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Iterative Abel Inversion of Optically Thick, Cylindrically Symmetric Radiation Sources

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This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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An inversion algorithm for obtaining the radial emission and absorption coefficients of a cylindrically symmetric radiation source from transverse profiles of monochromatic radiance and absorptance is described and compared with the algorithm of Elder et al. It is shown that the present method converges to the correct solution, whereas the Elder et al. method does not.

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ITERATIVE ABEL INVERSION OF OPTICALLY THICK, CYLINDRICALLY SYMMETRIC RADIATION SOURCES

The transverse monochromatic radiance and transmittance profiles for a cylindrically symmetric radiation source are related to the radial emission and absorption coefficients by the radiative transfer equations[1]

$$S(z) = 2\int_z^R J(r) \cosh G(z, r) \frac{r\,dr}{(r^2 - z^2)^{1/2}} \qquad (1)$$

and

$$-\ln \tau(z) = 2\int_z^R K(r) \frac{r\,dr}{(r^2 - z^2)^{1/2}} = 2\,G(z, R) \qquad (2)$$

where $S(z) = N(z)/\tau^{1/2}(z)$; $N(z)$, $\tau(z)$, and z are, respectively, the transverse radiance, transmittance, and coordinate; $J(r)$, $K(r)$, and r are, respectively, the radial emission coefficient, absorption coefficient, and coordinate; R is the source radius; and

$$G(z, r) = \int_z^r K(r') \frac{r'\,dr'}{(r'^2 - z^2)^{1/2}} \qquad (3)$$

These equations are valid for arbitrary optical thickness of the source. The problem addressed is the solution for $J(r)$ and $K(r)$ from $N(z)$ and $\tau(z)$ by inversion of Eqs. (1) and (2). No consideration is given here to the problem of random or bias error propagation in the inversion.

Regardless of optical depth, the transmittance equation is an Abel integral, and $K(r)$ can be found immediately from $K(r) = \mathscr{A}[-\ln \tau(z)]$ where $\mathscr{A}$ is the Abel transformation operator

$$\mathscr{A}[f(z)] = \frac{1}{\pi}\int_r^R \frac{df(z)}{dz} \frac{dz}{(z^2 - r^2)^{1/2}}. \qquad (4)$$

For optically thin sources, that is, for $RK(r) << 1$ for all r, $\cosh G(z, r) \simeq 1$ and the radiance equation also reduces to an Abel integral equation. Then, Eq. (4) can be used to obtain $J(r) = \mathscr{A}[S(z)]$. If the source is not optically thin, an iterative procedure can be used to obtain $J(r)$.

Elder et al.[1] proposed the following iterative solution

$$J(r) = J_0(r) + J_1(r) + J_2(r) + \ldots$$

where

$$J_0(r) = \mathscr{A}[S(z)]$$

and

$$J_n(r) = \mathscr{A}\left[2\int_z^R J_{n-1}(r)\left[\cosh G(z, r) - 1\right] \frac{r\,dr}{(r^2 - z^2)^{1/2}}\right] \qquad (5)$$

The summation is truncated when $J_n(r)$ is less then some preassigned minimum. The authors claimed rapid convergence with this method and quoted a 4% accuracy with only two iterations for values of the centerline transmittance $\tau(z=0)$ as small as ~0.2. Unfortunately, although the convergence is rapid, it is not to the correct solution. The flaw in the scheme is the implicit correction of J(r) based on an Abel inversion of the difference between the previous iteration result for S(z) and the <u>effective thin source</u> value of S(z), that is, the value of S(z) computed from Eq. (1) with J(r) taken as $J_0(r)$. Intuitively, the correction should be based on an Abel inversion of the difference between the previous iteration for S(z) and the <u>known</u> profile S(z). The iteration scheme based on this reasoning is

$$\left.\begin{aligned} J_n(r) &= J_{n-1}(r) - \mathscr{A}\left[2\int_z^R J_{n-1}(r)\cosh G(z,r)\frac{r\,dr}{(r^2-z^2)^{1/2}} - S(z)\right] \\ &\text{where} \\ J_0(r) &= \mathscr{A}[S(z)]. \end{aligned}\right\} \quad (6)$$

Here, $J_n(r)$ is the n<u>th</u> iteration result for J(r) and not a correction term as in Eq. (5).

Inversions with Eqs. (5) and (6) were carried out for simple example test cases in which both K and J are constants. From Eqs. (1) and (2), the transverse input profiles are then found to be

$$S(z) = \frac{2J}{K} \sinh [K (R^2 - z^2)^{1/2}]$$

and (7)

$$-\ln \tau(z) = 2K (R^2 - z^2)^{1/2} .$$

R and J were set to unity, and K was selected to give the center line transmittance $\tau(z = 0) = 0.9, 0.8, \ldots, 0.1$. Inversions of these profiles were then carried out with the two methods. Iteration was continued until $J(r = 0)$ differed by less than 1% between iterations. A numerical quadrature formula based on the method of Barr[2, 3] was used for both the iterative calculation of S(z) and the Abel transformation. The final J retrieved from the two inversion algorithms are shown in Fig. 1. It is evident that the method of Elder et al. does not converge to the correct solution, whereas the present method does. The rate of convergence is the same for the two methods as is indicated at the top of the figure by the number of iterations required to achieve the convergence criterion.

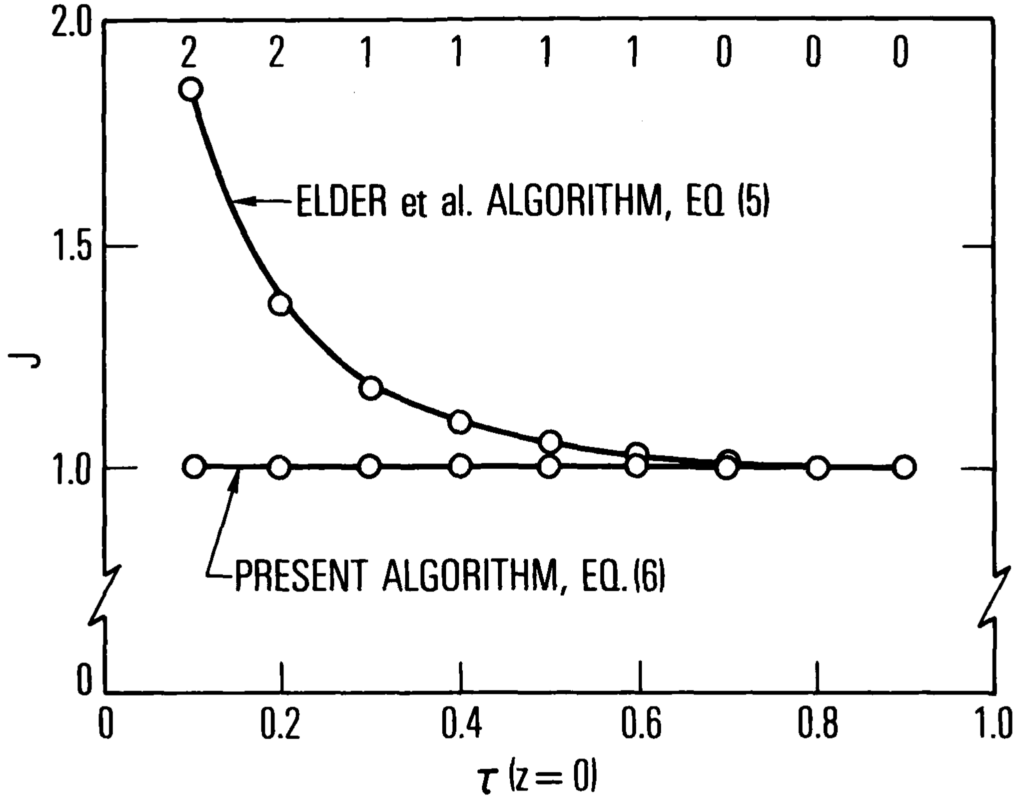

Fig. 1. **Results for retrieved emission coefficient J using two iterative inversion algorithms. The true value is J = 1. Numbers along the top are the number of iterations required for 1% convergence and are the same for each method. Zero indicates that only an Abel inversion was required.**

Extensive verification and use of the present inversion algorithm has been made within the context of band model radiation formulations for axisymmetric rocket plume sources.[3] In that application, the only essential differences from the present monochromatic application are that the kernel function $\cosh G(z, r)$ is replaced by a different (and more complicated) function, the radiance equation is formulated for $N(z)$ rather than $S(z)$, and the absorptance equation is no longer an Abel integral equation regardless of optical thickness. The significant consequence of the latter difference is that in the band model application, simultaneous iterative solutions must be carried out for both $J(r)$ and $K(r)$. The method was shown to be stable and accurate for a wide range of values and profiles for $J(r)$ and $K(r)$.

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