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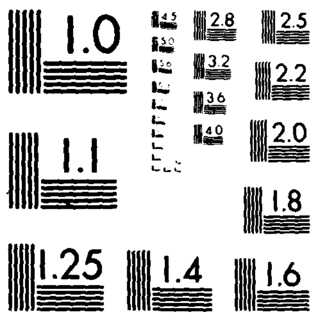

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Multivariate Harmonic New Better Than Used in Expectation Distributions

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University of Missouri-Columbia

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Multivariate Harmonic New Better Than
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By A. P. Basu and Nader Ebrahimi
University of Missouri, Columbia

Letters on file

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*This research was supported by the NSF Grant IV7 8009463 and ONR Grant N00014-78-C-0655.

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Multivariate Harmonic New Better Than Used in Expectation distributions*

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1. Introduction. In reliability theory various concepts of (univariate) aging or wearout have been proposed to study lifetimes of systems and components. The five most commonly studied class of distributions are the following:

1) The increasing failure rate class (IFR); 2) the increasing failure rate average class (IFRA); 3) the new better than used class (NBU); 4) decreasing mean residual class (DMRL); and 5) the new better than used in expectation (NBUE) class. For a description of some of these classes see Barlow and Proschan (1975). Recently Rolski (1975) proposed a new class of distributions called the harmonic new better than used in expectation (HNBUE) class which will be defined later. Each of the above six classes have their dual with standard nomenclature. The dual of HNBUE class is said to be harmonic new worse than used in expectation (HNWUE). Klefsjö (1980) has studied the properties of HNBUE (HNWUE) classes of distributions. He has proven several closure theorems for this class and the following chain of implication exists among the six classes of distributions.

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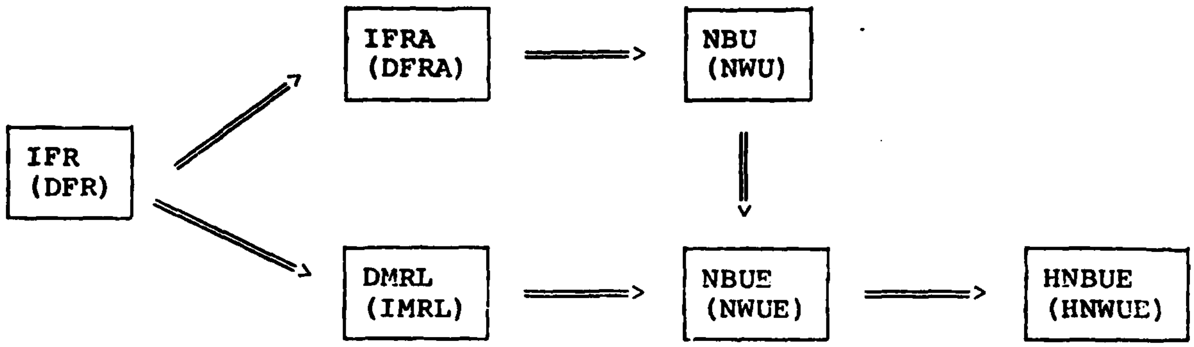

Thus HNBUE is the largest available class of distributions with aging property.

Recently attention has been directed towards extending the concepts of univariate aging to the multivariate case, (see for example Harris (1970), Basu (1971), and Brindly and Thompson (1972)), and using those multivariate concepts to define corresponding multivariate class of distributions. Buchanan and Singpurwalla (1977), Block and Savits (1980), and Ghosh and Ebrahimi (1980) have considered the cases for multivariate IFR, IFRA, NBU, DMRL, and NBUE distributions. The purpose of this note is to propose multivariate versions of HNBUE distributions (MHNBUE). It is shown that these are the largest available classes, and include the class of MHNBUE proposed by Klefsjö (1980) for the bivariate case.

In section 2 of this paper, we have introduced the various definitions of the MHNBUE involving a certain hierarchy, and have described their physical implications. Our definitions of the MHNBUE are different from Klefsjö's definitions of MHNBUE. We have compared our definitions of MHNBUE with Klefsjö definitions of MHNBUE. We have also examined in this section how several important classes of life distributions satisfy one or the other definition of MHNBUE.

The dual of the HNBUE is HNWUE. In section 2, multivariate HNWUE (MHNWUE) definitions are also given parallel to those for the MHNBUE.

Various closure properties of MHNBUE distributions under different definitions are studied in section 3. It is known that in the univariate case NBUE class is included in the HNBUE class. It is examined in section 4 how far the MHNBUE distributions as introduced by Buchanan and Singpurwalla (1977) and discussed in more detail by Ghosh and Ebrahimi (1980) lead to one or the other MHNBUE definition as introduced in section 2.

2. MHNBUE: definitions and example. Let $X_1,\ldots,X_p$ denote the survival (failure) times of p devices having a joint distribution function $H_p(x_1,\ldots,x_p)$. The joint survival function of these p devices is denoted by $\bar{H}_p(x_1,\ldots,x_p) = P(X_1 > x_1,\ldots,X_p > x_p)$. It is assumed that $\bar{H}_p(0,\ldots,0) = 1$. In the univariate case, a non-negative random variable X_1 is said to have a HNBUE (HNWUE) distribution if

$$\int_{t_1}^{\infty} \bar{H}_1(x)dx \leq (\geq)\mu \exp(-t_1/\mu) \text{ for all } t_1 \geq 0, \tag{2.1}$$

where it is assumed that $\int_0^{\infty} \bar{H}_1(x)dx = \mu < \infty$.

With the alternate representation of (2.1)

$$\frac{1}{\frac{1}{t_1}\int_0^{t_1} e_{H_1}(x)^{-1}dx} \leq (\geq)\mu \text{ for all } t_1 \geq 0 \tag{2.2}$$

where $e_{H_1}(x) = \dfrac{\int_x^\infty \bar{H}_1(y)\,dy}{\bar{H}_1(x)}$, it is easy to see that the condition is equivalent to saying that the integral harmonic mean value of the mean residual life of a unit at age x is less (greater) than or equal to the integral harmonic mean value of a new unit.

Definitions. Our definitions of MHNBUE are the natural multivariate extension of (2.2). Suppose for simplicity that $\bar{H}_p(x_1,\ldots,x_p) > 0$ and let

$$e_{H_p}(x_1,\ldots,x_p) = \frac{\int_{x_1}^\infty \cdots \int_{x_p}^\infty \bar{H}_p(y_1,\ldots,y_p)\,dy_1\ldots dy_p}{\bar{H}_p(x_1,\ldots,x_p)}, \qquad (2.3)$$

$$g_{H_p}(x,\ldots,x) = \frac{\int_x^\infty \bar{H}_p(y,\ldots,y)\,dy}{\bar{H}_p(x,\ldots,x)}\,. \qquad (2.4)$$

H_p is said to be

(i) MHNBUE-I (MHNWUE-I) if

$$\frac{\prod_{i=1}^{p} t_i}{\int_0^{t_1}\cdots\int_0^{t_p} e_{H_p}^{-1}(x_1,\ldots,x_p)\,dx_1,dx_2,\ldots,dx_p} \le (\ge)\, e_{H_p}(0,\ldots,0) \qquad (2.5)$$

for all $t_i \ge 0$ $(1 \le i \le p)$, and similar inequalities are assumed to hold for all subsets of random variables.

(ii) MHNBUE-II (MHNWUE-II) if

$$\frac{t^p}{\int_0^t \cdots \int_0^t e_{H_p}^{-1}(x_1,\ldots,x_p)\,dx_1\ldots dx_p} \leq (\geq)\, e_{H_p}(0,\ldots,0) \tag{2.6}$$

for all $t \geq 0$, and similar inequalities hold for all subsets of random variables.

(iii) MHNBUE-III (MHNWUE-III) if

$$\frac{t}{\int_0^t e_{H_p}^{-1}(x,\ldots,x)\,dx} \leq (\geq)\, e_{H_p}(0,\ldots,0) \tag{2.7}$$

for all $t \geq 0$, and similar inequalities hold for all subsets of random variables.

(iv) MHNBUE-IV (MHNWUE-IV) if

$$\frac{t}{\int_0^t g_{H_p}^{-1}(x,\ldots,x)\,dx} \leq g_{H_p}(0,\ldots,0) \tag{2.8}$$

for all $t \geq 0$, and similar inequalities hold for all subsets of random variables.

Next we give the physical interpretation of these four definitions. First note that

$$e_{H_p}(x_1,\ldots,x_p) = \int_{x_1}^{\infty} \cdots \int_{x_p}^{\infty} \prod_{i=1}^{p} (y_i - x_i)\,dH(y_1,\ldots,y_p), \tag{2.9}$$

for proof see Ghosh and Ebrahimi (1980). Now using (2.9) it follows that (2.5) is equivalent to the statement that the integral harmonic mean value of the conditional mean residual product lifetime of the components of a unit with the components surviving ages $x_1, \ldots, x_p$ respectively is less (greater) than or equal to the integral harmonic mean value of the mean product lifetime of the components of a new unit. Similar interpretation holds for (2.6). The definition (2.7) is equivalent to the statement that the integral harmonic mean value of the conditional mean residual product lifetime of the components of a unit when all the components have survived a certain time x is less (greater) than or equal to the harmonic mean value of the mean product lifetime of the components of a new unit. Finally, definition (2.8) is equivalent to the statement that a multivariate distribution is HNBUE (HNWUE) if the minimum of the components has a univariate HNBUE distribution.

It is trivial to check that MHNBUE-I $\Longrightarrow$ MHNBUE-II. However, the following examples show that

$$\text{MHNBUE-I} \not\Longrightarrow \text{MHNBUE-IV (so that MHNBUE-II} \not\Longrightarrow \text{MHNBUE-IV)}.$$

<u>Example 1</u>. Let X_1 and X_2 be iid with common survival function

$$\bar{F}(x) = \begin{cases} 1 & \text{if } 0 \le x < 3 \\ 1/4 & \text{if } 3 \le x < 7 \\ 0 & \text{if } x \ge 7 \end{cases}.$$

Then by using (p_2) of section 3, (X_1, X_2) is MHNBUE-I. But $\min(X_1, X_2)$ is not HNBUE, i.e., (X_1, X_2) is not MHNBUE-IV.

The next example shows that MHNBUE-I $\not\Longrightarrow$ MHNBUE-III (so that MHNBUE-II $\not\Longrightarrow$ MHNBUE-IV).

Example 2. Let X_1 and X_2 be independent with survival functions

$$\bar{F}_{X_1}(x) = \begin{cases} 1 & \text{if } 0 \le x < 3 \\ 1/4 & \text{if } 3 \le x < 7 \\ 0 & \text{if } x \ge 7 \end{cases} \quad \text{and} \quad \bar{F}_{X_2}(x) = \begin{cases} 1 & \text{if } 0 \le x < 1 \\ 1/2 & \text{if } 1 \le x < 5. \\ 0 & \text{if } x \ge 5 \end{cases}$$

respectively. By using (P_2) of section 3, (X_1, X_2) is MHNBVE-I. But (X_1, X_2) is not MHNBUE-III.

Finally, the following example shows that MHNBUE-IV $\not\Rightarrow$ MHNBUE-III.

Example 3. Let X_1, X_2 denote the survival times of 2 devices having a joint survival distribution function

$$\bar{H}_2(x_1, x_2) = \exp(-\max(\lambda_3 x_1, \lambda_4 x_2)),$$

where $\lambda_3, \lambda_4 > 0$. Then, (X_1, X_2) is MHNBUE-IV. But (X_1, X_2) is not MHNBUE-III.

Remark 1. Example 1 shows that MHNBUE-III $\not\Rightarrow$ MHNBUE-IV.

Klefsjö's (1980) definitions of the MHNBUE are the natural multivariate extension of (2.1).

We first introduce the following definitions of MHNBUE as given in Klefesjö (1980). A bivariate distribution function H_2 is said to be MHNBUE if

A. $$\int_0^\infty\int_0^\infty \bar{H}_2(x_1 + t_1, x_2 + t_2)dt_1dt_2 \le \int_0^\infty\int_0^\infty \bar{G}(x_1 + t_1, x_2 + t_2)dt_1dt_2$$

for $x_1, x_2 \ge 0$;

B. $$\int_0^\infty \overline{H}_2(x_1 + t, x_2 + t)dt \le \int_0^\infty \overline{G}(x_1 + t, x_2 + t)dt$$

for $x_1, x_2 \ge 0$;

C. $$\int_0^\infty\int_0^\infty \overline{H}_2(x + t_1, x + t_2)dt_1dt_2 \le \int_0^\infty\int_0^\infty \overline{G}(x + t_1, x + t_2)dt_1dt_2$$

for $x \ge 0$;

D. $$\int_0^\infty \overline{H}_2(x + t, x + t)dt \le \int_0^\infty \overline{G}(x + t, x + t)dt$$

for all $x \ge 0$.

In all cases $\overline{G}(t_1, t_2)$ is the bivariate exponential distribution proposed by Marshall and Olkin (1967) where

$$\overline{G}(t_1, t_2) = \exp(-\lambda_1 t_1 - \lambda_2 t_2 - \lambda_{12}\max(t_1, t_2)) \text{ for } t_1, t_2 \ge 0,$$

and

$$\lambda_1 = \frac{\mu_1 + \mu_2}{\mu_{12}} - \frac{1}{\mu_1}, \quad \lambda_2 = \frac{\mu_1 + \mu_2}{\mu_{12}} - \frac{1}{\mu_2}, \quad \lambda_{12} = \left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)\frac{\mu_{12} - \mu_1\mu_2}{\mu_{12}},$$

$$\mu_1 = \int_0^\infty \overline{H}_2(t_1, 0)dt_1, \quad \mu_2 = \int_0^\infty \overline{H}_2(0, t_2)dt_2, \text{ and } \mu_{12} = \int_0^\infty\int_0^\infty \overline{H}(t_1, t_2)dt_1dt_2 .$$

Remark 2. The Klefsjö's definitions of MHNBUE are restricted to the class of life distributions for which the following conditions hold among μ_1, μ_2, and μ_{12}:

$$\text{(i)} \quad \frac{\mu_1 + \mu_2}{\mu_{12}} - \frac{1}{\mu_1} \geq 0 \,, \quad \text{(ii)} \quad \frac{\mu_1 + \mu_2}{\mu_{12}} - \frac{1}{\mu_2} \geq 0, \quad \text{and}$$

$$\text{(iii)} \quad \left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)\left(\frac{\mu_{12} - \mu_1\mu_2}{\mu_{12}}\right) > 0. \tag{2.10}$$

The following example shows that Klefsjö's definitions of MHNBUE does not imply our definitions of MHNBUE.

Example 4. Let X_1, X_2 be iid and X_1, X_2 be HNBUE. Choose μ_1, μ_2, and μ_{12} which does not satisfy one of the conditions in (2.10). Then (X_1, X_2) satisfies our definitions, but it does not satisfy Klefsjö's definition of MHNBUE.

3. MHNBUE closure properties. In this section, we prove certain closure properties of MHNBUE distribution. Let β_1, β_2, β_3, and β_4 denote the classes of life distributions satisfying the definitions (i), (ii), (iii), and (iv) respectively of MHNBUE. Then we have the following theorem.

Theorem 1.

(P 1) If $(T_1,\ldots,T_m)\varepsilon\beta_j$, any subset of $(T_1,\ldots,T_m)\varepsilon\beta_j$ $(1 \leq j \leq 4)$;

(P 2) If $(T_1,\ldots,T_m)\varepsilon\beta_j$, $(T_1',\ldots,T_n')\varepsilon\beta_j$ and are independent, then $(T_1,\ldots,T_m, T_1',\ldots,T_n')\varepsilon\beta_j$ $(1 \leq j \leq 2)$. If $(T_1,\ldots,T_n)\varepsilon\beta_3$, $(T_1',\ldots,T_n')\varepsilon\beta_3$ are independent with identical distritution, then $(T_1,\ldots,T_n, T_1,\ldots,T_n')\varepsilon\beta_3$.

(P 3) If $(T_1,\ldots,T_m)\varepsilon\beta_j$, then $(c_1T_1,\ldots,c_mT_m)\varepsilon\beta_j$ $(1 \leq j \leq 2)$ for all $c_i > 0$ $(1 \leq i \leq m)$. If $(T_1,\ldots,T_m)\varepsilon\beta_j$, then $(cT_1,\ldots,cT_m)\varepsilon\beta_j$ $(2 \leq j \leq 4)$ for all $c > 0$.

<u>Proof.</u>

(P 1): This property follows immediately from the definitions.

(P 2): Let $\underline{a} = (a_1,\ldots,a_m)$, an m vector and $\underline{a}' = (a_1',\ldots,a_n')$, an n vector. Other vectors can be defined similarly. Let

$$I(\underline{x}) = \int_{\underline{0}}^{t} [P(\underline{T} > \underline{x}) / \int_{\underline{x}}^{\infty} P(\underline{T} > \underline{y}] d\underline{y}] dx, \text{ where } d\underline{x} = dx_1,\ldots,dx_m \text{ and}$$

$\int_{\underline{0}}^{t}$ is a multiple integral with m factors. $I(\underline{x}')$ and $I(\underline{x}, \underline{x}')$ are defined similarly. Then

$$\prod_{i=1}^{m} t_i \prod_{j=1}^{n} t_j' / I(x, \underline{x}')$$

$$[\prod_{i=1}^{m} t_i / I(\underline{x})] [\prod_{j=1}^{n} t_j' / I(\underline{x}')]$$

$$\le \int_{\underline{0}}^{\infty} P(\underline{T} > \underline{x}) d\underline{x} \int_{\underline{0}}^{\infty'} P(\underline{T}' > \underline{x}') d\underline{x}'$$

$$= \int_{\underline{0}'}^{\infty'} \int_{\underline{0}}^{\infty} P(\underline{T} > \underline{x}\ ,\ \underline{T}' > \underline{x}') d\underline{x}' d\underline{x}$$

Similar proof works for β_2. To prove β_3, use the Jensen inequality

(P 3): If $\underline{T} \in \beta_1$, $c_i > 0$ $(1 \le i \le m)$

$$\prod_{i=1}^{m} t_i / \int_{\underline{0}}^{t} [P(\underline{d} > \underline{x}) / \int_{\underline{0}}^{\infty} P(\underline{d} > \underline{y} + \underline{x}) d\underline{y}] d\underline{x}$$

$$= \prod_{i=1}^{m} t_i / \int_{\underline{0}}^{t} [P(\underline{T} > \underline{e}) / \int_{\underline{0}}^{\infty} P(\underline{T} > \underline{f}) d\underline{y}] d\underline{x}$$

$$\le \int_{\underline{0}}^{\infty} P(\underline{T} > \underline{e}) d\underline{x} = \int_{\underline{0}}^{\infty} P(\underline{d} > \underline{x}) d\underline{x}.$$

Here $\underline{d} = (c_1T_1,\ldots,c_mT_m)$, $\underline{y} + \underline{x} = (y_1 + x_1,\ldots,y_m + x_m)$, $\underline{e} = (x_1/c_1,\ldots,x_m/c_m)$ and $\underline{f} = (y_1/c_1,\ldots,y_m/c_m)$. Similar proofs work for β_2, β_3, and β_4.

Remark 3. Since it is known in the univariate case that the HNBUE is not closed under the formation of coherent system (see Klefsjö (1980)), the same cannot be expected for the MHNBUE under any of the four definitions.

Remark 4. Example 1 shows that (P 2) does not hold for β_4.

Remark 5. To prove that MHNBUE is closed under limits in distribution, we need an extra condition to guarantee the application of the dominated convergence theorem.

Theorem 2. Let $\{(T_{1k},\ldots,T_{mk}),\ k \geq 1\}$ be a sequence of MHNBUE random vectors belonging to β_j for each k. If $(T_{1k},\ldots,T_{mk}) \rightarrow (T_1,\ldots,T_m)$ weakly as $k \rightarrow \infty$ and $(T_{1k},\ldots,T_{mk}) \underset{st}{\leq} (S_1,\ldots,S_m)$ for all $k \geq k_0$ where $E(\prod_{i=1}^{m} S_i) < \infty$, then $(T_1,\ldots,T_m) \in \beta_j$ for each j.

Proof. Use the dominated convergence theorem and the appropriate definition of the MHNBUE.

It is known in the univariate case that HNBUE is closed under convolution. We have not been able to prove the same result for the multivariate case. Instead, we have proved the following theorem which is a special type of convolution.

Theorem 3. Let (a) $\underline{X} = (X_1,\ldots,X_n)$ and $\underline{Y} = (Y_1,\ldots,Y_n)$ be componentwise independent; (b) $\underline{X}$ and $\underline{Y}$ are independent; and (c) $\underline{X}$ and $\underline{Y} \in \beta_j$ for j = 1, 2. Then $W = \underset{\sim}{X} + \underset{\sim}{Y} \in \beta_j$ for j = 1, 2.

<u>Proof</u>. Use (P 2) and the fact that $X_i + Y_i$'s are HNBUE.

Remark 5. In Theorem 3 if $\underset{\sim}{X}$ and $\underset{\sim}{Y}$ have the same distribution, then $\underset{\sim}{W} = \underset{\sim}{X} + \underset{\sim}{Y} \in \beta_3$.

4. Relationship between MNBUE and MHNBUE.

In the univariate case, a life distribution function H_1 is said to satisfy the NBUE property if

$$\int_t^\infty \overline{H}(x)\,dx \le \overline{H}_1(t)\int_0^\infty \overline{H}_1(x)\,dx \text{ for all } t \ge 0.$$

Rolski (1975), and Klefsjö (1980) have shown in the univariate case that NBUE $\Longrightarrow$ HNBUE, and that the converse implication does not hold. It is of interest to know whether a similar implication holds in the multidimensional case. Buchanan and Singpurwalla (1977) have given several definitions of the MNBUE based on multivariate generalization of the univariate NBUE distributions.

We first introduce the following definitions of MNBUE as given in Ghosh and Ebrahimi (1980). A p-variate distribution function H_p is said to be MNBUE if

A.

$$\int_{t_p}^\infty \cdots \int_{t_1}^\infty \overline{H}_p(x_1,\ldots,x_p)\,dx_1,\ldots,dx_p \le \overline{H}_p(t_1,\ldots,t_p) \int \cdots \int \overline{H}_p(x_1,\ldots,x_p)\,dx_1,\ldots,dx_p$$

for all $t \ge 0$ $(1 \le i \le p)$, and similar inequalities are assumed to hold for all subsets.

B.

$$\int_t^\infty \cdots \int_t^\infty \overline{H}_p(x_1,\ldots,x_p)\,dx_1,\ldots,dx_p \le \overline{H}_p(t,\ldots,t)\int_0^\infty \cdots \int_0^\infty H_p(x_1,\ldots,x_p)\,dx_1,\ldots,dx_p$$

for all $t \geq 0$, and similar inequalities are assumed to hold for all subsets.

C.

$$\int_t^\infty \overline{H}_p(x,\ldots,x)\,dx \le \overline{H}_p(t,\ldots,t)\int^\infty H_p(x,\ldots,x)\,dx$$

for all $t \geq 0$, and similar inequalities hold for all subsets.

Ghosh and Ebrahimi (1980) have shown that the following implications hold: $A \Longrightarrow B$, $A \not\Longleftrightarrow C$, and $C \Longrightarrow B$.

We first explore the interrelationship of the A - C definitions of MNBUE with MHNBUE-I to IV.

It is trivial to check that MHNBUE-I $\Longrightarrow$ MHNBU-I $\Longrightarrow$ MHNBUE-II, MNBUE-II $\Longrightarrow$ MHNBUE-III, and MNBUE-III $\Longrightarrow$ MHNBUE-IV.

Example 5. (MNBUE-I $\not\Longleftrightarrow$ MHNBUE-IV). Consider once again example 1. Then (X_1, X_2) is MNBUE-I. But (X_1, X_2) is not MHNBUE-IV.

Remark 6. The example 4 shows that MNBUE-II $\not\Longleftrightarrow$ MNBUE-IV.

Remark 7. Let X_1 and X_2 have bivariate Marshal-Olkin (1967) exponential distribution with survival function

$$\overline{H}_2(x_1, x_2) = \exp[-\lambda_1 x_1 - \lambda_2 x_2 - \lambda_{12}\max(x_1, x_2)]$$

$x_1 \geq 0$, $x_2 \geq 0$, $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, $\lambda_{12} > 0$. Then $\overline{H}_2(x_1, x_2)$ is both MHNBUE-IV and MHNWUE-IV, MHNBUE-III, and MHNWUE-III.

Remark 8. To see the interrelationship between MHNBUE and other classes of life distribution see Ghosh and Ebrahimi (1980). It is interesting to mention that multivariate IFRA defined by Block and Savits (1980) implies MHNBUE-IV.

The next example shows that MHNBUE class of life distributions is larger than the class of MNBUE life distributions.

Example 6. Let X_1, X_2 are independent and identically distributed with the following survival distribution function,

$$\bar{F}(t) = \begin{cases} 1 & 0 \le t < 1 \\ \frac{1}{2} & 1 \le t < 2 \\ \frac{1}{8} & 2 \le t < 4 \\ 0 & t \ge 4 \end{cases}$$

Then (X_1, X_2) is MHNBUE-I, MHNBUE-II, MHNBUE-III, and MHNBUE-IV. But (X_1, X_2) is not MNBUE-I or MNBUE-II or MNBUE-III.

The following example shows that the MHNWUE class is also larger than the MNWUE class.

Example 7. Let X_1, X_2 are iid with the following survival distribution,

$$\bar{F}(t) = \begin{cases} 1 & 0 \le t < 1 \\ \frac{37}{192} & 1 \le t < 2 \\ \frac{21}{192} & 2 \le t < 3 \\ \frac{5}{192} & 3 \le t < 4 \\ (\frac{1}{4})^k & k \le t < k+1 \text{ for } k = 4, \ldots . \end{cases}$$

Then (X_1, X_2) is MHNWUE-I, MHNWUE-II, MHNWUE-III, and MHNWUE-IV. But (X_1, X_2) is not MNWUE-I or MNWUE-II or MNWUE-III.

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