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BASIC MECHANICS OF DE BANGE OBTURATOR SPLIT RING PRESTRESSING

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An analysis is presented to show the relationship between the maximum obtainable residual shear force in a split ring preform and the prestressing parameters (included angle between fixture grips and total angle of twist). Also included is an analysis of the section depth of the ring that is required to provide sufficient material for the finish machining operation. Application of the formulas derived would require the quantification of certain parameters by either experimental or numerical methods.

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NOMENCLATURE

E - Young's Modulus of elasticity

G - Elastic shear modulus

I - Ring cross-sectional moment of inertia

S_{max} - Combined maximum shear stress

S_y - Material yield stress in tension

T - Torque

T' - Torque in the shorter arc of the ring preform

T'' - Torque in the longer arc of the ring preform

V - Transverse shear force in the ring

ℓc_1 - Geometrical factor for elastic torsion (equivalent to the polar moment of inertia)

ℓc_2 - Geometrical factor for elastic-plastic torsion similar to the elastic torsion factor

r - Radius of the ring

x - Circumferential distance on the ring

y - Deflection normal to the plane of the ring

θ_o - Measure of one half the included angle between the grips

ϕ - Angle of twist between the grips

ϕ_E - Angle of twist between the grips at which the ring material reaches the elastic limit in the shorter arc

ϕ_P - Angle of twist between the grips beyond ϕ_E

ϕ_T - Total angle of twist between the grips

ϕ_R - Residual angle of twist between the grips

ρ - Radius in a circular cross-section of the ring

ρ_o - Outer radius of the ring cross-section

ρ_p - Elastic-plastic interface radius of the ring cross-section

INTRODUCTION

In a De Bange obturator the split rings have the function of preventing the pad material from being extruded out between the spindle and tube or between the disk and tube. This pad sealing function can only be accomplished if the rings are free to expand out to contact the tube; thus the requirement for the split in the split rings. However, unless a certain minimum residual force (or preload) is maintained on the split there will be a tendency for the obturator pad material to be extruded into the split when the ring dilates and then nibbled off when the ring is unloaded and allowed to close up again. If the required preload cannot be consistently obtained in the production of the rings then a problem will exist since rings which fail to meet specification cannot be reworked and must be scrapped.

Since the preloading (or "kinking") of the ring preforms is not a closely controlled process and indeed is largely a matter of the operator's judgment, it would seem desirable to study the mechanics of the process. The following analyses are an attempt to put the design of the fixture, the process, and the dimensioning of the preform on a rational basis. Specifically, we wish to know the included angle, θ_o, between the fixture grips, and the relative angular displacement, ϕ_T, of the grips that will give the greatest residual transverse shear force in the ring preform. We would also like to know what width the ring preform should have so as to minimize the amount of stock removal in the final machining process while still allowing for adequate material for the finished ring.

To see how torque and transverse shear are related, consider a segment of the ring of radius, r, which is subjected to an applied torque as shown in Figure 1. Taking moments about the radial axis:

$$2T \sin \theta = 2Vr \sin \theta$$

or

$$T = Vr \quad . \tag{1}$$

Thus gripping a ring preform at the locations indicated in Figure 2 and twisting in opposite directions to the extent that permanent deformation occurs in the shorter arc of the ring will induce a shear preload into the ring.

The process which the ring undergoes is shown in Figure 3 where the solid line represents the torsion in the shorter arc and the broken line represents the torsion in the longer arc. Examination of the figure will show that the greatest residual torque occurs when θ_o is chosen so that the slopes of the torque-twist plots are such that the longer arc is just about to yield when $\phi = \phi_T$ and the shorter arc is just about to reverse yield when $\phi = \phi_R$. These conditions are met if

$$\frac{\phi_E}{\phi_T} = \frac{\theta_o}{\pi - \theta_o} \,, \quad \text{and} \quad \phi_T - \phi_R = 2\phi_E \quad . \tag{2}$$

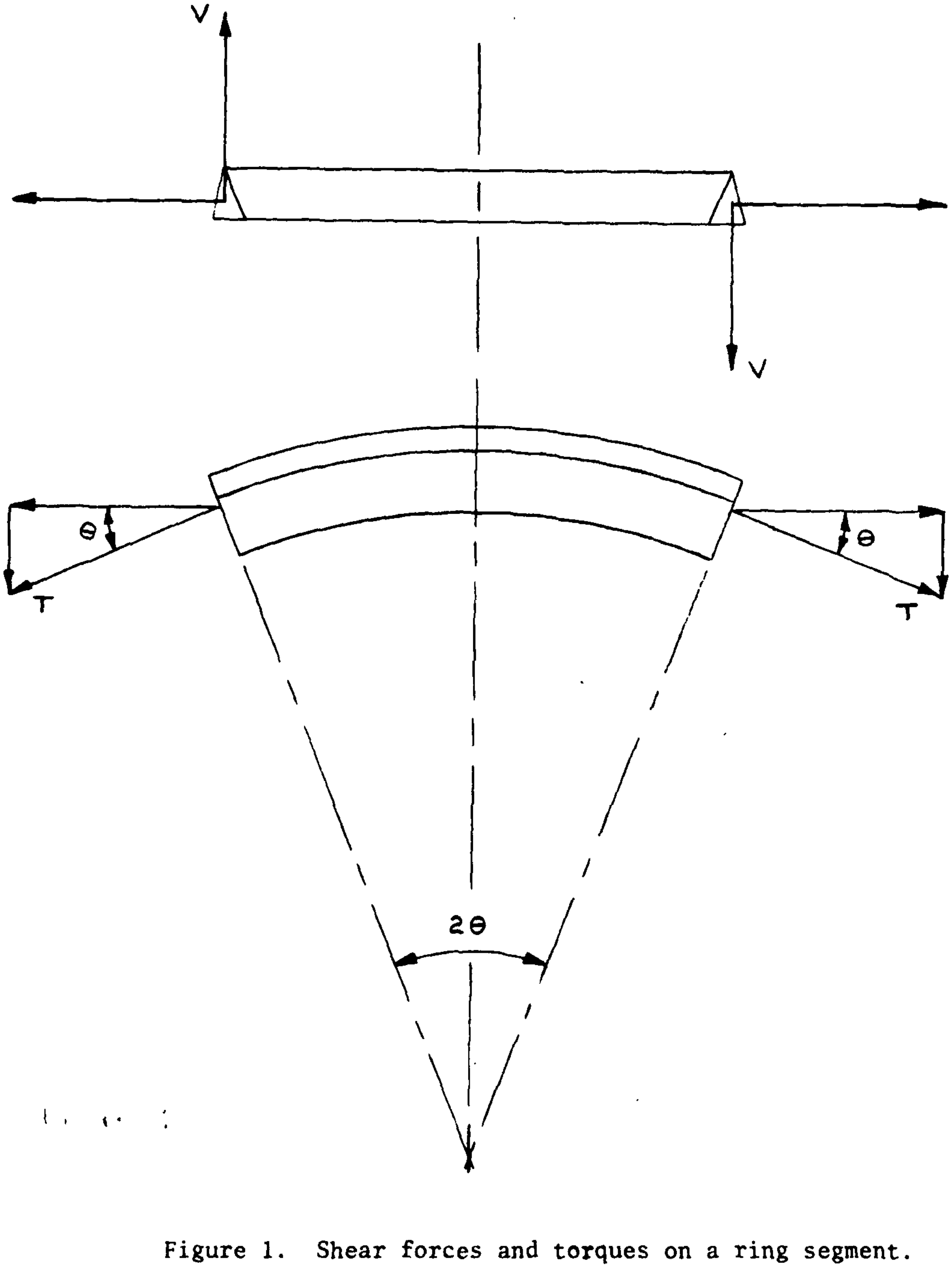

Figure 1. Shear forces and torques on a ring segment.

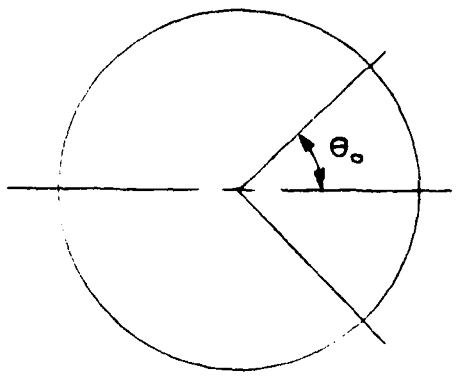

Figure 2. Grip positions on the ring preform (definition of θ_0).

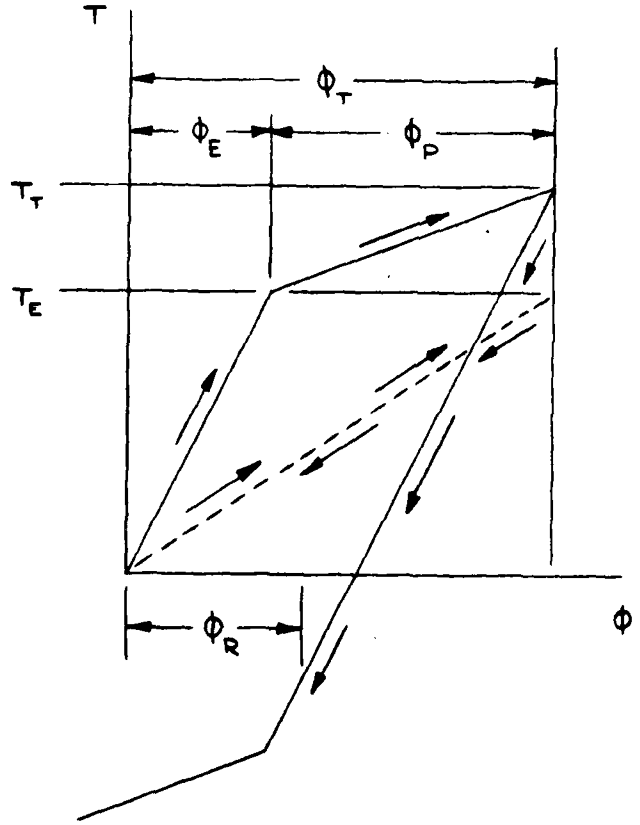

Figure 3. Torque-twist diagram for segments of the ring preform.

DETERMINATION OF θ_o AND ϕ_T

In order that a fixture may be constructed to provide the means for optimum twisting of the ring preforms, the angle θ_o must be determined. To do this we start by noting that

$$\frac{\theta_o}{\pi - \theta_o} = \frac{\phi_E}{\phi_T} = \frac{\phi_T - \phi_P}{\phi_T} = 1 - \frac{\phi_P}{\phi_T}$$

which readily gives

$$\frac{\theta_o}{\pi} = \left[\frac{1 - \dfrac{\phi_P}{\phi_T}}{2 - \dfrac{\phi_P}{\phi_T}} \right] . \qquad (3)$$

Also we have

$$\phi_T - \phi_R = 2\phi_E = 2(\phi_T - \phi_R)$$

which gives

$$1 - \frac{\phi_P}{\phi_T} = \frac{1}{2}\left(1 - \frac{\phi_R}{\phi_T}\right) ,$$

and

$$2 - \frac{\phi_P}{\phi_T} = \frac{1}{2}\left(3 - \frac{\phi_R}{\phi_T}\right) ,$$

so that

$$\frac{\theta_o}{\pi} = \left[\frac{1 - \dfrac{\phi_R}{\phi_T}}{3 - \dfrac{\phi_R}{\phi_T}} \right] . \qquad (4)$$

The torque on a twisted bar or section of ring is inversely proportional to the length of the twisted section and directly proportional to the angle of twist, shear modulus, and a geometrical factor. For elastic deformation the geometrical factor shall be designated ℓc_1, and for elastic-plastic deformation it shall be designated ℓc_2. This amounts to a bilinear idealization of the deformation process.

When the ring is twisted through an angle ϕ_T and then allowed to relax, the torque in the shorter arc is given by

$$T' = \frac{\phi_E}{2r\theta_o} \ell c_1 G + \frac{\phi_P}{2r\theta_o} \ell c_2 G - \frac{\phi_T - \phi_R}{2r\theta_o} \ell c_1 G$$

$$= \frac{\phi_R - \phi_P}{2r\theta_o} \ell c_1 G + \frac{\phi_P}{2r\theta_o} \ell c_2 G \qquad (5)$$

and on the longer arc by

$$T'' = \frac{\phi_R}{2r(\pi - \theta_o)} \ell c_1 G \quad . \qquad (6)$$

When the external torque is completely removed then

$$T' + T'' = 0 \quad ,$$

or

$$\frac{\phi_R - \phi_P}{\theta_o} + \frac{\phi_P}{\theta_o} \left(\frac{\ell c_2}{\ell c_1}\right) + \frac{\phi_R}{(\pi - \theta_o)} = 0 \qquad (7)$$

which after some manipulation becomes

$$\left(\frac{\phi_R}{\phi_P}\right) = \left(1 - \frac{\ell c_2}{\ell c_1}\right)\left(\frac{\pi-\theta_o}{\pi}\right)$$

so that

$$\left(\frac{\phi_R}{\phi_P}\right)\left(\frac{\phi_P}{\phi_T}\right) = \left(\frac{\phi_R}{\phi_T}\right) = \left(1 - \frac{\ell c_2}{\ell c_1}\right)\left(\frac{\pi-\theta_o}{\pi}\right)\left(1 - \frac{\theta_o}{\pi-\theta_o}\right)$$

which reduces to

$$\left(\frac{\phi_R}{\phi_T}\right) = \left(1 - \frac{\ell c_2}{\ell c_1}\right)\left(1 - \frac{2\theta_o}{\pi}\right) \quad . \tag{8}$$

Direct substitution then gives

$$\frac{\theta_o}{\pi} = \left[\frac{1 - \left(1 - \frac{\ell c_2}{\ell c_1}\right)\left(1 - \frac{2\theta_o}{\pi}\right)}{3 - \left(1 - \frac{\ell c_2}{\ell c_1}\right)\left(1 - \frac{2\theta_o}{\pi}\right)}\right] \quad . \tag{9}$$

The appropriate solution for $\frac{\theta_o}{\pi}$ follows as

$$\frac{\theta_o}{\pi} = \frac{-3\left(\frac{\ell c_2}{\ell c_1}\right) + \sqrt{\left(\frac{\ell c_2}{\ell c_1}\right)^2 + 8\left(\frac{\ell c_2}{\ell c_1}\right)}}{4\left(1 - \frac{\ell c_2}{\ell c_1}\right)} \quad . \tag{10}$$

For a value of $\frac{\ell c_2}{\ell c_1} = 0$ it is obvious that $\frac{\theta_o}{\pi} = 0$ while it can be shown that in the limit as $\frac{\ell c_2}{\ell c_1}$ approaches a value of 1, $\frac{\theta_o}{\pi}$ approaches $\frac{1}{3}$.

The maximum shear stress on the cross-section of the ring in the longer arc is proportional to the angle of twist up to the point of yielding at $\phi = \phi_T$. Using the Huber-Hencky-Von Mises distortion energy theory as the yield criterion allows us to write

$$\phi_T = \frac{1}{\sqrt{3}} \frac{S_y}{S_{max}} \phi \qquad (11)$$

where ϕ represents the angle of twist corresponding to the maximum shear stress S_{max}. Therefore, providing that the yield strength of the material is known and S_{max} can be determined as a function of ϕ, ϕ_T can be established.

Also we have

$$\phi_R = \phi_T \left(1 - \frac{\ell c_2}{\ell c_1}\right)\left(1 - \frac{2\theta_o}{\pi}\right) \qquad (12)$$

which may be used to calculate the residual shear force in the ring from

$$V = \frac{T_R}{r} = \frac{\phi_R}{2r^2(\pi - \theta_o)} \ell c_1 G \ . \qquad (13)$$

DETERMINATION OF PREFORM SECTION DEPTH

The residual deflection of the ring preform can be computed in order to determine if there is sufficient material to finish machine. Assuming that the curvature of the ring is not so great as to preclude approximation by prismatic beam theory we can say

$$\frac{d^3y}{dx^3} = \frac{V}{EI} = \frac{T}{rEI}$$

where on the longer (elastically deformed) arc

$$T = \frac{\phi R}{2r(\pi-\theta_o)} \ell c_1 G$$

so that

$$\frac{d^3y}{dx^3} = \frac{\phi R}{2r^2(\pi-\theta_o)} \frac{\ell c_1 G}{EI} \quad . \qquad (14)$$

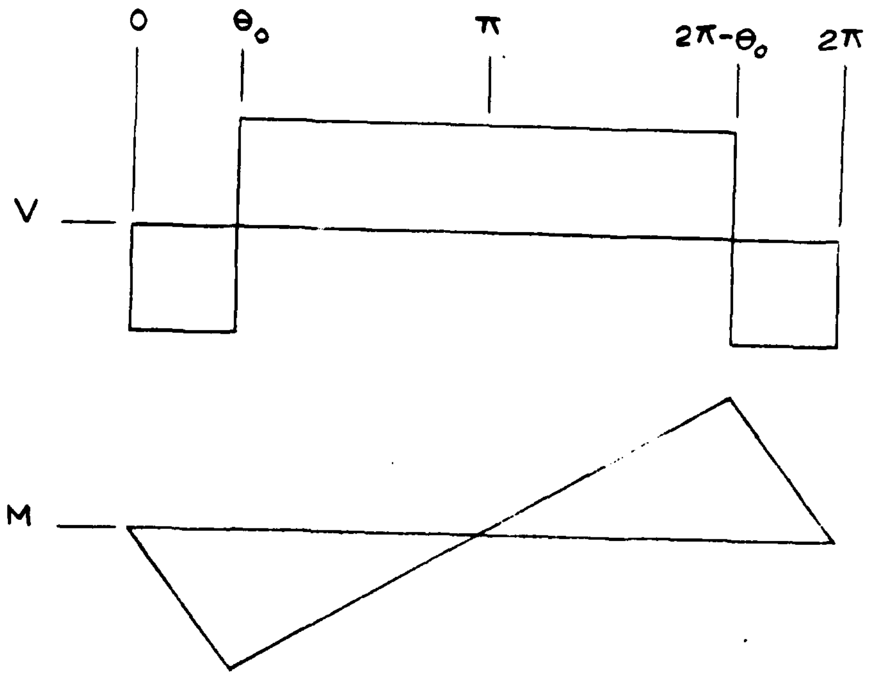

Figure 4. Shear and bending moment diagrams for the ring preform.

From Figure 4 is is seen that

$$\left.\frac{d^2y}{dx^2}\right|_{x=r\theta=r\pi} = 0 \quad . \tag{15}$$

Also it will be convenient to define the reference plane according to

$$\left.y\right|_{x=r\theta=r\theta_o} = \left.y\right|_{x=r\theta=r\pi} = \left.y\right|_{x=r\theta=r(2\pi-\theta_o)} = 0 \quad . \tag{16}$$

Integrating once gives

$$\frac{d^2y}{dx^2} = \frac{\phi_R}{2r^2(\pi-\theta_o)} \frac{\ell c_1 G}{EI} x + c_1''$$

$$= \frac{\phi_R}{2r^2(\pi-\theta_o)} \frac{\ell c_1 G}{EI} (r\theta) + c_1'' \quad . \tag{17}$$

Now since $M = 0$ at $\theta = \pi$, then $\frac{d^2y}{dx^2} = 0$ at $\theta = \pi$ and we have

$$0 = \frac{\phi_R}{2r^2(\pi-\theta_o)} \frac{\ell c_1 G}{EI} (r\pi) + c_1''$$

or

$$c_1'' = \frac{\phi_R}{2r^2(\pi-\theta_o)} \frac{\ell c_1 G}{EI} (-r\pi) \quad . \tag{18}$$

Therefore

$$\frac{d^2y}{dx^2} = \frac{\phi_R}{2r(\pi-\theta_o)} \frac{\ell c_1 G}{EI} (\theta-\pi)$$

$$= \frac{\phi_R}{2r(\pi-\theta_o)} \frac{\ell c_1 G}{EI} \left(\frac{x}{r} - \pi\right) \quad . \tag{19}$$

Integrating again gives

$$\frac{dy}{dx} = \frac{\phi_R}{2r(\pi-\theta_o)} \frac{\ell c_1 G}{EI} (\frac{x^2}{2r} - \pi x) + c_1' \quad , \tag{20}$$

and

$$y = \frac{\phi_R}{2r(\pi-\theta_o)} \frac{\ell c_1 G}{EI} (\frac{x^3}{6r} - \frac{\pi x^2}{2}) + c_1'x + c_1$$

$$= \frac{r\phi_R}{12(\pi-\theta_o)} \frac{\ell c_1 G}{EI} (\theta^3 - 3\pi\theta^2) + c_1'r\theta + c_1 \quad . \tag{21}$$

From the condition of zero displacement at $\theta = \theta_o$ and $\theta = 2\pi-\theta_o$ we get

$$0 = \frac{r\phi_R}{12(\pi-\theta_o)} \frac{\ell c_1 G}{EI} \left[(\theta_o)^3 - 3\pi(\theta_o)^2\right] + c_1'r\theta_o + c_1 \quad , \tag{22}$$

and

$$0 = \frac{r\phi_R}{12(\pi-\theta_o)} \frac{\ell c_1 G}{EI} \left[(2\pi-\theta_o)^3 - 3\pi(2\pi-\theta_o)^2\right] + c_1'r(2\pi-\theta_o) + c_1 \quad . \tag{23}$$

Expanding and subtracting the second equation from the first eliminates one of the unknown constants.

$$0 = \frac{r\phi_R}{12(\pi-\theta_o)} \frac{\ell c_1 G}{EI} \left[-4\pi^3 + 6\pi\theta_o^2 - 2\theta_o^3\right] + 2c_1'r(\pi-\theta_o) \quad . \tag{24}$$

Dividing and transposing yields

$$c_1'r = \frac{r\phi_R}{12(\pi-\theta_o)} \frac{\ell c_1 G}{EI} (2\pi^2 + 2\pi\theta_o - \theta_o^2) \tag{25}$$

so that

$$y = \frac{r\phi_R}{12(\pi-\theta_o)} \frac{\ell c_1 G}{EI} (\theta^3 - 3\pi\theta^2 + 2\pi^2\theta + 2\pi\theta_o\theta - \theta_o^2\theta) + c_1 \ . \tag{26}$$

Now since $y = 0$ at $\theta = \pi$ we have

$$c_1 = \frac{r\phi_R}{12(\pi-\theta_o)} \frac{\ell c_1 G}{EI} (\pi\theta_o^2 - 2\pi^2\theta_o) \tag{27}$$

and

$$y = \frac{r\phi_R}{12(\pi-\theta_o)} \frac{\ell c_1 G}{EI} (\theta^3 - 3\pi\theta^2 + 2\pi^2\theta + 2\pi\theta_o\theta - \theta_o^2\theta \quad \pi\theta_o^2 - 2\pi^2\theta_o) \ . \tag{28}$$

Also

$$\frac{dy}{dx} = \frac{r\phi_R}{12(\pi-\theta_o)} \frac{\ell c_1 G}{EI} (3\theta^2 - 6\pi\theta + 2\pi^2 + 2\pi\theta_o - \theta_o^2) \ . \tag{29}$$

The maximum deflection occurs where $\frac{dy}{dx} = 0$ which is to say where

$$3\theta^2 - 6\pi\theta + 2\pi^2 + 2\pi\theta_o - \theta_o^2 = 0$$

or

$$3\theta^2 - 6\pi\theta + 3\pi^2 = \theta_o^2 - 2\pi\theta_o + \pi^2 \ . \tag{30}$$

The above reduces to

$$3(\theta-\pi)^2 = (\theta o-\pi)^2 \tag{31}$$

or

$$\theta = \frac{\sqrt{3}\,\pi \pm (\theta_o - \pi)}{\sqrt{3}}$$

$$= \frac{(\sqrt{3}-1)\pi+\theta_o}{\sqrt{3}} \;, \quad \frac{(\sqrt{3}+1)\pi-\theta_o}{\sqrt{3}} \;. \tag{32}$$

Where

$$\theta = \frac{(\sqrt{3}-1)\pi+\theta_o}{\sqrt{3}}$$

we have

$$\theta^2 = \frac{2(2-\sqrt{3})\pi^2 + 2(\sqrt{3}-1)\pi\theta_o + \theta_o^2}{3} \;, \tag{33}$$

and

$$\theta^3 = \frac{2(3\sqrt{3}-5)\pi^3 + 6(2-\sqrt{3})\pi^2\theta_o + 3(\sqrt{3}-1)\pi\theta_o^2 + \theta_o^3}{3\sqrt{3}} \;. \tag{34}$$

Where

$$\theta = \frac{(\sqrt{3}+1)\pi-\theta_o}{\sqrt{3}}$$

we have

$$\theta^2 = \frac{2(2+\sqrt{3})\pi^2 - 2(\sqrt{3}+1)\pi\theta_o + \theta_o^2}{3} \;, \tag{35}$$

and

$$\theta^3 = \frac{2(3\sqrt{3}+5)\pi^3 - 6(2+\sqrt{3})\pi^2\theta_o + 3(\sqrt{3}+1)\pi\theta_o^2 - \theta_o^3}{3\sqrt{3}} \;. \tag{36}$$

Substituting $\theta = \dfrac{(\sqrt{3}-1)\pi+\theta_o}{\sqrt{3}}$ into the deflection equation gives

$$y = \frac{r\phi_R}{12(\pi-\theta_o)} \frac{\ell c_1 G}{EI} \left(\frac{2\pi^3 - 6\pi^2\theta_o + 6\pi\theta_o{}^2 - 2\theta_o{}^3}{3\sqrt{3}}\right)$$

$$= \frac{r\phi_R}{18\sqrt{3}} \frac{\ell c_1 G}{EI} (\pi-\theta_o)^2 \quad . \tag{37}$$

Substituting $\theta = \dfrac{(\sqrt{3}+1)\pi-\theta_o}{\sqrt{3}}$ into the deflection equation gives

$$y = \frac{r\phi_R}{12(\pi-\theta_o)} \frac{\ell c_1 G}{EI} \left(\frac{-2\pi^3 + 6\pi^2\theta_o - 6\pi\theta_o{}^2 + 2\theta_o{}^3}{3\sqrt{3}}\right)$$

$$= -\frac{r\phi_R}{18\sqrt{3}} \frac{\ell c_1 G}{EI} (\pi-\theta_o)^2 \quad . \tag{38}$$

Since the greatest deflections will occur in the longer arc it is clear that the amount by which the preform must exceed the finished ring in depth is

$$\frac{r\phi_R}{9\sqrt{3}} \frac{\ell c_1 G}{EI} (\pi-\theta_o)^2 \quad . \tag{39}$$

The relationship between the elastic constants, $E = 2G(1+\mu)$, allows the above to be written

$$\frac{\pi r\phi_R}{18\sqrt{3}(1+\mu)} \frac{\ell c_1}{I} \left(1 - \frac{\theta_o}{\pi}\right)^2 \quad . \tag{40}$$

APPLICATION TO DESIGN

The application of the formulas developed in the previous sections requires the evaluation of the ratio $\frac{\ell c_2}{\ell c_1}$. This is most easily accomplished by first making a change in variables. Referring to Figure 3 we see that

$$\frac{\ell c_2}{\ell c_1} = \frac{\dfrac{T_T - T_E}{\phi_T - \phi_E}}{\dfrac{T_E}{\phi_E}} = \frac{\dfrac{T_T}{T_E} - 1}{\dfrac{\phi_T}{\phi_E} - 1} \; . \tag{41}$$

In most cases the evaluation of the ratios $\frac{T_T}{T_E}$ and $\frac{\phi_T}{\phi_E}$ would be accomplished with the aid of numerical (i.e., finite element) methods. If the preform is of circular cross-section, however, numerical methods need not be resorted to. Although the use in practice of a circular cross-section is unlikely, the example given here will serve to demonstrate the application of the theory and may well serve as the starting point for the analysis of more practical shapes.

Figure 5 shows two plots of idealized stress vs. radius relationships for a circular cross-section subject to torsion. In the lower plot the material is just at the yield stress at the outer radius, ρ_o, while in the upper plot the material has yielded inward to the radius of the elastic-plastic interface, ρ_p.

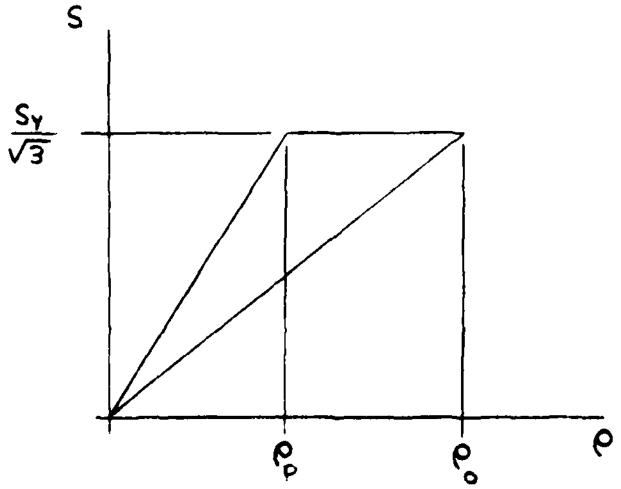

Figure 5. Shear stress-radius diagram for circular cross-section ring

The expression for the torque associated with the purely elastic strain as depicted by the lower plot in Figure 5 follows as

$$T_E = \int_o^{\rho_o} 2\pi\rho S\rho d\rho = \int_o^{\rho_o} (\frac{\rho}{\rho_o} \frac{S_y}{\sqrt{3}})\ 2\pi\rho^2 d\rho = \frac{\pi}{2\sqrt{3}} S_y\rho_o{}^3 \quad . \tag{42}$$

In a similar way the torque associated with the elastic-plastic strain as depicted by the upper plot in Figure 5 follows as

$$T_T = \int_o^{\rho_p} 2\pi\rho S\rho d\rho + \int_{\rho_p}^{\rho_o} 2\pi\rho S\rho d\rho$$

$$= \int_o^{\rho_p} (\frac{\rho}{\rho_p} \frac{S_y}{\sqrt{3}}) 2\pi\rho^2 d\rho + \int_{\rho_p}^{\rho_o} 2\pi \frac{S_y}{\sqrt{3}} \rho^2 d\rho$$

$$= \frac{\pi}{\sqrt{3}} S_y\ (\frac{2}{3} \rho_o{}^3 - \frac{1}{6} \rho_p{}^3) \quad . \tag{43}$$

Therefore

$$\frac{T_T}{T_E} - 1 = \frac{1}{3}\left(1 - \frac{\rho_p{}^3}{\rho_o{}^3}\right) \quad . \tag{44}$$

In a torsionally loaded member of given length and circular cross-section the angular deflection is proportional to the elastic stress at a radius and inversely proportional to that radius. And since the stresses at ρ_o and ρ_p are equal

$$\frac{\phi_T}{\phi_E} = \frac{\rho_o}{\rho_p} \quad . \tag{45}$$

Also because $\phi_T - \phi_R = 2\phi_E$ we have

$$\frac{\phi_R}{\phi_T} = 1-2\,\frac{\phi_E}{\phi_T} = 1-2\,\frac{\rho_p}{\rho_o} \quad . \tag{46}$$

Substituting equations (46), (41), and (4) into

$$\left(\frac{\phi_R}{\phi_T}\right) = \left(1 - \frac{\ell c_2}{\ell c_1}\right)\left(1 - \frac{2\theta_o}{\pi}\right) \quad . \tag{8}$$

gives

$$\frac{\rho_p{}^3}{\rho_o{}^3} + 6\,\frac{\rho_p}{\rho_o} - 1 = 0 \quad . \tag{47}$$

The single real root of this equation is $\frac{\rho_p}{\rho_o} = 0.1659$. This leads in turn to the results

$$\frac{\ell c_2}{\ell c_1} = 0.065996 \quad , \quad \frac{\theta_o}{\pi} = 0.142295 \quad , \quad \theta_o = 25.6° \text{ and } 2\theta_o = 51.2° \ .$$

Thus for any ring of circular cross-section the optimum spacing of the grips is 51.2 degrees. Completion of the design will require the specification of additional parameters. Let us assume the following values for the required dimensions and material properties:

$$\frac{r}{\rho_o} = 10, \ \rho_o = 8.255 \text{ mm } (0.325 \text{ in}) ,$$

$$S_y = 1100 \text{ MPa } (160 \text{ Kpsi}) \quad , \quad \frac{S_y}{G} = 14.5 \times 10^{-3} .$$

From the elementary theory of torsion of circular elastic rods we have

$$\frac{\phi}{S_{max}} = \frac{2r(\pi - \theta_o)}{\rho_o G} = \frac{2\pi(1 - \frac{\theta_o}{\pi})}{G} \frac{r}{\rho_o} . \tag{48}$$

Substituting into equations (11) and (12) gives

$$\phi_T = \frac{2\pi(1 - \frac{\theta_o}{\pi})}{\sqrt{3}} \frac{r}{\rho_o} \frac{S_y}{G} = 0.45115 \text{ radian} = 28.85° \tag{49}$$

and

$$\phi_R = 0.45115(1 - \frac{\ell c_2}{\ell c_1})(1 - \frac{2\theta_o}{\pi}) = 0.30145 \text{ radian} = 17.27° \tag{50}$$

Since the torque on a circular section is equal to the product of the angular displacement, the polar moment of inertia and the shear modulus divided by the length of the member we may compute

$$T_T = \frac{\phi_T \ell c_1 G}{2r(\pi-\theta_o)} = \frac{\pi\rho_o^{\,3} S_y}{2\sqrt{3}} = 563 \text{ N}\cdot\text{m (415 ft. lbs.)}\ , \tag{51}$$

and

$$T_R = \frac{\phi_R \ell c_1 G}{2r(\pi-\theta_o)} = \frac{\phi_T \ell c_1 G}{2r(\pi-\theta_o)} \left(1 - \frac{\ell c_2}{\ell c_1}\right)\left(1 - \frac{2\theta_o}{\pi}\right)$$

$$= 376 \text{ N}\cdot\text{m (277 ft. lbs.)} \tag{52}$$

The shear in the ring is computed from equation (13) as

$$V = \frac{T_R}{r} = \frac{T_R}{\rho_o \frac{r}{\rho_o}} = 4555 \text{ N (1024 lbs.)} \tag{53}$$

And finally the width allowance is given by

$$\frac{\pi r \phi_R}{18\sqrt{3}\ (1+\mu)} \frac{\ell c_1}{I} \left(1 - \frac{\theta_o}{\pi}\right)^2 = \frac{\pi r \phi_R}{18\sqrt{3}\ (1+\mu)} \frac{\frac{\pi\rho_o^{\,4}}{2}}{\frac{\pi\rho_o^{\,4}}{4}} \left(1 - \frac{\theta_o}{\pi}\right)^2 \tag{54}$$

$$= 2.84 \text{ mm (0.1117 in.)}$$

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