COHERENT SCATTER OF MICROWAVES
FROM MODERATELY ROUGH SURFACES

M. M. WEINER
G. A. ROBERTSHAW

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M. ZYMARIS, GS-13
Project Engineer

PAUL M. SULLIVAN, LT COL USAF
Deputy Director
Technical Surveillance Systems Directorate

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SECTION I

INTRODUCTION

The effect of surface roughness on coherent scatter of microwaves from terrain and sea surfaces, in a multipath mode of propagation, is most commonly modeled by assuming that the surface height is a zero mean independent random variable which is normally distributed. That model gives good agreement with experimental data points for slightly rough surfaces but considerably underestimates coherent scatter from moderately rough surfaces. The discrepancy has been attributed by various investigators to multiple-scattering, shadowing, and depolarization effects which are neglected by the model. It has also been speculated that the model may not be strictly valid because it is usually derived for a receiver in the Fraunhofer far-field of an illuminated surface, whereas receivers are actually in the Fresnel near-field of an extended scattering surface.

This memorandom reports an improved model for coherent scatter from moderately rough surfaces with a two-dimensional height profile. The surface height is assumed to be a zero mean random variable which is exponentially distributed. This model is found to give good agreement with experimental data for moderately rough as well as slightly rough surfaces. Multiple-scattering, shadowing, and depolarization effects are neglected, but the model is equally valid in both the near and far-fields of the illuminated surface. The model lends itself to an interesting physical interpretation of the stochastic process associated with the surface profile. In previous models of stochastically rough surfaces, the surface profile is usually described in terms of a height independent random variable and either a slope independent random variable or an arbitrarily assumed autocorrelation function. In the proposed model, the surface profile is characterized by a dependent height random variable which

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is a function of two independent base random variables and an
independent angle tangent random variable.

Nomenclature, the proposed model, and its comparison with
experimental data are discussed in that order in the following
sections.

SECTION II

NOMENCLATURE

The concepts of surface roughness reflection coefficient, different forms of scatter (specular, diffuse, coherent and incoherent), and the Rayleigh roughness parameter are defined in this section.

Consider an experiment (see Figure 1) in which a transmitter of wavelength λ illuminates a two-dimensional patch of rough surface (such as terrain or the ocean) and a receiver measures the intensity of the resultant field scattered by each point $P(x, y, z)$ of the patch with height profile $h(x, y)$ with respect to the mean surface level (MSL). A "patch" is the total surface area illuminated by the incident radiation at a given instant of time. The central ray of the illumination is incident on the MSL at a grazing angle ψ_i and is scattered to a receiver positioned at a grazing angle ψ_r. At the receiver, let $E_h (\psi_i, \psi_r)$ be the complex amplitude of the resultant field scattered by the patch of rough surface and let $E_o (\psi_i, \psi_r)$ be the complex amplitude of the resultant field if the surface were perfectly smooth. The effect of surface roughness is to reduce the field scattered in the specular direction ($\psi_r = \psi_i$) and to increase the field scattered in other directions. This effect of surface roughness may be characterized by a surface roughness amplitude reflection coefficient, ρ, defined by

$$\rho = \rho (\psi_i, \psi_r) = E_h (\psi_i, \psi_r)/E_o (\psi_i, \psi_i) \qquad (1)$$

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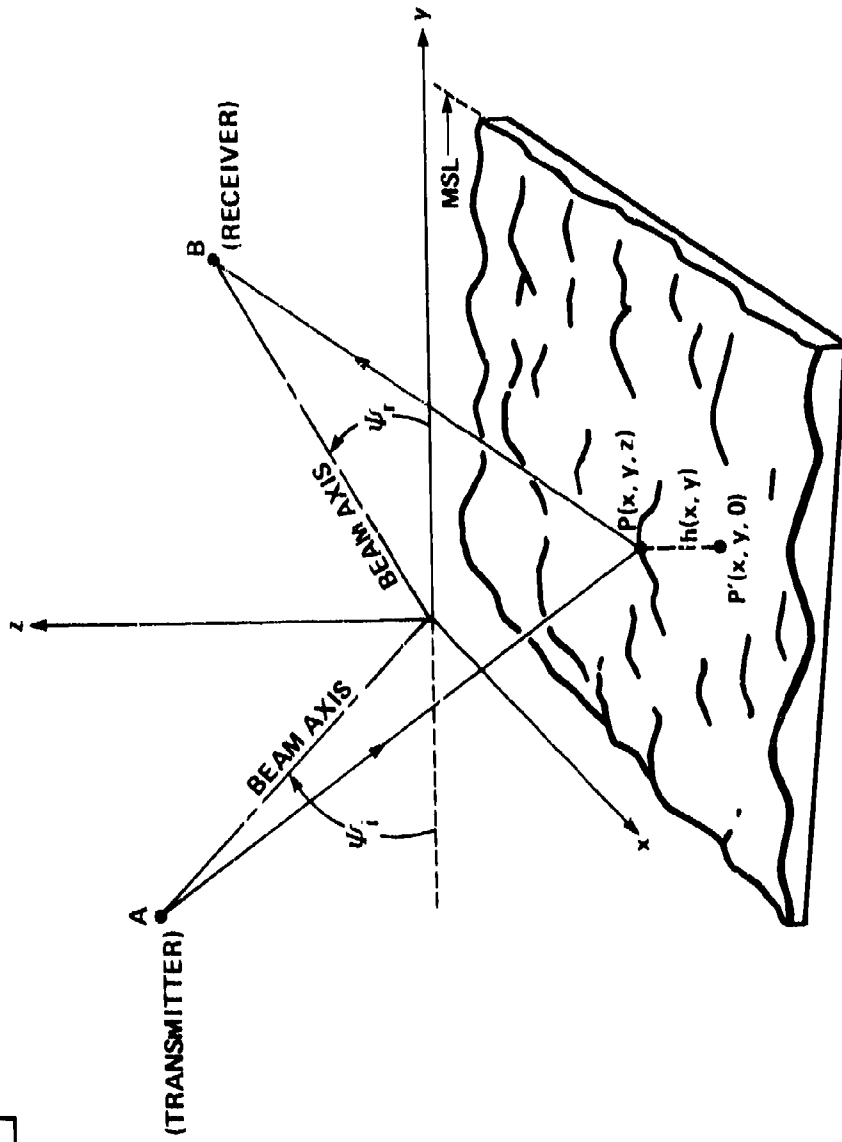
FIGURE 1. Rough Surface Scattering Geometry

The corresponding surface roughness power reflection coefficient is $\rho\rho* = |\rho|^2$. The overall amplitude reflection coefficient in the specular direction is given by the product of ρ, the Fresnel reflection coefficient, and a spherical earth divergence coefficient*.

If the experiment is performed on a single patch of rough surface whose surface profile is arbitrary but deterministic, then the intensity reflection coefficient may be written as the sum of specular and diffuse reflection coefficients defined by

$$|\rho|^2 = |\rho_s|^2 + |\rho_d|^2 \qquad (2)$$

where

$$|\rho_s|^2 = \left| \frac{E_h \, (\psi_i, \, \psi_i)}{E_o \, (\psi_i, \, \psi_i)} \right|^2 \left| \frac{E_o \, (\psi_i, \, \psi_r)}{E_o \, (\psi_i, \, \psi_i)} \right|^2 = \text{specular intensity reflection coefficient}$$

$$|\rho_d|^2 = \left| \frac{E_h \, (\psi_i, \, \psi_r)}{E_o \, (\psi_i, \, \psi_i)} \right|^2 - |\rho_s|^2 = \text{diffuse intensity reflection coefficient}$$

It should be noted that for $\psi_r = \psi_i$, $|\rho_d|^2 = 0$. The coefficient $|\rho_s|^2$ is a function of whether the surface is extended or non-extended. For an extended surface (i.e., the surface extends beyond the illumination incident upon it), the receiver is generally in the Fresnel near-field of the illuminated surface even though the surface may be in the Fraunhofer far-field of the transmitter and receiver antennas.[1] For a non-extended surface (i.e., the surface extent is less than the extent of the illumination incident upon it), the receiver is in the Fraunhofer far-field of the illuminated surface if the surface is in the Fraunhofer far-field of the

---

*It is assumed that the local Fresnel reflection coefficient does not vary appreciably over the surface patch. (See Reference 3, pp. 22, 68 and 244).

transmitter and receiver antennas.[1] The factor $\left|\dfrac{E_h\;(\psi_i,\;\psi_i)}{E_o\;(\psi_i,\;\psi_i)}\right|^2$

is known in optics as the Strehl intensity S for the case when the

receiver is in the Fraunhofer far-field of a non-extended surface.[2]

The factor $\left|\dfrac{E_o\;(\psi_i,\;\psi_r)}{E_o\;(\psi_i,\;\psi_i)}\right|^2$ is the smooth surface diffraction

intensity pattern which, for an extended surface, reduces to the

transmitter antenna pattern $G_t\;(\psi_r-\psi_i)/\;G_t\;(0)$.

Thus far, the discussion has been limited to an experiment
defining the reflection coefficient for a single, deterministic patch
of rough surface. Consider now the case in which the experiment is
repeated under the same measurement conditions for an ensemble of
many different patches of the rough surface. For a surface profile
characterized by a stochastic process, the expected value of the
intensity reflection coefficient averaged over the ensemble of
clutter patches may be written as the sum of coherent and incoherent
reflection coefficients defined by[3]

$$\langle \rho\rho*\rangle = |\langle\rho\rangle|^2 + \langle|\rho-\langle\rho\rangle|^2\rangle \qquad (3)$$

$$= \text{expectation of intensity reflection coefficient}$$

where

$$|\langle\rho\rangle|^2 = \langle\rho\rangle\langle\rho*\rangle = |\text{expectation of } \rho|^2 = \begin{array}{l}\text{coherent intensity} \\ \text{reflection coefficient}\end{array}$$

$$\langle|\rho-\langle\rho\rangle|^2\rangle = \langle(\rho-\langle o\rangle)\;(\rho-\langle\rho\rangle)*\rangle$$

$$= \text{variance of } \rho = \text{incoherent intensity reflection} \\ \text{coefficient.}$$

The terminology "incoherent" which is widely found in the
literature[4] for the variance of ρ is rather unfortunate and is
appropriate only for the case $\langle\rho\rangle = 0$ because for such a case
$\langle|\rho-\langle\rho\rangle|^2\rangle$ is proportional to the sum of the power reflection

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coefficients of each sample patch. For terrain and sea scatter in the forward direction, at shallow grazing angles of incidence, the coherent reflection coefficient is usually the dominant reflection coefficient. The incoherent reflection coefficient, which is generally not equal to zero for a rough surface even in the specular direction $(\psi_i = \psi_r)$, reduces to zero for a perfectly smooth surface or for the non-physical case of zero correlation length of the surface profile. The following sections are limited to a discussion of the coherent reflection coefficient.

A parameter commonly used to characterize the roughness of a surface is the Rayleigh parameter, $g^{1/2}$, defined by

$$g^{1/2} = (2\pi/\lambda) \ (\sin \psi_i + \sin \psi_r) \ \sigma_H \qquad (4)$$

where

$\sigma_H = \langle h^2 \rangle^{1/2} =$ standard deviation of the surface height
random variable H

$h(x, y) = z =$ height of the surface profile at a point
$P(x, y, z)$ above the MSL = value of the height
random variable H .

According to the Rayleigh criterion, surfaces may be characterized as being "smooth" if $g^{1/2} < \pi/2$ radians and "rough" if $g^{1/2} > \pi/2$ radians.[5] We shall arbitrarily characterize a surface as being "moderately rough" if $\pi/2 \leq g^{1/2} \leq 2\pi$ radians.

SECTION III

PROPOSED MODEL

The expected value $<\rho>$ of the amplitude reflection coefficient, averaged over an ensemble of many surface patches whose surface profiles are generated by a zero mean stochastic process, is given by[3], [4]

$$<\rho> = <\rho_o> [G_t (\psi_r - \psi_i)/G_t (0)]^{1/2} \tag{5a}$$

$$<\rho_o> = <\exp [-i (2\pi/\lambda) (\sin \psi_i + \sin \psi_r) h (x, y)]> \tag{5b}$$

$$= \int_{-\infty}^{\infty} f_H(h) \exp [-i (2\pi/\lambda) (\sin \psi_i + \sin \psi_r) h] dh \tag{5c}$$

where

ρ_o = normalized amplitude reflection coefficient

$[G_t (\psi_r - \psi_i)/ G_t (0)]^{1/2}$ = transmitter antenna pattern function

$f_H(h)$ = probability density function of the height random variable H.

Equation (5b) is derived assuming a physical optics approximation in which multiple-scattering, shadowing, and depolarization effects are neglected, $\cos (\psi_r - \psi_i) \approx 1$, and the beam width of the incident radiation is small. Shadowing and multiple-scattering effects are particularly important at very low grazing angles of incidence because not only is the surface no longer uniformly illuminated but, more importantly, because smooth earth spherical diffraction and knife-edge diffraction become the dominant mode of propagation rather than ground multipath propagation. Equation (5b) is valid for ensemble averages in either the near-field or far-field of the illuminated patches.*

---

*If the averaging is performed over a single surface patch whose profile is arbitrary but deterministic, Equation (5b) is valid only in the Fraunhofer far-field of the illuminated surface.[1], [2]

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In one of the most commonly used models, the height random variable H is assumed to be normally (Gaussian) distributed.[6],[7] That model is reported to give good agreement with experimental data for smooth surfaces but underestimates the reflected coherent power for moderately rough surfaces in the cases of scattering of microwaves from the ocean[7], from different terrain and ocean surfaces[8], and for scattering of acoustical waves from a surface model submerged in a water tank[4]. In this latter case, Boyd and Deavenport found that good agreement with acoustical data could be obtained for $g^{1/2}$ as large as 7 if the height irregularities were distributed with a probability density similar to a symmetrical exponential distribution but artificially modified to have a larger peak for h = 0. Boyd and Deavenport also acknowledged that their model was in better agreement with the microwave data of Beard than the normally distributed model.

Consider now a model based on a pure zero mean, symmetrical exponential probability density for the height random variable H. For this density, the random variable H shall be referred to as being "exponentially distributed". In the following section, it is shown that this model gives better agreement with microwave data than the model of Boyd and Deavenport and almost as good agreement with acoustical data for $g^{1/2} \leq 2\pi$. The exponentially distributed model lends itself to the following interesting physical interpretation of the stochastic process associated with the two-dimensional profile of a three-dimensional rough surface.

A three-dimensional rough surface consists of regions which extend above the MSL (labelled +), below the MSL (labelled -), and on the MSL (labelled 0), as shown in Figure 2. These regions intersect the MSL in base areas enclosed by perimeters shown in Figure 2a. Consider an arbitrary point P (x, y, z) of the surface profile.

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TAN ∝ = A = ANGLE TANGENT R.V.

$\overline{P'Q}$ = B = BASE R.V.

$\overline{P\,P'}$ = H = BA = HEIGHT R.V.
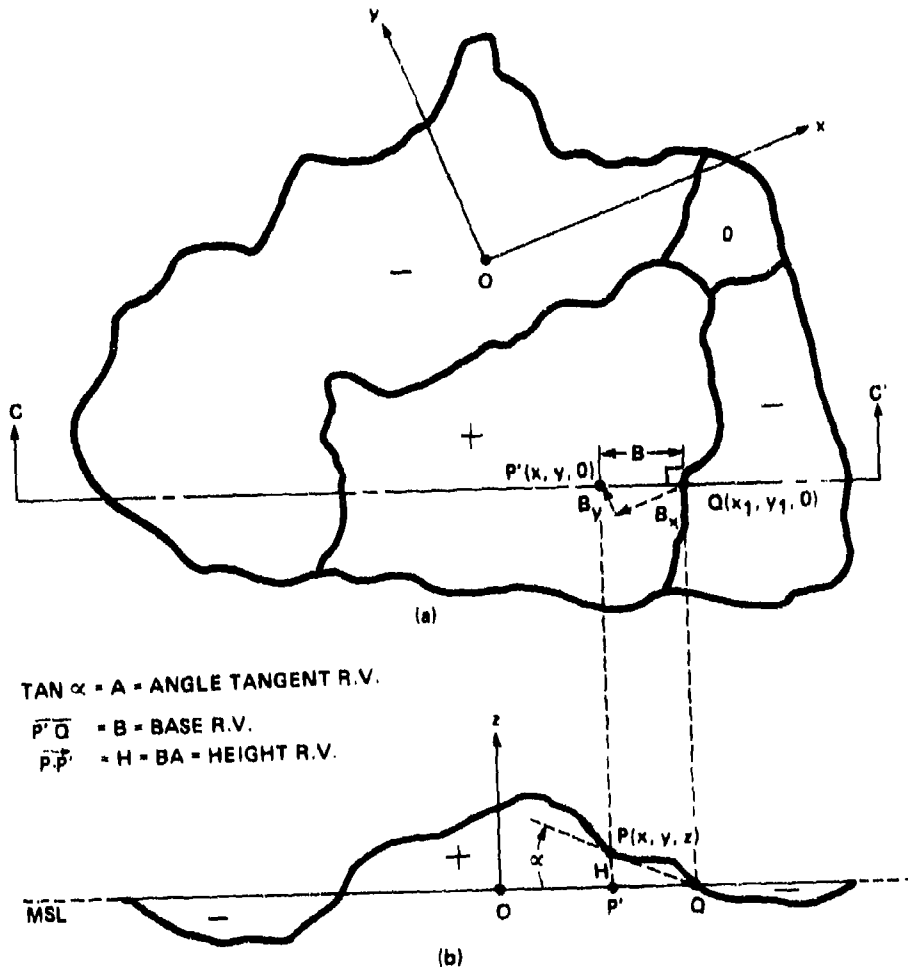
FIGURE 2. Surface Profile Random Variables
(a) Plan View
(b) Elevation View (Section C-C')

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Its projection P' (x, y, 0) onto the MSL is shown in Figure 2a.
Let Q (x_1, y_1, 0) be the point on the base perimeter which is
closest to the point P' (z, y, 0). The distance $\overline{P'Q}$ is defined
as the base random variable B whose components along the x and y
axes are the base random variables B_x and B_y , respectively, which
we shall assume to be independent and normally distributed with zero
mean and identical standard deviation σ_B. The base random variable
$B = \sqrt{B_x^2 + B_y^2}$ is therefore Rayleigh distributed. The values of
B_x, B_y, and B are given by $b_x = (x_1 - x)$, $b_y = (y_1 - y)$, b =
$[(x_1 - x)^2 + (y_1 - y)^2]^{1/2}$, respectively. Now consider the acute
angle α which $\overline{PQ}$ makes with respect to the MSL as shown in
Figure 2b. The angle α is positive, negative, or zero, depending
upon whether P is above, below, or in the MSL. The point
P (x, y, z) is at a height h (x, y) above the MSL given by

$$h(x, y) = z = b \tan \alpha \qquad (6)$$

Assume that TAN α = A is a zero mean, normally distributed, random
variable which is designated the "angle tangent" random variable.
The height random variable H is therefore given by

$$H = B A \qquad (7)$$

where H is exponentially distributed. The standard deviation σ_H
of the height random variable is related to the standard deviations
σ_B and σ_A of the base and angle tangent random variables respectively
by

$$\sigma_H = \sqrt{2} \, \sigma_B \, \sigma_A \qquad (8)$$

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Equations (7) and (8) follow from the statistical property that a symmetrical exponentially distributed random variable may be derived from the product of two independent random variables, one of which is normally distributed and the other is Rayleigh distributed.[9] These properties are reviewed in Appendix A. In previous models of stochastically rough surfaces, the surface profile is usually described in terms of a height independent random variable and either a slope independent random variable or an arbitrarily assumed autocorrelation function.[10] In the proposed model, the surface profile is characterized by a dependent height random variable which is a function of two independent base random variables and an independent angle tangent random variable. The standard deviation σ_B is related to the surface profile correlation length, but the exact relationship will be deferred to future discussions of "incoherent" scatter. For incoherent scatter analysis, a joint probability density function for multiple surface points is needed. This will require additional development of the model.

The probability densities, of the height random variable H for the three distributions which have been discussed, are given by

$$
f_H(h) = \begin{cases}
(2\pi\,\sigma_H^2)^{-1/2}\,\exp\left[-(h^2/2\,\sigma_H^2)\right], \text{ normally distributed} & (9a) \\[2ex]
\dfrac{(\frac{3}{2})^{5/8}\,h^{1/4}}{\sigma_H^{5/4}\,2^{1/4}\,\pi^{3/2}}\,\cos(\frac{\pi}{4})\,\Gamma(\frac{1}{4})\,K_{1/4}\left[(\frac{3}{2})^{1/2}\,h/\sigma_H\right], & (9b) \\[1ex]
\qquad\qquad\qquad\qquad\text{pseudo-exponentially distributed} \\[2ex]
(2\,\sigma_H^2)^{-1/2}\,\exp\left[-(\sqrt{2}|h|/\sigma_H)\right], \text{ exponentially distributed} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (9c)
\end{cases}
$$

where $K_n(x)$ is the modified Bessel function of order n and $\Gamma(x)$ is the gamma function.
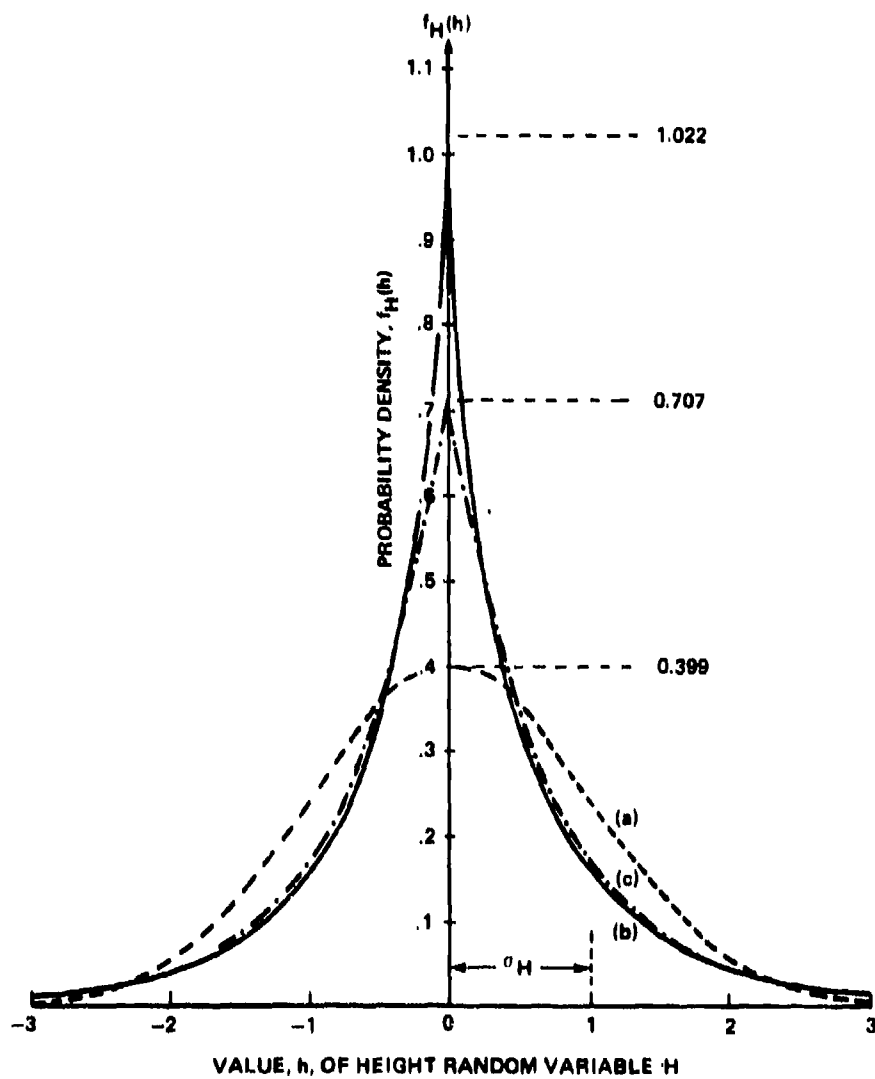
FIGURE 3. Probability Density of the Height Random Variable H for $\sigma_H = 1$.
(a) Normal, (b) Pseudo-Exponential, (c) Exponential.

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These probability densities are plotted in Figure 3 for $\sigma_H = 1$.
The exponentially distributed density is larger than the normally
distributed density for both very small and very large values of $|h|$
and is smaller for intermediate values of $|h|$. The exponential and
pseudo-exponential densities are almost identical except at small
values of h. At h = 0, the exponential density is approximately
equal to the average of the pseudo-exponential and normal densities.

The expected value $\langle \rho_o \rangle$ of the normalized surface roughness amplitude
reflection coefficient is found by substituting Equation (9) into
Equation (5c). Accordingly, the expected value $\langle \rho_o \rangle$ is given by

$$\langle \rho_o \rangle = \begin{cases} \exp(-g/2) & \text{, normally distributed} \quad (10a) \\ \\ (1 + \frac{2}{3}g)^{-3/4} & \text{, pseudo-exponentially distributed} \quad (10b) \\ \\ (1 + \frac{g}{2})^{-1} & \text{, exponentially distributed} \quad (10c) \end{cases}$$

where $g^{1/2}$ is the Rayleigh parameter defined by Equation (4).
Equation (10c) is derived in Appendix A. The corresponding expected
value of the power reflection coefficient, $|\langle \rho_o \rangle|^2$, is given by

$$|\langle \rho_o \rangle|^2 = \begin{cases} \exp(-g) & \text{, normally distributed} \quad (11a) \\ \\ (1 + \frac{2}{3}g)^{-3/2} & \text{, pseudo-exponentially distributed} \quad (11b) \\ \\ (1 + \frac{g}{2})^{-2} & \text{, exponentially distributed} \quad (11c) \end{cases}$$

The amplitude and power reflection coefficients are plotted on
a linear scale in Figures 4 and 5, respectively, as a function of
$g^{1/2}$. The power reflection coefficient is also plotted on a
logarithmic scale in Figure 6. For arbitrary values of $g^{1/2}$, the

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reflection coefficient is smallest for the normally distributed model, largest for the pseudo-exponentially distributed model, and intermediate for the exponentially distributed model. For smooth surfaces, the power reflection coefficient for the normal and pseudo-exponential models differ from the exponential model by a maximum at $g^{1/2} = \pi/2$ of 4.4 dB and 0.6 dB, respectively. However, for moderately rough surfaces, the power reflection coefficient for the normal and pseudo-exponential models differ from the exponential model at $g^{1/2} = \pi$ by 27 dB and 2.3 dB, respectively, and at $g^{1/2} = 2\pi$ by 145 dB and 4.8 dB, respectively.
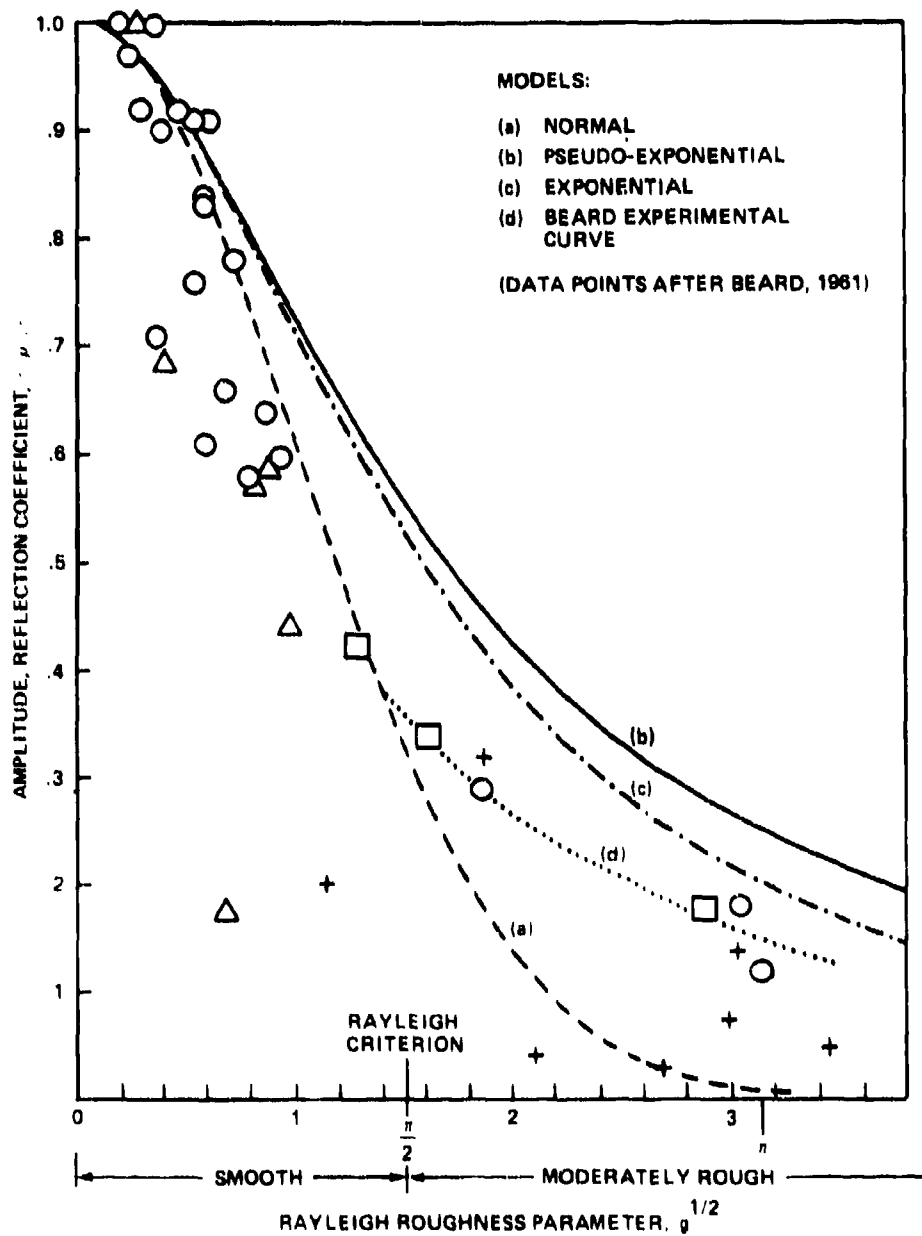
FIGURE 4. Coherent Amplitude Reflection Coefficient in the Specular Direction.
Comparison of Models with Experimental Data for Microwave Scatter from the Ocean.
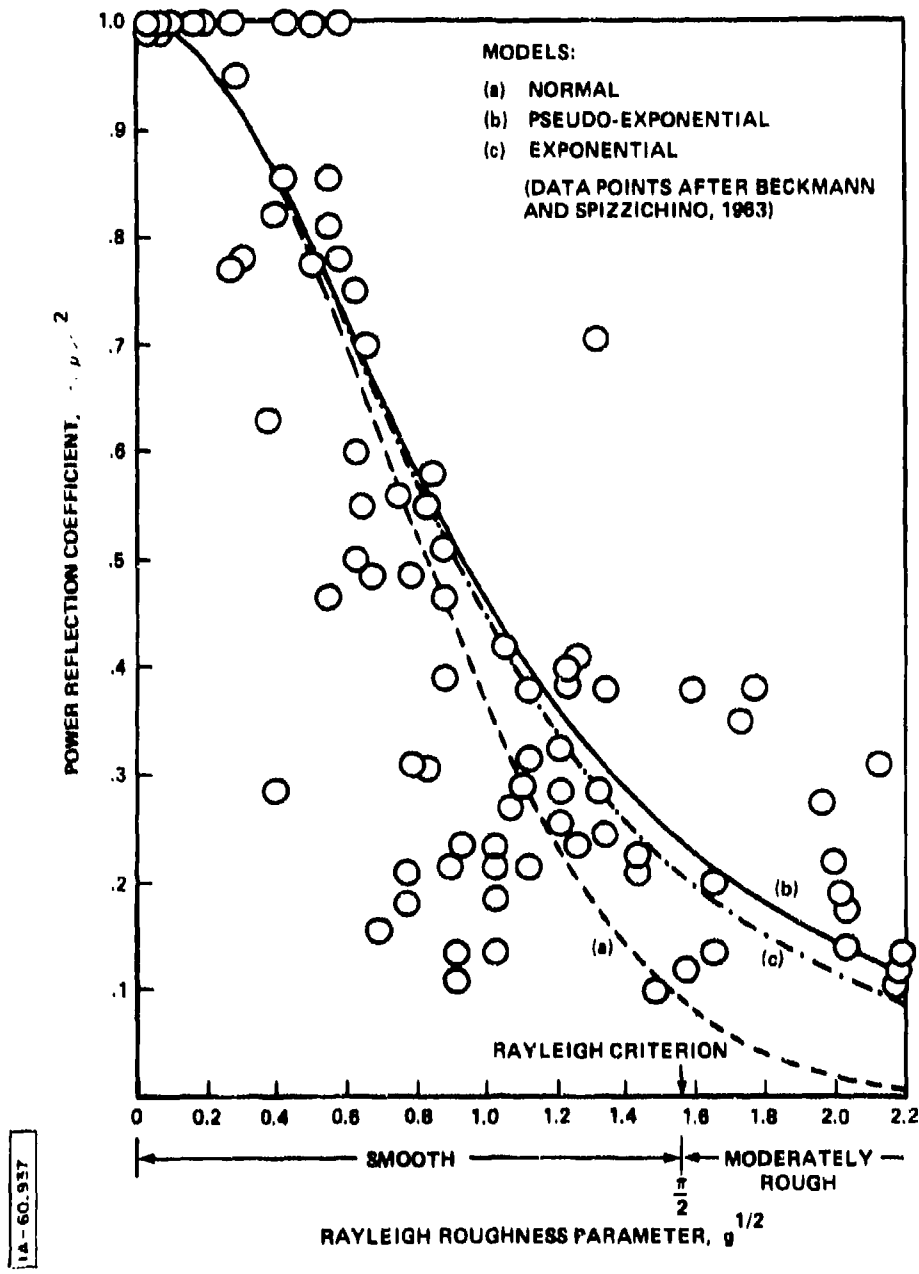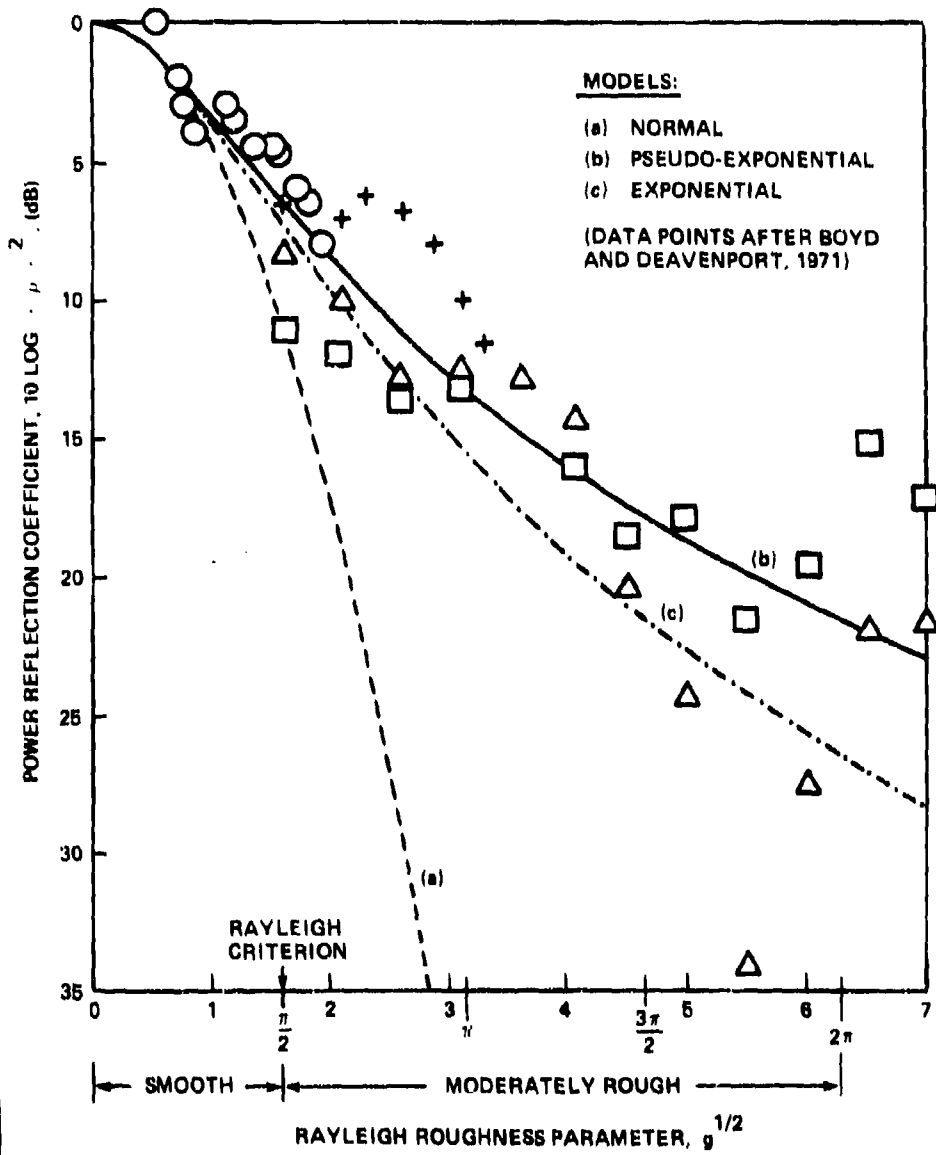
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FIGURE 5. Coherent Power Reflection Coefficient in the Specular Direction.
Comparison of Models with Data for Microwave Scatter from Terrain and Sea Surfaces.

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FIGURE 6. Coherent Power Reflection Coefficient in the Specular Direction. Comparison of Models with Experimental Data for Scatter of Acoustical Waves from a Surface Submerged in a Water Tank.

SECTION IV

COMPARISON WITH EXPERIMENTAL DATA

In Figures 4-6, the models are compared with experimental data for the coherent scattering coefficient in the specularly reflected direction. In the following discussion of Figures 4-6, the coherent amplitude and power reflection coefficients are reduced to the equivalent power reflection coefficient expressed in dB. Table 1 compares the power reflection coefficient predicted by the models with that given by the experimental data. For $g^{1/2} = 0$ and $\pi/4$, all three models are within approximately 1 dB of the data points. The comparison for moderately rough surfaces is examined below.

For microwave coherent scatter from the ocean (Figure 4), the normal, pseudo-exponential, and exponential models differ from Beard's experimental curve by -1.3 dB, +3.0 dB and +2.4 dB, respectively, for $g^{1/2} = \pi/2$ and by -25.6 dB, +6.9 dB, and +1.5 dB, respectively, for $g^{1/2} = \pi$. The exponential model differs from these data points by less than 3 dB, whereas the normal and pseudo-exponential models differ by more than 25 dB and 6 dB, respectively.

For microwave coherent scatter from several terrain and ocean surfaces (Figure 5), the data points have too large a spread for an arbitrary value of $g^{1/2}$ to allow a precise comparison with the models. However, for moderately rough surfaces, the exponential and pseudo-exponential models clearly give better agreement with data than the normal model, whereas for smooth surfaces the spread in data encompasses all the models. The wide scatter of experimental data from several different investigators results partly because of the inaccuracies associated with the measurements of both the reflection coefficient and $g^{1/2}$ and partly because incoherent scatter may have been included with the measurement of coherent scatter.

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TABLE 1

Coherent Power Reflection Coefficient in the Specular Direction,
Comparison of Models with Experimental Data.

Rayleigh Parameter, $g^{1/2}$	Coherent Power Reflection Coefficient $|\langle\rho\rangle|^2$ (dB)					
	Predicted by Models			Experimental Data		
	Normal	Pseudo-Exponential	Exponential	Beard	Beckmann and Spizzichino	Boyd and Deavenport
0	0	0	0	0	Wide	0
$\pi/4$	− 2.68	− 2.24	− 2.33	− 3.4 Averaged	Scatter	− 3.0
$\pi/2$	− 10.72	− 6.34	− 6.98	− 9.4	of	− 4.0
π	− 42.66	−13.19	−15.47	−17.1	Data	−12.0
2π	−171.45	−21.55	−26.36	−−		−23.0

For acoustical waves scattered from a surface model submerged in a water tank, (Figure 6), the normal, pseudo-exponential, and exponential models differ from the experimental data by -6.7 dB, -2.3 dB, and -2.7 dB, respectively, for $g^{1/2} = \pi/2$; by -30.7 dB, -1.2 dB, and -3.5 dB, respectively, for $g^{1/2} = \pi$; and by -148.5 dB, +1.5 dB, and -3.4 dB, respectively, for $g^{1/2} = 2\pi$. The exponential and pseudo-exponential models differ from these points by less than approximately 3 dB, whereas the normal model differs by more than 148 dB.

SECTION V

CONCLUSIONS

The exponentially distributed model predicts the coherent power
reflection coefficient for smooth and moderately rough surfaces
within approximately 3 dB of representative data points, whereas the
normally distributed model is within 3 dB for smooth surfaces but
underestimates the coefficient by 7 to 148 dB for moderately rough
surfaces. Furthermore, the exponentially distributed model lends
itself to an interesting physical interpretation of stochastic
process associated with the height random variable defined on a
three-dimensional rough surface. None of these models consider
multiple scattering, shadowing, or depolarization effects. Never-
theless, the exponentially distributed model gives a reasonable
estimate of the effect of surface roughness in reducing coherent
scatter for a wide variety of terrain and sea surfaces illuminated
by electromagnetic or acoustical radiation.

REFERENCES

1. In the earlier literature, scattering from extended surfaces is often treated as if the receiver were in the Fraunhofer far-field of a non-extended illuminated surface (see Reference 3, p. 19). In the more recent literature, the receiver is properly considered to be in the Fresnel near-field of the extended illuminated surface so that the transmitter image solution is obtained in the limit of a smooth surface (see Reference 4).

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10. Op. cit. 3, pp. 80-81.

APPENDIX A

Some Properties of an Exponentially Distributed Random Variable

Consider a random variable H defined by

$$H = B A \qquad (A-1)$$

where B and A are independent random variables whose probability densities are given by

$$f_B(b) = (\sigma_B)^{-2} b \exp(-b^2/2\sigma_B^2), \quad 0 \le b < \infty \qquad (A-2)$$

$$f_A(a) = (2\pi \sigma_A^2)^{-1/2} \exp[-a^2/2\sigma_A^2], \quad -\infty < a < \infty \qquad (A-3)$$

The probability densities $f_B(b)$ and $f_A(a)$ are Rayleigh and normally distributed respectively with standard deviations

$$\sqrt{2}\,\sigma_B = \langle b^2 \rangle^{1/2} \quad \text{and} \quad \sigma_A = \langle a^2 \rangle^{1/2} .$$

The probability density $f_H(h)$ is given by[9]

$$f_H(h) = \int_{-\infty}^{\infty} f_B(b)\, f_A(h/b)\, \frac{db}{|b|}$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma_A\,\sigma_B^2} \int_0^{\infty} e^{-[(b^2/2\sigma_B^2) + (h^2/2\sigma_A^2 b^2)]}\, db \qquad (A-4)$$

From standard tables of definite integrals, Equation (A-4) reduces to

$$f_H(h) = (2\,\sigma_A\,\sigma_B)^{-1} \exp[-|h|/\sigma_A\sigma_B] \qquad (A-5)$$

Therefore, $f_H(h)$ is exponentially distributed. Equating Equation (A-3) with Equation (9c), the standard deviation σ_H is given by

$$\sigma_H = \sqrt{2}\ \sigma_A\ \sigma_B \tag{A-6}$$

which is identical to that of Equation (8).

The characteristic function $\langle e^{-iuh} \rangle$ of the exponentially distributed random variable H is given by

$$
\begin{aligned}
\langle e^{-iuh} \rangle &= \int_{-\infty}^{\infty} e^{-iuh}\ f_H(h)\ dh \\
&= \frac{1}{\sqrt{2}\ \sigma_H} \left[\int_{-\infty}^{0} e^{(\frac{\sqrt{2}}{\sigma_H} - iu)h}\ dh + \int_{0}^{\infty} e^{-(\frac{\sqrt{2}}{h} + iu)h}\ dh \right. \\
&= \frac{1}{\sqrt{2}\ \sigma_H} \left[\left(\frac{\sqrt{2}}{\sigma_H} - iu \right)^{-1} + \left(\frac{\sqrt{2}}{\sigma_H} + iu \right)^{-1} \right]
\end{aligned} \tag{A-7}
$$

Substituting $g^{1/2}/\sigma_H = u$ into Equation (A-7),

$$\langle e^{-i(g^{1/2}/\sigma_H)h} \rangle = \left(1 + \frac{g}{2} \right)^{-1} \tag{A-8}$$

which is identical to Equation (10c).