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APPROXIMATE FLUID-STRUCTURE INTERACTION THEORIES FOR ACOUSTIC E--ETC(U)

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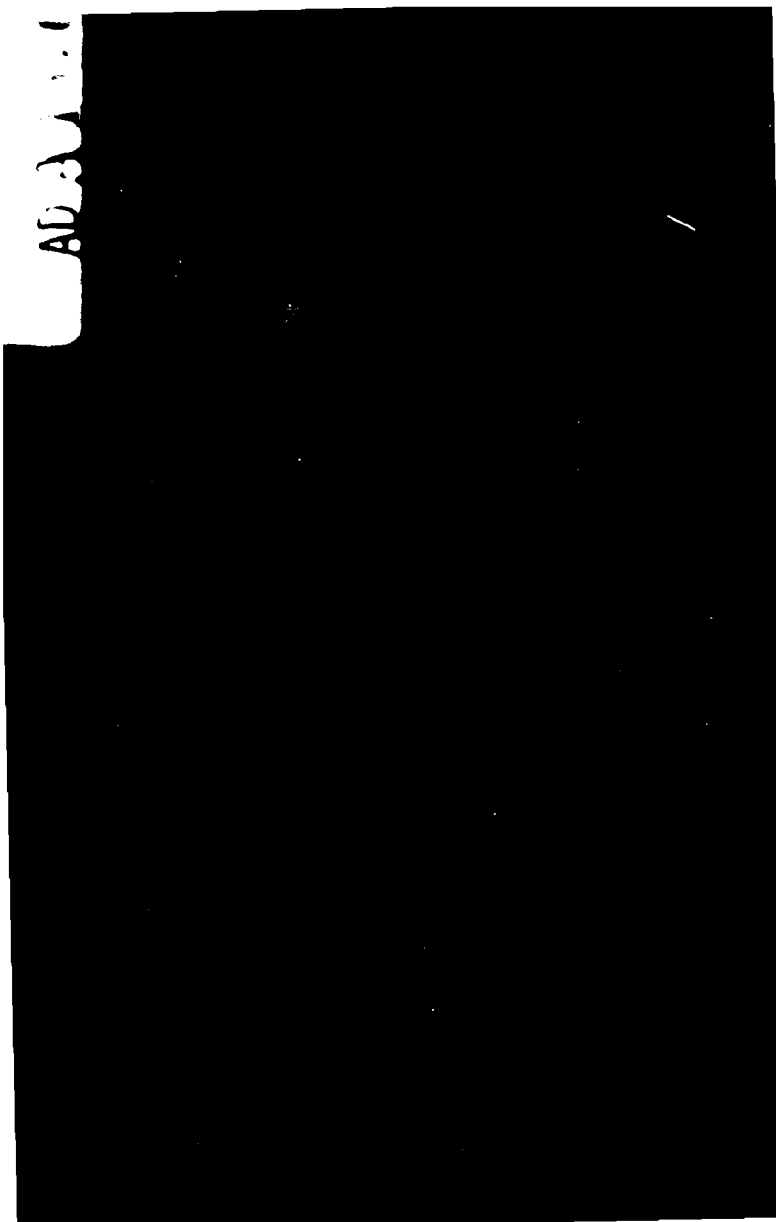
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20 ABSTRACT (Continued)

It is found that a theory termed the second order doubly asymptotic approximation is promising for this scheme. An optimum computation scheme could be a combination of the exact and various approximate theories depending on the wavelength of the incident pulse and the geometry and the dynamic characteristics of the structure.

## CONTENTS

INTRODUCTION .....	1
THE APPROXIMATE PREDICTION SCHEME .....	1
THE BENCHMARK PROBLEM .....	3
RESULTS AND DISCUSSION .....	6
CONCLUSIONS .....	7
ACKNOWLEDGMENT .....	13
REFERENCES .....	13

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## APPROXIMATE FLUID-STRUCTURE INTERACTION THEORIES FOR ACOUSTIC ECHO SIGNAL PREDICTIONS

### INTRODUCTION

To predict the acoustic echo signals from a submerged elastic structure of arbitrary shape impinged upon by an acoustic pulse, it is necessary to simultaneously solve the dynamic structural response and the acoustic scattering problems due to the interaction amongst the incident pulse, the structure and its surrounding acoustic medium. The scattered field here includes the pressure field due to the submerged body acting as an obstacle to the passage of the incident pulse and that due to the radiation by the vibratory structural response motion. The exact mathematical formulations of this problem often use the Helmholtz integral or the simple source integral representation of the solution to the wave equation for the surrounding acoustic medium. This integral equation together with the equation of motion of the structure form the simultaneous set to be solved numerically. The pressure and acceleration distribution on the fluid-structure interface have to be first obtained prior to the calculation of the far field quantities. The bulk of computation lies in obtaining the surface distributions. The far field calculation is rather straightforward. For extensive computation involving a complex shaped structure, the computer expense could be quite costly. Depending on the computation strategy used, numerical trouble could arise due to the so called internal resonance of the integral representation [1,2]. For some situation, e.g., in analyzing high frequency echo signals, the exact formulation is not only unnecessary but is also clumsy to use. Thus it appears that an optimum computation scheme could be a combination of various approximate theories of fluid-structure interaction and the exact formulation for the computation of distribution of pressure and acceleration on the fluid-structure interface and therefrom the far field is obtained by integrating the Helmholtz integral.

Herein this scheme, which was suggested by N. Basdekas [3], is explored using the problem of the echo signal from a spherical elastic shell [4] as the benchmark. It is found that a theory termed the second order doubly asymptotic approximation is particularly suitable.

### THE APPROXIMATE PREDICTION SCHEME

The mathematical formulation for acoustic scattering problem in the frequency domain is discussed here. For linear scattering in a isotropic homogeneous acoustic medium, the discussions also apply directly to the time domain formulation except for some computation procedures.

The pressure  $p$  in the acoustic field is governed by the Helmholtz wave equation

$$\nabla^2 p + k^2 p = 0 \quad (1)$$

where  $\nabla^2$  is the Laplace operator,  $k = \omega/c$ ,  $\omega$  is the angular frequency and  $c$  the sound speed of the acoustic medium. The pressure must satisfy the radiation condition at far field and the boundary condition at the fluid-structure interface  $S$

$$\frac{\partial p}{\partial n} = \rho \omega^2 w \quad (2)$$

where  $\rho$  is the mass density of the acoustic fluid medium,  $n$  is the normal at the fluid-structure interface directing into the medium and  $w$  is the normal component of displacement of the interface. The pressure can also be written as the sum of the incident pressure and the scattered pressure, i.e.,

$$p = p^{inc} + p^{sca} \quad (3)$$

Let  $\underline{r}$  be the position vector of a field point (observation point)  $P$  on  $S$  or in space,  $\underline{r}'$  be the position vector of a source point (integration point)  $Q$  on  $S$ ,  $\underline{r}_{PQ} = \underline{r}' - \underline{r}$  and  $r = |\underline{r}_{PQ}|$ . If  $S$  is piecewise smooth and  $P$  is exterior to  $S$ , it has been shown that, e.g. [2],

$$p^{sca}(P) = \frac{1}{4\pi} \iint_S \left[ p(Q) \frac{\partial}{\partial n} \frac{e^{ikr}}{r} - \frac{e^{ikr}}{r} \frac{\partial p(Q)}{\partial n} \right] ds. \quad (4)$$

For  $P$  on  $S$ ,

$$p^{sca}(P) = \frac{1}{2\pi} \iint_S \left[ p(Q) \frac{\partial}{\partial n} \frac{e^{ikr}}{r} - \frac{e^{ikr}}{r} \frac{\partial p(Q)}{\partial n} \right] ds. \quad (5)$$

For known distributions of the total pressure  $p$  and its normal derivative on  $S$  and the geometrical properties of  $S$ , the scattered pressure field  $p^{sca}$  can be readily evaluated by integrating the Helmholtz surface integrals in equations (4) and (5). For scattering problems in general, however, the surface distributions of the total pressure and its normal derivative are unknown and need be solved for. Equations (2), (3) and (5) constitute a relation between the pressure acting on the structure and the structural normal velocity (or acceleration) at the fluid-structure interface. This, together with the equation of motion of the elastic structure, form the governing system for the problem of scattering from an elastic body. For numerical computation, equation (5) is discretized into a matrix algebraic system. The coefficients of the matrices are dependent on  $k$ , therefore the computation effort could be quite large if the computation is to be swept through a wide range of frequency. This is equally true for the simple-source integral formulation [5]. Moreover, for high frequency situations, it is necessary to discretize  $S$  into a large number of elements and proportionally the sizes of the matrices are increased.

It is well known in both acoustic scattering and fluid-structure interaction theories that the mathematics can be much simplified for very high or very low frequency problems. Here, the high or low frequency is measured relative to the size of the scatterer and its fundamental natural frequencies. Recently, in analyzing structural response to the impingement of acoustic pulses, an approximation theory [6] emerged to bridge the high and low frequency approximations. This theory provides a hierarchy of formulae which approach exactness in the limit of low- and high-frequency and effect a smooth transition in the intermediate frequency range. This is therefore termed the DAA (Doubly Asymptotic Approximation) family [6]. The DAA family can also be derived directly from equation (5) [7]. Replacing equation (5) by DAA or other relating approximations, the computation effort could be much reduced for calculating the surface distributions of pressure and normal acceleration and hence for calculating the far field scattered pressure.

The equation of motion of an elastic structure can be written in the following matrix form [6]

$$M\ddot{\chi} + C\dot{\chi} + K\chi = f_{int} - GA_f(p^{sca} + p^{inc}) \quad (6)$$

where  $\chi$  is the structural displacement vector, a dot denotes time differentiation,  $M$ ,  $C$  and  $K$  are the structural mass, damping and stiffness matrices respectively,  $A_f$  is a diagonal area matrix for the fluid-structure interface,  $G$  is a transformation matrix which relates the forces on the structure to those on the interface, and  $f_{int}$  is the known internal force vector. Practical doubly asymptotic approximations are encompassed in the second order DAA2 formula [6]

$$M_f \ddot{p}^{sca} + \rho c A_f \dot{p}^{sca} + \rho c \Omega_f A_f p^{sca} = \rho c M_f (G^T \ddot{\chi} - \ddot{U}^{inc}) \quad (7)$$

$$\rho c [M_f (G^T \ddot{\chi} - \ddot{U}^{inc}) + \Omega_f M_f (G^T \ddot{\chi} - \ddot{U}^{inc})]$$

where  $M_f$  and  $\Omega_f$  are respectively the fluid added mass and frequency matrices pertaining to the fluid-structure interface  $G^T$  is the transpose of  $G$  and  $\ddot{U}^{inc}$  is the known normal fluid-particle velocity vector associated with the incident wave. The method for obtaining  $M_f$  and  $\Omega_f$  and their definition have been propounded in [6]. These matrices are time invariant, i.e., frequency independent. It is this property that equation (7) requires less computation effort compared to equation (5). If  $\Omega_f=0$ , equation (7) reduces to the first order DAA1 formula

$$M_f \ddot{p}^{sca} + \rho c A_f \dot{p}^{sca} = \rho c M_f (G^T \ddot{\chi} - \ddot{U}^{inc}). \quad (8)$$

For every early time (high frequency) interaction, equation (8) approaches the plane wave approximation, PWA,

$$p^{sca} = \rho c (G^T \dot{\chi} - \dot{U}^{inc}), \quad (9)$$

and for late time (low frequency) interaction, equation (8) approaches the added mass approximation

$$A_f p^{sca} = M_f (G^T \ddot{\chi} - \ddot{U}^{inc}). \quad (10)$$

The proposed approximate scheme here for analyzing the scattered acoustic field from an arbitrary elastic structure utilizes approximate fluid-structure interaction theories such as those embodied in equations (7) through (10) to first calculate the pressure and normal acceleration on the fluid-structure interface. These results are then used as inputs in equation (4) to determine the far field echo signals by numerical quadratures.

#### THE BENCHMARK PROBLEM

The effectiveness of this approximate scheme is investigated here by comparisons of results to an exact analysis of the echoes from a spherical elastic shell irradiated by a distant point source. Extensive exact numerical results were first published in reference [4] which treated the elastic shell by the three-dimensional theory of elasticity. The same problem has also been solved exactly without numerical result treating the shell by a shell theory [8]. To facilitate computations involving various approximate interaction theories, the shell solution is also used here.

Figure 1 sketches the uniform spherical elastic shell and the incident plane wave front.  $R$  and  $\theta$  are spherical radial and polar coordinates whose origin  $O$  coincides with the center of the spherical shell. Since the problem is axisymmetric, the azimuthal coordinate is not needed here. The shell is made of an isotropic elastic material and its geometric and material properties are its middle surface radius  $a$ , thickness  $h$ , Young's modulus  $E$ , Poisson's ratio  $\nu$  and mass density  $\rho_s$ . The radial deflection and pressure can be expanded in terms of series of Legendre functions as the following

$$\begin{aligned} w(\theta, t) &= \sum_{m=0}^{\infty} w_m P_m(\cos\theta) \exp(-i\omega t) \\ p(R, \theta, t) &= \sum_{m=0}^{\infty} p_m(R) P_m(\cos\theta) \exp(-i\omega t) \\ p_m &= p_m^{\text{inc}} + p_m^{\text{sca}} \end{aligned} \quad (11)$$

where  $P_m$  is the Legendre polynomial of the first kind and  $m$ th degree,  $t$  designates time and  $i = \sqrt{-1}$ . For brevity, the time factor  $\exp(-i\omega t)$  will be omitted hereafter.

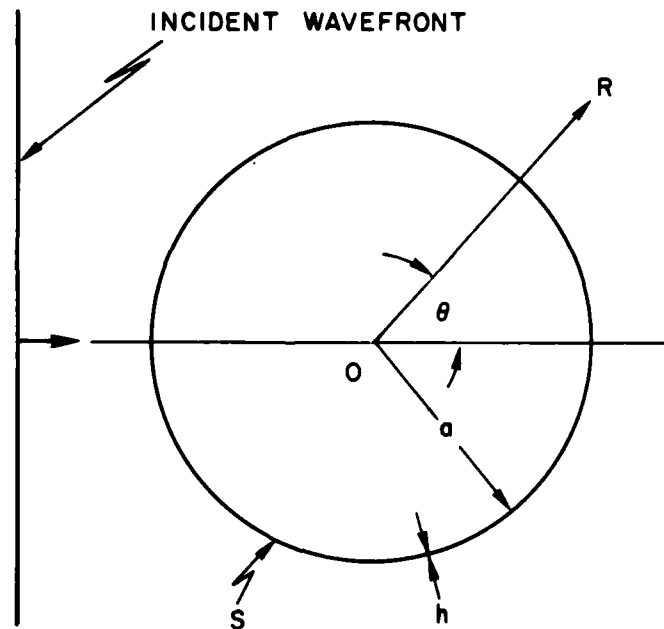


Fig. 1 — Spherical elastic shell irradiated by an incident plane wave

The incident plane wave front travels in such a direction that it first impinges on the shell at  $(R=a, \theta=\pi)$ . The incident wave can be represented by

$$\begin{aligned} p^{\text{inc}}(R, \theta) &= p^0 \exp(ikR \cos\theta) \\ &= p^0 \sum_{m=0}^{\infty} (2m+1) i^m P_m(\cos\theta) j_m(kR) \end{aligned} \quad (12)$$

where  $p^0$  is the incident pressure amplitude and  $j_m$  is the spherical Bessel function of the first kind. The boundary condition at the fluid-shell interface S is now

$$\frac{\partial p_m}{\partial R} = \omega^2 \rho w_m \quad \text{at } R=a \quad (13)$$

The equation of motion of the spherical shell can be written as [9]

$$\begin{aligned} \Gamma_m w_m &= -p_m(a) \\ &= -[p_m^{\text{inc}}(a) + p_m^{\text{sca}}(a)] \end{aligned} \quad (14)$$

where

$$\begin{aligned} \Gamma_m &= \frac{\rho C^2}{a} \frac{k^4 a^4 - (A_m + C_m) k^2 a^2 + (A_m C_m - B_m D_m)}{M(C_m - k^2 a^2)} \\ A_m &= C^2 \left\{ 2 + m(m+1) \frac{m(m+1) - (1-\nu)}{1+\nu} I \right\} \\ B_m &= m(m+1) C^2 \left[ 1 + \frac{m(m+1) - (1-\nu)}{1+\nu} I \right] \\ C_m &= (1+I) C^2 \frac{m(m+1) - (1-\nu)}{1+\nu} I \\ D_m &= B_m / [m(m+1)] \\ M &= \rho a / (\rho_s h) \\ C^2 &= E / [\rho_s (1-\nu) C^2] \\ I &= h^2 / (12a^2). \end{aligned} \quad (15)$$

Solving equation (1) with the boundary condition, equation (13), and using equation (11) and (12), the scattered pressure can be written as

$$p_m^{\text{sca}}(R) = \frac{-h_m'(kR)}{\omega h_m'(ka)} \rho C (\dot{w}_m - \dot{u}_m^{\text{inc}}) \quad (16)$$

where

$$\dot{u}_m^{\text{inc}} = -\frac{\omega}{\rho C} p^0 (2m+1) i^m j_m'(ka) \quad (17)$$

$h_m$  is the spherical Hankel function of the first kind and a prime denotes differentiation of the Bessel functions with respect to its argument. Evaluating equation (16) at  $R=a$  gives the exact relationship between the scattered pressure on S and the normal acceleration of the shell. Combining this and equations (12) and (14) yields the normal acceleration

$$-\omega^2 w_m = \frac{p^0 C^2 (2m+1) i^m + 1}{a^2 \Gamma_m h_m'(ka) + \rho C^2 k a^2 h_m(ka)} \quad (18)$$

Substituting this result back in equation (14) and (16) and performing summations as required in equation (11), the total pressure distribution on S and the far field scattered pressure can be determined. Alternatively, the resulting distributions of pressure and normal acceleration on S can be used in conjunction with equation (4) to calculate the scattered pressure. Following reference [4], the echo function is defined as

$$f_{\infty} = f_{\infty}(R, \theta, ka, C, M, h/a) = \frac{2R}{p^0} p^{sca}(R, \theta) e^{-ikR}$$

with the time factor  $\exp(-i\omega t)$  in  $p^{sca}$  omitted.

The approximate equations (7) through (10) are of the same nature as equation (16), i.e., they are relations between the scattered pressure  $p^{sca}$  and the normal acceleration at S. For a spherical surface, it has been shown in reference [6] that the values of M, A and  $\Omega$  for the mth mode are respectively  $4\pi a^3 \rho / [(2m+1)(m+1)]$ ,  $4\pi a^2 / (2m+1)$  and  $(m+1)c/a$ . Thereupon, the modal forms of equations (7) through (10) are respectively

$$\begin{aligned} \frac{\rho a}{m+1} \ddot{p}_m^{sca} + \rho c p_m^{sca} + \frac{\rho c (m+1)}{a} p_m^{sca} = \\ \rho c \frac{\rho a}{m+1} (\ddot{w}_m - \dot{U}_m^{inc}) + \frac{(m+1)c}{a} (\ddot{w}_m - \dot{U}_m^{inc}) \end{aligned} \quad (20)$$

$$\frac{\rho a}{m+1} \dot{p}_m^{sca} + \rho c p_m^{sca} = \rho c \frac{\rho a}{m+1} (\ddot{w}_m - \dot{U}_m^{inc}), \quad (21)$$

$$p_m^{sca} = \rho c (\dot{w}_m - U_m^{inc}) \quad (22)$$

and

$$p_m^{sca} = \frac{\rho a}{m+1} (\ddot{w}_m - \dot{U}_m^{inc}). \quad (23)$$

Each of these can be used with equation (14) for determining the distributions of pressure and normal acceleration on S.

## RESULTS AND DISCUSSION

Numerical calculations are carried out for the following material property values and thickness to radius ratio

$$\begin{aligned} \rho &= 1000 \text{ Kg/m}^3, \\ c &= 1410 \text{ m/sec}, \\ \rho_s &= 2.7 \times 10^3 \text{ kg/m}^3, \\ E &= 6.74822 \times 10^{10} \text{ Newton/m}^2, \\ \nu &= 0.35506, \\ h/a &= 2.5/98.75. \end{aligned} \quad (24)$$

Plots of the amplitudes of the backscattered echo function  $f_{\infty}$  at  $(R=20a, \theta=\pi)$  versus  $ka$  resulted from exact and approximate  $p^{sca}-\dot{w}$  relations are presented here.

The exact  $|f_{\infty}|$  values are computed using the Helmholtz integral, equation (4), and alternately the series summation of equation (16). Both results agree completely with each other and coalesce into one single curve as displayed in Figure 2. The maximum amplitude of this backscattering echo function  $|f_{\infty}|$  is about unity for the hard sphere case and is 0 for the soft sphere case [10]. Here the two sharp peaks of  $|f_{\infty}|$  between  $ka$  equals 1 and 2 with amplitudes close to 8 dramatically reveal the significant effect of elastic vibration. The convergence criterion for the summations of the Legendre series for the pressure and acceleration on  $S$  and the far field scattered pressure is

$$(\sum^{m+1} - \sum^m) / \sum^m < 0.00001 \quad (25)$$

where  $\sum^m$  represents a sum of  $m$  terms in equation (11). The number of terms required by this criterion depends on  $ka$ , e.g., 5 terms are needed for  $ka=1$  and 25 terms for  $ka=15$ . In the numerical integration of equation (4), the spherical surface  $S$  is divided into a number of rings the widths of which are always less than  $1/6$  of the incident wave length. All subsequent results for  $|f_{\infty}|$  are obtained by equation (4) using the same summation convergence criterion and integration requirement.

Figure 3 plots the  $|f_{\infty}|$  curve obtained using the added mass approximation (AMA), equation (23). As anticipated, the AMA agrees with the exact solution for small values of  $ka$  ( $ka < 1$ ). It correctly predicts the location of the first sharp peak but over predicts its amplitude by more than 50% attributable to the neglect of the radiation damping of the shell vibration. For higher  $ka$  values, the AMA results are erroneous.

Figure 4 is a juxtaposition of the exact and plane wave approximation (PWA), equation (22), solutions. They agree extremely well for  $ka > 5$ . Here, it is evident that the PWA should not be used for low frequency situations.

Figure 5 compares the exact and the first order doubly asymptotic approximation (DAA1), equation (21), solutions. It can be seen that the DAA1 is quite effective for  $ka < 1$  and  $ka > 5$ . However it fails to predict the two important sharp peaks in  $1 < ka < 1.5$ . This is also anticipated since it has been known that the DAA1 tends to overestimate fluid radiation damping in the intermediate frequency range [11,6].

Finally, figure 6 presents the  $|f_{\infty}|$  curve obtained using the second order doubly asymptotic approximation (DAA2), equation (20). Albeit the magnitude of the second peak is less satisfactorily predicted, the DAA2 result here is a very viable approximation to the exact solution for the entire  $ka$  range.

## CONCLUSION

It appears that the second order doubly asymptotic approximation (DAA2) is promising for predicting acoustic echo signals using the present scheme. For very low or very high frequency situations, simpler approximations can be employed. If the neighborhood of frequency in which the structural resonance effect is significant is known a priori, the exact method should be used in this neighborhood.

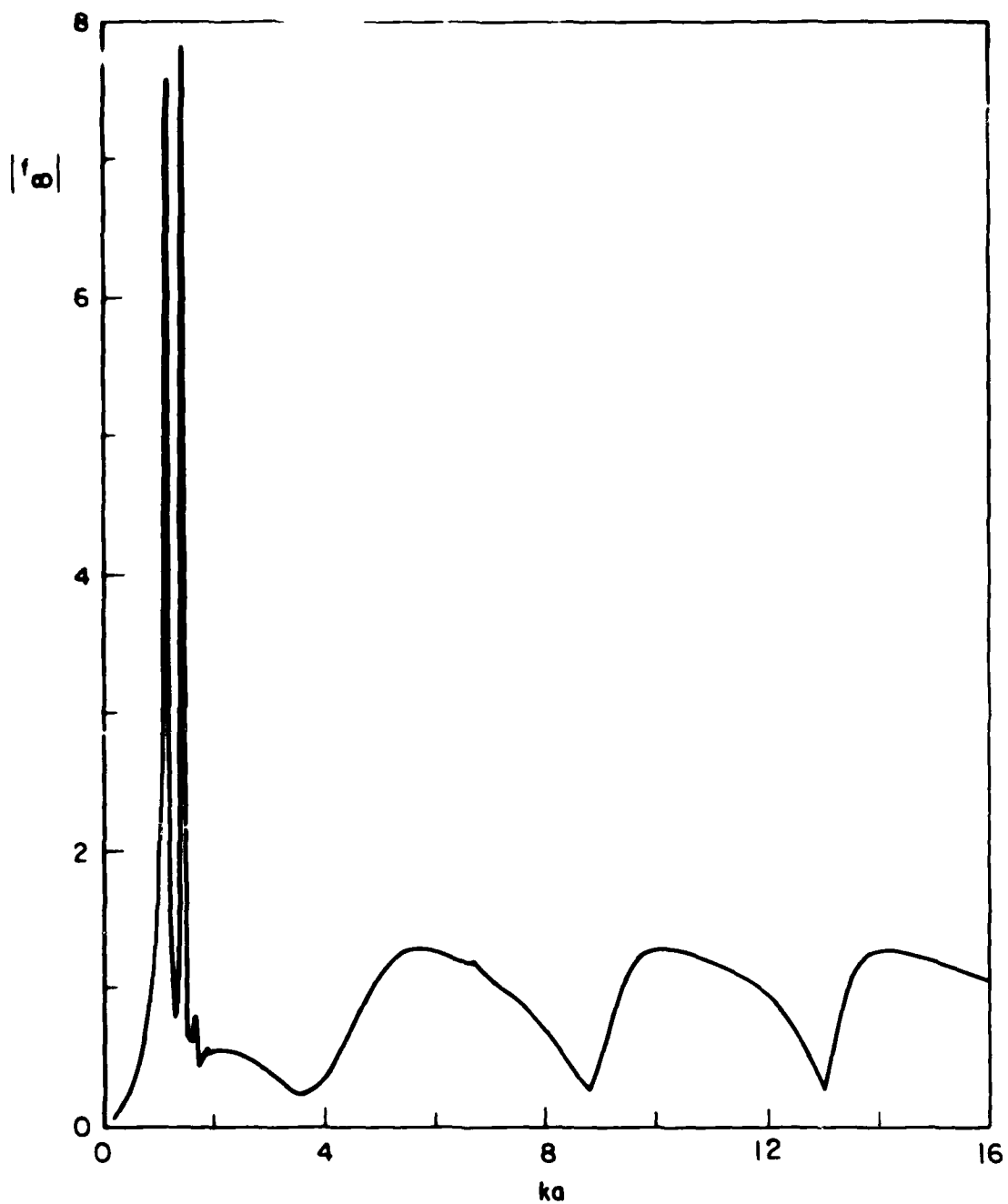


Fig. 2 — Exact pressure amplitude of the Echo returned by a spherical elastic shell

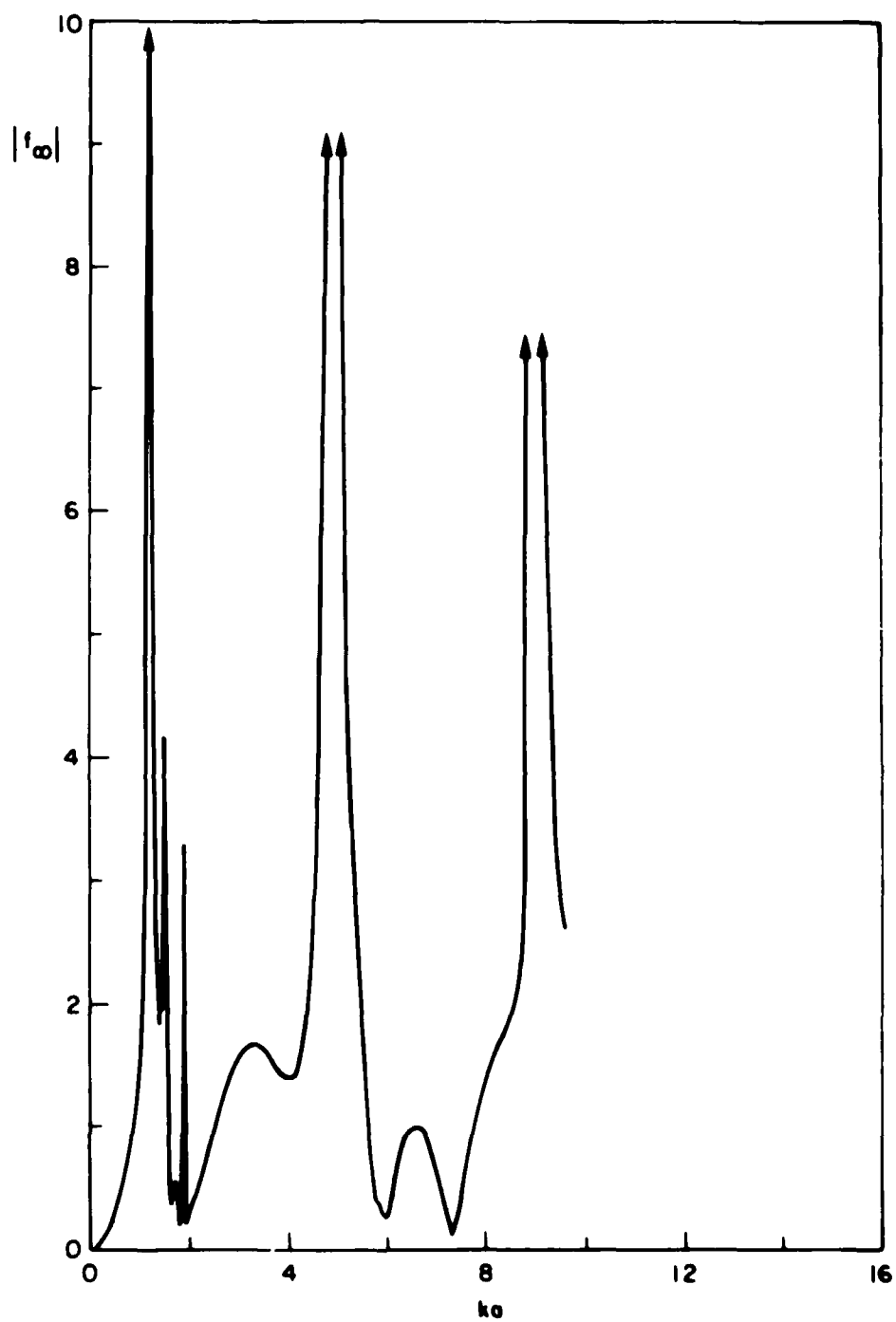


Fig. 3 — Pressure amplitude of the Echo returned by a spherical elastic shell calculated from surface pressure and acceleration obtained by the added mass approximation.

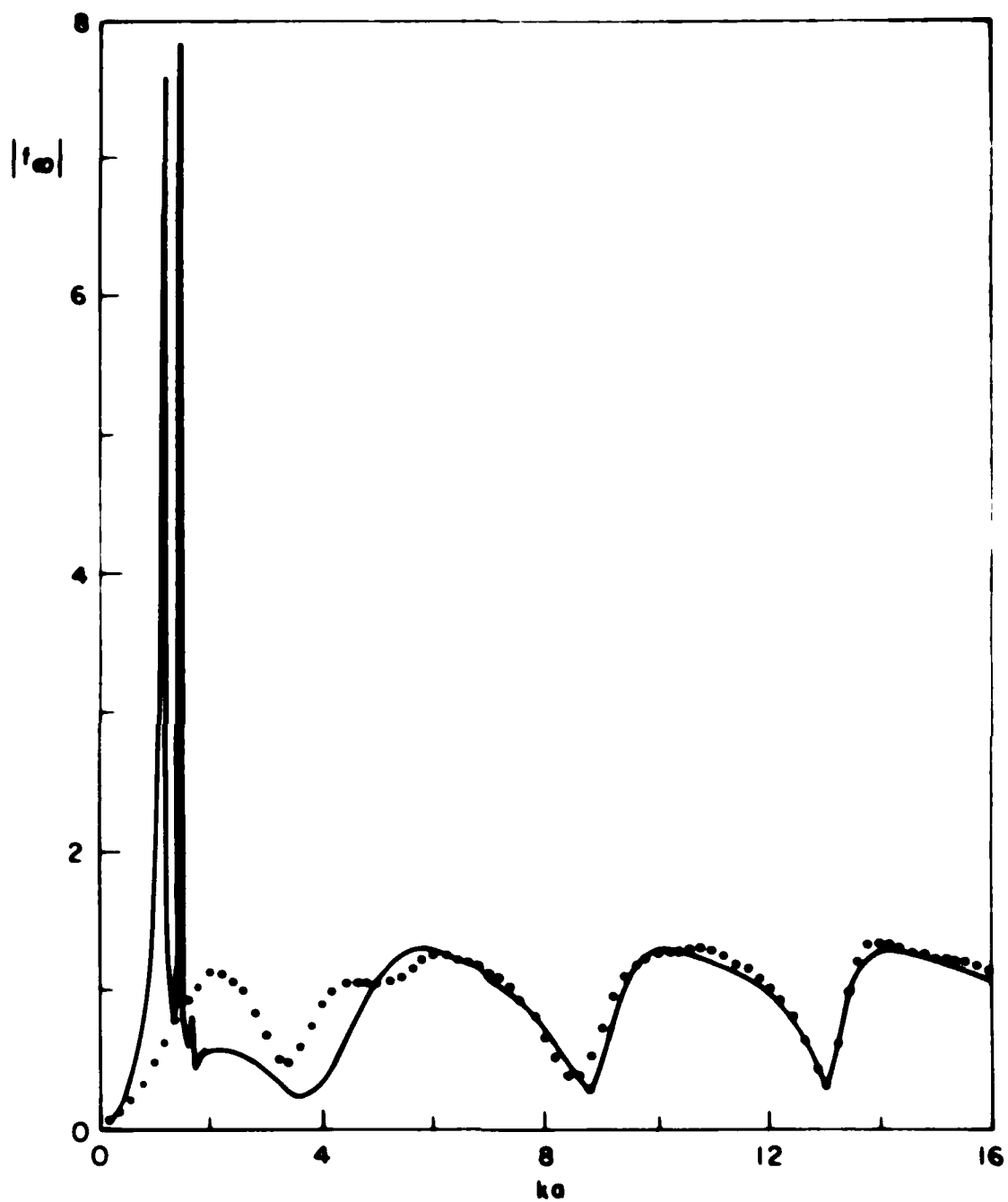


Fig. 4 — Pressure amplitude of the Echo returned by a spherical elastic shell. The solid and dotted lines are respectively calculated from the exact and the plane wave approximation of the surface pressure and acceleration.

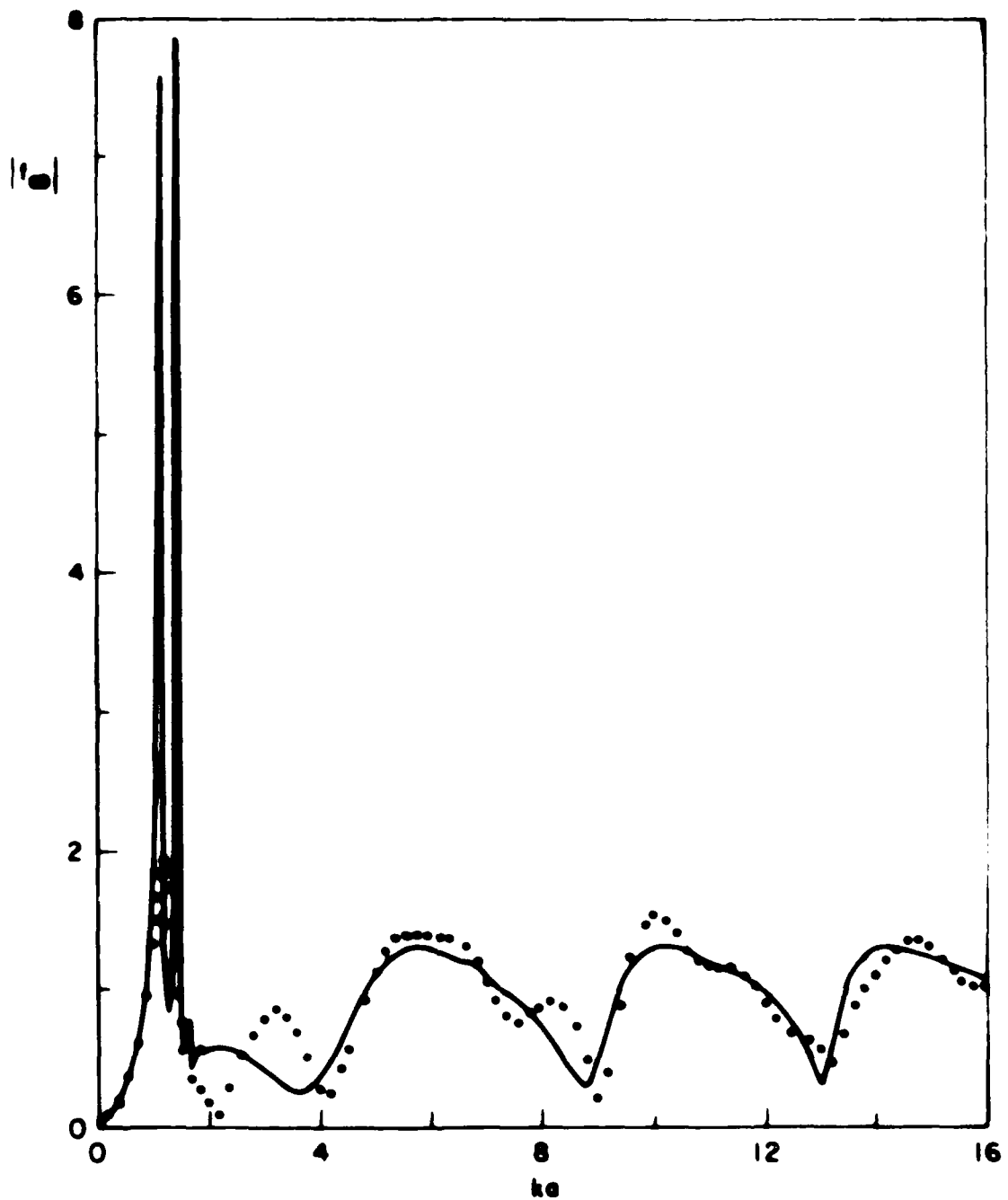


Fig. 5 — Pressure amplitude of the Echo returned by a spherical elastic shell. The solid and dotted lines are respectively calculated from the exact and the first order doubly asymptotic approximation of the surface pressure and acceleration.

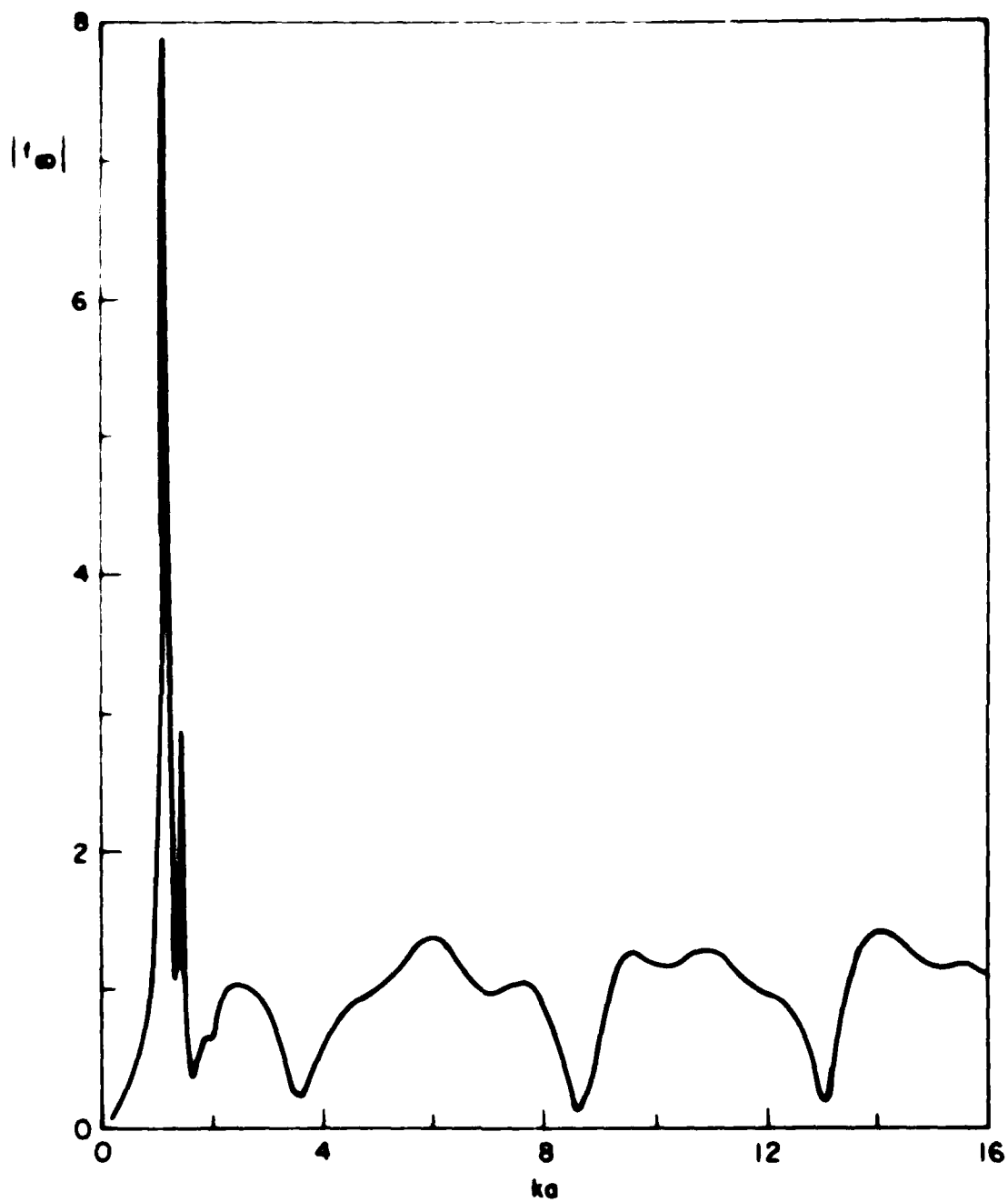


Fig. 6 — Pressure amplitude of the Echo returned by a spherical elastic shell calculated from surface pressure and acceleration obtained by the second order doubly asymptotic approximation.

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