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FTD-ID(RS)T-0826-81 16 Octo~~ber 19~~81

MICROFICHE NR: FTD-81-C-000957

ANTENNA (Selected Articles)

~~English pages: 122~~

~~Source:~~ Antenny~~, Nr. 11, 1971, pp. 1-~~59

Country of origin: USSR
This document is a machine translation
Requester: FTD/TQFE
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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

ANTENNAS.

Page 2 no Typing.

Page 3.

Excitation of radial resonator with annular slot.

V. A. Poperechenko.

Is solved the problem about the external and internal excitation of radial resonator with the narrow annular slot, which radiates into the half-space. Is more precisely formulated a question about the completeness of set of functions, which describe fields within the resonator during the excitation by its only alternating outside currents. Is applied the known method of addition and integrating of the badly/poorly converging series and integrals.

In this article based on the example of sufficiently typical antenna in the form of narrow annular slot with the radial resonator, excited by the arbitrarily distributed alternating outside currents (Fig. 1), are stated the method of the sufficiently strict solution of electrodynamic problem and the calculations of the fundamental parameters of the antenna: the distribution of voltage along the slot, the radiation pattern, polarizational characteristic, composite conductivity of radiation/emission and input resistance. In the literature [1, 2, 3, etc.] were examined similar problems for other special cases of the form of resonator, slot and exciting sources.

Let us turn directly to the solution of stated problem whose electrodynamic formulation is clear from Fig. 1. The walls of resonator and unlimited infinitely thin screen, which divides half-spaces, are accepted ideally conducting. The obtained below results in certain part are close to the series/row of the published works. Nevertheless the presentation of these results is useful from a systematic point of view for further improvement of the engineering methods of calculation of nonresonant slot antennas on the base of strict methods.

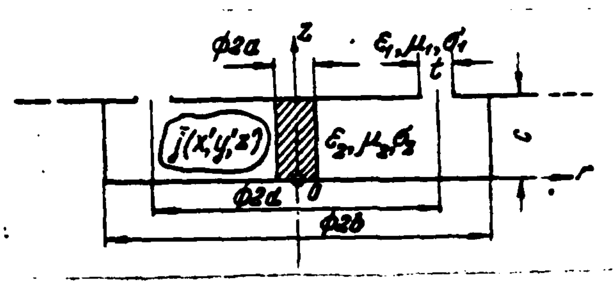

Fig. 1.

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The solution of stated here problem is realized by the method of partial regions with the imposition of condition of continuity on the average/mean in the width slots of the value of the tangential components of the vectors of field. In this case it is systematic convenient in accordance with the principle of equivalency the tangential component of electric field in the slot of each of the partial regions of representing as equivalent surface magnetic current [4], superimposed in the limits of slot on the fictitious ideally conducting screen, which completely divides partial regions from each other (Fig. 2).

Continuity conditions indicated for the average/mean values of field analytically are represented in the form

$$H_{\varphi_{\text{ср}}}^{\text{внешн}}(\varphi) = H_{\varphi_{\text{ср}}}^{\text{внутр}}(\varphi)$$

for $\bar{H}$,

where

$$H_{\varphi_{\text{cp}}}(\varphi) = \frac{1}{t} \int\limits_{d-\frac{t}{2}}^{d+\frac{t}{2}} H_{\varphi}(\varphi, r)\, dr|_{z=c}$$

and

$$I_{\varphi}^{\text{м внутр}}(\varphi) = -I_{\varphi}^{\text{м внешн}}(\varphi) \overset{(1)}{\text{для}}\ \bar{E},$$

Key: (1). for

where with an accuracy to the sign, which depends on the direction of normal to the surface of slot,

$$I_{\varphi}^{\text{м}}(\varphi) = \frac{1}{t} \int\limits_{d-\frac{t}{2}}^{d+\frac{t}{2}} E_r(\varphi, r)\, dr.$$

Are here for the narrow slot taken in the attention only respectively longitudinal and transverse components of field $\bar{H}$ and $\bar{E}$. Magnetic current $I_{\varphi}^{\text{м внеш}}(\varphi)$ is the unknown function, which is represented in the form of Fourier series

$$I_{\varphi}^{\text{м внеш}}(\varphi') = \sum_{n=-\infty}^{+\infty} u_n e^{in\varphi'}.$$

Let us pass directly to the solution of the formulated problem. For this let us record the expressions of field in the external space and within the resonator through components E_z and H_z. The solution of the equations of Maxwell for these components in the external space let us record in that form, as this is given in work [5].

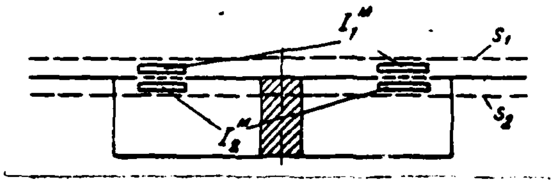

Fig. 2.

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Taking into account the mirror image in the infinite screen of the expression of full/total/complete fields E_z and H_z they will take the form

$$\left.\begin{matrix} E_z \\ H_z \end{matrix}\right\} = \sum_{n=-\infty}^{+\infty} \int_0^{\infty} \frac{e^{\sqrt{\varkappa^2-\kappa_1^2}(z-c)}}{4\pi} e^{-in\varphi} J_n(\varkappa r) \left[\left\{\begin{matrix} F_n^{э+} \\ F_n^{м+} \end{matrix}\right\} e^{\sqrt{\varkappa^2-\kappa_1^2}\, c} + \left\{\begin{matrix} F_n^{э-} \\ F_n^{м-} \end{matrix}\right\} e^{-\sqrt{\varkappa^2-\kappa_1^2}\, c} \right] \frac{\varkappa\, d\varkappa}{\sqrt{\varkappa^2-\kappa_1^2}}$$

for z>z'

$$\left\{\begin{matrix} E_z \\ H_z \end{matrix}\right\} = \sum_{n=-\infty}^{+\infty} \int_0^{\infty} \frac{\operatorname{ch}\left[\sqrt{\varkappa^2-\kappa_1^2}(z-c)\right]}{2\pi} e^{-in\varphi} J_n(\varkappa r) \left\{\begin{matrix} F_n^{э+} \\ F_n^{м+} \end{matrix}\right\} e^{\sqrt{\varkappa^2-\kappa_1^2}\, c} \frac{\varkappa\, d\varkappa}{\sqrt{\varkappa^2-\kappa_1^2}}$$

for z<z'.

Here coefficients $F^э$ and $F^м$ are equal to:

$$F_n^{э\pm} = \int_{v'} e^{\mp \sqrt{\varkappa^2-\kappa_1^2}\, z'} e^{in\varphi'} \left\{ \frac{\varkappa^2}{i\omega\varepsilon_1} j_z^э J_n(\varkappa r') + \right.$$

$$+ \left(\frac{\pm \sqrt{\varkappa^2-\kappa_1^2}}{i\omega\varepsilon_1} j_\varphi^э - j_r^м \right) \frac{in}{r'} J_n(\varkappa r') +$$

$$\left. + \left(\frac{\pm \sqrt{\varkappa^2-\kappa_1^2}}{i\omega\varepsilon_1} j_r^э + j_\varphi^м \right) \varkappa J_n'(\varkappa r') \right\} r' dr' d\varphi' dz',$$

$$F_\varkappa^{м\pm} = \int_{v'} e^{\mp i \sqrt{\varkappa^2-\kappa_1^2}\, z'} e^{in\varphi'} \left\{ \frac{\varkappa^2}{i\omega\mu'} j_z^м J_n(\varkappa r') + \right.$$

$$+ \left(\frac{\pm \sqrt{\varkappa^2-\kappa_1^2}}{i\omega\mu'} j_\varphi^м + j_r^э \right) \frac{in}{r'} J_n(\varkappa r') +$$

$$\left. + \left(\frac{\pm \sqrt{\varkappa^2-\kappa_1^2}}{i\omega\mu'} j_r^м - j_\varphi^э \right) \varkappa J_n'(\varkappa r') \right\} r' dr' d\varphi' dz',$$

where when $\varkappa<\kappa_1$ the sign of imaginary radical $\sqrt{\varkappa^2-\kappa_1^2}$ is taken by negative.

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Remaining field components at any point spaces are expressed as E_z and H_z and currents $j_r^э, j_\varphi^э, j_r^м, j_\varphi^м$ with the help of the relationships/ratios:

$$\kappa^2 E_r + \frac{\partial^2 E_r}{\partial z^2} = -\frac{i\omega\mu}{r}\frac{\partial H_z}{\partial\varphi} + \frac{\partial^2 E_z}{\partial r\,\partial z} - \frac{\partial j_\varphi^м}{\partial z} + i\omega\mu\, j_r^э;$$

$$\kappa^2 E_\varphi + \frac{\partial^2 E_\varphi}{\partial z^2} = i\omega\mu\frac{\partial H_z}{\partial r} + \frac{1}{r}\frac{\partial^2 E_z}{\partial\varphi\,\partial z} + \frac{\partial j_r^м}{\partial z} + i\omega\mu\, j_\varphi^э;$$

$$\kappa^2 H_r + \frac{\partial^2 H_r}{\partial z^2} = \frac{\partial^2 H_z}{\partial r\,\partial z} + \frac{i\omega\varepsilon}{r}\frac{\partial E_z}{\partial\varphi} + i\omega\varepsilon\, j_r^м + \frac{\partial j_\varphi^э}{\partial z};$$

$$\kappa^2 H_\varphi + \frac{\partial^2 H_\varphi}{\partial z^2} = \frac{1}{r}\frac{\partial^2 H_z}{\partial\varphi\,\partial z} - i\omega\varepsilon\frac{\partial E_z}{\partial r} + i\omega\varepsilon\, j_\varphi^м - \frac{\partial j_r^э}{\partial z}.$$

The function of unknown magnetic current $I_{\varphi}^{M}(\varphi)$ enters into data of expression, being expressed through the bulk density of current in the following analytical form:

$$j_{\varphi}^{M}=\frac{I_{\varphi}^{M}}{t}\frac{d}{r^{4}}\delta(z'-c)\int\left[\delta\left(r'-d+\frac{t}{2}\right)-\delta\left(r'-d-\frac{t}{2}\right)\right]dr'.$$

Fields within the resonator through the outside currents are determined by the method of solution of the nonhomogeneous wave equations, valid in the absence of outside static charges and steady currents:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\partial E_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 E_z}{\partial\varphi^2}+\frac{\partial^2 E_z}{\partial z^2}+\kappa_2^2 E_z=$$
$$=\mathrm{rot}_z\,\overline{j}^{M}+i\,\omega\mu_2 j_z^{э}-\frac{1}{i\,\omega\varepsilon_2}\,\mathrm{grad}_z\,(\mathrm{div}\,\overline{j}^{э}) \qquad (1)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\partial H_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 H_z}{\partial\varphi^2}+\frac{\partial^2 H_z}{\partial z^2}+\kappa_2^2 H_z=$$
$$=-\mathrm{rot}_z\overline{j}^{э}+i\,\omega\varepsilon_2 j_z^{M}-\frac{1}{i\,\omega\mu_2}\,\mathrm{grad}_z\,(\mathrm{div}\,\overline{j}^{M})\,. \qquad (2)$$

Similar vector wave equations for the electrical sources are given in [6].

The full/total/complete unknown solutions of equ. (1) and (2) (see the appendix) can be represented in the form:

$$E_z=\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty}F_{lmn}^{э}\cos\frac{l\pi z}{c}\,\mathrm{sn}_n(\varkappa_m^{e}b,\ \varkappa_m^{e}r)\,e^{-in\varphi}, \qquad (3)$$

$$H_z=\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=-\infty}^{+\infty}F_{lmn}^{M}\sin\frac{l\pi z}{c}\,\mathrm{cs}_n(\varkappa_m^{h}b,\ \varkappa_m^{h}r)\,e^{-in\varphi}, \qquad (4)$$

where through $sn_n(\varkappa_m b, \varkappa_m r)$ and $cs_n(\varkappa_m b, \varkappa_m r)$ are designated respectively sine and cosine small radial of n order [7]:

$$sn_n(\varkappa_m b, \varkappa_m r) = N_n(\varkappa_m b) J_n(\varkappa_m r) - N_n(\varkappa_m r) J_n(\varkappa_m b),$$
$$cs_n(\varkappa_m b, \varkappa_m r) = N_n'(\varkappa_m b) J_n(\varkappa_m r) - N_n(\varkappa_m r) J_n'(\varkappa_m b).$$

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Parameters $\varkappa_m^e$ and $\varkappa_m^h$ are the n roots of the equations

$$sn_n(\varkappa_m^e b, \varkappa_m^e a) = 0 \quad \text{при} \quad m = 0, 1, 2, \ldots \quad (\varkappa_0^e = 0)$$

and

$$\varkappa_m^h Sn_n(\varkappa_m^h b, \varkappa_m^h a) = 0 \quad m = 0, 1, 2, \ldots \quad (\varkappa_0^h = 0),$$

where $Sn_n(\varkappa_m^h b, \varkappa_m^h a)$ - sine large radial is equal to:

$$Sn_n'(\varkappa_m b, \varkappa_m a) = N_n'(\varkappa_m b) J_n'(\varkappa_m a) - N_n'(\varkappa_m a) J_n'(\varkappa_m b).$$

For determining the coefficients F_{lmn}^e and F_{lmn}^m in the source function we will use the relationships/ratios of the orthogonality:

$$\int_0^{2\pi} e^{-in\varphi} e^{i\nu\varphi}\, d\varphi = \begin{cases} 2\pi & \text{при } \nu = n, \\ 0 & \text{при } \nu \neq n, \end{cases}$$

$$\int_0^c \sin\frac{l\pi z}{c}\sin\frac{L\pi z}{c}\, dz = \begin{cases} \frac{c}{2} & \text{при } L = l, \\ 0 & \text{при } L \neq l, \end{cases}$$

$$\int_0^c \cos\frac{l\pi z}{c}\cos\frac{L\pi z}{c}\, dz \begin{cases} \frac{c}{2} & \text{при } L = l,\ l \neq 0, \\ c & \text{при } L = l,\ l = 0, \\ 0 & \text{при } L \neq l, \end{cases}$$

$$\int_a^b sn_n(\varkappa_m^e b,\ \varkappa_m^e r)\, sn_n(\varkappa_\mu^e b,\ \varkappa_\mu^e r)\, r dr =$$

$$= \begin{cases} \frac{2}{(\pi\varkappa_m^e)^2}\left\{1 - \left[\frac{\pi\varkappa_m^e a}{2} cs_n(\varkappa_m^e a,\ \varkappa_m^e b)\right]^2\right\} & \text{при } \mu = m, \\ 0 & \text{при } \mu \neq m, \end{cases}$$

$$\int_a^b cs_n(\varkappa_m^h b,\ \varkappa_m^h r)\, cs_n(\varkappa_\mu^h b\, \varkappa_\mu^h r)\, r dr =$$

$$= \begin{cases} \frac{2}{(\pi\varkappa_m^h)^2}\left\{1 - \left(\frac{n}{\varkappa_m^h b}\right)^2 - \left[1 - \left(\frac{n}{\varkappa_m^h a}\right)^2\right]\left[\frac{\pi\varkappa_m^h a}{2} cs_n(\varkappa_m^h b,\ \varkappa_m^h a)\right]^2\right\} & \text{при } \mu = m, \\ 0 & \text{при } \mu \neq m. \end{cases}$$

Key (1). with.

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Substituting predicted solution (3) and (4) into wave equ. (1) and (2) and taking into account the recorded above relationships/ratios of orthogonality, it is not difficult to arrive at the values of the unknown coefficients:

$$F^{э}_{lmn}=\frac{\int\limits_{0}^{2\pi}\int\limits_{0}^{c}\int\limits_{a}^{b}\left[\mathrm{rot}_z\bar{j}^{м}+i\omega\mu_2 j^{э}_z-\frac{1}{i\omega\varepsilon_2}\mathrm{grad}_z(\mathrm{div}\,\bar{j}^{э})\right]\times}{\kappa_l\frac{2c}{\pi\varkappa^{e2}_m}\left[\kappa_2^2-\varkappa^{e2}_m-\left(\frac{l\pi}{c}\right)^2\right]\times}\to$$

$$\to\frac{\times e^{in\varphi'}\cos\frac{l\pi z'}{c}\,\mathrm{sn}_n(\varkappa^e_m b,\ \varkappa^e_m r')\,r'dr'd\varphi'dz'}{\times\left\{1-\frac{\pi\varkappa^e_m a}{2}\mathrm{cs}_n(\varkappa^e_m a,\varkappa^e_m b)\right]^2\right\}}, \quad (5)$$

$$F^{м}_{lmn}=\frac{\int\limits_{0}^{2\pi}\int\limits_{0}^{c}\int\limits_{a}^{b}\left[-\mathrm{rot}_z\bar{j}^{э}+i\omega\varepsilon_2 j^{м}_z-\frac{1}{i\omega\mu_2}\mathrm{grad}_z(\mathrm{div}\,\bar{j}^{м})\right]\times}{\kappa_l\frac{2c}{\pi\varkappa^{h2}_m}\left[\kappa_2^2-\varkappa^{h2}_m-\left(\frac{l\pi}{c}\right)^2\right]\left\{1-\left(\frac{n}{\varkappa^h_m b}\right)^2-\right.}\to$$

$$\to\frac{\times e^{in\varphi'}\sin\frac{l\pi z'}{c}\,\mathrm{cs}_n(\varkappa^h_m b,\ \varkappa^h_m r')\,r'dr'd\varphi'dz'}{\left.-\left[1-\left(\frac{n}{\varkappa^h_m a}\right)^2\right]\left[\frac{\pi\varkappa^h_m a}{2}\mathrm{cs}_n(\varkappa^h_m b,\ \varkappa^h_m a)\right]^2\right\}}, \quad (6)$$

where κ_l - symbol of Kronecker: $\kappa_0=2;\ \kappa_l|_{l\neq 0}=1$.

The zero terms of solution (3)-(6) describe the fields, which have in one, two or three coordinate planes the instantaneous structure of permanent field, and they are the solutions of the corresponding one-, two- or three-dimensional equations of Poisson. The zero terms of single multiplicity ($\varkappa_m=0$, or n=0, or l=0) correspond to waves of the type E and H without variations in the field on one of the transverse coordinates. The zero terms of dual multiplicity ($\varkappa_m=0$, n=0, or n=0, l=0 or $\varkappa_m=0$, $l=0$) correspond to waves electric wave mode. And finally the zero terms of triple multiplicity ($\varkappa_m=0$, n=0, l=0) correspond to potential waves. Let us designate them PEM.

In connection with this problem for the region out of the sources it is possible to ascertain that from a number of lowest transmission modes (PEM and TEM) they become zero, independent of the form of sources, field of both of waves PEM, both azimuthal waves TEM, also, on one of the cylindrical and plane waves of TEM. In order to be convinced of this, it suffices to use passage to the limit $\varkappa \to 0$, $n \to 0$, $l \to 0$ in (3) - (6) and in the relationships/ratios of field components E_r, E_φ, H_r, H_φ.

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Let us return to our problem and will pass to composition and solution of equations for determining the magnetic current in the slot. Let us write out for this coefficients $F^{e\pm}$, $F^{m\pm}$, F^{e}_{lmn}, F^{m}_{lmn}:

$$F_n^{e\pm} = a_n \frac{2\pi e^{\mp\sqrt{\varkappa^2-\kappa_1^2}\,c}}{t} \int\limits_{d-\frac{t}{2}}^{d+\frac{t}{2}} \varkappa\, dJ_n'(\varkappa r')\,dr',$$

$$F_n^{m\pm} = a_n \frac{2\pi e^{\mp\sqrt{\varkappa^2-\kappa_1^2}\,c}}{t}\,\frac{\pm\sqrt{\varkappa^2-\kappa_1^2}\,n}{\omega\mu_1} \int\limits_{d-\frac{t}{2}}^{d+\frac{t}{2}} \frac{d}{r'} J_n(\varkappa r')\,dr',$$

$$F_{lmn}^{e} = a_n \frac{2\pi d(-1)^l}{t}\,\frac{\mathrm{sn}_n\left[\varkappa_m^e b,\ \varkappa_m^e\left(d-\frac{t}{2}\right)\right] - \mathrm{sn}_n\left[\varkappa_m^e b,\ \varkappa_m^e\left(d+\frac{t}{2}\right)\right]}{\kappa_l \frac{2c}{\pi\varkappa_m^{e2}}\left[\kappa_2^2\varkappa_m^{e2} - \left(\frac{l\pi}{c}\right)^2\right]\left\{1-\left[\frac{\pi\varkappa_m^e a}{2}\,\mathrm{cs}_n(\varkappa_m^e a, \varkappa_m^e b)\right]^2\right\}},$$

$$F_{lmn}^{m} = a_n \frac{\frac{2\pi d(-1)^l}{t}\,\frac{l\pi n}{\omega\mu_2 c}\int\limits_{d-\frac{t}{2}}^{d+\frac{t}{2}} \frac{\mathrm{cs}_n(\varkappa_m^h b,\ \varkappa_m^h r')}{r'}\,dr'}{\kappa_l\frac{2c}{\pi\varkappa_m^{h2}}\left[\kappa_2^2-\varkappa_m^{h2}-\left(\frac{l\pi}{c}\right)^2\right]\left\{1-\left(\frac{n}{\varkappa_m^h b}\right)^2-\left[1-\left(\frac{n}{\varkappa_m^h a}\right)^2\right]\times \left[\frac{\pi\varkappa_m^h a}{2}\,\mathrm{cs}_n(\varkappa_m^h b,\ \varkappa_m^h a)\right]^2\right\}},$$

In the latter/last two equalities during the integration for z' and r' is used the relationship/ratio, known from the theory of δ-function [10]:

$$\int f(x)\,\delta^{(n)}(x-\xi)\,dx = f^{(n)}(\xi)(-1)^n$$

is taken into consideration, that when one δ component of magnetic current is present, the right sides of the equations of Maxwell take the form:

$$\mathrm{rot}_z\,\bar{j}^{м} + i\,\omega\mu_2 j_z^{э} - \frac{1}{i\,\omega\varepsilon_2}\,\mathrm{grad}_z\,(\mathrm{div}\,\bar{j}^{э}) = \frac{1}{r'}\,\frac{\partial(r' j_\varphi^{м})}{\partial r'},$$

$$-\mathrm{rot}_z\,\bar{j}^{э} + i\,\omega\varepsilon_2 j_z^{м} - \frac{1}{i\,\omega\mu_2}\,\mathrm{grad}_z\,(\mathrm{div}\,\bar{j})^{м} = -\frac{1}{i\,\omega\mu_2 r'}\,\frac{\partial^2 j_\varphi^{м}}{\partial\varphi'\,\partial z'},$$

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Substituting obtained coefficients $F_l^{м,э}$ into the expressions of field E_z and H_z and writing/recording continuity conditions $H_{\varphi_{cp}}$ in the region out of the sources in the expanded/scanned form, we will obtain the equation, from which is determined the value of the coefficients of Fourier series for unknown magnetic current a_n:

$$a_n = -\frac{1}{i\kappa_1 t \Delta_n}\Bigg\{\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^l\sqrt{\frac{\varepsilon_1\mu_1}{\varepsilon_2\mu_2}}}{\left[1-\left(\frac{l\pi}{\kappa_2 c}\right)^2\right]}\Bigg[\frac{\pi l n}{\kappa_2 c}\int\limits_{d-\frac{t}{2}}^{d+\frac{t}{2}}\frac{cs_n\left(x_m^h b,\ x_m^h r\right)}{r}\times$$

$$\times dr F_{lmn}^{м} - \sqrt{\frac{\varepsilon_2}{\mu_2}}\left(sn_n\left[x_m^e\left(d+\frac{t}{2}\right),\ x_m^e b\right] - sn_n\left[x_m^e\left(d-\frac{t}{2}\right),\right.\right.$$

$$\left.\left. x_m^e b\right] F_{lmn}^{э}\right]-\int\limits_0^{\infty}\frac{\kappa_1 e^{\sqrt{x^2-\kappa_1^2}\,c}}{2\pi x}\Bigg[n\int\limits_{d-\frac{t}{2}}^{d+\frac{t}{2}}\frac{J_n(x r)}{r}dr F_n^{м+} - \sqrt{\frac{\varepsilon_1}{\mu_1}}\times$$

$$\times\frac{J_n\left[x\left(d+\frac{t}{2}\right)\right]-J_n\left[x\left(d-\frac{t}{2}\right)\right]}{\sqrt{\frac{x^2}{\kappa_1^2}-1}}F_n^{э+}\Bigg]d x\Bigg\},$$

where

$$\Delta_n=\frac{i\kappa_1 d}{(\kappa_1 t)^2}\sqrt{\frac{\varepsilon_1}{\mu_1}}\left\{\int_0^\infty\left[n^2\sqrt{\frac{x^2}{\kappa_1^2}-1}\left(\int_{d-\frac{t}{2}}^{d+\frac{t}{2}}\frac{J_n(xr')}{r'}\,dr'\right)^2-\right.\right.$$

$$\left.-\frac{\left(J_n\left[x\left(d+\frac{t}{2}\right)\right]-J_n\left[x\left(d-\frac{t}{2}\right)\right]\right)^2}{\sqrt{\frac{x^2}{\kappa_1^2}-1}}\right]\frac{\kappa_1}{x}\,dx+\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\frac{n^2\frac{\mu_1}{\mu_2}}{\kappa_l c\left[1-\left(\frac{l\pi}{\kappa_2 l}\right)^2\right]}\times$$

$$\times\left[\frac{\left(\frac{x_m^h\pi ln}{\kappa_2^2 c}\right)^2\left(\int_{d-\frac{t}{2}}^{d+\frac{t}{2}}\frac{cs_n(x_m^h b,\ x_m^h r')}{r'}\,dr'\right)^2}{1-\left(\frac{n}{x_m^h b}\right)^2-\left[1-\left(\frac{n}{x_m^h a}\right)^2\right]\left[\frac{\pi x_m^h a}{2}cs_n(x_m^h b,\ x_m^h a)\right]^2}\right\}\times\rightarrow$$

$$-\frac{\cdot}{\times\left[1-\left(\frac{x_m^h}{\kappa_2}\right)^2-\left(\frac{l\pi}{\kappa_2 c}\right)^2\right]}-\left(\frac{x_m^e}{\kappa_2}\right)^2\times$$

$$\times\frac{\left(sn_n\left[x_m^e\left(d+\frac{t}{2}\right),\ x_m^e b\right]-sn_n\left[x_m^e\left(d-\frac{t}{2}\right),\ x_m^e b\right]\right)^2}{\left\{1-\left[\frac{\pi x_m^e a}{2}cs_n(x_m^e a,\ x_m^e b)\right]^2\right\}\left[1-\left(\frac{x_m^e}{\kappa_2}\right)^2-\left(\frac{l\pi}{\kappa_2 c}\right)^2\right]}\Bigg]\Bigg\}.$$

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Expression Δ_n is the recording of the total of the external and internal composite conductivities of the radiation/emission of annular slot per the unit of its length for the appropriate azimuthal harmony of field, by definition equal to

$$y_{1n}=\frac{H_{\varphi n}}{E_{rn}t}=\frac{J_m^э}{U_n},\quad \frac{мо}{м},$$

where $J_m^э$ - surface electric current density of displacement in the slot, U_n - voltage on the slct.

The external conductivity cf radiation/emission in expression Δ_n by integral on $\varkappa$, moreover the range of integration from 0 to $\varkappa=\varkappa_1$ corresponds to the real part cf the conductivity, and remaining interval - its imaginary part. To internal conductance corresponds dual sum on l and m, which is in the absence of heat losses in the resonator pure imaginary value.

If we trace the made abcve linings/calculations, then it is not difficult to be convinced of the fact that Δ_n actually/really corresponds to conductivity y_{1n}.

Utilizing straight/direct physical conformity between Δ_n and the linear conductivity of slct y_{1n}, it is possible to approximately take into account the effect of the final thickness of screen, without resorting to complicated linings/calculations. For this it suffices to supplement to conductivity y_{1n} identical for all azimuthal ones to harmonic $e^{-in\varphi}$ supplementary linear capacity susceptance y_{1n_C} of the capacitor/condenser, formed by the opposite edges of screen. This conductivity approximately ccmprises

$$y_{1n_C} \approx \frac{i\omega\varepsilon\delta}{t},$$

where δ- thickness of screen, t - width of slot, ε-the absolute dielectric constant of medium in the gap between the ends/faces of screen.

Obtained expression a_m, determining magnetic current in the slot, together with the expressions of field in the resonator and outside it in fact are the completed formal solution of stated problem. However, the direct use of these results, especially for the very narrow slots, meets with on its path the computational difficulties, connected with the slow convergence of series on m and integrals on $\varkappa$.

For facilitating the engineering calculations during the computation of these integrals and series/rows let us produce some conversions of the obtained solution.

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For this let us decompose the intervals of integration and addition respectively into two parts from 0 to $\varkappa_\xi$ (from 0 to $\varkappa_{m\xi}$) (I) and from $\varkappa_\xi$ to ∞ (from $\varkappa_{m\xi}$ to ∞) (II) so that in the limits of entire II interval would be possible the replacement of cylindrical functions by their asymptotic expressions with the help of the first members of the series/rows of Hankel. This means that in the II interval is necessary fulfilling of the inequalities

$$\varkappa_{m\xi} \gg \frac{n}{a}, \quad \varkappa_\xi \gg \frac{n}{d} \text{ (or) или } \frac{\varkappa_\xi}{\kappa_1} \gg n,$$

Key: (1). or.

if we have in mind that a radius of the annular slot d and a radius of resonator b are commensurated with the wavelength or compose several wavelengths, i.e.

$$\left.\begin{array}{c} d \\ b \end{array}\right\} \gtrsim \frac{1}{\kappa_1}.$$

Then in comparatively short interval the I computation of integrals and sums can be implemented respectively graphically and by term-by-term addition, and in interval II - with the use of the approximations:

$$\mathrm{cs}_n(\varkappa b, \varkappa r) \approx \frac{2\cos[\varkappa(b-r)]}{\pi\varkappa\sqrt{br}}, \quad \mathrm{sn}_n(\varkappa b, \varkappa r) \approx \frac{2\sin[\varkappa(b-r)]}{\pi\varkappa\sqrt{br}},$$

$$\mathrm{Sn}_n(\varkappa a, \varkappa b) \approx \frac{2\sin[\varkappa(a-b)]}{\pi\varkappa\sqrt{ab}}, \quad \varkappa_m^e \approx \varkappa_m^h \approx \frac{m\pi}{b-a},$$

$$J_n(\varkappa r) \approx \sqrt{\frac{2}{\pi\varkappa r}} \cos\left[\varkappa r - \left(n + \frac{1}{2}\right)\frac{\pi}{2}\right],$$

$$\sqrt{\left(\frac{\varkappa}{\kappa_1}\right)^2 - 1} \approx \frac{\varkappa}{\kappa_1}, \qquad 1 - \left(\frac{n}{\varkappa_m r}\right)^2 \approx 1,$$

$$1 - \left(\frac{\varkappa_m}{\kappa_2}\right)^2 - \left(\frac{l\pi}{\kappa_2 l}\right)^2 \approx -\left(\frac{\varkappa_m}{\kappa_2}\right)^2 - \left(\frac{l\pi}{\kappa_2 c}\right)^2$$

and tabular sums and integrals.

The introduced demarcation of the intervals of integration and addition makes it possible to also simplify the problem of the preliminary computation of integrals on the radial coordinates r and r'. So when $\varkappa<\varkappa_t$ in view of condition t<<d it is possible to ccunt functions frcm $\varkappa r$ and $\varkappa r'$ within the limits of slot by those by slowly changing and to calculate these integrals according to the formula

$$\int\limits_{d-\frac{t}{2}}^{d+\frac{t}{2}} f(r)\,dr \approx tf(d)$$

independent of the complexity of integrand.

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For second section $(\varkappa>\varkappa_t)$ the integration for r and r' after the carrying out of the slcwly changing factors also presents no difficulties:

$$\int\limits_{d-\frac{t}{2}}^{d+\frac{t}{2}} J_n(\varkappa r)\,dr \approx t\sqrt{\frac{2}{\pi\varkappa d}}\,\frac{\sin\frac{\varkappa t}{2}}{\frac{\varkappa t}{2}}\cos\left[\varkappa d-\left(n+\frac{1}{2}\right)\frac{\pi}{2}\right].$$

$$\int\limits_{d-\frac{t}{2}}^{d+\frac{t}{2}} cs_n(\varkappa b,\ \varkappa r)\,dr \approx t\,\frac{2}{\pi\varkappa\sqrt{bd}}\,\frac{\sin\frac{\varkappa t}{2}}{\frac{\varkappa t}{2}}\cos[\varkappa(b-d)].$$

The obtained as a result of integrating the complicated trigonometric functions integral function si from argument $\varkappa d$ in interval $\varkappa>\varkappa_t$ can be represented by the approximately asymptotic expression

$$\mathrm{si}(\varkappa d)\approx-\frac{\cos\varkappa d}{\varkappa d}.$$

The same path can be used for the addition of simpler series/rows on 1. In this case as sign/criterion for demarcation of two intervals of addition $l<l_{\xi}$, and $l>l_{\xi}$ serve the conditions of the approximate representation of functions from 1

$$\left(\frac{l_{\xi}\pi}{\kappa_{\xi}c}\right)^{2}\gg1-\left(\frac{\varkappa_{m}}{\kappa_{\xi}}\right)^{2} \text{ и } \left(\frac{l_{\xi}\pi}{\kappa_{\xi}c}\right)^{2}\gg1.$$

Appendix.

A question about the completeness of solutions for the fields within the resonators from a systematic point of view deserves supplementary examination. The fact is that in the literature, which concerns the solutions of such problems in the generalized form (for example, [8], [9]), are indications about the methods of obtaining the full/total/complete solutions by the inclusion in of them vortex/eddy and potential field component.

However, during the solution of specific electrodynamic problems the completeness of solution can be achieved/reached, also, without the special separation of electromagnetic field to these two parts. For this it suffices to ensure the completeness of the spectrum of the solutions of the system of wave equations for the fields or the potentials with the observance of the conditions of the validity of these equations. As is known [6], these conditions are uniformity, isotropy and linearity of medium, and for the potentials even and the relationship/ratio:

$$\operatorname{div}\overline{A}^{э} = -\varepsilon\frac{\partial\varphi^{э}}{\partial t}, \quad \operatorname{div}\overline{A}^{м} = -\mu\frac{\partial\varphi^{м}}{\partial t},$$

eliminating full/total/complete arbitrariness $\varphi^{э}$ and $\varphi^{м}$.

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For the outside sources, which satisfy continuity conditions:

$$\operatorname{div}\overline{j}^{э} = -\varepsilon\frac{\partial\rho^{э}}{\partial t}, \quad \operatorname{div}\overline{j}^{м} = -\mu\frac{\partial\rho^{м}}{\partial t},$$

the system of four wave equations indicated for potentials $\overline{A}^{э}(\overline{j}^{э}), \overline{A}^{м}(\overline{j}^{м}), \varphi^{э}(\rho^{э}), \varphi^{м}(\rho^{м})$ is reduced to the system of two equations for $\overline{A}^{э}(\overline{j}^{э}), \overline{A}^{м}(\overline{j}^{м})$, and in the system of two equations for $\overline{E}(\overline{j}^{э}, \overline{j}^{м}, \rho^{э}, \rho^{м}), \overline{H}(\overline{j}^{э}, \overline{j}^{м}, \rho^{э}, \rho^{м})$. in their right side charges $\rho^{э}$ and $\rho^{м}$ are eliminated and remain only currents $\overline{j}^{э}$ and $\overline{j}^{м}$. A number of sources, which do not satisfy continuity conditions, in particular, includes steady currents $\overline{j}_0^{э}$ and $\overline{j}_0^{м}$ and static charges $\rho_0^{э}$ and $\rho_0^{м}$, which, being constant/invariable in the time, in the general case are not in any way connected.

The completeness of the spectrum of the solutions of the system of wave equations is ensured, if in it are taken into consideration all terms with the eigenvalues from 0 to ∞ (for the spectrum of the erect ones of will). Thus, the full/total/complete solution of the equations of Maxwell in the resonator in the general case should be considered such, which contains the full/total/complete spectrum of the solutions of the system of wave equations for $\vec{A}^{э}(\vec{j}^{э}), \vec{A}^{м}(\vec{j}^{м})$, or $\vec{E}(\vec{j}^{э}, \vec{j}^{м}), \vec{H}(\vec{j}^{э}, \vec{j}^{м})$ in the form of waves E, H, electric wave mode and PEM, the electro- and magnetostatic fields of charges $\rho_0^{э}, \rho_0^{м}$ and currents $\vec{j}_0^{э}, \vec{j}_0^{м}$ and field of that part of the time-varying of charges $\rho^{э}, \rho^{м}$, which do not for some reason or other satisfy continuity conditions. In problem examined above in view of the conditions accepted outside sources satisfy continuity conditions. Therefore expressions (3), (4) together with the expressions of the transverse components of field are the full/total/complete solutions of the equations of Maxwell for this resonator.

In conclusion the author expresses appreciation to the Candidate of Technical Sciences L. S. Benenson for the useful observations and meaningful discussion.

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Article was received by the editcrial staff on 6 March, 1969, after treatment/processing - on 24 December, 1969.

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Synthesis of nonequidistant line-source antenna by the statistical methods of search.

O. I. Levin.

Are examined questions of the use/application of methods of statistical optimization to the tasks of the synthesis of the nonequidistant rarefied gratings. Is comprised the system stochastic differential equations, the point of stable equilibrium of which is the point of the location of the unknown extremum of the functional of the disagreement/mismatch of preset and synthesized diagrams. Are given the results of the calculations of nonequidistant antenna arrays by the method in question.

Formulation of the problem.

Is examined the task of the construction of the discrete/digital line-source antenna, which realizes the preset radiation pattern $f(u)$.

The radiation pattern of the linear grating

$$F(u)=\sum_{n=1}^{N} A_n \exp(\mathrm{i}\, d_n u + \mathrm{i}\,\psi_n), \qquad (1.1)$$

where u=w sin θ, θ - the angle, calculated off the normal to the grating; A_n, d_n, ψ_n — unknown amplitude, position and the phase of the n emitter, moreover positions are measured in the halflengths of waves;

N - number of emitters in the grating.

For a precise realization of the preset diagram f(u) by the sum of form (1.1) it is necessary that f(u) would belong functions (see [1]); but if $f(u)\bar{\in} B^1_\sigma$ the solution of problem consists in the determination of such values of controlling parameters A_n, d_n, ψ_n, n=1, 2, ..., N so that the obtained diagram F*(u) as possible better would approach the given one. The variation formulation of this problem consists in the determination of such vectors $\vec{A}, \vec{d}, \vec{\psi}$ (where $\vec{A}=\{A_1, A_2, \ldots, A_N\}$, $\vec{d}=\{d_1, d_2, \ldots, d_N\}$, $\vec{\psi}=\{\psi_1, \psi_2, \ldots \psi_N\}$), which they supply the minimum to the functional of disagreement/mismatch ρ [f(u), F(u)], where ρ - distance between f(u) and F(u).

Depending on the selection of the function space, elements/cells of which are functions f(u), F(u), is determined the concrete/specific/actual form of the functional of disagreement/mismatch ρ.

The minimization of this functional leads to the determination of those optimum values of the controlling parameters which make it possible to most accurately obtain the required diagram.

Thus, the task of synthesis is reduced in terms of the "functionals" to the task of mathematical programming.

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To this formulation of the problem of synthesis is turned the attention in work [2], where are constructed the concrete/specific/actual forms of the functionals of disagreement/mismatch and are solved several tasks with the help of the optimization of such functionals.

Unfortunately, in the majority of the in practice interesting cases ρ [f(u), F(u)] it is nonlinear and, as a consequence of this, multi-extremum functional, what considerably complicates the solution of stated problem - the determination of the global minimum of the functional of disagreement/mismatch. Is constructed the iterative process of the search for the optimum values of the controlling paramaters, corresponding to the point of the global minimum.

For an improvement in the probability of the convergence of the process of search at the point of the global extremum of functional it is possible to recommend several methods of organizing the search for the optimum values of the paramaters. Among such methods, indicated in [2], it is possible to note:

1. Selection of random initial conditions in the combination with the local search. This glotal search is the statistical expansion of the usual method of local search.

Search from the different initial positions consists in the random sampling of initial position with the subsequent motion, determined by any of the known local methods of descent, to the extremum and by the selection of the main thing.

This algorithm of the statistical search for global extremum is in fact the algorithm of random sorting the local minimums and because of this is effectively applicable with their small number.

It is necessary to note that the a priori information, which escape/ensues from the analysis of the physical structure of process, makes it possible to immediately reject/throw the "hopeless" regions of the initial conditions that considerably accelerates the search, and to also so organize the probability distribution function of the random initial conditions, in order to increase substantially the probability of the assignment of the initial conditions, which lead to the point of global extremum.

2. Method of preliminary approximate solution consists in construction of rough solution of stated problem on the basis of methods, on requiring high expenditures of machine time (for example, use of method of steady state, resolution into Fourier series in terms of systems of different functions, etc.). In this case it is very probable that approximate solution gravitates to the global extremum (or at least to the very deep extremum, which unessentially differs from global). The subsequent local search can then lead to the determination of global extremum. Some methods of the rough estimate of initial approximation/approach will be in more detail investigated subsequently.

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3. Method of fitting (utilizing terminology [2]). This method consists in the fact that first the problem is solved for this small number of parameters n, with which is feasible full/total/complete sorting. Then is added an an even smaller number of emitters and is

sought the extremum of functional by the method of the "adequate/approaching directions" from the initial values of n of the arguments, which correspond to the global extremum, found full/total/complete countershaft for the n emitters. Thus, each time is conducted the search for the optimum parameters the narrowed space, that begins from the "bed", obtained in the preceding/previous stage. The advisability of applying this method, which is one of the variaties of the group of methods of the type of dynamic programming, increases with an increase in the number of parameters, since in this case sharply grow/rise the expenditures of machine time and the difficulties, connected with the multi-extremality of the optimizable functional. Therefore the multistage process of optimization in the space of a considerably smaller number of measurements is very promising. Despite the fact that the direct use/application of methods of dynamic programming to the tasks of the synthesis of antennas incorrectly [3], results of works [3], [4] make it possible to judge the sufficiently good quality of the obtained solution with the help of these methods.

In this work the task of the determination of extremum is reduced to the solution of the system of inequalities and to finding of zero functions. Is comprised the system stochastic differential equations, the point of stable equilibrium of which is the point of the location of the unknown extremum.

The method in question is borrowed from [5].

Stochastic method of the search for the global extremum of the function of many variable/alternating.

Let in region R of n-dimensional be euclidean space it is determined the differentiated function $y=\rho(X)\geq 0$, $X=\{x_1, x_2, \ldots, x_n\}$, where R region is determined by the system of the inequalities

$$\varphi_i(X)\leqslant 0, \quad i=1, 2, \ldots, m.$$

If R is preset and by equations $\varphi_j(X)=0$, the each such condition is reduced to two inequalities:

$$\varphi_j(X)\leqslant 0, \quad -\varphi_j(X)\leqslant 0.$$

Is required to construct the algorithm of determination from given by accuracy ε point $X^*\in R$, in which function $\rho(X)$ it reaches global extremum. for example the minimum [for the determination of maximum it is necessary to examine function $-\rho(X)$].

Let us construct the sequence of monotonically decreasing positive numbers $\{\lambda_k\}$ such, that $\lambda_{k-1}-\lambda_k=\varepsilon>0$. To the system of inequalities let us supplement one additional inequality

$$\varphi_0^{(k)}(X)=\rho(X)-\lambda_k\leqslant 0.$$

This system of inequalities is satisfied under condition $\lambda_{\kappa} \geqslant \min_{X \in R} \rho(X)$ and is determined point $X \in R$, which is the point of the local extremum of function $\rho(X)$, moreover the depth of this extremum is determined λ_{κ}.

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Then we take the following value of $\{\lambda_{\kappa}\}$ we seek point $X \in R$, being determining a deeper local minimum of function $\rho(X)$. With certain λ_{κ} can seem that $\lambda_{\kappa} < \min_{X \in R} \rho(X)$, then preceding/previous value $\lambda_{\kappa-1}$ with accuracy ε will be equal to minimum value $\rho(X)$, and the corresponding point X--X* - to the point of the minimum.

Thus the task of the search for the minimum of function $\rho(X)$ is replaced by the task of solving the system of inequalities.

We further construct function $F(X) = \frac{1}{2}\sum_{i=0}^{m} H_i(X)$, where

$$H_i(X) = \varphi_i(X)\, p_i \operatorname{sign} \varphi_i(X),\ i = 0, 1, \ldots, m;\ \operatorname{sign} x = \begin{cases} 0, & x \leqslant 0, \\ 1, & x > 0, \end{cases}$$

p_i — weight coefficients of limitations (account of the importance of different limitations) $\sum_{i=0}^{m} p_i = 1$.

Function F(X) is equal to zero for those X, for which the system of inequalities is satisfied. Consequently, the task of determination $\min_{X\in R} \rho(X)$ is reduced to the determination of zero functions F(X) with given one λ_{κ}. As it is not difficult to see, $F(X)\geq 0$. Consequently, zero functions F(X) are its minimums. Zero functions F(X) we will search with the help of the "stray" random search, which is the statistical development of the regular method of gradient and consists in the fact that for the purpose of imparting to search global character on the gradient descent are superimposed the random "jerks/impulses" $\vec{\xi}(t)$, which create the mode/conditions of the random walk. This motion of point under the effect of the determined removal/drift to the side of antigradient and random "jerks/impulses" is determined by the following system stochastic differential equations:

$$\frac{dx_m}{dt}=-\mathrm{grad}_{x_m}F(X)+\psi_m[F(X)]\xi_m(t),\ m=1,2,\ .\ .\ .\ ,\ n,\ (1.2)$$

where $\vec{\Psi}(X)=\{\Psi_1(X),\Psi_2(X),\ldots,\Psi_n(X)$ —the vector function of scalar argument. $\vec{\xi}(t)=\{\xi_1(t),\xi_2(t),\ldots,\xi_n(t)\}$ — the vector random process whose components - the normally distributed random processes with the zero mathematical expectations and the correlation matrix/die of the form

$$M\xi_i(t)\xi_j(t-\tau)=\kappa_{ij}\delta(\tau), \qquad (1.3)$$

where $\delta(\tau)$ - the generalized function of Dirac, κ_{ij} — the Kronecker delta.

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Generally speaking, as $\vec{\varepsilon}(t)$ it is possible to take any with the symmetrical relative to zero densities of distribution of probability process. $\bar{\Psi}[F(X)]$ — the noise intensity which is the higher, the further the representative point X from X*, which corresponds to minimum value $\rho(X)$.

The selection of fan-junction $\bar{\Psi}[F(X)]$ determines, actually, "damping" of the process of search, i.e., the relationship/ratio between the determined descent and the random search. As function $\bar{\Psi}[F(X)]$ were investigated the functions of the type exp [F(X)] -1, $F^2(X)$ and so forth.

It is necessary to note that successful selection $\bar{\Psi}[F(X)]$ in many respects contributes to the rapid search for extremum, but for this selection it is necessary to produce a sufficiently large number of preliminary experiments. The process, determined by equ. (2.2), with $t \to \infty$ leads to the most probable position of point to a X-position of global extremum [6].

Different statistical methods of the search for global extremum with the adaptation in the process of search and evaluation/estimate of the parameters of these methods are given in [6].

It would be interesting to use different statistical methods for the tasks of the synthesis of optimum antennas and to compare between themselves obtained by these methods results.

Examples of the synthesis of nonequidistant line-source antennas.

By the method stochastic search for the global extremum of the functional of disagreement/mismatch $\rho(X)=\rho[f(u), F(u; X)]$, where vector X - the vector of the controlling parameters, was calculated a large number of diagrams. For the economy of machine time are examined only the cases of real diagrams; transition/junction to the optimization of the complex-valued diagrams adds no fundamentally new difficulties.

As f(u) was utilized the diagram with the preset form of major lobe and the zero lateral radiation; for example,

$$f(u)=\begin{cases} 1-u^2, & |u|\leqslant 0{,}04\pi, \\ 0, & 0{,}04\pi<|u|\leqslant\pi. \end{cases} \quad (1.4)$$

As the functional of disagreement/mismatch let us take divergence of f(u) from F(u; X) in space $L_{p[-\pi\pi]}$:

$$\rho_{L_p}(X)=\rho\big(f(u),\ F(u;\ X)\big)=\left[\int_{-\pi}^{\pi}|f|u-F(u;\ X)|^p\,du)\right]^{\frac{1}{p}}. \quad (1.5)$$

In the case of the complex-valued function F(u; X) it is possible to produce the optimization of the modulus/module of the diagram:

$$\rho_{L_p}(X)=\rho[f(u),\ F(u;\ X)]=\left(\int_{-\pi}^{\pi}|f(u)-|F(u;\ X)||^p\,du\right)^{\frac{1}{p}}. \quad (1.6)$$

Thus, the functional of disagreement/mismatch is nothing else but the norm of divergence f(u) from F(u; X) in space L_p.

Page 20. Despite the fact that occurs the minimization of divergence of f(u) from F(u; X) in L_p, which guarantees the integral nearness of diagrams; however, according to the known theorem of functional analysis with $p\to\infty$ norm in L_p approaches the uniform norm (space C). Therefore with sufficiently large p (in the calculations was utilized $p\geq 6$) it is possible to speak of the "almost" uniform approximation/approach of the preset diagram f(u).

Functionals (1.5) and (1.6) provide approximation/approach taking into account both fundamental ($|u|\leq 0.04\pi$ and the side lobes of diagram ($0.04\pi\leq|u|\leq\pi$). The more general/more common/more total functional

$$\rho_{L_p}(X)=\alpha\left(\int_{|u|<0.04\pi}|f(u)-|F(u;\ X)||^p\,du\right)^{\frac{1}{p}}+$$
$$+\beta\left(\int_{0.04\pi<|u|<\pi}|f(u)-|F(u;\ X)|^p\,du\right)^{\frac{1}{p}}, \quad (1.7)$$

where $\alpha\beta\geq 0$ it makes it possible to provide the required relationship/ratio between the accuracy of the approximation/approach of the main thing and side lobes depending on concrete/specific/actual requirements for the optimizable antenna.

Are calculated the following optimum diagrams:

1. Is produced synthesis in positions and amplitudes of nine element, by aperture 19λ, symmetrical cophasal grating.

The form of the obtained diagram is shown in Fig. 1, and the values of amplitudes and positions of emitters are given in Table 1. Maximum side lobe has a value - 5.45 dB.

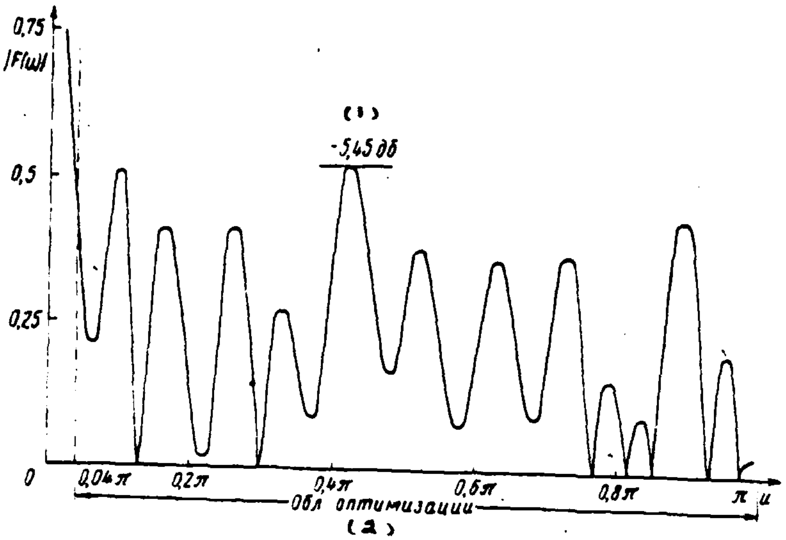

Fig. 1. Key: (1). dB. (2). Region of optimization.

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2. Is produced synthesis on positions of equal-amplitude symmetrical broadside antenna array, which consists of 25 elements/cells, by aperture 50λ. Optimum diagram is given in Fig. 2, and the sequence of decreasing the side lobes in the process of search - in Table 2. Maximum side lobe - 10.5 dB.

In the cases of optimizations examined as the limitations to the region of the determination of the controlling parameters were utilized the following conditions:

1) the equipment of emitters with the aperture: $d_n \leqslant R$, n=1, 2,... N; R - value of aperture;

2) the determination of emitters at a distance from each other not lass than given one: $d_{n+1} - d_n \geqslant d_{\text{зад}}$;

3) standardization of the amplitudes of the emitters: $\sum_{n=1}^{N} A_n = 1$.

Due to the high expenditures of machine time for the realization stochastic search the high value acquire the methods of determination relative to gross initial approximation/approach which supposedly gravitates to the global extremum, from which then is conducted local search by any of the known methods.

Table 1.

n	A_n	d_n
0	0,15503	0
1	0,14591	3.70316
2	0,11081	5,39699
3	0.04281	17,18713
4	0,11111	19,0

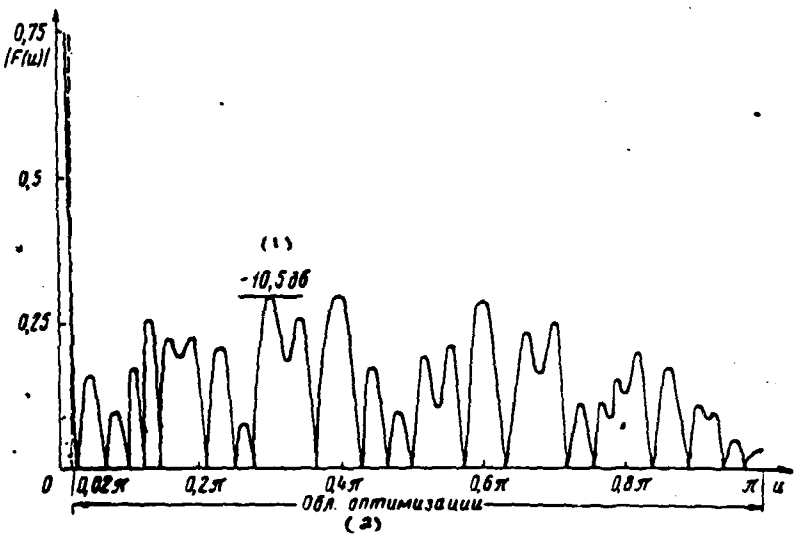

Fig. 2.

Key: (1). dB. (2). Region of optimization.

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Some from the methods, that make it possible to obtain gross initial approximation/approach, are described in [7].

Among them it is possible to note:

1. Method of dynamic programming with the large sample between the adjacent elements/cells.

2. Location of elements/cells according to any a priori selected law, which depends on small number of parameters:

$$d_n = f(n, a_1, a_2, \ldots, a_k), \text{ где}^{(1)}\ k \ll n.$$

Key: (1). where.

In particular, distances can be arranged/located along the power law. The asymptotic evaluations/estimates of the side-lobe level of this grating are given in work [8].

As the a priori selected laws of the location of elements/cells it is possible to note logarithmic (law of prime numbers), law of the increase of the distances between the elements/cells on the arithmetical progression, etc.

The results of calculating the gratings with such laws of the location of elements/cells are given in [7].

3. Method of "rarefaction/evacuation" of equidistant grating. As the initial approximation/approach is taken the equidistant, completely "filled" grating of the preset aperture; then from it they begin to reject on one emitter, until remains the preset number of elements/cells, moreover the number of the ejected emitter is determined at each step/pitch of process by the optimum of the functional in question. This process was used for the synthesis of the equal-amplitude cophasal symmetrical antenna with $N=21$ and $D=40\lambda$. Began from the equidistant grating $\Delta d=0.5\lambda$ and $N=81$; after in the grating it remained 21 elements/cells, maximum side lobe had a level - 8.9 dB, and after the use/application of a method of gradient it fell to - 10.1 dB. It is interesting that approximately the same result was obtained, if distances were arranged/located according to the parabolic law with the optimum selection of the parameters of parabola and the subsequent gradient descent. The corresponding positions of elements/cells are given in Fig. 3.

Table 2.

(1) Номер локального экстремума	(2) Величина локального экстремума	(1) Номер локального экстремума	(2) Величина локального экстремума
1	0,3417	9	0.449
2	0,4329	10	0.5156
3	0,3582	11	0,446
4	0,337	12	0.4
5	0,384	13	0,585
6	0.336	14	0,474
7	0,373	15	0,3124
8	0,327	16	0.3

Key: (1). Number of local extremum. (2). Value of local extremum.

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For the evaluation/estimate of the properties of the optimizable functional was used another, more rapid method of local descent, it is much better than the gradient, fitted out to the ravine character of function, method of conjugated/combined gradients [9]. However, the point of extremum significantly was not changed, although a number of iterations decreased doubly.

Summarizing that presented, it is possible to draw the conclusion that the methods, connected with the rough determination of initial approximation/approach and the subsequent local descent are considerably more economical, than the global statistical methods of search without the a priori assumptions; however, the proof of the gravity of initial approximation/approach to a point of global extremum is the difficult-to-solve mathematical task.

Table 2 depicts the values of the passable local extrema (i.e. the value of maximum side lobes). After reaching/achievement of the level - 10.5 dB, in spite of the passed 100 steps/pitches, the value of extremum they were not reduced also on the basis of this, probably, it is possible to consider that with an accuracy to 1 dB was obtained the global extremum.

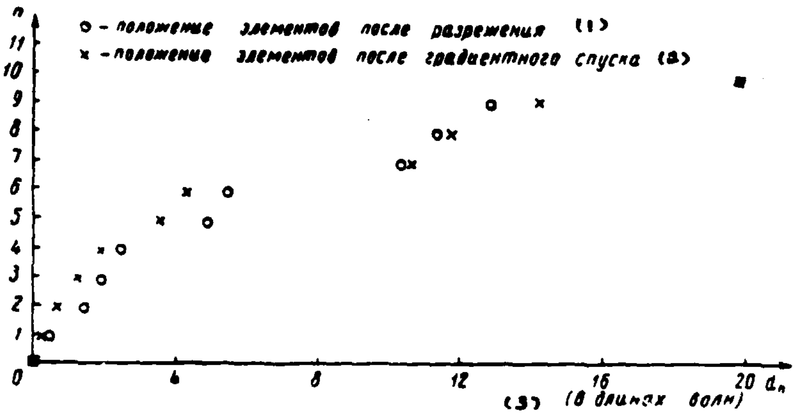

Fig. 3.

Key: (1). the position of elements/cells after rarefaction/evacuation. (2). position of elements/cells after gradient descent. (3). In wavelengths.

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In the calculations conducted it was assumed that

$$\sum_{n=1}^{N} A_n = 1, \quad A_n \geqslant 0, \quad d_{k+1} - d_k \geqslant 1.$$

For the evaluation of the effect of the input parameters were carried out the calculations for N=25 with the aperture 100λ, with interelement distances of not less than 1λ, with different regions of optimization according to the side-lobe level and according to different number of elements/cells in the gratings. The obtained results are represented in tables 3, 4, 5, 6.

Table 3 shows the effect of the region of optimization on the attainable side-lobe level (number of elements/cells of grating - 25, the value of aperture - 50λ, interelemental distances - not less 0.5λ).

Table 4 shows the effect of interelemental distances on the sidelobe level (number of elements/cells - 25, the value of aperture - 50λ, the region of optimization - [0.02π, π]).

Table 5 gives the results of the calculations of antenna arrays with different number of elements/cells (value of aperture - 50λ, interelemental distances - not less 0.5λ).

Table 3.

(1) Область оптимизации	0.02π; π	0.04π; π	0.08π; π	0.02π; 0.5π	0.02π; 1.98π
(2) Величина на макс. бок. лепестка, дб	—10,5	—10,8	—20,2	—14,2	—6

Key: (1). Region of optimizaticn. (2). Value on max. side. of lobe/lug, dB.

Table 4.

Δd	0.5λ	1λ
(1) Величина на макс. бок. лепестка, дб	—10,5	—7,6

Key: (1). Value on max. the side. of lobe/lug, dB.

Table 5.

(1) Число элементов	9	17	25	33	41	51
(2) Величина на макс. бок. лепестка, дб	—5,45	—6,3	—10,5	—11,4	—13,35	—14,8

Key: (1). Number of elements/cells. (2). Value on max. side. of lobe/lug. dB.

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Table 6 gives the data about the effect of the value of aparture on the sidelobe level (number of elements/cells - 25, the interelemental distances - not less 0.5λ).

In conclusion it is necessary to note that stochastic method of the determination of global extremum used without the use of any a priori information about the assumed region of the determination of global extremum is sufficiently to labor-consuming ones in the practical realization and requires the considerable expenditures of the time of computer(s).

The advantage of method as generally the methods of mathematical programming, is great flexibility to a change in the input parameters and, as a consequence of this, rapid transition/junction from calculation of one version to another.

Conclusion.

The synthesis of line-source antennas with the help of the methods of mathematical programming makes it possible to avoid a whole series of the difficulties, which appear during the use of analytical methods. The task of the synthesis of antennas in this case is placed as the task of the optimization of the complicated self-tuning system, which makes it possible to use the numerous ideas of technical cybernetics, developed for similar systems, appears the possibility of the equipment synthesis of antennas with the help of the analog technology. Vast bibliography from this question can be found in [6].

The results of the calculations of the specific problems of the synthesis of antennas, obtained with the help of the methods of mathematical programming, successfully compete with the analogous results, obtained by analytical methods.

The methods examined easily can be propagated to the solution of other problems of electrodynamics with a large number of complicated nonlinear limitations.

The essential advantage of the methods of mathematical programming is the absence of limitations to the form of the synthesized diagram (equipment with its defined class of functions).

This setting of the reverse tasks theory of antennas in the terms of the optimization of functionals makes it possible to obtain the virtually realizable solutions, connected with the functional limitations on them.

Table 6.

Величина раскрыва (1)	50λ	100λ
Величина (2) макс. бок. лепестка, дб	—10,5	—8,6

Key: (1). Value of aperture. (2). Value max. side. of lobe/lug, dB.

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A rapid increase in the means of computer technology, an increase in high speed and memory of contemporary computers make it possible to effectively apply different statistical methods of the search for the optimum controlling parameters of complicated antenna systems, moreover the circle of the applicability of these methods ever more is expanded (discrete/digital and continuous phase synthesis, phase scanning, etc.).

It is necessary to also note that these methods make it possible to obtain the solutions of such problems, use/application to which the detailed analytical methods is extremely difficult.

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Received 6 February 1969; revised - 19 December, 1969.

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Principles of the construction of systems, which possess potential frequency independence.

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From the equations of Maxwell, which contain outside currents, by the methods of theory of similitude are obtained sufficient conditions of the similarity of field. On their basis are formulated the general/common/total principles of the construction of the systems, which possess potential frequency independence.

Introduction.

The development of the broadband and super wide-band devices/equipment of different designations/purposes is constantly/invariably one of the most important problems of theory and of antennas technique.

The practical absence of the sufficiently general methods of the construction of such antennas leads often to the high expenditures of time for the determination of the adequate/approaching broadband structures with empiricism.

The goal of present article is the examination of the general/common/total principles of the construction of antennas with the weak frequency dependence.

To this group let us relate all varieties of antennas, whose parameters are virtually stable in the more than twofold frequency band, including the antennas, called frequency-independent.

In the works on the frequency-independent antennas are formulated the known semi-empirical principles of their construction fundamental from which is considered the geometric scale principle, which is, actually, the condition of simulation ([1], [2], [3]).

The existing samples/specimens of the frequency-independent antennas, constructed in accordance with the mentioned above principles, according to the fundamental idea belong to one group of the antennas which can be defined as auto-commutation ones.

In the principle, each of these antennas consists of the chain/network of the geometrically similar radiating elements/cells, increasing sizes/dimensions, which consecutively/serially resound at frequencies of similarity. The remaining part of the structure remains virtually nonradiating.

As it is easy to establish/install, the structures indicated, global, including outside source, actually, they do not answer the principle of simulation, although it is accepted as the initial.

Let us note that fundamental principle itself it is obtained from the homogeneous equations of Maxwell, which in connection with the goals of radiation/emission, strictly speaking, are not adequate, since they do not contain the initial cause of radiation/emission - outside sources.

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Some of the utilized at present principles of the construction of the frequency-independent antennas can be somewhat more strictly obtained by the methods of the theory of the synthesis of antennas ([4], [5]; however, with the help of this theory there can be obtained only necessary spatial distribution of sources, while the concrete/specific/actual structure of antenna it remains unknown.

Is feasible another approach to the resolution of the problem of the construction of antennas with the weak frequency dependence.

In this case is placed the task of determining all possible varieties of the unlimited electromagnetic structures, the only general requirement for which consists in, at any or specific discrete/digital values of the frequency of outside source, the distribution of electromagnetic field in them being remained similar.

In such structures angular field distribution at large distances from the outside sources, and also impedances do not depend on frequency, i.e., by definition they will be frequency-independent.

The finite segments of the frequency-independent structures of the unlimited sizes/dimensions are characterized by weak frequency dependence.

Of this type broadband devices/equipment we will call systems with the potential frequency independence, since, in the principle, they become frequency-independent with the unlimited increase in the sizes/dimensions.

The process of the determination of the electromagnetic structures, which possess the general/common/total preset property, can be named the synthesis of structures. In contrast to the known theory of the synthesis of antennas, in which through the diagram is located only the source distribution of field, by the task of the synthesis of structures is in our case the determination of the possible geometric forms of ideally conducting bounding surfaces and of outside current distributions in the electromagnetic systems, which possess with the unlimited sizes/dimensions frequency independence.

Are examined below some questions of the theory of synthesis and principles of the construction of structures with the potential frequency independence.

Formulation of the problem.

It is necessary to determine the class of the structures of the electromagnetic systems, excited by the harmonic sources, only general/common/total property of which is a similar change in the field with the frequency.

Similar we will consider the fields whose vectors can be combined everywhere by changing in the graphic scale and amplitude of the current of outside source.

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In the completely determinate structures must be satisfied the condition of the uniqueness of the solutions of equations, which describe processes in the system. In connection with this we will proceed from the equations of Maxwell, which contain outside currents, assuming/setting by the given ones bounding surfaces and distribution on them of surface impedance.

With the dependence on time $e^{i\omega t}$ initial equations take the form:

$$\mathrm{rot}\,\overline{H} - i\omega\varepsilon_a\overline{E} = \overline{j}^{\,\text{эст}}(u_1', u_2', u_3'), \quad (1)$$
$$\mathrm{rot}\,\overline{E} + i\omega\mu_a\overline{H} = -\overline{j}^{\,\text{мст}}(u_1', u_2', u_3'), \quad (2)$$

where E, H - composite amplitudes of the vectors of electromagnetic field; ε_a, μ_a — absolute dielectric and magnetic permeability respectively:

$$\overline{j}^{\,\text{эст}}(u_1', u_2', u_3'),\ \overline{j}^{\,\text{мст}}(u_1', u_2', u_3') -$$

some fields of the composite vectors of the densities of outside electrical and magnetic currents; u'_1, u'_2, u'_3 - orthogonal coordinates of the points of spaces with the currents.

Bounding surface of system let be determined by the equation

$$F(u_1, u_2, u_3) = 0. \quad (3)$$

Since energy of real outside sources must be final, we will assume that the vector functions, which are determining outside currents, are integrated squared of space V', in which they are preset:

$$W \approx \int_{(V')} |j^{ст}|^2 dV' < \infty. \quad (4)$$

Let us name this relationship/ratio the condition of physical realizability.

For the establishment of the conditions for a similar change in the fields with the frequency let us use the methods of theory of similitude ([6], [7]).

Passing in equ. (1), (2) and (3) to the dimensionless quantities, let us introduce the designations:

$$\left.\begin{array}{l} \bar{E} = e\bar{E}; \quad \bar{H} = h\bar{H}; \quad \omega = \omega_0\nu \\ \varepsilon_a = \varepsilon_0 x_э; \quad \mu_a = \mu_0 x_м \end{array}\right\}; \quad (5)$$

$$\bar{j}^{(э,м)ст}(u_1', u_2', u_3') = i^{э,м}\bar{j}^{(э,м)ст}(u_1', u_2', u_3') = i^{э,м}\bar{j}_0^{(э,м)ст}\left(\frac{u_1'}{l_0}; \frac{u_2'}{l_0}; \frac{u_3'}{l_0}\right); \quad (6)$$

$u_i'/l_0 = u_{0i}'$ - dimensionless i-th coordinate within the limits of space with the current;

$$\overset{(1)}{\text{длина}} = l_0 L. \quad (7)$$

Key: (1). length.

The first factors in the right sides of expressions (5)-(7) are scale and have the appropriate dimensionalities; the second - are dimensionless and numerically equal to the values of fields, currents and parameters in equ. (1) and (2) the system for which all first factors are equal to one.

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Vector functions $\overline{j}^{\text{ст}}(u_1', u_2', u_3')$ и $\overline{j}_0^{\text{ст}}\left(\frac{u_1'}{l_0}, \frac{u_2'}{l_0}, \frac{u_3'}{l_0}\right)$ represent the fields of the dimensionless vectors of outside currents, correspondingly, in the real and dimensionless coordinates.

At the mutually appropriate points (u'_1, u'_2, u'_3) and $\left(\frac{u_1'}{l_0}, \frac{u_2'}{l_0}, \frac{u_3'}{l_0}\right)$ vectors $\overline{j}^{\text{ст}}$ and $\overline{j}_0^{\text{ст}}$ are equal, since upon transfer to the dimensionless coordinates changes only graphic scale l_0.

Taking into account designations (5)-(7) we obtain equ. (1), (2) and (3) in a dimensionless form:

$$\operatorname{rot}\overline{H}-i\varkappa_{э}\frac{\omega_0 l_0 \varepsilon_0 e}{h}\overline{E}=\frac{l_0 i^{э}}{h}\overline{j}_0^{\,э\,\text{ст}}\left(\frac{u_1'}{l_0}, \frac{u_2'}{l_0}, \frac{u_3'}{l_0}\right), \tag{8}$$

$$\operatorname{rot}\overline{E}+i\varkappa_{м}\frac{\omega_0 l_0 \mu_0 h}{e}\overline{H}=-\frac{l_0 i^{м}}{e}\overline{j}_0^{\,м\,\text{ст}}\left(\frac{u_1'}{l_0}, \frac{u_2'}{l_0}, \frac{u_3'}{l_0}\right), \tag{9}$$

$$F\left(\frac{u_1}{l_0}, \frac{u_2}{l_0}, \frac{u_3}{l_0}\right)=0. \tag{10}$$

The invariance of the solutions of $\overline{E}$, $\overline{H}$ of the system of equ.

(8)-(10) in the dimensionless coordinates with a change in the frequency ω is a sufficient condition of similar a change of fields $\overline{E}$, $\overline{H}$ in the real coordinates, since a change in scale l_0 upon transfer to them corresponds to similarity transformation.

In turn, solutions $\overline{E}$, $\overline{H}$ in the dimensionless space they do not change with the frequency, if are executed the following requirements:

1). do not change with the frequency coefficients in equ. (8) and (9);

2). remain constant/invariable dimensionless vector functions $\overline{j}_0^{\,э\,ст}$ and $\overline{j}_0^{\,м\,ст}$;

3). they remain the fixed shape of bounding surface (10) dimensionless coordinates and the distribution of surface impedance on it.

We will determine the general conditions of electrodynamic similarity, which ensure fulfilling the three requirements indicated.

Let us consider at first the case of the free space when bounding surfaces are absent.

Conditions of electrodynamic similarity in a free space.

In this case the fields are described by the system of equ. (8) and (9).

From the requirement of independence from the frequency of the coefficients of equations escape/ensue the following similarity criteria:

$$\frac{\omega l_0 \varepsilon_a e}{h} = c_1, \tag{11}$$

$$\frac{\omega l_0 \mu_a h}{e} = c_2, \tag{12}$$

$$\frac{l_0 i^{э}}{h} = c_3, \tag{13}$$

$$\frac{l_0 i^{м}}{e} = c_4. \tag{14}$$

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Let ε_a and μ_a not depend on frequency, then from expressions (11) and (12) we obtain the known condition of geometric similarity

$$l_0/\lambda_0 = \mathrm{const}, \tag{15}$$

where λ_0 - length of transmitting wave in the free space.

Thus, the necessary condition of the invariability of the solutions of the system of equ. (8) and (9) is the inverse proportionality of frequency and scale of lengths l_0.

Eliminating from expressions (13) and (14) l_0, we have

$$\frac{i^{э}}{i^{м}} = \text{const}\,\frac{h}{e}. \tag{16}$$

Since for the fields, which are changed similarly, regarding $h/e = \text{const}$, we obtain the supplementary condition of the similarity, which relates to the amplitudes of the outside currents:

$$i^{э}/i^{м} = \text{const}, \tag{17}$$

on the strength of which, with a change in frequency the ratio of the amplitudes of these currents should remain constant.

Let us note that condition (17) corresponds to the requirement of the invariability of the relation of essential forces in similar systems in the known postulate of similarity method ([7]).

When only one form of currents is present, condition (17) is satisfied automatically.

Let us pass to the examination of the requirement of independence from the frequency of the distributions of dimensionless outside currents $\bar{j}_0^{э\,ст}$ and $\bar{j}_0^{м\,ст}$, located in the right sides equ. (8) and (9).

The independence of these current listributions from the frequency, and consequently, from the value of the scale factor l_0 occurs, if simultaneously with l_0 is changed spatial distribution of outside currents in the real coordinates.

In this special case the distribution of the dimensionless vectors of currents in the real and dimensionless coordinates can be represented in the form of the functions

$$\bar{j}^{\text{ст}}(l_0u'_{01},\ l_0u'_{02},\ l_0u'_{03}) = \bar{j}_0^{\text{ст}}(u'_{01},\ u'_{02},\ u'_{03}), \tag{18}$$

where u'_0, u'_{02}, u'_{03} - dimensionless orthogonal coordinates of the points, at which are preset outside currents.

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Equal sign in this expression indicates equality dimensionless vectors at the mutually appropriate points of real and dimensionless spaces.

Vector function on the left side of equality (18) represents real spatial distribution of the vectors of the composite amplitudes of outside currents, which changes similarly with a change in the scale of lengths l_0.

Thus, in the absence of bounding surfaces the conditions of the invariability of fields in the dimensionless space and, consequently, also a similar change of the fields in the real space with a change in the frequency are the following:

1. Spatial distribution of vectors $\bar{j}^{\text{эст}}$ and $\bar{j}^{\text{м ст}}$ must change with the frequency similarly. The scale factor l_0 of the coordinate system, in which the specified distribution of the currents indicated, must in this case vary in proportion to to wavelength λ_0, which escape/ensues from condition (15).

2. Ratio of amplitudes of densities of electrical and magnetic currents with change in frequency must remain constant [condition (17)].

Fulfilling of given conditions provides a similar change of electromagnetic field with the frequency in entire space, including the points, which lie out of the space with the currents where equ. (1) and (2) are uniform, since the field in the dimensionless space remains constant/invariable in entire space.

Conditions indicated above can be related both to the primary outside currents, flowing in the sections where operate outside emf and to secondary currents on bounding surfaces, connected with the fields, which appear in the system as a result of acting of the applied electromotive forces.

As is known, secondary currents can be assumed/set by outside ones, given in free space, when their distribution is accurately known, for example, as a result of a strict solution of problem taking into account the effect of bounding surfaces [8].

In turn, conditions for a similar change with the frequency, obtained for the currents, they make it possible to rate/estimate the forms of bounding surfaces of the systems in which the fields change with the frequency similarly.

As shown below, is more expedient to determine requirements for the form of bounding surfaces directly from the system of equ. (5)-(7).

Some similar current distribution.

General/common/total characteristic of similar sources.

We will call similar currents, or similar sources of current distribution, that satisfy the first of the conditions indicated above, i.e., changing with the frequency similarly.

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After taking $l_0=\lambda_0$, it is possible to present such distributions in the following general view:

$$\bar{j}^{\text{ст}}(\lambda_0 u'_{01}, \lambda_0 u'_{02}, \lambda_0 u'_{03}). \qquad (19)$$

After transition into the dimensionless space to the coordinates

$$u'_{0i}=\frac{u'_i}{l_0}=\frac{u'_i}{\lambda_0}$$

the distribution of form (19) is expressed by vector function $\overline{j_0}(u'_{01}, u'_{02}, u'_{03})$, not containing scale factor.

Thus, current distribution for which the transition to the dimensionless coordinates is accompanied by the exception/elimination of scale factor from the function, with the help of which is expressed the distribution, it belongs to similar currents or similar sources.

We will call the centers of similitude of point, that remain motionless ones with a change in the graphic scale.

For example, in spherical coordinates (R, Θ, φ) center of similitude is point R=0. Respectively in dimensionless spherical coordinates $(R/\lambda_0, \Theta, \varphi)$ center of similitude is arranged/located at point $R/\lambda_0=0$.

If on certain initial transmitting wave spaces with similar outside currents lie/rest beside the center of similitude, for example in the manner that spherical spaces 1 and 2 with the centers at points $(R_{11}, \theta_1, \phi_1)$ and $(R_{21}, \theta_2, \phi_2)$ on Fig. 1, then with an increase in the wave distances R_{12} and R_{22} of the centers of spheres will increase proportional to new wavelength.

A similar source is not displaced with a change of the frequency only in such a case, when it is arranged/located in the center of similitude.

Elementary similar source.

Let at the point, combined since the beginning of the orthogonal coordinate system, be arranged/located the elementary outside source, which in the limits of certain frequency band can be approximately represented in the form of the three-dimensional δ-function:

$$j^{\text{ст}}(u_1', u_2', u_3') = \bar{\alpha}\delta(u_1' - 0)\delta(u_2' - 0)\delta(u_3' - 0), \qquad (20)$$

where $\bar{\alpha}$ - composite constant vector.

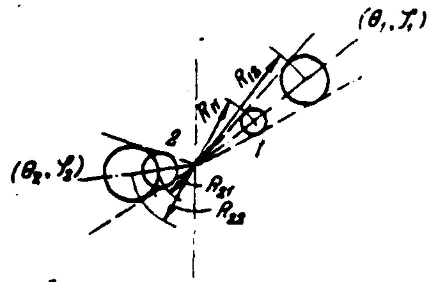

Fig. 1.

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Upon transfer to the dimensionless coordinates this outside current distribution will be with an accuracy to constant factor determined also by the δ-function:

$$j_0^{\text{ст}}(u'_{01}, u'_{02}, u'_{03}) = \bar{a}\,\delta(u'_{01}-0)\,\delta(u'_{02}-0)\,\delta(u'_{03}-0),$$

since it is obvious that a change in the length scale will not change the maximum form of this function, determined asymptotically.

Since in this case the function, which expresses outside current distribution in the dimensionless coordinates, does not contain wavelength, the distribution of outside current (20) satisfies the conditions of similarity.

The simplest elementary similar sources are electrical and magnetic dipoles or concentrated sources outside emf, the arranged/located in the center cf similitude.

Similar current filament.

Let us consider the proceeding from the center of similitude straight/direct, semi-infinite current filament whose cross section is small in comparison with any wavelength of certain range.

Let the current along the filament in the general case be the traveling wave with the distribution of the composite amplitudes of the following form:

$$\bar{J}^{\text{ст}}(R, \Theta_1, \varphi_1) = \bar{J}_0 \frac{e^{-\frac{\kappa_l}{\kappa}\kappa R}}{R^{1-\nu}} e^{\pm i\xi\kappa R}, \qquad (21)$$

where $\xi = \frac{\lambda_0}{\lambda_a}$; λ_a — the length of the traveling wave; κ_l — positive value; $0<\nu<1$.

With the independence from the frequency of parameters ξ, ν and κ_l/κ distribution (21) in the dimensionless coordinates does not contain wavelength, i.e., it is similar.

If we exclude the elementary section of current, which adjoins point R=0, then integral (4) for distributing form (21) will be that converging, therefore, will be satisfied the condition of physical realizability.

By direct computation it is easy to show that the radiation pattern for similar current (21) with ξ, λ_m, κ_l and ν, not depending on the frequency, is frequency-independent.

The condition of physical realizability is similar to the known "principle of the cutoff", formulated as one of the conditions, by which they must satisfy frequency-independent antennas (1).

As it was noted above, accurately known secondary currents, flowing on bounding surfaces, can be examined as the outside currents, which form the field of electromagnetic system.

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Since the field in the frequency-independent system changes with the frequency similarly, secondary currents in this system are similar outside sources.

Thus, similar sources are the distributions of secondary currents in any frequency-independent system.

For example, at the specific frequencies a similar source is current in any log-periodic system, in particular, in the flat/plane or conical isogonal spirals and in the vibrator ones and in the vibrator log-periodic antennas. φ Intensely the radiating "active regions" in these antennas, as is known, are moved with the frequency in such a way that at frequencies the similarities retain constant/invariable position in the dimensionless space.

Conditions of similarity in the presence of bounding surfaces and the principles of the construction of systems with the weak frequency dependence.

Let the system, which contains bounding surfaces, be excited by a similar outside source.

In this case for guaranteeing a similar change with the field frequency of the system of the requirement of independence from the frequency of coefficients and current distribution in dimensionless equ. (8) and (9) it is necessary to supplement with the requirements of independence from the scale of lengths $l_0=\lambda_0$ of the equation of bounding surface (10) and invariability of the distribution on it of surface impedance in the dimensionless coordinates.

Scale factor λ_0 is eliminated from the equation of bounding surface (10) in such a case, when this surface in the real coordinates changes with the frequency similarly and its equation takes the following form:

$$F(\lambda_0 u_{01}, \lambda_0 u_{02}, \lambda_0 u_{03}) = 0, \quad (22)$$

where λ_0 - wavelength; u_{0i} — dimensionless orthogonal coordinates, which contain lengths.

With satisfaction of other conditions of similarity the register of bounding surfaces in the dimensionless coordinates at different frequencies is a sufficient condition of the similarity of the fields of electromagnetic system at these frequencies. However, this is accurate coincidence not always necessarily.

The structure of boundary in the dimensionless coordinates can change with the frequency in some limits, if this virtually is not accompanied by changes in value and distributing the surface impedance.

The example to this bounding surface is sufficiently dense foil lattice from the straight/direct parallel conductors.

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In this case the transition to the dimensionless space at different frequencies is accompanied by a change in the cascade density and thickness of conductors; however, under specific conditions this virtually is not accompanied by a change in surface impedance [9].

Consequently, the systems, which satisfy the conditions of similarity in certain frequency band, can contain boundaries with the anisotropic structures whose parameters sufficiently slowly change with the frequency.

Let us note finally that for retaining/preserving/maintaining the similarity of fields, besides a similar change in the currents and bounding surfaces with the frequency separately, is required the coincidence of their centers of similitude in the real coordinates.

As it is easy to show, only with satisfaction of this supplementary condition the mutual location of outside currents and bounding surfaces in the dimensionless space does not change with the frequency.

Thus, for the systems, excited by the outside currents of one form, sufficient conditions for a similar change in the fields with the frequency are the following:

1. System is excited by similar outside sources.

2. Form and structure of bounding surface are such, that upon transfer to dimensionless coordinates at different frequencies either are obtained accurately coinciding surfaces or is retained constant/invariable location of boundaries and distribution of surface impedance on them.

3. Centers of similitude of bounding surface and similar outside sources coincide.

Let us consider the common principles of the construction of the electromagnetic systems, which satisfy these conditions.

Any combination of outside source and bounding surface, separately satisfying the conditions of similarity, under the condition of the coincidence of their centers represents the system in which the field changes with the frequency similarly.

Depending on the type of outside source and bounding surface the similarity can be retained with a continuous change in the frequency, at some discrete/digital frequencies or it will be only approximate in certain limited frequency range.

In connection with this let us conditionally divide all possible varieties of systems with the potential frequency independence into three groups:

1) continuously similar;

2) it is discrete or periodically similar;

3) approximately or almost similar systems.

For the characteristic of the special features/peculiarities of the structure of different types of devices/equipment, which satisfy three conditions indicated above, it suffices to describe the form of bounding surface and the method of excitation used.

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For this in the examination of the diverse variants of systems with the potential frequency independence are utilized the block diagrams, which contain only indicated fundamental data.

As similar sources we will utilize an elementary source and a current filament. Thus, for the construction of the possible types of systems it remains to determine the acceptable forms of bounding surfaces.

Continuously similar systems.

In spherical coordinates (R, Θ, φ) the equation of the surface, which satisfies the conditions of similarity, takes in accordance with expression (22) the following form:

$$F(\lambda_0 R_0, \Theta, \varphi)=0, \quad (23)$$

where R_0 - nondimensional distance from the center of similitude.

In the particular case when surface is determined only by angles, into its equation scale factor does not enter; therefore the conditions of similarity are satisfied independent of wavelength.

Such surfaces satisfy the known "principle of angles" [1].

Examples of continuously similar systems, during the appropriate location of similar outside sources, they are:

- any conical surface with locked or open guide, in particular, internal surface of any cone whose finite segment serves as bounding surface of horn antenna;

- any system from several conical surfaces, which have the overall apex/vertex: biconical systems of the round cones, the conical surfaces of any form above the plane, biconical systems from the wedge-shaped or flat/plane elements/cells, conical surfaces with the sector grooves;

- ideally conducting key with any angle, in particular, half-plane;

- plane or half-plane with any sector grooves, which have overall apex/vertex and, etc.

Each of the surfaces indicated can be combined with any form of a similar outside source.

The determination of the centers of similitude of bounding surfaces does not represent difficulties. For the plane this is any point of it, for the key - any point of edge/fin and for the conical surfaces, and also the sector grooves - their apex/vertex.

It must be noted that the systems with continuously similar bounding surfaces little are studied.

One of the possible reasons for this is, apparently, failure from biconical V- antenna, being according to all signs/criteria classical frequency-independent antenna.

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They assume that the diagram of this antenna at the finite length of cones diverges from the frequency, although in this case it remains the unknown such as the structure of field at the infinite length of elements/cells.

Discretely or periodically similar systems.

This form includes all devices/equipment, in boundary structures of which the distances between the elements/cells and their sizes/dimensions grow/rise in the geometric progression. The sections of such structures I am utilized in log-periodic antennas [1].

Simplest flat/plane bounding surface of this form is depicted in Fig. 2. On Z-axis, which is the line of similarity, is shown elemantary similar source 1.

Structure can be characterized by the parameters:

$$d_{n-1}/d_n = c_1 < 1; \quad b_n/d_n = c_2. \qquad (24)$$

On certain wave λ_{01} distance r_n/λ_{01} from the line of similarity to the edge of the n-band is the sum of the infinite geometric progression:

$$\frac{r_n}{\lambda_{01}} = \frac{d_n}{\lambda_{01}} \frac{1}{1-c_1}. \tag{25}$$

On certain another wave λ_{02}, $\frac{d_{n-1}}{\lambda_{02}} = \frac{d_n}{\lambda_{01}}$ consequently, on this wave $\frac{r_{n-1}}{\lambda_{02}} = \frac{r_n}{\lambda_{01}}$.

It is easy to show that on the waves λ_{01} and λ_{02} in the dimensionless coordinates all structural elements coincide precisely, including outside source.

The full/total/complete coincidence of structures in the dimensionless coordinates is obtained on any waves whose lengths are connected with the known relationship/ratio:

$$\lambda_{0n} = \lambda_0 c_1^n. \tag{26}$$

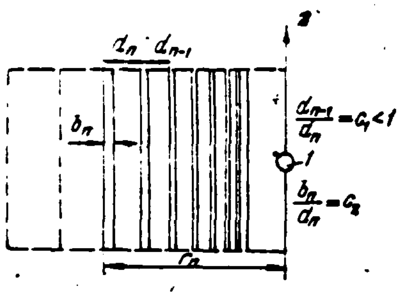

Fig. 2.

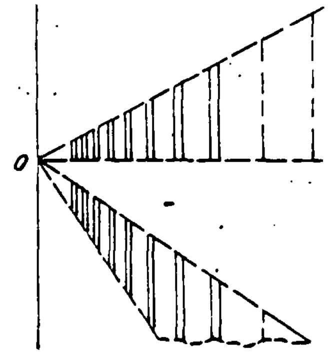

Fig. 3.

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Discretely similar will be any part of the structure in question, limited by the angle whose apex/vertex lies/rests on the line of similarity (Fig. 3). Any combination of continuously similar surface and discretely similar structure so represents the periodically similar surface (Fig. 4). The center of similitude of outside source 1 must in this case lie/rest on the line of the similarity of continuously similar surface of 2 (key, half-plane, etc.), also, at the apex/vertex of the angle, which limits periodically similar structure 3.

Analogously it is possible to construct the periodically similar structures, which coincide with the boundaries of the continuously similar surfaces of any forms.

The group of periodically similar structures includes also all surfaces, described by the equation, obtained by V. Rumsey. [1]:

$$R = e^{a(\varphi+\varphi_0)} F(\Theta). \qquad (27)$$

On the waves of similarity these structures correspond to the general/common/total equation of similar surfaces (23).

A special case of surfaces (27) is, as is known, isogonal conical spiral.

Let us note that in the log-periodic systems and the isogonal spirals the location of elementary similar source in the center of similitude is not accurately necessary, since in these systems emits the small region near the resonance element/cell, while the remaining part of the structure plays the role of feeder line; therefore the displacement of source along the system changes only initial phase of field in the remote zone.

Thus, the coincidence of the centers of similitude of source and boundary structure is not in a number of cases the necessary condition of similarity.

At the same time during the construction of complicated systems from frequency-independent elements/cells the coincidence of the centers of similitude of boundary structures is the necessary condition for a similar change in the field of system with the frequency.

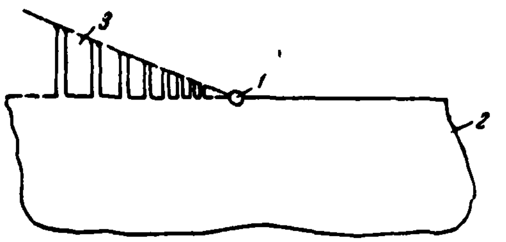

Fig. 4.

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Approximately or almost similar systems.

The example to approximately similar surface is semi-infinite foil lattice from the straight/direct, parallel conductors, arranged/located at a distance of $d<<\lambda_0$ one from another and having diameter small in comparison with distance of d.

The straight line of the shear/section of this grating, which is the line of similarity, can be arranged/located both perpendicular to the conductors and at angle to them.

In certain frequency band the grating indicated in the dimensionless coordinates has fixed shape and virtually constant/invariable distribution of anisotropic surface impedance, consequently, satisfies the conditions of similarity.

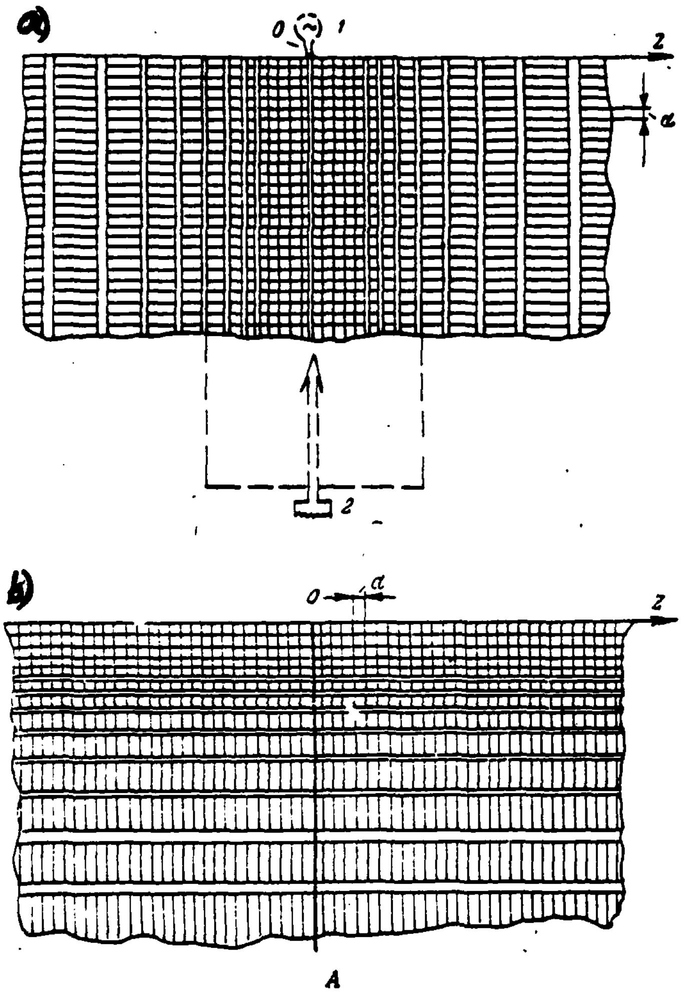

Fig. 5.

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Angular groove from the grating in question so is boundary similar surface.

Is possible the use of combination by uniform and periodically similar of gratings. Fig. 5a and 5b shows two possible forms of such approximately periodically similar structures.

In both cases at point 0 is located the center of similitude. Line OA represents a similar current filament. The finite segment of structure, shown in Fig. 5a by dotted line, excited by narrow slot with generator 1 and matched impedance to 2, is the analog of traveling-wave antenna with the anisotropic radiating fabric.

Almost similar bounding surface is also key on surface of which it is arranged/located wedge-shaped rack/comb[1] (Fig. 6).

FOOTNOTE [1] The claim "wedge-shaped rack/comb" No. 1260620/26-9 of 7 August, 1968. ENDFOOTNOTE.

It is assumed that the plates of rack/comb 2 are arranged/located at equidistance of $d \ll \lambda_0$. Wedge angle ψ can be arbitrary. The center of similitude of source must be arranged/located on the edge/fin of key, as shown in figure for the case of elementary source 1.

In this case upon transfer to the dimensionless coordinates at different frequencies are obtained the wedge-shaped racks/combs, which are characterized by only the denseness of the location of plates, that at sufficiently low values of d/λ_0 it does not lead to a substantial change of distributing the surface impedance.

The examples examined show possibility on the basis of simple geometric considerations to arrange, or to synthesize a larger number of diverse electromagnetic systems with the potential frequency independence.

At the finite length each of such systems is antenna with the weak frequency dependence whose possible operating range it grows/rises with an increase in the sizes/dimensions.

Together with the known frequency-independent antennas into this class enter continuous-similar/such systems, which possess sufficiently high directivity.

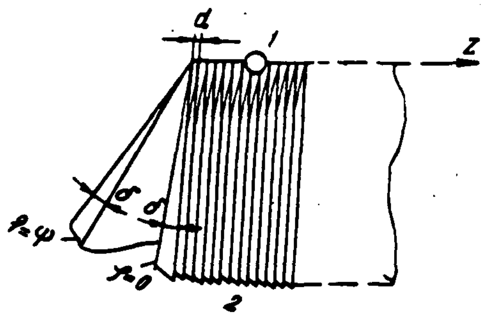

Fig. 6.

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Conclusion.

The use/application of methods of theory of similitude to the examination of the boundary-value problems of electrodynamics makes it possible to formulate the sufficiently general conditions of the electrodynamic similarity whose use gives the possibility to synthesize the broad class of electromagnetic structures without the losses, which possess potential frequency independence.

In the discussion of work took part G. T. Markov, Ya. S. Shifrin and F. B. Chernyy, for which the author expresses to them his gratitude.

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Received 31 October, 1969.

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Natural and mutual conductivities of half-wave slots, arbitrarily oriented on the conducting surface of circular cylinder.

I. F. Dobrovol'skiy, V. M. Klyuyev, A. I. Rogachev.

On the base of the representation of the functions of Hankel with Airy's functions are found the asymptotic expression of improper integral, giving tangential components of field on the surface of the ideally conducting cylinder, excited by magnetic current.

The obtained analytical expressions are applied for the calculation by the method of the induced by emf external their own and mutual conductivities of the arbitrarily oriented half-wave slots on the cylinder of comparatively large radius (ka$\gg$10).

Are given the results of numerical calculations for different orientations of slots.

Introduction.

A precise calculation of the radiation pattern of the gratings of slots, evaluation/estimate cf agreement and broad-band character of system and decoupling are far not the full/total/complete enumeration of the problems whose solution requires the determination of the full/total/complete its cwn and mutual conductivities of slots, often taking into account of their curvature and firite dimensions of screen. Known resclts about the conductivity of slcts on the cylindrical surface in essence concern the active component of the conductivity of the radiaticn/emission of single slot either they relate to the limiting cases of lcngitudinal or transverse slots.

Are obtained below expressions for the external conductivities of the half-wave slcts of arbitrary orientation, arranged/located on the surface of the conducting circular cylinder of a large radius.

Initial relationships/ratios.

Determining the slot of the constant width d and the length 1 in accordance with [1], on the basis of method of MDS we will obtain

$$Y_{mn}=G_{mn}+i\,B_{mn}=\frac{\int_{S_m}\left\{\int_{S_n}[\bar{n}\bar{E}^n(u',v')]\Gamma(a,u,v,u',v')dS_n\right\}[\bar{n}\bar{E}^m(u,v)]\,dS_m}{V_mV_n}, \quad (1)$$

where G(a, u, v, u', v') - the tensor of Green of the exterior of cylinder;

u, v, u', v' - arc length alcrg the appropriate geodetic lines;

V_m, V_n - voltage on the slots;

$\overline{n}$ - the unit vector of external normal to the surface of cylinder.

Let centers n- and the m-th of slots be located, correspondingly, at points (a, c, o) and (a, ϕ, z) cylindrical coordinate system (ρ, ϕ, z).

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Counting slot sufficient narrow (d<<l) and being limited to the most important for the practice case of "inclined" slots ($l=\lambda/2$),

$$\overline{n}E(u, v)=\overline{\rho}_0\overline{u}_0E(u, v)=\overline{v}_0V\Phi(u)\cos\kappa v, \qquad (2)$$

$$\Phi(u)=1/\pi[(\frac{d}{2})^2-u^2]^{-1/2} —$$

where $\kappa=\frac{2\pi}{\lambda}$ — wave number; Λ - electrostatic function, which describes the transverse dependence of tangential field on the slot; $\overline{u}_0, \overline{v}_0$ — the unit vectors of the positive direction of geodetic lines, $\overline{u}_0$ coinciding with the direction of electric field to the slot and the vectors $\overline{\rho}_0$, $\overline{u}_0$, $\overline{v}_0$ is formed the right-handed triad. Then for mutual conductivities [1]

$$Y_{mn}=\frac{\int_{l_m}V_m\left\{\int_{l_n}V_n\cos\kappa v'\,\overline{v}_{0n}\Gamma(a,v,v')\,dv'\right\}\overline{v}_{0m}\cos\kappa v dv}{V_mV_n}. \qquad (3)$$

Tangential magnetic field of slot on the surface of cylinder.

The vector of the full/total/complete magnetic current $\bar{J}$, which flows along the slot (segment of the geodetic line of circular cylinder), can be represented in the form

$$\bar{J} = V\cos\alpha\cos\kappa v\,\bar{\varphi}_0 + V\sin\alpha\cos\kappa v\,\bar{z}_0, \qquad (4)$$

where α - angle between the positive direction of helix and plane z=const, positive reference direction α - counterclockwise.

Substituting (4) into internal integral (3), in accordance with the determination of the tensor function of Green [2] we will obtain for the tangential magnetic field on the cylinder the following expression:

$$\bar{H}_t(\varphi, z) = H_z\bar{z}_0 + H_\varphi\bar{\varphi}_0, \qquad (5)$$

where

$$H_z = V_n \int_{l_n} \cos[\kappa v'(z', \varphi')]\,[\sin\alpha_n\Gamma_{zz} + \cos\alpha_n\Gamma_{z\varphi}]\,dv'(z', \varphi'),$$
$$H_\varphi = V_n \int_{l_n} \cos[\kappa v'(z', \varphi')]\,[\sin\alpha_n\Gamma_{\varphi z} + \cos\alpha_n\Gamma_{\varphi\varphi}]\,dv'(z', \varphi').$$

It is possible to show that on the surface of cylinder (with $\rho=a$)

$$\Gamma_{zz} = -\frac{i}{4\pi^2\omega\mu}\left(\kappa^2 + \frac{\partial^2}{\partial z^2}\right)\Psi, \quad \Gamma_{\varphi z} = \Gamma_{z\varphi} = -\frac{i}{4\pi^2\omega\mu}\,\frac{\partial^2}{\partial(a\varphi)\partial z}\Psi, \qquad (6)$$
$$\Gamma_{\varphi\varphi} = -\frac{i}{4\pi^2\omega\mu}\left[\kappa^2 + \frac{\partial^2}{\partial(a\varphi)^2}\right]\Psi,$$

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where

$$\Psi=\frac{1}{a}\int_{-\infty}^{\infty}\frac{e^{-ih(z-z')}}{(\kappa^2-h^2)^{1/2}}\sum_{n=-\infty}^{\infty}e^{-in(\varphi-\varphi')}\frac{H_n^{(2)}\left(a\sqrt{\kappa^2-h^2}\right)}{H_n^{(2)\prime}\left(a\sqrt{\kappa^2-h^2}\right)}dh.$$

A precise value for the tangential magnetic field of the slct of arbitrary orientation on the surface of cylinder, obtained from (5) and (6), inconveniently fcr the numerical calculations due to are slow convergences of series. After using results [3] for the functions Ψ, it is possible to cbtain the approximation, valid fcr large radii of cylinder (ka>10): where

$$\Psi=-2\pi\kappa\frac{e^{-iR_1}}{R_1}f(y),$$
$$R_1=\kappa\sqrt{(z-z')^2+[a(\varphi-\varphi')]^2}, \qquad (7)$$
$$\text{где } f(y)=\begin{cases}1-\sqrt{\frac{\pi i}{2}}\frac{y}{4}+\frac{7i\,y^2}{120}+\sqrt{\frac{\pi}{2i}}\frac{7y^3}{1024}+\ldots & \text{для малых } \tau,\\ \sqrt{\frac{\pi}{i}}\left(\frac{y}{\sqrt{2}}\right)^{1/3}\sum_{m=1}^{\infty}\frac{\exp\left[-i\left(\frac{y}{\sqrt{2}}\right)^{2/3}t_m\right]}{t_m} & \text{для больших } \tau,\end{cases}$$
$$\tau=\left(\frac{\kappa a}{a}\right)^{1/3}(\varphi-\varphi'),\quad y=\frac{[\kappa a(\varphi-\varphi')]^2}{\kappa a\sqrt{R_1}},\quad t_m-\text{корни } W'(t)=0.$$

Key: (1). where. (2). for small cnes τ. (3). for large ones τ. (4). roots.

W'(t) - the derivative cf Airy's function. Substituting (6) taking into account (7) in (5), introducing the replacement of the variable/alternating

$$\kappa z = Z, \quad \kappa a \varphi = X, \quad \kappa z' \csc \alpha_n = \kappa a \varphi' \mathrm{sc}\, \alpha_n = Y \tag{8}$$

and considering integrals in (5) as line integrals l-ro of the type, for the half-wave slot we will obtain, by producing integration in parts and taking into account that f(y) - the slowly varying function:

$$H_z = \frac{i\kappa V_n}{2\pi W_0}\Bigg\{\sin\alpha_n\left[\frac{e^{-iR_{11}}}{R_{11}} f(y_1) + \frac{e^{-iR_{12}}}{R_{12}} f(y_2)\right] + \\ + \cos\alpha_n\left[\frac{e^{-iR_{11}}}{R_{11}} f(y_1)\,\frac{Z\sin\alpha_n + X\cos\alpha_n + \frac{\pi}{2}}{X\sin\alpha_n - Z\cos\alpha_n} + \\ + \frac{e^{-iR_{12}}}{R_{12}} f(y_2)\,\frac{Z\sin\alpha_n + X\cos\alpha_n - \frac{\pi}{2}}{X\sin\alpha_n - Z\cos\alpha_n}\right]\Bigg\}, \tag{9a}$$

$$H_\varphi = \frac{i\kappa V_n}{2\pi W_0}\Bigg\{\cos\alpha_n\left[\frac{e^{-iR_{11}}}{R_{11}} f(y_1)\;\frac{e^{-iR_{12}}}{R_{12}} f(y_2)\right] - \\ - \sin\alpha_n\left[\frac{e^{-iR_{11}}}{R_{11}} f(y_1)\,\frac{Z\sin\alpha_n + X\cos\alpha_n + \frac{\pi}{2}}{X\sin\alpha_n - Z\cos\alpha_n} + \\ + \frac{e^{-iR_{12}}}{R_{12}} f(y_2)\,\frac{Z\sin\alpha_n + X\cos\alpha_n - \frac{\pi}{2}}{X\sin\alpha_n - Z\cos\alpha_n}\right]\Bigg\}, \tag{9б}$$

(1) где $R_{11,12} = \sqrt{\left(Z\sin\alpha_n + X\cos\alpha_n \pm \frac{\pi}{2}\right)^2 + (X\sin\alpha_n - Z\cos\alpha_n)^2}$

$$y_{1,2} = \frac{\left(X \pm \frac{\pi}{2}\cos\alpha_n\right)^2}{\kappa a\sqrt{R_{11,12}}}; \quad W_0 = 120\pi\ [\text{ом}].$$

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Key: (1). where.

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In the limiting cases of longitudinal and transverse slots those obtained expressions coincide with appropriate expressions [3].

Mutual conductivities of half-wave slots, arbitrarily oriented on the surface of round cylinder.

Introducing the new system of coordinates (Fig. 1),

$$Z = Z_{mn} + \xi; \quad X = X_{mn} + \xi;$$
$$Z_{mn} = R_{mn} \cos \Theta_{mn};$$
$$X_{mn} = R_{mn} \sin \Theta_{mn}; \quad \xi \csc \alpha_n = \xi \sec \alpha_m = \eta \qquad (10)$$

(where Z_{mn} and X_{mn} - coordinate of the center of the m slot in the old coordinate system), we will obtain from (3):

$$Y_{mn}=\frac{i}{2\pi W_e}\left\{\left[R_{mn}\cos(\alpha_m+\Theta_{mn})-\frac{\pi}{2}\sin(\alpha_m-\alpha_n)\right]\times\right.$$

$$\times\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\eta\,\frac{e^{-iR_{11}}}{R_{11}}f(y_1)\frac{d\eta}{R_{mn}\cos(\alpha_m+\theta_{mn})+\eta\sin(\alpha_m-\alpha_n)}+$$

$$+\left[R_{mn}\cos(\alpha\ \ +\Theta_{mn})+\frac{\pi}{2}\sin(\alpha_m-\alpha_n)\right]\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\eta\,\frac{e^{-iR_{12}}}{R_{12}}f(y_2)\times$$

$$\times\frac{d\eta_i}{R_{mn}\cos(\alpha_m+\theta_{mn})+\eta\sin(\alpha_m-\alpha_n)}, \qquad (11)$$

where

$$R_{11,12}=\sqrt{\left[R_{mn}\sin(\alpha_n+\Theta_{mn})+\eta\cos(\alpha_m-\alpha_n)\pm\frac{\pi}{2}\right]^2+\rightarrow}$$

$$\overline{\rightarrow+[R_{mn}\cos(\alpha_n+\Theta_{mn})+\eta\sin(\alpha_m-\alpha_n)]^2}$$

$$y_{1,2}=\frac{\left[R_{mn}\sin\Theta_{mn}+\eta\cos\alpha_m\pm\frac{\pi}{2}\cos\alpha_n\right]^2}{\kappa a\sqrt{R_{11,12}}}.$$

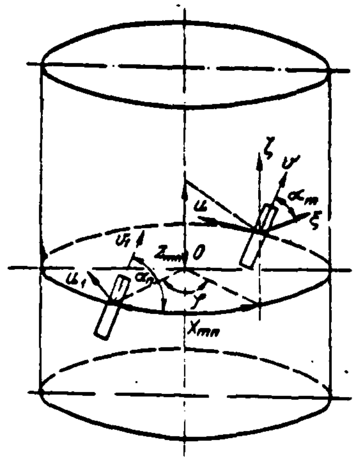

Fig. 1.

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With the help of the replacements of the variable/alternating of the type $t=R_{11}\pm\eta$ and the resolution of the obtained integrand into the common fractions finally we will obtain, dividing the real and imaginary parts:

$$G_{mn}=\frac{1}{4\pi W_0}[\mathrm{Re}\,f(y_{11})Q_{11}+T_{11}\,\mathrm{Jm}\,f(y_{11})+Q_{12}\mathrm{Re}f(y_{12})+$$
$$+T_{12}\,\mathrm{Jm}\,f(y_{12})+Q_{21}\,\mathrm{Re}\,f(y_{21})+T_{21}\,\mathrm{Jm}\,f(y_{21})+Q_{22}\,\mathrm{Re}\,f(y_{22})+T_{22}\,\mathrm{Jm}\,f(y_{22})] \quad (12)$$

$$B_{mn}=\frac{1}{4\pi W_0}[Q_{11}\,\mathrm{Jm}\,f(y_{11})-T_{11}\,\mathrm{Re}\,f(y_{11})+Q_{12}\,\mathrm{Jm}\,f(y_{12})-$$
$$-T_{12}\,\mathrm{Re}\,f(y_{12})+Q_{21}\,\mathrm{Jm}\,f(y_{21})-T_{21}\,\mathrm{Re}\,(y_{21})+Q_{22}\,\mathrm{Jm}\,f(y_{22})-T_{22}\,\mathrm{Re}\,f(y_{22})]. \quad (13)$$

where

$$y_{11,12} = \frac{\left[R_{mn}\sin\theta_{mn} \pm \frac{\pi}{2}\cos\alpha_m \pm \frac{\pi}{2}\cos\alpha_n\right]^2}{\kappa a\sqrt{B_{1,2}}},$$

$$y_{12,21} = \frac{\left[R_{mn}\sin\theta_{mn} \mp \frac{\pi}{2}\cos\alpha_m \pm \frac{\pi}{2}\cos\alpha_n\right]^2}{\kappa a\sqrt{A_{1,2}}},$$

$$B_{1,2} = \sqrt{R_{mn}^2 + \frac{\pi^2}{2}[1+\cos(\alpha_m-\alpha_n)] \pm \pi R_{mn}[\sin(\alpha_n+\theta_{mn}) + \sin(\alpha_m+\theta_{mn})]},$$

$$A_{1,2} = \sqrt{R_{mn}^2 + \frac{\pi^2}{2}[1-\cos(\alpha_m-\alpha_n)] \pm \pi R_{mn}[\sin(\alpha_n+\theta_{mn}) - \sin(\alpha_m+\theta_{mn})]},$$

$$D_{1,2} = R_{mn}\frac{\cos(\alpha_n \mp \theta_{mn}) \pm \cos(\alpha_m \mp \theta_{mn})}{\sin(\alpha_m-\alpha_n)},$$

$$\begin{aligned} Q_{11,22} = -\{&\cos D_1[\mathrm{Ci}(|B_{1,2}-D_1|) + \mathrm{Ci}(|B_{1,2}+D_1|)] + \\ &+\cos D_2[\mathrm{Ci}(|B_{1,2}+D_2 \pm \pi|) + \mathrm{Ci}(|B_{1,2}-D_2 \mp \pi|)] + \\ &+\sin D_1[\mathrm{Si}(B_{1,2}+D_1) - \mathrm{Si}(B_{1,2}-D_1)] + \\ &+\sin D_2[\mathrm{Si}(B_{1,2}+D_2 \pm \pi) - \mathrm{Si}(B_{1,2}-D_2 \mp \pi)]\}, \end{aligned}$$

$$\begin{aligned} T_{11,22} = -\{&\cos D_1[\mathrm{Si}(B_{1,2}-D_1) + \mathrm{Si}(B_{1,2}+D_1) - \pi] + \\ &+\cos D_2[\mathrm{Si}(B_{1,2}-D_2 \mp \pi) + \mathrm{Si}(B_{1,2}+D_2 \pm \pi) - \pi] + \\ &+\sin D_1[\mathrm{Ci}(|B_{1,2}-D_1|) - \mathrm{Ci}(|B_{1,2}+D_1|)] + \\ &+\sin D_2[\mathrm{Ci}(|B_{1,2}-D_2 \mp \pi|) - \mathrm{Ci}(|B_{1,2}+D_2 \pm \pi|)]\}, \end{aligned}$$

$$\begin{aligned} Q_{12,21} = \{&\cos D_1[\mathrm{Ci}(|A_{1,2}-D_1 \pm \pi|) + \mathrm{Ci}(|A_{1,2}+D_1 \mp \pi|)] + \\ &+\cos D_2[\mathrm{Ci}(|A_{1,2}-D_2|) + \mathrm{Ci}(|A_{1,2}+D_2|)] + \sin D_1[\mathrm{Si}(A_{1,2}+D_1 \mp \pi) - \\ &-\mathrm{Si}(A_{1,2}-D_1 \pm \pi)] + \sin D_2[\mathrm{Si}(A_{1,2}+D_2) - \mathrm{Si}(A_{1,2}-D_2)]\}, \end{aligned}$$

$$\begin{aligned} T_{12,21} = \{&\cos D_1[\mathrm{Si}(A_{1,2}-D_1 \pm \pi) + \mathrm{Si}(A_{1,2}+D_1 \mp \pi) - \pi] + \\ &+\cos D_2[\mathrm{Si}(A_{1,2}-D_2) + \mathrm{Si}(A_{1,2}+D_1) - \pi] + \sin D_1[\mathrm{Ci}(|A_{1,2}-D_1+\pi|) - \\ &-\mathrm{Ci}(|A_{1,2}+D_1 \mp \pi|)] + \sin D_2[\mathrm{Ci}(|A_{1,2}-D_2 \pm \pi|) - \mathrm{Ci}(|A_{1,2}+D_2|)]\}, \end{aligned}$$

$$\mathrm{Ci}(x) = -\int_x^\infty \frac{\cos t}{t}\,dt, \quad \mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\,dt.$$

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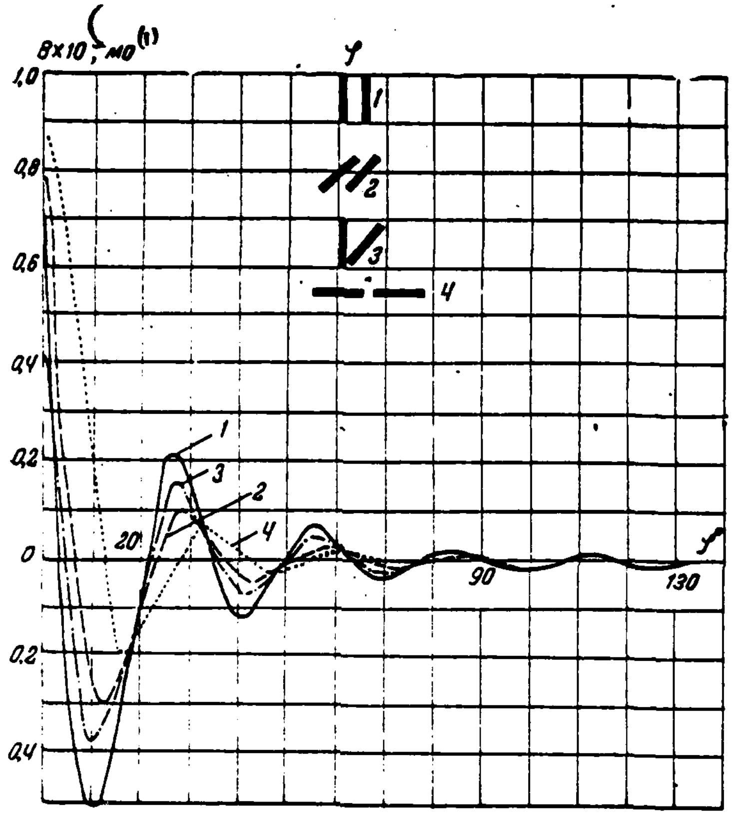

Fig. 2.

Key (1). mho.

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Function f(y) is determined by expression (7). Numerical calculations according to (12), (13) were performed for value of ka=12. Fig. 2.3 shows dependences G_{mn} and B_{mn} on the angular distance between centers of the slots, located in one section (z=const) of cylinder.

In the extreme case of parallel longitudinal slots $(\alpha_m = \alpha_n = \frac{\pi}{2})$, centers of which are arranged/located on one line z=const $|\theta_{mn} = \frac{\pi}{2}|$ of infinite plane $|ka=\infty|$, (12) and (13) they coincide with the well known results for the one-sided slots on infinite plane [4].

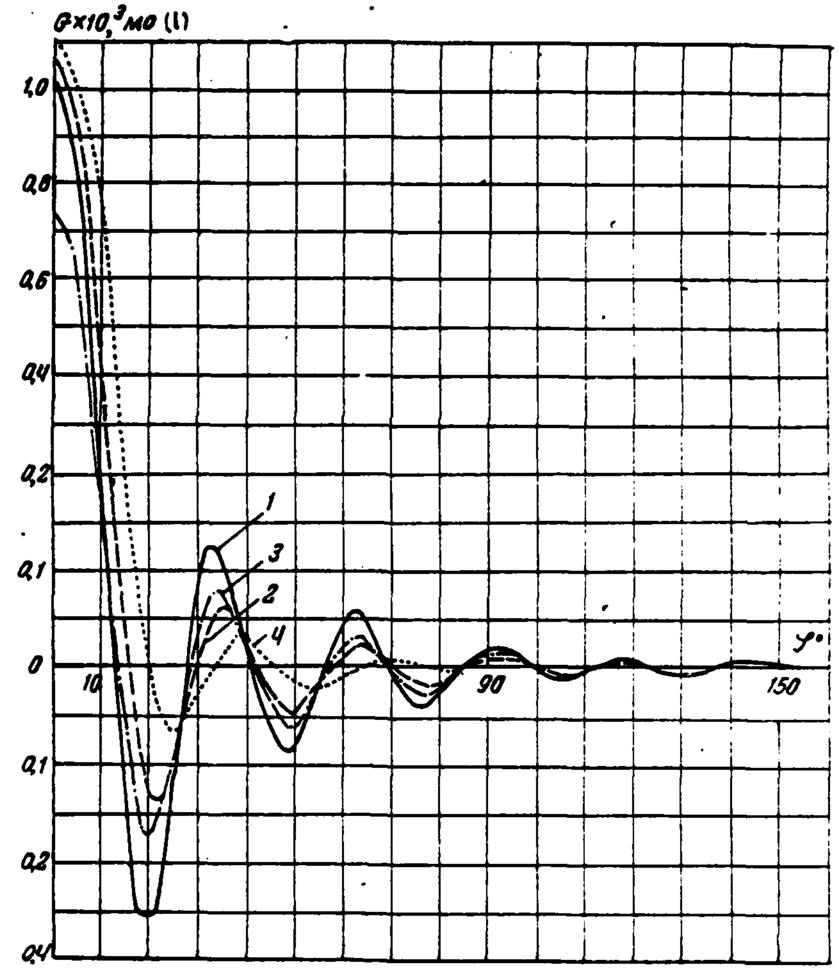

Fig. 3.

Key: (1). mho.

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Intrinsic conduction of half-wave slots, arbitrarily oriented on the surface of circular cylinder.

For obtaining the intrinsic conduction of slots it is necessary to consider the final width of slot [1].

Substituting (2), (6), (7) in (1) and taking into account that u, u', v, v' - arc length of helixes, is possible to easily show that for the half-wave slot

$$Y_{mn}=\frac{i}{2\pi W_0}\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int\limits_{-\frac{d'}{2}}^{\frac{d'}{2}}\cos V\Phi(U)\left(1+\frac{\partial^2}{\partial V^2}\right)\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int\limits_{-\frac{d'}{2}}^{\frac{d'}{2}}\cos V'\Phi(U')\times$$
$$\times\frac{e^{-i\sqrt{(V-V')^2+(U-U')^2}}}{\sqrt{(V-V')^2+(U-U')^2}}f(y)\,dV\,dV'\,dU\,dU', \quad (14)$$

where

$$V=\kappa v,\quad V'=\kappa v',\quad U=\kappa u,\quad U'=\kappa u',\quad d'=\kappa d,$$
$$y=\frac{[(V-V')\cos\alpha_n-(U-U')\sin\alpha_n]^2}{\kappa a\sqrt{(U-U')^2+(V-V')^2}}.$$

Differentiating twice under the internal integral and taking into account that

$$\frac{\partial}{\partial V}\left[\frac{e^{-i\sqrt{(V-V')^2+(U-U')^2}}}{\sqrt{(V-V')^2+(U-U')^2}}f(y)\right]=-\frac{\partial}{\partial V'}\left[\frac{e^{-i\sqrt{(V-V')^2+(U-U')^2}}}{\sqrt{(V-V')^2+(U-U')^2}}f(y)\right],$$

after the necessary conversions, dividing real and imaginary parts, we will obtain:

$$G_{nn}\simeq\frac{1}{2\pi W_0}\left[C+\ln 2\pi-\mathrm{Ci}(2\pi)+\frac{\pi^2}{8\kappa a}\cos^2\alpha\,\mathrm{Si}(2\pi)\right],$$
$$B_{nn}\simeq\frac{1}{2\pi}W_0\left\{\mathrm{Si}(2\pi)+\frac{\pi^2}{8\kappa a}\cos^2\alpha\left[\ln 2\pi-C+\mathrm{Ci}(2\pi-\ln\frac{\kappa d}{4}\right]\right\}. \quad (15)$$

where C=0.5772.

Passing in (15) to the limit with ka->∞ (infinite plane), we will obtain well-known expression for the half-wave slot on the infinite plane, which radiates into the hemisphere.

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Article was received by the editorial staff on 18 August 1969.

Characteristics of antennas with frequency beam swinging.

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In the work are determined the characteristics of the line-source antennas with frequency beam swinging, intended for the use in the flat antennas with wide-angle beam swinging in two planes.

It is claimed that for obtaining the high electrical characteristics in the flat/plane antenna arrays it is expedient to utilize the line-source antennas, constructed on the base of the waveguide kite circuits with the waveguide bends in the plane of magnetic field.

One of the methods of the construction of flat/plane antenna arrays with wide-angle electrical beam swinging is the use of a frequency response method of beam swinging in one plane and "phase" (with the help of the electrically controlled phase inverters) - in another. In this case antenna fabric is assembled from the separate line-source antennas with frequency swinging of beam (rules), each of which is excited through individual phase inverter [1].

For beam swinging in the cone with the apex angle of 40-90° necessary is the low pitch of emitters and, therefore, a small transverse size/dimension of rule - 0.7λ-0.58λ, what is one of the serious requirements, presented to the rule.

Another obvious group of the requirements, usually presented to the rule, are the requirements of high electrical characteristics (large throughput, low losses, the linearity of angular-frequency characteristic), moreover for each of these characteristics it is possible to indicate potential, maximally high value.

Maximally high throughput of line-source antenna with frequency beam swinging it is logical to consider the power, passed by flat rectangular waveguide of the corresponding wave band.

It will be shown below that it is possible to propose such rules circuits of which they possess throughput, which virtually coincides with the maximum.

The installation/setting up of emitters leads to certain weakening of dielectric strength of circuit, but not in the larger measure than in waveguide-slot type usual antennas. Speaking in other words, a decrease in throughput can and not be the specific character of antenna with frequency beam swinging.

Maximally low losses (or maximally high efficiency) has the rule, constructed according to the parallel diagram, in which the excitation of each emitter is realized from the single input with the help of the segments of waveguides with the progressively increasing length.

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The diagram of this rule (waveguide prism) is shown in Fig. 1, where it is marked:

1, 2, 3, ..., n - emitters;

nF - unguided phase inverters;

L =nl - segments of the feeder lines, which connect N of the outputs of adder Σ with the emitters. Without examining a question about the practical realizability of a similar diagram (by the way, very doubtful in the case of antenna to any extent large length), let us find its efficiency, disregarding losses in the adder and considering that the length of the first, shortest waveguide L_1 (see Fig. 1), is equal to zero.

With the symmetrical adder when losses in the segments of waveguides form geometric progression, it is easy to obtain the following expression for efficiency:

$$\eta = \frac{1 - e^{-\beta L_N}}{\beta L_N}, \qquad (1)$$

where β - attenuation in the coupling waveguides;

$L_N = Nl$ - maximum length of coupling waveguide;

N - total number of emitters in the rule;

l - difference in the lengths of two adjacent waveguides.

Difference in the lengths of adjacent waveguides - l and their width - a determine the angular-frequency sensitivity of waveguide prism and the sector of frequency beam swinging. From the phase relationships/ratios for the boundaries of the sector of oscillation it is possible to obtain the following relationship/ratio, which connects sizes/dimensions l and a with the boundaries of sector θ_{f_1} and θ_{f_2} and the limits of a change in frequency f_1, f_2:

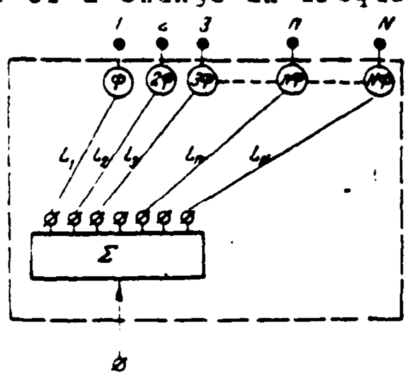

FIG. 1.

$$\frac{a}{\lambda_0}=\frac{1}{2}\frac{f_0}{f_2}\sqrt{\frac{B-1}{B-\left(\frac{f_1}{f_2}\right)^2}}, \quad (2)$$

$$\frac{l}{\lambda_0}=\frac{\left(m-\frac{\Phi}{2\pi}\right)\frac{f_0}{f_2}\frac{d_f}{\lambda_0}\sin\theta_{f2}}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2\left(\frac{f_0}{f_2}\right)^2}}, \quad (3)$$

where

$$B=\left[\frac{m-\frac{\Phi}{2\pi}-\frac{d_f}{\lambda_0}\frac{f_1}{f_0}\sin\theta_{f1}}{m-\frac{\Phi}{2\pi}-\frac{d_f}{\lambda_0}\frac{f_2}{f_0}\sin\theta_{f2}}\right]^2;$$

f_0 - midband frequency;

m - 1, 2, 3, 4 ...;

F - supplementary phase displacement between the adjacent emitters, created due to connection of phase inverters into the channels of emitters;

d_f - step/pitch of emitters along the axis of rule.

Subsequently is utilized also designation d_v - the distance between centers of rules in the plane of antenna fabric.

Maximally small nonuniformity of angular-frequency sensitivity (UChCh) has the antenna, assembled on parallel or series circuit with the use of the air twin-lead or coaxial lines, for which there is no dispersion (group and phase speeds are equal). Let us show this, for which let us record UChCh in the following form:

$$A=\frac{d\theta_f}{\frac{d_f}{f}}=-\frac{1}{\cos\theta_f}\left[\left(m-\frac{\Phi}{2\pi}\right)\frac{\lambda_0}{d_f}\frac{f_0}{f}+\gamma_r\frac{d\gamma}{\frac{d_f}{f}}\right]. \qquad (4)$$

where $\gamma_r=\frac{l}{d_f}$ - geometric delay/retarding/deceleration, equal to the relation of the length of the segment of the waveguide line, connected between two adjacent emitters, to the step/pitch of emitters;

γ - phase delay/retarding/deceleration.

Expressing value $\left(m-\frac{\Phi}{2\pi}\right)$ through the group and phase delays/retardings/decelerations in the normal position of ray/beam ($f=f_H$):

$$\sin\theta_f|_{f=f_H}=\left(m-\frac{\Phi}{2\pi}\right)\frac{\lambda_0}{d_f}\frac{f_0}{f_H}-\gamma_r\gamma_H=0, \qquad (5)$$

$$m-\frac{\Phi}{2\pi}=\gamma_r\gamma_H\frac{d_f}{\lambda_0}\frac{f_H}{f_0}, \qquad (6)$$

let us rewrite (4) in the following form:

$$A=-\frac{\gamma_H}{\cos\theta_f}\frac{l}{\lambda_0}\left[\frac{\lambda_0}{d_f}\frac{f_H}{f}+\frac{\lambda_0}{d_f}\frac{1}{\gamma_H}\frac{d\gamma}{\frac{d_f}{f}}\right]. \qquad (7)$$

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From (7) it is evident that UChCh of line-source antenna is composed of two members, who depend on the frequency and, therefore, that contributes its contribution to the nonuniformity UChCh with beam swinging. However, on the type of circuit depends only second component/term/addend, which can be equal to zero when $\frac{d\gamma}{d_f}=0$, i.e. in the absence of dispersion.

Nonuniformity UChCh usually is computed according to the formula

$$\Delta A = \pm 100\% \frac{(A_1 - A_2)}{2A_{cp}} = \pm 100 \frac{(A_1 - A_2)}{A_1 + A_2}\%. \qquad (8)$$

where A_1 and A_2 - UChCh on the boundaries of the sector of frequency beam swinging.

From formulas (8) and (9) it is possible to calculate minimum nonuniformity UChCh for the symmetrical relative to standard sector of beam swinging:

$$\Delta A_{min} = \pm 100 \frac{\Delta f}{f_0}\%. \qquad (9)$$

In this case

$$\frac{\Delta f}{f_0} = \frac{1}{2} \frac{f_1 - f_2}{f_1 + f_2}. \qquad (10)$$

In paragraph A) Table 1 are given all enumerated maximum characteristics for two wave bands at the length of line-source antenna to ~100 λ and average/mean angular-frequency sensitivity:

$$A_{cp}=5\frac{(1)\ \text{град}}{(2)\ \text{процент частоты}}\ (\theta_{f1}=\theta_{f2}=17{,}5^\circ;\ f_1=0{,}965\ f_0;\ f_2=1{,}035\ f_0).$$

Key: (1). Deg. (2). percentage cf frequency.

During the calculation of losses according to formula (1) it was assumed that the waveguides were prepared from aluminum and have the height b=0.45λ. This height of waveguide provides the possibility of the order of rules with the step/pitch $d_\varphi=0.5\ \lambda$[1], that it makes it possible to obtain the sector of the phase oscillation $2\Delta\theta_\varphi=180^\circ$[2].

FOOTNOTE 1. In this case the waveguides in the aperture of prism will be joined by narrow walls.

2. This value $2\Delta\theta_\varphi$ is obtained only from condition of single-ray beam swinging; under actual conditions it is hardly expedient to approach sector of scanning, greater than +-45°, for which is sufficient step/pitch of emitters $0.58\ \lambda_{min}$. ENDFOOTNOTE.

Table 7.

(1) Сектор фазового качания луча, *град*	(2) Разрядная мощность, *Мвт*, при различной λ, *см*		(3) кпд при различной λ, *см*		(4) Неравномерность УЧЧ, %
	10	3	10	3	
а) Предельные характеристики					
±90	13	1,3	0,92	0,86	±4
(b) Характеристики линейной антенны с Е-ЗВ					
21	1,2	0,11	0,62	0,45	±9,6
(c) Характеристики линейной антенны с Н-ЗВ					
±90	10,6	0,96	0,84	0,75	±9,6

Key: (1) the sector of phase swinging of beam, deg. (2). discharge power, MW, with different λ, cm. (3). efficiency with different λ, cm. (4). Nonuniformity UChCh, o/o. (a). Maximum characteristics. (b) and (c) Characteristics of line-source antenna.

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During the calculation of discharge power was assumed [1] that rectangular waveguide has a section 0.72 λx0.45 λ, and dielectric strength of air is equal to 29 kV/cm.

FOOTNOTE [1]. The passed (worker) and discharge power differ to the safety factor, equal to 1.5-4 [5]. ENDFOOTNOTE.

Types of rules with the characteristics, close to the maximum ones.

We will be bounded to the examination only of series circuit of the construction of rule (with the use of a circuit) and will show that in a number of cases of the characteristic of circuit (and, consequently, of rule as a whole) they can be close to the maximum ones.

The angular-frequency sensitivity of rule, as it follows from works [2], is determined by angle of radiation and by group delay/retarding/deceleration γ_{rp}, which can be recorded in the form

$$\gamma_{rp} = \frac{l}{d_f}\left(\gamma + \frac{d\gamma}{\frac{d_f}{f_1}}\right),$$

where γ - phase delay/retarding/deceleration in the waveguide of lines;

$\frac{d\gamma}{d_f/f}$ - term, which characterizes dispersion;

$\gamma = \frac{l}{d_f}$ - geometric delay/retarding/deceleration;

1 - segment of the waveguide line [2], connected between two adjacent emitters.

FOOTNOTE [2]. By waveguide here and throughout is understood any canalizing system. ENDFOOTNOTE.

Using the method of obtaining the assigned group delay/retarding/deceleration the circuits can be somewhat conditionally decomposed into the classes strong and weakly dispersive ones.

In the highly dispersive systems the necessary delay/retarding/deceleration is composed of phase delay/retarding/deceleration and dispersion with $1/d_f \sim 1$ [3]; in the weakly dispersive systems the value of group delay/retarding/deceleration determines factor $1/d_f >> 1$; therefore such circuits occasionally referred to as geometric [3].

The highly dispersive ones include the systems with ground waves, and also periodic waveguide systems, whose each period is formed by the strongly reflecting waveguide heterogeneity (for example, flanged waveguides, system "finger/pin into the finger/pin" and the like).

The weakly dispersive circuits include the periodic waveguide systems whose each period is formed by the waveguide heterogeneity (curvature) with the negligible coefficient of reflection; necessary group delay/retarding/deceleration is here obliged to the simple geometric elongation of the path of motion of electromagnetic energy (for example, waveguide spirals and coils during the appropriate selection of the curvatures of the ensuring smallness reflection coefficients).

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In the highly dispersive circuits, in contrast to the weakly dispersive ones, are always regions with the increased concentration of electrical and (or) magnetic fields and consequently, with the increased concentration of surface currents. This is the direct consequence of the increased dispersion of system and imply an increase in the ohmic losses, a decrease in the uniformity UChCh and a sharp decrease in throughput in comparison with the analogous characteristics of the weakly dispersive circuit during the same average group delay/retarding/deceleration.

The only useful property of the highly dispersive circuits (but sometimes by very important), which also there is a consequence of the presence of regions with the increased concentration of fields, is the decrease of overall sizes.

Thus, circuit with the high electrical characteristics should be sought only in the class of the weakly dispersive circuits - kite and spiral waveguides. By fundamental difficulty during the construction of a similar system is obtaining low reflection coefficient from the separate curvature with the observance previously the size limitations indicated;, other conditions being equal, it is better than the characteristics of that system, in which most completely are utilized the overall sizes, i.e., for the assigned step/pitch of emitters d_f and d_φ and assigned angular-frequency sensitivity are used maximally possible sizes/dimensions of waveguides, and the distribution of electromagnetic energy in them is maximally even, without the local concentrations.

Taking into account these considerations let us attempt to design circuit with the characteristics, close to the maximum ones. Preliminarily let us measure off, that the systems on the base of lines with the wave TEM, although they possess the maximally small nonuniformity of angular-frequency sensitivity and easily satisfy all size limitations, they are distant from the circle of the searches for circuit it was throttled/tapered - from the optimum according to such most important indices as losses and throughput. This is the consequence of the fact that the wave TEM can be only transmission mode in the systems whose transverse sizes/dimensions are much lower than the wavelength, the at the same time assigned step/pitch of emitters $(d_f=d_\varphi=0.55\lambda-0.7\lambda)$ can be ensured, utilizing circuits on the base of the lines whose transverse sizes/dimensions are commensurated with the wavelength - usual rectangular waveguides with wave H_{01}. From the waveguide weakly dispersive systems it is possible to immediately reject/throw waveguide spirals, since during sufficiently large delay/retarding/deceleration ($\gamma_{rp}>3.5$) their transverse overall size does not make it possible to obtain the required step/pitch of emitters d_{φ}.

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Thus, the circle of the searches for circuit from which most natural for the use/application in the antennas with frequency beam swinging seems the coil with the 180° curvatures in the plane of electric field and with the emitters, excited of each curvature. However, Fig. 2a, where is shown the diagram of this snake, immediately shows deficiencies/lacks in a similar system: for obtaining the sufficiently low pitch of emitters d_{φ} the size/dimension of waveguide "b" must be considerably less than the same size/dimension in the standard waveguides of the corresponding wave band. Thus, with the sector of frequency swinging of beam of +-20° step/pitch of emitters $d_{\varphi}=0.71\lambda$, and $b\sim0.12\lambda$. So low a value b leads to comparatively large losses in the system and decrease in throughput. However, throughput in the larger measure is limited to the second deficiency/lack in this system - by small bending radius of waveguide in the plane of electric field. Thus, with step/pitch

d_f=0.71λ bending radius $r \approx d_f/b = 0.12\lambda$. So small a bending radius and, therefore, high reflection coefficient from it they do not make it possible to hope for obtaining of the to any extent satisfactory characteristics of this system.

The best characteristics possesses the kite system, shown in Fig. 2b, which is characterized by from the only fact that the bending radius is here made maximally possible for the assigned step/pitch of emitters. Latter/last system exhausts, apparently, the possibilities of kite waveguides (ZV) with the curvatures in the plane of electric field (E-ZV); its design characteristics for three wave bands are given in Table 1.

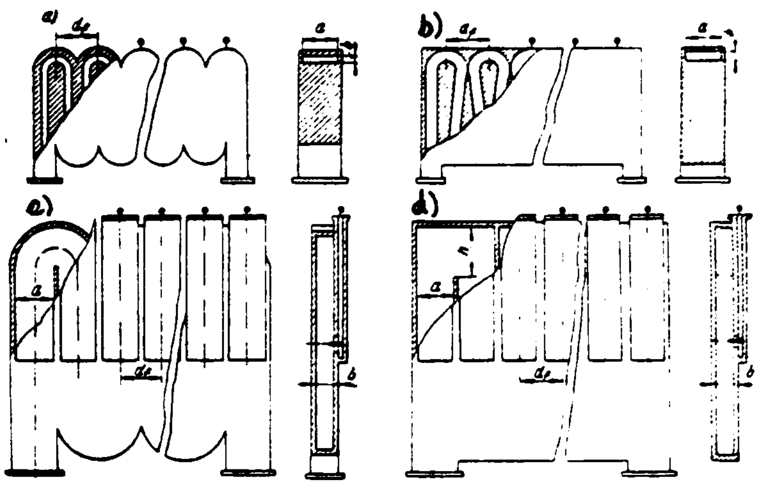

Fig. 2.

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An overall deficiency/lack in the rules on the basis of kite waveguides with the curvatures in the plane of electric field is the impossibility of order their sufficiently closely to each other for the purpose of obtaining the low pitch of emitters d_φ and, therefore, larger sector of phase beam swinging. Taking into account all design features of the antennas in question obtaining step/pitch d_φ, smaller than 0.7λ, and therefore the sector of the phase deflection of the beam, greater than +-22.5°, with the use E-ZV, apparently, it is impossible.

At the same time the decrease of step/pitch d_φ and the maximum expansion of the sector of the phase beam deflection is considered by desirable and more advisable than the decrease of step/pitch d_f for the purpose of the expansion of the sector of frequency beam swinging. The latter in the real systems in which is included the antenna, usually is limited either by the deviation of frequency or by the angular-frequency sensitivity whose improvement blocks the requirement of small linear ohmic losses in the circuit of antenna. However, the latter increase with an increase in the angular-frequency sensitivity not more weakly than it is directly proportional [2]. However, the expansion of the sector of the phase beam deflection can block only steep pitch of emitters d_φ.

All noted deficiencies/lacks prove to be sharply weakened in the rules, constructed on the base of kite waveguides with the curvatures in the plane of magnetic field (N-ZV) [6] to the latter/last not examined by us weakly dispersive circuit.

At first glance the possibility of use N-ZV in the antenna seems doubtful because its period, in contrast to the E-ZV, more than wavelength. Also can seem by doubtful the possibility of obtaining the sufficiently high electrical characteristics in view of a small bending radius of waveguide.

However, the first doubt drops off, if we excite emitters from the middles of waveguide channels, as shown in Fig. 2c; the step/pitch of emitters d_1 is equal to half of the period of coil and can comprise 0.65λ-0.7λ, what it is sufficient for frequency beam swinging in sector of +-20-+-30°.

The sector of the phase beam deflection in the foil lattices, formed by the set of rules on the base N-ZV, can be led to +-45 and even it is more, since with the common for standard waveguides size/dimension of narrow wall (B≤0.5λ) the step/pitch of the order of rules in the fabric can be close to the half-wave. On the other hand, the possibility of using the waveguide circuit whose size/dimension b is close to the standard, i.e., two or three times more than in the E-ZV, makes it possible to significantly decrease the linear losses.

But, perhaps, even more important advantage N-ZV before the E-ZV is the fact that waveguide bend in the plane of magnetic field, even with so small a bending radius (see Fig. 2c), has low reflection coefficient and virtually (with the accuracy approximately 20o/o) does not affect throughput. The special features/peculiarities of waveguide curvatures indicated in the plane of magnetic field to a certain degree are known from literature [5], and were also in detail investigated both theoretically and experimental [5-7].

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By the same high characteristics, although into the somewhat smaller frequency band, possesses the N-ZV even with the rectangular curvatures (Fig. 2d). Due to insignificant worsening/deterioration in the range properties this system obtained the series/row of essential structural/design advantages in comparison with the system Fig. 2c.

Latter/last two systems, in the principle, differ little from each other, exhaust the possibilities of using the kite waveguides with the curvatures in the plane of magnetic field in the antennas with frequency beam swinging.

All mentioned kite systems are depicted in Fig. 2 with the exemplary/approximate observance of scale. Among these systems N-ZV most completely is utilized the transverse overall size, determined by the step/pitch of emitters d_{φ} and d_{θ}. Simultaneously the N-ZV has the greatest height.

Design characteristics of rule on base N-ZV are given in the latter/last column Table 1, from which it is clear that these characteristics are very close to the maximum ones.

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Article was received by the editorial staff on 1 December 1969.

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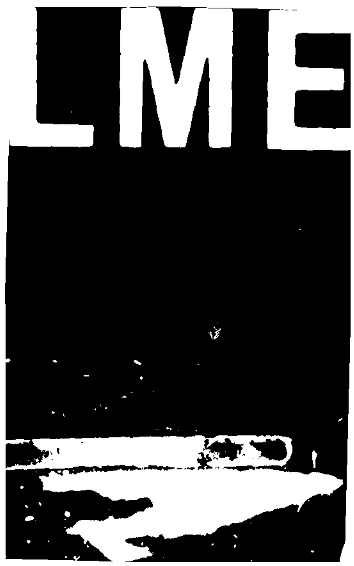
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