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ON THE SEQUENTIAL DIAGNOSIBILITY OF A CLASS OF DIGITAL SYSTEMS,(U)
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On the Sequential Diagnosibility of a Class of Digital Systems

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1. Introduction

We study in this paper a problem concerning diagnosis of digital systems. We use a model that was first introduced by Preparata, Metze, and Chien [1]. In this model, a digital system is partitioned into a certain number of units, each of which can be at one of two possible states, fault-free ($\bar{F}$) and faulty (F). A configuration of a system is an assignment of either the fault-free or the faulty state to each unit in the system. We assume that each unit in the system possesses a certain amount of computational resources to enable it to test one or more of the other units in the system. The outcome of a test is a binary signal which depends on the state of the testing and the tested units. In particular, we assume that:

(i) if a fault-free unit is tested by a fault-free unit, a signal 0 will be generated;

(ii) if a faulty unit is tested by a fault-free unit, a signal 1 will be generated;

(iii) if a fault-free or a faulty unit is tested by a faulty unit, either a signal 0 or a signal 1 will be generated. (In other words,

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the signal generated by a faulty testing unit is completely unreliable.) A diagnosis experiment is one in which every unit tests all the units it is capable of testing once. The outcomes of the tests are referred to as a syndrome.

In graph theoretic terms, a digital system can be described by a directed graph G = (V, E) where the vertices represent the units of the system. An edge (v_i, v_j) in G indicates that unit v_i is capable of testing unit v_j. A configuration is an assignment of the values $\bar{F}$ and F to the vertices in V. A syndrome is an assignment of the values 0 and 1 to the edges in E. A syndrome is said to be consistent with a configuration if conditions (i)(ii)(iii) above are not violated. We note that a given configuration might yield a number of different syndromes, and a given syndrome might be consistent with a number of different configurations. (However, because of (iii) in our assumption above, any syndrome is consistent with at least one configuration, namely, the configuration in which all units are faulty.)

The goal of a diagnosis experiment is to identify one or more of the faulty units in the system. A one-step diagnosis is one in which all faulty units in the system are identified. A sequential diagnosis is one in which at least one faulty unit, if there is any, is identified. For any system, both one-step diagnosis and sequential diagnosis are possible, provided that the number of faulty units does not exceed certain critical value. The one-step diagnosibility of a digital system, t_0, is defined to be the maximum number of faulty units in the system such that for any syndrome corresponding to a configuration with more than t_0 faulty units, one-step diagnosis is not possible. The sequential diagnosibility, t_r, is defined to be the maximum number of faulty units in the system such that for any syndrome corresponding to a configuration with more than t_r faulty units, sequential diagnosis is possible. In graph theoretic terms both t_0 and t_r are invariants of the graph $G = (V, E)$. The problem of determining t_0 and t_r is, in general, a difficult one [2]. In this paper, we show a useful technique for obtaining lower bounds on the value of t_r for a class of digital systems.

2. A General Result

Throughout our discussion, we shall assume G to be a strongly connected graph. In this case, for a given syndrome S, sequential diagnosis is possible if we can unambiguously identify a certain unit to be faulty or fault-free. (Clearly, our goal is achieved if a unit is identified as faulty. On the other hand, if a unit is identified as fault-free then any unit tested by this unit will be fault-free if a 0 signal results and any unit tested by this unit will be faulty if a 1 signal results. Repeating such an argument if necessary, because G is strongly connected, either a faulty unit is identified eventually or all units in the system are confirmed to be fault-free.)

For a given syndrome S, for a vertex v in G, we use $G_0^S(v)$ to denote the minimum number of faulty units in the configuration(s) that are consistent with S with v being fault-free. Also, we use $G_1^S(v)$ to denote the minimum number of faulty units (excluding v)[†] in the configuration(s) that are consistent with S with v being faulty. If

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[†] It is simply a matter of convenience that we exclude v in computing the value $G_1^S(v)$.

$$t_r \leq \max(G_0^S(v) - 1,\ G_1^S(v))$$

then v can be identified unambiguously as faulty if $G_0^S(v) - 1 > G_1^S(v)$, and as fault-free if $G_0^S - 1 < G_1^S(v)$.† Consequently, the sequential diagnosibility of a graph G can be computed as

$$t_r = \min \left\{ \max_{v \in V} \left[\max(G_0^S(v) - 1,\ G_1^S(v)) \right] \right\}$$

A directed graph T is called a <u>2-star</u> if

(i) T is a rooted tree with all the edges directed toward the root v.

(ii) With the exception of v, all internal nodes have indegree 1.

(iii) The height of T is atmost 2.

Figure 1 shows an example of a 2-star. The size of a 2-star T is defined to be the number of vertices in T minus 1.

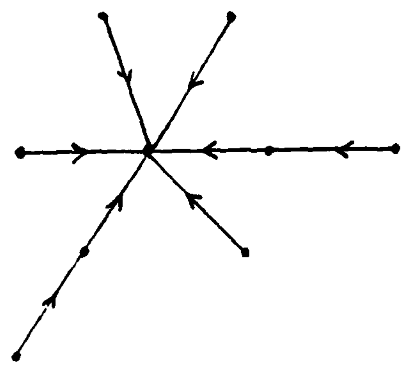

Figure 1.

<u>Theorem 1</u>: Let G be a directed graph that contains a 2-star of size k, then

$$t_r \geq \left\lceil \frac{k-1}{2} \right\rceil$$

Proof: Let S be a given syndrome. Let x and v be two vertices in G. The vertex x is said to be <u>non-fault-free with respect to</u> v if

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† If $G_0^S(v) - 1 = G_1^S(v)$ then S corresponds to a configuration with more than t_r faulty units.

for any two configurations C_1 and C_2 that are consistent with S (i) v is fault-free in C_1 and is faulty in C_2 (ii) x is faulty in at least one of C_1 and C_2.

Let T be a 2-star of size k in G. Let v be the root of T. Let v_i be a vertex of distance 1 from v in T. For any given syndrome S, v_i must be non-fault-free with respect to v. (If the test signal in (v_i, v) is 0, for any configuration in which v is faulty, v_i must be faulty also. If the test signal in (v_i, v) is 1, for any configuration in which v is fault-free, v_i must be faulty.) Let C_1 be a configuration such that

(i) C_1 is consistent with S,

(ii) C_1 contains a minimum number of faulty units,

(iii) v is a fault-free in C_1.

Let C_2 be a configuration such that

(i) C_2 is consistent with S,

(ii) C_2 contains a minimum number of faulty units,

(iii) v is faulty in C_2.

Consider a path of length 2 $(v_j, v_i)(v_i, v)$ in T. We have two cases.

Case 1: v_i is faulty in both C_1 and C_2.

Case 2: v_i is faulty in one of C_1 and C_2. In this case, v_j must be faulty in at least one of C_1 and C_2 (since v_j is non-fault-free with respect to v_i).

In either case, v_i and v_j will contribute a count of at least 2 in $G_0^S(v) + G_1^S(v)$. Thus, we have

$$G_0^S(v) + G_1^S(v) \geq k$$

or

$$\max\left(G_0^S(v) - 1,\ G_1^S(v)\right) \geq \left\lceil \frac{k-1}{2} \right\rceil$$

Thus,

$$t_r \geq \left\lceil \frac{k-1}{2} \right\rceil \qquad \square$$

As an immediate application of Theorem 1, we note that for the graph H shown in Figure 2(a), because H contains a 2-star as shown in Figure 2(b), we must have

$$H_0^S(v) + H_1^S(v) \geq 2p$$
$$t_r \geq \left\lceil \frac{2p-1}{2} \right\rceil = p$$

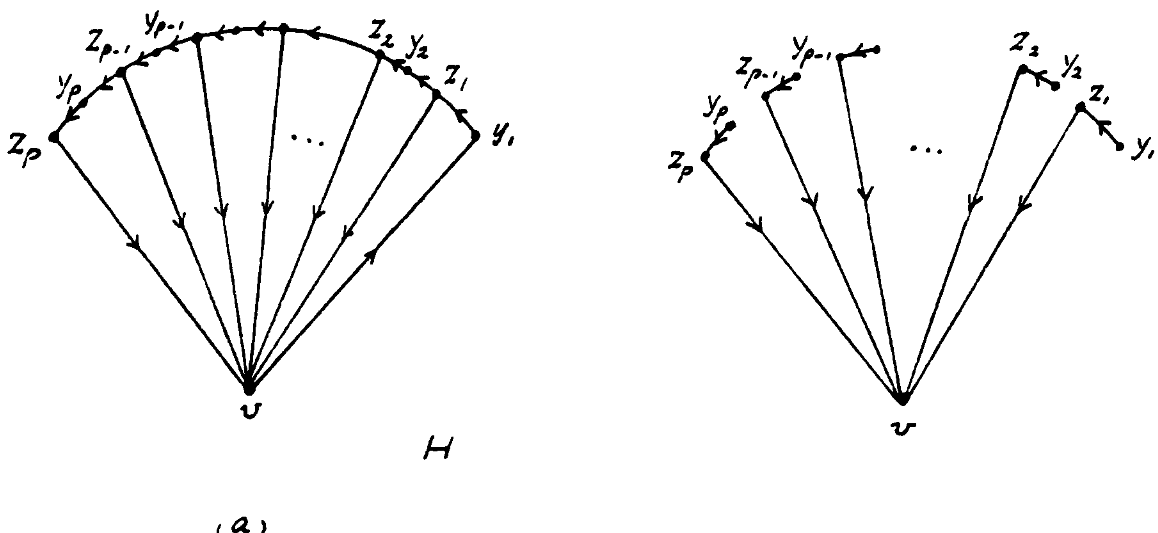

Figure 2

Furthermore, let R be a graph obtained by putting c copies H together at a common vertex v as shown in Figure 3. Then for any syndrome S

$$R_0^S(v) + R_1^S(v) \geq 2\ cp$$

$$t_r \geq cp$$

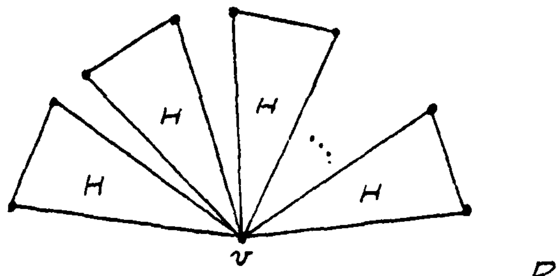

Figure 3

3. A Generalization

Consider the graph B shown in Figure 4 in which there is a cycle of m units. At each unit in the cycle, a copy of the graph R is attached.

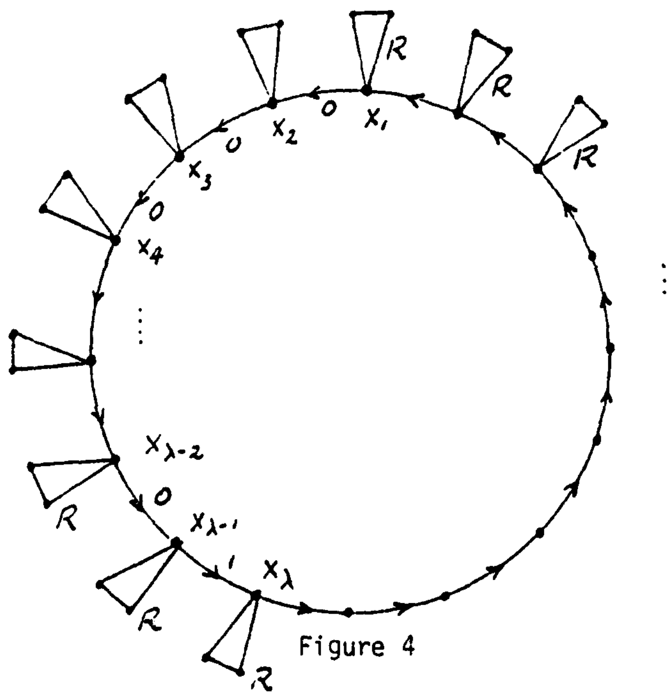

Figure 4

It is well-known that for a given syndrome S, the test signals in the edges in the cycle can be partitioned into sequences of the form $\cdots 00001$. Let there be ν sequences, and let λ be the number of units in the longest sequence(s). As in Figure 4, let the test signals at (x_1, x_2), (x_2, x_3), (x_3, x_4), ..., $(x_{\lambda-2}, x_{\lambda-1})$, $(x_{\lambda-1}, x_\lambda)$ be $\cdots 00001$. We note first that if x_1 is fault-free in a configuration that is consistent with S, then x_2, x_3, ..., $x_{\lambda-1}$ must also be fault-free and x_λ must be faulty in that configuration. On the other hand, if x_λ is fault-free in a configuration that is consistent with S, then x_1, x_2, ..., $x_{\lambda-1}$ must be faulty in that configuration. Furthermore, using the known fact that corresponding to any $\cdots 0001$ sequence of test signals along the cycle, there must be at least one faulty unit in any configuration consistent with S. We thus have

$$B_0^S(x_1) \geq R_0^S(x_1) + R_0^S(x_2) + \ldots R_0^S(x_{\lambda-1}) + R_1^S(x_\lambda) + 1 + \nu-1$$

$$B_0^S(x_\lambda) \geq R_1^S(x_1) + R_1^S(x_2) + \ldots R_1^S(x_{\lambda-1}) + R_0^S(x_\lambda) + \lambda-1 + \nu-1$$

or

$$B_0^S(x_1) + B_0^S(x_\lambda) \geq \lambda \cdot 2cp + \lambda + 2\nu - 2^{\dagger}$$

or

$$\max\left(B_0^S(x_1) - 1,\ B_0^S(x_\lambda) - 1\right) \geq \left\lceil \frac{(2cp+1)\lambda}{2} \right\rceil + \nu-2$$

Thus, we obtain

$$t_r \geq \min_{\text{all } S} \left(\left\lceil \frac{(2cp+1)\lambda}{2} \right\rceil \right) + \nu-2$$

$$\geq \sqrt{\frac{2cp+1}{2}}\,\sqrt{m}-2 \qquad (1)$$

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†We remind the reader that $R_1^S(v)$ does not include the vertex v.

4. A Further Generalization

It is often the case that the units in a digital system can be divided into subsets such that units in one subset are more reliable than units in another subset. For a digital system represented by the graph $G = (V, E)$, let V_1 be a subset of V, let t be a positive integer less than or equal to $|V_1|$. We define the sequential diagnosibility t_r of G with respect to (V_1, t) to be the maximum number of fault units in V such that for any syndrome corresponding to a configuration with no more than t_r faulty units, furthermore, with no more than t of them in the units in V_1, then sequential diagnosis is possible. Clearly, to determine t_r is an even more complex task. However, our result in Section 3 provides at least an example of results of such nature. Let V_1 be the set of units in the cycle. If it is known that V_1 will not contain more than t faulty units, then

$$\lambda \geq \left\lceil \frac{m}{t_i} \right\rceil - 1 \qquad\qquad t_i \leq t$$

$$\nu = \left\lceil \frac{t_i}{2} \right\rceil$$

$$t_r \geq \min_{\text{all } S} \left(\frac{2cp+1}{2} \cdot \frac{m}{t_i} \right) + \left\lceil \frac{t_i}{2} \right\rceil + \nu - 2$$

which can be larger than the result in (1) for small t_i.

5. Another model

Note that our results apply immediately to the following model:

(i) If a fault-free unit is tested by a fault-free unit, a signal 0 will be generated,

(ii) if a fault-free unit is tested by a faulty unit, a signal 1 will be generated.

(iii) if a faulty unit is tested by a fault-free or by a faulty unit, either a signal 0 or a signal 1 will be generated.

References

[1] F. P. Preparata, G. Metze and R. T. Chien, "On the Connection Assignment Problem of Diagnosable Systems". IEEE Trans. Electron. Comput., Vol. EC-16, pp. 848-854, Dec. 1967.

[2] H. Fujiwara and K. Kinoshita, "On Computational Complexity of System Diagnosis". IEEE Trans. on Comput., Vol. C-27, pp. 379-384, April 1978.