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FINAL REPORT

on

UNSTABLE RESONATORS

by

L. B. Felsen

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Unstable laser resonators; waveguide analysis of optical resonators; ray-optical treatment of optical resonators.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
This report summarizes accomplishments pertaining to two new methods for analyzing unstable resonators: a) a waveguide method, and (b) a ray-optical method. The waveguide method has been found effective for calculating eigenvalues, but not eigenmodes, in symmetric configurations with sharp-edged or round-edged mirrors, and with misaligned mirrors. The ray-optical method has been found effective not only for these same configurations but also for non-symmetric structures containing internal axicons. Moreoever, it yields the eigenvalues as well as the eigenmode fields. Therefore, the ray method promi

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to provide a useful tool for studying features introduced by successive complication of the resonator model.

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ACKNOWLEDGMENT

I. Technical Summary

A. Background and Accomplishments

The research performed under this grant had as its objective the development of a ray-optical theory for the behavior of eigenvalues and eigenmodes in unstable laser resonators. The detailed program was determined in consultation with Drs. J. Hanlon, R. Butts, and G. Dente at the Air Force Weapons Laboratory, Albuquerque, New Mexico, who served successively as technical project monitors. The investigation proceeded along two principal lines.

1. Waveguide Analysis

Further exploration of the waveguide analysis of Chen and Felsen[1] and Santana and Felsen[2] was undertaken to explain, on a coupled waveguide mode basis, the structure of the eigenmode charts obtained by the conventional "brute-force" numerical solution of the resonator integral equation. Here, the resonator is regarded as a waveguide perpendicular to its axis, with the mirrors representing the waveguide boundaries (Fig. 1). Coupling between selected modes in this highly overmoded structure is introduced by the mirror edges which represent waveguide discontinuities. The scattering from these discontinuities is taken into account by a ray-optical analysis incorporating the constructs of the geometrical theory of diffraction (GTD).[3] These edge-diffracted fields are then converted into modal coupling coefficients to permit the formulation of a <u>waveguide</u> mode resonance equation ("transverse resonance" equation) that specifies the eigenvalues of the <u>composite</u> resonator modes. Thus, the method involves a combination of waveguide mode and ray-optical techniques.

The method had previously been applied successfully to sharp-edged, aligned, symmetrical strip and circular mirror configurations.[2] Its extension to misaligned sharp-edged strip mirrors[4] (Fig. 2) and to aligned strip or circular mirrors with rounded edges (Fig. 3) was undertaken under this grant.[5] The results for misalignment were compared to numerical data based on the resonator integral equation, and were found to be interpretable in terms of coupling between waveguide modes with even and odd symmetry with respect to the resonator axis. The coupling disappears when the mirrors are perfectly aligned. New results for <u>very small</u> misalignment, inadequately explored in the literature, were found from a careful study of the transverse

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resonance equation for different ranges of equivalent Fresnel numbers, N_{eq}.[6]

The results obtained for round-edged mirrors were entirely new.[5] Of particular interest was the demonstrated existence of an azimuthally symmetric low-loss detached mode for spherical mirror structures; mode detachment here is not observed in the sharp-edged mirror case. For strip mirrors (two-dimensional case), edge rounding was shown to introduce detachment of the azimuthally symmetric mode at lower values of N_{eq}, and increased mode separation (see Appendix A). The waveguide method was shown to be effective for simple calculation of the eigenvalues, and for cogent interpreation of the complicated eigenmode charts. This is due to the fact that coupling between only two systematically selected modes is adequate to explain the resonances. However, the more sensitive eigenmode fields on the mirrors are not well enough expressed in terms of this simple model; coupling to many modes would be required to yield adequate mode shapes. For this reason, the waveguide method is not regarded as an attractive alternative for computation of unstable resonator modal fields.

2. Ray-Optical Analysis

By an alternative approach, an eigenmode in the unstable resonator may be regarded as being established by self-consistent treatment of the mirror-edge-diffracted ray-optical fields that find their way back into the resonator by direct and multiply reflected paths between the mirrors. This direct ray tracing method, performed according to the rules of GTD, is hampered by the existence of ray fields with very many reflections, thereby making direct summation over these fields impractical and inaccurate. A breakthrough in our analysis of this problem has been the ability to deal collectively with the higher order reflected rays.[7] Thus, the resonant field is represented in terms of a selected number of lower-order reflected rays plus a collective ray that accounts for the cumulative effect of all higher order reflections. This procedure was found to yield not only a numerically attractive and accurate alternative to the "brute-force" method of integrating the resonator integral equation, but it also furnishes some penetrating insights into the behavior of the unstable resonator in purely ray-optical terms, without recourse to the resonator integral equation.

This study was motivated by the asymptotic theory for the resonator integral equation developed by Horwitz[8] and applied by him[9] and others[10] to various resonator configurations. Here, it was shown that the solution

of the eigenvalue problem could be reduced to solution of a polynomial equation, and that the corresponding eigenmode fields can be represented in terms of a set of certain wave functions. The method, which was found to yield remarkable agreement with eigenvalues and eigenmode shapes obtained by the direct numerical procedure, is based on a postulated form of the solution (ansatz). While the postulates were qualitatively identified with various ray-optical phenomena, no quantitative correspondence and interpretation was established.

Our ray-optical method provides a complete and quantitative ray-optical theory of the unstable resonator. The results of the Horwitz method emerge from our analysis as a special case, and each element in the Horwitz ansatz is derived deductively. To test our method, it was first applied (with complete success) to symmetric, aligned, sharp-edged strip[7] and spherical mirror[11] structures. It was also applied to the round-edged mirror case.[12] From the understanding of the physical phenomenology gained thereby, it appeared feasible to attack more systematically other resonator configurations that depart from the idealized shapes investigated originally. A first step in this direction was an application of the ray method to half-symmetric resonators with internal axicon (HSURIA),[13] which provided an understanding of the effect of the internal axicon reflector, and of the influence of shielding of the axicon tip.

B. Summary of the Ray Method

As noted in Section A2, a key feature in our procedure has been the ability to show that ray-optical phenomena descriptive of high-order multiple reflections can be treated collectively. Thus, our approach involves the simultaneous use of individually tracked ray fields as well as collective ray fields that resemble modal congruences. Modal ray congruences describe self-consistent fields in resonators with infinite mirrors,[2] and they are exemplified in a special case by the two self-replicating cylindrical wave fronts of Siegmann. More generally, these wave fronts, and the associated normal ray congruences, are generated by elliptical caustics when the infinite mirrors have hyperbolic shape.

A second key feature has been our ability to construct the resonance equation for the eigenvalues and eigenmodes by invoking self-consistency of the modal fields without recourse to the integral equation.[7] Thus, both

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the fields per se and the resonance condition have been formulated entirely within a deductive ray-optical framework.

Our procedure for the symmetrical strip resonator involves the following steps:[13]

1. Determining the collective (modal) ray fields (self-replicating wave fronts and ray congruences) that describe the fields in a resonator with infinite hyperbolic mirrors. This incident modal field, which can be found by direct ray-optical techniques or by the WKB method, provides the excitation. Its precise form is determined by the elliptic caustic which is characterized by the initially assumed caustic parameter, p (Figs. 4(a) and (b)).

2. Accounting for edge diffraction, when the mirrors are finite, by postulating equivalent edge line currents with non-isotropic radiation pattern (Fig. 5). The pattern function $f(\theta, \theta_i)$ in the direction θ is determined from the known solution of edge diffraction by a half-plane when an incident plane wave arrives at the angle θ_i. The local modeling of the curved mirror near the edge by a plane surface and of the modal ray field incident on the edge by a plane wave is legitimized by the rules of GTD. The starting amplitude of the field on each edge-diffracted ray thus depends on θ_i which, in turn, depends on the caustic parameter p.

3. Tracking of edge-diffracted ray fields into the resonator. This tracking takes place via the well-known rules of geometrical optics, relying on optical path lengths to establish phase (Fig. 6(a)) and on conservation of energy in a ray tube to establish amplitude (Fig. 6(b)). Various species of multiply reflected rays are identified in Fig. 6(a). Some of these are reflected out before they reach the resonator axis while others pass through the resonator from one side to the other.

4. Summing of edge-diffracted ray fields with many reflections into a collective closed form. These collective rays appear to emanate from the foci of the hyperbolic mirror system and thus resemble the self-replicating modal ray fields that provided the initial excitation (Fig. 7).

5. Determining the total singly diffracted ray-optical field. The field due to single diffraction at each edge, as observed at a point within the resonator, consists of 2N individually reflected ray fields plus the field

carried along the collective ray, which accounts for all ray fields having undergone more than N reflections (Fig. 8). The choice of N is not critical but must be large enough to legitimize the collective treatment.

6. Determining the ray-optical field due to double diffraction. When the observation point is chosen at an edge, the (2N+1) singly diffracted ray fields in Fig. 8 provide the excitation for double diffraction. The resulting doubly diffracted ray fields are tracked through the resonator in precisely the same fashion as the singly diffracted fields. However, the excitation amplitude of each ray species now depends on the angle of incidence θ_n of each of the (2N+1) singly diffracted rays, thereby requiring the pattern function $f(\theta, \theta_n)$ instead of $f(\theta, \theta_i)$ (Fig. 9).

7. Establishing the resonance condition. Resonant solutions for the unstable open resonator require self-sustaining fields in the absence of excitation. Thus, the field incident on an edge after a certain number of diffractions must be identical with the field incident on the same edge before the last diffraction took place. Therefore, with reference to the above, the singly diffracted fields incident on an edge must be the same as the doubly diffracted fields (Figs. 8 and 9). Imposition of this condition yields the equation for the resonant eigenvalues. These eigenvalues are directly related to the caustic parameter p in Fig. 4a. In essence, the resonant p values adjust the initially assumed incident ray congruences so that the resulting field in the finite mirror structure is self-resonant.

8. Determining the eigenmode fields. These are given by the (2N+1) ray-optical fields when the resonant eigenvalues are inserted for the caustic parameter p that appears in each of the ray field expressions.

The details of the above-described formulation have been given in references 7 and 13. It has also been shown there how each of the wave functions and recursion relations in the Horwitz ansatz[8] can be interpreted ray-optically, and how the ray-optically derived resonance condition reduces to that of Horwitz after certain approximations. The same correspondence between the ray-optical approach and the procedure of Butts and Avizonis[10] for symmetric circular mirror configurations has also been demonstrated.[11] Here, the simple ray-optical method requires correction near the resonator axis where edge diffracted fields due to the circular rim accumulate.

C. Implications of the Ray Method

The successful ray-optical solution of the ideal symmetric strip and circular mirror resonators has demonstrated two features with major implications for more general problems:

1. The ability to construct the field within the resonator by _local_ tracking along geometric-optical ray paths, with the possibility of summing ray fields with many reflections into a _collective_ form.

2. The ability to construct a resonance equation by invoking self-consistency on the ray fields before and after a successive diffraction event.

Feature 1 implies that modifications of the ideal geometry can be accounted for by corresponding modifications of the appropriate ray fields. This is particularly relevant when these modifications occur locally (as when the medium is inhomogeneous, or when there are local perturbations on the mirrors) since the rays describe locally tracked fields. The ray tracking also clarifies obscuration and blocking effects introduced by axicons in annular resonator configurations, by scraper mirrors or by coupling apertures.

Feature 2 implies that once the ray structure and the ray fields have been determined, these fields can be used _directly_ to construct the resonance equation without the need for going through the resonator integral equation. In fact, by appropriate categorization of various ray species, one may ascertain the influence of each on the resonance behavior.

These aspects are illustrated for the HSURIA (half symmetric unstable resonator with internal axicon). Various possible ray species are depicted in Fig. 10. They are ordered like those in the equivalent symmetric circular mirror configuration, which can be constructed from the HSURIA by imaging and unfolding. Due to the blocking effect of the axicon, some of the rays of the unfolded structure are eliminated. The categorization into existing and blocked rays can be undertaken directly in the symmetric resonator if the position of various rays relative to the axicon tip location is observed (Figs. 11 and 12). The resulting resonance condition is the same as for the symmetric unfolded resonator provided that one eliminates therefrom the blocked rays. However, fields on rays reflected near the axicon tip should be modified to account for the blocking ("reflected-ray

boundary" effect). Tip diffracted rays could be included to refine the resonance condition.

D. Conclusion

The ray method has been developed sufficiently to warrant application to various problem areas related directly to high-power laser systems now under development, and to provide alternatives to, and independent checks of, the purely numerical procedures now in use. Some topics for investigation have been suggested by us, which address remaining questions in the conventional symmetrical mirror unstable resonator, and also in annular resonators with conical elements, which are gaining increased importance. However, these studies, which include effects of scraper mirrors, coupling apertures in injection-locked resonators, medium inhomogeneities, etc., could not be performed within the grant period.

II. Personnel

The following individuals were associated with the research performed under this grant:

Dr. L. B. Felsen, Institute Professor (Principal Investigator)

Dr. S. H. Cho, Research Assistant Professor

Dr. S. Y. Shin, Post-doctoral Fellow[*]

Dr. C. Santana, Consultant[**]

Dr. G. Whitman, Consultant

III. Publications

The following publications resulted from research conducted under this grant:

A. Technical Journals

C. Santana and L. B. Felsen, "Unstable Strip Resonators with Misaligned Circular Mirrors," Appl. Opt. 17 (1978), p. 2352.

C. Santana and L. B. Felsen, "Eigenvalues for Unstable Resonators with Slightly Misaligned Strip Mirrors," Appl. Opt. 19 (1980), p. 3189.

C. Santana and L. B. Felsen, "Mode Losses in Unstable Resonators with Rounded Edges," Appl. Opt. 17 (1978), p. 2239.

S. H. Cho, S. Y. Shin and L. B. Felsen, "Ray-Optical Analysis of Cylindrical Unstable Resonators," J. Opt. Soc. Am. 69 (1979), p. 563.

S. H. Cho and L. B. Felsen, "Ray-Optical Analysis of Unstable Resonators with Spherical Mirrors," J. Opt. Soc. Am. 69 (1979), p. 1377.

C. Santana, "Ray-Optical Calculation of Eigenmode Behavior of Unstable Laser Resonators with Rounded Edges," submitted to Appl. Opt. (see Appendix A).

S. H. Cho and L. B. Felsen, "Ray Method for Unstable Resonators," Appl. Opt. 19 (1980), p. 3506.

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[*]Now at the Korean Advanced Institute of Science, Seoul, Korea.

[**]Formerly at the Polytechnic Institute of New York, now at the Institute for Space Research (INPE), San Jose dos Campos, San Paulo, Brazil.

B. Reports and Memoranda

S. H. Cho and L. B. Felsen, "Ray Analysis of Unstable Laser Resonators -- A User's Manual," Department of Electrical Engineering and Computer Science, Polytechnic Institute of New York, Farmingdale, New York 11735.

G. M. Whitman and L. B. Felsen, "Axial Focusing Due to Reflection at an Interior Cone Surface," Department of Electrical Engineering and Computer Science, Polytechnic Institute of New York, Farmingdale, New York 11735.

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1. L. W. Chen and L. B. Felsen, "Coupled Mode Theory of Unstable Resonators," IEEE J. Quantum Electronics QE-9 (1973), p. 827.

2. C. Santana and L. B. Felsen, "Unstable Open Resonators: Two-Dimensional and Three-Dimensional Losses by a Waveguide Analysis," Appl. Opt. 15 (1976), p. 1470.

3. C. Santana and L. B. Felsen, "Ray-Optical Calculation of Edge Diffraction in Unstable Resonators," IEEE Trans. Microwave Theo. and Techniques MTT-26 (1978), p. 101

4. C. Santana and L. B. Felsen, "Unstable Strip Resonators with Misaligned Circular Mirrors," Appl. Opt. 17 (1978), p. 2352.

5. C. Santana and L. B. Felsen, "Mode Losses in Unstable Resonators with Rounded Edges," Appl. Opt. 17 (1978), p. 2239.

6. C. Santana and L. B. Felsen, "Eigenvalues for Unstable Resonators with Slightly Misaligned Strip Mirrors," Appl. Opt. 19 (1980), p. 3189.

7. S. H. Cho, S. Y. Shin and L. B. Felsen, "Ray-Optical Analysis of Cylindrical Unstable Resonators," J. Opt. Soc. Am. 69 (1979), p. 563.

8. P. Horwitz, "Asymptotic Theory of Unstable Resonator Modes," J. Opt. Soc. Am. 63 (1973), p. 1529.

9. P. Horwitz, "Modes in Misaligned Unstable Resonators," Appl. Opt. 15 (1976), p. 167.

10. R. R. Butts and P. Avizonis, "Asymptotic Analysis of Unstable Laser Resonators with Circular Mirrors," J. Opt. Soc. Am. 68 (1978) p. 1072.

11. S. H. Cho and L. B. Felsen, "Ray-Optical Analysis of Unstable Resonators with Spherical Mirrors," J. Opt. Soc. Am. 69 (1979), p. 1377.

12. C. Santana, "Ray-Optical Calculation of Eigenmode Behavior of Unstable Laser Resonators with Rounded Edges," submitted to Appl. Opt. (see Appendix A).

13. S. H. Cho and L. B. Felsen, "Ray Method for Unstable Resonators," Appl. Opt. 19 (1980), p. 3506.
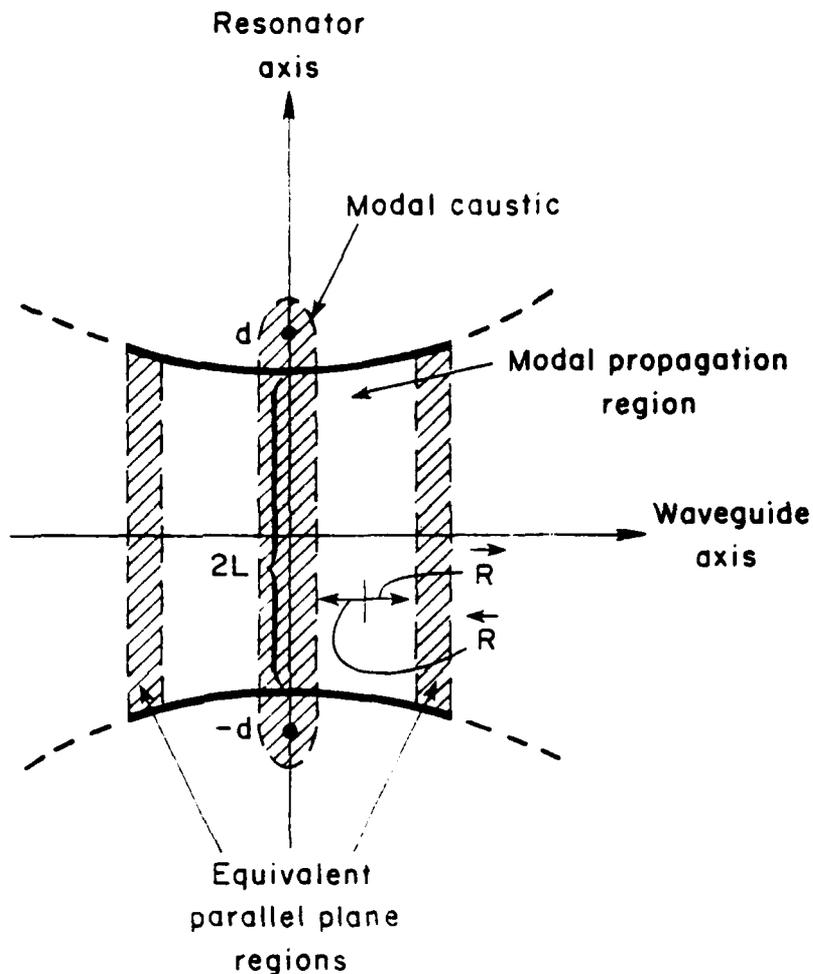
Fig. 1. Waveguide model of the unstable resonator with hyperbolic
mirrors. A waveguide mode in the infinite mirror structure
(solid plus dashed curves) is described by an elliptic caustic
with foci at $\pm d$ and caustic parameter p. Truncation of the
mirrors (solid curves only) causes waveguide mode coupling
and reflection. To determine the mode coupling and reflection
coefficients, the local regions near the edges may be regarded
either as equivalent parallel plane regions (for small Fresnel
numbers), or as individual equivalent half-plane scattering
centers (for large Fresnel numbers). Resonance defining the
resonator eigenmodes is established by invoking a "transverse
resonance" condition on the waveguide modes, which ensures
self-consistent reflection of modal fields traveling to the
right ($\vec{R}$) and left ($\overleftarrow{R}$).
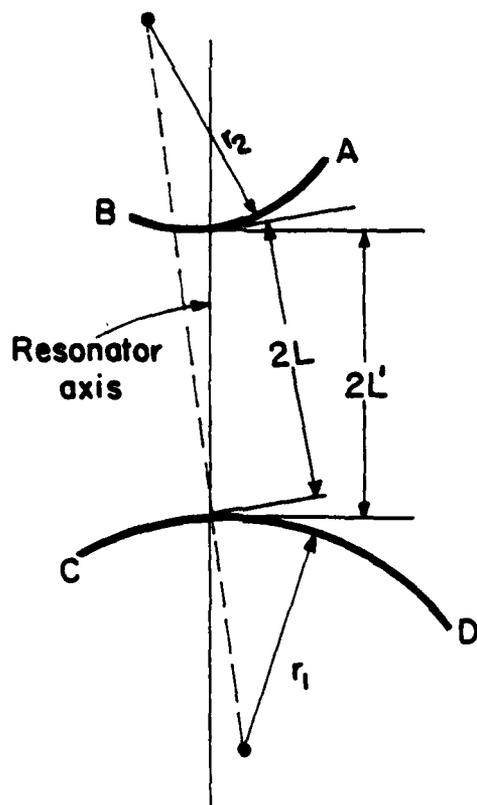
a) misaligned resonator
(no symmetry with
respect to axis)
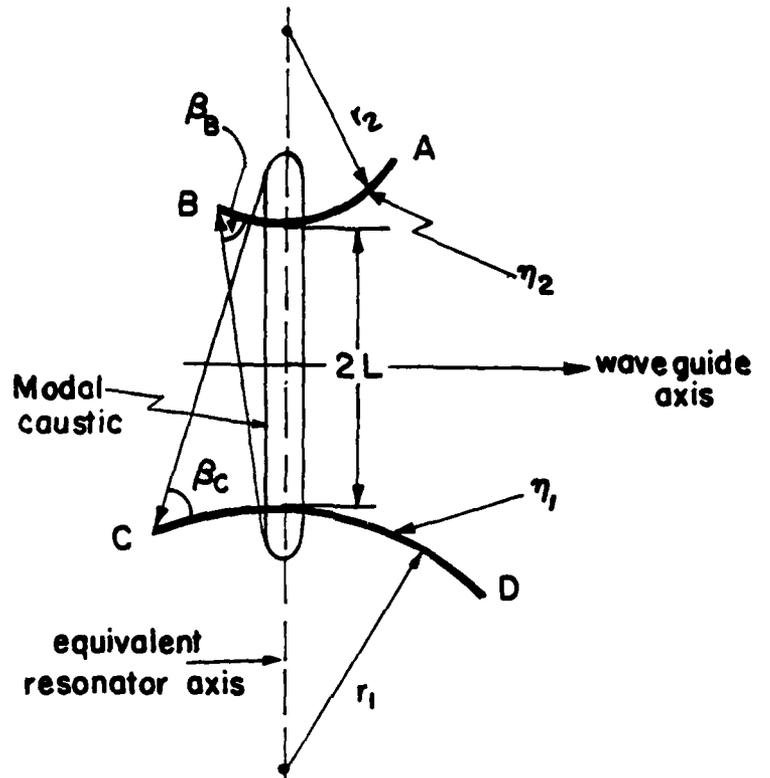
b) equivalent resonator with unequal mirror length (symmetry with respect to axis); η_1 and η_2 are hyperbolic surfaces in an elliptic coordinate system. β_B and β_C are the angles made by the modal ray at the edges B and C, respectively.

Fig. 2 - Resonator with misaligned circular strip mirrors
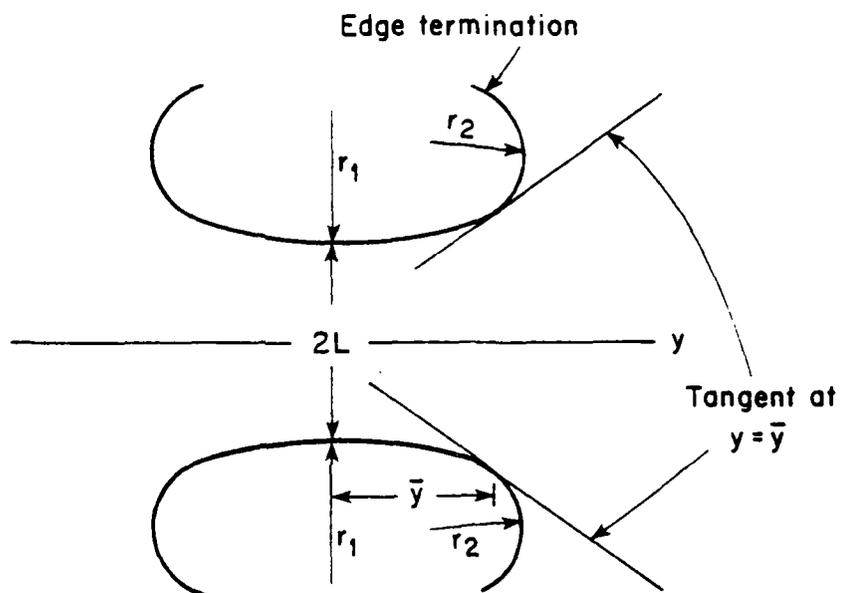
Fig. 3. Resonator with rounded edges. At $y = \bar{y}$, a section of waveguide with circular cylindrical boundaries of radius r_2 is joined smoothly to a cylindrical resonator which otherwise would be sharp-edged.
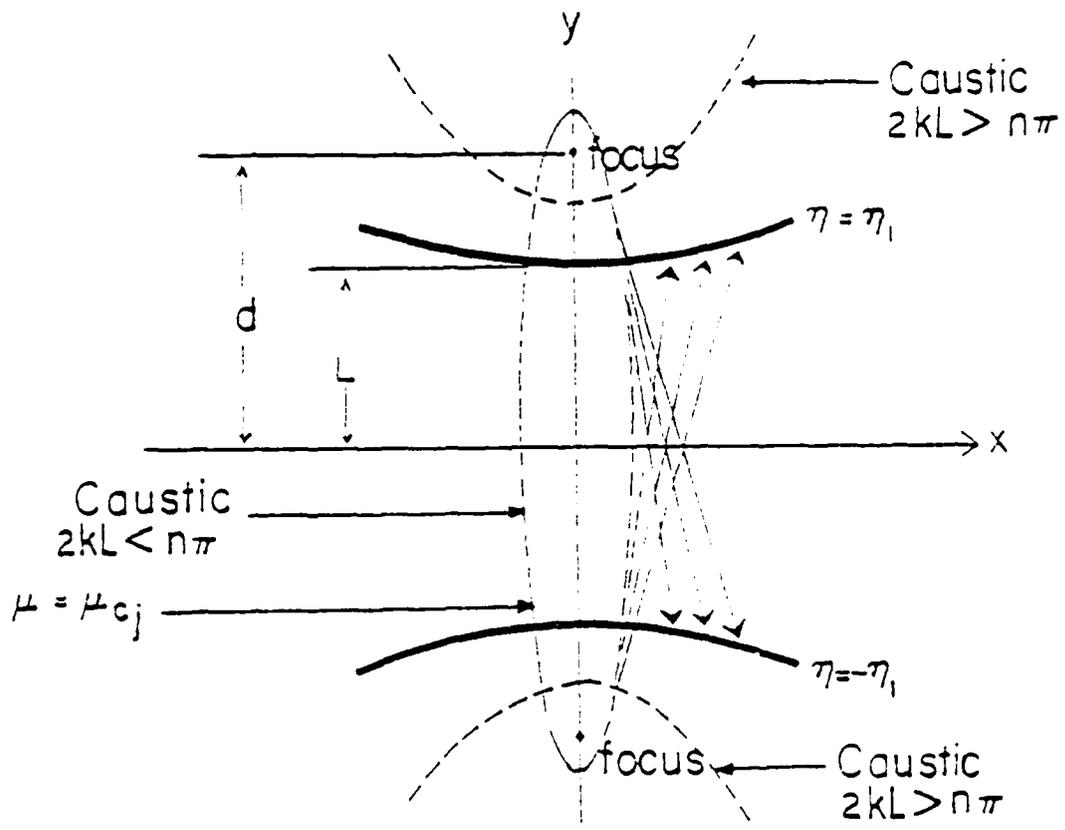
Fig. 4(a). Ray congruences for self-consistent field between infinite, hyperbolic strip mirrors: caustic for $2kL \neq n\pi$. The caustic parameter p is related to the ellipse $\mu = \mu_{cj}$.
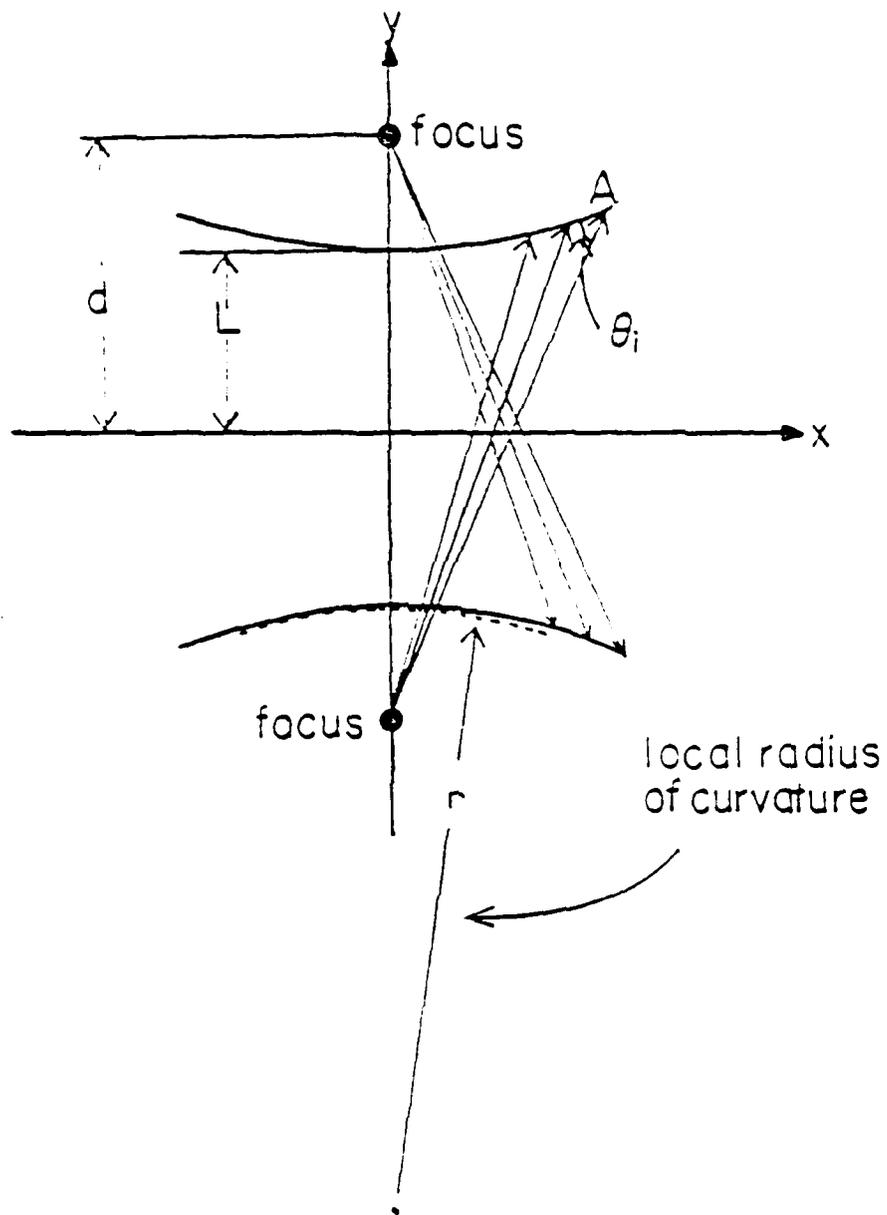
Fig. 4(b). Ray congruences for self-consistent field between infinite hyperbolic strip mirrors: caustic for $2kL = n\pi$.
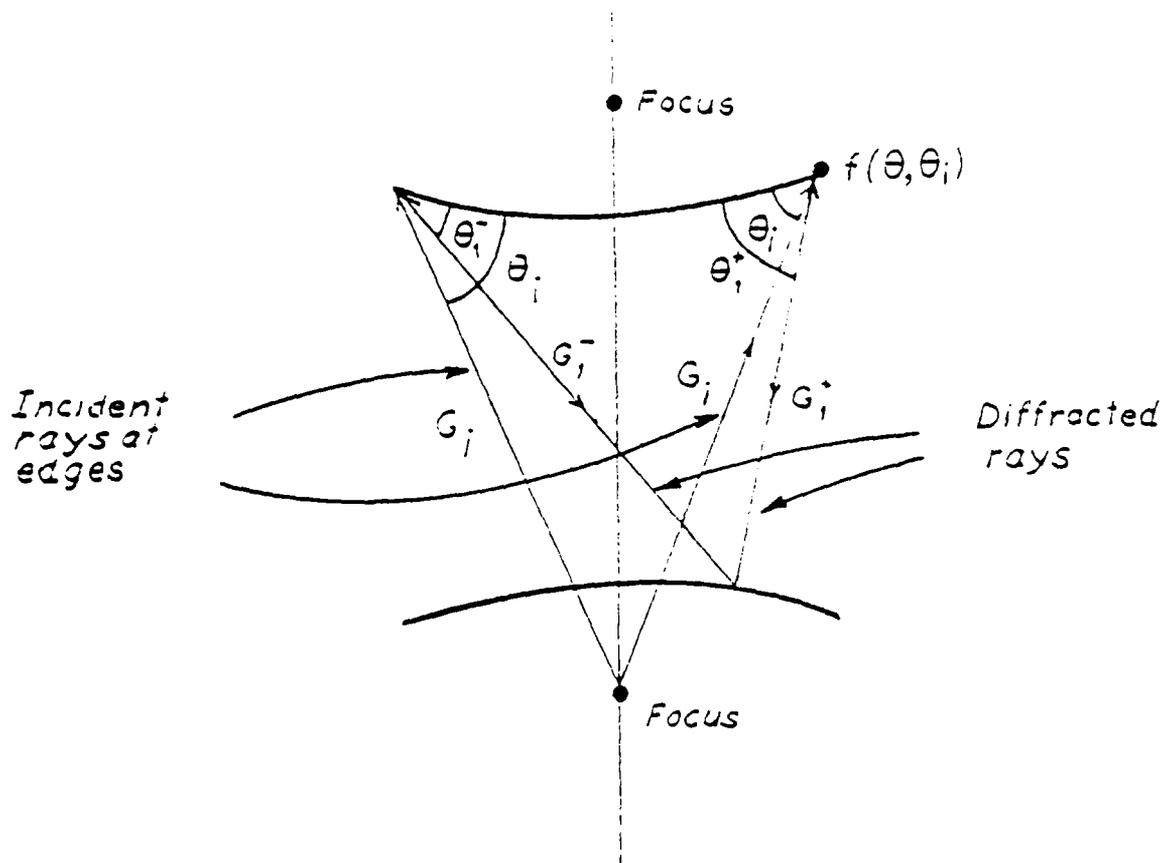
Fig. 5.   Diffracted rays from mirror edges.

Fig. 6(a) Reflection points x_m^- and x_m^- of multiply reflected rays originating at A and A', respectively.
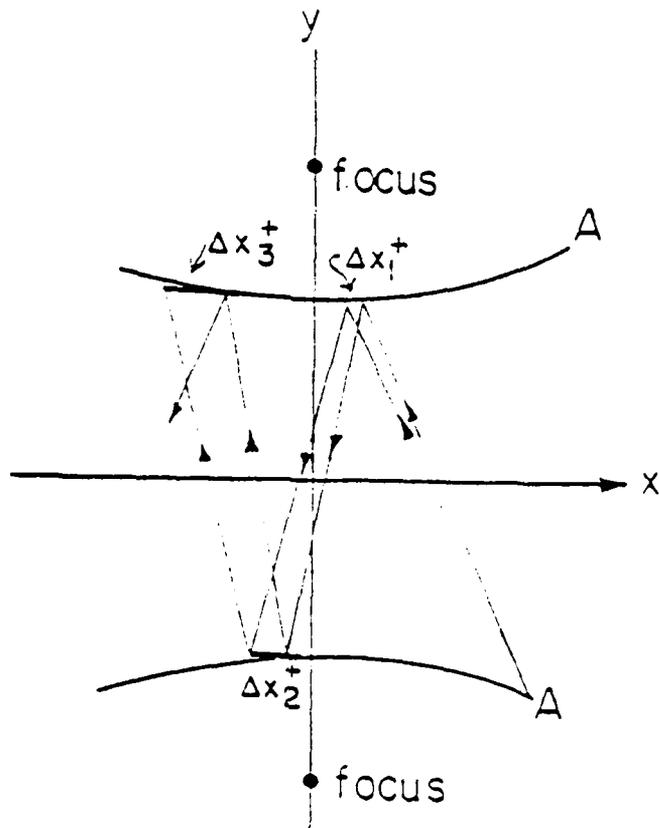
Fig. 6(b).   Divergence of ray tube due to reflection from the curved mirror surfaces.
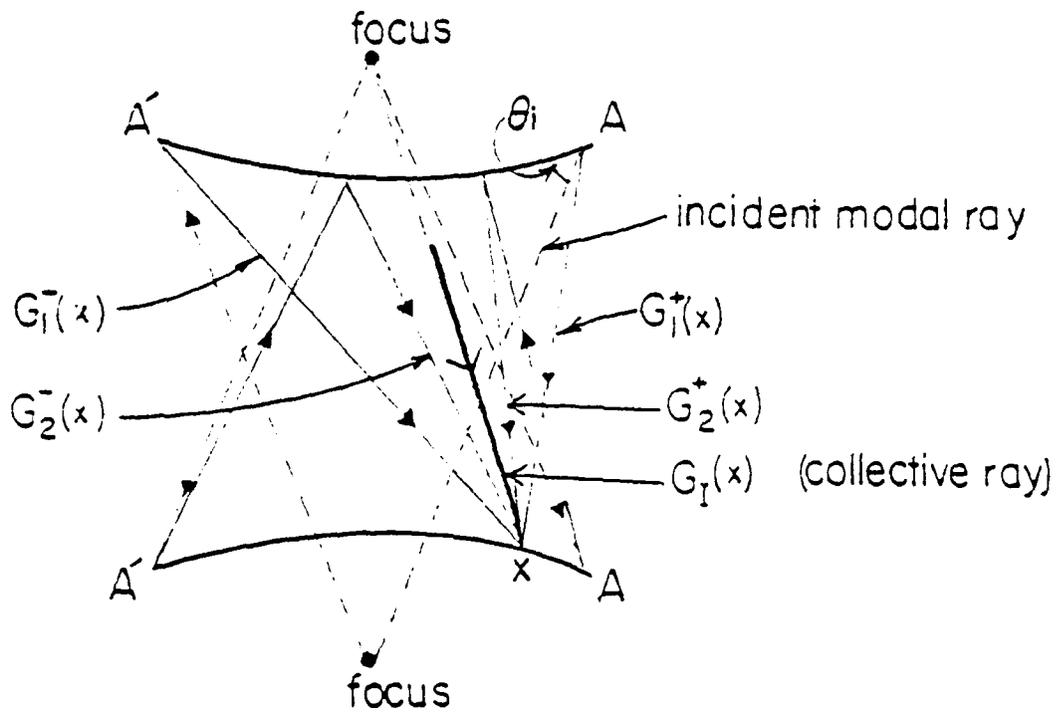
Fig. 7. Singly diffracted ray fields $G_n^{\pm}(x)$ reaching an observation point x on the lower mirror after $(n-1)$ reflections; the incident rays (shown dashed) belong to the modal field between infinite hyperbolic mirrors. $G_I(x)$ represents the collective ray field for $n > N$.
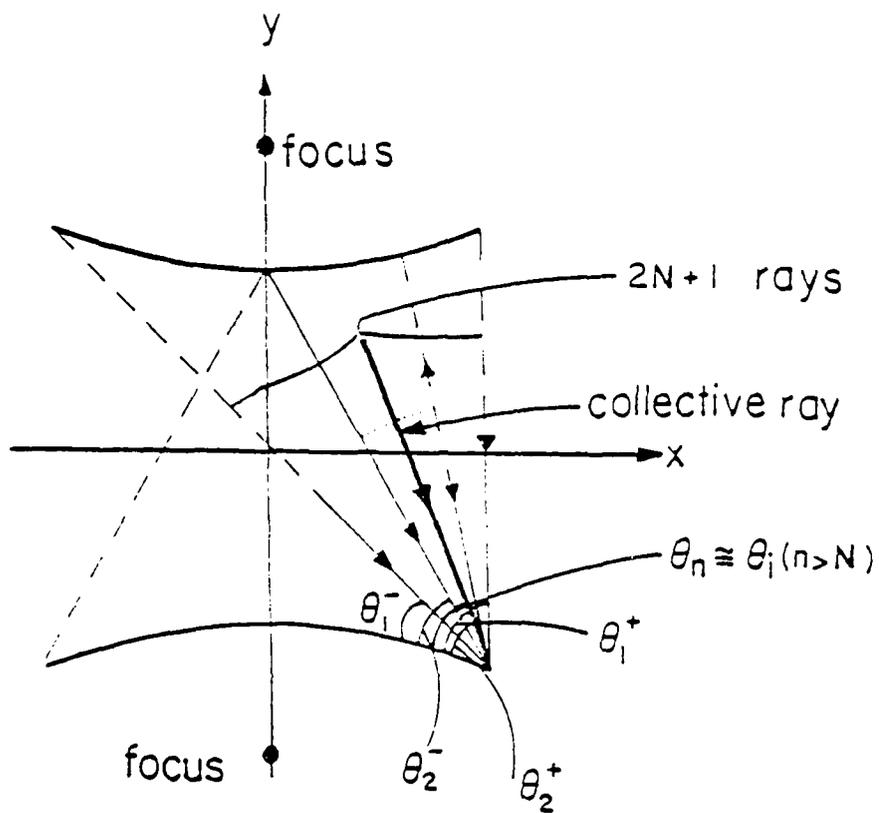
Fig. 8.   Total ray field at lower right-hand edge due to primary diffraction of incident field.
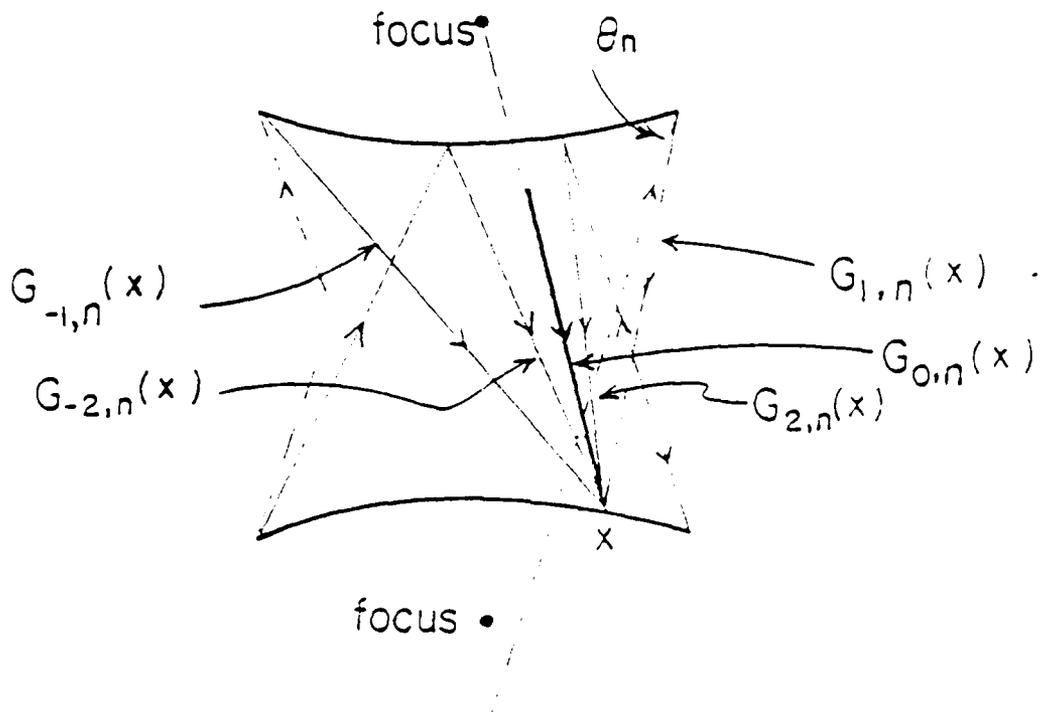
Fig. 9. Doubly diffracted ray fields $G_{\ell,n}(x)$ reaching an observation point x on the lower mirror after $(|\ell|-1)$ reflections, the excitation being a unit amplitude incident ray field (shown dashed) making the angle θ_n at the edge. $G_{0,n}(x)$ represents the collective ray field for $|\ell| > N$.
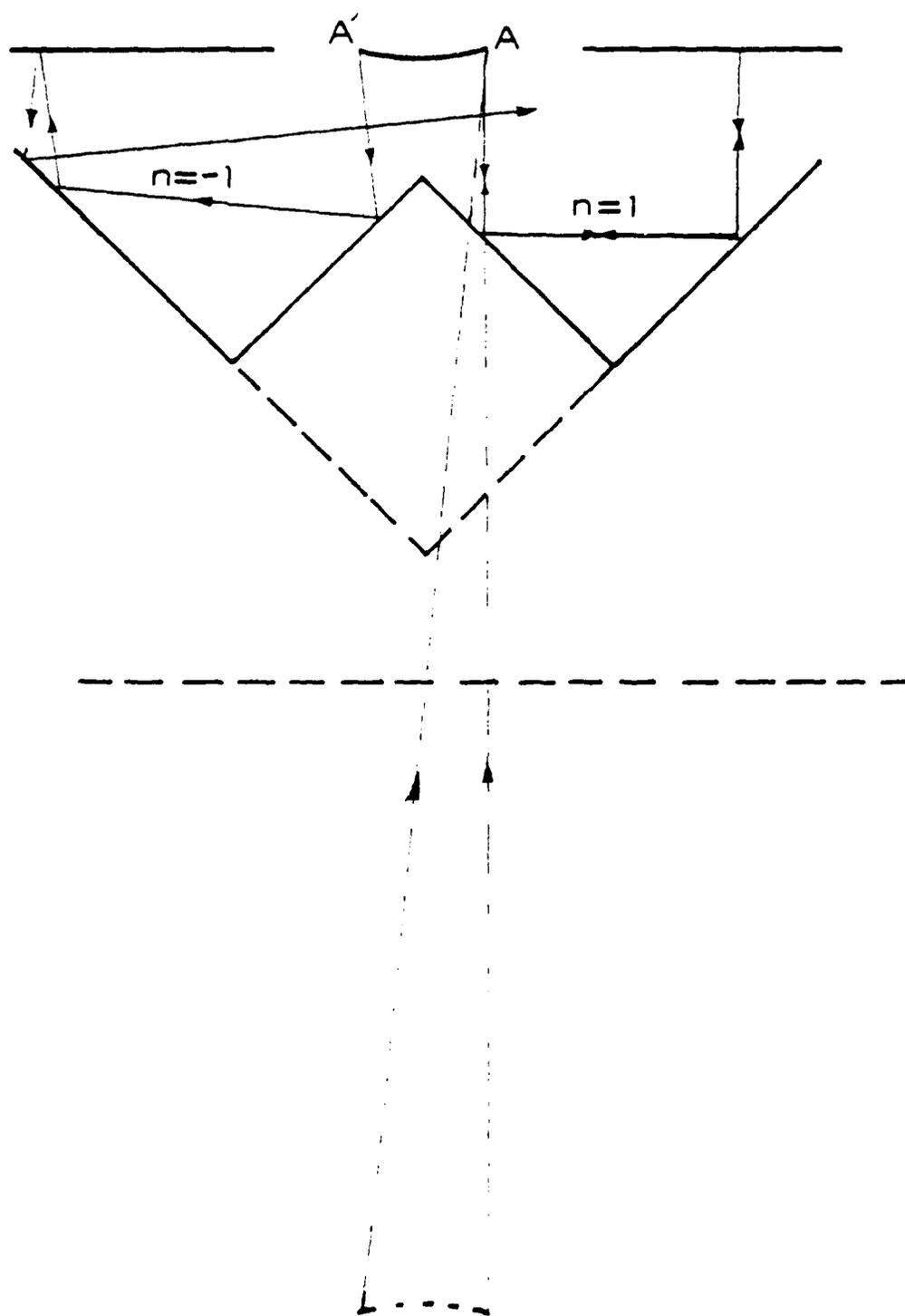
Fig. 10(a). Trajectories for ray species $n = \pm 1$ in the HSURIA. The $n = -1$ ray returns to the point A on the edge where it originates but the $n = -1$ ray originating at A′ does not reach A. Equivalent trajectories in the unfolded symmetrical resonator are shown dashed.

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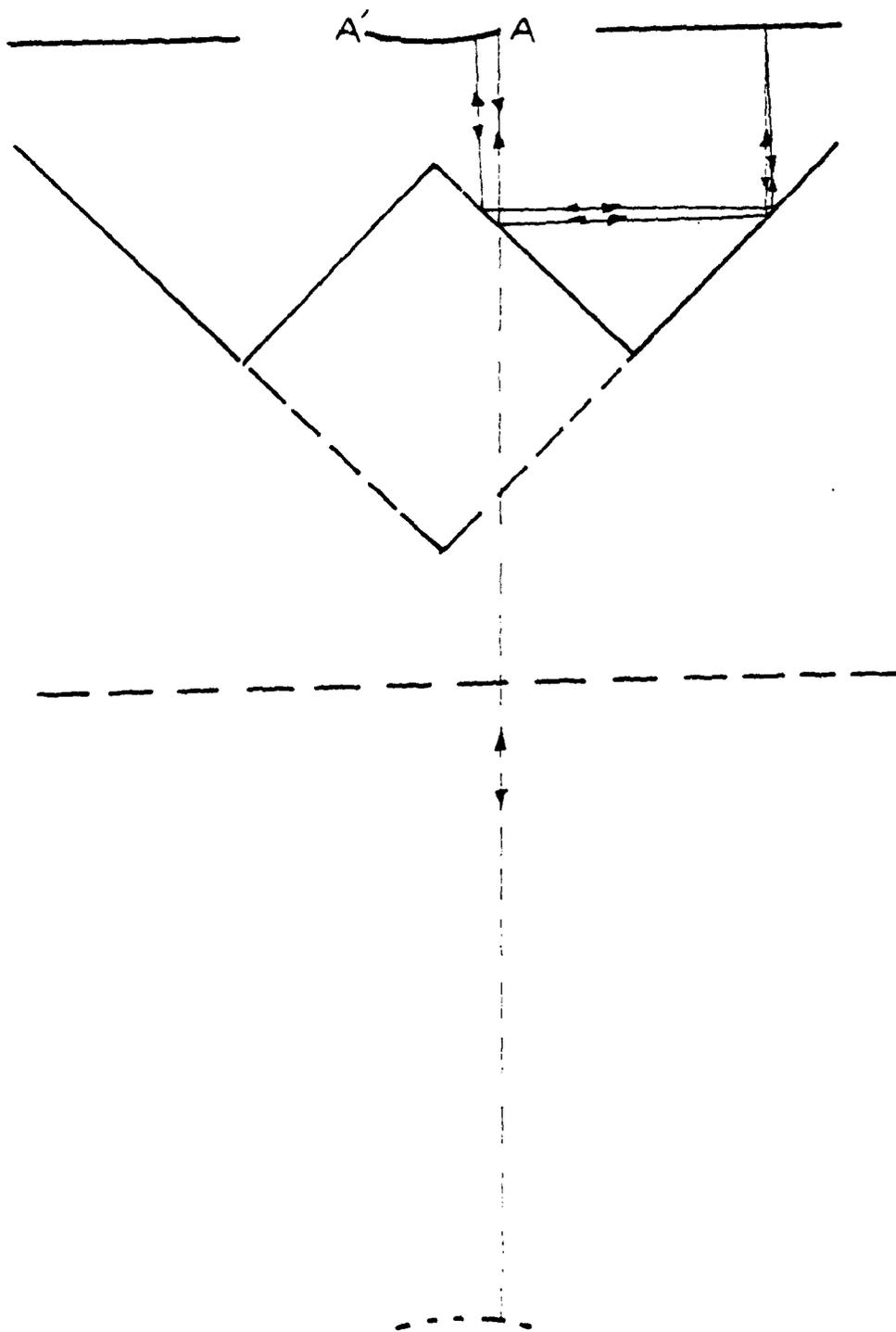
Fig. 10(b).   Another trajectory to A for the n = 1 ray species.
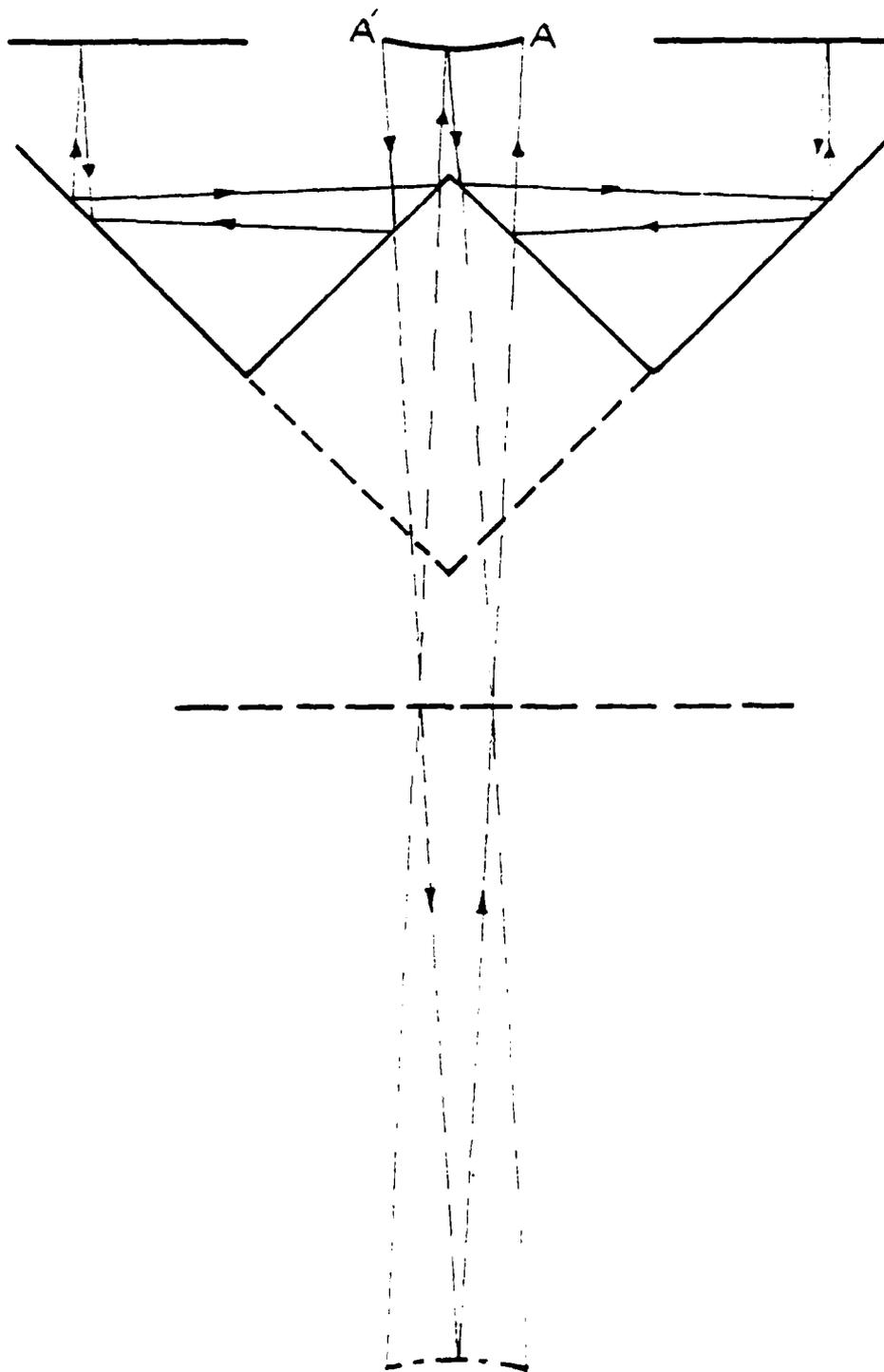
Fig. 10(c).   Trajectory from A′ to A for n = -2 rw species.
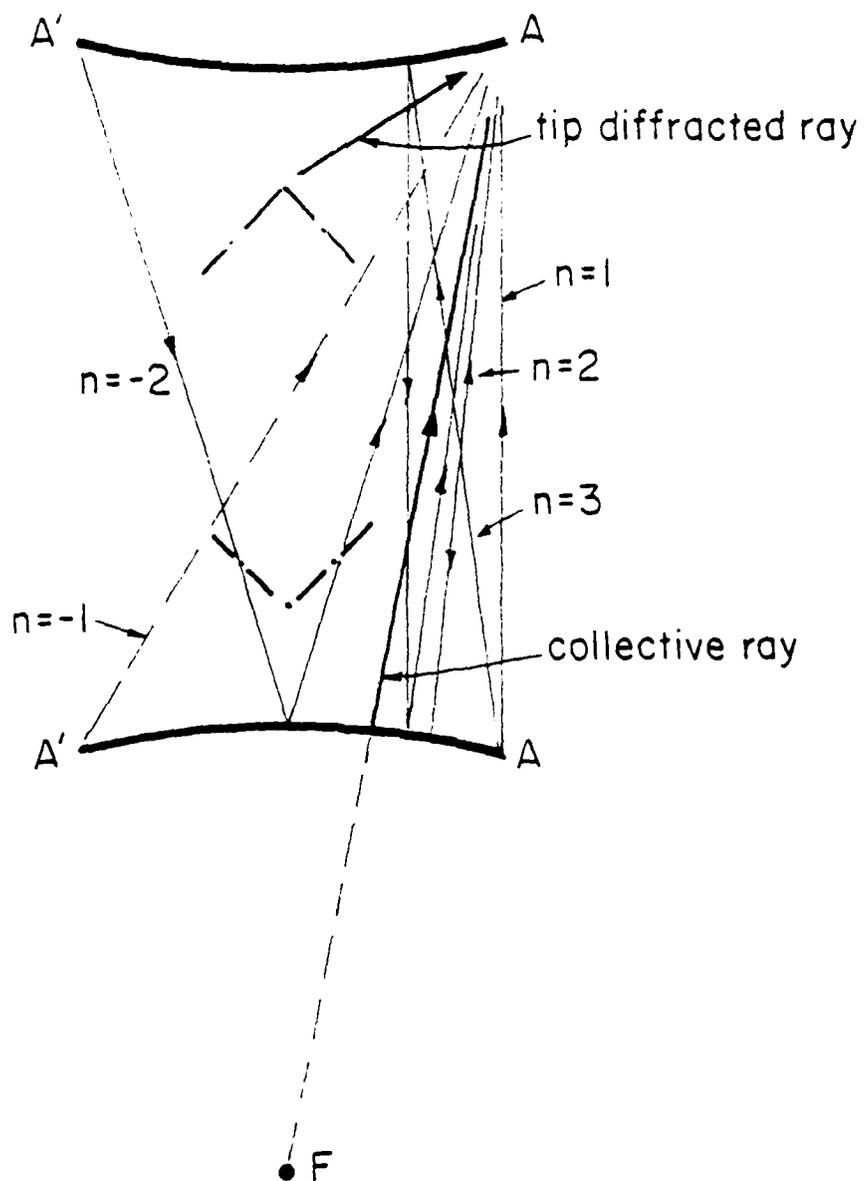
Fig. 10(d). Trajectories to A for various ray species, dealing directly with the
unfolded structure. The dashed ray n = -1 and others in this category
(not shown) are eliminated by the axicon.
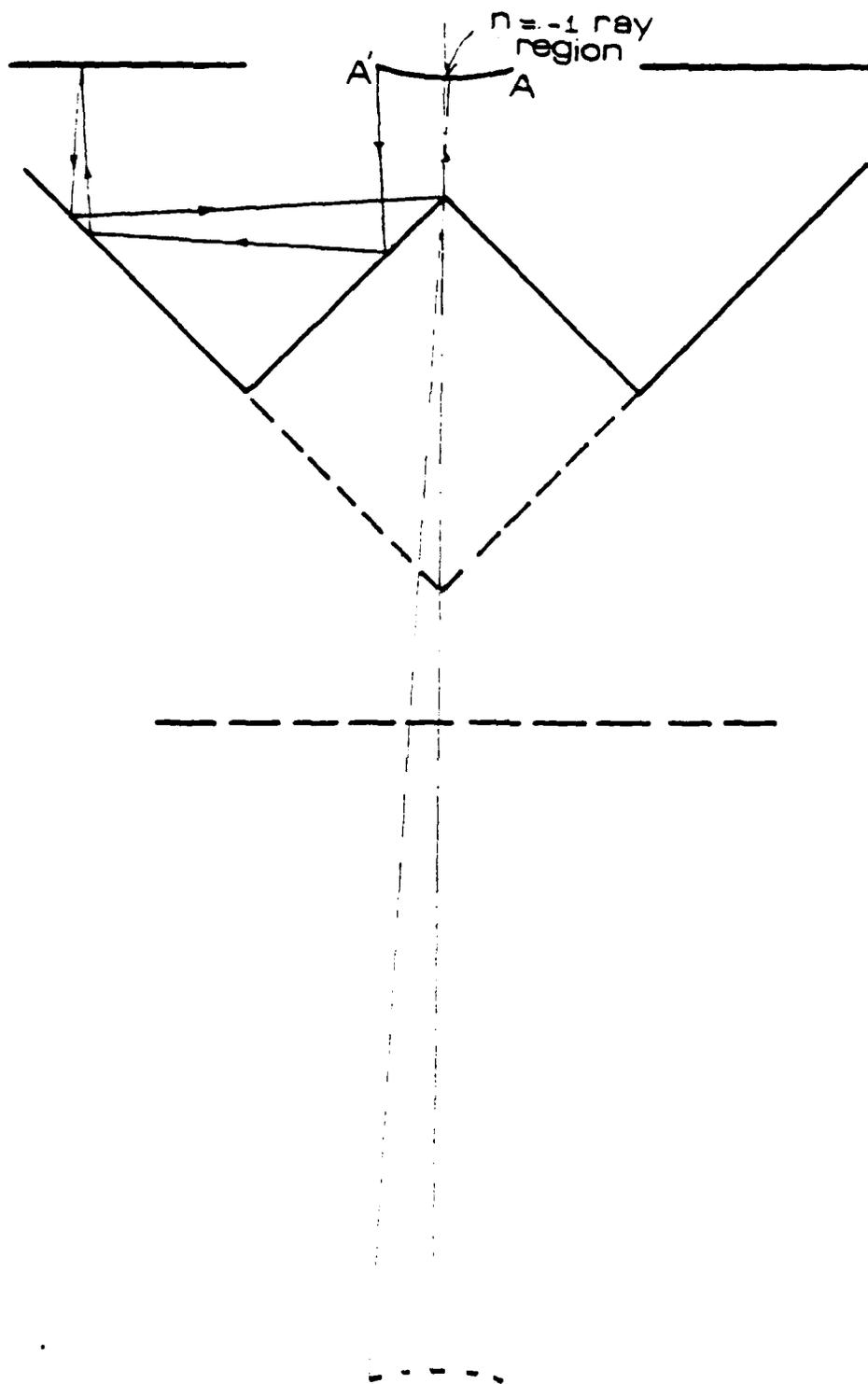
Fig. 11(a). Domain of existence of n = -1 ray to the right of the resonator axis.
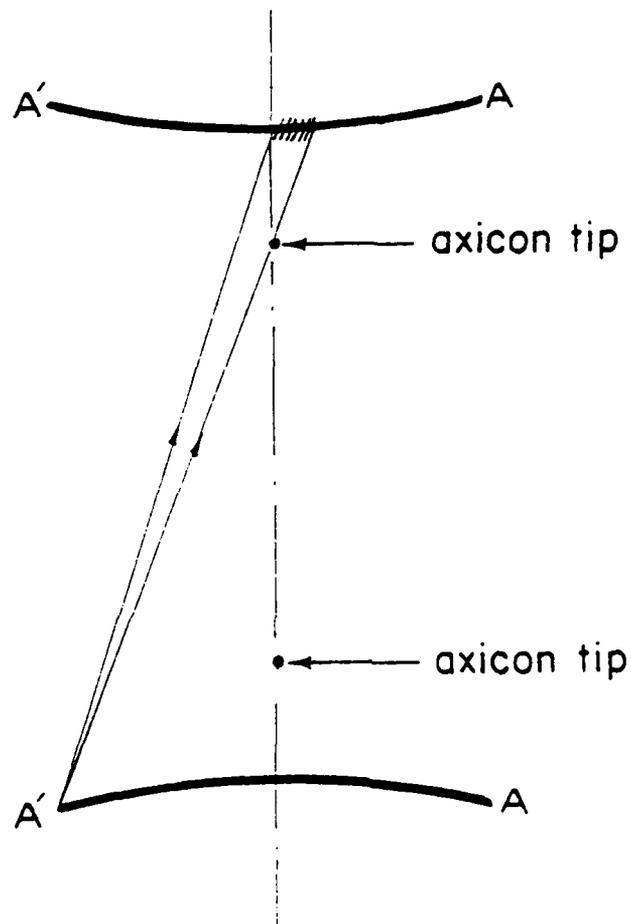
Fig. 11(b). Equivalent construction of the domain in Fig. 11(a) (shown
shaded) directly in the unfolded configuration, noting placement
of the axicon tip.
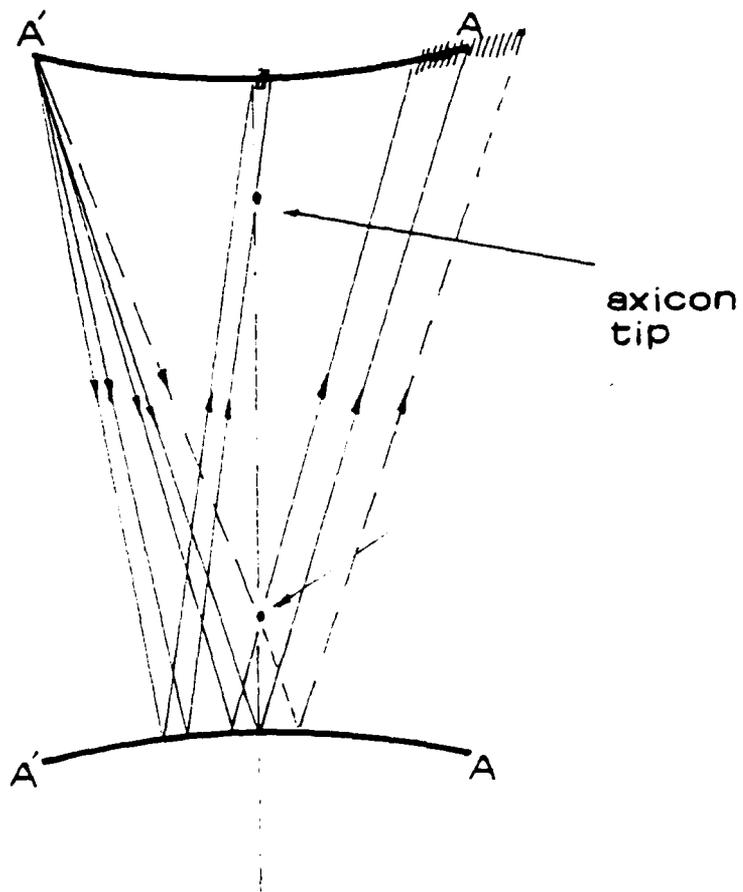
Fig. 12. Construction of the domain of the n = -2 ray (shown shaded) by the procedure in Fig. 11(b).

APPENDIX A

Submitted to Applied Optics

RAY-OPTICAL CALCULATION OF EIGENMODE BEHAVIOR OF UNSTABLE

LASER RESONATORS WITH ROUNDED EDGES

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ABSTRACT

The eigenmode resonance equation previously developed by the ray-optical method of analysis for unstable, symmetric, bare resonators with strip and spherical mirrors is employed here to establish the basic properties of resonators with rounded mirror edges. The numerical results obtained in this manner are far more comprehensive than those presented earlier from a waveguide model approach to the resonator problem. As in this earlier study, edge rounding is found to improve mode separation and to shift the onset of low-loss mode detachment to lower Fresnel numbers. It is also found that a very simple closed form expression predicts quite accurately the low-loss mode behavior after it has detached from the remaining mode spectrum.

I. Introduction and Summary

Edge rounding of strip mirrors in unstable symmetric laser resonators initiates detachment of the low-loss eigenmode at small values of the equivalent Fresnel number. For the spherical mirror geometry, rounded edges produce a low-loss detached mode that is not found in the sharp-edged configuration. Mode discrimination is improved in both cases. Because of these attractive features which have been pointed out in a previous publication[1], it is desirable to generate detailed eigenmode charts that exhibit the dependence of the resonance on the edge radius. The resonator edges are modeled by an abrupt change in the mirror curvature as shown in Fig. 1. The termination of the added mirror section is assumed to be constructed so as to render its effect negligible.

Some preliminary results obtained via the waveguide approach to the resonator problem have already been presented[1]. The waveguide resonance equation is convenient for predicting the physical mechanism responsible for the resonance and for dealing with the lowest-order detached mode, but it is less easily implemented for determination of the behavior of the higher order modes. An alternative view of the resonant wave process is obtained by a ray-optical analysis[2-4] which leads to a resonance equation that is equally applicable to the lowest order and higher-order modes. The ray-optical resonance equation for round-edged mirrors[2] is employed for the generation of the data in this communication.

II. Strip Resonators

A. Resonance Equation

From the analysis in Ref. 2, the eigenvalues λ for the even and odd symmetric modes in unstable strip resonators are given by the polynomial equation

$$(\lambda - 1)\lambda^N \left[1 + \epsilon_2 \sum_{\ell=1}^{N} \lambda^{-\ell} (1 - M^{-2\ell})^{5/2} \cdot \left\{ \frac{E_\ell(1)}{(1-M^{-\ell})^3} + \frac{E_\ell(-1)}{(1+M^{-\ell})^3} \right\} \right] +$$

$$+ 2\epsilon_2 \exp(i2\pi N_{eq}) = 0 \tag{1}$$

for the even modes, and

$$(\lambda M - 1)\lambda^N \left[1 + \epsilon_2 \sum_{\ell=1}^{N} \lambda^{-\ell} (1 - M^{-2\ell})^{5/2} \cdot \left\{ \frac{E_\ell(1)}{(1-M^{-\ell})^3} - \frac{E_\ell(-1)}{(1+M^{-\ell})^3} \right\} \right] +$$

$$+ 2\epsilon_2 M^{-N} \exp(i2\pi N_{eq}) = 0 \tag{2}$$

for the odd modes. The notation is the same as in Ref. 2, with

$$\epsilon_2 = \frac{(r/r_2 - 1)}{4\pi^2} \frac{M-1}{M+1} \left[\frac{\exp(i\pi/2)}{2 N_{eq}} \right]^{3/2} . \tag{3}$$

The power loss per transit is related to the eigenvalue λ and the magnification M by

$$PL = 100 \left(1 - |\lambda|^2/M\right) . \tag{4}$$

For the N_{eq} values for which the resonator loss approaches the geometrical optics value (corresponding to $|\lambda| \approx 1$), an approximate equation for the eigenvalue of the even symmetric mode, which is the lowest-loss mode[5], can be found by iteration as in Ref. 3:

$$\lambda \approx 1 - \frac{2\epsilon_2 \exp(i2\pi N_{eq})}{1 - 2\epsilon_2 N \exp(i2\pi N_{eq})} \tag{5}$$

provided that

$$\ln(4\pi N_{eq})/\ln M \ll N \ll \ln M/2|\epsilon_2| . \tag{6}$$

This simplified equation developed from the ray-optically derived polynomial resonance equation is in agr ment with that obtained from the waveguide analysis.

B. Numerical Results

The full resonance equation (1) has been solved numerically for the eigenvalues of an unstable strip resonator with rounded edges having a magnification factor M = 2.9. Figure 2 shows the magnitude of λ versus N_{eq} for an edge rounding factor $r/r_2 = 100$. It is seen that, in conformity with the results of Ref. 1, the essential effect of edge rounding in strip resonators is the formation of a low-loss detached mode at lower Fresnel numbers than for the sharp edge case. In a sharp edged strip resonator with M = 2.9, mode detachment occurs only for $N_{eq} > 9$, [5] in contrast to $N_{eq} > 4$ as seen from Fig. 2. A comparison of Fig. 2 with the plot of $|\lambda|$ for a sharp edged resonator with the same parameter values[5] reveals that edge rounding also enhances mode discrimination.

Figure 3 shows a plot of $|\lambda|$ versus N_{eq} for the same resonator as in Fig. 2, but with $r/r_2 = 30$. We note that as the edge effect is deemphasized by decreasing r/r_2, the geometrical optics value is attained for smaller N_{eq} values and the separation between the low-loss detached mode and the next higher order modes is increased.

The simplified eq. (5) can be utilized for the calculation of λ in the N_{eq} region where $|\lambda| \approx 1$, which generally corresponds to the detached mode region. In fact, for $N_{eq} > 2.5$, the $|\lambda|$ values for the lowest-loss modes of Figs. 2 and 3 if calculated via eq. (5), deviates negligibly from the values calculated by the full resonance equation. Equation (5) is also in excellent agreement with the power loss calculations effected by modal analysis[1].

In the numerical computations, it has been found that the parameter N, which characterizes the number of non-modal ray fields can be set in accordance with Horwitz's criterion $M^N = 250 N_{eq}$.

For plotting the eigenvalue cur.es, the roots of the full resonance equation
(1) were found at each N_{eq} value, in steps of 0.1, except in the vicinity of
those N_{eq} values where mode detachment was to be ascertained. There, the step size
was reduced to 0.01 and the detailed evolution of the complex values of λ was
determined in order to show which modes evolved into which. It is found that if
the step size is not reduced, one can easily arrive at a false conclusion about
the onset of mode detachment.

III. Spherical Mirrors Resonators

A. Resonance Equation

The polynomial equation for the eigenvalues λ of an unstable resonator with
round-edged circular hyperbolic mirrors can be found from the results of Ref. 2:

$$(\lambda M^m - 1)\lambda^N \left[1 + \frac{\overline{\delta}}{2} (-i)^m \sum_{\ell=1}^{N} \lambda^{-\ell}(1 - M^{-2\ell})^2 F_{\ell,m}(1) \right] +$$

$$+ \frac{\overline{\delta}}{m!} (-i)^m \exp(i2\pi N_{eq}) \left(\frac{2\pi N_{eq}}{M^N} \right)^m = 0 \tag{7}$$

where

$$\overline{\delta} = \frac{i(r/r_2 - 1)}{4\pi N_{eq}} \cdot \frac{M - 1}{M + 1}, \tag{8}$$

$$F_{\ell,m}(x) = 2 \exp\left[\frac{i2\pi N_{eq}}{1-M^{-2\ell}} \left(1 + \frac{x^2}{M^{2\ell}} \right) \right]$$

$$\cdot \left\{ \frac{J_m(Y_\ell) \left[1 + 3x^2 M^{-2\ell} \right] - i J_{m+1}(Y_\ell) xM^{-\ell} \left[3 + x^2 M^{-2\ell} \right]}{(1 - x^2 M^{-2\ell})^3} \right\}, \tag{9}$$

and the remaining terms are in conformity with the definitions of Ref. 4.

The power loss per tran .. is related to λ and M via

$$PL = 100 \left[1 - (|\lambda|/M)^2 \right]. \tag{10}$$

As in the cylindrical mirror case an approximate equation for the eigenvalues, valid when the 'ss approaches the geometrical optics limit $\lambda = 1$ for the azimuthally symmetric modes ($m=0$), can be found by iteration of eq. (7):

$$\lambda \approx 1 - \frac{\overline{\delta}\exp(i2\pi N_{eq})}{1 - \overline{\delta} N \exp(i2\pi N_{eq})} \tag{11}$$

with the restriction

$$\ln(4\pi N_{eq})/\ell nM << N << \ell nM/\overline{\delta} \tag{12}$$

B. Numerical Results

To assess the effects of edge rounding in spherical mirror unstable resonators, the full resonance equation (7) was solved numerically for various resonator parameters. Figures 4 and 5 show plots of $|\lambda|$ versus N_{eq} for a resonator with $M=2$ and $r/r_2 = 100$. The most obvious difference between these plots and their correspondents for a sharp edged resonator[6] is the appearance of a detached mode for $N_{eq} > 59$, thereby confirming the prediction made by modal analysis, that mode detachment does take place in three-dimensional resonators with rounded edges[1]. One may also observe that the eigenvalue approaches the geometrical optics limit $|\lambda| = 1$ faster than in the sharp-edged configuration. Another characteristic of edge rounding is illustrated in Fig. 6, which is a plot of $|\lambda|$ versus N_{eq} for the same resonator parameters as in Figs. 4 and 5, but this time for the lowest order asymmetric ($m=1$) modes. Figure 6 should be compared with Fig. 5 of Ref. 6 to conclude that edge rounding improves the discrimination between the dominant modes for $m=0$ and $m=1$, even in the N_{eq} range where mode detachment for $m=0$ has not yet been attained. The 'igner losses achieved by edge rounding for the $m=1$ modes are welcome since these modes are influential in beam deterioration for not having a maximum at the center. All of these features caused by edge rounding are

in agreement with the predictions made by modal analysis,[1] where it was anticipated that the eigenvalue for the m=1 modes should approach the geometrical optics limit $|\lambda_1| = 1/M$ only for very large N_{eq} values.

Depending on the edge rounding parameter r/r_2, mode detachment for three--dimensional resonators may occur for low Fresnel numbers, as is the case illustrated in Fig. 7, where $|\lambda|$ is plotted versus N_{eq} for a resonator with $M = 2.5$ and $r/r_2 = 16.7$. Here, detachment starts for $N_{eq} \sim 6$.

The simplified equation (10) gives good accuracy in the detached mode region and is also in very good agreement with the power loss per transit calculations by modal analysis.[1]

ACKNOWLEDGMENT

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FIGURE CAPTIONS

Fig. 1 – Physical configuration. For strip or circular mirror resonators, the rounded edge is modeled by a circular section with radius r_2, while the radius of curvature of the main mirror is r.

Fig. 2 – Modulus of λ versus N_{eq} for a strip mirror resonator with M = 2.9 and r/r_2 = 100. Even symmetric modes.

Fig. 3 – Modulus of λ versus N_{eq} for a strip mirror resonator with M = 2.9 and r/r_2 = 30. Even symmetric modes.

Fig. 4 – Modulus of λ versus N_{eq} for a circular mirror resonator with M = 2 and r/r_2 = 100. Azimuthally symmetric modes.

Fig. 5 – Modulus of λ versus N_{eq} for a circular mirror resonator with M = 2 and r/r_2 = 100. Azimuthally symmetric modes (m=0). The stationary roots have been suppressed.

Fig. 6 – Modulus of λ versus N_{eq} for a circular mirror resonator with M = 2 and r/r_2 = 100. First asymmetric modes (m=1).

Fig. 7 – Modulus of λ versus N_{eq} for a circular mirror resonator with M = 2.5 and r/r_2 = 18.0. Azimuthally symmetric modes (m=0).

Figure 1.
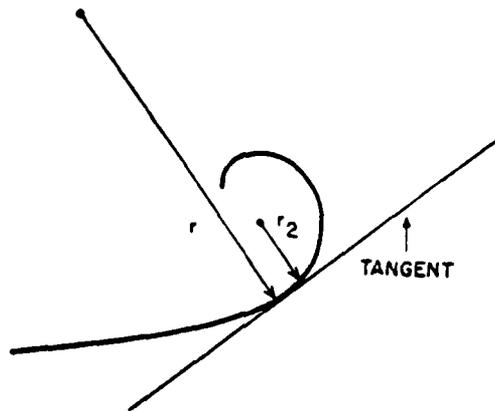
Figure 2.

Figure 3.

Figure 4.

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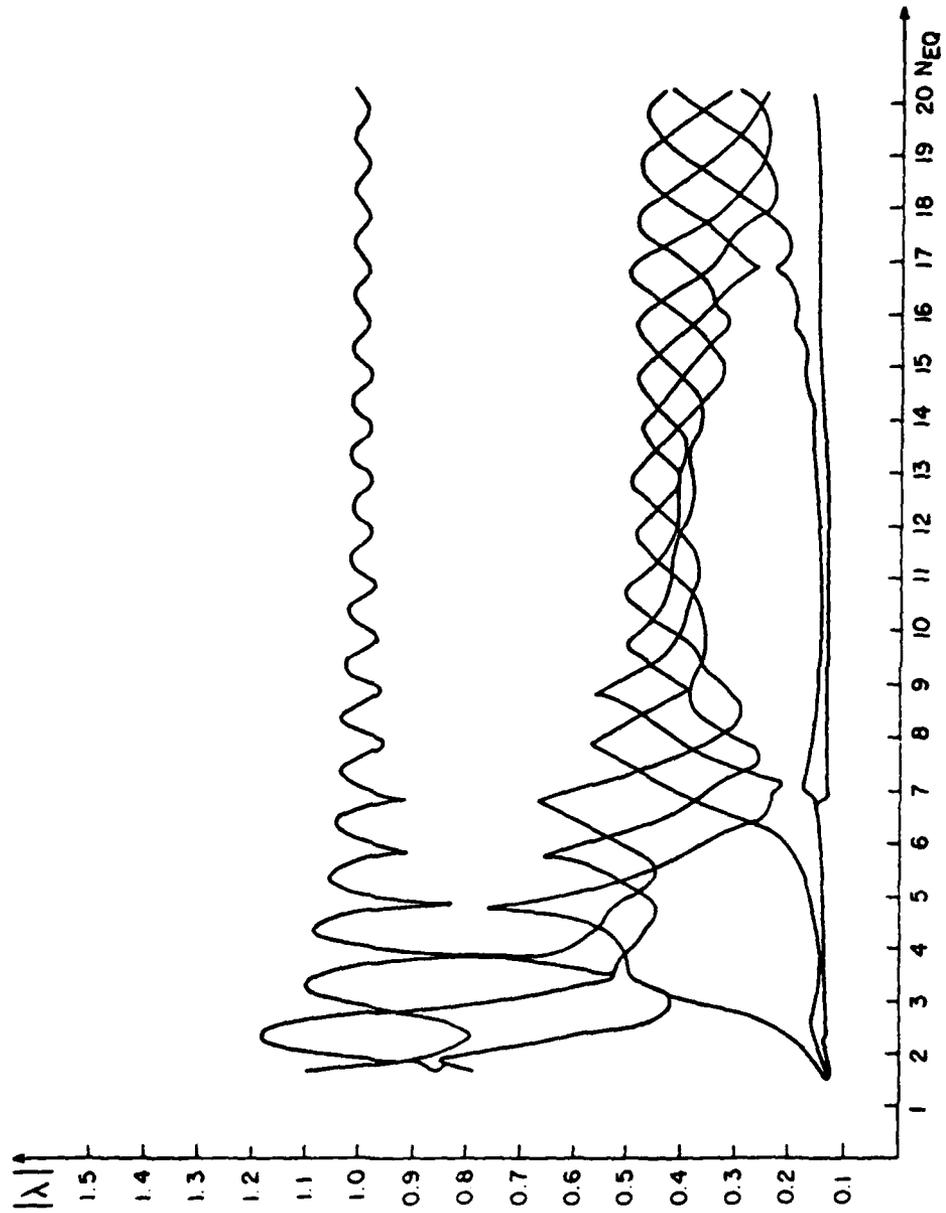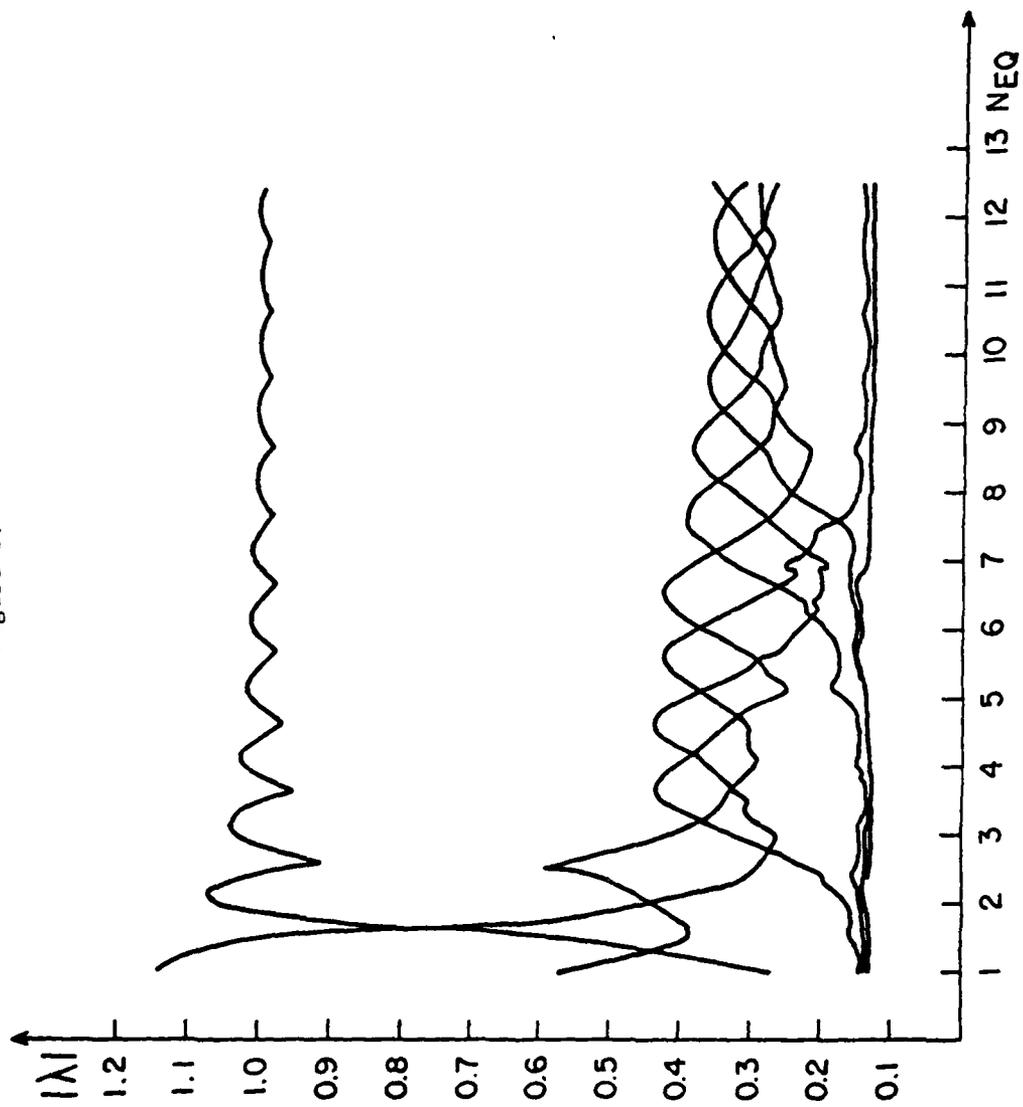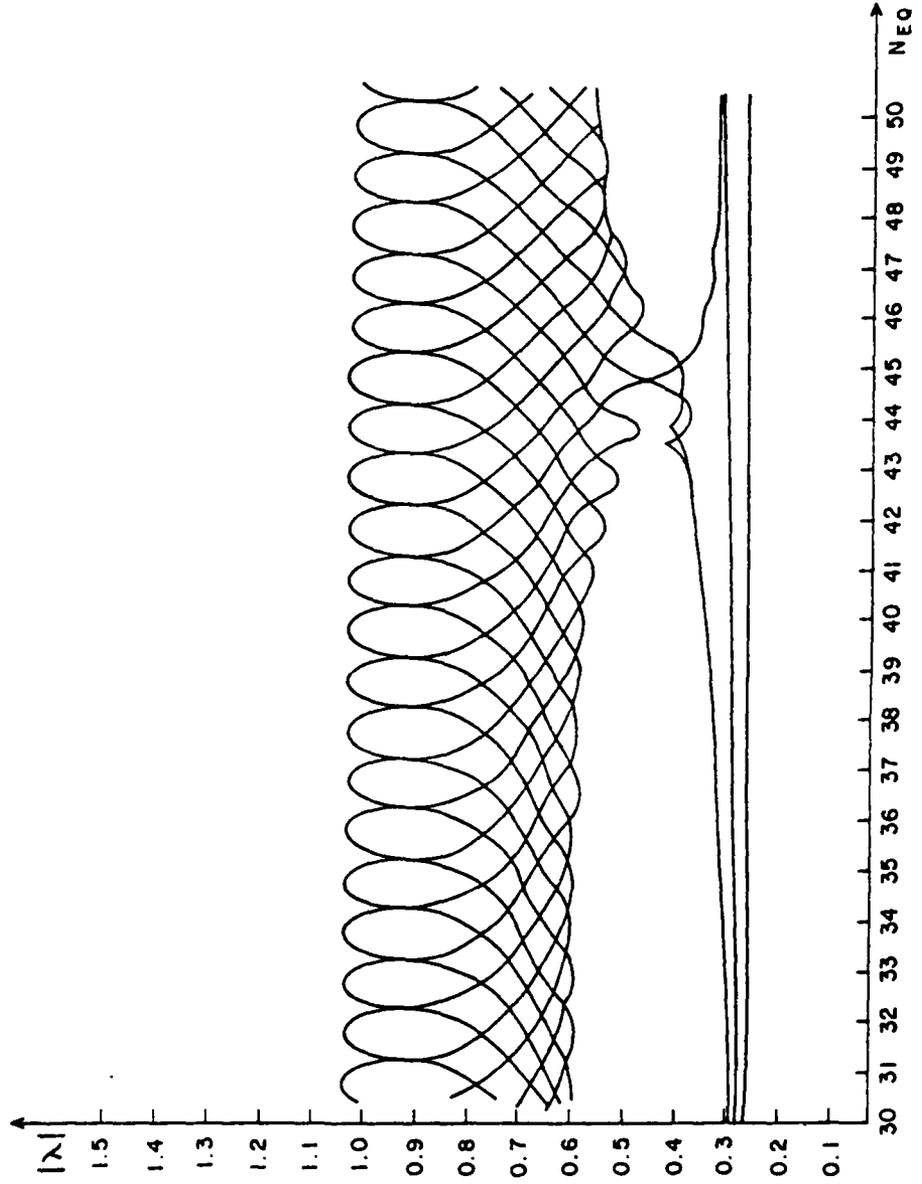
Figure 5.

Figure 6.

Figure 7.

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