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OPTIMAL MISSILE AVOIDANCE AND IMPROVED AIR COMBAT MODELS

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The research efforts under this contract were oriented to solve two different air combat problems: optimal missile avoidance and medium range air-to-air interception. In both topics closed form solutions were obtained using either optimal control or differential game formulations. Application of the technique of singular perturbation for nonlinear differential games has shown to be very promising.

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TABLE OF CONTENTS

LIST OF TABLES

I. INTRODUCTION

The research performed under the present contract (F49620-79-C-0135) has been composed of two independent, but closely related activities. The investigation efforts were aimed towards two air-combat oriented pursuit-evasion problems, namely:

(i) missile vs. aircraft engagement,

(ii) air to air interception between airplanes.

The research objectives outlined in Section E of the contract were the following:

1. Extending the results of a previous investigation (performed under AFOSR Grant 77-3458) which dealt with optimal avoidance of proportionally guided missiles based on a linearized kinematic model. The extension included beam-rider type guidance laws as well as generalized pay off functions.

2. Analysis of the impact of imperfect information on optimal missile avoidance. Two particular avoidance problems were addressed as characteristical examples of eventual situations. (a) The parameters of the missile are known but the evading airplane has no information on the relative state. (b) The guidance law of the missile is unknown but some of its physical limitations are assumed.

3. Application of the technique of singular perturbations to analyse air combat problems (interception for example) as nonlinear zero-sum differential games.

The ultimate goal in all research topics has been to derive approximating algorithms which are suitable for real-time airborne applications

Detailed descriptions of the investigation efforts as well as the results achieved during the period of this research contract are presented in a set of six separate Interim Scientific Reports (Refs. 1-6). These Interim Reports were issued and forwarded to the USAF immediately as they became available.

This Final Report intends to present a summarizing viewpoint on the main topics of the investigation and some recommendations for future research In Section II the results relating to the problem of optimal missile avoidance with perfect information are summarized Section III deals with the effect of information imperfections as derived from two particular examples. In Section IV an approximate closed-form solution is outlined for the problem of medium range aircraft interception this solution was obtained applying the approach of forced singular perturbations to this nonlinear pursuit-evasion game Conclusions of the research effort and recommendations for further investigations are presented in Section V

II. OPTIMAL MISSILE AVOIDANCE WITH PERFECT INFORMATION.

A. Effect of Limited Aircraft Roll-Rate

The analysis of optimal avoidance from a proportionally guided missile in three-dimensional space was presented in Ref. 7. This work indicated that optimal missile avoidance can be reduced to an optimal roll-position control problem of the following nature: orienting the lateral acceleration vector of the evading aircraft into the plane of optimal evasion (determined by interception geometry) and changing the direction of this acceleration, which has to be of maximal amplitude, by rapid roll maneuvers of 180° timed by an optimal switch function. The roll-rates required to execute properly this maneuver sequence exceed considerably current and even predicted future aircraft capabilities. Including the constraints of admissible roll-rates leads to formulate a singular optimal control problem. The formal analytical solution indicates that the optimal evasion strategy is composed by an alternating sequence of regular subarcs of maximum roll-rate and singular subarcs of almost zero roll-rate. The effects of roll-rate limitation on the optimal missile avoidance are the following:

1. The start of the rapid roll maneuvers of 180° using maximum admissible roll-rate has to be advanced relatively to the optimal switch time in order to allow the airplane to complete a 90° roll position change. A proper timing is most important for the last maneuver before the estimated time of intercept.

2. Bounded aircraft roll-rates produce some reduction of the maximum attainable miss distance.

In order to express these qualitative results in a quantitative form the singular optimal control problem has to be solved numerically. The difficulties to perform such a solution are well known. In the present research a special variable metric (quasi-Newton) algorithm, based on Broyden parameter optimization method, was developed as reported in Ref. 1. This algorithm was used to solve a very large number of numerical optimization problems covering the following parameter space of interest:

a. Effective proportional navigation ratio $3 \leq N \leq 5$.

b. Normalized time of flight (τ being the guidance time constant) $10 \leq (t_f/\tau) \leq 20$

c. Missile-aircraft maneuver ratio $\left(\mu = \frac{a_M}{a_T}\right)$ $2 \leq \mu \leq 100$

d. Normalized roll-rate constraint, expressed by the number of missile time constants to perform a 180° roll maneuver $\left(\Theta_\varphi = \frac{180}{\tau(\dot{\varphi})_{max}}\right)$ $0 \leq \Theta_\varphi \leq 6$

The extensive numerical investigation has allowed to express the dependence of the normalized miss distance (defined by $M^* \triangleq m/\tau^2 a_T$) on the normalized roll-rate constraint (Θ_φ) by a simple approximate formula of the form

$$M^* = M_0^*\left(1 - B_\varphi \Theta_\varphi^2\right) \tag{1}$$

where M_0^* (the miss distance attainable without roll-rate constraint) and the coefficient B_φ, are both functions of the parameters μ,N and (t_f/τ)

$$M_0^*(\mu,N',t_f/\tau) = E\left[1 + e_1/\mu + e_2/\mu^2\right]^{-1} \tag{2}$$

$$B_\varphi(\mu,N',t_f/\tau) = B\left[1 + b_1/\mu + b_2/\mu^2\right] \tag{3}$$

where

$$E(N',t_f/\tau) = E_0\left[1 + e_{01}/N' + e_{02}/N'^2\right]^{-1} \tag{4}$$

$$e_1(N',t_f/\tau) = e_{10}\left[1 + e_{11}/N' + e_{12}/N'^2\right]^{-1} \tag{5}$$

$$e_2(N',t_f/\tau) = e_{20}\left[1 + e_{21}/N' + e_{22}/N'^2\right]^{-1} \tag{6}$$

and

$$B(N',t_f/\tau) = B_0\left[1 + b_{01}N' + b_{02}N'^2\right] \tag{7}$$

$$b_1(N',t_f/\tau) = b_{10}\left[1 + b_{11}/N' + b_{12}/N'^2\right] \tag{8}$$

$$b_2(N',t_f/\tau) = b_{20}\left[1 + b_{21}/N' - b_{22}/N'^2\right] \tag{9}$$

The set of coefficients (E_0, e_{ij} ... B_{01}, b_{ij}) are tabulated for different values of (t_f/τ) in Table 1.

From the numerical results the following qualitative information can be summarized:

1. If the normalized roll-rate constraint is not too large, i.e., 180° roll position change can be performed during 2 missile time constants or less, the decrease of miss distance due to the roll-rate constraint is negligible.

Normalized Time of Flight $t_f/\tau = 10$

$E_0 = 1.32$	$e_{01} = -5.70$	$e_{02} = 14.28$
$e_{10} = 1.49$	$e_{11} = -9.72$	$e_{12} = 15.90$
$e_{20} = -3.70$	$e_{21} = -14.55$	$e_{22} = 27.81$
$B_0 = 0.055$	$b_{01} = -0.685$	$b_{02} = 0.079$
$b_{10} = -0.52$	$b_{11} = -16.8$	$b_{12} = 39.3$
$b_{20} = -35.3$	$b_{21} = -7.17$	$b_{22} = 14.25$

Normalized Time of Flight $t_f/\tau = 20$

$E_0 = 2.51$	$e_{01} = -5.26$	$e_{02} = 19.30$
$e_{10} = -2.5$	$e_{11} = -6.82$	$e_{12} = 21.34$
$e_{20} = -3.0$	$e_{21} = -14.05$	$e_{22} = 24.26$
$B_0 = 0.023$	$b_{01} = -0.678$	$b_{02} = 0.15$
$b_{10} = -0.29$	$b_{11} = -14.4$	$b_{12} = 35.7$
$b_{20} = -47.2$	$b_{21} = -7.94$	$b_{22} = 17.05$

TABLE 1. List of Coefficients in Eqs. (4)-(9).

2. If the normalized roll-rate limitation is important (for example $\Theta_{\varphi} = 6$) a considerable decrease in the miss distance (up to 20-30%) can be expected.

3. The higher the effective navigation ratio N' and the shorter the time of flight, the stronger is the sensitivity of the miss distance to roll-rate constraints.

4. If, due to the slow admissible roll-rate of the evading airplane, one of the 180° roll maneuvers cannot be completed a rather serious loss of miss distance (50% or more) can be expected.

B. Optimal Evasion from Beam Rider Missiles (Ref. 2)

Since many operational ground to air missile systems use the command to line of sight (or three point) guidance law or its derivatives, it seemed to be important to analyse optimal avoidance from this type of missiles. The beam rider concept implemented in these guidance systems require from the missile to follow the line of sight between the illuminating radar (the "beam") and the target.

The kinematical equations of such missiles are strongly nonlinear which made analysis rather difficult. However, in new command guidance "beam-rider" missiles, used in point defence missions, the line of sight rotation can be neglected and a linearized kinematical model can be used for analysis. Such an analysis was reported in Ref. 2. In this report the effect of beam "lead" modification was also investigated. The modified guidance law, used in some operational systems, can be expressed by the following equation

$$\alpha_{T_L} = \alpha_T + \psi(t_f - t)\dot{\alpha}_T \qquad (10)$$

where α_{T_L} is the corrected line of sight angle, α_T is the actual one, $\dot{\alpha}_T$ is the line of sight rate and ψ is the lead parameter $(0 < \psi < 1)$.

The results of this preliminary investigation can be summarized as follows:

1. The optimal evasion strategy, as predicted by the linearized kinematical model, has a "bang-bang" structure similar, but not identical, to the one used against proportionally guided missiles.

2. The exact timing of this optimal maneuver and the resulting miss distance depend very strongly on the guidance system parameters and target characteristics.

3. The effect of the "lead" parameter ψ on the performance of beam rider missiles can be summarized in qualitative terms by:

a. a decrease in the sensitivity to target maneuvers

b. an increase in the sensitivity to noise

c. a requirement for more rapid evasive maneuvers.

A more detailed quantitative analysis was not in the scope of the present research contract.

C Extension of the Validity of Linearized Kinematics Using a Generalized Pay-off (Ref. 4).

Optimal missile avoidance was analysed in previous works [Refs 7, 1, 2, etc.] using a linearlized kinematic model. The validity of trajectory linearization, the very core of such models, is valid only if the geometry of the engagement does not change considerably. Since the geometrical conditions mainly depend on the evaders' trajectory it should not deviate much from its initial conditions. This requirement can be satisfied if:

(i) The dynamic similarity parameter of the problem $\left(\gamma \triangleq \frac{a_T \tau}{V_T}\right)$, defined as the direction change of the evader during the period of the missile's time constant, is a small quantity;

(ii) excessively long turns in one direction are not performed.

The first condition can and has to be examined before trajectory linearization is adopted. The second one, however, can be verified only after the solution is known. Due to the alternating "bang-bang" structure of the optimal missile avoidance strategy this second condition is satisfied in a large majority of the cases of interest. A recent previously reported investigation (Ref. 11) revealed that there exists a range of parameters (long flight times, small values of effective proportional navigation constants, low missile-target maneuver ratios) for which long turns in one direction are predicted by the linarized kinematic model. In other works (Ref. 7), it was shown that the sensitivity of the miss distance to target maneuvers which are performed far away from the intercept is relatively low. These inefficient long maneuvers, which invalidate the linearized kinematic assumption, can be eliminated by modifying the performance index of the optimal missile avoidance problem by augmenting it by a control penalization term as proposed in Ref. 4. The augmented cost function is

$$J = m^2(t_f) - K \int_0^{t_f} u^2 \, dt \tag{11}$$

By proper choice of the weighting coefficient K the difference between the resulting miss distance and the optimal one can be made sufficiently small. Results of three-dimensional complet simulation (program developed in a previous research phase reported in Ref. 12) has confirmed that the optimal control strategy obtained by minimizing the modified performance index leads to miss distances which are equal or even slightly larger than the ones predicted by the linearized model.

It can be thus summarized that using the above described modification the domain of validity of trajectory linearization for optimal missile avoidance has been largely extended.

III. OPTIMAL MISSILE AVOIDANCE WITH IMPERFECT INFORMATION

A. Introductory Discussion

All previously reported studies dealing with optimal missile avoidance were formulated as deterministic optimal control problems. Such formulation assumes implicitly the existence of perfect information on the state variables as well as on the parameters of the problem. As a consequence the results predicted in these works can be achieved only if the pilot of the evading airplane is allerted in time whenever a missile of known type is launched against his aircraft and he can measure or at least estimate the instant of the interception Unfortunately in real air combat environment these conditions are not satisfied.

There are several sources of information imperfections:

(i) lack of intelligence data;

(ii) lack of real-time threat identification;

(iii) lack of threat warning;

(iv) measurement errors and/or noise

Each of these topics deserves a separate analysis and an extensive research effort, which are out of the scope of the reported contract. In the present frame the impact of imperfect information on optimal missile avoidance is shown by two particular examples. The first one relates to an eventual "near future" application, dealing with optimal evasion from a known missile without having any information on the relative state (Ref. 3). The second example analyses the problem of avoidance from a "future" missile using an unknown, probably optimal guidance law [Refs. 8 and 6].

B. Missile Avoidance Without State Information (Ref 3)

Deterministic optimal missile avoidance requires a "bang-bang" maneuver strategy governed by a switch function which depends on the "time to go" estimated by measured range and range-rate. If those measurements are not available the stochastically optimal avoidance has to be based on randomly varying maximum maneuvers. In this case the optimal missile avoidance can be transformed to the problem of a homing missile fired against a randomly maneuvering target. Between the three types of random maneuvers of interest:

(i) Random Telegraph Manuever (of Poissonian probability distribution),

(ii) periodical maneuver with random starting time,

(iii) periodical maneuver with random phase.

The last one seems to be the most efficient. It can be shown that for a given maneuver energy the periodical one is indeed optimal and the optimal maneuver frequency can be determined as a function of problem parameters (Ref. 3). Assuming unlimited missile maneuverability the normalized optimal frequency "u" is the function of the effective proportional navigation ratio (N'),

$$u = \omega\tau = \left(\frac{N'}{2} - 1\right)^{1/2} \tag{12}$$

and the normalized R.M.S. miss distance for a random sinusoidal maneuver of this frequency is given by

$$\sqrt{\overline{M^{*2}_{\sin}}} = \frac{\sqrt{\overline{m^2}}}{\tau^2 a_T} = \sqrt{2}\left(\frac{(N'-2)^{(N'-2)}}{N'^{N'}}\right)^{1/2} \tag{13}$$

τ being the missile's first order time constant and a_T is the amplitude of the lateral target acceleration. For a random square wave type maneuver of the same frequency the R.M.S. miss distance is about 30% larger.

Considering limited missile maneuverability leads to predict lower maneuver frequencies and considerable larger R.M.S. miss distances.

Comparing the results of such stochastic optimization to the case of perfect information reveals that the R.M.S. miss distance obtained by a random maneuver of the optimal period can reach 60-80% of the optimal deterministic value. This comparison indicates that the degradation of missile avoidance capability due to lack of accurate state information may not be as serious as is generally estimated.

C. Evasion from a Missile of Unknown (Optimal) Guidance Law (Ref. 6)

The analysis of this problem, which may be of major importance in future air combat, was carried out using the formulation of a zero-sum (perfect information) linear differential game [Refs. 6 and 8]. In this formulation it is assumed that the relative state and the time to go are perfectly measured and the physical limitations of the missile (maximum acceleration, speed) and its dynamics (time constants) are known. The unknown is the missile's actual guidance strategy and it is assumed that this strategy can be optimal in a differential game sense. Using a linearized kinematical model this differential game was solved in a closed form. The solution included the optimal guidance law of the missile, the optimal evasive strategy and the value of the miss distance obtained by these optimal strategies.

The conclusions of the analysis for missile avoidance are not encouraging. The game solution predicts that if the following inequality

$$\left(a_P\right)_{max} > \left(a_E\right)_{max}\left(\frac{\tau_P}{\tau_E}\right) \tag{14}$$

(where $\left(a_P\right)_{max}$, $\left(a_E\right)_{max}$ are maximal lateral accelerations and τ_P, τ_E are first order time constants of the pursuing missile and the evading airplane respectively) is satisfied, an optimally guided missile can guarantee <u>zero miss distance</u> for most initial conditions in its firing envelope against <u>any</u> evasive maneuver. Since a well designed missile can easily satisfy the requirements imposed by the inequality, the success of an interception depends mainly on the capability to implement the "optimal" guidance law. This guidance strategy is based on perfect measurements of the state variables including the acceleration of the evading airplane. Only by denying such perfect information from the missile can aircraft survivability be enhanced.

If the missile cannot measure or accurately estimate evader accelerations a simple avoidance strategy can be used. However, the miss distance guaranteed by such evasive maneuver may not be sufficient to exceed the lethal radius of the warhead. Large miss distances can be expected only if missile measurements are very noisy or jammed. For analysis of such situation a stochastic differential game formulation is required.

IV. MEDIUM RANGE AIR TO AIR INTERCEPTION GAME SOLVED BY THE TECHNIQUE OF FORCED SINGULAR PERTURBATIONS

A Problem Formulation and Qualitative Analysis

The medium range air to air interception appears to be one of the basic elements of future air combat. Assuming that the roles of the participating airplanes are determined as pursuers (interceptors) and evaders (targets) by the pertinent operational conditions, such an engagement can be formulated as a nonlinear zero-sum differential game. Medium range interceptions are characterized by large initial distances of separation. Termination of the interception is by firing an air to air missile near to its maximum range, which is larger than the turning radius of the airplanes. As a consequence of these geometrical features the rotation of the line of sight is very slow and terminal maneuvers of the evader are not effective. The objective of the interceptor is to fire its guided weapon as soon as possible and the evading target tries to escape from the firing envelope of the missile. It can be intuitively seen that the engagement has two phases:

(i) the main "pursuit" phase, in which each airplane tries to accelerate to its maximum speed; this is a straight line "tail chase", which follows the previous

(ii) initial "acquisition" phase, in which both participants concentrate to correct the unfavourable initial conditions of the engagement and reach the optimal position for the "pursuit" phase.

The turning maneuver in the acquisition phase has to be an optimal compromise between the "fastest" turn (requiring generally loss of speed) and the best acceleration. Due to the inherent nonlinear nature of the problem it could not be solved in a closed form in the past. In the frame of the present research contract an approximate analytical solution was obtained using the technique of forced singular perturbation. This technique, which was only recently adopted for nonlinear zero-sum differential games (Ref. 9), is based on the assumption that there exists a time scale separation between the variables.

B. Zero-Order Solution.

The application of forced singular perturbation technique (FSPT) to medium range air to air interception yielded a closed form zero-order approximation as reported in Ref. 5

In this paper the following time scale separation was assumed.

(i) range and line of sight orientation are the slowest variables,

(ii) aircraft velocities are next in the hierarchy,

(iii) aircraft turning dynamics are the "fastest".

The zero-order solution of this FSPT model can be expressed in a "feedback" form determining the required turning rate of each aircraft based on its own current speed and the angular difference between his present direction and the line of sight. Such control strategy is very attractive for real-time airborne implementation since it is based on variables which are easily measurable onboard.

To evaluate the accuracy of the zero-order approximation and the eventual necessity for higher order correction a comparison to the exact numerical solution was required.

C. Comparison to an Exact Numerical Example

The required comparison was made very recently and has not yet been reported. The numerical solution was obtained from Dr. Bernt Järmark of Saab-Scania, Sweden, using a differential dynamic programming (DDP) algorithm. Due to the inherent difficulties only a single, relatively simple, but characteristic example was solved. The initial conditions of the engagement and the aircraft data are given in Tables 2 and 3.

TABLE 2. Engagement Conditions

Combat altitude [m]	0
Initial range [m]	4000
Capture radius [m]	2680
Initial line of sight orientation	0°
Initial pursuer velocity [m/sec]	150 0
Initial evader velocity [m/sec]	100 0
Initial pursuer direction	100°
Initial evader direction	10

TABLE 3. Aircraft Data

	Pursuer	Evader
Weight [kg]	20,000	5,000
Wing area [m^2]	50	30
Max. load factor	7 5	6 0
*Max. lift coefficient	0.88	0 88
*Zero lift drag coef.	0.02	0.02
*Induced drag coef.	0.157	0.157
*Max. sea level thrust [kg]	5500	2500
Maximum velocity [m/sec]	290.0	252.5
Corner velocity [m/sec]	233.3	180.7
Minimum turning radius [m]	747	563

* Assumed to be independent of Mach Number.

The results of the comparison as presented in the following table are very encouraging.

TABLE 4. Comparison of Computational Results

Algorithm	Zero-order FSPT	Järmark's DDP
Capture time [sec]	87.7	85.0
Final line of sight angle	-32°	-22.7°
Final Pursuer Mach Number	0.704	0.690
Final Evader Mach Number	0.618	0.614

Comparison of the pursuer turning and velocity time histories indicate that the difference of 2.7 sec. in capture time is probably due to the slightly higher initial turning rates in the numerical DDP solution. Evader velocity profiles are almost identical in both solutions

Since only a single numerical comparison was made, it has been difficult to evaluate the necessity of higher order correction. The additional computational effort is not very important. It has to be pointed out, however, that such correction requires an interative approach and therefore cannot be implemented in a feedback form as the zero-order approximation.

V. CONCLUSIONS AND RECOMMENDATIONS

A. Optimal Missile Avoidance

The extensive and systematical investigation of optimal missile avoidance, which included several different aspects of the problem, has lead to the following conclusions:

(i) Optimal evasive maneuvers from currently used guided missiles can be determined by a relatively simple methodology based on a linearized kinematical model.

(ii) The validity of the linearized kinematics can be extended by a simple modification, and the results of the analysis are confirmed by three-dimensional nonlinear simulation.

(iii) Since the analysis assumed "perfect information" the implementation of the optimal missile avoidance strategy depends on measurements (or accurate estimation) of the line of sight, range and range-rate as well as the knowledge of missile parameters.

(iv) The simple semi-analytic formulae, derived from the detailed numerical analysis, which are presented in Ref. 11 and in this report, can serve for the real-time airborne implementation of the optimal evasive strategies as well as for the assessment of their effectiveness.

(v) If measurements of the state variables are not available for the evading airplane a random periodical maneuver strategy can be used. The effectiveness of such random maneuvers can reach in some conditions 60-80% of the optimal deterministic maneuver.

(vi) Avoidance of future missiles, probably guided by an "optimal" guidance law will not be possible if the missile will be provided perfect (or even relatively precise) measurements on the relative state. In this case aircraft survivability can be enhanced only by denying the missile such "perfect" information.

(vii) Future problems of missile guidance and avoidance with "imperfect" information can be analysed by the methodology of stochastical differential games. This area of research has to be motivated by the predicted feasibility of technical solutions as state of art optimal filtering and jamming. Investigations in this direction are necessary to define the "system concepts" of future aircraft survivability, and deserve focused attention and strong support.

B. Improved Air Combat Models

The first step to develop improved air combat model was made by applying the method of forced singular perturbation, which had been used in the past for aircraft performance optimization, in air combat oriented zero-sum nonlinear differential games.

The first example to be investigated was the medium range air to air interception using variable speed aircraft models with realistic aerodynamic and thrust data. The results of this effort can be summarized by the following:

(i) The FSPT model of medium range air to air interception yielded a closed form zero-order solution expressed in a "feedback" form.

(ii) This approximate "feedback" control strategy seems to be very attractive for "real-time" airborne implementation in future interceptors.

(iii) Comparison to results of an "exact" numerical solution indicates that the accuracy of the zero-order approximation is satisfactory.

(iv) Accuracy of the FSPT solution can be further approved, but the correction terms are not expressed in a feedback form and require off-line computation.

(v) Current FSPT methodology cannot deal with problems where the relative speed of the variables change during the engagements. Neither can this technique determine "feedback" control strategies for problems of "terminal boundary layer". These topics will be subjects of further investigation.

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