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6 Probabilistic Structures of Modern Lottery Games.

by

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An introduction to the basic structure of lottery games is given. Several examples from modern state lotteries are included. Statistical considerations such as tests of randomness as required in the conduct of lottery games are discussed.

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1. Introduction and Historical Remarks

Lotteries have existed in some form or the other from the beginning of civilization. In the modern form, it is believe that Romans invented them for entertainment. Roman emperors Nero and Augustus used lotteries to distribute the slaves. In Europe, lotteries were used as early as 1539 to collect state revenue in France and Italy and were also used in England, Germany, and Austria. Many public and private lotteries were prevalent in the American colonies. Continental Congress also sponsored a lottery in 1777. Lotteries were used to finance certain projects by schools like Harvard, Columbia, Dartmouth and Williams. Many American states including Massachusetts, New York, Pennsylvania, Ohio, Vermont, Maine, New Jersey, New Hampshire and Illinois had legal lotteries in the nineteenth century. Louisiana Lottery Company got a charter in 1869 to raise money for a hospital.

Due to inefficient administration and prevalent fraud, considerable opposition developed to the running of lotteries, and they were made illegal in 1895 by federal laws. After seven decades, lotteries again surfaced legally in 1963, the first being the New Hampshire State Lottery. The number of state lotteries is increasing every year. Presently there are states having legal lotteries and several are seriously considering to institute lotteries. Several Canadian provinces and countries all over the world have lotteries to collect revenues.

American laws have recently been changed so as to allow various facilities for the development of state lotteries such as the use of television and mails. Excise federal taxes have been exempted from legal lotteries. Most of the present lottery games in the United States and abroad have been designed by private companies, and are essentially similar in nature. The introduction of instant lottery games and legal numbers game have increased participation by the general public.

Lottery is a game of chance and it is like any other gambling game. However, in lotteries the stakes are low and the winnings are very high which make them especially appealing to the average player. Gambling problems have been studied by mathematicians since the seventeenth century and this effort has resulted in the introduction of Probability Theory. Two famous mathematicians, Pascal and Fermat are generally credited with the development of probability theory. However, it is now believed that an Italian physician gambler, Cardano had given the elements of the theory of games of chance a hundred years earlier.

One of the earliest problems of probability theory is concerned with the gambler's ruin. The implications of the solution are that if the game is fair, then, the player is sure to be ruined against a rich adversary. More recently an abstract theory of gambling has been developed by Dubins and Savage and gives insight into many aspects of gambling. For example, if a player is playing against a casino with a cut, and the game is fair, mathematical derivation shows that it is preferable to bet in smaller amounts rather than to bet the whle fortune at once.

In this paper, probabilistic structure of many lottery games is described. Some of the lottery games of the states of New Jersey, New York, Michigan and Ohio are given as examples. There are commonly three types of lottery games being played simultaneously in a given state. There are weekly games where the drawings are made weekly to announce a winner. The daily number games announce winning numbers and the associated prizes every day is the evening. Instant lottery games allow a player to win instantly when he buys a ticket, the tickets have been manufactured so that they contain a specified number of winners. Some statistical considerations as required in the conduct of the lottery games are discussed.

2. Lotteries and Gambling

Gambling is the act of playing a game of chance with stakes. Lotteries are special cases of gambling games where the stakes are low and winnings are high. Many important areas of business such as speculation in stocks and commodities are nothing but special types of gambling. Another form of lottery is insurance. The preminum of an insurance policy is like the value of a ticket of a lottery. Tontine is another example of a lottery. It was introduced in 1653 by a banker, Lorenzo Tonti in Italy, as a scheme in which members subscribing to a common fund share an annuity with the benefit of survivorship. The share of the survivors are increased as the subscribers die until the whole goes to the last survivor. Other forms of lottery games played all over the world are Bingo, numbers, policy, and Bolita.

Gambling has been mentioned in almost all ancient religions. There are stories in the Hindu epics, of a king who put his whole kingdom as a stake in a game of chance and lost. Mention of gambling has been made in the Bible, the Talmud and the Koran. For some reason, it has been denounced as immoral in most of the societies. Hindu epics denounce it, Talmudic law declared acceptance of winnings as a thievery, the Koran also has prohibitions aginst gambling.

There have been rulings in the courts of the United States to the effect that "Gambling is injurious to the morals and welfare of the people". Recently the thesis that a real gambler is a neurotic person with an unconscious wish to lose has been developed by Bergler (1957) who has classified a gambler into one of three categories:

a. Racketeer gambler who is a conscious player,

b. Gambler who plays for small stakes for entertainment,

c. Compulsive gambler who is really a sick individual and is unaware that he is sick.

The cumpulsive gambler now can be medically treated.

Public participation in legal and illegal betting is quite extensive. A recent survey on adult betting in the New York City is given in Table I, based on Oliver Quayle Survey, given by Beale and Goldman (1974).

Table I

Extent of Gambling

Legal Betting	
Bingo	22 percent
Cards	33 percent
Lottery	74 percent
Sports Betting	
Football	14 percent
Basketball	14 percent
Baseball	30 percent
Horse Racing	30 percent
Other Sports	4 percent
Illegal Betting	
Numbers, Policy	24 percent

Total adult participation in betting was estimated to be 81 percent in New York City by the same survey. Some rough guesses about the gross revenue from gambling are put around five hundred billion dollars a year, only one tenth of which is available legally.

The basic problem in any gambling game is to develop an optimal strategy. Suppose a gambler has $1000 to stake where he has some chance to win $10,000. At each stage of the game, he is confronted with the decision whether to stop or to continue and how much to put for stakes. Abstractly such considerations lead to the formulation of gambling as a probability measure on a set of fortunes. The theory of how to play optimally then merges with the statistical theory of sequential decision making: Its recent formulation in the form of dynamic programming is due to Bellman (1957).

Gambler's Ruin problem is one of the earliest problems in the development of the theory of probability. This problem was discussed by Pascal and Fermat and was solved by Montemart in the seventeenth century. The discussion of the problem may be found in many introductory books on probability, such as Chung (1974). The game involves betting between two players, Peter and Paul. They have initial wealth of a and b dollars respectively. If Peter wins, he receives one dollar from Paul, otherwise he loses one dollar to Paul. The game continues till one of the players is ruined. The problem is to find the probability of Peter being ruined. This probability for Peter turns out to be $b/(a+b)$ in case the game is fair. The result is rather disturbing since if b is quite large, the probability of Peter's ruin is almost one. In other words, Peter will surely be ruined playing against a rich adversary.

Gambling provides many other interesting problems to the probabilist and the statistician. In a recent monograph, Dubins and Savage (1965) have studied the theory of optimal stopping in various gambling situations. There are several interesting results in their theory and we give here a problem which has a direct bearing on lotteries. Suppose a gambler has fortune f, $0 < f < 1$, and is playing against a casino with infinite fortune. The object of the gambler is to increase his fortune to one. Suppose further that the casino takes a cut equal to c times the fortune bet at any given time. If the game is fair, and there is no cut,

$$w(1 - f) + (1 - w)(-s) = 0$$

Let

$$w = \text{probability of winning } 1 - f$$

$$1 - w = \text{probability of losing the state } s$$

That is, $w = s/(1 - f + s)$.

Now if the gambler puts all his fortune at stake, that is, $s = f$, we have $w = f$, implying that the probability of winning is proportional to his fortune under bold play.

Suppose there is a cut by the casino equal to cf, $0 < f < 1$, then the gambler's probability of winning is reduced to w_c where

$$w_c = f(1 - c) \ . \tag{2.4}$$

Notice that as f tends to one, w_c tends to $1 - c$ and the probability of winning is strictly less than one, even though the gambler is ready to bet one. Another betting strategy is obtained by trial and error and proves to be better in the sense that it increases his probability of winning. This strategy requires that he divide his fortune in n equal parts, and bet equal amounts n times. Details are given by Dubins and Savage. The above strategy provides only one of the many available strategies and there may exist other strategies, which are still better. The implication this example has in lotteries can be seen from the fact that a lottery player has small amount to play and there is always a cut since the states require only 40-50 percent of the gross to be paid in prizes.

Returns in gambling games differ extensively starting with lotteries at the lowest level at 40-50%.

Irish sweepstakes are the only other games which pay back 37.5%, lower than the lotteries. Some of the returns from popular gambling games are given in Table II.

Table II

Present Returns on Various Gambling Games

Game	Return
Roulette	94.7 percent
Black Jack	94.1 percent
Slot Machines	75 - 97 percent
Dice Games	83.4 - 98.6 percent
Horse Racing	82 percent
Off Track Betting	77 percent
Lotteries	40 - 50 percent
Irish Sweepstakes	37.5 percent

There is extensive literature on lotteries and gambling, including abstract mathematical developments for example in Breiman (1961) and Halmos (1939). Popular writings are available by Scarne (1974), Ezell (1960), Landau (1965) and Maistrov (1974).

3. Present Lotteries

Gambling casino games are generally designed so that the probability of win by a player is less than half. In lotteries, the probability of wins is quite low. Present lotteries use random numbers generated by computers. Either printed tickts or computer terminals are used in present lotteries. Most of the present lottery games in the United States are designed by private lottery manufacturing companies and the designs tend to quite similar.

Most of the lottery games in the United States can be classified into the following three categories:

a. Weekly Games

b. Instant Games

c. Legalized Numbers Games

The weekly games are the oldest games in existence. They are played on computer generated tickets for which certain probabilities of winning are known to the public. Once a week, the drawings for winners are held and the prizes are awarded. Sometimes additional drawings are held after a certain number of winners have been awarded. These occasional drawings are made to keep the interest in the game. A few variations of these games have been used where the first time losers also have a chance of winning on a second try through occasional drawings for losers.

The instant lottery games were introduced recently to develop interest into a different segment of the public so that the player knows instantly whether he wins a prize or not. These games required a complicated design so as to preserve the nature of the ticket for security. The cost of printing tickets with fool-proof design and the nature of the game made them available only for a short period of time. They were generally used when the regular weekly games are not doing too well.

The legalized number games depend heavily on computer technology. The computer terminals are used in this game for making ticket and these terminals are attached to a central unit where records are kept. Once a day, a number is announced and prizes are given accordingly.

The concept is essentially copied from illegal numbers games which use people as runners and illegal games generally depend on some well known set of numbers available to the public such as in newspapers or radio, for the winning combination. These recently introduced games have rekindled interest in otherwise depressed lottery sales and people are playing lottery games in larger numbers.

Lottery Ticket Design

Consider the case that the lottery ticket has only a single digit out of 0, 1, 2, ..., 9. The probability of chosing a single digit 'at random' from the above 10 digits is 0.1. If one wants the probability of win to be 0.1, or that only 10% of the players are given prizes, we need only a single digit lottery. The winner can be found by matching a randomly drawn digit out of the given 10 digits.

A two-digit lottery ticket can be used to assign probability 0.01 to a winner. Also by matching the first or second digit in a two-digit lottery, and other prize can be awarded. This will have probability of winning equal to 0.09. To make games interesting and to provide a variety of winning probabilities, several digits may be used. For example, in a six-digit lottery, the probabilities may be described as follows:

Match all six with probability 0.000,001

Match first five but not the sixth with probability 0.000,009

Match the first four but not the fifth with probability 0.000,09

and so on.

Most of the state lotteries use six and seven-digit numbers. Various other ways in which the games are designed include choosing one out of several three digit numbers, and so on. In the following, a few games of the weekly type are discussed, with various types of assigned probabilities of winning.

Weekly Lottery Games

We discuss a few lottery games which have weekly prizes. The New Jersey lottery game, Weinstein and Deitch (1974), was designed to use a single six-digit number.

The prizes depended on matching all, last five, last four and last three digits. The prize structure in such games is designed in such a way that the percentage of the prize corresponds to the amount fixed by law of the state. Table III gives the prize structure of this game.

Table III

Prize Structure of a New Jersey Lottery Game

Ticket Number	Prize	Prizes per million
123456	$50,000	1
X23456	4,000	9
XX3456	400	90
XXX456	40	900

Additional drawings are usually held in weekly lotteries. In this lottery, a prize of a million dollars was given after 20 million tickets were sold using the tickets which won on the last three digits. There were also other consolation prizes, e.g. $100 prize was given to these matching the last two of the three digits.

Another example is of a game of the New York State Lottery, where seven digit numbers on the ticket were used. The winners are awarded prizes according to the Table IV.

Table IV

Prize Structure of a New York State Lottery Game

Winner	Prizes	Number of Prizes
Match all digits	$250,000	1
Match last six digits	25,000	3
Match last five digits	100	1,998
Match last four digits	25	5,994

Examples of games where more than one set of numbers is matched in weekly lotteries are given below. The Michigan weekly lottery used two three-digit numbers in one of its early games.

123	456

The game required matching at least one number when a three-digit number is drawn weekly.

The probability of matching one three-digit number in the above lottery is 3992/1,000,000 and for both is 2/1,000,000

Matching	Probability of win	Prize
One three-digit number	$\frac{3,992}{1,000,000}$	$25
Both three-digit numbers	$\frac{2}{1,000,000}$	Additional Drawing

Superdrawing is held after 5 million tickets have been sold, and another drawing called "millionaire drawing" is held occasionally when the number of tickets sold is about 30 million. The superdrawing is held to give 5 prizes when the expected number of tickets sold is 2.5 million. These prizes are given to the 5 who are holders of both 3-digits matches. The prize structure for superdrawing is given in Table V.

Table V

Super Drawing Prizes

Prizes	Number of prizes
$20,000 for 10 years	1
10,000 for 5 years	1
10,000	3

For the millionaire drawing, 120 finalists are selected from 120,000 winners of $25 prizes. Since the probability that a $25 winner qualifies for a millionaire drawing is 1/1000, on the average it requires about 30 million tickets to have been sold before the drawing. The prize structure is given in Table VI.

Table VI

Prize Structure for Millionaire Drawing

Prizes	Number of Prizes
$50,000 for 20 years	1
20,000 for 5 years	1
10,000 for 5 years	1
5,000	7
1,000	110
	120

Matching of several numbers of three digits was used in a Ohio Lottery game. This game was called "Buckeye 300" and it was based on a weekly prize of $300,000 which was the highest prize given weekly in any state lottery in 1974. A millionaire drawing was held whenever thirty million tickets had been sold. In this lottery, there were five three-digit numbers, two on green color and three on blue color.

Blue 123	Green 456	Blue 789	Green 012	Blue 345

A single three-digit number and a double three-digit number was drawn weekly to declare various prize. In Table VII, we give the prize structure of "Buckeye 300" and the associated probability structure.

Table VII

Prize Structure of "Buckeye 300"

Winner	Probability	Prize
Single number	$\frac{5}{1000}$	$20 and a chance on a million
Double green numbers	$\frac{2}{999,000}$	Qualifies for super drawing
Double blue numbers	$\frac{6}{999,000}$	1,000
Mixed Numbers	$\frac{12}{999,000}$	500

Since there are 2 winning double green numbers out of every million tickets, the expected number of double green after 3 million tickets are sold is 6. The prize structure followed is given in Table VIII.

Table VIII

Prize Structure for a Weekly Drawing

Prize	Number of Prizes
\$15,000 over 20 years	1
15,000 for 2 years	1
15,000	4

This structure is modified depending on the number of winning tickets sold. The probability of winning a \$300,000 prize depends on the number of persons in the super drawing and in the above case of six double green winners, the probability is $1/6 \times 2/999{,}000 = 1/2{,}997{,}000$.

The finalists for a millionaire drawing are selected out of \$20 winners. One out of each 1500 winners qualifies for the drawing. As soon as 100 such qualifiers are selected, a millionaire drawing is held. The prizes are given as described in Table IX.

Table IX

Prize Structure for a Millionaire Drawing

Prizes	Number of Prizes
$50,000 for 20 years	1
10,000 for 10 years	1
10,000 for 5 years	1
10,000	7
2,000	10
1,000	80

The involved structure of such weekly games were found to be too complicated for an average player and resulted in simpler games in weekly lotteries. The present weekly games are much simpler in form.

Instant Lotteries

A game in which the winner is determined at once rather than after a week, has many advantages. Small prizes are redeemed immediately by the vendor and the buyer is able to invest his winnings again for more tickets resulting in higher sales of instant lottery tickets. Most of the state instant lotteries are expected to run only for a short period of time as the public seems to get bored easily.

In an instant lottery ticket, the winning combination is covered with a security proof film. As soon as this film is uncovered, the buyer knows if he is the winner. Fraud proof design and secure manufacturing procedures have to be used. The probabilities of winning are arbitrarily assigned and the games are very simple. Winners can be determined in several ways. They may check a winning digit, come up with a winning total or simply match a combination of digits and so on. The frequency of small prizes is kept high. The lowest prize has been two dollars. In recent games, the prize of a free ticket has also been added. A recent game in the Ohio Lottery had the structure given in Table X.

Table X

Prize Structure of Ohio Instant Game

Prize	Probability of winning	Expected number of prizes per million
$ 2	1/10	100,000
5	3/200	15,000
200	1/20,000	50
1,000	1/100,000	10
10,000	1/100,000	10
Finalist	1/1,000,000	1

In this game, the final drawing was held after the sale of fifty million tickets and we give in Table XI its prize structure.

Table XI

Prize Structure of Final Drawing

Prize	Number of Prizes
$ 15,000	47
250,000	1
500,000	1
1,000,000	1

(c) Daily Numbers Game

Many lotteries have introduced legalized numbers games through an extensive computer network. The game by its very nature, is similar to illegal numbers games, played extensively by the gambling public. Three or four digits are chosen by the player and a few different kinds of bets are available in a game. In Massachusetts Numbers Games, for example, a four digit number is used. The bets can be placed on any order of the number, on first three digits in any order, in last three digits in any order, on first two digits, on a single digit, and so on. In Ohio Numbers Game, only a three digit number is used. The bets can be placed on three digits in any order, and combinations of one and two digits. Other states have similar numbers games.

Studies of statistical nature related to modern lottery games are few. Recently Chernoff (1980) has studied in detail the behavior of prizes for the Massachusetts numbers game. He studied the data of the game for 1320 plays through sophisticated statistical analysis. He has concluded that one can not do really well, in the sense of increasing the expected prize money, by developing a certain optimal strategy. The "regression to the mean" phenomenon takes over when unpopular numbers are being used and high payoffs are awarded.

The returns from Ohio Numbers Games are given in Figures 1 and 2. They provide the payout percentages for the first few weeks. The random fluctuations of the payout is evident from Figure 1.

4. Statistical Considerations

In the successful administration of lotteries, there are several stages where statistical questions arise. The major questions of statistical nature arise right when one wants to test the honesty and integrity of a designed game. To test the manufacturers' claim about the frequency and randomness of a certain prize structure, usual statistical techniques are utilized. We give here examples of testing randomness in various lottery settings.

Example 1. <u>Test of Randomness of Occurance of Small Prizes</u>

Consider the situation that the probability of getting a prize is 0.01. Then the hypothesis of randomness means that in the independent trials resulting in losers and winners, the winners occur with probability 0.01. The test statistic in this case can be obtained from goodness of fit test for the geometric distribution with probability p = 0.01. In a game designed for a state lottery, the frequency distri- of the number of winning tickets in a given sequence of 1500 tickets generated, is given in Table XII. The test for randomness in this case is the usual Chi-squared test for goodness of fit. The Chi-squared statistic is 4.98 in this case and the comparison is made with the tabulated Chi-squared value of 9.24 with 5 degrees of freedom at 10% level of significance, accepting the hypothesis of randomness of the occurance of prizes.

The following argument shows the geometric distribution is the discrete analog of the exponential distribution. Consider the exponential distribution with parameter λ. The discretized probability between [X] - 1 and [X] is

$$\int_{[x]-1}^{[x]} \lambda e^{-\lambda t} dt = e^{-\lambda[x]-\lambda}[1-e^{-\lambda}] .$$

Let $p = 1-e^{-\lambda}$, then we have for $[x] = y$, a positive interger, the probability

$$= p(1-p)^{y-1}$$

which is the geometric distribution.

Table XII

Frequence of Occurrence of Prizes

Number of winning tickets	Observed frequency	Expected frequency
1	51	54
2	40	43
3	27	35
4-5	51	50
6-7	37	32
8-11	35	34
12 or more	32	25
	273	273

Example 2. Randomness of Occurrence of Large Prizes

The number of losers between successive winners is designated as waiting time. In Table XIII, we provide a list of 180 waiting times for certain large tier prizes in a state lottery game. Here the probability of this prize is very small. The average waiting time is 106.67.

Table XIII

Waiting times for high tier denomination prizes in a state lottery game (given in multiples of 300 and arranged in increasing order in every column.

33	5	1	6	13	20	4	5	5	8
35	15	7	24	32	21	7	6	15	18
47	18	9	28	34	43	17	25	17	22
49	29	26	29	43	43	24	29	27	26
60	29	40	42	47	62	37	38	57	47
63	38	42	45	65	68	42	40	68	52
72	55	50	55	66	75	60	46	70	59
77	58	55	68	71	75	79	60	89	84
86	77	72	70	74	77	80	63	106	83
91	92	93	77	80	80	88	70	108	121
106	112	94	81	94	89	96	79	113	137
120	120	105	87	98	115	102	94	113	140
126	122	119	109	130	116	156	118	117	141
132	172	153	119	134	155	189	129	147	143
154	195	164	190	163	162	206	134	148	162
160	195	166	232	163	219	210	151	212	198
235	253	282	266	191	227	219	259	219	216
274	335	442	392	422	273	307	574	289	258

We test the hypothesis that the waiting times are exponentially distributed since the continuous analog of the geometric distribution is the exponential distribution. The frequency distribution is given in Table XIV for these waiting times. The Chi-squared value for the table under the exponential model is 6.S8 and we again accept the hypothesis of randomness at 10 percent level of significance as the tabulated Chi-squared value is 7.78 for 4 degrees of freedom.

Table XIV

Frequency Distribution of Waiting Times for Large Tie. Prizes

Interval	Observed Frequency	Expected Frequency
0 - 60	64	77
61 - 120	59	45
121 - 180	27	29
181 - 240	16	12
241 - 300	8	10
301 or more	6	7
Total	180	180

Example 3. Randomness of Digits in a Daily Lottery Numbers Game

The test of randomness for numbers generated for lottery games as well as for awarding of prizes in drawings are made in the same manner. Table XV gives a sequence of three-digit random numbers are from a state lottery "Numbers Game" for 100 drawings. The frequency test for the random digits is obtained with the help of Table XVI. Testing for uniformity provides the confirmation of the hypothesis of randomness at 10 percent level of significance with the Chi-squared value obtained as 7.20 from the table.

Table XV

Drawings in a Number Game in a State Lottery

308	967	521	492	407
646	514	559	458	145
554	991	751	259	730
804	657	432	972	407
098	109	986	261	748
130	743	551	167	682
037	691	717	002	688
709	146	544	706	909
089	503	163	758	710
613	340	081	114	036
876	758	972	580	738
519	123	568	854	760
810	351	742	392	810
892	983	988	415	460
392	623	533	743	454
726	190	714	750	407
516	951	024	253	107
953	080	035	988	798
969	547	158	472	216

Table XVI

Frequency Distribution of Digits

digit	frequency	expected
0	34	30
1	32	30
2	21	30
3	25	30
4	29	30
5	33	30
6	22	30
7	34	30
8	30	30
9	30	30
	300	300

5. Miscellaneous Issues in Lotteries

(i) Probability Games in Instant Lotteries

An instant lottery game can be designed in such a way that every ticket can be a winner with a preassigned probability. This is different from the usual games where certain tickets are definite winners and certain one's are losers. The attempt to introduce such probability games in the United States has not been successful. In 1976, Maine State Lottery introduced an instant lottery game with twelve covered panels on a ticket. The player was supposed to select any three panels and if the numbers under the panels added to twenty one, he/she was a winner. A vendor studied a few hundred tickets for the spatial distribution of winning combinations thereby improving his chances of winning. Publications of his findings in newspapers increased the winners six times more than expected. This resulted in a great loss to the Maine State Lottery and the designer of the game. Such games have not been tried ever since. The concept that the player can use his skill for improving the chances of his winning is quite attractive to a gambler. New games of this type with some control about the total number of winners are likely to be of great interest to the public.

(ii) Marketing of Lottery Games

Lottery games present special problems in marketing strategies. Lottery is a very special type of gambling where one may not see other people playing a game simultaneously. The tickets can be purchased in the privacy of a public grocery counter and one can wait for a week or longer to get a prize. To attract a continuous flow of players, the lotteries have designed several strategies such as sensational prizes, redemptions of losing tickets for a second try at winning, running several games simultaneously and so on.

Sensational prizes given at intervals attract sometimes worldwide attention. One of the prizes, $1776 per week for life, announced by New Jersey lottery a few years ago received wide coverage in press and media. Hence most of the weekly and instant lotteries include a major drawing during the period of the game. Such prizes, such as, one million dollars occur with very small probability. In some of the recent games, the probability has been 1/30,000,000. These prizes are quite useful in attracting the consumer. The present value of such prizes depends on the average age of the player and the life expectancy. The data obtained by the Ohio Lottery provided the estimate of age distribution of the lottery players. The average age was 45 years. The expected value of the prize at the time it is won, can be obtained, using various discounting factors. In Table XVII, the present value of $1,000 paid monthly for various periods at 6.5 and 7% per anum rates of interest is provided.

Table XVII

Present Value of $1,000 Paid Monthly

Period in Years	Interest Rate 6 1/2 % Compounded Annually	Interest Rate 7% Compounded Annually
35	169,150	160,350
40	174,800	165,100
45	178,950	168,500
50	181,950	170,900
55	184,150	172,650
60	185,750	173,650
65	186,950	174,750
70	187,800	175,350
75	188,450	175,800
80	188,850	176,150

Hence, using average life expectancy of the lottery player, the present value of this prize will be expected to be not more than $169,500.

Redemption of losing tickets for a prize has been tried in many lotteries. It seems that people after losing several times give up on playing the lottery. To keep their interest alive in a lottery, drawings are arranged to select a winner from losing tickets sent by the players. Another strategy used is that several games are run simultaneously so as to appeal to different segments of the lottery players. In general, weekly games are supplemented by instant games for a given period of time. In many states, daily numbers games have increased the total volume of betting in lotteries.

Some games are designed so that to win a prize, a player has to buy several tickets such as in a Bingo type game. In some games, winners are required to spell a certain word and so on.

The study of playing habits of lottery players has convinced most of the lottery game manufacturers to have a "concave" prize structure. That is, small denomination prizes such as $2, 5 with high probability and high denomination prizes such as $500, $10,000 with small probability. Middle size prizes are not considered useful in lotteries at all.

(iii) Drawing Procedures

Questions about the integrity and honesty of drawing procedures are always raised in the administration of lotteries. Recent actions against members of the staff and media involved in rigging the drawings in the state of

Pennsylvania received worldwide attention. The drawings to select a winning number use Bingo machines with ping-pong balls having digits zero through nine. Various other procedures are also used to declare winning numbers. In this case, water was injected into all balls except numbered 4 and 6 and various combinations of these numbers were bet by those involved in the Pennsylvania daily numbers game. The winning number that day was 666, and it raised serious objections which resulted in the investigation and apprehension of those involved.

Similarly in a Canadian Provincial lottery, a question of bias in the drawing procedures was pointed out by Bellhouse (1979) and it is reported that corrective measures were taken as a result of this objection.

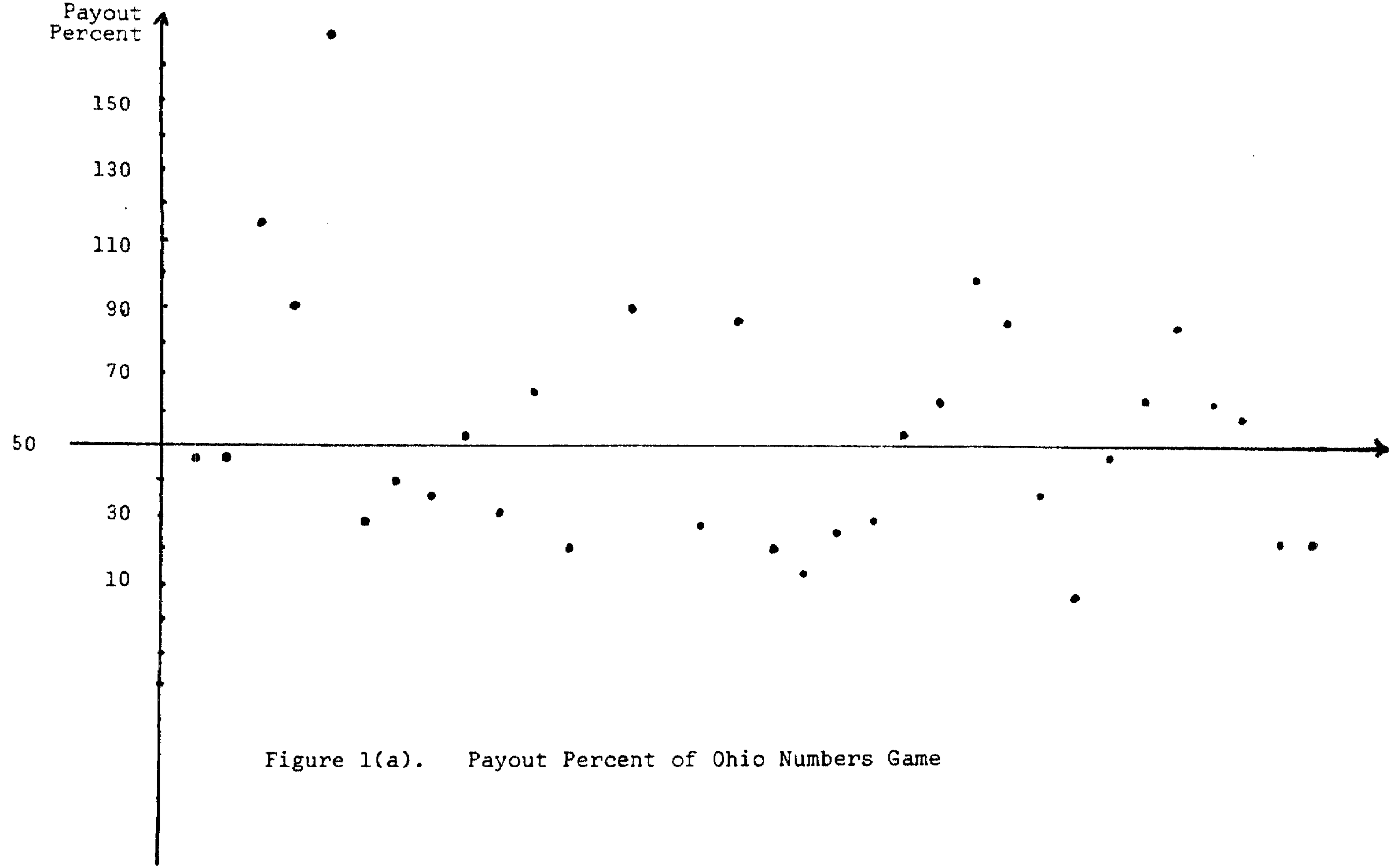

Figure 1(a). Payout Percent of Ohio Numbers Game

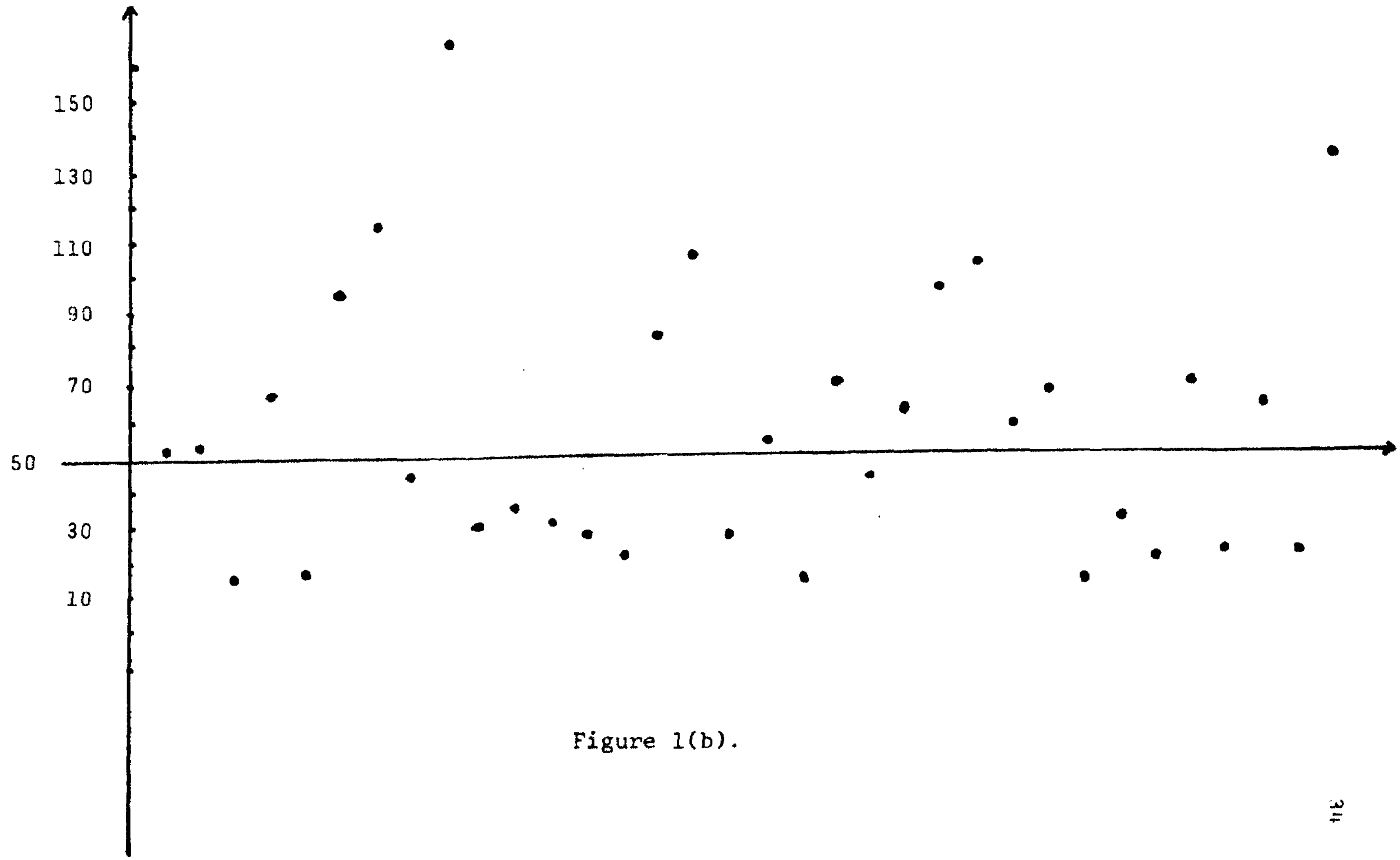

Figure 1(b).

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