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DEFLAGRATION AND DETONATION FOR SMALL HEAT RELEASE.(U)
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6 DEFLAGRATION AND DETONATION FOR

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9 Technical Report No. 121

10 D.S. Stewart G.S.S. Ludford

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Contents

1. Introduction

Recently Stewart and Ludford (1980) and Lu and Ludford (1980) have put forward a theory of one-dimensional steady combustion waves that travel faster than justifies the use of the combustion approximation; where it is assumed that the Mach number, flame speed divided by a characteristic sound speed, is vanishingly small. When the combustion approximation is abandoned the momentum equation is retained and coupled with the energy equation. This coupling is essential in the description of any combustion phenomena where the Mach number is not small. Among the topics treated in these papers are fast deflagrations and detonations respectively and are described using large activation-energy asymptotics. The aim is to lay down the groundwork for a theoretical treatment of the general problem of transition from deflagration to detonation.

In particular Stewart and Ludford show that the solution of fast deflagrations has a very simple form when the heat release during reaction is small. This limit is useful in the sense that it allows explicit formulas to be developed, whereas, otherwise numerical integrations must be performed. The present paper adds to Stewart and Ludford's original results and uses the small heat release limit of the results for large activation energy to analyze the combustion waves discussed in the previous papers; fast deflagrations and weak and strong detonations. An interesting result from detonation theory, made explicit in the small heat release limit, is that the minimum wave speed of a detonation is almost always greater than the Chapman-Jouget value.

2. The governing equations

The present paper will cite some details from the two mentioned papers. Stewart and Ludford and Lu and Ludford will be referred to as I and II respectively. The equations used here are those in I and are in fact valid for the description of both steady waves of combustion theory, deflagrations and detonations. In particular ρ, v, T and Y are the dimensionless density, fluid velocity, temperature and mass fraction of the deficient reactant, so that ρ, v, T, $Y \to 1, 0, 1, Y_{-\infty}$; or quiescent values; as the steady wave frame coordinate $s \to -\infty$. The problem of one-dimensional steady combustion has a special formulation whenever the Prandlt and Lewis numbers are set equal to one and is governed by

$$\gamma M_o^2 dV/ds = \gamma M_o^2 (V-1) + (TV^{-1}-1) \tag{1}$$

$$d\tau/ds = d^2\tau/ds^2 + \alpha\Lambda Y e^{-\theta/T} \tag{2}$$

$$T = \tau - (\gamma-1)M_o^2 V^2/2 \tag{3}$$

$$\tau + \alpha Y = 1 + \beta + (\gamma-1)M_o^2/2 \quad \text{with} \quad \beta = \alpha Y_{-\infty} \tag{4}$$

$$\rho V = 1 \quad . \tag{5}$$

In the present formulation V and τ are the independent variables and the temperature T and mass fraction Y serve as auxillary variables defined by (3) and (4). Equation (5) defines the density ρ and the pressure has been eliminated by use of the ideal gas law. The other parameters that appear, γ, M_o^2, α, Λ, θ are explained in Table 1.

Thus equation (1) and (2) are to be solved under the condition that $(V,\tau) \to (1, 1+(\gamma-1)M_o^2/2)$ [corresponding to $(T,Y) \to (1, Y_{-\infty})$ as $s \to -\infty$] and that the solution is bounded as $s \to +\infty$.

3. Activation-energy asymptotics

In this section we summarize the results of I and II that describe the solutions for deflagrations and detonations in the limit of large activation energy, $\theta \to \infty$. First a review of the basic properties and differences of deflagration and detonation waves is appropriate.

Both waves represent transition solutions of equations (1) and (2) that leave the point $(V,\tau) = (1, 1+(\gamma-1)M_o^2/2)$ at $s = -\infty$ and flow into the point $(V,\tau) = (V_\infty, \tau_\infty)$ at $s = +\infty$. In determining the fixed points of equation (1) and (2) we find that

$$\tau_\infty = 1 + \beta + (\gamma-1)M_o^2/2 \quad , \tag{6}$$

however there are two possible choices for V_∞ given by

$$V_\infty = V_\pm = [(\gamma M_o^2+1) \pm \sqrt{(1-M_o^2)^2 - 2(\gamma+1)\beta M_o^2}]/(\gamma+1)M_o^2 \quad . \tag{7}$$

The reality of V_∞ requires that for a given β the wave speed be restricted so that

$$\frac{(1-M_o^2)^2}{2(\gamma-1)\beta M_o^2} \geq 1 \quad . \tag{8}$$

Equality refers to the Chapman-Jouget wave speeds. Equation (8) represents a quadratic in M_o^2 and thus there are two possible steady waves

$$0 \leq M_o^2 \leq M_{ocJ-}^2 = [1+(\gamma+1)\beta - \sqrt{[1+(\gamma-1)\beta]^2-1}] < 1 \quad , \tag{9}$$

corresponding to deflagrations and

$$1 < M_{ocJ+}^2 = [1+(\gamma+1)\beta + \sqrt{[1+(\gamma-1)\beta]^2-1} \leq M_o^2 \quad , \tag{10}$$

corresponding to detonations.

As we shall see for deflagrations, the point (V_-, τ_∞) is associated with a weak deflagration and is accessible, while (V_+, τ_∞), corresponding to strong deflagrations is not. For detonations (V_-, τ_∞) corresponds to strong detonations and is almost always accessible while (V_+, τ_∞) corresponding to weak detonations is accessible for only very special conditions.

3.a The asymptotic structure of fast deflagrations

It was shown in I, that a description of fast deflagration waves for $0 \leq M_o^2 < M_{ocJ-}^2$ was uniformly valid in the limit $\theta \to \infty$ for Damköhler numbers D of the form

$$D = \beta^2 \frac{M_o^2 m^2}{2T_*^4} \theta^2 \exp(\theta/T_*) \quad , \tag{11}$$

where $T_* = T_*(M_o^2)$ is a determined function of M_o^2, implicitly defining the flame speed M_o^2 in terms of the flame temperature T_*.

By expanding all dependent quantities as

$$u = u_o + \theta^{-1} u_1 + \ldots \tag{12}$$

it was shown that the leading order solutions for τ_o, and V_o in the θ^{-1} expansion, herein denoted by the zero subscript, could be constructed by solving reactionless equations (1) and (2) subject to jump conditions across the flame sheet located at $s = 0$. The conditions derivable from a flame sheet analysis are that τ_o and V_o and hence dV_o/ds are continuous and that $T_o = T_*$ at $s = 0$. Thus

$$\tau_o = 1 + \beta e^s + (\gamma-1)M_o^2/2 \quad \text{for} \quad s < 0 \tag{13}$$

and

$$\tau_o = 1 + \beta + (\gamma-1)M_o^2/2 \quad \text{for} \quad s > 0 \quad . \tag{14}$$

V_o satisfies the differential equations

$$\frac{dV_o}{ds} = \frac{\gamma+1}{2\gamma} \frac{1}{V_o} \left(V_o - \frac{(\gamma-1)M_o^2+2}{(\gamma+1)M_o^2}\right)(V_o-1) + \frac{\beta e^s}{\gamma M_o^2 V_o} \quad \text{for} \quad s < 0 \quad , \tag{15}$$

and

$$\frac{dV_o}{ds} = \frac{\gamma+1}{2\gamma} \frac{1}{V_o} (V_o-V_+)(V_o-V_-) \quad \text{for} \quad s > 0 \quad . \tag{16}$$

The problem has been reduced to solving equations (15) and (16) under the condition that $V_o(-\infty) = 1$ and $V_o(0^+) = V_o(0^-)$.

Integrating the latter equation is straightforward and we find that for $s > 0$

$$\frac{2^\gamma}{\gamma+1}\frac{1}{V_+-V_-}\ \ln\frac{|V_o-V_+|^{V_+}}{|V_o-V_-|^{V_-}} = s + c \quad , \tag{17}$$

where c is a constant. Since $V_+ - V_-$ is positive $V_o \to V_-$ as $s \to +\infty$ which is appropriate for weak deflagrations.

As noted in I, we cannot integrate (15) without further assumption because the right hand side explicitly contains e^s. However we discern some information by replacing e^s in (15) with ϕ. Then the solution to

$$d\phi/ds = \phi \tag{18}$$

and equation (15) and (16) for $s < 0$ and $s > 0$ respectively define a trajectory in the (V_o,ϕ) phase plane. The trajectory starts at $(1,0)$ at $s = -\infty$. The local solutions in the neighborhood of the starting point are easily seen to be a linear combination of

$$\begin{pmatrix} \beta/[1+(\gamma-1)M_o^2] \\ 1 \end{pmatrix} e^s \quad , \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{\frac{M_o^2-1}{\gamma M_o^2}\, s} \quad . \tag{19}$$

Thus $(1,0)$ is a saddle point (since $M_o^2 < 1$) and we leave along a unique integral curve. Then $s = 0$ corresponds to a point $(V_*,1)$ accordingly. (See Fig. 1.) Since $V_o(0) = V_*(\gamma,\beta,\ M_o^2)$ is determined by the integration for $s < 0$, and hence as a function of the parameters γ,β and M_o^2, constant c is fixed and $T_o(0) = T_*$ has the value

$$T_* = 1 + \beta + (\gamma-1)M_o^2(1-V_*^2)/2 \quad , \tag{20}$$

which determines the flame temperature T_* as a unique function of M_o^2.

3.b The asymptotic structure of detonations

The general theory of detonations developed in II is completely analogous to the theory of fast deflagrations except for one important aspect; there is not necessarily a unique wave speed corresponding to an acceptable D. As before D is considered specified generally as

$$D = C\theta^2 \exp(\theta/T_*) \quad , \quad C \sim O(1) \tag{21}$$

where C and T_* characterize D.

Then the construction of the zeroth order solution follows as before. Equations (13) and (14) are correct for τ_o and V_o satisfies (15) and (16) respectively. However for detonations $(M_o^2 > 1)$ the character of the initial point $(V_o, \phi) = (1,0)$ is that of a source. There is not a unique curve, but in fact a one-parameter family of curves leaving the point. In particular the curves that intersect $\phi = 1$ at some $V_o(0) = V_*$ can be assigned a flame temperature T_* which serves as a parametrization. (See Fig. 2.) For a given wave speed, $M_o^2 \geq M_{ocJ+}^2$, the velocity structure in front of the flame sheet is not uniquely determined as it is for deflagrations. Instead, for a given fixed M_o^2, specification of T_* identifies V_* uniquely from equation (20) and hence the appropriate velocity structure.

The restrictions on the existence of detonation structure are essentially those from asking, for fixed M_o^2, what values of V_* are attainable, and for this set of V_* does there exist a velocity adjustment behind the flame $(s > 0)$? In II it was shown that V_* has the permissible values

$$V_{*min} < V_* \leq V_+ \quad ; \quad V_{*min} < V_- \quad . \tag{22}$$

V_{*min} corresponds to the value of V_* found by assuming the velocity given by the adiabatic shock, which adjusts the solution from $(V_o,\phi) = (1,0)$ to $(V_s,0) = ([(\gamma-1)M_o^2+2]/(\gamma+1)M_o^2,0)$, followed by the integration from the end state of the shock $(V_s,0)$ to $(V_{*min},1)$. The last integration is uniquely determined as $(V_s,0)$ is a saddle point for detonations.

The weak detonation has $V_* = V_+$ and $V_o = V_+$ is the solution for the velocity for $s > 0$. In II it is shown that $V_* = V_+$ represents the maximum possible velocity at the flame sheet and that there are no solutions possible for $V_* > V_+$. Again this observation can be made simply by examining the nature of the point $(V_+,1)$.

Thus for every $M_o^2 \geq M_{cJ+}^2$, V_{*min} must be determined by a numerical integration from the saddle point $(V_s,0)$. For a given T_* then (22) represents a complicated restriction on M_o^2 rewritten as

$$V_{*min}(M_o^2) < 1 - \frac{2[T_*-(1+\beta)]}{(\gamma-1)M_o^2} \leq V_+(M_o^2) \quad . \tag{23}$$

4. The limit of small heat release; $\beta << 1$

The smallness of the heat release may be due either to a small heat of reaction or to a small amount of reactant. The main advantage from a theoretical point of view is that the limit allows analytical expressions to be developed for the structure of deflagration and detonations as outlined in Section 2, which otherwise have to be derived by numerical integration.

For $\beta << 1$, conditions (9) and (10) become

$$0 \leq M_o^2 \leq M_{ocJ-}^2 = 1 - \sqrt{2(\gamma+1)}\,\beta^{1/2} + \ldots \tag{24}$$

for deflagrations, and

$$M_o^2 > M_{ocJ+}^2 = 1 + \sqrt{2(\gamma+1)}\ \beta^{1/2} + \dots \tag{25}$$

for detonations. τ_∞ is still given by (6), where as

$$V_+ = \frac{(\gamma-1)M_o^2+2}{(\gamma+1)M_o^2} - \frac{\beta}{1-M_o^2} + \dots, \tag{26}$$

$$V_- = 1 + \beta/(1-M_o^2) + \dots \tag{27}$$

for deflagrations, and for detonations we interchange V_+ and V_- in the above formulas. The apparent non-uniformity near $M_o^2 = 1$ is resolved by noting that there are no steady solutions possible for

$$M_{ocJ-}^2 < M_o^2 < M_{ocJ+}^2 \quad . \tag{28}$$

Thus M_o^2 is bounded away from one by $O(\beta^{1/2})$ and $V_\pm$ are uniformly close to one as $\beta \to 0$ as required.

Also we note that if we were to set $\beta \equiv 0$, the theory presented so far collapses to that of the adiabatic shock. The solution of which is given by equation (17) where $V_\pm$ are found by setting $\beta = 0$ appropriately. With the condition that $V_o(-\infty) = 1$ there is a non-constant solution only if $M_o^2 > 1$. In which case (17) predicts that $V_o(\infty) = [(\gamma-1)M_o^2+2]/(\gamma-1)M_o^2$. Taylor's classical analysis of the weak shock wave assumed that

$$M_o^2 - 1 = \varepsilon << 1 \quad \text{and} \quad V_o = 1 + \varepsilon v' + \dots \quad . \tag{29}$$

Directly from equation (8) we can anticipate Taylor-like shock structure only when the heat release is sufficiently small. We will see that this is the case.

4.a Deflagrations

Following I, for M_o^2 not close to one, we write

$$V_o = 1 + \beta V' + \dots \quad , \quad \tau_o = 1 + (\gamma-1)M_o^2/2 + \beta\tau' \quad , \tag{30}$$

so that

$$T_o = 1 + \beta(\tau' - (\gamma-1)M_o^2 V') + \ldots \tag{31}$$

and

$$\gamma M_o^2 dV'/ds = -(1-M_o^2)V' + \tau' \quad . \tag{32}$$

The solution which vanishes at $s = -\infty$ and is continuous at $s = 0$ corresponding to deflagrations is

$$V' = \begin{cases} [1+(\gamma-1)M_o^2]^{-1}e^s & \text{for } s < 0 \ , \\ [1-M_o^2]^{-1}\{1-\gamma M_o^2[1+(\gamma-1)M_o^2]^{-1}e^{-(1-M_o^2)s/\gamma M_o^2}\} & \text{for } s > 0 \ . \end{cases} \tag{33}$$

For small heat release the flame temperature must be represented as

$$T_* = 1 + \beta t_* \tag{34}$$

and the condition that $T_o(0) = T_*$ then leads to

$$t_* = 1/(1+(\gamma-1)M_o^2) \quad . \tag{35}$$

For a given t_* equation (35) serves to determine the wave speed, i.e., M_o^2.

However as we have indicated, the formula (33) clearly shows a non-uniformity as $M_o^2 \to 1$. From the expansion of the Chapman-Jouget Mach number, equation (24) we see that to resolve this difficulty we must take

$$M_o^2 = 1 + \sigma\beta^{1/2} \quad , \quad \sigma \leq -\sqrt{2(\gamma+1)} < 0 \quad . \tag{36}$$

We consider the expansion

$$V_o = 1 + \beta^{1/2}v_1 + \beta v_2 + \ldots \tag{37}$$

and substitution of (37) into (15) and (16) leads to the conclusions that

$$v_1 = 0 \quad , \quad v_2 = e^s/\gamma \quad \text{for} \quad s < 0 \quad \text{and} \tag{38}$$

$$v_1 = 0 \quad , \quad v_2 = (s+1)/\gamma \quad \text{for} \quad s > 0 \quad . \tag{39}$$

The unboundedness of v_2 as $s \to \infty$ suggests that we consider a change of scale for $s > 0$ (also suggested by the Taylor shock wave). So let

$$\beta^{1/2} s = \eta \tag{40}$$

and then there is a nontrivial balance for v_1 given by

$$\frac{dv_1}{d\eta} = \frac{\gamma+1}{2\gamma} (v_1 - v_+)(v_1 - v_-) \tag{41}$$

where

$$v \pm = - \frac{\sigma}{\gamma+1} [1 \pm \sqrt{1-2(\gamma+1)/\sigma^2}] \quad . \tag{42}$$

The only solution to equation (41) that satisfies the condition $v_1(0) = 0$ is found to be

$$v_1 = \frac{v_+ - v_- e^{\delta\eta}}{1-(v_+/v_-)e^{\delta\eta}} \tag{43}$$

where $$\delta = \frac{4\gamma}{(\gamma+1)^2} \sqrt{\sigma^2 - 2(\gamma+1)} \quad . \tag{44}$$

We note as a consequence of solution (43), a strong deflagration (i.e. $v_1 \to v_+$ as $\eta \to +\infty$) is not a possibility.

Finally for the Chapman-Jouget value $\sigma_{cJ} = - \sqrt{2(\gamma+1)}$, $v_+ = v_-$, and solving (41) again gives

$$v_1 = \sqrt{\frac{2}{\gamma+1}} \left(1 - \frac{1}{\sqrt{(\gamma+1)/2}\eta/\gamma+1}\right) \quad . \tag{45}$$

4.b Detonations

If we proceed to analyze detonations using expansions (30) then equation (32) is valid. The corresponding solution for V' that vanishes at $s = -\infty$ and is continuous at $s = 0$ is then

$$V' = \begin{cases} \dfrac{[\gamma M_o^2 \exp(\frac{-(M_o^2-1)}{\gamma M_o^2} s) + (1-M_o^2)e^s]}{[1+(\gamma-1)M_o^2](1-M_o^2)} & \text{for } s < 0 , \\ \dfrac{1}{1-M_o^2} \quad \text{for } s > 0 . & \end{cases} \tag{46}$$

Again the flame temperature is represented as in (34) and

$$t_* = (1-\gamma M_o^2)/(1-M_o^2) \quad . \tag{47}$$

Thus the detonation structure described here has a unique speed determined by the flame temperature exactly like the deflagration. In fact the deflagration and detonation analyzed in this way correspond to the weak deflagration and weak detonations, i.e.

$$V_o \to 1 + \beta/(1-M_o^2) + \ldots \quad \text{as} \quad s \to \infty \quad .$$

The fact that the above analysis does not yield a description of the strong detonation, (strong deflagration) is not suprising since it would require the strong end point lie close to $V = 1$, or

$$[(\gamma-1)M_o^2+2]/(\gamma+1)M_o^2 \sim 1 \quad , \tag{48}$$

which is only true if $M_o^2 \sim 1$.

Thus to recover the description of the strong detonation it is again necessary to consider the distinguished limit given by (36) where

$$\sigma \geq \sqrt{2(\gamma+1)} > 0 \quad . \tag{49}$$

Expanding V_o as in (37) we find in particular that

$$v_1 = v_* \quad , \quad v_2 = \frac{\gamma+1}{2\gamma} v_*(v_* + \frac{2\sigma}{\gamma+1})s + \frac{e^s}{\gamma} + v_{2*} \quad \text{for} \quad s < 0 \quad , \tag{50}$$

where v_* is a constant.
We note that for $v_* \neq 0$, then we must have a Taylor adjustment region downstream as well as the upstream adjustment that we encountered earlier. We assume the stretch (40) and the equation for v_1 becomes

$$\frac{dv_1}{d\eta} = \frac{\gamma+1}{2\gamma} v_1(v_1 + \frac{2\sigma}{\gamma+1}) \quad \text{for} \quad \eta < 0 \quad , \tag{51}$$

and the equation (41) holds for $\eta > 0$.

The solution of (51) is given by

$$v_1 = \frac{-2\sigma}{\gamma+1} (1+Ae^{-(\sigma/\gamma)\eta})^{-1} \quad . \tag{52}$$

Since $\sigma > 0$, $v_1 \to 0$ as $\eta \to -\infty$ satisfying the required boundary condition for any value of $A \neq 0$. In fact (52) is Taylor's solution of the weak adiabatic shock mentioned at the end of section 3. The value of the constant A is not determined and its choice then defines the strength of the shock, i.e. the value of the velocity v_* at the flame sheet. Clearly to describe the shock transition we require

$$A > 0 \quad ; \tag{53}$$

otherwise we will encounter an infinity for v_1 at some negative but finite value of η. Matching requires that

$$v_* = \frac{-2\sigma}{\gamma+1} \frac{1}{1+A} \quad , \tag{54}$$

hus (53) implies that

$$-\frac{2\sigma}{\gamma+1} < v_* < 0 \quad . \tag{55}$$

The downstream adjustment region, $\eta > 0$, is again found by solving (41), so that the solution is generally given by

$$v_1 = \frac{v_+ - [(v_+ - v_*)/(v_- - v_*)]v_- e^{\delta\eta}}{1 - [(v_+ - v_*)/(v_- - v_*)]e^{\delta\eta}} \quad . \tag{56}$$

In order to ensure a uniformly bounded solution as $\eta \to \infty$ it is necessary to restrict v_* so that

$$[(v_+ - v_*)/(v_- - v_*)] > 1 \quad . \tag{57}$$

Since $v_- < v_+ < 0$, equation (57) implies that

$$v_* < v_+ \quad . \tag{58}$$

We note that, as one would expect, that (56) also contains the simplest solutions of (41) namely

$$v_1 = v_* = v_- \text{ or } v_+ \text{ for } \eta > 0 \quad . \tag{59}$$

Finally the Chapman-Jouget detonation occurs when

$$v_+ = v_- = \sqrt{\frac{2}{\gamma+1}} \text{ and } \sigma_{cJ} = \sqrt{2(\gamma+1)} \quad . \tag{60}$$

Its solution downstream, $\eta > 0$, is found to be

$$v_1 = \left[\frac{-1}{v_* + \sqrt{\frac{2}{\gamma+1}}} - \frac{\gamma+1}{2\gamma}\eta \right]^{-1} - \sqrt{\frac{2}{\gamma+1}} \quad . \tag{61}$$

Thus we have found that if

$$-\frac{2\sigma}{\gamma+1} < v_* \leq v_+ \quad , \tag{62}$$

we have a proper description of the velocity structure and hence the entirety of the detonation wave. We note that the lower limit in (62) represents the maximum velocity difference that the adiabatic shock can attain while the upper limit represents the maximum velocity allowable at the flame sheet such that there is a velocity adjustment region behind the flame sheet.

Equation (62) then implies a restriction of the flame temperature $T_* = 1 + \beta^{1/2} t_*$, where t_* is related to v_* simply by

$$t_* = -(\gamma+1)v_* \tag{63}$$

In the present theory, t_* is assumed to be specified so that (62) should in fact be interpreted as a restriction on σ (i.e. the Mach number corresponding to the wave speed). The two inequalities expressed in (62) lead to the conclusion that for a given t_*

$$\sigma \geq \frac{(\gamma-1)}{2t_*}\left(2 + \frac{(\gamma+1)}{(\gamma-1)^2} t_*^2\right) \quad . \tag{64}$$

The minimum wave speed is found by differentiating the right hand side of (64) and setting the result equal to zero. Thus we find that when

$$t_* = (\gamma-1)\sqrt{2/(\gamma+1)} \quad , \quad \sigma_{min} = \sigma_{cJ} = \sqrt{2(\gamma+1)} \quad . \tag{65}$$

Thus the minimum wave speed is precisely the Chapman-Jouget speed. (See Fig. 3.)

Since specification of T_* characterizes the Damköhler number D, we are lead to the conclusion that in the context of the small heat release assumption, detonations nearly always travel faster than the Chapman-Jouget wave speed. And for a given gas, (i.e. t_*) the minimum wave speed possible is nearly always greater than the Chapman-Jouget wave speed.

Finally, the work presented here can be extended to Pr, Le $\neq$ 1. Full details of this extension concerning deflagrations are given in I. We have carried out the work for deflagrations near the Chapman-Jouget wave speed and for all detonations which can be described in the small heat release limit and we have found the differences minor enough as to regard the discussion presented here as quite general.

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Lu, G.C. & Ludford, G.S.S., 1980, Asymptotic analysis of plane steady detonations. Submitted for publication.

TABLE 1

γ	Ratio of specific heats.
M_o	Mach number of flame with respect to the undisturbed speed.
α	Nondimensional heat of reaction.
θ	Activation energy.
D	Damköhler number.
Λ	$= DM^{-2}$
M	= Reference mass flux.

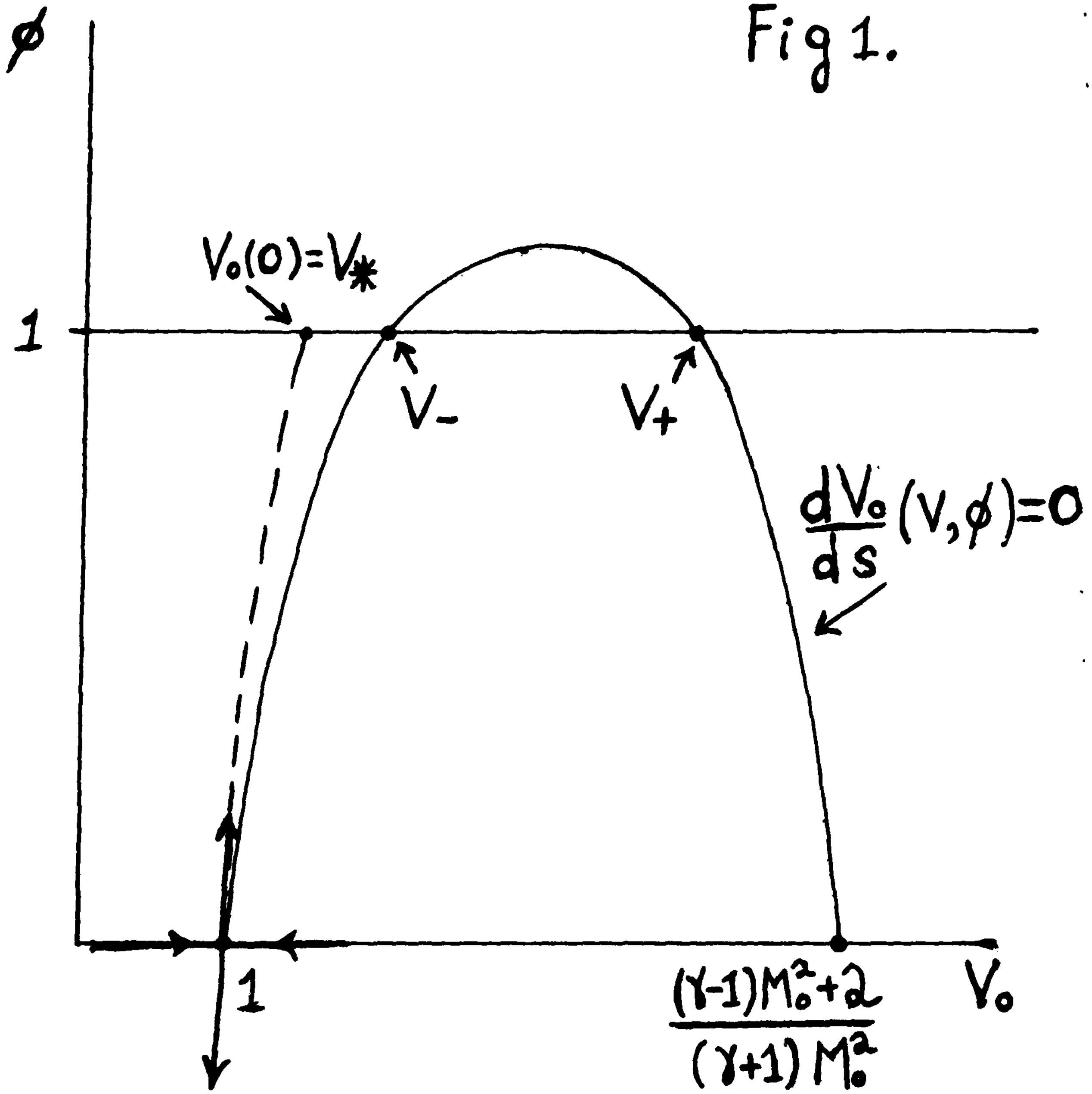
CAPTIONS

Fig. 1. The V_o, ϕ phase plane shown for deflagrations. Note that equation (15) with $dV/ds = 0$ defines a parabola whose intersection with $\phi = 0$ and $\phi = 1$ define the fixed points of the system.

Fig. 2. The V_o, ϕ phase plane shown for detonations. Permissible V_* lie in a range $V_{*min} < V_* < V_+$. The weak detonation goes directly into V_+ and has no downstream adjustment.

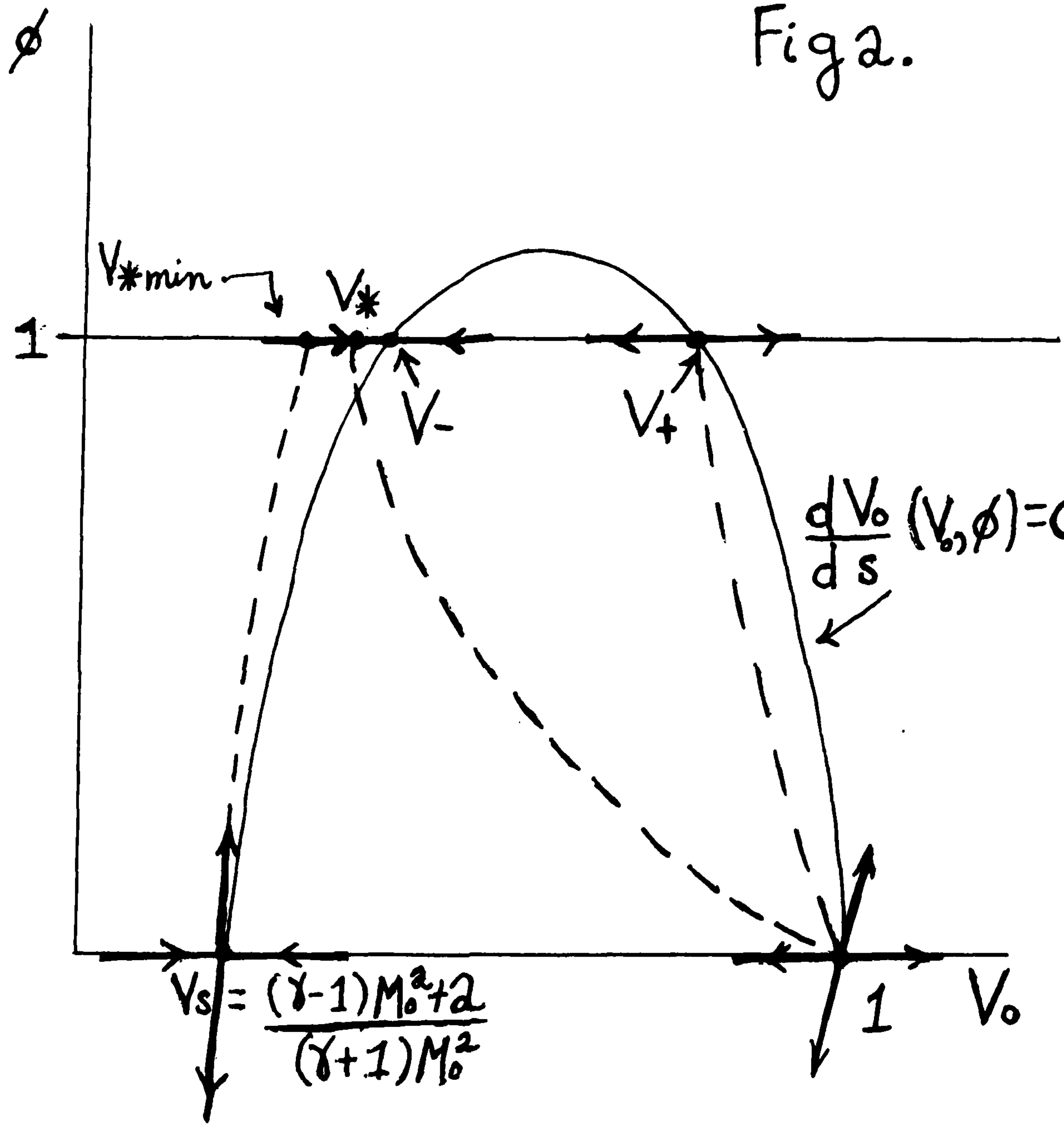
Fig. 3. Shaded region shows permissible wave speeds σ. For a given t_*, $\sigma_{min} < \sigma$ where σ_{min} is nearly always greater than σ_{cJ}.

Fig 1.



ϕ
$V_0(0)=V_*$
1
V_-
V_+
$\frac{dV_0}{ds}(V,\phi)=0$
1
$\frac{(\gamma-1)M_0^2+2}{(\gamma+1)M_0^2}$
V_0

Fig 2.


ϕ
1
V_{*min}
V_*
V_-
V_+
$\frac{dV_0}{ds}(V_0,\phi)=0$
$V_s = \frac{(\gamma-1)M_0^2+2}{(\gamma+1)M_0^2}$
1
V_0

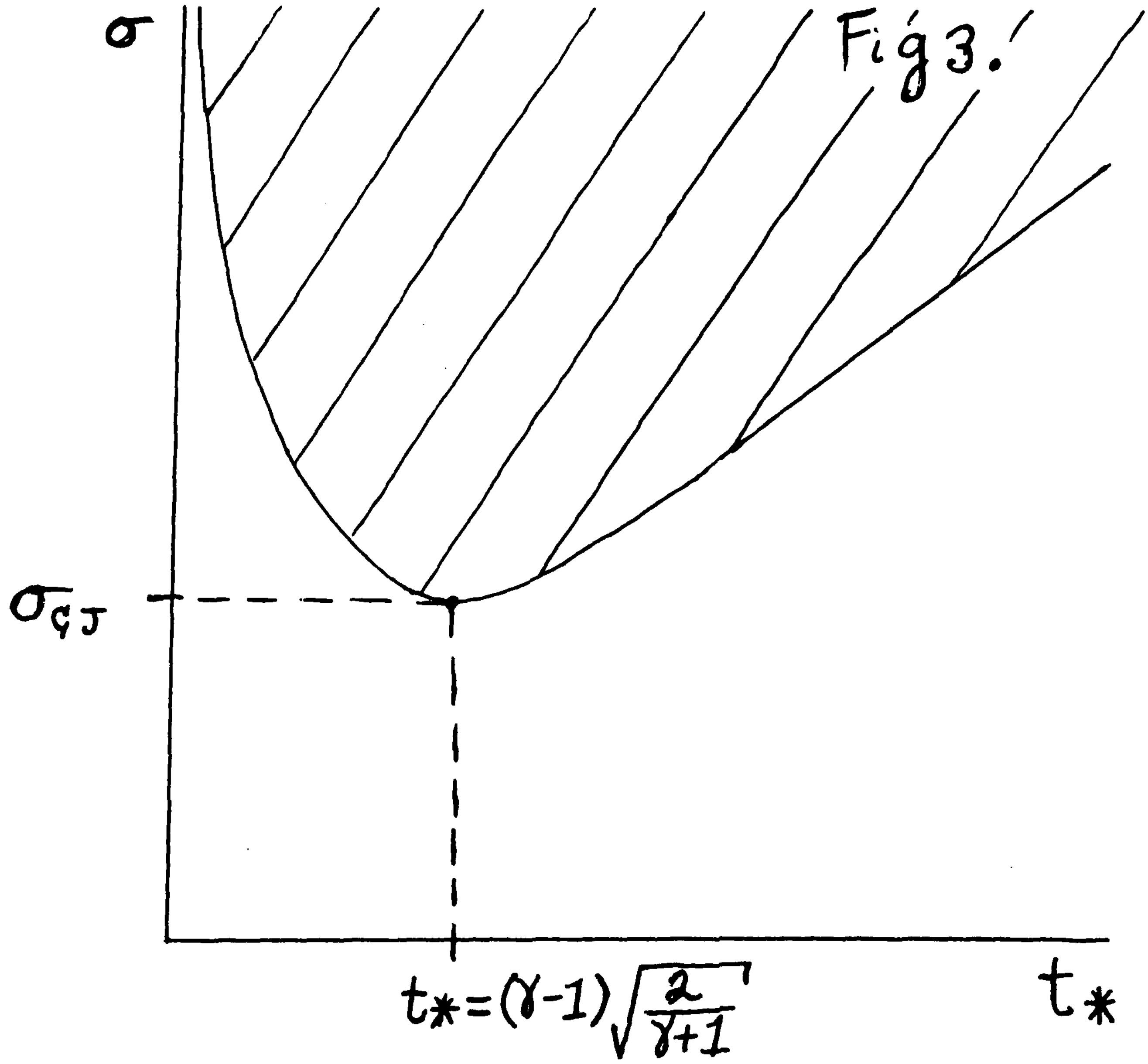


Fig 3.
σ
σ_{cJ}
$t_* = (\gamma-1)\sqrt{\frac{2}{\gamma+1}}$
t_*

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Technical Reports 117 and 118 develop the solutions (using activation-energy asymptotics) of one-dimensional steady combustion waves, deflagrations and detonations repectively, when the Mach number is not small. Even with the great simplifications afforded by the limit of large activation-energy some numerical calculations are necessary. However a completely analytical description of these solutions is possible whenever the heat released during reaction is small. In this paper we give these explicit

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) analytical solutions for the fast deflagration wave and a simple expression for its speed of propagation. As the speed of propagation approaches the lower Chapman-Jouget wave speed (slightly less than sonic velocity) we show that the velocity structure in front of the flame adjusts to a classical Taylor shock. We also give an explicit analytical solution for detonations traveling at speeds greater thatn the upper Chapman-Jouget velocity (slightly greater than sonic velocity); in particular, such strong detonations are characterized by Taylor-like velocity adjustments both in front of and behind the flame. (For detonations the speed is not determined.)

This work serves as the basis for a completely analytical treatment of the transition form deflagration to detonation.