing plane) let us use the plane $\phi=\pi/2$ and let the zero direction of the difference pattern coincide with the axis $Z(\theta=0^{\circ})$ of the spherical system of coordinates.

After using an explicit representation of the vector eigenfunctions in the spherical system of coordinates [4] and considering that the difference pattern is equal to zero when $\theta=0^{\circ}$, and considering the behavior of γ in the direction $\theta=0^{\circ}$, we may demonstrate that the index of summation in (1) should be put at $m=2$ for $\bar{E}^{e}_{mn}$ and $m=0$ or 2 for $\bar{E}^{\mu}_{mn}$. Then, for the difference pattern $\bar{F}^{\Delta}(\theta\phi)$ from (1) we shall have:

$$\bar{F}_\Delta(\theta,\varphi)=\sum_{n=1}^{N}\left[C_{2n}^{e}\frac{\partial P_n^2(\cos\theta)}{\partial\theta}+C_{2n}^{\mu}\frac{2}{\sin\theta}P_n^2(\cos\theta)\right]\sin 2\varphi\,\bar{i}_\theta - $$
$$+\left[C_{2n}^{e}\frac{2}{\sin\theta}P_n^2(\cos\theta)+C_{2n}^{\mu}\frac{\partial P_n^2(\cos\theta)}{\partial\theta}\right]\cos 2\varphi\,\bar{i}_\varphi + C_{0n}^{\mu}P_n^1(\cos\theta)\,\bar{i}_\varphi \cdot \quad (2)$$

where $P_n^m(\cos\theta)$ are the associated Legendre polynomials; i_θ and $\bar{i}_\phi$ are the unit vectors of the spherical system of coordinates.

Let us note that, in order to exclude the superdirectional mode, we shall restrict the upper index of summation in (2) to the condition $N=ka$ (ka is the electrical radius of the antenna) [4].

We shall select the coefficients C_{2n}^{e}, C_{2n}^{μ}, C_{0n}^{μ} such that the spatial difference pattern is optimal in the sense of Dolf and Chebyshev.

As is known [1,2], a spatial amplitude direction pattern $|\bar{F}_\Delta(\theta,\phi)|$ in the plane $\phi=0$, perpendicular to the plane $\phi=\pi/2$, should be equivalent to zero, while in the plane $\phi=\pi/2$ a perfectly natural condition is imposed on $|\bar{F}_\Delta(\theta,\phi)|$, namely that $|\bar{F}_\Delta(\theta,\pi/2)|$ should satisfy the requirements of an optimal difference pattern, which we shall designate as $Y_N(\theta)$. Thus,

$$\left|\bar{F}_\Delta\left(\theta,\frac{\pi}{2}\right)\right| = |Y_N(\theta)|;\quad |\bar{F}_\Delta(\theta,0)| \equiv 0. \quad (3)$$

Then, from (2) and (3) we may find that:

$$\sum_{n=1}^{N} C_{0n}^{\mu}P_n^1(\cos\theta) = \frac{1}{2}Y_N(\theta), \quad (4a)$$

$$\sum_{n=1}^{N}\left[C_{2n}^{e}\frac{2}{\sin\theta}P_n^2(\cos\theta)+C_{2n}^{\mu}\frac{\partial P_n^2(\cos\theta)}{\partial\theta}\right]=-\frac{1}{2}Y_N(\theta). \tag{4b}$$

It can be shown that, if additional conditions $C_{2n}^{e}=C_{2n}^{\mu}$ are imposed on the excitation coefficients, then the spatial vectorial difference pattern is represented in the form:

$$\overline{F}_{\Delta}(\theta,\ \varphi)=Y_N(\theta)\sin\varphi\,[\sin\varphi\,\overline{i}_{\varphi}-\cos\varphi\,\overline{i}_{\theta}], \tag{5}$$

while the amplitude pattern is:

$$|\overline{F}_{\Delta}(\theta,\varphi)|=|Y_N(\theta)\sin\varphi|, \tag{6}$$

i.e. it has a practically satisfactory dependence of the azimuth on the coordinate ϕ [1].

Thus, the above-formulated problem for $\overline{F}_{\Delta}(\theta,\phi)$ has been reduced to two unidimensional problems (4a) and (4b) under the condition $C_{2n}^{e}=C_{2n}^{\mu}$.

In order to solve these, let us examine the properties of the functions:

$$F_{N1}^{\Delta}=\sum_{n=1}^{N}C_{2n}^{\mu}P_n^1(\cos\theta), \tag{7a}$$

$$F_{N2}^{\Delta}=\sum_{n=1}^{N}C_{2n}^{\mu}\left[\frac{2}{\sin\theta}P_n^2(\cos\theta)+\frac{\partial P_n^2(\cos\theta)}{\partial\theta}\right]. \tag{7b}$$

Considering the known properties of the associated Legendre polynomials [8], we can represent $F_{N1}^{\Delta}(\theta)$ and $F_{N2}^{\Delta}(\theta)$ in the form:

$$F_{N1}^{\Delta} = (1-x^2)^{1/2} \sum_{n=1}^{N} C_{0n}^{\mu} K_n(\theta), \tag{8a}$$

$$F_{N2}^{\Delta} = (1-x^2)^{1/2}(1+x) \sum_{n=1}^{N} C_{2n}^{\mu} L_n(\theta). \tag{8b}$$

Here,

$$x = \cos\theta,$$

$$K_n(\theta) = \sum_{\kappa=0}^{n-1} \varkappa_{\kappa n} x^{\kappa}, \quad n = 1, 2, \ldots N, \tag{9a}$$

$$L_n(\theta) = \sum_{\kappa=0}^{n-2} l_{\kappa n} x^{\kappa}, \quad n = 2, 3, \ldots N, \tag{9b}$$

where κ_{kn} and l_{kn} designate the coefficients which are easily determined from (7a) and (7b), respectively.

From (7a)-(8b) it is not difficult to obtain:

$$F_{N1}^{\Delta} = (1-x^2)^{1/2} \sum_{\kappa=0}^{N-1} a_{\kappa} x^{\kappa}, \tag{10a}$$

$$F_{N2}^{\Delta} = (1-x^2)^{1/2}(1+x) \sum_{\kappa=0}^{N-2} b_{\kappa} x^{\kappa}, \tag{10b}$$

where

$$a_{\kappa} = \sum_{n=\kappa+1}^{N} C_{0n}^{\mu} \varkappa_{\kappa n}, \tag{11a}$$

$$b_{\kappa} = \sum_{n=\kappa+1}^{N} C_{2n}^{\mu} l_{\kappa n}. \tag{11b}$$

Let us further replace the variables in (10a) and (10b) (the necessity of which will become clear from the following):

$$x = 2\tilde{x} - 1. \tag{12}$$

Then, $F_{N1}^{\Delta}(\theta)$ and $F_{N2}^{\Delta}(\theta)$ can be converted to:

$$F_{N1}^{\Delta}(\theta) = 2[\tilde{x}(1-\tilde{x})]^{1/2} \sum_{p=0}^{N-1} \tilde{a}_p \tilde{x}^p, \tag{13a}$$

$$F_{N2}^{\Delta}(\theta) = 2[\tilde{x}(1-\tilde{x})]^{1/2}\, 2\tilde{x} \sum_{p=0}^{N-2} \tilde{b}_p \tilde{x}^p. \tag{13b}$$

Here, $\tilde{a}_p$ and $\tilde{b}_p$ designate the new coefficients $F^{\Delta}_{N1}(\theta)$ and $F^{\Delta}_{N2}(\theta)$, related to the old coefficients by:

$$\tilde{a}_p = \sum_{\kappa=p}^{N-1} a_\kappa (-1)^{\kappa-p} 2^p C^p_\kappa, \tag{14a}$$

$$\tilde{b}_p = \sum_{\kappa=p}^{N-2} b_\kappa (-1)^{\kappa-p} 2^p C^p_\kappa, \tag{14b}$$

while C^p_k is the number of combinations of k elements taken p at a time.

Thus, as is seen from (13a) and (13b), $F^{\Delta}_{N1}(\theta)$ and $F^{\Delta}_{N2}(\theta)$ represent quasipolynomials of the form

$$F^{\Delta}_N(\theta) = 2[\tilde{x}(1-\tilde{x})]^{1/2} Q_{N-1}(\tilde{x}), \tag{15}$$

where $Q_{N-1}(\tilde{x})$ is a (N-1) degree polynomial in $\tilde{x}=\cos^2\theta/2$, represented by:

$$Q_{N-1}(\tilde{x}) = \begin{cases} \sum_{p=0}^{N-1} \tilde{a}_p \tilde{x}^p & \text{when} \quad F^{\Delta}_{N1}(\theta), \ (13a) \\ 2\tilde{x} \sum_{p=0}^{N-2} \tilde{b}_p \tilde{x}^p & \text{when} \quad F^{\Delta}_{N2}(\theta). \ (13b) \end{cases} \tag{16}$$

The problem further consists in constructing such a quasipolynomial $F^{\Delta}_N(\theta)$ (or, which is the same thing, in selecting the coefficients of the polynomial $Q_{N-1}(\tilde{x})$) so that the spatial difference pattern $\bar{F}^{\Delta}_N(\theta,\phi)$ - (5) and (6) - is optimal in the sense of Dolf and Chebyshev on the interval $0\leqslant\tilde{x}\leqslant 1$ $(0\leqslant\theta\leqslant\pi)$.

CHEBYSHEV-AKHIYEZER POLYNOMIALS IN THE SOLUTION OF THE PROBLEM OF MINIMIZATION OF THE SIDE RADIATION IN DIFFERENCE PATTERNS OF SPHERICAL ANTENNAS

Let us try to convert the above problem to an analogous problem for linear antennas [3], for which we shall change the variables:

$$\tilde{x} = 1 - \tilde{y}^2, \tag{17}$$

which converts the interval $0 \leqslant \tilde{x} \leqslant 1$ to the interval $-1 \leqslant \tilde{y} \leqslant +1$ (the selected direction of bearing $\theta = 0°$ corresponding to the value $\tilde{y} = 0$), and the quasipolynomial (15) to a quasipolynomial of the form:

$$F_N^\Delta(\theta) = (1 - \tilde{y}^2)^{1/2}\, \Gamma_{2N-1}(\tilde{y}), \tag{18}$$

where

$$\Gamma_{2N-1}(\tilde{y}) = \sum_{p=0}^{N-1} d_p y^{2p+1} \text{ when } F_{N1}^\Delta(\theta), \tag{19a}$$

$$\Gamma_{2N-1}(\tilde{y}) = (1 - \tilde{y}^2) \sum_{p=0}^{N-2} e_p \tilde{y}^{2p+1} \text{ when } F_{N2}^\Delta(\theta), \tag{19b}$$

while d_p and e_p designate new coefficients, related to the old coefficients $\tilde{a}_p$ and $\tilde{b}_p$ by formulas of the type (14a) and (14b).

It is known [3] that, of all the quasipolynomials $F_N^\Delta(\theta)$ (18) of degree (2N-1), having an identical value of the principal maximum, the quasipolynomial

$$Y_N(\theta) = (1 - \tilde{y}^2)^{1/2} G_{2N-1}(\tilde{y}, \xi) \tag{20}$$

assures that the other two are extreme when one of the three parameters (R, γ, ξ) is given; where $G_{2N-1}(\tilde{y}, \xi)$ designates a second order Chebyshev-Akhiyezer of degree (2N-1), which does not contain even degrees and has the least deviation from zero on the two segments $[-1, -\xi]$ and $[\xi, 1]$ with a weight $\omega=(1-\tilde{y}^2)^{1/2}$ [3,5].

Let us observe the following. The representation of $F^{\Delta}_{N2}(\theta)$ of (18) and (19b) in the form of the quasipolynomial $Y_N(\theta)$ of (20), i.e. the satisfaction of the identity:

$$F^{\Delta}_{N2}(\theta) \equiv Y_N(\theta), \tag{21}$$

uniquely determines the coefficient e_p. However, moving on to determine the excitation coefficients C^{μ}_{2n}, we arrive at an incompatible set of N equations with (N-1) unknowns. The condition of incompatibility becomes more explicit by noting that one of the roots of the polynomial $Y_{2N-1}(\tilde{y})$ (19b) is the point $\tilde{y}=\pm 1$. On the other hand, for the Chebyshev-Akhiyezer polynomial $G_{2N-1}(\tilde{y},\xi)$ the point $\tilde{y}=\pm 1$ is not a root for any values of N or ξ. Therefore, condition (21) cannot be made identical in respect of $\tilde{y}$ and the solution of the above problem is sought in the form:

$$F^{\Delta}_{N2}(\theta) = (1-\tilde{y}^2)(1-\tilde{y}^2)^{1/2}\sum_{p=0}^{N-2} e_p \tilde{y}^{2p+1} \equiv (1-\tilde{y}^2)(1-\tilde{y}^2)^{1/2} Q_{2N-3}(\tilde{y}, \xi), \tag{22}$$

where $Q_{2N-3}(\tilde{y},\xi)$ is a certain polynomial which, with an appropriate weighting function in (22) and one of the three parameters (R, γ, ξ) given, assures that the other two are extreme.

The finding of such a polynomial would lead to additional trouble in the solution of our problem, and therefore instead of this polynomial we shall use the second order Chebyshev-Akhiyezer polynomial $G_{2N-3}(\tilde{y},\xi)$, the properties of which have been discussed above.

Furthermore, returning to (18) and (19a) and taking into account (4a) and (4b), we shall find it necessary to search for $F_{N1}^{\Delta}(\theta)$ as well in the form:

$$F_{N1}^{\Delta}(\theta)=(1-\tilde{y}^2)^{1/2}\sum_{p=0}^{N-1} d_p\tilde{y}^{2p+1}=(1-\tilde{y}^2)(1-\tilde{y}^2)^{1/2}G_{2N-3}(\tilde{y},\xi). \qquad (23)$$

In view of the complexity of the closed representation of Chebyshev-Akhiyezer polynomials, we may calculate the coefficients d_p and e_p by the method of exponential polynomials or the method of trigonometric interpolation [8], and then find the excitation coefficients $C_{0n}^{0\Pi T}$ and $C_{2n}^{0\Pi T}$ which provide a spatial direction pattern of the type (5); here, $Y_N(\theta)$ is determined from (22) and (23).

The required distribution of the magnetic current $\bar{j}^{\mu}(\theta'\phi')$ at the surface of a spherical antenna of radius a can be found by the method explained, e.g., in [4]. With an accuracy down to a constant, this equals:

$$\begin{aligned}\bar{j}^{\mu}(\theta',\varphi')=\sum_{n=1}^{N}\Big\{ i^n C_{2n}^{\text{опт}}\left[\frac{\partial\zeta_n(x)}{\partial x}\Big|_{x=\kappa a}\frac{2}{\sin\theta'}P_n^2(\cos\theta')+i\zeta_n(\kappa a)\frac{\partial P_n^2(\cos\theta')}{\partial\theta'}\right]\times\\ \times\cos 2\varphi'\bar{i}_\theta+i^{n+1}C_{0n}^{\text{опт}}\zeta_n(\kappa a)P_n^1(\cos\theta')\bar{i}_\theta\Big\}-\sum_{n=1}^{N} i^n C_{2n}^{\text{опт}}\times \qquad (24)\\ \times\left[\frac{\partial\zeta_n(x)}{\partial x}\Big|_{x=\kappa a}\frac{\partial P_n^2(\cos\theta')}{\partial\theta'}+i\zeta_n(\kappa a)\frac{2}{\sin\theta'}P_n^2(\cos\theta')\right]\sin 2\varphi'\bar{i}_\varphi.\end{aligned}$$

where $\zeta_n(x)=\sqrt{\frac{\pi x}{2}}H^{(1)}_{n+1/2}(x)$; $H^{(1)}_{n+1/2}(x)$ is the Henkel function of the first order; (θ',ϕ') are spherical coordinates of the point at the surface of the spherical antenna.

The use of Chebyshev-Akhiyezer polynomials involve certain computational difficulties [2,5], and therefore it is of interest to consider so-called quasioptimal Δ direction patterns, i.e. those similar to patterns of the type (22) and (23).

QUASIOPTIMAL DIFFERENCE DIRECTION PATTERNS

Quasioptimal [in the sense of approaching patterns of the type (22) and (23)] difference patterns will be sought in the class of direction patterns obtained by differentiation of the amplitude patterns of non-superdirectional (N=ka) spherical antennas, representing a body of revolution with respect to the maximum of radiation, with optimal summary characteristics (in the sense of Dolf and Chebyshev) in an arbitrary section of the space pattern by a plane passing through the radiation maximum which, by analogy with [6] and allowing for (12), we shall represent in the plane $\phi=\pi/2$ in the form:

$$F^{s}_{N}(\theta)=T_N(2\alpha\tilde{x}-1), \qquad (25)$$

where $T_N(2\alpha\tilde{x}-1)$ is a Chebyshev polynomial of the first order; α is a parameter $(\alpha\geq 1)$, $\tilde{x}=\cos^2\frac{\tilde{\theta}}{2}$.

We note that this approach was apparently used for the first time in [7].

A differentiation of (25) with respect to the variable θ, with an accuracy down to a constant, yields:

$$Y_N(\theta) = \frac{dF_N^{\Delta}(\theta)}{d\theta} = 2[\tilde{x}(1-\tilde{x})]^{1/2} U_{N-1}(2\alpha\tilde{x}-1). \qquad (26)$$

where $U_{N-1}(2\alpha\tilde{x}-1)$ is a second order Chebyshev polynomial.

Turning further to (15) and (16) and taking into account the above remark as to the impossibility of satisfying the identity $F_N^{\Delta}(\theta) \equiv 2[\tilde{x}(1-\tilde{x})]^{1/2} U_{N-1}(2\alpha\tilde{x}-1)$ with respect to $\tilde{x}$, we shall represent the quasioptimal difference pattern in the plane $\phi=\pi/2$ $-Y_N(\theta)$ in the form

$$Y_N(\theta) = 2[\tilde{x}(1-\tilde{x})]^{1/2} \tilde{x} U_{N-2}(2\alpha\tilde{x}-1). \qquad (27)$$

Since the spatial direction pattern in this case is determined by expression (5), for its analysis we shall examine the properties of the quasipolynomial $Y_N(\theta)$ (27) on the interval $0 \leqslant \tilde{x} \leqslant 1$.

All the zeroes of the quasipolynomial $Y_N(2\alpha\tilde{x}-1)$ are real and situated on the interval [0,1]. The quasipolynomial has a maximum lobe corresponding to the value $\tilde{x}=\tilde{x}_{m1}$ (cf. Fig. 1); the value of the side lobes decreases smoothly away from the value $\tilde{x}_{m1}$, and therefore the nearest to the principal lobe ($\tilde{x}=\tilde{x}_{m2}$) has the maximum value. The values of $\tilde{x}_{m1}$ and $\tilde{x}_{m2}$ can be determined from the following transcendental equation:

$$(-\tilde{x}+2-3\alpha\tilde{x}+2\alpha\tilde{x}^2) U_{N-2}(2\alpha\tilde{x}-1) = (N-1)(1-\tilde{x}) T_{N-1}(2\alpha\tilde{x}-1). \qquad (28)$$

supplemented by the appropriate inequalities:

$$1<\widetilde{x}_{m1}<\frac{1+\cos\frac{\pi}{N-1}}{2\alpha}, \tag{29a}$$

$$\frac{1+\cos\frac{\pi}{N-1}}{2\alpha}<\widetilde{x}_{m2}<\frac{1+\cos\frac{2\pi}{N-1}}{2\alpha}. \tag{29b}$$

The maximum level of the side lobes in this case is:

$$R=\frac{|\bar{F}_N^{\Delta}(\theta,\varphi)|}{|\bar{F}^{\Delta}_{N\text{макс}}|}\leqslant\frac{[\widetilde{x}_{m2}(1-\widetilde{x}_{m2})]^{1/2}\,x_{m2}U_{N-2}(2\alpha\widetilde{x}_{m2}-1)}{[\widetilde{x}_{m1}(1-\widetilde{x}_{m1})]^{1/2}\,\widetilde{x}_{m1}U_{N-2}(2\alpha\widetilde{x}_{m1}-1)}, \tag{30}$$

while the slope of the normalized characteristic difference curve is:

$$\gamma=\frac{\frac{\partial}{\partial\theta}\left|\bar{F}_N^{\Delta}\left(\theta,\frac{\pi}{2}\right)\right|_{\theta=0}}{|\bar{F}^{\Delta}_{N\text{макс}}|}=\frac{U_{N-2}(2\alpha-1)}{[\widetilde{x}_{m1}(1-\widetilde{x}_{m1})]^{1/2}\,U_{N-2}(2\alpha\widetilde{x}_{m1}-1)}. \tag{31}$$

For the width $\Delta\theta_0$ of the quasioptimal difference pattern we obtain:

$$\Delta\theta_0=2\arccos\left[\frac{\cos\frac{\pi}{N-1}+1-\alpha}{\alpha}\right]. \tag{32}$$

However, by analogy with [6], a more convenient characteristic for the analysis of the difference patterns is the ratio of ($\Delta\theta_0$) to the corresponding width of both principal lobes ($\Delta\theta_{00}$) of the equivalent planar aperture with a difference pattern which assures a maximum value of the parameter μ (slope of the non-normalized difference pattern) [1], determined in accordance with (34). Then, we obtain:

$$\Delta = \left(\frac{\Delta\theta_0}{\Delta\theta_{00}}\right) = \frac{\arccos\left[\dfrac{\cos\dfrac{\pi}{N-1}+1-\alpha}{\alpha}\right]}{\arcsin\dfrac{5,14}{N}}. \tag{33}$$

Figure 2 shows curves for the maximum level of the side lobes R, while Fig. 3 shows the parameter Δ, as functions of α for different values of ka(N=ka), constructed from (30) and (33), respectively.

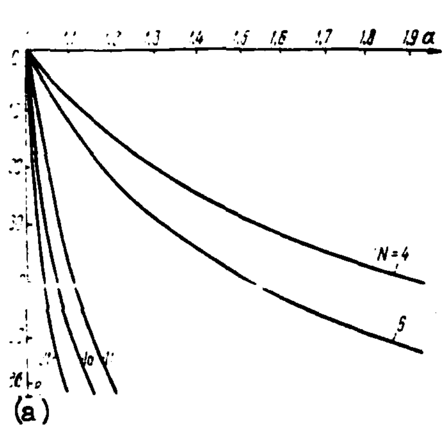


Fig. 2.
Key: (a) Decibels.

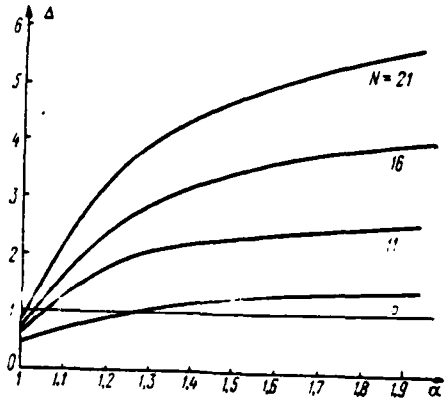


Fig. 3.

The above-obtained relations (30)-(32) enable the finding of the value of the parameter α from a given value of R for non-superdirectional spherical antennas and then the determination of γ and $\Delta\theta_0$, corresponding to the given R or, vice versa, the finding of R for a given value of γ (or $\Delta\theta_0$).

An important characteristic for the analysis of antennas with difference direction patterns is the slope of the difference pattern in the equal-signal direction μ [1], defined as:

$$\mu = \frac{\partial}{\partial\theta}\left\{\frac{\left|\bar{F}_N^{\Delta}\left(\theta, \frac{\pi}{2}\right)\right|}{\left|\bar{F}_{N\max}^{\Delta}\right|}\right\}_{\theta=0}\left[G_N^{\Delta}\left(\frac{\pi}{2}, \theta_0\right)\right]^{1/2}. \tag{34}$$

where $G_N^{\Delta}(\pi/2, \theta_0)$ is the directivity of the pattern in the direction of its principal maxima.

Inserting in (34) the expression for $\bar{F}_N^{\Delta}(\theta, \phi)$ from (5), where $Y_N(\theta)$ is determined by (27), we have after simple transformations:

$$\mu = \frac{U'_{N-2}(2\alpha - 1)}{\sqrt{2}\left\{\int\limits_0^1 \tilde{x}^3(1-\tilde{x})\, U_{N-2}^2(2\alpha\tilde{x} - 1)\, d\tilde{x}\right\}^{1/2}}. \tag{35}$$

Since the integrand is a polynomial of degree 2N, the process of integration can be carried out rather easily in each particular case.

Let us now turn to the determination of the conditions of excitation of the antenna which assure the formation of difference patterns of the type (5) with $Y_N(\theta)$ as determined by expression (27), i.e. when:

$$\bar{F}_N^{\Delta}(\theta, \varphi) = \cos^2\frac{\theta}{2}\sin\theta\, U_{N-2}\left(2\alpha\cos^2\frac{\theta}{2} - 1\right)\sin\varphi\,[\sin\varphi\,\bar{i}_{\varphi} - \cos\varphi\,\bar{i}_{\theta}]. \tag{36}$$

From (4a) and (4b), allowing for (15), (16), and (27), we obtain:

$$\sum_{p=0}^{N-1} \widetilde{a}_p \widetilde{x}^p \equiv \frac{1}{2} \widetilde{x} U_{N-2}(2\alpha \widetilde{x} - 1),$$
$$\sum_{p=0}^{N-2} \widetilde{b}_p \widetilde{x}^p \equiv -\frac{1}{4} U_{N-2}(2\alpha \widetilde{x} - 1). \tag{37}$$

For the purposes of our problem, it is more convenient to represent $U_{N-2}(2\alpha\tilde{x}-1)$ in the form:

$$U_{N-2}(2\alpha \widetilde{x} - 1) = \sum_{p=0}^{N-2} u_p \widetilde{x}^p, \tag{38}$$

where u_p are coefficients of a corresponding displaced Chebyshev polynomial of the second order.

We point out that the coefficients of the first 13 displaced Chebyshev polynomials of the second order are given in [8].

In order to satisfy the identities (37), it is necessary and sufficient that:

$$\left.\begin{array}{l} 2\widetilde{a}_p = u_p, \quad p = 0, 1, 2, \ldots, N-1 \\ 4\widetilde{b}_p = -u_p, \quad p = 0, 1, 2, \ldots, N-2 \end{array}\right\}. \tag{39}$$

Considering that the coefficients $\tilde{a}_p$ and $\tilde{b}_p$ are expressed linearly by the coefficients of excitation of the natural waves, we find that each of the conditions (39) represents a set of N (or N-1) simultaneous linear algebraic equations with N (or N-1) unknowns for the coefficient of excitation C^{μ}_{0n}, C^{μ}_{2n}. We shall designate the solution of system (39) by C^{corr}_{0n}, C^{corr}_{2n}.

In this case, the spatial difference pattern has the appearance (36), while the amplitude-phase and polarization distribution of the current which excites the spherical antenna can be found from (24).

RESULTS OF THE NUMERICAL CALCULATION

Let us illustrate the above remarks by the example of a spherical non-superdirectional (N=6) antenna of electrical radius ka=6 and a difference pattern $\bar{F}_6^{\Delta}(\theta,\phi)$ of the type (36).

Figures 4a and 4b show the difference patterns of this particular spherical antenna ka=6 of the type (36) (without allowing for the factor $\cos^2 \frac{\theta}{2}$) in the plane $\phi=\pi/2$ for R=-20 and -30 dB; here also for comparison are given a non-lobed direction pattern (R=-∞ dB), as well as a pattern with maximum μ. As can be seen, contrary to the difference patterns of Dolf and Akhiyezer [3], the side lobes are not identical: the value of the first side lobe is a maximum, the others decreasing smoothly away from the major lobe. The factor $\cos^2 \frac{\theta}{2}$ in (36) merely influences the level of the remote side lobes in the discussion of pencil-beam spherical antennas, producing a monotonic decrease of these lobes, with virtually no influence on the level of the first side lobes.

Figures 5a and 5b show the amplitude-phase distributions of the magnetic current in the plane $\phi'=\pi/2$ on the surface of our spherical antenna, realizing the difference patterns $\bar{F}_6^{\Delta}(\theta,\phi)$ (36) with different levels of side radiation, calculated by (24).

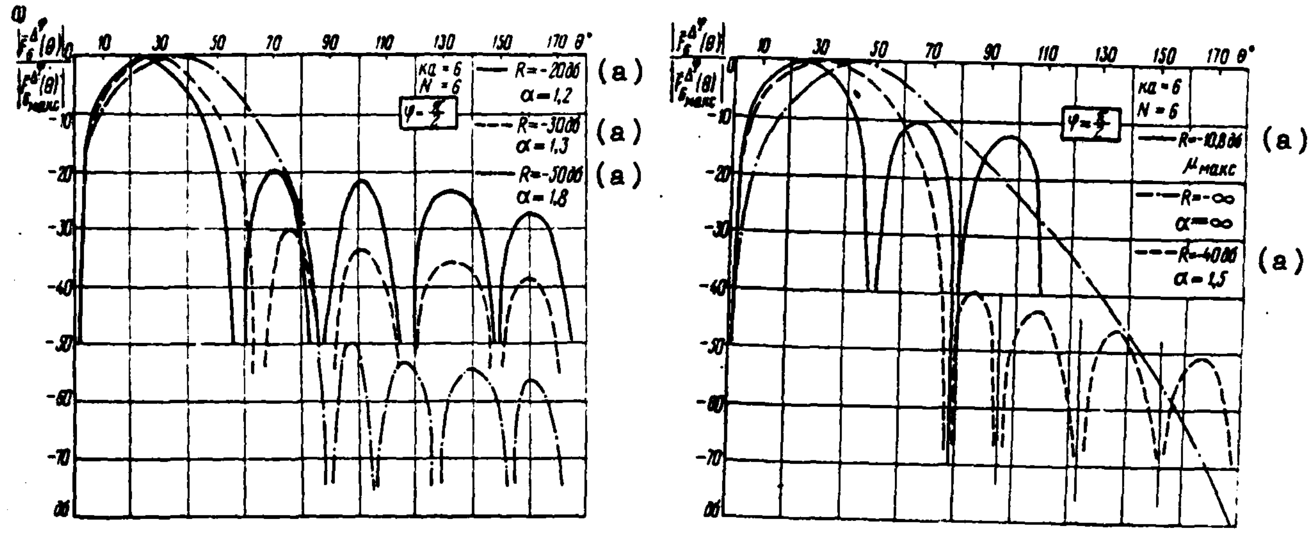
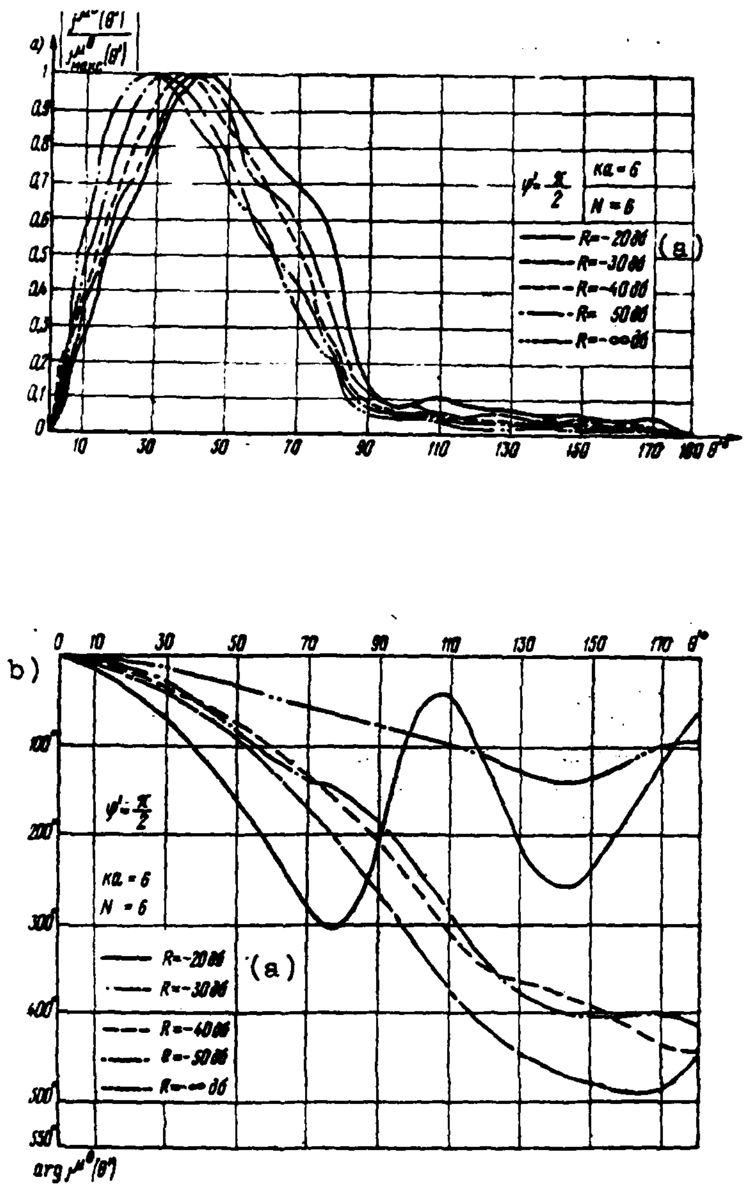


Fig. 4.
Key: (a) dB.

Fig. 5.
Key: (a) dB.



a)
$\left|\frac{I^{\mu\theta}(\theta')}{I^{\mu\theta}_{макс}(\theta')}\right|$
$\psi' = \frac{\pi}{2}$
$ka = 6$
$N = 6$
R=-20 дб
R=-30 дб
R=-40 дб
R=-50 дб
R=-∞ дб
(a)
b)
$\psi' = \frac{\pi}{2}$
$ka = 6$
$N = 6$
R=-20 дб
R=-30 дб
R=-40 дб
R=-50 дб
R=-∞ дб
(a)
$\arg I^{\mu\theta}(\theta')$

Fig. 5.
Key: (a) dB.

As can be seen, even in the case of a significant variation of R the necessary amplitude distribution (Fig. 5a) is almost unaffected, especially in the antenna region which is "illuminated" with respect to the equal-signal direction. On the other hand, the necessary phase excitation (Fig. 5b), contrary to planar non-superdirectional antennas, depends considerably on the required level of side radiation, the less this level the smaller the rate of change in the required phase distribution over the antenna.

Figure 6 shows the dependence on R of the normalized (to the maximum possible) value of μ (35) of our spherical antenna ka=6. When R increases, the curve approaches the level determined from (35) when $\alpha\to\infty$ ($R\to\infty$).

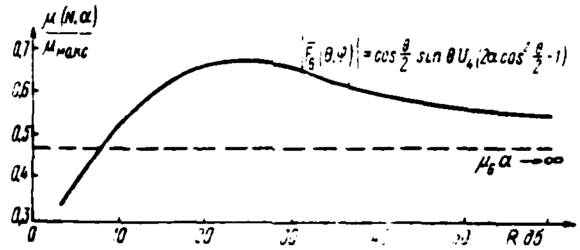


Fig. 6.
Key: (a) dB.

In conclusion let us note that the expressions for patterns (5) and (22) and for the quasioptimal difference patterns (36) of non-superdirectional spherical antennas coincide only for the two limit cases: a non-lobed pattern (R=-∞ dB) and a pattern with identical level of major and side lobes (R=0 dB). In all other cases, the difference patterns which employ

Chebyshev-Akhiyezer polynomials have a somewhat smaller level of side radiation than the quasioptimal patterns for a given value of γ.

CONCLUSIONS

The problem of synthesis of spherical non-superdirectional antennas with spatial difference patterns, optimal in the sense of Dolf and Chebyshev, has been formulated.

The problem of minimization of the spatial side radiation of the difference patterns of spherical antennas, similar to those of Dolf and Chebyshev [cf. relations (22) and (23)], has been solved by converting it to two similar unidimensional problems, the solutions of which are represented in the form of Chebyshev-Akhiyezer polynomials, multiplied by a certain weakly-directional factor.

A consideration is made for the problem of synthesis of quasioptimal difference direction patterns, greatly simplifying the numerical calculations. The obtained relations determine the correlation between the level of side radiation, the slope, and the width of both major lobes at the zeroes.

Results of the numerical calculations are given.

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THE CALCULATION OF THE INPUT RESISTANCE OF A VIBRATOR WITH DIELECTRIC COVER BY THE METHOD OF INDUCED EMF

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The paper discusses the radiation of a vibrator with dielectric cover, placed in a semiconducting medium. It analyzes the field in the dielectric.

The discussion, along with the results obtained [6], enable the finding of the complete value of the input resistance under the assumption of a sinusoidal current distribution.

In [6] the problem of calculating the flux of electromagnetic energy across the cylindrical surface of a vibrator was considered in the case of a given distribution of current along it by the law sin $\gamma(l-z)$, when the quantity γ is not equal to the wave number of the external medium k. Such a current distribution in a first approximation obtains for a vibrator placed in a semiconducting medium and possessing a dielectric cover.

For the complete solution of the problem of calculating the input resistance by the method of induced EMF it is further necessary to find the value of the z-component of the electric field in the immediate vicinity of the transmitting conductor, i.e. inside the insulating shell.

Before beginning the solution of this problem, let us point out the well-known fact that the field of a symmetrical vibrator can be represented by a superposition of the fields of three point sources, situated at the edges and in the center of the vibrator.

In the case of $\gamma \neq k$, it is easy to demonstrate this fact by representing the Hertz vector of the vibrator field in the form:

$$\Pi_z = \frac{i I_0 \omega\mu}{4\pi\kappa^2} \int_{-l}^{+l} \left\{ \sin\gamma(l-\zeta) \int_{-\infty}^{+\infty} K_0\left(r\sqrt{p^2-\kappa^2}\right)\left[e^{-i|z-\zeta|p} + e^{-i|z+\zeta|p}\right] dp \right\} d\zeta \tag{1}$$

where K_0 is a modified Bessel function.

After integrating with respect to the variable ζ, we immediately obtain the sum of the fields of sources situated at the points $z=\pm l$ and $z=0$:

$$\Pi_z = \frac{i I_0 \omega\mu}{4\pi\kappa^2} \int_{-\infty}^{+\infty} \frac{\gamma}{\gamma^2-p^2} K_0\left(r\sqrt{p^2-\kappa^2}\right)\left[e^{-i|z+l|p} + e^{-i|z-l|p} - 2\cos\gamma l\, e^{-i|z|l}\right] dp. \tag{2}$$

This circumstance allows us to confine our treatment to the field of a point source, situated in the cylindrical cavity, and to utilize the available solution for this problem. This problem has been treated by many authors [1,2, et al.], and we shall touch briefly here on the solution given in [2], somewhat modifying the final results.

The Hertz vector of the field of a point source, situated in a cylindrical cavity of radius r_1 with a moment $\bar{m}$ directed along the axis of the cavity, can be represented in the form:

$$\Pi_z^{(1)} = \int_{-\infty}^{+\infty} \left[c(p) K_0(rv_1) + a(p) J_0(rv_1)\right] e^{-ip|z|} dp. \tag{3}$$

The expression for the Hertz vector outside the cylindrical cavity has the appearance:

$$\Pi_z^{(2)} = \int_{-\infty}^{+\infty} b(p) K_0(rv) e^{-|p||z|} dp, \qquad (4)$$

where $v = \sqrt{p^2 - \kappa^2}$, $v_1 = \sqrt{p^2 - \kappa_1^2}$

c(p) is the arbitrary function of the source;

k is the wave number of the external medium;

k_1 is the wave number of the material of the cylinder;

r is the radial coordinate;

K_0 and J_0 are modified Bessel functions of the zero order.

The boundary conditions at the boundary between the cylinder and the external medium lead to the following expression for a(p) (when $r=r_1$):

$$a(p) = c(p) \frac{\kappa_1^2 v K_0(r_1 v) K_1(r_1 v_1) - \kappa^2 v_1 K_1(r_1 v) K_0(r_1 v_1)}{\kappa^2 v_1 K(r_1 v) J_0(r_1 v_1) - \kappa_1^2 v K_0(r_1 v) J_1(r_1 v_1)}. \qquad (5)$$

It is possible to considerably simplify this relation by employing the expressions for Bessel functions of small arguments. The fundamentals of such a replacement have been given in [2] and reduce to the satisfaction of the conditions: $|k_1 r_1| \ll 1, z \gg r$, and $k > k_1$.

Then, disregarding the second term in the denominator of expression (5), we obtain:

$$a(p) = c(p) \left[\frac{\kappa_1^2 v}{\kappa^2 v_1} K_0(r_1 v) \frac{K_1(r_1 v_1)}{K_1(r_1 v)} - K_0(r_1 v) \right]. \qquad (6)$$

Considering the fact that $E_z^{(1)} = \kappa_1^2 \Pi_z^{(1)} - \frac{\partial^2 \Pi_z^{(1)}}{\partial z^2}$ (z is the component of the electric field inside the cylinder), and also the fact that $\frac{K_1(r_1 v_1)}{K_1(r_1 v)} \approx \frac{v}{v_1}$, , we find that:

$$E_z^{(1)} = \int_{-\infty}^{+\infty} c(p) \left[v_1^2 K_0(r v_1) + \frac{\kappa_1^2}{\kappa^2} v^2 K_0(r_1 v) - v_1^2 K_0(r_1 v_1) \right] e^{-ip|z|} dp. \tag{7}$$

The function for the source of the incident wave c(p) for a vibrator of finite dimensions and for the case $\gamma \neq k$, in accordance with expression (2), has the appearance:

$$c(p) = \frac{i I_0 \mu \omega \gamma}{4\pi(\gamma^2 - p^2)\kappa_1^2} = \frac{c_0(p)}{\kappa_1^2},$$

where $c_0(p)$ does not depend on the parameters of the material of the cavity (the magnetic permeability of the material of the cavity, as well as of the external medium, will be assumed to equal μ_0 here and below).

Then:

$$E_z^{(1)} = \int_{-\infty}^{+\infty} c_0(p) \left[\frac{v_1^2}{\kappa_1^2} K_0(r v_1) + \frac{v^2}{\kappa^2} K_0(r_1 v) - \frac{v_1^2}{\kappa_1^2} K_0(r_1 v_1) \right] e^{-ip|z|} dp. \tag{8}$$

It follows from expression (8) that the field of a source inside a cylindrical cavity can be represented by the following sum:

$$E_z^{(1)} = E(r_1, \kappa) + E(r, \kappa_1) - E(r_1, \kappa_1). \tag{9}$$

where

$E(r_1,k)$ is the field of the source at the radius r_1 in a homogeneous medium with wave number k;

$E(r_1,k_1)$ is the field of a source at the radius r_1 in a homogeneous medium with wave number k_1;

$E(r,k_1)$ is the field of a source at an arbitrary radius $r<r_1$ in a homogeneous medium with wave number k_1.

We note that, if $k \gg k_1$, then with no additional assumptions it follows from (3) and (5) that

$$E_z^{(1)} = E(r, \kappa_1) - E(r_1, \kappa_1). \tag{10}$$

There is no field $E(r_1,k)$ in this case, as it is screened by the external medium. This case corresponds to a passage to the limit for conditions of a coaxial cable.

Expression (9) on the basis of (2) is also valid for a vibrator of finite dimensions.

In order to calculate the value of the energy flux through the lateral surface of the vibrator, let us multiply expression (9) by the quantity:

$$H_\varphi^* = \frac{I_0}{2\pi r_0} \sin \gamma^* (l - z).$$

Setting $r=r_0$ in the obtained expression and performing an integration over the cylindrical surface of the vibrator, we find that:

$$W = \int_{-l}^{+l}\int_{0}^{2\pi} r E_z^{(1)} H_\varphi^* \, dz \, d\varphi = \int_{0}^{l} 2I_0 \sin\gamma^* (l-z) E(r_1, \kappa)\, dz + + \int_{0}^{l} 2I_0 \sin\gamma^* (l-z) [E(r_0, \kappa_1) - E(r_1, \kappa_1)]\, dz, \tag{11}$$

where

$$E_z(r_1, \kappa) = \kappa^2 \Pi_z(r_1, \kappa) + \frac{\partial^2 \Pi_z(r_1, \kappa)}{\partial z^2}, \tag{12}$$

$$\Pi(r_1, \kappa) = \frac{l}{i\, 4\pi\omega\varepsilon_0\varepsilon'} \int_0^l I_0 \sin\gamma^*(l-\zeta) \left[\frac{e^{-i\kappa\sqrt{r_1^2 + (z-\zeta)^2}}}{\sqrt{r_1^2 + (z-\zeta)^2}} + \frac{e^{-i\kappa\sqrt{r_1^2 + (z+\zeta)^2}}}{\sqrt{r_1^2 + (z+\zeta)^2}} \right] d\zeta \tag{12a}$$

ε' is the relative dielectric permeability of the external medium.

The expression for $E_z(r_1,k_1)$ is obtained from (12) and (12a) by the replacement of k_1 for k; an expression for $E_z(r_0,k_1)$ is obtained from the expression for $E_z(r_1,k_1)$ by substituting r_0 for r_1.

The calculation of the first term in (11) has been considered in [6]. The value of the radiation resistance corresponding to this term is determined by formulas (19) and (20) in that work.

The other term appearing in (11) is calculated by means of the same formulas under the already-mentioned substitutions for k and r.

The last term in (11) specifies an additional component of the radiation resistance ΔZ_Σ, the calculation of which in accordance with expression (11) and expressions (20) and (22) of [6] leads to the relation:

$$\Delta Z_\Sigma = -i \frac{30}{\sqrt{\varepsilon_1'}} \frac{1}{\kappa_1} \left[\frac{c_2'}{\beta} \sin 2\beta l + \frac{c_1'}{\alpha} \operatorname{sh} 2\alpha l \right] \ln \frac{r_1}{r_0}, \tag{13}$$

where

$$c_1' = \alpha^2 + \beta^2 - \kappa_1^2, \quad c_2' = \alpha^2 + \beta^2 + \kappa_1^2,$$

ε_1' is the relative dielectric permeability of the material of the shell.

The complete value of the radiation resistance of the vibrator, found on the surface of the conductor, is determined by the sum:

$$Z_\text{э}^n = Z_\text{э} + \Delta Z_\text{э}. \tag{14}$$

We can further show that, in the presence of several shells surrounding the conductor, with small dimensions as compared to the length of the wave, it is sufficient in formula (13) to replace r_1 by r_n and ε_1, k_1 by $\varepsilon_{\text{эKB}}$, $k_{\text{эKB}}$, as determined by the relations:

$$\frac{1}{\varepsilon_{\text{экв}}} \ln \frac{r_n}{r_0} = \sum_{m=1}^{n} \frac{1}{\varepsilon_m} \ln \frac{r_m}{r_{m-1}}, \quad \kappa_{\text{экв}} = \omega \sqrt{\mu_0 \varepsilon_0 \varepsilon_{\text{экв}}}, \tag{15}$$

n is the number of the last shell.

Let us consider several partial cases of the resulting solution.

At present the theory of long conductors is widely used for the calculation of vibrators with dielectric coating. We shall also begin our comparison with the premises of this theory.

From expression (22) of [6], by passing to the limit $\alpha l \to \infty$ with a consideration of the formula for the rescaling of Z_Σ for the input of the vibrator, we obtain:

$$Z_{\text{вх}} = \frac{Z_\Sigma}{\sin^2 \beta l + \operatorname{sh}^2 \alpha l} = \frac{30}{\sqrt{\varepsilon'}} \frac{1}{\kappa} \left[\frac{c_2}{\beta} \ln \frac{\kappa + \gamma}{\kappa - \gamma} - i \frac{c_1}{\alpha} \ln \frac{1,12}{i r_1 \sqrt{(\kappa + \gamma)(\kappa - \gamma^*)}} \right], \tag{16}$$

where

$$c_1 = \alpha^2 + \beta^2 - \kappa^2, \quad c_2 = \alpha^2 + \beta^2 + \kappa^2. \tag{17}$$

Similarly, after converting the value of ΔZ_Σ to the input and under the condition of $\alpha l \to \infty$, we obtain:

$$\Delta Z_{\text{вх}} = -i \frac{60}{\sqrt{\varepsilon_1'}} \frac{1}{\kappa_1} \frac{c_1'}{\alpha} \ln \frac{r_1}{r_0}. \tag{18}$$

The expressions for c_1 and c_1' can be represented in the form:

$$-i \frac{c_1}{\alpha} = -i \frac{\gamma^2 - \kappa^2}{\alpha} + 2\gamma, \quad -i \frac{c_1'}{\alpha} = \frac{-i(\gamma^2 - \kappa_1^2)}{\alpha} + 2\gamma. \tag{19}$$

Using the relationship:

$$\ln \frac{1,12}{i r_1 \sqrt{(\kappa + \gamma)(\kappa - \gamma^*)}} = \ln \frac{1,12}{r_1 \sqrt{\gamma^2 - \kappa^2}} + \ln \sqrt{\frac{\kappa - \gamma}{\kappa - \gamma^*}} \tag{20}$$

and adding expressions (16) and (18), after inserting (19) and (20) into these, we obtain:

$$
\begin{aligned}
Z^{n}_{\text{вх}} &= \frac{30}{\sqrt{\varepsilon'}}\,\frac{1}{\kappa}\,\frac{c_2}{\beta}\ln\frac{\kappa+\gamma}{\kappa-\gamma^{*}} + \frac{120}{\sqrt{\varepsilon_1}}\,\frac{\gamma}{\kappa}\times \\
&\times\left[\frac{\varepsilon'}{\varepsilon_1'}\ln\frac{r_1}{r_0} + \ln\frac{1,12}{r_1\sqrt{\gamma^2-\kappa^2}}\right] - i\,\frac{60}{\sqrt{\varepsilon'}}\,\frac{1}{\kappa}\,\frac{c_1}{\alpha}\ln\sqrt{\frac{\kappa-\gamma}{\kappa-\gamma^{*}}} - \\
&- i\,\frac{60\kappa_0}{\alpha}\left[\frac{\gamma^2-\kappa_1^2}{\kappa_1^2}\ln\frac{r_1}{r_0} + \frac{\gamma^2-\kappa^2}{\kappa^2}\ln\frac{1,12}{r_1\sqrt{\gamma^2-\kappa^2}}\right].
\end{aligned}
\tag{21}
$$

where $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$.

By virtue of the dispersion equation for γ [4], [5], the last term in (21) vanishes.

The third term in (21) shall be transformed for the case of $|k-\beta| \gg \alpha$. Then, with an accuracy down to terms on the order of $\frac{\alpha 2}{k-\beta}$, we find:

$$
i\,\frac{c_1}{\alpha}\ln\sqrt{\frac{\kappa-\gamma}{\kappa-\gamma^{*}}} = -(\gamma+\kappa+i\alpha), \tag{22}
$$

and expression (21) becomes:

$$
\begin{aligned}
Z^{n}_{\text{вх}} &= \frac{60}{\sqrt{\varepsilon'}}\left[\frac{c_2}{2\beta\kappa}\ln\frac{\kappa+\gamma}{\kappa-\gamma^{*}} - \left(1+\frac{\alpha}{\kappa}\right)\right] + \\
&+ \frac{120}{\sqrt{\varepsilon'}}\,\frac{\gamma}{\kappa}\left[\frac{\varepsilon'}{\varepsilon_1'}\ln\frac{r_1}{r_0} + \ln\frac{1,12}{r_1\sqrt{\gamma^2-\kappa^2}} - \frac{1}{2}\right].
\end{aligned}
\tag{23}
$$

The second term in (23) coincides with the expression for the wave resistance of a conductor in an insulator, obtained by the method of expanding the excitation function by a continuous spectrum of cylindrical waves [4,5]. The solution in this case is represented in the form of a contour integral and likewise contains two terms, one of which corresponds to the remainder of the integrand and determines the wave resistance of the conductor;

the other term is represented by an integral over the cross section and it has not yet been possible to obtain this in an explicit form. However, estimates suggest that, for the case of $\gamma \ll k$, this term is much less than the wave resistance.

For this particular case we shall also estimate the expression (23). When $\gamma \ll k$:

$$\frac{60}{\sqrt{\varepsilon'}}\left[\frac{c_2}{2\beta\kappa}\ln\frac{\kappa+\gamma}{\kappa-\gamma^*}-\left(1+\frac{\alpha}{\kappa}\right)\right]\approx\frac{60}{\sqrt{\varepsilon'}}\left(\frac{\gamma}{\kappa}\right)^2. \qquad (24)$$

Thus, when the wave resistance of the vibrator is on the order of $1000\Omega/\sqrt{\varepsilon'}$ and when $|\gamma/k|=0.7$, the first term in (23) is no more than 3% of the second.

For the case of short vibrators with $\gamma l \ll 1$, in accordance with expression (13) as well as expression (24) of [6], we have:

$$Z_\Sigma^n=-i\frac{120}{\sqrt{\varepsilon'}}\frac{(\alpha^2+\beta^2)\,l}{K}\left(\frac{\varepsilon'}{\varepsilon_1'}\ln\frac{r_1}{r_0}+\ln\frac{l}{r_1}-1\right). \qquad (25)$$

For the case of $k \gg k_1$, we can disregard Z_Σ and the formula for the doubled input resistance of a coaxial cable follows from expression (13), after converting ΔZ_Σ to the input of the vibrator.

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ON THE CALCULATION OF THE H-PLANE STEPPED JOINT OF RECTANGULAR WAVEGUIDES

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The paper considers a block diagram of a computer program and presents numerical results of a calculation by the program for the scattering matrix of a step in a rectangular waveguide. It is shown that the balance of active powers of the waveguide structures should be carried out independently of the number of higher types of waves for which an allowance is made in the waveguides between heterogeneities.

INTRODUCTION

Structures with coordinate-plane discontinuities in a rectangular waveguide with H_{m0} waves are very common in UHF techniques. This also includes the relatively simple designs of a slit bridge, inductive diaphragm, waveguide turn, H-plane step, and more complicated structures such as a zig-zag waveguide with retarded wave, step junctions, a pair of waveguides coupled by rectangular slits in a common narrow wall, etc. [1]. In [2] a method is proposed for as accurate a design as desired for all the above and other structures of this class, in the form of a combination of an accurate method of factorization of the functions (the method of Wiener, Hopf, and Foch) of a unitary switch function [3] and of matrix transformations, including inversion.

The fundamental formula in the method determines the scattering matrix of the system in terms of familiar scattering matrices of the elements. This signifies that the space of the vector functions is a portion of the space of the vector arguments and not every vector argument has a corresponding vector function. Consequently, the above matrix transformations only in part correspond to the conventional definition of an operator [4]. On the basis of the above, we shall designate the method proposed in [2] the method of scattering quasioperators.

The successful use of the method of scattering quasioperators for the design of structures with coordinate-plane discontinuities largely depends on how accurately the scattering matrix of the H-plane nonsymmetrical step is calculated in the available machine time. This is due to the fact that the step, along with waveguide branching (in the switch problem), is an elementary discontinuity which figures in almost all structures of this particular class.

On the other hand, in the method of scattering quasioperators, the step is regarded as a partial form of the structure in the form of a branching of waveguides with a short circuit in one narrow channel. The nature of the discontinuity near the sharp edge (near field) differs considerably for the zero and nonzero separation of the short circuit from the edge. (In the former case there is also a waveguide junction step.) In [5] a proof is given for the validity of the passage to the limit of a zero length of the short circuited waveguide for matrix transformations similar to those used in the method of scattering quasioperators.

However this feature of a step junction may make it necessary to invert an excessively large matrix when using the method of scattering quasioperators for the calculation. This would make a numerical realization of the algorithms of the method difficult or impossible for complicated structures.

The goal of the present work is to demonstrate the possibility of computing the scattering matrix of a step by the method of scattering quasioperators, using the computer, and of predicting, on the basis of these calculations, the rate of convergence of the method in calculations for complicated structures with coordinate-plane discontinuities. The paper discusses a block diagram of a computer program and the numerical results obtained from a M-20 type computer.

THE COMPUTER ALGORITHM

A H-plane nonsymmetrical step junction of rectangular waveguides is shown in Fig. 1. The vector E is parallel to the edge of the step.

In the method of scattering quasioperators, the design of the step junction is regarded as a partial case in the design of the structure, the general layout of which is shown in Fig. 2.

Here, S is the discontinuity in the form of a branching of three regular waveguides A, B, T; the waveguides A and B are semi-infinite, while waveguide T is short circuited by the distance l from the discontinuity S. In the waveguide step, the

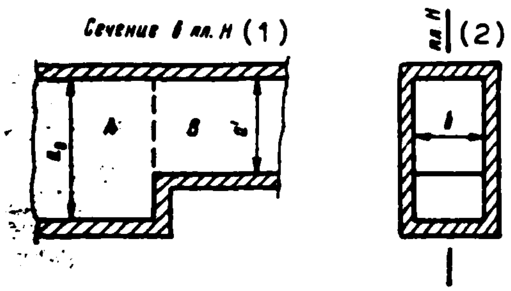

Fig. 1.
Key: (1) Section in the H plane; (2) H plane.

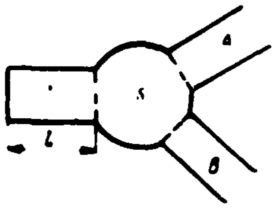

Fig. 2.

length l is equal to zero.

Figure 3 shows the H-plane sections of structures which can be presented in the generalized form of Fig. 2.

I - a waveguide turn, II and III - junctions of two rectangular waveguides with a stub. In the partial case, when l=0, structures II and III are converted to the H-plane nonsymmetrical step.

The scattering matrix of structures I, II, III contains four infinite components:

$$\overline{S}=\left[\begin{array}{c|c}\{aA\} & \{aB\}\\ \hline \{bA\} & \{bB\}\end{array}\right].$$

here, {aA},{bB} are scattering submatrices of channels A and B, respectively; {aB},{bA} are submatrices for the passage from waveguide B into waveguide A and vice versa. The elements of the

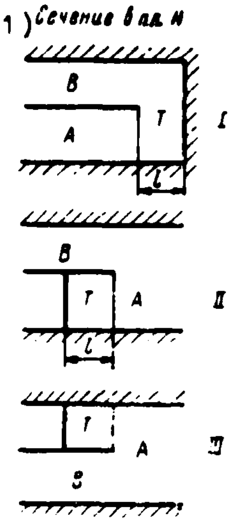

Fig. 3.
Key: (1) Section
in the H plane.

submatrices have additional indexes, for example $\{aB\}_{ij}$, which designates the element of the i-th row of the j-th column of component {aB}, being the amplitude of the E-field of the H_{i0} wave, excited in waveguide A by the wave H_{j0}, incident on the waveguide junction from the direction of waveguide B.

In accordance with the method of scattering quasioperators, the submatrices of the structures shown in Fig. 3 can be calculated from the following exact matrix formula:

$$\bar{A} = \bar{S}^{(1)} - \bar{S}^{(2)} [\bar{I} + \bar{T}\bar{T}\bar{S}^{(3)}]^{-1} \bar{T}\bar{T}\bar{S}^{(4)}, \tag{1}$$

where

$\bar{A}$ is the symbol for any given cell of the matrix $\bar{S}$;

$\bar{S}^{(1)}$-$\bar{S}^{(4)}$ are the cells of the scattering matrix of the waveguide branching which is part of the particular structures;

T is a diagonal matrix with elements T_{ij}:

$$T_{ij} = \begin{cases} 0 & \text{at } i \neq j, \\ e^{i\Gamma_j l} & \text{at } i = j. \end{cases}$$

Here, l and Γ_j are the geometrical length and the lengthwise wave number: H_{j0} is the mode of the communication waveguide. When l=0 and $\bar{T}$=1, formula (1) is simplified:

$$\bar{A} = \bar{S}^{(1)} - \bar{S}^{(2)} [\bar{I} + \bar{S}^{(3)}]^{-1} \bar{S}^{(4)}. \qquad (2)$$

Thus, the scattering matrices of structures I, II, III can be calculated by a single computer program, the machine time for the calculation of any of the structures depending solely on the dimensions of the required matrix and the given accuracy. As initial data for the program, the operator should enter in the machine the values of the geometrical parameters (a_0, a'), the frequency (or wavelength in free space λ), the length l, the accuracy, the dimensions, and the designator (version) of that submatrix of structures I, II, III which is to be computed. There are four submatrices in each of the three structures, and therefore a total of 12 versions is possible.

Each of the versions is characterized by a set of four conventional codes, corresponding to the specific submatrices $\bar{S}^{(i)}$

of the waveguide branching. The conventional codes of all 12 versions can be entered in the operational memory of the computer as octuple constants.

In the following section we discuss the block diagram of the computer program, formulated on the basis of the above-described computational algorithm.

Formulas of the type (1) and (2) have been obtained in [2]. The computational algorithms, estimates of accuracy, and the program for computer calculation of submatrices $\bar{S}^{(1)}$-$\bar{S}^{(4)}$ have been considered in [3].

THE BLOCK DIAGRAM OF THE COMPUTER PROGRAM

Let us discuss the basic elements of the program formulated for computer calculation of the scattering matrices of the devices in Fig. 3, using formulas (1) and (2). The block diagram of the program is shown in Fig. 4. We distinguish foremost in the program the unit for calculation of the submatrix $\bar{A}$ (independently of the specific values of the component matrices $\bar{S}^{(1)}$-$\bar{S}^{(4)}$, $\bar{T}$) and shall call this the matrix program unit.

Another unit of the program should implement the preparation for calculation of the specific submatrices of the waveguide branching, corresponding to the given cell of the matrix of the selected structure. We shall call this part of the program the analysis unit. The calculation of the values of the elements of submatrices $\bar{S}^{(1)}$-$\bar{S}^{(4)}$ is carried out by the program described in [3].

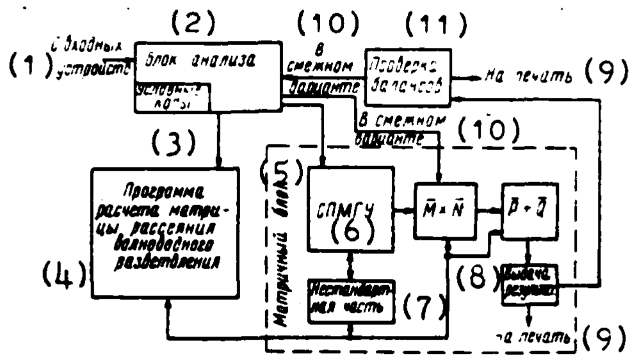

Fig. 4.
Key: (1) From the input structures; (2) Analysis unit; (3) Conventional codes; (4) Program for calculation of the scattering matrix of the waveguide branching; (5) Matrix unit; (6) SPMGU (standard program of Moscow State University); (7) Nonstandard portion; (8) Output of results; (9) To printer; (10) In adjacent version; (11) Check of balances.

Thus, the complete program for calculation of the scattering matrices of structures I, II, III (Fig. 1) consists of an analysis unit, a matrix unit, and a program for calculation of the scattering matrix of the waveguide branching [3]. Let us consider the structure of the first two units.

The analysis unit. As shown in [3], the scattering matrix of a waveguide branching consists of 9 cells. We shall mark each of these by a serial number, located in a single address or copy of an operative storage location (in a three-address computer):

$$\{nN\} \to 1, \quad \{nM\} \to 4, \quad \{nR\} \to 7,$$
$$\{mN\} \to 2, \quad \{mM\} \to 5, \quad \{mR\} \to 8_{10}\,(10_8),$$
$$\{rN\} \to 3, \quad \{rM\} \to 6, \quad \{rR\} \to 9_{10}\,(11_8).$$

Now, to each of the submatrices determined by formula (1) or (2) there corresponds an individual set of four code numbers, which may be stored at a single location of the operational storage of a three-address computer: the number of the submatrix $\bar{S}^{(1)}$ in the copy, the numbers of submatrices $\bar{S}^{(2)}$, $\bar{S}^{(3)}$, $\bar{S}^{(4)}$ in the three addresses of the same storage location, as shown in Table 1.

Table 1.

(a) Номер устройства	Условные коды (b)				(c) Обозначение субматрицы	(d) Номер варианта
	$\bar{S}^{(1)}$	$\bar{S}^{(2)}$	$\bar{S}^{(3)}$	$\bar{S}^{(4)}$		
I	001	0004	0005	0002	$\{aA\}$	10
	003	0006	0005	0002	$\{bA\}$	11
	011	0006	0005	0010	$\{bB\}$	12
	007	0004	0005	0010	$\{aB\}$	13
II	011	0003	0001	0007	$\{bB\}$	14
	010	0002	0001	0007	$\{aB\}$	15
	005	0002	0001	0004	$\{aA\}$	16
	006	0003	0001	0004	$\{bA\}$	17
III	001	0007	0011	0003	$\{bB\}$	18
	002	0010	0011	0003	$\{aB\}$	19
	005	0010	0011	0006	$\{aA\}$	20
	004	0007	0011	0006	$\{bA\}$	21

Key: (a) Number of structure; (b) Conventional codes; (c) Designator of submatrix; (d) Number of version.

The conventional codes in Table 1 are stored in the register of the operative memory in succession, as the initial octuple constants. To each sought submatrix of these particular structures there is assigned a serial number of the version - from 10 to 21 (cf. the right column of Table 1). The number of the version is entered in the operative storage from punched card, along with the other decimal constants of the problem.

The analysis unit operates in the following sequence. The conventional code corresponding to the given version is entered in the working register. Addresses are formed for the calculation of the cells of the matrix of the waveguide branching, corresponding to the code numbers at the copy and at the addresses of the working register with conventional code. In another working register there is recorded the quantity $(2\kappa/\lambda)^2$, where κ is the width of the communication waveguide, corresponding to the contents of the second address of the working register with conventional code, for calculation of the T matrix. After this, the control is transferred to the matrix unit.

The matrix unit of the program, in accordance with formula (1) or (2), carries out:

- the inversion of the matrix with complex elements;
- the multiplication of the complex matrices;
- the algebraic addition of the complex matrices.

In order to obtain results with high precision, it may be necessary to carry out all or certain of the operations on matrices of an elevated order, as the latter cannot be fully entered in the operative computer storage (MOZU). Therefore, in formulating the program, a provision should be made for the possibility of reading during the recording in the MOZU of one row of complex matrices which take part in the operations.

Since there is no appropriate standard program included in the IS-2 of M-20 type computers, we employed the program of the Computer Center of MGU (Moscow State University) [6], which enables the combination of the inversion of the complex matrix and the subsequent multiplication of the result by another complex matrix. During this process, only a single row of the first and second matrix is recorded at the same time in the MOZU.

In addition to the standard program of MGU [6], the matrix unit includes subprograms for the multiplication and addition of two complex matrices, as well as a subunit for the output of the results, which controls the operation of a printer. The calculation results are printed out in the form of individual files of a decimal printing, represented by consecutive lines of the sought submatrices.

In order to check the correct functioning of the program and to evaluate the accuracy of the intermediate results, the program provides for the calculation of the balance of the active powers. In order to determine the balance in the matrix unit there is an additional calculation of the submatrix of an "adjacent" version, which is characterized by the same excitation waveguide as the initial version. Versions which are "adjacent" to each other are 10 and 11, 12 and 13, etc. After calculating the initial and the "adjacent" versions and printout of their corresponding submatrices, there is a printout for a file of an even number of decimal values which are equal in pairs to the left and right portions of the balance equations for the active power of the particular device. The quantity of pairs of numbers in the file is equal to the quantity of balance equations of active powers which is possible in the problem.

In a type M-20 computer, this program requires a slight computer time. Thus, the calculation of two submatrices of the 10-th order with an allowance for 25 modes of the communication waveguide is carried out in not more than 15 minutes.

ESTIMATION OF THE COMPUTATIONAL ACCURACY. DISCUSSION OF THE RESULTS

Let us estimate the accuracy of the calculation of the scattering matrix of a step junction, obtained by the described computer program. For the step, the length of the communication region 1 is equal to zero and, consequently, the calculations should be carried out by formula (2). It is apparent from formula (2) that the calculation accuracy for any given submatrix $\bar{A}$ depends, in the first place, on the accuracy of calculating the submatrices $\bar{S}^{(i)}$ of the waveguide branching and, in the second place, on the order of the matrix which is to be inverted (i.e. on the number of waves considered in the communication region). The potential accuracy of the calculation for $\bar{A}$ by formula (2) does not exceed the precision of finding the elements of $\bar{S}^{(i)}$. As shown in the appendix to this paper, the potential accuracy of the calculation for the submatrices $\bar{A}$ can be evaluated by the balance of active powers which, however, cannot be used to estimate the resulting precision, as it should be carried out independently of the number of waves considered in the communication waveguide.

In order to estimate the accuracy of the solution determined by formula (2), let us find a series of solutions for different values for the order of the inverted matrix (it is necessary to draw up the balance of the active powers at all the points). In the case of a waveguide step, the elements of the inverted matrix vary monotonically with respect to the modulus and the argument (cf. [3]). Therefore, as the order of the inverse matrix increases, the elements of the sought matrix $\bar{A}$ should vary smoothly, asymptotically approaching the most accurate solution. Table 2 shows such a series of solutions for several values of the geometrical and frequency parameters of the step.

Table 2.

n	$\{bB\}_{II}$	$\{bB\}_{II}$	n	$\{aA\}_{II}$
	$a_0/\lambda = \sqrt{1,2}$; $a'/a_0 = 0,475$	$a_0/\lambda = \sqrt{1,2}$; $a'/a_0 = 0,4999$		$a_0/\lambda = 1/1,4$; $a'/a_0 = 0,575$
3	$-0,3364 + 0,1591$	$-0,1653 + 0,1284$	2	$-0,7993 + 0,6009$
6	$-0,3331 + 0,1622$	$-0,1621 + 0,1301$	4	$-0,8087 + 0,5880$
9	$-0,3323 + 0,1634$	$-0,1611 + 0,1309$	6	$-0,8112 + 0,5849$
12	$-0,3319 + 0,1640$	$-0,1607 + 0,1313$	7	$-0,8119 + 0,5838$
13	$-0,3318 + 0,1642$	$-0,1606 + 0,1315$	8	$-0,8125 + 0,5829$

Analyzing the data of Table 2, we see that it is sufficient to take an order of n=10 for the matrix which is to be inverted, so that the difference between the solutions by formula (2) for the preceding and following value of n is not more than 0.5% of the latter. Upon further increase in n, this difference will decrease monotonically to zero when $n \to \infty$, upon which an accuracy will be obtained which is determined solely by the accuracy of the calculation for $\bar{S}^{(i)}$. In the methods which provide a series of solutions which asymptotically approach the exact solution, the error is estimated by the relative size of the difference between the last two solutions of the series [4]. Thus, the error of the results does not exceed 0.5%. The data of Table 2 permits an assessment of the "rate of convergence of the solution" to the "exact", i.e. the speed of reduction of the difference between two adjacent solutions with increase in n. From the standpoint of computer realization of the calculations, this speed is fully acceptable.

The results of the calculations by the method of scattering quasioperators are in good agreement with the theoretical data, obtained by other methods. Figure 5 shows curves from [7], calculated by means of the equivalent statistical method, using two quasistatic waves. The points indicated by small circles on these graphs have been calculated by the method of scattering quasioperators and are in good agreement with the curves (agreement not worse than 3-4 of the first decimal figures).

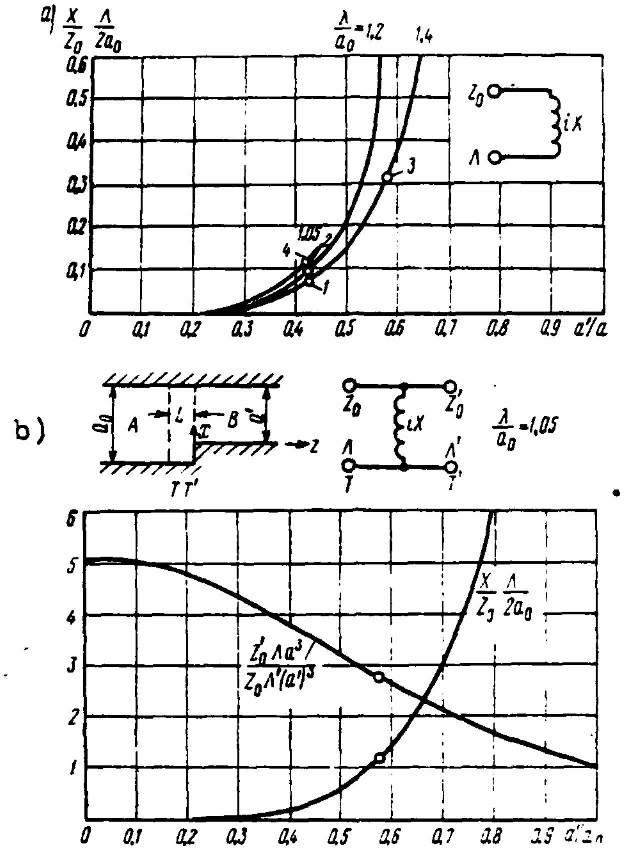

Fig. 5.

Figures 6-9 show the various submatrices of a H-plane waveguide step as a function of the number p in the first column and the number K in the first row. The figures indicate the number n of modes of the communication waveguide which were taken into account. The geometrical and frequency relationships for the calculation were chosen so that, in a wide waveguide, two propagating waves (H_{10}, H_{20}) could exist in all the particular cases, while in a narrow waveguide could exist either one propagating H_{10} wave or only attenuating modes. The former case occurs with $a'/a_0>0.5$, and the latter when $a'/a_0<0.5$ (a' and a_0 are the width of the narrow and broad articulated waveguides, respectively). .

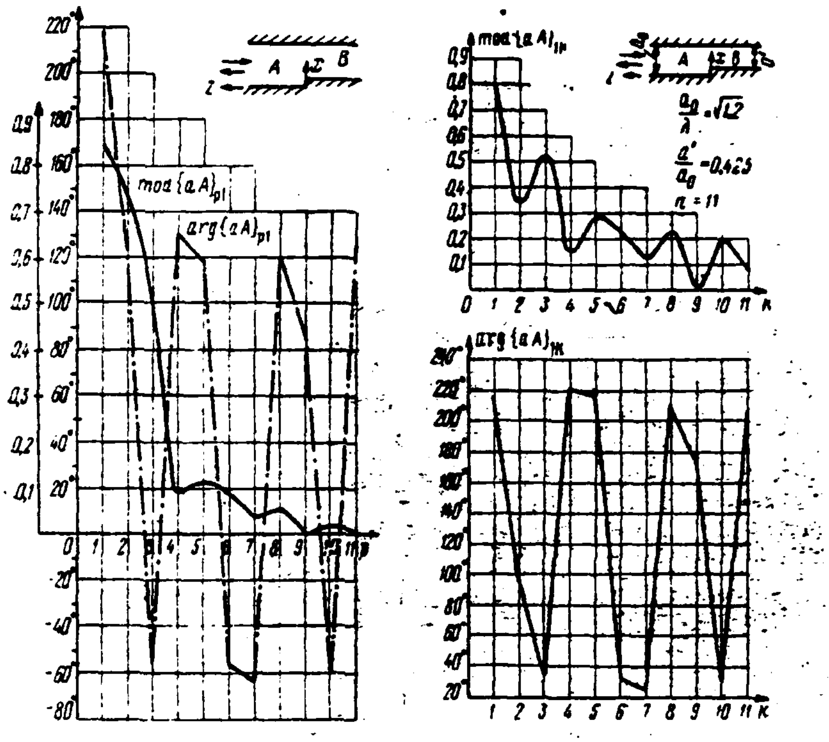

Fig. 6.

The curves shown in Figs. 6-9 permit an idea as to the nature of the dependence of the moduli and arguments of the elements of the step submatrices on the indexes in the row and column. It can be seen that, starting with a certain number in the sequence, in the row and in the column there is a decrease in the moduli of the elements, while the arguments of two adjacent elements differ from each other by an angle close to 0 or 180°. In all the submatrices of the step, except for {bB}, there are oscillations of the moduli of the elements, the minima corresponding to those p or K for which the quantities $\frac{a'}{a_0}\pi K$ or $\frac{a'}{a_0}\pi p$ are multiples of the number π.

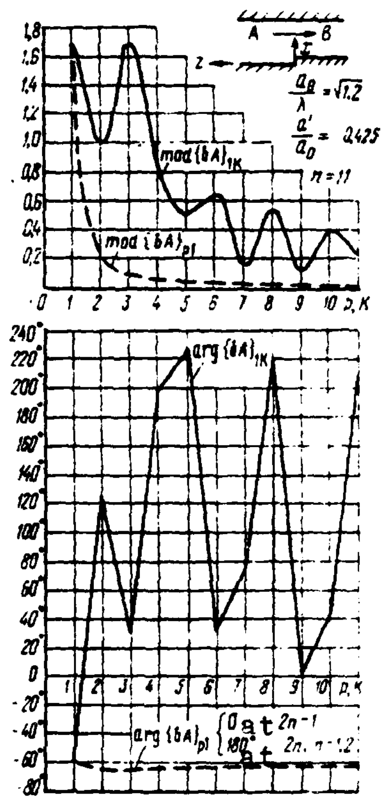

Fig. 7.

It is interesting to compare the graphs in Figs. 6-9 with the data given in [3] for the scattering matrix of a waveguide branching, forming a part of the step. It may be noted that the nature of the dependence of the elements in the rows and columns of the submatrix of the step resembles the dependence in the particular waveguide branching. In this case, the difference between the elements of the scattering matrices of the step and of the initial branching is the greater as the ratio between the width of the narrow and that of the wide waveguides, comprising the step, is smaller. At the limit, when this ratio approaches zero, the scattering matrix of the step is converted to the scattering matrix of an ideal short circuit.

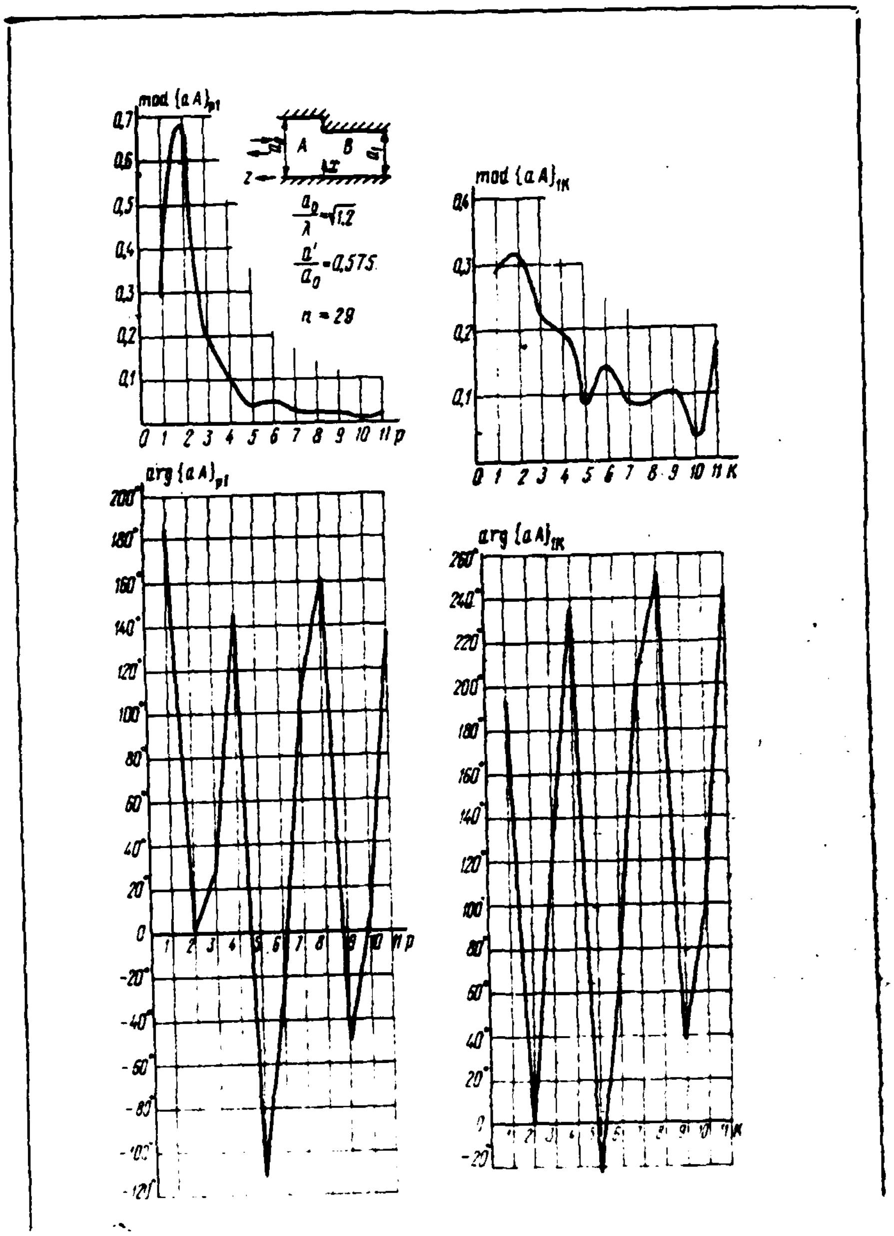

Fig. 8.

From these graph comparisons it is apparent that the rate of decrease in the moduli of the elements in the rows and columns of the matrix of the step is not less than that in the matrix of the corresponding waveguide branching (cf. the graphs of [3]). This fact allows us to suppose that, when designing more complicated structures by the algorithms of the method of scattering quasioperators, the use of the matrix of the step will not impair the rate of convergence of the solution. Therefore, in order to obtain an identical accuracy when repeatedly using the formulas of the method of scattering quasioperators, the order of the inverted matrices need not be increased significantly.

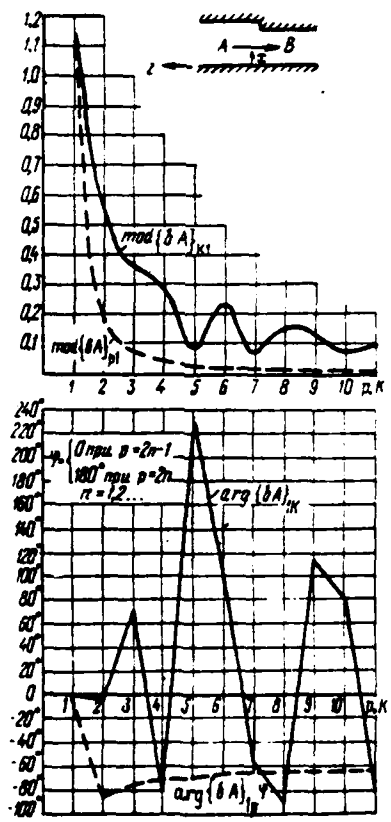

Fig. 9.

CONCLUSIONS

The materials of the calculation of the scattering matrix of a H-plane step junction, given in this paper, demonstrate that the method of scattering quasioperators can be used with success for the computer calculation of structures of varying complexity with coordinate-plane computer discontinuities, with a high degree of accuracy. A reliable criterion for the correctness of the intermediate results and the absence of computer malfunction is the drawing up of balances of the active powers.

APPENDIX

ON THE BALANCE OF THE ACTIVE POWERS IN WAVEGUIDE STRUCTURES

In the design of waveguide structures which consist of regular waveguides and discontinuities, the condition of drawing up balances of the active powers is often used as the solitary criterion for the exact solution of a problem for the fields in the output waveguides of the device. Rough errors may occur in this process, one of which, overlooked in [Ap1], has been discused in [Ap2]. In this work [Ap1], the electrical parameters of the slit bridge with short slit were incorrectly determined as a result of failing to allow for the upper modes in the communication region, even though the balance of the active powers was carried out with high precision.

In numerous calculations of waveguide structures by type M-20 computers, performed by the author of this paper, a balance of the active powers was drawn up at all points under the most diverse numbers of waves, considered in the communication waveguide - from merely the propagating waves (1,2,3) up to the consideration of 32 modes of the communication waveguide. The discussion gives reason to suppose that the drawing up of a balance of active powers is a necessary, but not sufficient condition for obtaining an exact solution of the waveguide problem. We present below an analytic proof of this supposition.

Let us consider a waveguide structure, consisting of two discontinuities S_1 and S_2, joined by a regular communication waveguide T, and two semi-infinite regular waveguides I and II (Fig. Ap.1). In the waveguide T there may propagate m types of waves. We shall first prove that the balance of active powers for this particular structure does not allow for all the attenuating waves of the waveguide T. By P_I and P_{II} we shall designate the exact values of the powers produced by channels I and II when a wave arrives from the direction of waveguide I. The power of the incident wave shall be designated by P_0. The balance of active powers for the structure shown in Fig. Ap.1 will be written in the form:

$$P_0 = P_I + P_{II}. \qquad \text{(Ap.1)}$$

Equation (1) is valid for any given values of the length of the communication waveguide l and takes into account all types of waves in this waveguide.

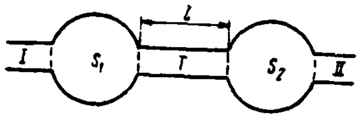

Fig. Apl.

Now let the length l be so large that all the higher types of waves in waveguide T are attenuated within a length l. By $\mathscr{P}_I$ and $\mathscr{P}_{II}$ we shall designate the powers produced from channels I and II; when l is large, we shall write the equation for the balance of the active powers as:

$$\mathscr{P}_I + \mathscr{P}_{II} = P_0. \tag{Ap.2}$$

Equation (Ap2) only allows for the propagating types of waves.

The dependence of $\mathscr{P}_I$ and $\mathscr{P}_{II}$ on l of the amplitudes of these waves is determined by the functions $e^{ia_j T_j l}$, where j is the number of the propagating wave from 1 to m; a_j is the number of a natural series; T_j is the real constant of propagation of the j-th mode. The powers $\mathscr{P}_I$ and $\mathscr{P}_{II}$ do not change if the quantities $a_j T_j l$ increase or decrease by $2\pi b_j$ (b_j are the numbers of the natural series). Consequently, the balance of active powers (Ap2) - without allowing for the attenuating modes of the wave-guide T - should be carried out for any given values of the length l direction down to zero.

It is now necessary to prove that the balance of active powers of the device shown in Fig. Ap1 should take into account all the propagating modes and any given finite number of attenuating modes of waveguide T.

Let the length l be small and therefore the exact values of the powers produced from channels I and II (P_I and P_{II}) differ from the quantities $\mathscr{P}_I$ and $\mathscr{P}_{II}$, calculated without allowing for the attenuating waves of the waveguide T. The differences

$$\left.\begin{array}{l} P_I - \mathscr{P}_I = \Delta P_I \\ P_{II} - \mathscr{P}_{II} = \Delta P_{II} \end{array}\right\} \qquad \text{(Ap.3)}$$

represent the summary active powers, transferred by the attenuating modes of the waveguide T to waveguides I and II, respectively. The possibility of transferring the active power by attenuating modes in a waveguide between two discontinuities has been proved in [Ap3].

From equations (Ap1)-(Ap3) it is not difficult to obtain:

$$\Delta P_I + \Delta P_{II} = 0. \qquad \text{(Ap.4)}$$

The left side of equation (Ap4) will be written in the form of a sum:

$$\Delta P_I + \Delta P_{II} = \Delta' + \Delta'' + \Delta''' + \ldots = 0, \qquad \text{(Ap.5)}$$

where $\Delta^{(j)}$ is the summary active power transferred to channels I and II by the j-th attenuating mode.

Since the active powers in a passive system without losses cannot be negative, while their sum is equal to zero, each of the quantities $\Delta^{(j)}$ should be equal to zero. On this basis, we shall alter the balance equation:

$$P_0 = P_{Iq} + P_{IIq}, \tag{Ap.6}$$

where P_{Iq} and P_{IIq} are the active powers transferred by the outgoing waves in waveguides I and II, calculated with an allowance for any given finite number q of attenuating modes in the junction waveguide T. It is obvious that

$$\begin{aligned} &\text{when} \quad q = 0 \quad P_{Iq} = \mathcal{P}_I, \quad P_{IIq} = \mathcal{P}_{II}, \\ &\text{when} \quad q \to \infty \quad P_{Iq} = P_I, \quad P_{IIq} = P_{II}. \end{aligned}$$

The significance of equation (Ap6) is the following.

An allowance for the active power transferred by any given number of attenuating types of waves in a waveguide between two discontinuities, without disrupting the balance of the active powers, produces a change in the calculated (but not the actual) active power in the output branches of the waveguide system. Thus, the validity of the balance of active powers in the design of systems of discontinuities, closely arranged in regular waveguides, may be considered necessary, but not a sufficient condition for the correctness of finding the active powers in the output channels of the waveguides.

This proof employs the condition of rationality

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of the longitudinal wave numbers of the propagating waves of the junction waveguide (when $m>1$), since, in the opposite case, it is not possible to find values l which can satisfy the system

$$a_j \Gamma_j l = 2\pi b_j, \quad j=1 \ldots m.$$

The longitudinal wave numbers are defined as the square root of a certain number and, as a rule, are irrational numbers, the ratios of which are likewise irrational. However, for a manual or computer calculation, we always employ numbers with a certain finite number of decimal figures, i.e. rational numbers. Therefore, the calculation may require a mandatory equalization of the left and right halves of the balance equation of the active powers with a certain given accuracy for any given values of the length of the junction waveguide.

This proof is valid for any given waveguide systems, consisting of nonuniform regions, joined by segments of regular waveguides. It is easy to be convinced of this, as a similar proof can be given at first for more simple parts, treating these as ideally disconnected. The proof should then be repeated for the entire connection as a whole, under the condition that the more simple parts will be treated as partial discontinuities.

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Submitted
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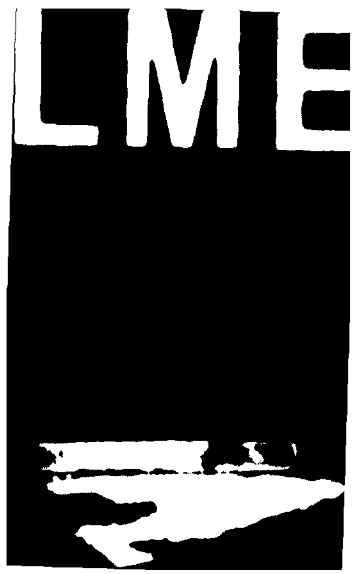
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