LEVEL II

12

AD A099227

UNIVERSITY of PENNSYLVANIA

PHILADELPHIA, PENNSYLVANIA 19104

JUN. 1979

DTIC
ELECTE
MAY 19 1981
S
C

This document has been approved
for public release and sale; its
distribution is unlimited.

81 4 23 035

81 4 23 035

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A099227	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) FAULT ANALYSIS of Analog Electronic Systems: Algorithms Based on Fuzzy Sets.		5. TYPE OF REPORT & PERIOD COVERED INTERIM rept.
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Jonghee Lee		8. CONTRACT OR GRANT NUMBER(s) ONR Contract N00014-75-C-0768
9. PERFORMING ORGANIZATION NAME AND ADDRESS Systems Engineering Department University of Pennsylvania Philadelphia, PA 19104		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE Jun 1979
193		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Office of Naval Research Code 427 800 North Quincy St. Arlington, VA 22217		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Ph.D. dissertation in Systems Engineering
supervised by Professor S. D. Bedrosian

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Fault analysis; Analog electronic systems; Fuzzy sets; Fuzzy algorithms
Fault diagnosis; Automated test equipment (ATE)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

There are essentially three fundamental problems involved in achieving effective automatic generation of fault isolation tests for analog electronic systems: feature extraction, fault classification and diagnosis.

For practical electronic circuits having component drifts and measurement noise, how are we able to introduce fuzzy set concepts and provide methods to achieve fault classification

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

278950

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20.

and diagnosis?

Along with the feature extraction problem, given an electrical network of known topology, what are the conditions for testability?

To attack the long standing fault isolation problem in analog electronic circuits, we have focused on two of the major problems. One is the presence of uncertainties such as indeterminacy, vagueness, randomness, and so on that naturally arise during the solution procedure of analog fault isolation. The other is the presence of topological restrictions inherent in specific circuit configurations.

Our main attention was focused on dealing with the fault isolation problem involving various kinds of uncertainties such as indeterminacy or vagueness. We show that such problems lend themselves very well to and in fact can be solved by adopting fuzzy set concepts. In particular this line of research has produced a modified fuzzy set technique applicable to automatic fault isolation. Topological aspects utilizing graph theory may be used effectively to assist in preanalysis of faulty analog electronic circuits. As a spin off of a consideration of these problems, we developed some new theorems for element value solvability. It should be made clear however that effective fault isolation can be accomplished with or without this pre-analysis to assist in resolving the more fundamental problem incurred by uncertainty.

As a consequence, this research yields the following specific results:

1. A base line automatic isolation system which can be used to deal with various kinds of uncertainties. A fuzzy automation model served as a point of departure for the base line system. Various fuzzy relations are used to select and update the parameters and structures of the system.

2. Set of algorithms and new decision criteria which can be implemented easily and used for effective fault isolation. A fuzzy distance measure and a fuzzy entropy measure are used for decision making in the fault isolation algorithms. The results are shown to be generally more effective than existing techniques.

3. Ample illustrative examples and simulation studies are included to back up these new methods. Several examples such as low pass filter, band pass filter, and communcation I/O circuits are used to illustrate the simulation studies. The results of simulation studies demonstrate the applicability of a fuzzy set technique.

ABSTRACT

FAULT ANALYSIS OF ANALOG ELECTRONIC SYSTEMS: Algorithms based on Fuzzy Sets

Jonghee Lee
Samuel D. Bedrosian

There are essentially three fundamental problems involved in achieving effective automatic generation of fault isolation tests for analog electronic systems : feature extraction, fault classification and diagnosis.

For practical electronic circuits having component drifts and measurement noise, how are we able to introduce fuzzy set concepts and provide methods to achieve fault classification and diagnosis?

Along with the feature extraction problem, given an electrical network of known topology, what are the conditions for tes[illegible]ty?

To attack the long standing fault isolation problem in analog electronic circuits, we have focused on two of the major problems. One is the presence of uncertainties such as indeterminacy, vagueness, randomness, and so on that naturally arise during the solution procedure of analog fault isolation. The other is the presence of topological restrictions inherent in specific circuit configurations.

Our main attention was focused on dealing with the fault isolation problem involving various kinds of

uncertainties such as indeterminacy or vagueness. We show that such problems lend themselves very well to and in fact can be solved by adopting fuzzy set concepts. In particular this line of research has produced a modified fuzzy set technique applicable to automatic fault isolation. Topological aspects utilizing graph theory may be used effectively to assist in preanalysis of faulty analog electronic circuits. As a spin off of a consideration of these problems, we developed some new theorems for element value solvability. It should be made clear however that effective fault isolation can be accomplished with or without this preanalysis to assist in resolving the more fundamental problem incurred by uncertainty.

As a consequence, this research yields the following specific results:

1. A base line automatic isolation system which can be used to deal with various kinds of uncertainties. A fuzzy automaton model served as a point of departure for the base line system. Various fuzzy relations are used to select and update the parameters and structures of the system.
2. Set of algorithms and new decision criteria which can be implemented easily and used for effective fault isolation. A fuzzy distance measure and a fuzzy entropy measure are used for decision making.

Accession For
NTIS
DTIC T
Unann
Justif
By
Distr
Avail
Dist
A

in the fault isolation algorithms. The results are shown to be generally more effective than existing techniques.

3. Ample illustrative examples and simulation studies are included to back up these new methods. Several examples such as low pass filter, band pass filter, and communication I/O circuits are used to illustrate the simulation studies. The results of simulation studies demonstrate the applicability of a fuzzy set technique.

FAULT ANALYSIS OF ANALOG ELECTRONIC SYSTEMS:

Algorithms based on Fuzzy Sets

Jonghee Lee

A DISSERTATION

in

SYSTEMS ENGINEERING

Presented to the Graduate Faculties of the University of Pennsylvania in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy.

1980

Supervisor of Dissertation

Graduate Group Chairperson

COPYRIGHT

Jonghee Lee

1980

ACKNOWLEDGEMENTS

I wish to express my sincere thanks to Dr. S. D. Bedrosian, my dissertation supervisor (and along the way English teacher) for his interest, encouragement and advice. I am also deeply indebted to Dr. D. W. C. Shen for his warm interest and encouragement over the whole spectrum of graduate studies at the Moore School.

Without the support of the Office of Naval Research on Contract N00014-75-C-076~~2~~98, this research would not have been done.

Mcst of all I want to thank my wife, Iljoo, for her belief, hope, and love.

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

$\mu_A(.)$: A fuzzy membership function of a fuzzy set A. 14

$\mu_A(x)$: The grade of membership of x in a fuzzy set A. 14

A, B, C,..., A_1, A_2,... : Fuzzy sets 13

$\square$: A rule of combination 16

Δ : Operator choosing the membership value close to ½. 16

∇, (ext) : Operator choosing the membership value far from ½. 16

med : Operator choosing the median value of membership values and ½. 17

α, β, γ, η : Constants between 0 and 1. 18

rf(.) : State transition performance function. 24

rg(.) : output performance function. 25

ω_i : ith fault pattern 50,52

χ : Training matrix formed by X. 54

χ^+ : Generalized inverse of χ. 54

H : Hyperplane 56

$d_k(. \, , \, .)$: kth discriminant function. 57

E_o^i : Threshold for ith measurement in reference vector. 59

Th_i : ith threshold for quantization. 67

R_i : ith fault pattern vector. 61

C_1, C_2 : Nominal values of capacitors. 63

U_1, U_2 : Operational amplifiers. 63

G_1,G_2 : Gains of operational amplifiers for U_1, U_2. 63

$Z_i(s)$: Transfer function of feed forward components. 64

Table of Contents

GLOSSARY OF TERMS

A. Definitions

Analog Circuit: Electronic subsystem, component, or printed circuit board.

Analog System: Set of analog circuits (boards) processing analog signals.

Fault: Physical defect causing a failure.

Failure: Effect of fault.

Fault Feature Extraction: Process that simplifies the fault isolation problem sufficiently to render it tracable for the fault feature selection.

Fault Feature Selection: Selection of effective fault features from a given set of feature measurements.

Fault Diagnosis: Determination of the cause of a fault (e.g., the exact value of off-tolerance parameters).

Test points: Connections or nodes of a circuit to which it is possible to connect a measuring equipment.

Test program: Definition of the test procedure.

Testability: Capability to find out if the unit under test is operational or not as well as the ability to find out which component or group of components have failed.

Stimuli: Signals applied to the system's inputs.

B. Faults

Deviation Fault: The value of a parameter deviates in a continuous manner with time or with environmental conditions up to an unacceptable value.

Catastrophic Faults: Those faults caused by a sudden and large variation of a parameter (e.g., short, open, break-down).

Single Faults: Those faults concerning only one parameter or a component at a time.

Multiple Faults: Those fault concerning simultaneously several parameters or components.

C. Types of Tests

Functional Test: Verification of the function of modules for nominal characteristics and conditions.

Parametric Test: Verification of analog characteristics within specified tolerance (voltages, currents, impedances, load conditions, etc.).

Static Test: Verification of stable states of unit under test.

Dynamic Test: Verification of dynamic characteristics of the unit under test, for normal use conditions (in particular for transient analysis).

Exhaustive Test: Verification of all modes of operation for all types of faults.

Partial Test: Verification is limited to certain characteristics or to a limited number of faults.

Off-line Test: Test of which operation of the system under test is interrupted.

On-line Test: Test of which operation of the system under test is not interrupted.

CHAPTER 1 Introduction and Summary

1.1 Motivation

The development of automatic test generation (ATG) for analog systems (AS) lags far behind that for digital systems partly because even under ideal conditions the complex interaction of the many components affects the response signals. Herein we confine our attention to analog electronic systems even though the general approach is not necessarily that limited. Due to the imprecision and indeterminacy of the complex structure of faulty networks, it is usually difficult to obtain exact solutions. It should be stressed that for fault isolation it is unnecessary to seek the exact solutions. Nevertheless, in the realistic situations component drifts and measurement noise must somehow also be taken into consideration. We can for convenience interpret such a system as a fuzzy system so that the fuzzy set concept of Zadeh becomes the basis for a fault isolation method.

It is well known that the topology of an electrical network with only a limited number of accessible terminals limits the testability of the unit under test (UUT). Testability refers to the capability to find out if the UUT is operational or not as well as the ability to find out which component or group of components have failed. To accomplish this effect, access to suitable test terminals is necessary. Berkowitz (74) introduced the concept of element value solvability as necessary conditions for solving the values of lumped network elements given limited access. Using graph theoretical aspects of this element value solvability, we can determine the sufficiency of the access terminals, or the solvability of the network.

1.2 Statement of the Problem

There are essentially three fundamental problems involved in achieving effective automatic generation of fault isolation tests for analog electronic systems : feature extraction, fault classification and diagnosis.

For practical electronic circuits having component drifts and measurement noise, how are we able to introduce fuzzy set concepts and provide methods to achieve fault classification and diagnosis?

Along with the feature extraction problem, given an electrical network of known topology, what are the conditions for testability?

1.3 Scope

For practical analog electronic circuits having component drifts and measurement noise, we adopt a fuzzy system model of faulty analog electronic circuits. Considering fuzzy system models of faulty analog circuits, the following specific objectives are sought: (1) To develop criteria for fault diagnosis and to optimize the level of diagnosability via learning algorithms in order to reduce the computational effort. (2) To develop efficient computational algorithm for assessing the power of discrimination in the test signals among fault conditions via information theoretical point of view. (3) To verify and assess the efficiency of the proposed test method by carrying out computer simulations on typical circuits. This approach lends itself to treatment of integrated circuits.

Introduction to the state of the art in network element value solvability and effort toward its solution are to be found in papers by Berkowitz (74), Bedrosian (78), Gayer (11),and Navid and Willson (14). In this dissertation we extend their work by providing a method to determine the necessary and sufficient conditions for the solvability of the network for single and two-element-kind networks. This includes an algorithm to determine the network solvability and the necessary theorems.

1.4 Summary

In Chapter 2 the state of the art is reviewed. A summary of Zadeh's fuzzy set theory is introduced. Some clarification is given of its suitability and relationship to faulty analog circuits.

In Chapter 3 a fuzzy automaton model (FAM) is formulated as a basis for a fault isolation method. Various properties of three fuzzy relations in fuzzy automata are examined. Especially learning properties of fuzzy automata are studied and applied to analog fault isolation problem. Section 3.5 deals with the problem of selecting the best set of parameters for fault tests using the fuzzy automaton model. An example of this learning technique is applied to simulated faults on a simple active circuit. Section 3.6 deals with the application of fuzzy relations to the highly overlapped fault patterns. Fault pattern classes are first separated into non-fuzzy and fuzzy parts corresponding to non-overlapping and overlapping regions obtained by sensitivity analysis. The grade of membership of the fuzzy parts are then modified according to simulation results and the decision based on fuzzy relations.

In Chapter 4 two classes of information measures are defined as measures of information content in the fuzzy system, namely fuzzy distance measure (FDM) and fuzzy entropy measure (FEM). Their properties are discussed and their applications to fault diagnosis with FAM have been made. Section 4.4 utilizes a special form of Tellegen's theorem to get the necessary values of port currents and voltages for diagnostic purposes. A fault isolation algorithm using fuzzy distance measure is developed. A simulation of part of a communication I/O circuit is used as an example. Section 4.5 discusses a fuzzy measure to facilitate analog fault diagnosis having nonlinearity, component drift and noise. The algorithm presented herein makes use of the available measured data on port responses to isolate the faulty component based on fuzzy set concepts.

Chapter 5 includes a summary and conclusions for the present study followed by some suggestions for futher work.

In Appendices necessary and sufficient conditions for solvability of single and two element kind network are given. An algorithm to determine the network solvability is developed. Detailed examples are included. Summary of NAP2 Nonlinear Analysis Program is introduced.

CHAPTER 2 Analog System Failure in the Context of Fuzzy Sets

2.1 Introduction

In view of the declining availability of skilled manpower, it becomes important to develop and adopt systematic means for analyzing faulty analog electronic systems. The development of fault isolation techniques for analog systems lags behind that for digital systems partly because the complex interaction of many components affects the response signals. Practical analog systems are exposed to noisy environments. Under fault conditions such systems in general become nonlinear. The behavior of faulty systems can conveniently be considered in the context of fuzzy systems (12,35,37).

Fuzzy system denotes a system with vague inputs, vague states, and vague outputs for a given system structure interacting with the fuzzy environment. We call those variables such as vague inputs, vague outputs, and vague states as "informal" variables (43).

The rationale for the development of fuzzy system can be described as follows. Informal variables suffer from vagueness or indeterminacy, so that a deterministic system is hopelessly inadequate to represent them. The traditional response can be interpreted to mean that the informal variables must be constrained , so that the deterministic system will apply. The fuzzy system approach proposes, instead, to loosen up, or "fuzzify" the deterministic system to obtain a new system which is directly applicable to constrained informal variables.

Fuzzy set theory itself has been developed since 1965 (35). The theory has been applied to the various fields such as pattern recognition, formal languages, medical diagnosis, automata theory, and so on. The primary part of this study focuses on formulation and solution of some of the specific real problems in the area of analog fault analysis by applying fuzzy set theory in conjunction with other related theories. For convenience the subjects of the study are grouped into three parts: 1) Analog system failure in the context of fuzzy set theory, 2) A fuzzy automaton model and its application to fault analysis, 3) Fuzzy measures and their applications to fault analysis.

Since some of the pattern recognition aspects play major roles in the area of fault analysis, fuzzy measures, such as fuzzy entropy and fuzzy distance, are defined. Above measures are used as effective measures of fuzziness. And based on the measure, we develop the criteria for fault feature selection as well as fault feature classification and diagnosis. Fuzziness is a type of imprecision which stems from a grouping of objects into classes which do not lend themselves to sharply defined boundaries. A basic difference between a fuzzy algorithm and a heuristic program is that the instructions in a fuzzy algorithm are themselves fuzzy whereas in a heuristic program they are not.

The secondary part of this study focuses on a review and extension of some of the graph theoretical aspects, especially initiated by Berkowitz's element value solvability.

2.2 Literature Survey and Theoretical Background

Until now, efforts at producing effective algorithms for automatically generating test programs have been confined mainly to digital circuits for which more or less satisfactory solutions have been reached. Today, industry uses computer programs for developing and analyzing test sequences for printed circuit boards built with MSI and LSI circuits. Analog circuits, on the other hand, have received much less attention and effort.

The main difference of development between digital and analog systems might be due to several of the following reasons.

1) Fault categories as well as their statistical distributions and correlations are not known with precision.

2) Even though theorectically measurable, conventional automatic test equipment has its own limitation of measurement range.

3) Relations among input and output signals in analog circuits are heavily depending on the network structure.

4) Particularly under fault conditions, analog systems are

frequently nonlinear, and involves measurement noise and component drifts.

Therefore the rest of this section is devoted to a review of existing methods with answers to the following questions. Characterization of method. What kind of theoretical background does it have? To which analog system can it be applied? The following classification methods are from Duhamel and Rault (15). For convenience we consider only three categories.

Estimation Methods: Two general classes of methods belong to this category; deterministic methods and probabilistic methods. The first ones consist in determining, from measurements, the actual values of the parameters of the UUT; determination may be purely analytic (based on analytic relations between input stimuli and responses) or based on estimation criteria (here both physical and mathematical conditions are taken into account). The following detailed methods belong to this first class; the least squares criterion method (16,17), the minimal deviation vector method (18), the quadratic programming method (19), the voting method (3,5), and the minimal distribution functions method (20). The second ones are probabilistic methods. In probabilistic methods, the distribution laws of measured responses are determined from the tolerances on the parameter values and their associated statistical distributions. The inverse probability method is one of the typical probabilistic methods. Theoretically, estimation methods have as main advantages the fact that test-points are the input and output connections of the UUT, fewer measurements are needed than parameters, no need for omission of particular faults (unlike taxonomical methods); furthermore, they allow savings in computer memory due to the use of analytic relations. Nevertheless, these advantages are obtained at the expense of the computations to be done at the moment of actual test.

Thus introducing the fuzzy set concept in this estimation method and application to fault isolation is essential to the satisfactory integration of various fault isolation methods.

Topological Methods: The basic data to be handled are the system's structure and, possibly, analytic relations between input variables and measured responses. The information path analysis method (21,22), the maximal current method (23), the inverse simulation method (24), and the graph analysis method (25) are in this category. The main advantage of these approaches is allowing to test a single system or large portions of a single system in a few steps. Moreover, they may be used as a preanalysis to probe technique. In our study, certain graph theoretical aspects are intensively examined. It could be used to enhance preanalysis to applications for fuzzy system.

Taxonomical Methods: They are based on a fault dictionary in which are stored the system's reference responses corresponding to each potential fault condition. During actual testing, measurement results are compared to the responses recorded in the dictionary; the detected fault is the one for which the set of measurements differs the least, according to a predetermined criterion, from its corresponding response vector in the fault dictionary. Obviously, the accuracy of such methods is directly dependent on how comprehensive one is able to make the fault dictionary. Main advantages of taxonomical methods are several levels of description (components, functions, boards) and diagnosis capability, independence with respect to technology (several types of measurements), test signals correspond to normal operation, capability for trend analysis, off-line or on-line testing, no assumption on the type of systems (linear or nonlinear). Their main drawback lies in the large volume of data to be processed, fuzziness in the definition of fault signatures, and the risk of overlooking faults not included in the fault dictionary.

2.3 Fuzziness in Analog Electronic Networks

Recall that the main problem is isolating fault pattern classes whose analog functions are degraded by the faulty components under additional constraints. In particular the main condition is that all the nonfaulty components are subject to drift within prescribed tolerances and the measurements are assumed to be corrupted by noise. Our main thesis is that such a system can be modelled by adopting the fuzzy set theory; in particular the fuzzy membership function.

On examining this concept, some researchers simply assumed that, since the membership function takes values in the closed interval (0,1), the theory of fuzzy subsets is a variant of probability theory. In fact, this is not the case. Probability theory is often viewed as a part of a general theory of measure. By contrast, fuzzy set theory falls within the theory of "valuation" (in this sense, "fuzzy measure" should be distingushed from usual use of the term) (49). A basic property of measure is its additivity. A valuation, on the other hand, exhibits a weaker property of monotonicity with respect to inclusion and thus is a more general notion than that of measure (51).

Many of the electronic and feedback control systems of interest are designed to perform certain specific analog functions. These are continuous functions and the deviations of a system function from its nominal value may range between an upper and lower bound established by the physical nature of the system. For such systems a slight out of tolerance condition yields a partial failure whereas large deviations usually result in a complete failure. The difficult problem is to diagnose the fault when a component exceeds the nominal value to an extent that the performance of the system is just outside the specification while all nonfaulty components are subject to drift within tolerances. The difficulties are compounded by measurements corrupted by noise. Since this case represents the "real world" situation, it is recognized as the diagnostic test case. Two classes of well known analog fault diagnostic methods, namely, the parameter estimation methods (70, 71) and the bilinear transformation methods (1, 2) are very sensitive to the noise and the nonlinearity of the components.

Thus we can identify the problem of fault analysis as involving two highly interactive stages; one is establishing the set of test measurements to characterize a fault pattern and the other is the construction of an optimum design procedure to classify a fault pattern based on these measurements.

2.4 Fuzzy Set Theory

The fuzzy set concept was originated by Zadeh (35). Instead of taking on only two values 0 or 1 depending on "included in" or "not included in" the set, the basic idea involves taking on values in the range (0,1) depending on the degree of belonging to the set. For clarity, some of the definitions are repeated in the following discussions (37, 39).

Definition 2.1: Let E be a set, denumerable or not, and let x be an element of E. Then a fuzzy subset A of E is a set of ordered pairs

$\{x, \mu_A(x)\}$, $x \varepsilon E$

where $\mu_A(x)$ is the grade of membership of x in A. Thus, if $\mu_A(x)$ takes its values in a set M, called the membership set, one may say that x takes its values in M through the function $\mu_A(x)$. Let us write

$$x \xrightarrow[\mu_A]{} M . \tag{2.4.1}$$

This function will likewise be called the membership function. Three major operations are defined as follows.

Definition 2.2: Let E be a set and M=(0,1) its associated membership set and let A and B two fuzzy subsets of E;

$\forall x \varepsilon E : \mu_B(x) = 1 - \mu_A(x)$: A and B are complementary denoted by $B=\bar{A}$, (2.4.2)

$\forall x \varepsilon E : \mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$: The intersection of A and B denoted by $A \cap B$, (2.4.3)

$\forall x \varepsilon E : \mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$: The union of A and B denoted by $A \cup B$. (2.4.4)

Since (0,1) is a complete lattice (39), we can define in M(x), unions and intersections of arbitrary families:

$$\mu_{\underset{i \varepsilon I}{\wedge} Ai}(x) = \inf_{i \varepsilon I} \mu_{Ai}(x) \tag{2.4.5}$$

$$\mu_{\underset{i \varepsilon I}{\vee} Ai}(x) = \sup_{i \varepsilon I} \mu_{Ai}(x) \tag{2.4.6}$$

However the lattice M(x) is not a Boolean algebra because

$A \cap A \neq \phi$, $A \cup A \neq E$

ϕ stanas for empty subset of E.

Definition 2.3: Fuzzy implication (Fuzzy conditional statement) is defined as follows.

If A then B,

where $A \subset X$, and $B \subset Y$, which has a membership function defined by

$$\mu_S(y, x) = \min(\mu_A(x), \mu_B(y)). \tag{2.4.7}$$

We are mapping an input A to an output in a fuzzy way. That is, input is big then output is medium, given $A \subset X$, $B \subset Y$.

Let's procede two description of a system in fuzzy implication. If input is big then output is medium, or if input is medium then output is small. i.e.,

If A1 then B1, or if A2 then B2, which has a membership function defined by

$$\mu_S(y, x) = \max(\min(\mu_{A1}(x), \mu_{B1}(y)), \min(\mu_{A2}(x), \mu_{B2}(y))). \tag{2.4.8}$$

These of course can be extended to more than two fuzzy implications. An example of an implication extension is as follows.

If A then (if B then C), whose membership is defined by

$$\begin{aligned} \mu_S(x, y, z) &= \min(\mu_A(x) ; \min(\mu_B(y) ; \mu_C(z))) \\ &= \min(\mu_A(x) ; \mu_B(y) ; \mu_C(z)). \end{aligned} \tag{2.4.9}$$

<u>Definition 2.4</u>: Fuzzy Inference; To calculate the inferred fuzzy subset, given a certain implicand fuzzy subset.

We know the rule : If the input is big, then output is medium. The question is : If the input is very big, what will be the output knowing that the preceding statement. The compositional rule of inference is as follows.

Given a fuzzy implication S : If A then B, the fuzzy subset B', inferred from a given fuzzy input set A' ($A, A' \subset X$ and $B, B' \subset Y$), has a membership function defined by

$$\mu_{B'}(y) = \max \min (\mu_{A'}(x), \mu_S(y, x)). \qquad (2.4.10)$$

Silvert developed some more fuzzy operators based on symmetry under complementation(84). Let $A_1 \circ A_2$ represent the combination of two fuzzy sets A_1 and A_2 under some rule, and let $\bar{A}$ be the complement of A. If μ_A is the membership function for A, then the membership function for $\bar{A}$ is $\mu_{\bar{A}} = 1-\mu_A$. The requirement that the rule of combination be independent of whether we deal with a set or its complement is equivalent to the condition

$$\overline{(A_1 \circ A_2)} = \bar{A}_1 \circ \bar{A}_2 \ . \qquad (2.4.11)$$

If the membership function μ_{12} for the set $A_1 \circ A_2$ is given by an equation of the form $\mu_{12} = C(\mu_1, \mu_2)$, then eq. (2.4.11) is equivalent to

$$1 \qquad {}_1,\ \mu_2) = C(1-\mu_1, 1-\mu_2) \ . \qquad (2.4.12)$$

A symmetric sum will be considered stable if and only if

$$\mu_1 \wedge \mu_2 \le C(\mu_1 \ \mu_2) \le \mu_1 \vee \mu_2 \ . \qquad (2.4.13)$$

Now we define two operators Λ and ∇. Operator Λ is defined as choosing the membership value close to 1/2, operator ∇ is defined as choosing the membership value far from 1/2. If we confine ourselves to add the associativity and stability of the symmetric sum, above operators satisfies the symmetry under complementation.

Theorem 2.1: $\wedge$ and $\vee$ are operators satisfying the symmetry under complementation with the associativity and stability.

We call the $\vee$ operator as the extremum (ext) operator. We can also denote the median of two fuzzy membership values and 1/2 by med (μ_1, μ_2, 1/2). Instead of using $\vee\wedge$ relations we can also use ext-med relations.

2.5 Classes of Failures in Analog Electronic Systems

We can categorize the potential failures according to the degree of system failure into "soft" failures (deviation failures) and "hard" failures (catastrophic failures). A "soft" failure indicates that some components exceed their nominal values to an extent that the performance of the system falls outside the specification. A "hard" failure on the other hand indicates a large deviation in the system performance due to the catastrophic change of components such as open or short.

There is a finite set of nominal measurements $X = (x_1, x_2, \ldots, x_i, \ldots, x_n)$ in the system with a tolerance Δx_i for each x_i, where x_i stands for the ith port measurement. A_i is the fuzzy set of all possible measurements at the ith port with fuzzy membership values. The membership value for a specific measurement means the degree of fault assigned to that specific port measurement. A fuzzy membership value of "1" indicates a definite hard fault while a fuzzy membership value of "0" indicates no fault. Hence an intermediate fuzzy membership value represents the degree of soft fault indicated by the measurement. We associate fuzzy membership values corresponding to the test measurements $X^o = (x_1^o, x_2^o, \ldots, x_i^o, \ldots, x_n^o)$ by

$$\mu_{X^o} = (\mu_{x_1^o}, \mu_{x_2^o}, \ldots, \mu_{x_i^o} \ldots \mu_{x_n^o}).$$

If the maximum fuzzy membership value of the test measurements exceeds α, then we say the system under test is at least in α-failure.

Another important distinction is between the single component failure case and the multiple component failure case. If only one component in any circuit is outside the tolerance limit to such an extent that the response measurements are out of specification, then we denote the faulty circuit as a single fault case. Similarly if more than two components in any circuit are sufficiently outside their tolerance limits so that the response measurements are out of specification, then we denote it as a multiple fault case. Throughout this dissertation, the single fault case will be the focus of study. Until now the multiple fault case has not been studied extensively, mainly because of its additional complexity. Whenever multiple fault cases are known a priori, we can extend our methods to isolate multiple fault cases.

CHAPTER 3 Fuzzy Automaton Model (FAM) as a Basis for Fault Isolation

3.1 Introduction

The concept of fuzzy automaton has been introduced by Wee and Fu (61) based on Zadeh's composition of binary fuzzy relations. Their main interest was the application to pattern classification as a model of learning systems in connection with the nonsupervised learning problems in automatic control and pattern recognition systems. Until now fuzzy automata are largely used as models of learning control systems (61), linguistics (45,46) and medical diagnosis (50).

The fuzzy automaton developed in (61) is basically an algebraic system, that merely replaces a deterministic input-output relation or a transition of states by a fuzzy relation. To impact on actual operational systems, we propose a baseline system whose parameters and structures may be updated by learning models. Specifically inputs, states, and outputs are first subjectively assigned using fuzzy membership functions and then the automaton model is updated through the use of operators on the fuzzy sets.

Fault isolation techniques for analog electronic systems are by nature imprecise. Hence it is quite natural to consider a fuzzy automaton as a model of fault isolation or automatic fault testing in analog networks. In this way, methods of handling fault isolation based on the fuzzy automaton become much broader and more general than by use of the conventional method reviewed in chapter 2. We frequently encounter situations in fault isolation in which the procedure is not precisely specified. Therefore, it is of interest to investigate algorithms that show how to achieve from imprecise procedures for reasonable fault diagnosis results under the framework of fuzzy automata.

Our detailed study includes the following topics :

1. Development of simple learning methods using various fuzzy relations based on the fuzzy automaton model.

2. The conditions for convergence, monotonicity in the above approach.

3. The learning behavior of the algorithms using fuzzy distance measure and fuzzy entropy measure assuming little statistical information is available.

4. Applicability of the automatic fault testing and fault isolation methods.

3.2 A Formulation of Fuzzy Automata

When a fuzzy automaton is used as a model of a learning system, the elements of a fuzzy state transition matrix and a fuzzy output matrix are varied via a linear reinforcement scheme or fuzzy relations. In this manner, the fuzzy automaton exhibits a variable structure. Wong and Shen (62) have modified the membership functions of the state directly to learn the parameter values that maximize the expected value of a noisy multi-model response function. The advantages are computational economy and analytical convenience.

When we consider the system whose structure and states are imprecise, a finite fuzzy automaton can be described by a sextuple (X,S,Y, h,f,g) where

X : a set of inputs $(x_1, x_2, \ldots, x_e, \ldots, x_q)$

S : a set of states $(s_1, s_2, \ldots, s_i, \ldots, s_m)$

Y : a set of outputs $(y_1, y_2, \ldots, y_j, \ldots, y_r)$

h : membership function that maps (X,S) into closed interval (0,1) (initial fuzzy state membership)

f : membership function that maps (X, S, S) into closed interval (0,1) (state transition membership)

g : membership function that maps (S,Y) into closed interval (0,1) (fuzzy output membership).

The above is a formulation close to the Moore type fuzzy automaton. The fuzzy membership function h is the state membership function. h assigns to each pair $(x_e, s_i) \in X \times S$ a certain fuzzy membership value. It is a fuzzy mapping from X x S into (0, 1) such that for $(x_e, s_i) \in X \times S$, e= 1,2,...,q, i= 1,2,...,n, and is abreviated as $h(x_e, s_i) = h_{ei}$, $0 \leq h_{ei} \leq 1$. The fuzzy membership function g is the output membership function. The output may be the decision to classify an object as belonging to the jth fault class in a fault classification problem. It should be noted that g is a fuzzy mapping from the state s_i to y_j represented by $g(s_i, y_j)$. The fuzzy membership function f is the state transition membership function. f defines the transition operator characterizing a learning automaton. The assumption here is as follows. 1) The degree of fuzziness in the input affects the change in the degree of fuzziness in the state through the state transition function by fuzzy relations such as max-min, max-product, linear-product, and extremum-median (ext-med) fuzzy relations. 2) The degree of fuzziness in the output is affected by the change of the degree of fuzziness in the state through the output fuzzy function by fuzzy relations. 3) The state choosing scheme is refined by the reinforcement scheme using the penalty and reward information.

Usually the membership functions f and g are represented by transition matrices Qn (x_e) and Gn respectively. When the input at the time instant n is x , the elements of fuzzy state transition matrix Qn (x_e) is given by

$f(n\ ;\ x_e,\ s_i,\ s_k) = f(\ x(n) = x_e,\ s(n) = s_i,\ s(n+1) = s_k\ ).$

where $i,k = 1,2,\ldots,m$.

Similarly, the element of the output fuzzy matrix Gn at time t is given by

$g(n\ ;\ s_k,\ y_j) = g(s(n) = s_k,\ y(n) = y_j),$

where $j = 1,2,\ldots,r$.

To be strictly correct, variables denoting the particular fuzzy sets should be attached to f and g explicitly but they may be omitted if they are self-evident. The main implication of the fuzzy state membership is as an index of vagueness in the state at a given input. The main implication of the fuzzy transition membership and the fuzzy output membership is as an index of vagueness in the procedure due to the degree of interaction between the input and the state, and between the state and output respectively constrained by the fuzzy operators. Fuzzy membership close to one indicates the certainty of the element being included in the class while an element value close to zero indicates the certainty of the element being excluded from the class. At the time instant n, suppose that the fuzzy automaton is in a state s_i with the grade membership $h(n\ ;\ x_e,\ s_i) = h(\ x(n) = x_e,\ s(n) = s_i\ )$ and that the input to the automaton is x_e. Then, the choice of the next state given that the previous state is represented as a state transition performance function rf, is given by

$$rf(n\ ;\ x_e,\ s_k) = \bigvee_i \wedge (h(n\ ;\ x_e,\ s_i),\ f(n-1\ ;\ x_e,\ s_i,\ s_k)) \tag{3.2.1}$$

where $\wedge$ is an inter operator and V is an intra operator. Inter operator $\wedge$ is defined as an operator between the corresponding fuzzy set elements, while intra operator V is defined as an operator among the chosen set of elements using inter operator. We often select the state s_k which satisfies

$$rf(n;\ x_e, s_{k_{max}}) \geq rf(n;\ x_e, s_k) \tag{3.2.2}$$

where $k_{max} \neq k$. The state $s_{k_{max}}$ indicates the most possible choice.

The next state transition membership function can be updated using a linear reinforcement scheme.

$$f(n+1;\ x_e,\ s_i,\ s_k) = \alpha f(n;\ x_e, s_i, s_k) + (1-\alpha)\phi(s_k) \qquad (3.2.3)$$

$$\begin{cases} \text{if correctly classified } \phi(s_k) = 1 \\ \text{otherwise } \phi(s_k) \neq 0, \end{cases}$$

where $0 < \alpha < 1$.

Simlarly, the choice of the output reoresented as a output performance function rg

$$rg(n\ ;\ x_e,\ y_j) = \bigvee_i \wedge(\ g(n\ ;\ s_i, y_j),\ h(n\ ;\ x_e,\ s_i)). \qquad (3.2.4)$$

The next output membership function is updated using linear reinforcement scheme,

$$g(n+1;\ s_k, y_j) = \alpha g(n;\ s_k,\ y_j) + (1-\alpha)\psi(y_j) \qquad (3.2.5)$$

$$\begin{cases} \text{if correctly classified } \psi(y_j) = 1 \\ \text{otherwise } \psi(y_j) \neq 0, \end{cases}$$

where $0 < \alpha < 1$.

In order to simplify the problem, when the set of states directly exhibits the set of outputs, we can choose

$$g(n;\ s\ ,y\ )=\begin{cases}1 & k=j\\ 0 & k\neq j.\end{cases}$$

Therefore the modification of h or f or g gives the fuzzy automaton a variable structure. This feature of variable structure results in the learning behavior of the automaton.

Depending on the choice of intra-inter operators, namely fuzzy relations, a fuzzy automaton exhibits a different variable structure. The method of direct modification of the state membership functions proposed by Wong and Shen (62) can be included as a special case of the modification of the state transition membership function when the fuzzy transition membership is an identity matrix. It is interesting to note that according to the definition of membership function, "h" does not reveal too much about the nature of the function. Thus h is a "grade of membership" function keeping the order of state, called the ordinal information that is defined for each $x \epsilon X$ and $h \epsilon (0,1)$. A special class of fuzzy automata would have the row sum of all transition matrices equal to unity. This type of automata can be called the normalized fuzzy automata which may retain the cardinal information during transformation and have the same structure as the stochastic automata.

The learning model (12) applied to fault classification is formulated as follows: Let the fault classifier consist of several sets of preselected discriminant functions. These are characterized by sets of parameters, for instance, values of the threshold E_0's and values of the tolerance on performance used to detect a fault mode. Depending upon whether or not external supervision (a teacher) is required, the process of learning is classified as being off-line or on-line learning respectively. Initial assignments of fuzzy membership values are "subjective" and "local" (43). By calling the values subjective, it simply means assigning arbitrarily which values of the "degree of fault" (base logic) belong to "degree of truth values to the above statements" (which linguistic truth-values to what degree). By local values we mean that the assignments to the primary term are defined only for a specified set of propositions. In the learning process with external supervision, the correct fault condition corresponding to a measurement is usually considered to be known exactly. Then the teacher directly varies the fuzzy automaton structure such that the decision is based on the maximum membership grade i.e.

$$\text{decide jth class if } g(n\ ;\ s,\ y_j) = \max_i g(n\ ;\ s_i,\ y_k)$$

Either with supervision or with a proper specification of the performance evaluation, the model adapts itself to the best solution. Here the best solution means the set of the discriminant functions that gives the minimum number of misrecognized faults among the given sets of discriminant functions within the set of learning samples. On every arrival of input x a transition may be executed from a state s_i to another state s_k or the same state s_i via the state transition performance function, and then an output may be sent out according to the branch in which the transition has been executed.

In this section, the formulation of a fuzzy automaton is described and basic learning diagnostic scheme is presented. In the subsequent section it is shown how fuzzy relations and linear reinforcement schemes contribute to the learning of the best solution.

3.3 Various Fuzzy Relations in Fuzzy Automata

To model a fuzzy system, we can choose the fuzzy automaton as a baseline model. We consider inputs, states, and outputs as fuzzy variables, and updating the structure by using fuzzy relations and a linear reinforcement scheme.

There are many ways in which one could modify a given concept, including the concept of automata to make it fuzzy. One of the most popular way is to use the maximum and minimum operators as a fuzzy relation. The model of fuzzy automata obtained in this manner often turns out to be similar to the extensions of existing deterministic ones (63). For these reasons, we investigate various fuzzy automata depending on the corresponding fuzzy relations utilized.

The membership function for a path in which a higher order transition may be executed from a state s_i to another state s_j or the same state via serial branches is calculated by one of several fuzzy relations.

Santos (63,64) discussed classes of automata obtained from the pseudo automaton by a rule of extension and a set of constraints. In his discussion, the pseudo automaton is defined as a single length input with a state transition function. Rules of extensions such as max-min operators, max-product operators, linear product operators are used to generate an atomaton which has a multiple length input. We can make the interpretation that each new fault condition in fault isolation corresponds to a single length input. Therefore our FA model will be restricted to a set of single length inputs as possible inputs.

The problem of fault isolation using various response deviation measurements is basically a problem of vector optimization. For example

$$Q(c) = \{Q_1(c), Q_2(c), \ldots, Q_i(c), \ldots, Q_m(c)\} \rightarrow \min,$$

where $Q_i(c)$ $(i=1,2,\ldots,m)$ are the elements of vector loss function which represents the loss incurred by the decision as ith single fault, and c is unknown parameter vector.

We are interested in determining some of the possible combinations of operators as fuzzy composite relations that may yield better fault isolation for the application of fault analysis. They are max-min operators, max-product operators, linear product operators , and ext-med operators.

3.3.1 Max-min Composite Relations

Following a general formulation of FA given in section 3.2, we now show how effective max-min relations are when used to direct the learning of the fuzzy automaton. We generally denote the composition of the two fuzzy set as $A^{o}B$. Max min fuzzy relations are defined as follows.

$$\mu_{A \circ B}(x,y) = \max_{z} \min (\mu_A(x,z), \mu_B(z,y)), \qquad (3.3.1)$$

where A and B are both fuzzy sets.

When we apply the above equation in the composition of a fuzzy set $A \circ A$, we get

$$\mu_{A \circ A}(x,y) = \max_{z} \min (\mu_A(x,z), \mu_A(z,y)). \qquad (3.3.2)$$

This fuzzy relation is explained as follows (61). (The pessimistic case is being considered when the minimum function is selected between $\mu_A(x,z)$ and $\mu_A(z,y)$ and the maximal grade of this minimum is being searched through z.) It is easy to prove that the result of using max min relations is equivalent to the use of min max relations having a monotonicity property in the fuzzy set A through variable z.

Since the state transition function f, the initial state membership function h, and the output membership function g may be interpreted as the grade of membership functions of fuzzy sets, we can define the state transition performance function of the automaton as follows.

$$rf(n: x_e, s_k) = \max_i \min (h(n; x_e, s_i), f(n; x_e, s_i, s_k)) \tag{3.3.3}$$

Likewise, the output performance function of the automaton is defined as follows.

$$rg(n; x_e, y_j) = \max_i \min (h(n; x_e, s_i), g(n; s_i, y_j)). \tag{3.3.4}$$

3.3.2 Max-product Composite Relations

The idea of max-product fuzzy relation is to choose the transition path which yields the maximum of the product of the fuzzy memberships between the two fuzzy sets. Max-product fuzzy relations of the state transition performance function rf and the output performance function rg can be defined as follows.

$$rh(n; x_e, s_k) = \max_i (h(n; x_e, s_i) * f(n; x_e, s_i, s_k)) \tag{3.3.5}$$

and

$$rg(n; x_e, y_j) = \max_i (h(n; x_e, s_i) * g(n; s_i, y_j)) \tag{3.3.6}$$

<u>Theorem 3.1)</u> Max-product operator is not

log-interchangable.

Proof.

$$\log h(n+1\ ;\ x) = \log \max_i\ (h(n\ ;\ x)*f(n\ ;\ x,\ s_i,\ s_k))$$

$$= \max_i \log\ (h(n\ ;\ x)*f(n\ ;\ x,\ s_i,\ s_k))$$

$$= \max_i(\log h(n\ ;x)+\log f(n\ ;\ x,\ s_i,\ s_k))$$

$$\neq \max_i \log h(n\ ;\ x)*\log f(n\ ;\ x,\ s_i,\ s_k)$$

Using above relations we are able to choose the maximum of the two product fuzzy membership sets. The important point is that the set can be ordered under the above structure. It is easily noticed that the value of fuzzy membership for $rh(n;\ x_e,s_k)$ conveys the information of the average (central tendency) in the sense of geometry.

3.3.3 Linear Product Composite Relations

As another possible fuzzy relation we can choose the transition path which yields the average of the product of the fuzzy memberships between the two fuzzy sets. Linear product fuzzy relations of the state transition performance function rh and the output performance function rg can be defined as follows:

$$rh(n;\ x_e,s_k) = \sum_i (h(n;\ x_e,s_i) * f(n;\ x_e,s_i,s_k)) \qquad (3.3.7)$$

and

$$rg(n;\ x_e,y_j) = \sum_i (h(n;\ x_e,s_i) * g(n;\ s_i,y_j)), \qquad (3.3.8)$$

where λ is used as an average operator. This relation is very similar to that of stochastic automata.

3.3.4 Max-topology Composite Relations

One other interesting class of fuzzy composite relations is defined as max-topology composite relations, when we have restrictions between the two fuzzy set given by some topological relations. We can choose the transition path which yields the maximum while satisfying the topological restrictions. Max-topology fuzzy relations for rh and rg are defined as follows.

$$rh(n;\ x_e, s_k) = F(h(n;\ x_e, s_i),\ f(n;\ x_e, s_i, s_k)) \qquad (3.3.9)$$

and

$$rg(n;\ x_e, y_j) = G(h(n;\ x_e, s_i),\ g(n;\ s_i, y_j) \qquad (3.3.10)$$

where fuzzy function F and G are induced by the restrictions of the topology of the two fuzzy sets.

3.4 Properties of Fuzzy Relations in Fuzzy Automata

3.4.1 Simplicity

Applications of fuzzy relations can be evaluated by the required time or space to solve a given problem. We would like to formulate the problem as seeking the lower bound of complexity given by the fuzzy relation

$$f_{A \circ B}(x,y) = \underset{z}{\vee\wedge} \left(f_A(x,z),\ f_B(z,y) \right) \qquad (3.4.1)$$

where the number of elements of the set A is mxn, and while that of set B is $n \times \ell$. The criterion used is the time needed by an algorithm expressed as a function of the size of the problem. We assume that operands are real numbers and the basic operations are +, x, max, min, Σ , $\wedge$, ∇ , where $\wedge$, and ∇ are defined in the previous chapter. For the brevity of comparisons we further assume that $m = \ell = 1$.

(i) max min fuzzy relations

$$f_{A \circ B}(x,y) = \max_{z} \min \left(f_A(x,z),\ f_B(z,y) \right) \qquad (3.4.2)$$

The above fuzzy relational equation has n comparisons between the two values to get the minimum values and one comparison among the n values to get the maximum value. In the worst case, one comparison among the n values is equivalent to (n-1) comparisons between the two values. In sum, max-min fuzzy relational equation has less than or equal to (2n-1) comparisons.

(ii) max product fuzzy relations

$$f_{A \circ B}(x,y) = \max_{z} \left(f_A(x,z) * f_B(z,y) \right) \qquad (3.4.3)$$

We need n products and one comparison among the n values to get the maximum value. In sum max product fuzzy relational equation has less than or equal to n products and (n-1) comparisons.

(iii) linear product fuzzy relations

$$f_{A \circ B}(x,y) = \sum_{z} f_A(x,z) * f_B(z,y) \qquad (3.4.4)$$

We need n products and (n-1) additions.

(iv) $\nabla\Delta$ fuzzy relations

$$f_{A \circ B}(x,y) = \nabla_{z}\Delta \, (f_A(x,z),\ f_B(z,y)) \qquad (3.4.5)$$

Recall that ∇ operator is the operator which picks the value far from 1/2 and operator Δ is the operator which picks the value close to 1/2. To perform the Δ operation, we need one addition and two comparisons. First we add two values $f_A(x,z)$ and $f_B(z,y)$. Comparison of the two values, comparison of the sum of the two values, and one addition will perform the operator. Therefore, in total, we need 2n comparisons and n additions to perform operators. To perform the ∇ operation, we need (n-1) comparison to pick the maximum and minimum values. We need one addition and two comparisons to pick one of the above values. In total we need (n+1) comparisons and one addition to finish ∇ operations. $\nabla\Delta$ fuzzy relational equation requires (3n+1) comparisons and (n+1) additions. Notice that in the microprocessor, addition and comparison take two cycles while multiplication has to have few more cycles. We can condense the above results into the following table.

	Number of Additions	Number of Multiplications
i	2n	-
ii	n	n
iii	n	n
iv	4n	-

Roughly speaking, we can order the fuzzy relations on the basis of needed calculation time. Thus we have the above table entries ordered as follows:

(i) < (iv) < (ii), (iii).

3.4.2 Convergence in Linear Reinforcement Scheme

We are dealing with the problem of "learning" in an unknown environment, i.e., where the function to be "learned" is known only by its form over the observation space. This implies that any desired solution which needs the knowledge of the function to be approximated is reached gradually by methods relying on experimentation and observations. If on the other hand we can assume that the form of the function to be approximated is known precisely, then we can approach the problem with stochastic approximation technique. There are cases when no assumption can be made or the possible form of the function to be "learned". For the solution to be found by stochastic approximation technique, the existence of the estimated or approximated unknown quantities must be assumed.

When the unknown environment is dominated by the vagueness rather than randomness, we have to make use of observed information mainly to unveil the vagueness. In some cases, fuzzy relations with linear reinforcement scheme can be used to remove some of the vagueness in the unknown environment. Based on the fuzzy automaton described in the section 3.2 and 3.3, we can compare the convergence property of various fuzzy relations. We have defined the state transition performance function rf as the result of the set operated on the state membership functions and the state transition membership functions.

From eq. 3.2.1,

$$rf(n;x_e,s_k) = \bigvee_i \wedge (h(n;\ x_e,s_i),\ f(n-1;\ x_e,s_i,s_k))$$

where $i = 1,2,\ldots,m$

$$= \bigvee_i \ (\wedge (h(n;x_e,s_i),\ f(n-1;\ x_e,s_i,s_i)),\ (h(n;x_e,s_i),\ f(n-1;\ x_e,s_i,s_k)))$$

where $i = 1,2,\ldots,m,\ i \neq k$.

In order to have learning behavior in the fuzzy transition function f, the fuzzy transition matrix must exhibit nonstationary behavior (61). As an example, let

$f(n;\ x_e,s_i,s_k) = c(n;\ x_e,s_k)$ for all $i \neq k$

and $f(n;\ x_e,s_k,s_k) = \alpha_k f(n-1;\ x_e,s_k,s_k) + (1-\alpha_k)\lambda_k$

with $f(n;\ x_e,s_i,s_k) = c(n;\ x_e,s_k) = \begin{cases} 0 \text{ if } n \text{ is odd} \\ f(n-1;\ x_e,s_k,s_k) \text{ if } n \text{ is even,} \end{cases}$

where $0 < \alpha_k < 1$, $0 < \lambda_k < 1$, $k = 1,2,\ldots,m$.

Furthermore, let

$$h(n+1;\ x_e,s_k) = rf(n;\ x_e,s_k) \text{ if } \begin{cases} f(n;\ x_e,s_k,s_k) - rf(n;\ x_e,s_k) * bn \\ f(n;\ x_e,s_k,s_k) + an, \text{ otherwise.} \end{cases}$$

where $|an| < |bn|$, and bn is bounded sequence such that $bn \to 0$ as $n \to \infty$.

Therefore $an \to 0$ as $n \to \infty$. With this assumptions, $h(n+1;\ x_e,s_k)$ is always between 0 and 1. When we have a perfect teacher, fuzzy state membership function $h(n+1,\ x_e,s_k)$ with max-min relations, max-product relations, and linear product relations converges to λ_k, $k=1,2,\ldots,m$.

As an example for the comparison of various fuzzy relations, when

$$a_n = \frac{1}{4n}, \quad b_n = \frac{1}{2n},$$

$$h_1 = \begin{bmatrix} 0.5 \\ 0.7 \\ 0.8 \end{bmatrix}, \quad f_1 = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \quad \lambda = \begin{bmatrix} 0.9 \\ 0.8 \\ 0.6 \end{bmatrix}.$$

The learning curves for three fuzzy relations are showed in Fig.3.1. After 13 steps all the learning curves converges to λ with an error within 1%.

Above convergence property can be varied, as $h(n+1;\ x_e, s_k)$, depends on $f(n;\ x_e, s_k, s_k)$ and $c(n;\ x_e, s_k)$. The typical learning curves with an unreliable teacher are shown in Fig. 3.2, 3.3, 3.4. In this example, the asumptions made are the same as in the perfect teacher case except λ_k is estimated and updated by the success or failure of the decision.

We showed in this section that the state membership function together with various fuzzy relations will converge using the linear reinforcement scheme.

3.4.3 Monotonicity property

One of the important aspect of using fuzzy membership function is the monotonicity property. Usually the absolute values of fuzzy membership themselves have relatively little meaning, while the order of the values is rather significant. Most of the decision is based on the cummulative aspects of the ordering of the fuzzy membership values. New fuzzy set is generated by the composition of

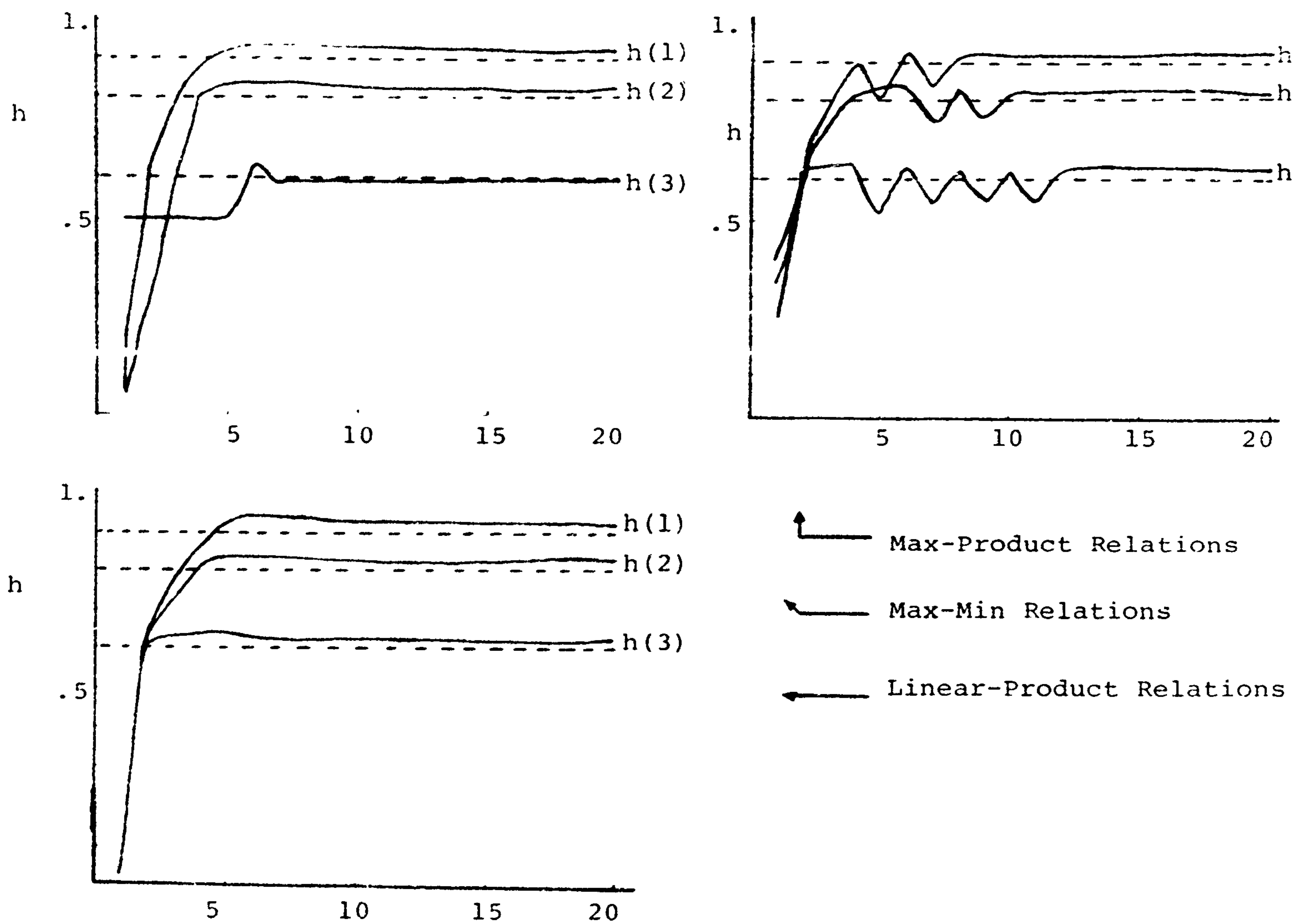


Figure 3.1 Learning Curves with Reliable Teacher

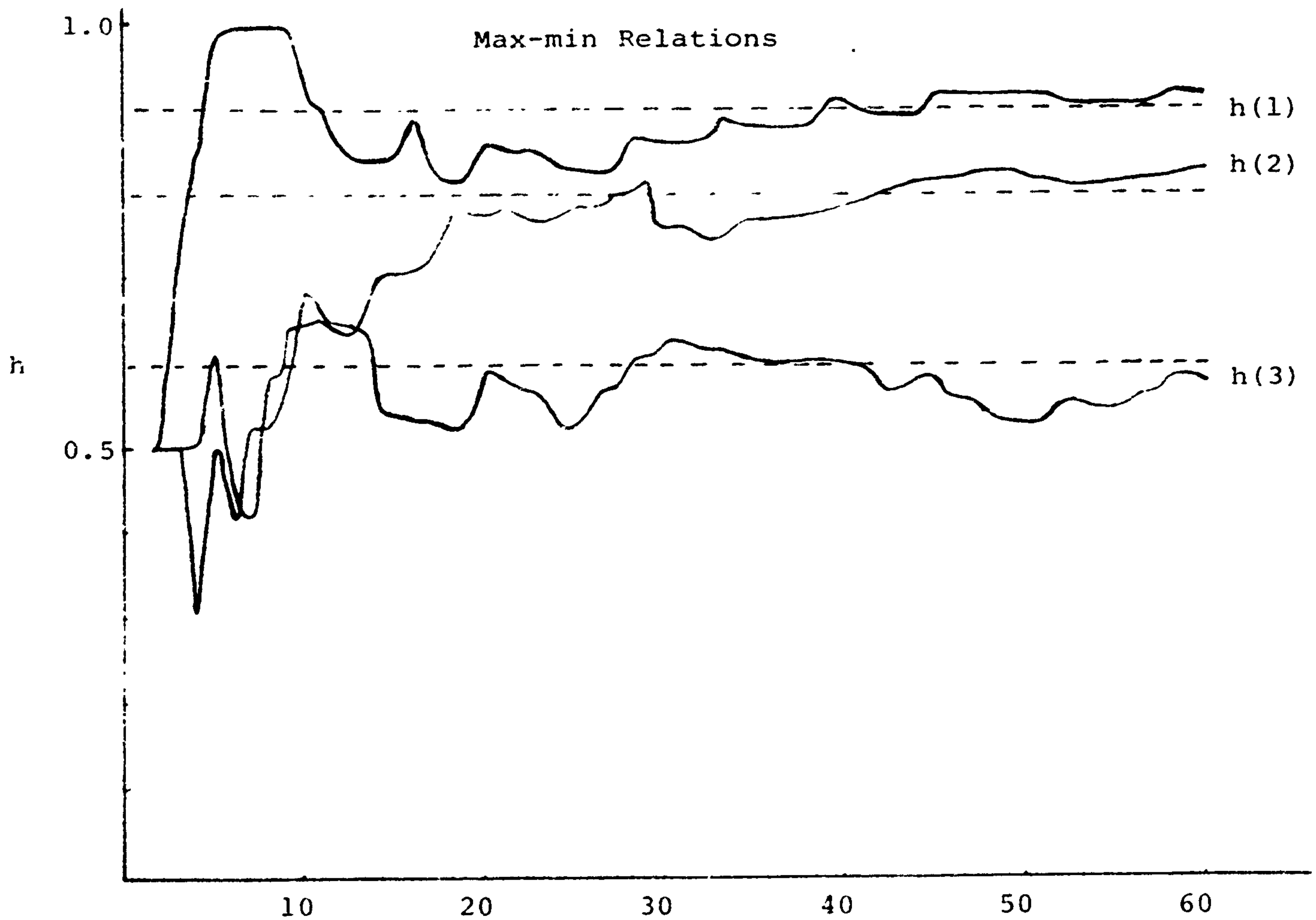


Figure 3.2 Learning Curves with Unreliable Teacher

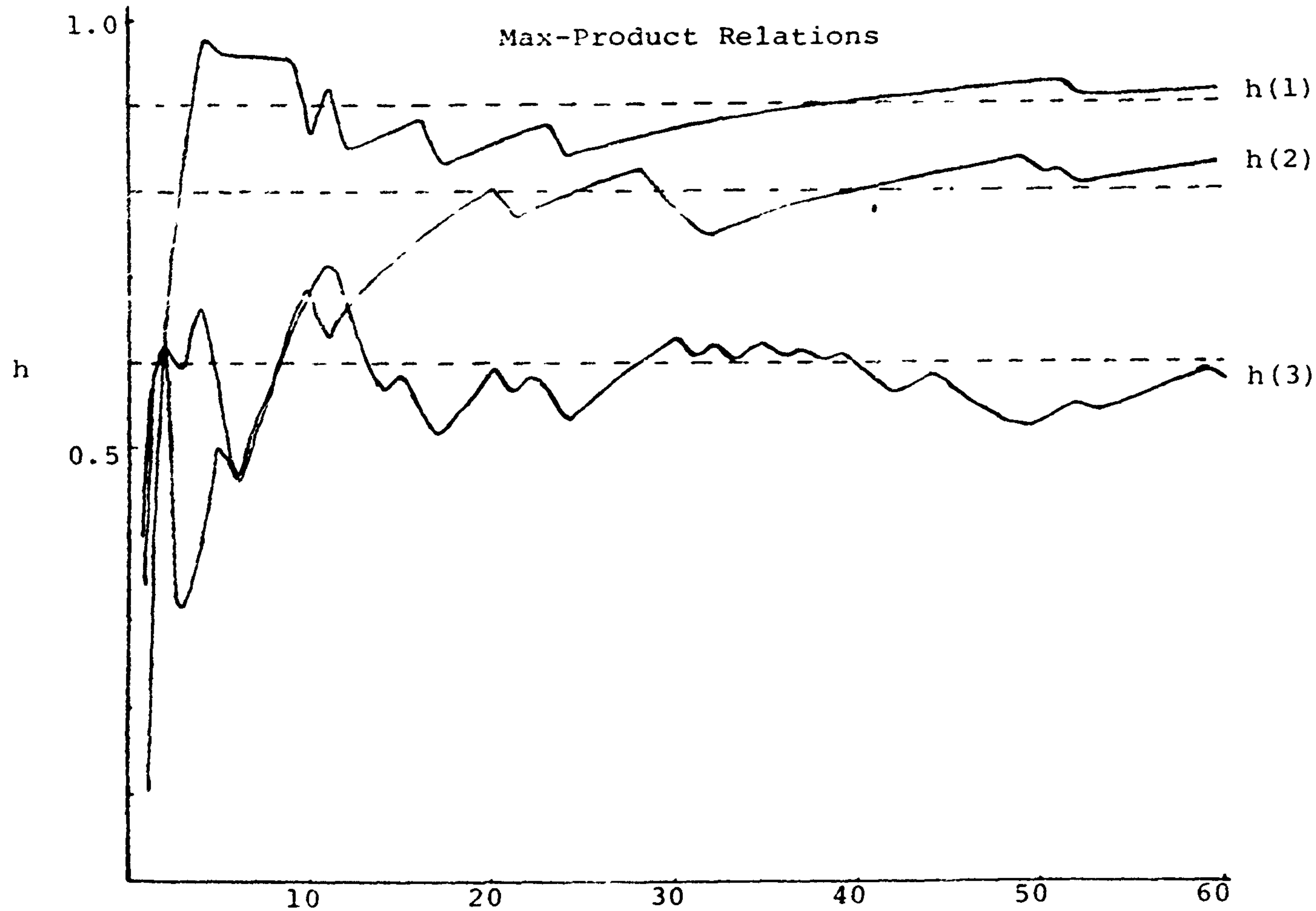


Figure 3.3 Learning Curves with Unreliable Teacher

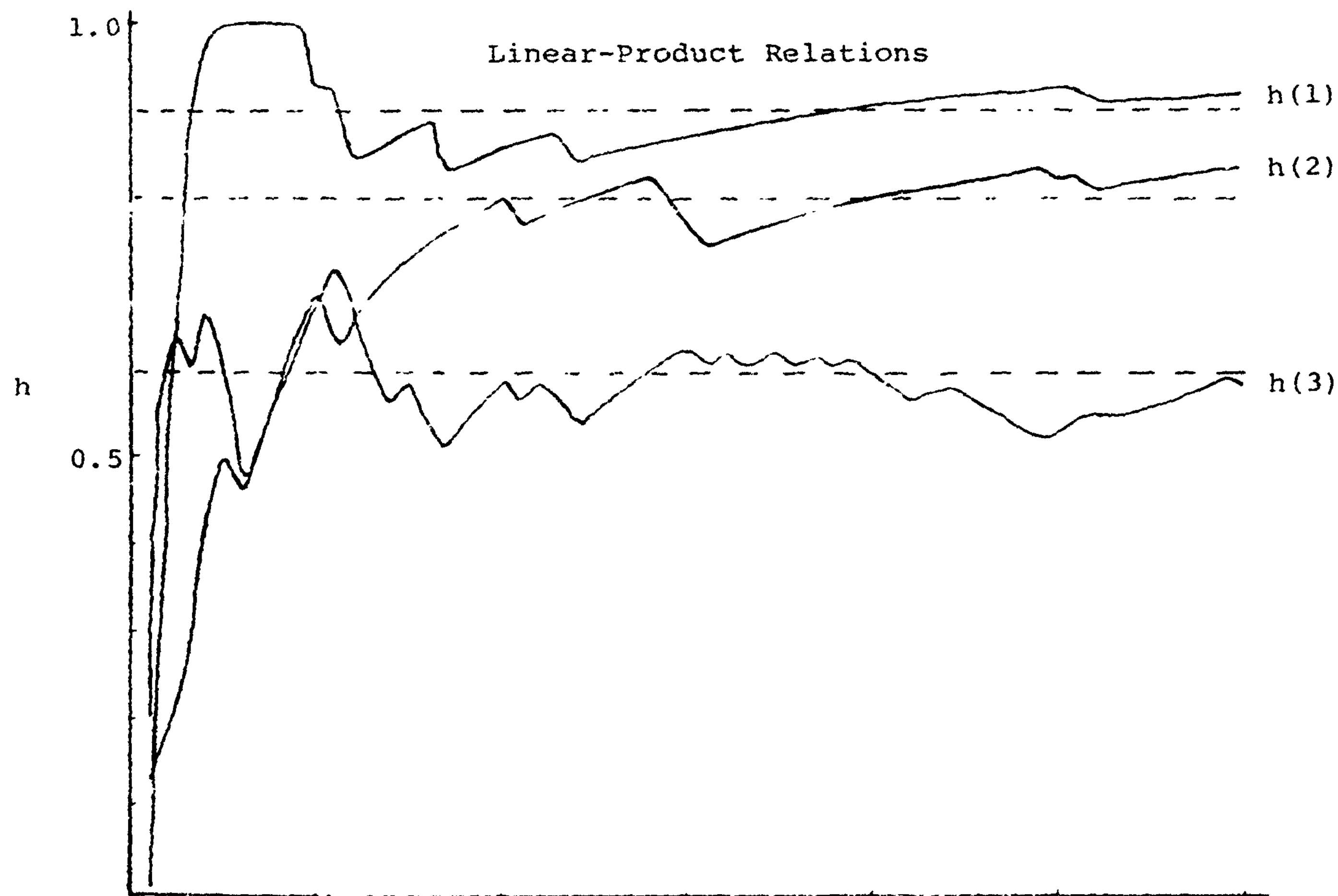


Figure 3.4 Learning Curves with Unreliable Teacher

two fuzzy sets using a fuzzy relation.

$$f: A \times B \rightarrow C$$

with $f_A \circ f_B \rightarrow f_C$,

i.e., a fuzzy relation f maps two-tuples of elements in the fuzzy set A and two-tuples of elements in the fuzzy set B into two-tuples of elements in the fuzzy set C. Several types of fuzzy relations are discussed in the earlier sections, such as max-min, max-product, and linear-product relations. We can extend these binary fuzzy relations to n-ary relations by successively applying the binary fuzzy relations. When the fuzzy membership is drastically restricted to only one and zero, this problem reduces to a certain type of the switching theory problem (87).

<u>For max-min composite relations</u>

$$f_{A \circ B}(x,y) = \max_z \min \left(f_A(x,z),\ f_B(z,y)\right),$$

$f_{A \circ B}(x,y)$ is a nondecreasing function of $f_A(x,z)$ and $f_B(z,y)$ for all f_A, $f_B \in (0,1)$.

Proof. If $f_A(x,z) \geq f_{A*}(z,y)$ then

$$f_{A \circ B}(x,y) - f_{A* \circ B}(x,y)$$

$$= \max_z \min \left(f_A(x,z),\ f_B(z,y)\right) - \max_z \min \left(f_{A*}(x,z),\ f_B(z,y)\right)$$

$$= \max_z \left(\min (f_A(x,z),\ f_B(z,y)) - \min (f_{A*}(x,z),\ f_B(z,y)) \right)$$

For all z

1) if $f_A(x,z) < f_B(z,y)$

   then $T(z) = f_A(x,z) - f_{A*}(x,z) \geq 0$,

2) if $f_A(x,z) \geq f_B(z,y)$

   then $\min (f_A(x,z),\ f_B(z,y)) = f_B(z,y)$,

   a) if $f_{A*}(x,z) \geq f_B(z,y)$,

      then $\min (f_{A*}(x,y),\ f_B(z,y)) = f_B(z,y)$

         and $T(z) = f_B(z,y) - f_B(z,y) = 0$,

   b) if $f_{A*}(x,z) < f_B(z,y)$,

      then $T(z) = f_B(z,y) - f_{A*}(x,z) > 0$.

Therefore $f_{A \circ B}(x,z) \geq f_{A* \circ B}(x,z)$.

Similarly if $f_B(z,y) \geq f_{B*}(z,y)$

then $f_{A \circ B}(z,y) \geq f_{A \circ B*}(z,y)$ qed.

<u>For max-product composite relations</u>

$f_{A \circ B}(x,y) = \max_z f_A(x,z) * f_B(z,y)$,

$f_{A \circ B}(x,y)$ is a nondecreasing function of $f_A(x,z)$ and $f_B(z,y)$ for all f_A, $f_B \in (0,1)$.

Proof If $f_A(x,z) \geq f_{A*}(x,z)$ then

$f_{A \circ B}(x,y) - f_{A* \circ B}(x,y)$

$$= \max_z f_A(x,z) * f_B(z,y) - \max_z f_{A*}(x,z) * f_B(z,y)$$

$$= \max_z ((f_A(x,z) - f_{A*}(x,z)) * f_B(z,y))$$

Since $0 \le f_B(z,y) \le 1$, for any z,

$$(f_A(x,z) - f_{A*}(x,z)) * f_B(z,y) \ge 0.$$

Therefore if $f_B(z,y) \ge f_{B*}(z,y)$ then

$$f_{AoB}(x,y) - f_{AoB*}(x,y) \ge 0. \quad \text{qed.}$$

<u>For linear product relations</u>

$$f_{AoB}(x,y) = \sum_z f_A(x,z) * f_B(z,y),$$

where $\sum_z$ is used as an averaging operator, $f_{AoB}(x,y)$ is a nondecreasing function of $f_A(x,z)$ and $f_B(z,y)$ for all f_A, $f_B \in (0.1)$.

Proof is similar to the max-product case.

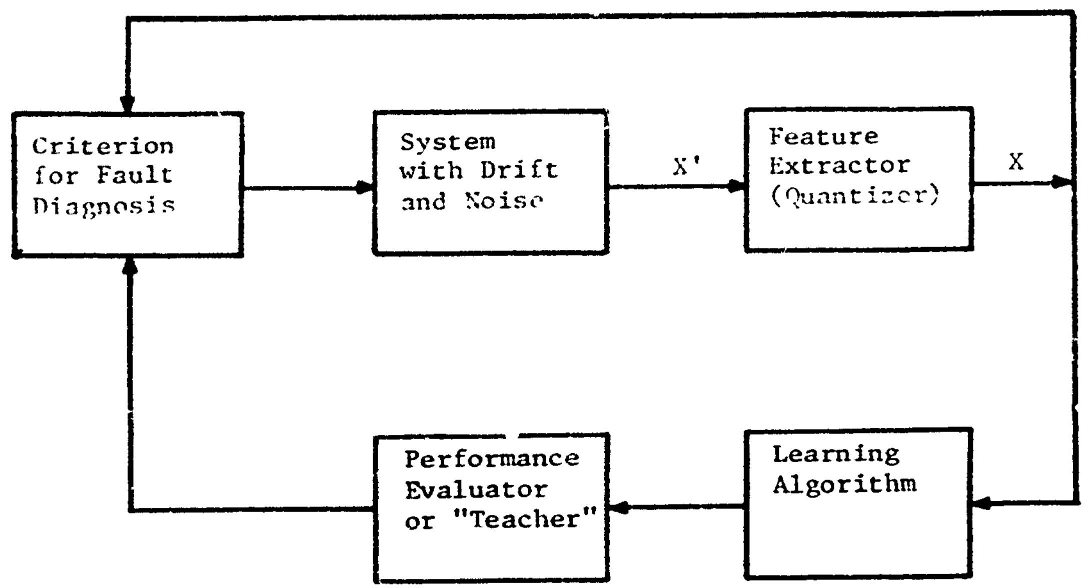


Fig. 3.5 Block Diagram of a Learning Fault Diagnostic Scheme

3.5 Learning Techniques Applied to Active Linear Networks using Frequency Domain Analysis

3.5.1 Introduction

This section addresses the problem of applying learning techniques to fault diagnosis in realistic situations where component drifts and measurement noise must also taken into consideration. In particular, the fuzzy concept of Zadeh is used as a framework for fault classification. Further, fuzzy automata are used as learning models to select the best set of parameters for fault tests, because they have the advantage of simplicity and straightforward computation. An example of this learning technique is applied to simulated faults on a simple active circuit.

There are essentially three fundamental problems involved in achieving effective automatic generation of fault isolation tests for analog systems: feature extraction, fault classification and diagnosis.

(1) The feature extraction problem.

Given a faulty pattern, say a set of input-output signals, we must be able to extract from these signals the information called signal attributes, $x_1, \ldots, x_n$, which not only properly characterize the fault, but also are amenable to automatic processing and computation. In fact, the development of an automatic test generation system is greatly influenced by the type of features or fault signature selected for use. A well known signature describing the state of the system in time domain testing is the input-output cross correlation function (3,26,27,28). Pseudo random binary sequences are shown to have approximately the required impulse autocorrelation. They have an important advantage over white noise perturbation methods (30) in that such sequences are not subject to statistical variation and can be easily realized by using feedback shift registers. In frequency domain testing, the use of gain and phase measurements at selected frequencies is widespread (5,29,30,31,32). It has been suggested that the transfer function parameters can also be used as a fault signature but it has the disadvantage of requiring complex computations to convert input-output samples to transfer function parameters (33).

(2) The fault classification problem.

A set of r test features, which characterize a number of identifiable failure modes, is chosen to form the coordinates of a feature space. Assume each faulty pattern is represented by a binary vector whose components are depending on whether each feature is present or missing in that pattern. The vertices of a set of a typical feature space are labeled and a set of hyperplanes is used to partition the feature space so that no region contains vertices corresponding two or more different fault types. Unknown fault patterns are then classified as belonging to the type corresponding to the prearranged pattern in the region in which they fall. The correlation method starts from the same presentation in that a set of typical patterns represented by binary vectors in the feature space are taken as references. The unknown patterns are then correlated with these references and classified as belonging to a particular reference pattern according to the highest degree of correlation.

(3) The diagnosis problem.

The ability of the fault classifier to determine correctly the type of new patterns of unknown classification is most appropriately stated in terms of probability of correct diagnosis. Put another way, we wish to determine $p(\omega_i|X)$ = probability that the given test data or feature X belongs to fault type ω_i. The measurements are assumed to have certain distributions $p(X|\omega_i)$, i=1,...m. Furthermore, there is a certain probability of occurrence of type ω_i patterns, $P(\omega_i)$. The key information required to make a diagnosis for a pattern is clearly contained in the function $p(\omega_i|X)$ which is determined by the application of Bayes' rule:

$$p(\omega_i|X) = \frac{P(\omega_i)\ p(X|\omega_i)}{\sum_{j=1}^{m} P(\omega_j)p(X|\omega_j)} \tag{3.5.1}$$

In the simple case of ω_1 and ω_2, the likelihood ratio test which is optimal under various assumptions on the cost of misclassification is given by

$$\begin{aligned} p(\omega_1|X) &> \eta \quad \text{classify } \omega_1 \\ p(\omega_2|X) &< \eta \quad \text{classify } \omega_2 \end{aligned} \tag{3.5.2}$$

where is some suitably chosen constant. The crucial step is the determination of $\rho(\omega_i|X)$ since it directly determines the chance and cost of future correct diagnosis. The main difficulty is that the computation and information required for equation (3.5.1) in a practical situation involving component drift and measurement noise are extremely difficult to acquire. If the probability densities are not analytically expressible, their values at each point in the feature space must be stored and tabulated. However this process requires excessive storage space.

The so called "template matching technique" for fault diagnosis has a lack of flexibility since it rarely tolerates noise and distortion due to drift of components. A fault diagnostic scheme incorporating learning will be more effective and more flexible. In this section we discuss the fuzzy set concept of Zadeh and its application to fault classification with the help of learning algorithms. Further we propose that the fuzzy automaton will serve as a convenient learning model for fault diagnosis.

3.5.2 Fuzzy Sets and Fault Classification

The essential function of a fault classifier is to recognize the membership of samples which belong to one fault type and to distinguish among them the different fault types even though the boundaries between alternatives is not sharply defined. Hence the task of classifying samples into a finite number of fault types can be conveniently established around the notion of "belonging" in the case of fuzzy sets (48). A fuzzy set (class) ω_i in the space Ω_X is represented by a characteristic function $f_{\omega_i}(X)$ which associates each point in a value in the interval [0,1], with the value of $f_{\omega_i}(X)$ at X representing the "grade membership" of X in ω_i. In order to generate a set of discriminant functions for fault types, it is convenient to introduce a single level, or two levels which lead to two-valued logic or three-valued logic. Fo simplicity, consider two fault types ω_1 and ω_2. At this point, we introduce two levels α and β ($0 < \alpha < 1$, $0 < \beta < 1$, $\alpha > \beta$). At level α, the two types may be disjoint or separable and at level β ($<\alpha$), they may be joint or not separable in the sense of ordinary sets. We then decide that (a) $X \in \omega_1$ if $f_{\omega_1}(X) \geq \alpha$ and $f_{\omega_2}(X) < \alpha$ and (b) $X \in \omega_2$ if $f_{\omega_1}(X) < \alpha$ and $f_{\omega_2}(X) \geq \alpha$.

Notice that when the level falls below β , X has an indeterminate status relative to ω_1 and ω_2 . This fuzzy intersection when ω_1 and ω_2 are not separable must be identified and the grade of membership estimated for each X in this section. This task can be accomplished by adopting learning procedures to generate the discriminant functions.

(1) Generation of discriminant function by learning.

If we use small $p(\omega_i \mid X)$ as a discriminant function and identify it with $f_{\omega_i}(X)$, the problem is one of reconstructing a function from a knowledge of its values over a collection of samples or observations. To do so, one must have a priori information about a type of functions to which $f_{\omega_i}(X)$ belongs. Then this information in combination with the learning samples would be sufficient to enable one to construct a good estimate of $f_{\omega_i}(X)$ (48). Let us assume that $f_{\omega_i}(X)$ can be represented by

$$f_{\omega_i}(x) = \sum_{j=1}^{n+1} W_{ij} X_j = X^T W_i \qquad (3.5.3)$$

where X_{n+1} is one and X_j is the j-th element of sample X. Again consider two types ω_1 and ω_2. The problem is to determine a solution with vector W such that the cross product of vector X and W, $X^T W > 0$ for all patterns of type ω_1 and $X^T W < 0$ for all patterns of ω_2. Let N be the total number of augmented samples. Also let the matrix whose elements are generated by test samples:

$$X = \begin{bmatrix} X_1^T \\ X_2^T \\ \vdots \\ X_N^T \end{bmatrix} \tag{3.5.4}$$

and assume that all X's belonging to ω have been multiplied by -1. The problem then reduces to determining W such that $XW>0$.

According to Ho and Kashyap (34), a minimum mean square error hyperplane can be generated even if the samples are not linearly separable by minimizing the criterion function

$$J = \frac{1}{2} \| XW - b \|^2 \tag{3.5.5}$$

with respect to both W and b, where b is an N-vector whose components are all positive where $\| \;\; \|$ stands for a distance criterion. Setting to zero the gradient of J with respect to W yields $W = (X^T X)^{-1} X^T b = X^+ b$ where X^+ is the generalized inverse of . The positivity constraint on b is fulfilled by the following iteration of b.

$$b(k+1) = b(k) + \delta b(k) \tag{3.5.6}$$

where

$$\delta b_i(k) = \begin{cases} 2c[XW(k)-b(k)]_i & \text{if}[XW(k)-b(k)]_i > 0 \\ 0 & \text{if}[XW(k)-b(k)]_i \leq 0 \end{cases} \tag{3.5.7}$$

the index i refers i-th component of the vector, and c is a constant such that $0 < c \leq 1$.

In vector form, equation (3.5.7) becomes

$$\delta b(k) = c[XW(k)-b+|XW(k)-b|] \tag{3.5.8}$$

The iterative learning algorithm is given by

$$W(k+1) = W(k) + cX^{+}\delta b(k) \tag{3.5.9}$$

and

$$\begin{aligned} b(k+1) &= b(k) + c\delta b(k) \\ W(1) &= X^{+}b(1),\ b(1)>0, \end{aligned} \tag{3.5.10}$$

otherwise arbitrary.

(2) Separation of the fuzzy section.

Once the minimum mean square error hyperplane H has been determined, separating boundaries are generated to contain only the learning samples belonging to the complete fuzzy section. This is accomplished by a search among the misclassified samples for the minimum and the maximum distances from H. Let the hyperplane H be represented by

$$d(X) \triangleq X^T W + w_{n+1}(H) = 0 \tag{3.5.11}$$

and the two separating boundaries by

$$\begin{aligned} X^T W + w_{n+1}(H_1) &= 0 \\ X^T W + w_{n+1}(H_2) &= 0 \end{aligned} \tag{3.5.12}$$

the distances from the origin to H_1 and H_2 are given by

$$\begin{aligned} \frac{w_{n+1}(H_1)}{|W|} &= \frac{w_{n+1}(H)}{|W|} - d_{max} \\ \text{and} \quad \frac{w_{n+1}(H_2)}{|W|} &= \frac{w_{n+1}(H)}{|W|} - d_{min} \end{aligned} \tag{3.5.13}$$

it follows that the two equations of H_1 and H_2 are

$$\begin{aligned} H_1: \; & X^T W + w_{n+1}(H) - |W| d_{max} \\ H_2: \; & X^T W + w_{n+1}(H) - |W| d_{min} \end{aligned} \tag{3.5.14}$$

Now the samples belonging to ω_1 and the samples belonging to ω_2 are separated from those whose status are indeterminate relative to ω_1 and ω_2. In other words, the nonfuzzy section is described by

$$\begin{aligned} X^T W + w_{n+1}(H) &> |W| d_{max}, \quad X \in \omega_1 \\ X^T W + w_{n+1}(H) &< |W| d_{min}, \quad X \in \omega_2 \end{aligned} \tag{3.5.15}$$

and the fuzzy section contains samples given by

$$|W|d_{min} < X^TW + w_{n+1}(H) < |W|d_{max} \qquad (3.5.16)$$

These fuzzy samples are then mapped into a feature space Ω_Y by the following transformation

$$Y_i = \begin{bmatrix} \dfrac{|X_i^TW + w_{n+1}(H_1)|}{|W|} \\ \dfrac{|X_i^TW + w_{n+1}(H_2)|}{|W|} \end{bmatrix} \qquad (3.5.17)$$

and the process is continued to separate the fuzzy section from the nonfuzzy section from the Ω_Y space until the whole Ω_X space is partitioned into two regions.

(3) Multiclass generalization.

For multiclass ω_i: i=1,...m, the classification procedure is to decide $X\varepsilon\omega_i$, if d (X) greater than $d_j(X)+M$ for all i=j, M>0. This is equivalent to decide $X\varepsilon\omega_i$ if $d_i(X) > d_j(X) + 1$, or simply $d_i(X) - \frac{M}{2} > 0$ and d (X) + $\frac{M}{2} \leq 0$.

Clearly, from multiclass discriminant functions the learning process can be used pairwise by using Ho and Kashyap's procedure. Ho and Kashyap's algorithm has been generalized to multiclass discriminant functions by Wee (61). The generalized algorithm has the advantage of requiring less computation than pairwise learning.

3.5.3 Fuzzy Automaton as a model of Analog Fault Isolation

Typically, available information at the early stage of FA, which can be represented using the fuzzy set idea, is given by the form of fuzzy membership function. The values of the fuzzy membership function might be subjective and local. Assuming those initial values are the best educated guess, we proceed to update fuzzy membership values through the fuzzy relations and/or various reinforcement schemes. The fuzzy relations and/or various reinforcement schemes act as training operators.

It is known that most of the existing diagnostic methods are sensitive to the presence of even minor drift in the nonfaulty components. A diagnostic scheme can learn to improve classification accuracy of observed input samples, if the weights in a set of discriminant functions can be adjusted according to the preselected criteria. For example, a criterion based on sample averages and the average deviations, when a set of test samples is available. These sets of weights which we call the reference vectors depend on the choice of the thresholds.

A basic learning model for fault diagnosis is shown in Fig. 3.5. During each time interval, the fault diagnoser

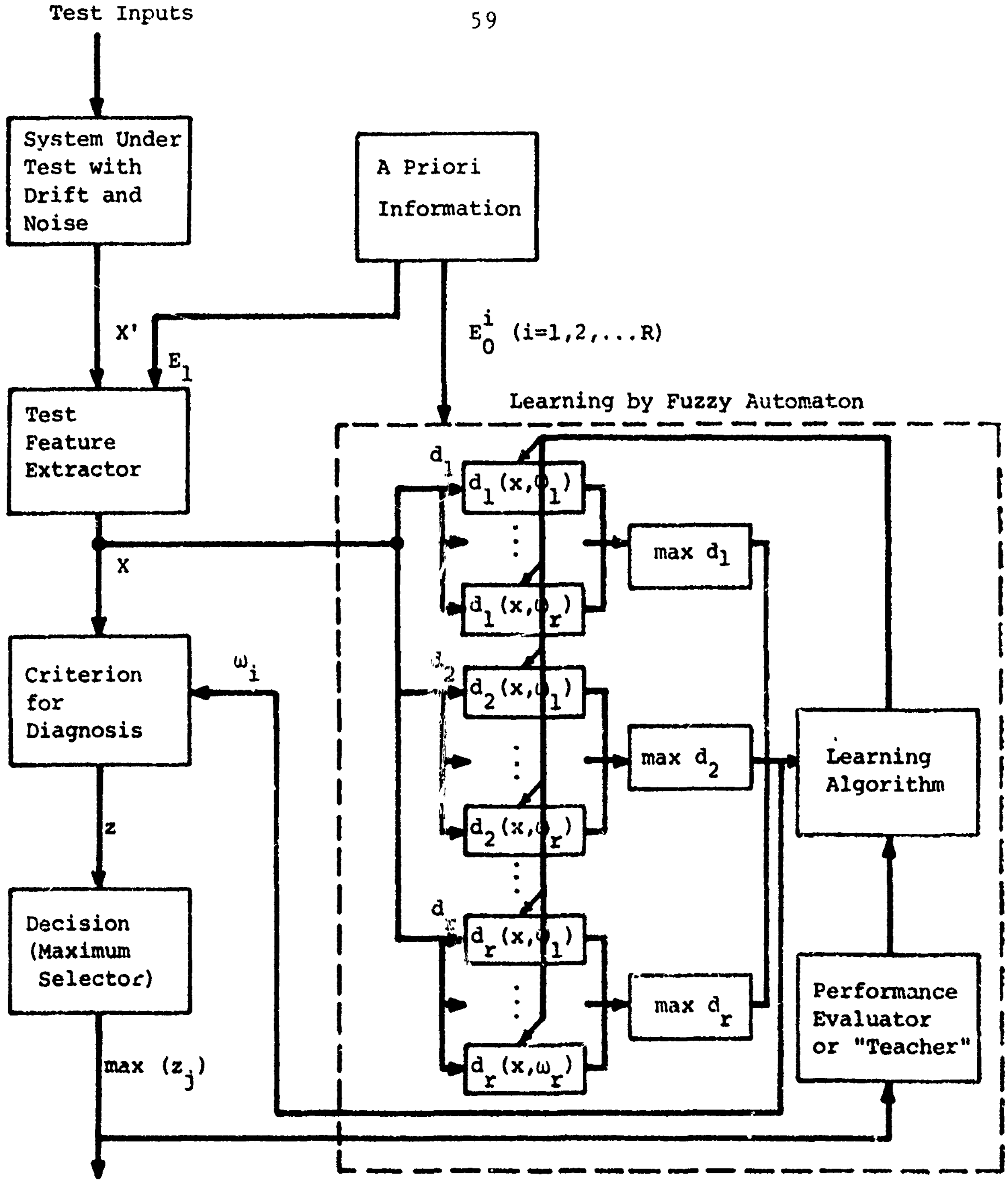


Fig. 3.6 Flow diagram for a learning diagnostic scheme in fault testing.

receives a new quantized sample X from the faulty system with drift and noise. Quantized X is fed to both fault diagnoser for classification and performance evaluator or teacher for performance evaluation. Teacher then directs the learning of the diagnoser using a linear reinforcement scheme. The learned information is considered as an experience of the fault pattern classifier and experience will be used to improved the quality of the diagnosis whenever similar situations recur. The new information extracted from recurring pattern is used to update the estimation or the experience associated with that fault pattern.

Fig. 3.6 shows a flow diagram for a proposed learning diagnosis scheme (12). In this scheme it is assumed that the classifier has at its disposal a set of discriminant functions characterized by a set of parameters such as the threshold levels. When there are m differnt fault classes, each class has ℓ learning samples, the jth sample of ith class can be represented by vector X_{ij} in the signal space of n dimensions providing that signal has n components. The samples may represent gain and phase deviations or impulse deviations. When the sets of discrimination functions characterized by sets of parameters such as thresholds of quantizations, selected frequencies or time delays are presented to a fuzzy automaton, the system adapts itself to the best solution. The best solution denotes the set of discriminant functions that gives the maximum recognition among the sets of discriminant functions within the set of test samples. Clearly the best set of discriminant functions contains the best set of parameter values for the generation of test programs.

Quantization of test samples, deviation measurements are as follows: If each component is to be quantized into three levels, say, zero, one, minus one according to a preassigned threshold, then X_{ij} is represented by

$$X_{ij}^T = (x_1, x_2, \ldots, x_n)_{ij} \tag{3.5.18}$$

which is a row vector of n random variables which assume the value of zero or one or minus one. A set of fault reference vectors is obtained by the sample averages of the training set, i.e.

$$R^T = (R_1, R_2, \ldots, R_n) \tag{3.5.19}$$

and $R_j = \frac{1}{m} \sum_{i=1}^{m} X_{ij}$ (3.5.20)

The correlation coefficient between X_{ij} and each of the fault references are determined to form a correlation vector

$$\Phi_{ij}^T = (\Phi_1, \Phi_2, \ldots, \Phi_m)_{ij}$$

The rules for deciding in which fault type in an unknown pattern should be classified are

$$\text{if } X^T R_j > X^T R_K \quad \text{all } k \neq j$$

and $|X-R_j| \leq d_j$, where d_j = a positive number, then the jth fault type is selected.

The diagnosis phase begins by applying a pattern of unknown fault type to the correlator so as to determine its correlation coefficient with respect to all references. Once decision is made, its corresponding type mean is modified so that the reference is updated. This operation has the advantage that, on the average, the diagnostic performance is also improved during the recognition phase. Consider the flow chart shown in figure 3.5 wherein sets of discriminant functions characterized by sets of parameters.

In particular the threshold of quantization, and selected frequencies or time delays are presented to a fuzzy automata for learning the best set. This implies the minimum misrecognition within the set of test samples. These sets of discriminant functions can well be the sample mean of fault references associated with the correlation process for decision making. Instead of sample mean references, one may employ Towill's voting techniques (3,5) which are heuristic rules and have been shown by actual problem simulation to be superior to the template-matching method. It should be clear that the best set of discriminant functions selected by the automaton contains the best set of parameter values necessary for the generation of a test program.

3.5.4 A Systematic Learning Procedure

This discussion is based on section 3.5.5. This procedure isolates the faulty components and obtains the optimum threshold by our learning method.

1. We start with a given active circuit description having one input port and one output port. The input is one Volt AC at different frequencies.

2. We are given the 14 following postulated fault conditions:

   $R_1 \rightarrow 2\times R_1$, $R_2 \rightarrow 2\times R_2$, $R_3 \rightarrow 2\times R_3$, $R_4 \rightarrow 2\times R_4$,
   $R_1 \rightarrow R_1/2$, $R_2 \rightarrow R_2/2$, $R_3 \rightarrow R_3/2$, $R_4 \rightarrow R_4/2$,
   $C_1 \rightarrow 2\times C_1$, $C_2 \rightarrow 2\times C_2$, $C_1 \rightarrow C_1/2$, $C_2 \rightarrow C_2/2$,
   $G_1 \rightarrow G_1/10$, $G_2 \rightarrow G_2/10$.

3. We assume that each nonfaulty component is varied under 3% normal distribution to simulate component drift. Also the output measurement noise with 1% normal distribution added.

4. Frequency domain analysis is based on 16 specified frequencies at which 14 gain deviation measurements and 11 phase measurements. In sum, we are using 25 frequency measurements.

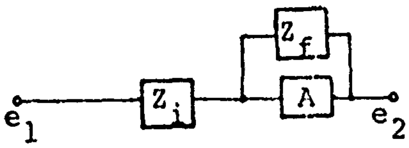
5. We assume that the amplifier of the active circuit

has one pole lowpass filter characteristics with gain of 5×10^5 and role off frequency at 10 Hz. The slope is -20dB/decade, and modeled by the following transfer function.

$$A_1 \leftarrow \frac{G_1}{1 + \frac{s}{62.8}}, \quad A_2 \leftarrow \frac{G_2}{1 + \frac{s}{62.8}} \tag{3.5.21}$$

6. The transfer function e_2/e_1 is given by

$$\frac{e_2}{e_1} = \frac{Z_f(s)}{Z_i(s)} * \frac{1}{1 + \frac{1}{A}\left(1 + \frac{Z_f(s)}{Z_i(s)}\right)} \tag{3.5.22}$$



7. We denote the two stages of the circuit as A_1, A_2 and substitute into Eq. 3.5.22. This yields two transfer functions e_2/e_1, and e_3/e_2 obtained similarly. When we multiply these two functions together, we will get the overall transfer function. This transfer function is a function of component values and frequencies.

8. We defined the 14 fault patterns according to the 14 fault conditions (given in the step 2) with component drifts and measurement noise. $(\omega_1, \omega_2, \ldots, \omega_{14})$

represents 14 fault types.

9. For computational efficiency as well as convenience we use the quantized values {1, 0, -1} of deviation measurements to form a fault recognition matrix (R_{ij}). An element R_{ij} of ith column and jth row of recognition matrix is the quantized value {1, 0, -1} of ith fault type and jth frequency measurement where $i = 1, 2, \ldots, 14$, $j = 1, 2, \ldots, 25$.

10. We arbitrarily assume one of 7 thresholds to quantize the total range of deviation measurements.

11. For each specific threshold, there corresponds one fault recognition matrix.

12. The choice of a specific threshold is made based on the fuzzy automaton model. For our illustrative example the input to the fuzzy automaton is either {1, 0} depending on the success or failure of the diagnosis. In general, we can use the fuzzy relations studied in section 3.3 to select the other threshold.

13. The computer model of the system is used to generate the simulated faults as follows:

    a) One fault condition is picked from among the 14 fault conditions randomly, and then all the other component values are picked ramdomly within the prescribed tolerance limit.

b) Generate output responses at the selected frequencies using the computer simulations (25 measurements). Add random noise to the measurements (given in step 3).

c) Calculate the deviation measurements.

d) Calculate the quantization deviation vector, $X = (X_1 , \ldots , X_j , \ldots , X_{25})$ corresponding to the threshold selected in step 12. Where X_j represents the quantized value of incoming jth deviation frequency measurement.

14. Fault Recognition Matrix

Once a specific threshold is chosen, we produce the corresponding recognition matrix (R_{ij}).

15. A classification rule for fault isolation is a minimum distance criterion. Thus the ith pattern yielding

$$\min_i \sum_j^{25} |R_{ij} - X_j| \quad \text{is chosen.}$$

16. Above choice is compared with the choice of teacher, and the decision as to the correctness of the classification for updating the membership values of given thresholds in the step 18.

17. We assign an initial fuzzy membership value to

ea. n c given thresholds. This value indicates the degree of correctness or the percentage of the diagnosis based on the selected threshold. The thresholds and the corresponding membership values are represented as

$$\{(Th_1, f_1), (Th_2, f_2), \ldots, (Th_7, f_7)\}.$$

18. The membership value to be assigned for a threshold is learned from the information as to the correctness of the fault isolation decision of an incoming set of measurements in step 16. The learning scheme used is known as the linear reinforcement scheme.

$$\begin{aligned} f_i(n+1) &= \alpha f_i(n) + (1-\alpha)\phi(X, \omega_i) \\ f_j(n+1) &= \alpha f_j(n) + (1-\alpha)(1-\phi(X, \omega_i)) \end{aligned} \qquad (3.5.23)$$

$$\text{where } 0 < \alpha < 1, \; \phi(X,\omega_i) = \begin{cases} 1 & \text{if } X \in \omega_i \\ 0 & \text{if } X \notin \omega_i \end{cases}$$

These learned fuzzy membership values will affect the choice of the next threshold in step 12.

19. If the maximum fuzzy membership value achieved based on the seven thresholds meets the design requirements, then stop the learning process and use this particular threshold.

20. If not, we select another set of thresholds close

to the one which achieved the maximum membership value. Then repeat the procedure starting at step 10.

21. Implementation of this fault isolation procedure is relatively simple and straightforward since in the field we only use the selected optimum threshold.

3.5.5 Illustrative example using an Active Linear Network

We start with a given active circuit in Figure 3.7 having one input port and one output port. The input is 1 Volt AC at different frequencies. Frequency domain analysis based on 16 specified frequencies ranging from 10 rad/sec to 10^7 rad/sec at which 14 gain deviation measurements and 11 phase measurements. In sum, we are using 25 frequency measurements.

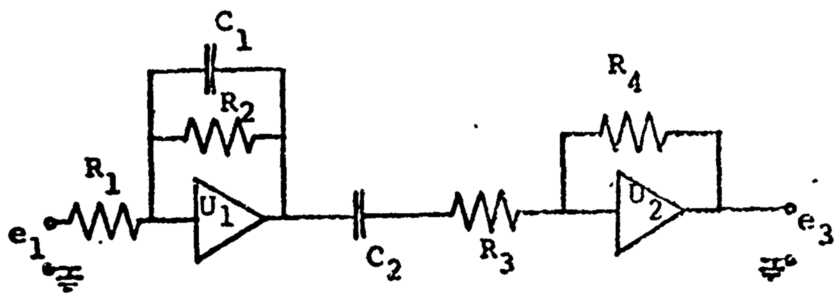


Fig. 3.7 Circuit Diagram

Figure 3.7 represents the functional model of a simple analog circuit consisting of four resistors, two capacitors and two operational amplifiers which are treated as functional models. Postulated fault conditions are at step 2 of section 3.5.4.

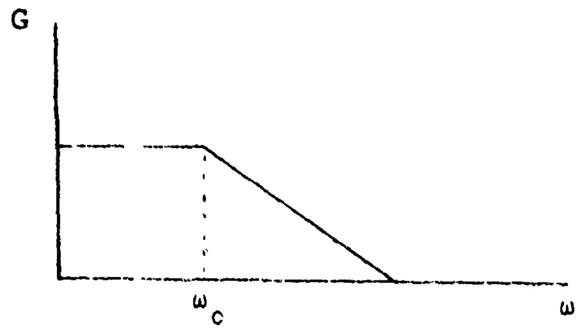


Fig. 3.8 Amplifier Characteristic

Figure 3.8 shows the characteristics of the two identical op. amps. The functional model of these op. amps. can be approximated as

$$A_1 \leftarrow \frac{G_1}{1 + \frac{s}{62.8}}, \qquad A_2 \leftarrow \frac{G_2}{1 + \frac{s}{62.8}}$$

Table 3.1 Nominal Values and Analytical Expressions for Linear Circuit

$R_1 = 1,000$	$R_3 = 10,000$
$R_2 = 10,000$	$R_4 = 10,000$
$C_1 = 1.6 \times 10^{-9}$	$C_2 = 1.6 \times 10^{-7}$
$G_1 = 2 \times 10^{5}$	$G_2 = 2 \times 10^{5}$

$f_1 = 10 \qquad\qquad f_2 = 10$

$$\frac{e_3}{e_1} = \frac{C_o s}{(a_o s^2 + a_1 s + a_2)(b_o s^2 + b_1 s + b_2)}$$

$$a_o = \frac{C_1 R_1 R_2}{2\pi f_1} \qquad\qquad b_o = \frac{C_2(R_3 + R_4)}{2\pi f_2}$$

$$a_1 = \frac{R_1 + R_2}{2\pi f_1} + R_1 C_1 R_2 + C_1 R_2 G_1 R_1 \qquad\qquad a_2 = G_1 R_1 + R_1 + R_2$$

$$b_2 = G_2 + 1$$

$$b_1 = \frac{1}{2\pi f_2} + C_2 R_3 + C_2 R_4 + C_2 R_3 G_2 \qquad\qquad C_o = G_1 R_2 G_2 C_2 R_4$$

Table 3.1 lists the nominal values of the elements together with the analytic expression of the transfer function. This circuit has previously been studied for fault diagnosis based on the choice of transfer function parameters as a fault signature (33). Herein we use gain and phase deviations between faulty and nominal response at a set of selected frequencies to isolate a single fault in the system under test. The nonfaulty components are assumed to have 3% tolerance while the measurements are contaminated by 1% noise. The Bode plot of the nominal gain response as shown in figure 3.9 has break away frequencies at 625 rad/sec, 5.95×10^4 rad/sec, 6.28×10^6 rad/sec and 1.319×10^7 rad/sec.

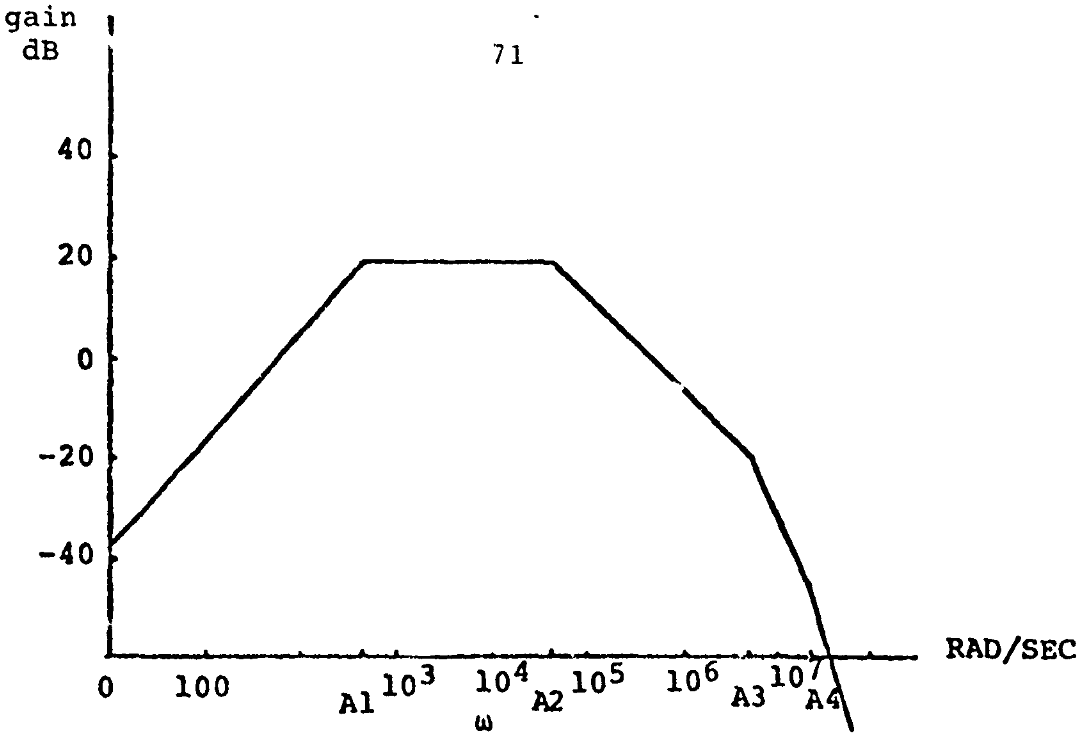


Fig. 3.9 Bode Plot

According to Sriyananda, Towill, and Williams (5) experience suggests that the number of selected measurements should be approximately three times the number of fault cases if all gain and phase data are useful. They also pointed out that the threshold for quantization of the deviations is a major influencing factor for the generation of the best set of features in analog testing. 16 frequencies, ranging from 10 rad/sec to 10^7 rad/sec, are selected. Six frequencies have both gain and phase measurements. Altogether there are 14 phase measurements and 11 gain measurements. Seven quantization thresholds are chosen to generate seven sets of fault reference vectors.

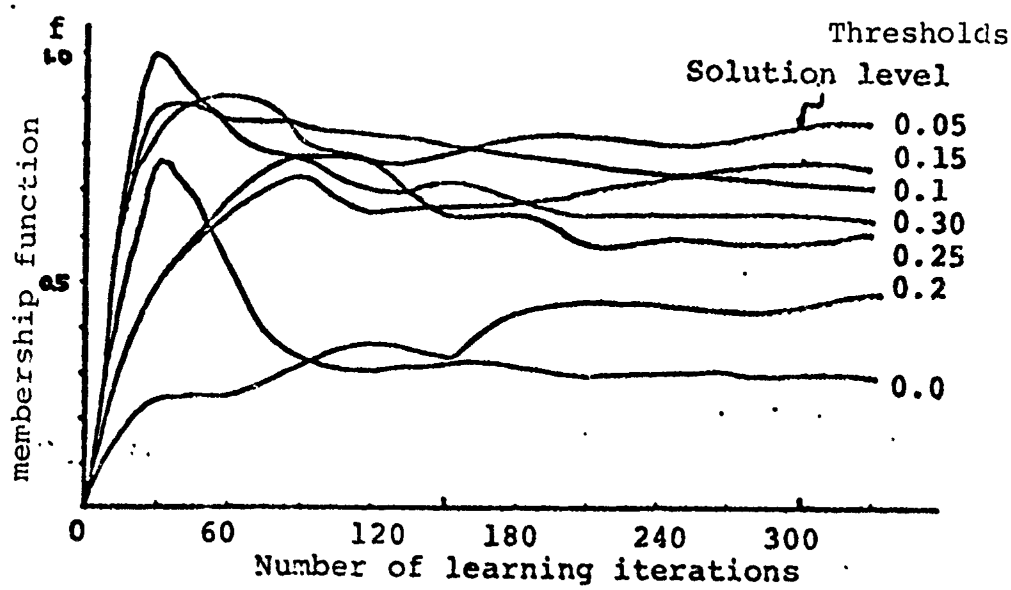


Fig. 3.10 Learning Curves for Sets of Discriminant Functions

Figure 3.10 shows the learning curves of the membership functions versus the number of training samples for the 14 types of faults in a computer simulation study using Eq. 3.5.23. Even though the number of training samples is not large the trend is already evident. The value 0.05 is the best among the set chosen, and fuzzy membership function approaches).85.

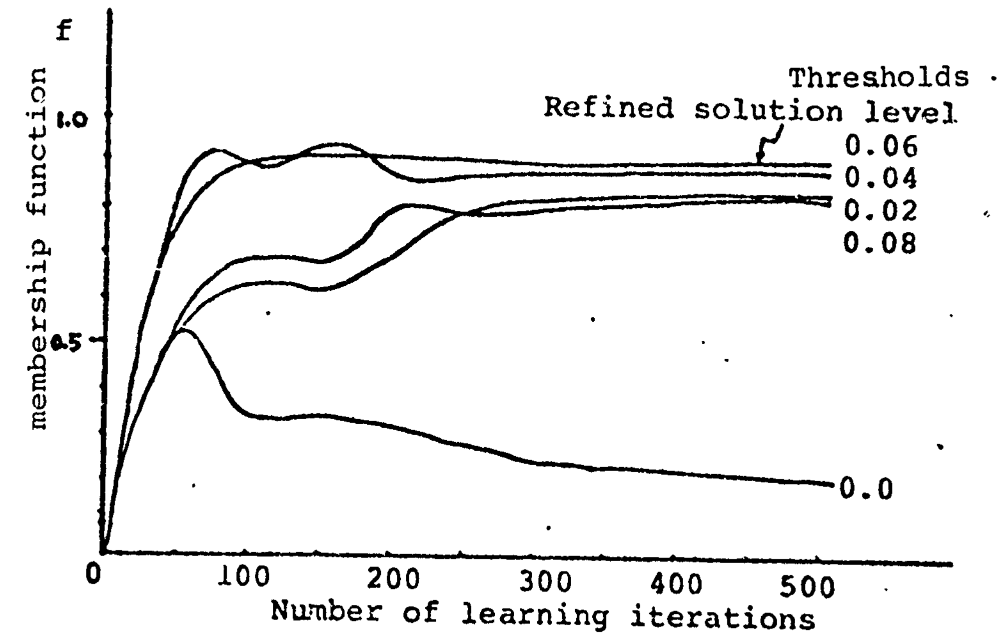


Fig. 3.11 Learning Curves for Refined Sets of Discriminant Functions

Figure 3.11 illustrates several thresholds in the neighborhood of 0.05, 0.06 has the highest grade of membership which is 0.9. When the selected frequencies are chosen from 10 rad/sec to 10^7 rad/sec and a single fault is isolated to a group of components such as R_1, R_2 and C_1, U_1, U_2, the learning curves are shown in figure 3.12 and figure 3.13. Note that the set of reference vectors with 0.1 threshold attains the grade of membership of 0.94.

The next simulation studies involve comparisons between learning with fuzzy automaton model and learning with fuzzy

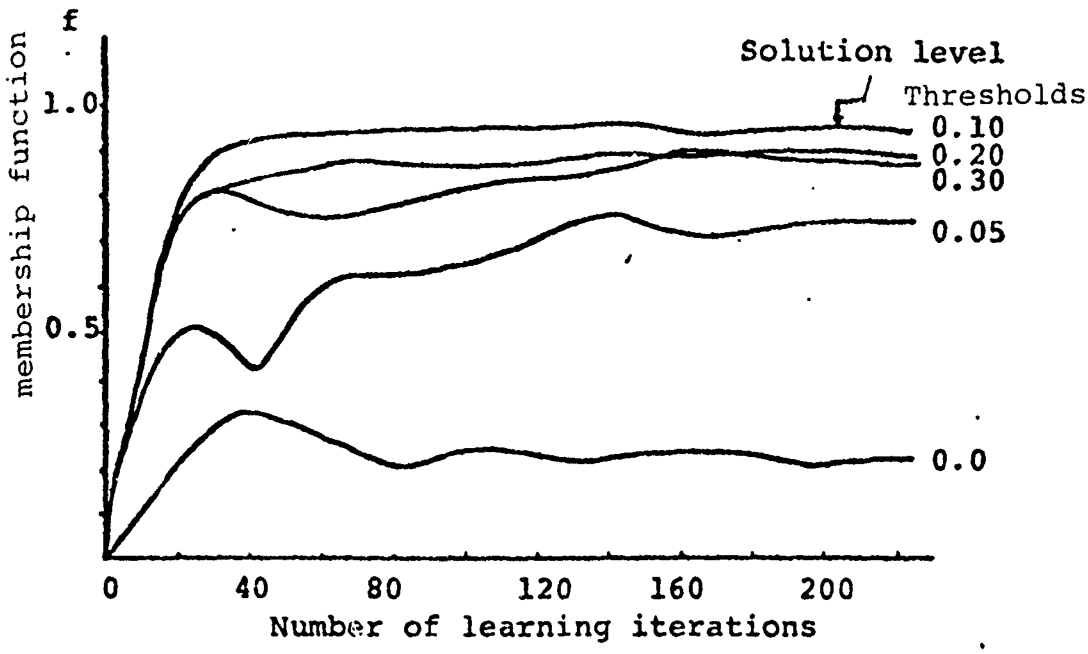


Fig. 3.12 Learning Curves for Sets of Discriminant Functions with Resolution

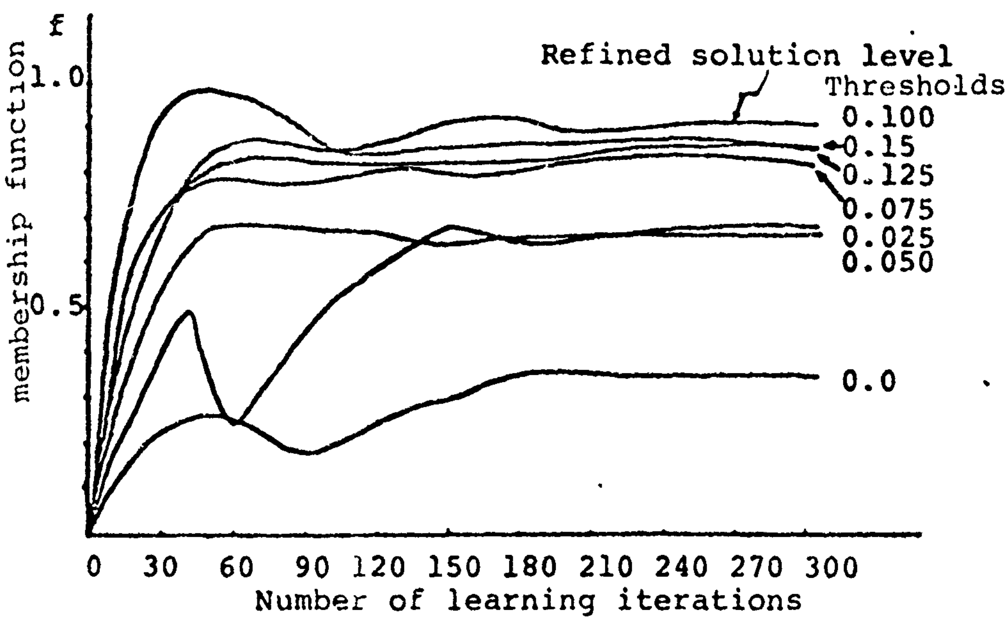


Fig. 3.13 Learning Curves for Refined Sets of Discriminant Function with Resolution

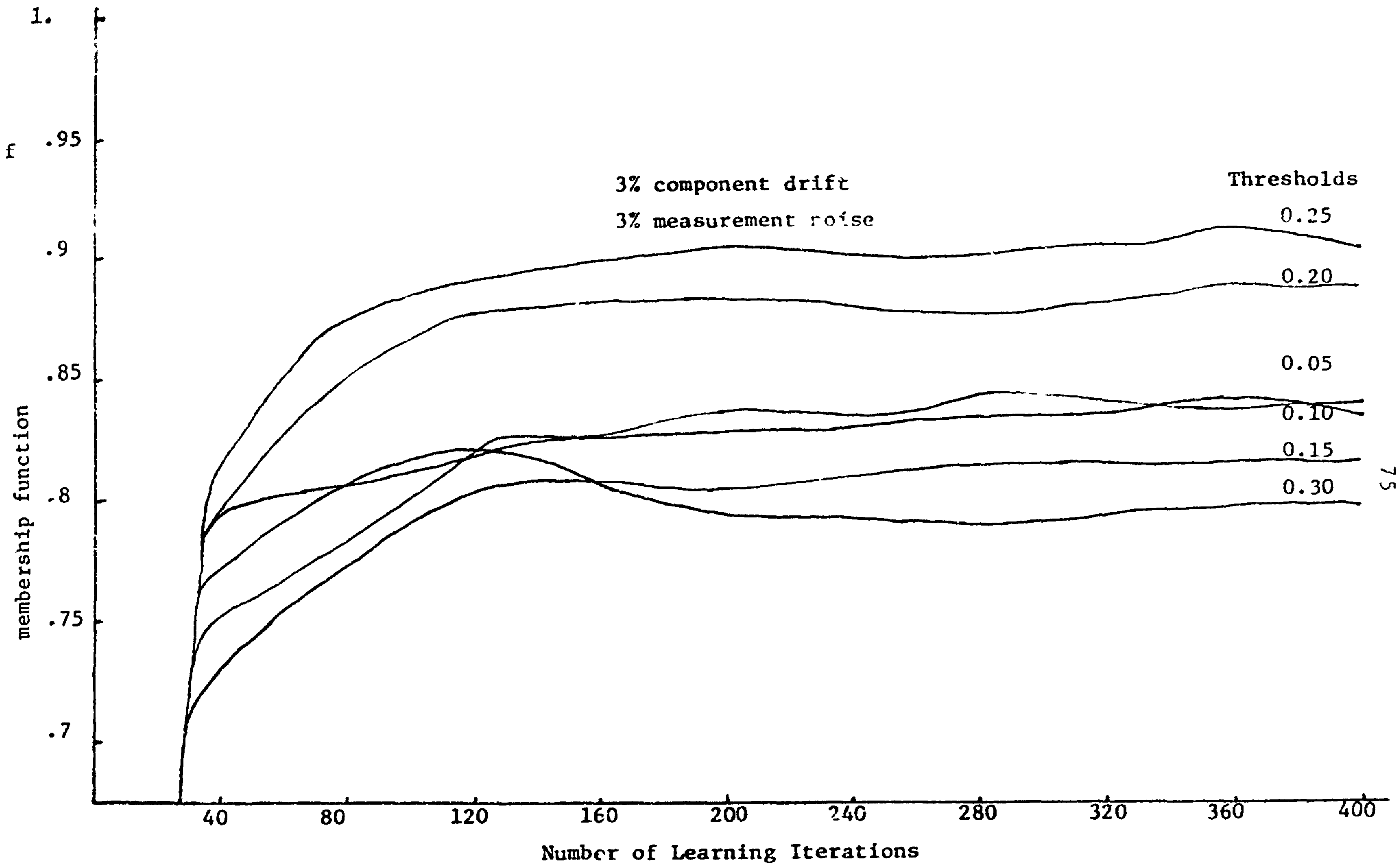


Fig. 3.14 Membership functions vs. k using distance measure

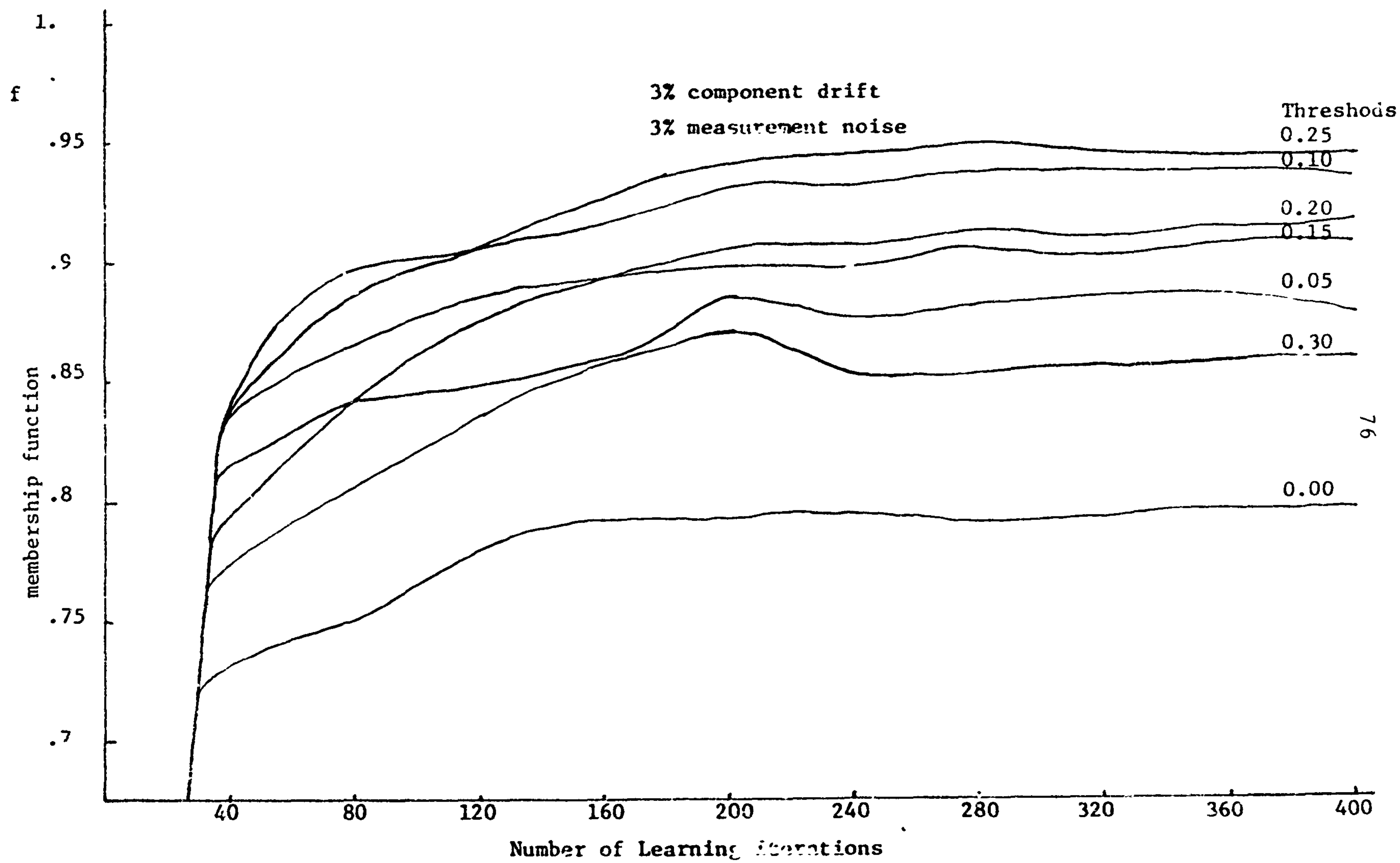


Fig. 3.15 Membership functions vs k using distance measure and refined reference vectors

automaton model using sample mean references while changing component drift and measurement noise. Assumputions are the same as in the previous example, except that 17 frequencies are selected in the same ranges. Eight frequencies have both gain and phase measurements. Seven quantization thresholds are chosen to generate 7 sets of fault references vectors. Figure 3.14 shows the learning curves of membership functions against the number of training samples for the 8 types of single faults in a computer simulation study. In this figure we assume that the component drift is 3% while the measurement noise is 3%. The reference vectors used in this case are from the deviations when the nonfaulty components are at their nominal values and the measurements are noise free. Although the number of training samples is not large the trend is again evident. The quantizing level 0.25 is the best among those chosen, and fuzzy membership approaches 0.9. Figure 3.15 shows the learning curves of the membership functions against the number of training samples with mean reference vectors. During learning, the mean reference vector is updated. This has the advantage that on the average, diagnostic performance is also improved even during the recognition phase. It turns out that a quantization level of 0.25 is again the best among the given set. Here the fuzzy membership function increases from 0.9 to 0.94. Figure 3.16 shows the case when component drift and measurement noise both increase to 10%. The reference vectors used in this simulation are the same ones used in Figure 3.13. In this case 0.15 level turned to be the best and the highest membership function is only 0.67. Level 0.25 is in third place.

Instead of using the given reference vectors, we can also use the mean reference vectors. The adjusting rule is:

$$R_{ij}(n+1) = \frac{n-1}{n} R_{ij}(n) + \frac{1}{n} X_j(n)*\psi(n)$$

$$\text{where } \psi(n) = \begin{cases} 1 & \text{if X is correctly classified} \\ -1 & \text{if X is not correctly classified.} \end{cases}$$

One of the advantages of this is to increase the percentage of correct fault diagnosis. A significant drawback is that one must revert to making analog measurements instead of the simple 1, 0, and -1 indicated in the procedure of sec. 3.5.4. Figure 3.17 has the same condition as Figure 3.16 except that we used the mean reference vectors. The best quantization threshold turns out to be the same as for figure 3.16 but the membership value at that threshold increases to 0.725. This is about 5% higher than in the previous case. Instead of the sample mean reference, one may use Towill's voting techniques. These are heuristic rules that have been shown by actual problem simulation to be superior to all existing template matching methods. Table 3.2 shows a comparison of the distance criteria and voting tecniques. We specifically use

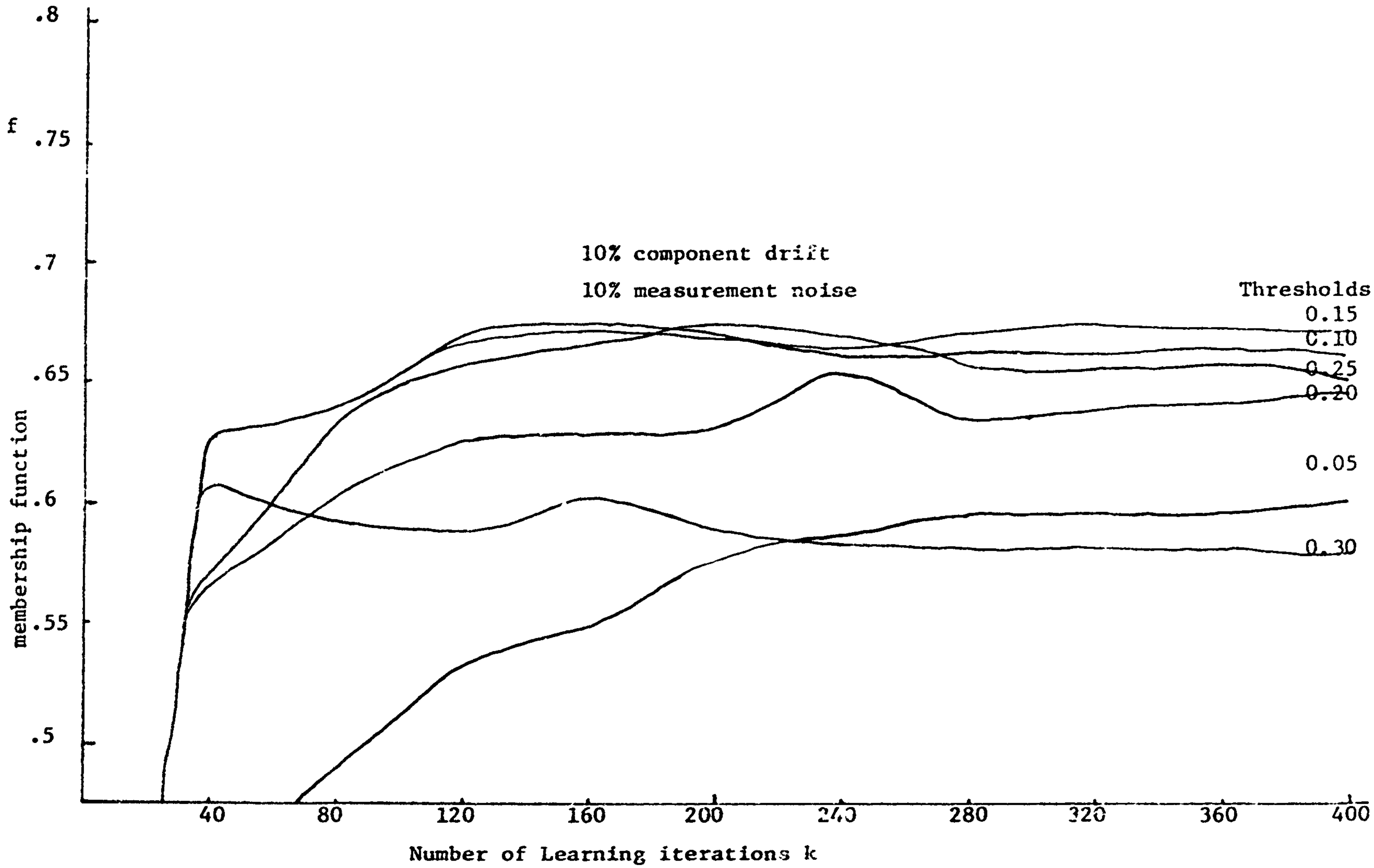


Fig. 3.16 Membership function vs. k using distance measure

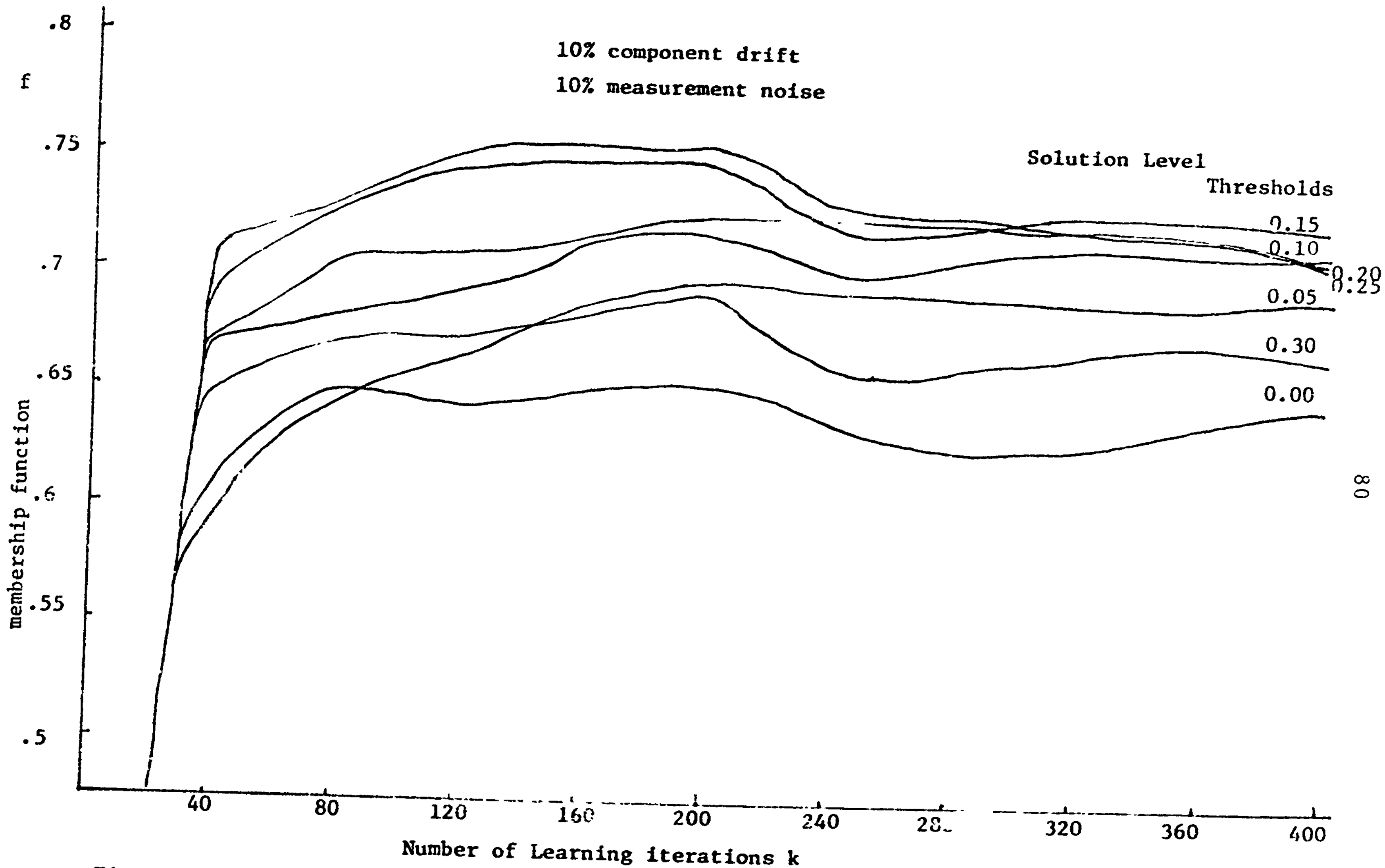


Fig. 3.17 Membership function vs. k for using distance measure and refined reference vectors

Drift, Noise \ Quantization Level		0.00	0.05	0.1	0.15	0.2	0.25	0.3	
3%	V	155	330	334	323	358	369	321	2190
	D	155	341	339	331	361	368	325	2290
	V A	281	334	366	365	367	387	351	2451
	D A	326	358	379	369	372	384	354	2542
5%	V	153	306	318	305	332	327	291	2032
	D	153	322	324	317	339	329	294	2078
	V A	265	316	335	333	340	349	315	2253
	D A	304	339	350	351	344	351	323	2362
8%	V	137	248	275	270	266	274	255	1725
	D	137	264	289	283	279	281	254	1787
	V A	235	271	280	273	295	294	274	1942
	D A	283	297	307	310	311	303	290	2101
10%	V	126	229	254	257	244	255	237	1602
	D	126	244	268	273	257	263	234	1665
	V A	227	254	262	275	273	271	250	1812
	D A	263	281	289	294	288	288	271	1974
SUM	V	571	1113	1181	1155	1200	1255	1104	7549
	D	571	1171	1220	1204	1236	1271	1107	7750
	V A	1008	1175	1243	1266	1278	1301	1190	8458
	D A	1176	1275	1325	1324	1315	1326	1238	8979

Table 3.2 Comparison of Different Criteria

the seven thresholds (0, 0.05, 0.1, 0.15, 0.20, 0.25, 0.30) dB. The four cases of tolerance and measurement noise are 3%, 5%, 8%, and 10% within the normal distributions. We compared voting technique, voting technique with mean reference vector adjustment, distance criterion, and distance criterion with mean reference vector. For small tolerances of nonfaulty components and measurement noise such as the 3% case, they perform equally well. As tolerance and measurement noise increase to 10% the distance criteria appears to be somewhat more effective. Although we have only used a preselected set of frequencies, we can also "learn" the best set from a given set of frequency measurements.

3.5.6 Discussion

Our objective is to select a specific threshold so as to permit simple implementation of ATE for easy field use. For analog electronic systems with drift and noise, the measured set of responses for different fault types often exhibits highly overlapping patterns. Depending on the choice of a specific threshold from among the seven thresholds, we can discriminate the best among the fault patterns. Since the assumed test frequencies are fixed, the method of choosing the specific threshold is the most important factor determining the optimum achievable discrimination. The concept of fuzziness is involved because one tries to determine how effective a discriminant function is obtained by use of a specific threshold. The application of learning techniques to reduce the degree of fuzziness has been presented. We emphasize the importance of selection of features to be measured in analog testing to achieve effective fault diagnosis.

Herein we have demonstrated that a fuzzy automaton learning model can be applied to select an optimal set of such features.

3.6 Fault Isolation Method using the Fuzzy Logic

For nonlinear analog circuits subject to drift and noise, the resulting measurement patterns of faults are in general fuzzy so that ad hoc specification of fault isolation tests is inadequate. Considered in the context of fuzzy systems, fault pattern types are first separated into non-fuzzy and fuzzy parts corresponding to non-overlapping and overlapping regions obtained by sensitivity analysis. The grade of membership of the fuzzy parts are then modified according to simulation results and the decision based on fuzzy relations (35). Thus, a sequence of input-access point responses with highest membership value is selected as the basis for generation of automatic fault isolation tests.

3.6.1 Introduction

The design of functional and fault isolation tests is now recognized as an essential task needed at the design and quality-assurance stages of analog circuits in electronic systems. Practical analog systems are subject to drift and are exposed to noise. Furthermore under fault conditions such systems in general become nonlinear. It is not suprising therefore that the design of an automatic fault isolation test based on a deterministic approach fails to give satisfactory results. Therefore the measurement responses yield highly overlapped and scattered fault patterns (4,12). The statistical approach also fails whenever it is impossible to represent highly overlapped and scattered fault patterns by known distribution functions. The statistical approach will also fail when there is no a priori information available. Then one can not expect the distribution of fault patterns to correspond precisely to an assumed distribution. Furthermore, it is often unreasonable to make the convenient statistical independence assumption for the components of fault pattern vectors. For systems with realistic component tolerances and noisy environment, therefore, we consider fault isolation is essentially fuzzy. Our approach uses the fuzzy concept to develop a systematic way of generating automatic fault isolation tests for practical circuits (49). The procedure for the design of automatic fault isolation tests is shown in Fig. 3.18. The essential steps are

(1) Simulation of responses at available access points of a unit under test (UUT) by means of computer aided network analysis program for a set of prespecified fault conditions.

(2) Estimation of the upper and lower bounds of fault pattern types by means of sensitivity analysis.

(3) Modification of the grade membership of the fault pattern belonging to certain fault type using fuzzy relations on a set of training samples assuming a specified distri-

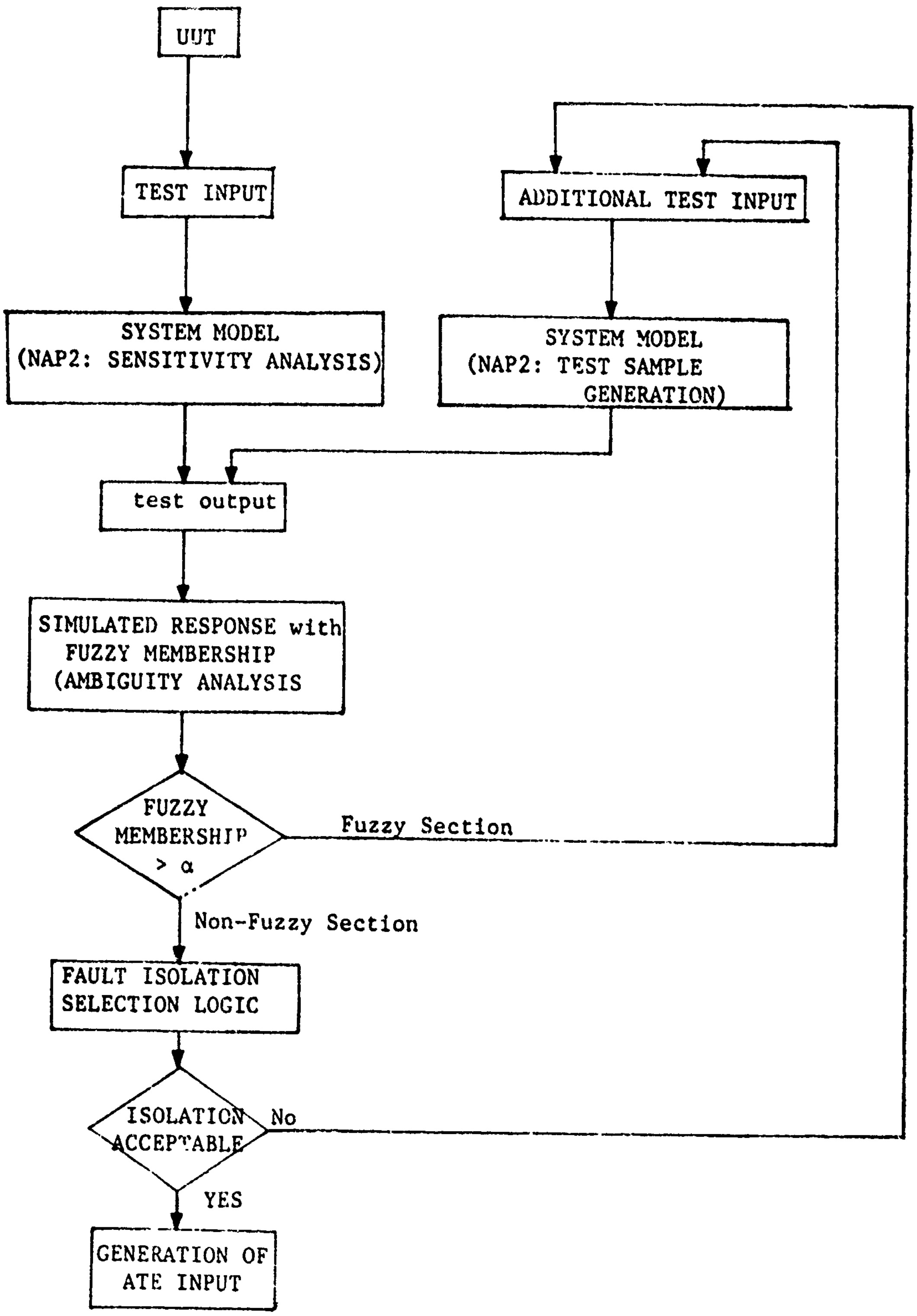


Fig. 3.18 A Design Procedure for Automatic Fault Isolation Tests

bution of the nonfaulty component tolerances.

(4) Selection of tests yielding the highest grade of membership functions for the discrimination among fault pattern types.

3.6.2 Fault Response Simulation

Instead of physically changing a component or module to introduce a specific fault in hardware, it is much more convenient to use a computer aided network analysis (CANA) program. For example, a library containing transistors and diodes may be found in a model library on the extended SCEPTRE (89) program tape. A CANA program uses the topological description and the component values of a circuit to formulate network equations which are then solved by numerical methods. To perform a fault isolation test simulation, the required information includes (1) the topological description, the nominal values of the components and their tolerances, (2) the description of the input and accessible test points, (3) the definition of the failure modes. In a nonlinear network analysis program for lumped circuits, the response calculation is based on the formulation of network equations and sparse matrix technique. For nonlinear circuits the Newton- Raphson method is used. From the standpoint of cost effectiveness it is advisable to model each integrated circuit type as a functional element because many internal failures of the integrated circuit package are indistinguishable at the external terminals. In a computer aided test design system it is desirable to reduce the amount of calculations for setting up a test program which leads to an acceptable degree of fault diagnosis. A simple fault isolation technique using binary logical conjunction of failure response regions has the advantage of reducing the amount of calculation but often fails to achieve an acceptable diagnostic level (7). Fuzzy set theory seems able to overcome some of these difficulties.

3.6.3 Response Sensitivity With Respect To Tolerances

To determine the upper and lower bounds of the response at an accessible point for a specified failure mode while nonfaulty components have reasonable tolerances would require a large amount of simulation work. To reduce the effort, one can use a sensitivity analysis to estimate the approximate bounds as follows. Let ΔC_i be the maximum per unit tolerance of non-faulty component C_i. The response sensitivities with respect to n non-faulty component at an access point are $\frac{\partial P}{\partial C_{io}}$, $i = 1,\cdots,n$, where P is the response and the C_{io} is the nominal value of C_i. As a first approximation we may compute the upper and lower bound according to the sign of $\frac{\partial P}{\partial C_{io}}$. If $\frac{\partial P}{\partial C_{io}}$ is positive for $i = 1,\cdots,m$ and $\frac{\partial P}{\partial C_{io}}$ is negative for $j = m+1,\cdots,n$, then

$$P_{max} = P_o + \sum_{i=0}^{m} \delta_i C_{io} \frac{\partial P}{\partial C_{io}} - \sum_{j=m+1}^{n} \delta_j C_{jo} \frac{\partial P}{\partial C_{jo}} \tag{3.6.1}$$

and

$$P_{min} = P_o - \sum_{i=0}^{m} \delta_i C_{io} \frac{\partial P}{\partial C_{io}} + \sum_{j=m+1}^{n} \delta_j C_{jo} \frac{\partial P}{\partial C_{jo}} \tag{3.6.2}$$

where P_o is the fault response when the non-faulty components have their nominal values. To take into account a nonlinear effect, one normally would have to compute second and higher order sensitivities and calculation becomes laborious. A simple way to include nonlinear effects is to use the deviations $\delta_i C_{io}$ and $\delta_j C_{jo}$ from Eq's (3.6.1) and (3.6.2) as the worst case to compute directly the response by means of CANA program (69). In practical cases this simple method gives the upper and lower bounds quite close to results obtained by extensive simulation. In some cases where the dominant variation of the response with respect to some components is quadratic, the simple method is not satisfactory. However, this can be remedied by using Max (P_{max}, $2P_o - P_{min}$) and Min (P_{min}, $2P_{max} - P_o$) as the upper and lower bound, respectively.

3.6.4 Fault Isolation Using the Fuzzy Logic

The essential task of fault isolation in a fuzzy environment is to recognize the membership of responses which belongs to a designated fault class and to distinguish among memberships which belong to different fault classes. This can be conveniently established around the notion of the "belonging" in the case of fuzzy sets if the boundaries between fault pattern classes are not sharply defined. Let $\omega^1, \omega^2, \ldots, \omega^r$ fuzzy fault pattern classes in the fault response space P, and p be the generating element of P. We define $f_{\omega}i(p)$ at p to be the grade of membership p in ω^i and $f_{\omega}i(p)$ associates each point p with a real number in the interval (0,1). When ω^i is a set in the ordinary sense of the term, then its membership function can take only one and zero according as p does or does not belong to ω^i.

Suppose the upper and lower bounds of the responses at available access points for a set of specified fault conditions have been estimated by the sensitivity analysis. The regions between these bounds may or may not overlap. The non-overlapping regions, wherein fault isolation becomes very simple, are easily distinguished from the fuzzy regions. Using binary logic some fault can also be isolated from the overlapping regions.

The grade of fuzzy membership for a particular fault condition when a response lies in a certain region may be assessed according to some a priori information. As an example Fig. 3.19 shows the response regions of 5 single faults measured at access points k, k=1,2,3. Let ${}_k\omega_j^i$ be the response due to the ith fault measured at access point k that falls in the region R_j, j=1,2,..., 5, and k=1,2,3. Initially we assign the membership value of ${}_k\omega_j^i$ as one only if i=j. Notice that because of overlapping regions, the response of a fault other than the ith one may also fall in the region R_i. To estimate the grade of membership for the case that only the response of ith fault falls in the region R_i and no other response can appear in the same region. To do this we use a set of additional samples for estimating the membership functions of the related faults in the overlapping regions together with fuzzy relations. The membership of the overlapping regions are shown in Table 3.3, in which $f_{{}_k\omega_j^i}$ represents the grade membership of ith fault response at kth access point in jth overlapping region or simply jth region.

(1) Fuzzy Relations

The commonly used modes of composition of two fuzzy relations are (a) conjunctive, involving the connective "and", (b), disjunctive, involving the connective "or". The membership function of the union of two fuzzy sets with respective membership functions $f_A(p)$, $f_B(p)$ is

$$f_{A\cup B}(p) = \text{Max}\ (f_A(p),\ f_B(p)),\quad p \in P \tag{3.6.3}$$

The membership function of the intersection of the above two fuzzy sets is given by

$$f_{A \cap B} = \min\ (\ f_A(p),\ f_B(p)),\quad p \in P \qquad (3.6.4)$$

In the case of binary fuzzy relations the composition of two fuzzy relations A and B is denoted by B∘A, and is defined as a fuzzy relation in P whose membership function is related to those of A and B by

$$f_{B \circ A}(p,q) = \operatorname*{Sup}_{v} \min\ (\ f_A(p,v),\ f_B(v,q)). \qquad (3.6.5)$$

(2) Membership Function of Refined Sets

Let ${}_k\omega_i$ be the response of the ith fault measured at access point k that falls in the region R_i under the condition that no other fault response appears in R_i. We call ${}_k\omega_i$ a refined set of ${}_k\omega_j^i$. To estimate the membership function $f_{{}_k\omega_i}$, it is convenient to use the composition of fuzzy relations similar to Eq. (3.6.5) as follows:

$$f_{{}_k\omega_i} = \operatorname*{Sup}_{v} \min\ (\ f_{{}_k\omega_v^i},\ f'_{{}_k\omega_i^v}\ ) \qquad (3.6.6)$$

where "'" denotes the complement. It may be noted that "conjunctive" and "disjunctive" modes in Eq. (3.6.6) also appears in Zadeh's possibility theory. Table 3.4 shows the values of $f_{{}_k\omega_i}$ calculated by Eq. (3.6.6) using the values given in Table 3.3. From Table 3.4, the overlapped regions can be eliminated by a pairwise comparison among the values

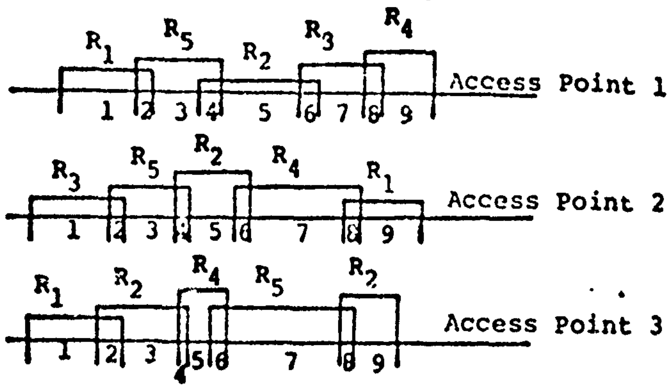


Fig. 3.19 Overlapped Response Regions

(1) Access Point 1

i \ j	1	2	3	4	5
1	1.00	0.00	0.00	0.00	0.15
2	0.00	1.00	0.40	0.00	0.01
3	0.00	0.20	1.00	0.20	0.00
4	0.00	0.00	0.25	1.00	0.00
5	0.20	0.02	0.00	0.00	1.00

(2) Access Point 2

i \ j	1	2	3	4	5
1	1.00	0.00	0.00	0.20	0.00
2	0.00	1.00	0.00	0.30	0.02
3	0.00	0.00	1.00	0.00	0.20
4	0.14	0.35	0.00	1.00	0.00
5	0.00	0.01	0.26	0.00	1.00

(3) Access Point 3

i \ j	1	2	3	4	5
1	1.00	0.00	0.30	0.00	0.00
2	0.00	1.00	0.00	0.00	0.02
3	0.35	0.00	1.00	0.05	0.00
4	0.00	0.00	0.10	1.00	0.20
5	0.00	0.01	0.00	0.25	1.00

Table 3.3 Fuzzy Membership $f_{\omega_k}{}^{i}_{j}$ for the Overlapping Region

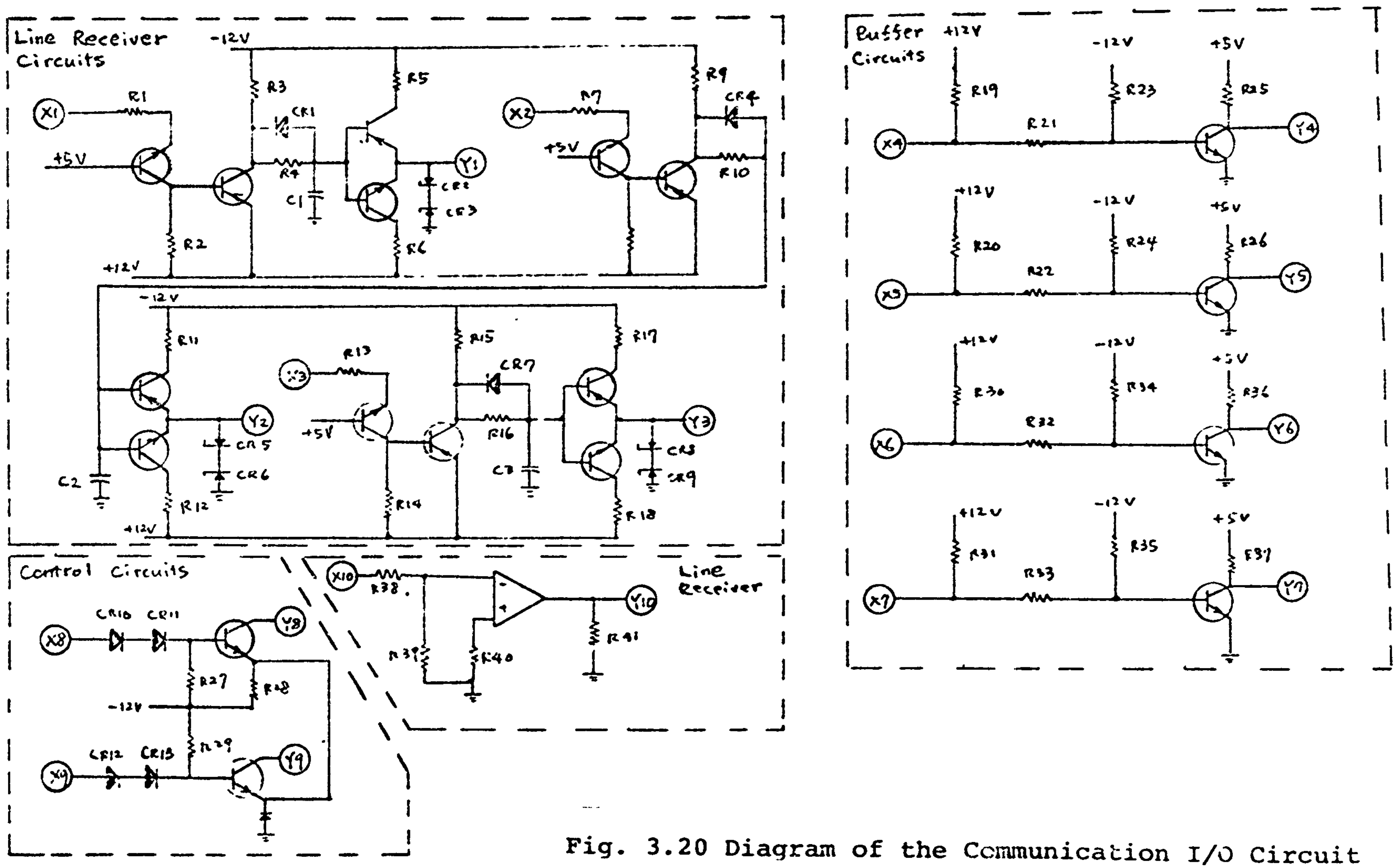


Fig. 3.20 Diagram of the Communication I/O Circuit

i \ k	1		2		3	
1	0.85	1,2	0.80	8,9	0.70	1,2
2	0.60	5	0.70	5,6	0.98	8,9
3	0.80	6,7,8	0.80	1,2	0.65	3
4	0.75	9	0.65	7	0.80	4,5,6
5	0.80	3,4	0.74	3,4	0.75	7

Table 3.4 Maximum Fuzzy Membership for ${}_k\omega_i$

K	I	L. LIMIT	U. LIMIT	F(I)
1	1	-5.571E 00	-5.285E 00	0.000
1	2	8.368E-05	8.394E-05	0.999
1	7	-5.571E 00	-5.285E 00	0.000
1	9	-5.501E 00	5.216E 00	0.060
1	12	-4.005E-01	3.427E-01	0.000
2	1	5.077E 00	5.339E 00	0.000
2	2	5.156E 00	5.442E 00	0.926
2	7	5.077E 00	5.339E 00	0.000
2	9	5.077E 00	5.339E 00	0.000
2	12	5.431E 00	5.732E 00	0.998

Table 3.5 A Partial Overlapped Response with Fuzzy Membership

of $f_{k^{\omega}i}$. The modified non-overlapped regions are also shown. Further, the value of k that yields the highest value of $f_{k^{\omega}i}$, k=1,2,3 is chosen as the input for ATPG.

3.6.5 Fault Isolation Procedure

1. We start with a given circuit description having 10 input ports, 10 output ports, and 72 components.

2. DC measurements at selected ports are used.

3. We generate upper and lower bound of the responses for each fault pattern using senitivity analysis of NAP2 Nonlinear Analysis Program.

4. Fuzzy membership for each interval is given as

$$\frac{\text{The length of ith nonoverlapping interval}}{\text{The length of total interval}}.$$

The implication is that if entire region is nonoverlapped then membership equals to one and if entire region is overlapped then membership equals to zero.

5. Whenever there is a simulated test sample, we assign the fuzzy membership values according to the step 4.

6. We use the fuzzy selection logic to diagnose the fault using Eq. 3.6.6.

7. If the fuzzy membership value of the selected fault is above certain value α $(0 < \alpha < 1)$, then use the selected measurements. If the fuzzy membership

value of the selected fault is below certain value α, we need additional access points to generate response measurements.

8. If fault isolation is acceptable, then use the set of fuzzy membership values for future fault isolation. Otherwise add the access points and increase the number of measurements.

3.6.6 An Illustrative Example

Figure 3.20 is a diagram of a communication I/O circuit board having 40 resistors, 10 diodes, 3 capacitors, 18 trnsistors, one operational amplifier, and 3 DC power sources. We assume 38 single fault cases including 33 catastrophic or "hard" failures and 5 "soft" fault cases in which failure component has twice its nominal value. The circuits can be subdivided into 3 line driver subcircuits, 4 buffer circuits, line receiver, and control circuits. In each subcircuit, one input point and one output access point are used. DC voltage inputs of 0 and 5 volts are used and the output voltages are measured at all access points. One output current is measured in each line driver subcircuit. The tolerances are 5% for the resistors and 1% for the forward current of the modeled transistors.

The worst case analysis of NAP2 (85,86) program provides us the approximate upper and lower bounds on the given simulated responses. These upper and lower bound are

```
/*************************************************************************************/
 TEST   50 ;
    STIMULI       64(  50) ;
       CONJUNCTION (  64) :
         ( < J1_X1 , J1_G > VOLT = ESUPPLY ( 5 VOLT) ;
    MEASUREMENT  65(  50) ;
       CONJUNCTION (  65) :
         ( < J1_Y1 , J1_G > VOLT = VOLTMETER (VJ1_Y1 VOLT) ;
          TARGET : VJ1_Y1 ;
       ASSERTION : VJ1_Y1 > 5.431E00 ;                    /* TEST LOWER LIMIT */
       ASSERTION : VJ1_Y1 < 5.732E00 ;                    /* TEST upper LIMIT */

       ASSERTION    63(  62) :       /* SAVE THE OUTCOME OF THE TEST MODULE  */
          IF VJ1_Y1 > 5.431E00 & VJ1_Y1 < 5.732E00
            THEN OUTCOME_50 = 0.998 /* OUTCOME OF THE TEST IS TRUE   */
            ELSE OUTCOME_50 = 0     /* OUTCOME OF THE TEST IS FALSE  */
          TARGET : OUTCOME_50 ;
/*************************************************************************************/
```

Fig. 3.21 A Partial Listing of the Input to the NOPAL changed by using Max (P_{max}, $2P_o - P_{min}$), Min (P_{min}, $2P_{max} - P_o$) when necessary. Starting with 38 fault cases, 27 fault cases are isolated using binary logic. Using fuzzy logic, we can distinguish four more failure cases for about 10% improvement.

Table 3.5 is a partial list of the overlapped response regions of ith fault at access point k and the membership functions computed from test samples with the method discussed in section 3.6.4. From fuzzy membership calculations with a preselected threshold 0.7, four more test regions with the highest fuzzy memberships are added into the test data. A partial listing of the input to the NOPAL is given in Fig. 3.6.4. This program is implemented in FORTRAN on the Moore School UNIVAC 90/70. Most of the cpu time was consumed to simulate the fault cases. Generation of the worst case analysis took about 600 sec. cpu time and a test sample simulation took about 1000 sec. cpu time.

3.6.7 Discussions

It has been shown that the application of the fuzzy concept together with the worst case analysis and test samples can reduce the number of simulations required for the design of fault isolation tests. Only single faults using DC signals were considered. The method presented herein should be refined in order to extend it for handling multiple faults of frequency or transient data.

3.7 Chapter Summary

The situation often encountered is that the system under consideration has very little information available, and the formulation of an optimal control or recognition policy needs accumulation of this information. Learning system is defined as a system which accumulates the information for certain improvements of the system. FA model we described is a good candidate for a learning system under fuzzy environments. Fuzzy environment refers to the unknown environment which tries to give the system having maximum vagueness or indeterminacy. As we discussesd in the earlier chapter, we can categorize the uncertainties into two different engineering discipline. One is randomness which can be handled by probability theory and the other is indeterminacy or vagueness which can be handled by fuzzy set theory. We are mainly concerned the case when indeterminacy and vaguenss are the major portion of the uncertainties. We established the fuzzy automaton model for learning systems. Properties of various fuzzy relations were explored.

Two analog fault isolation algorithms are studied. An active lowpass filter and a communication I/O circuit are used as examples.

CHAPTER 4 Fuzzy Distance Measure and Fuzzy Entropy Measure in Fault Isolation

4.1 Introduction

Mathematical modelling is typically based on the system which is developed by a set of axioms. An important observation is that the logical structure of a fuzzy set is essentially Boolean, except for the violation of the law of excluded middle and the violation of the law of contradiction where fuzziness remains (41). Care must be taken if one departs from an axiomatic system or tries to use several axiomatic systems which may be muatually inconsistent. The best hope one can have is that each axiomatic system approximates the other under specified conditions. Then we might be able to use these several axiomatic systems as an approximation to the real system.

Aspects of fuzzy set theory are close to but distinct from that of probability theory. Consequently the concept of a valuation differs an important ways from that of an ordinary measure. Valuations ot a fuzzy set may be restricted to subsets of reference space, as it is done in measure theory using the notions of Borel fields. In effect, valuations are defined on a reference space which form a convenient structure for use with the given operation. Such an approach to the theory of fuzzy subsets has recently been described in a highly significant work by Sugeno (51).

4.2 Fuzzy Distance Measure (FDM)

4.2.1 Definition

It has been pointed out recently (43) that the important terms in fuzzy set theory, "equivalence", "implication" can be expressed through the definition of metric terms. First the lattice L_j, $j = 1,\ldots,m$ is defined on each measurement space with "ℓ" fixed points which are given by the preset conditions. The binary operations on the fuzzy set are defined as the maximum and minimum of the two operands. A metric on the lattice giving a measure of the "distance" apart of two propositions under a valuation is defined as follows: (4.2.1)

$$\forall x_{1j},\ x_{2j} \in L_j,\quad d(x_{1j},\ x_{2j}) = \log_\ell(\text{ rank order}(x_{2j} | x_{1j})).$$

Thus the distance of the two measurements are defined by the logarithmic value of the rank order of x_{2j} from x_{1j}. Distance d satisfies a quasimetric on L_j such that

$$d(x_{1j}, x_{2j}) = 0 \tag{4.2.2}$$

$$0 \le d(x_{1j}, x_{2j}) \le 1 \tag{4.2.3}$$

$$d(x_{1j}, x_{2j}) + d(x_{2j}, x_{3j}) \ge d(x_{1j}, x_{3j}). \tag{4.2.4}$$

Proofs of eq. (4.2.2) and (4.2.3) are an immediate extension of the definition. Equation (4.2.4) can be proved as follows:

where n, m, k is the associated rank order.

$$n \le \ell,\ m \le \ell,\ k \le n+m \le \ell$$

$$d(x,y)+d(y,z)-d(x,z) = \log_\ell n + \log_\ell m - \log_\ell k \ge \log_\ell nm - \log_\ell (n+m)$$

$$= -(\log_\ell \frac{1}{n} + \log_\ell \frac{1}{m}) \ge 0 . \qquad \text{qed.}$$

It is also reasonable to define a measure of equivalence as

$$\forall\, y_j,\ x_{ij} \in L_j, \quad \mu(y_j \equiv x_{ij}) = 1 - d(y_j,\ x_{ij}) . \qquad (4.2.5)$$

4.2.2 FDM as a Fault Isolation Criterion

For our purposes this distance criterion assumes that inserted faults lie within a predetermined deviation of the "typical" single faults. With this in mind we turn to definition of a fuzzy distance for fault measurements.

We define a measure of the "distance" or apartness of two output measurements x_{1j} and x_{2j} as follows,

$$\forall\, x_{1j},\ x_{2j} \in L_j$$
$$d(x_{1j},\ x_{2j}) = \log_\ell(\text{rank order } (x_{2j} \mid x_{1j})) \qquad (4.2.6)$$

where "ℓ" is the number of fault classes.

Then the problem of fault isolation can be interpreted as the need to find the minimum distance between the measurement y_j of UUT Y and preset values x_{ij}'s, where the

x_{ij}'s are elements of a typical fault pattern vector X_i, and where subscript i stands for different fault types and subscript j stands for number of measurements to be made.

4.3 Fuzzy Entropy Measure

Kolmogorov argued that the basic information theory concept must and can be found without recourse to probability theory and in such a manner that "entropy" and "mutual information" concepts are applicable to individual values. Furthermore he pointed out that by using probability theory, we might need to resort to considerably rougher generalization. In their arguments, studies of Kolmogorov (54) and Löf (53) in randomness, probability, and information connected with the concept of calculation complexity provided for a new insight into the concept of information. Cerny and Brunovsky (57) have taken information as a basic concept, defining it axiomatically. Their definition, however, requires a special operator instead of using probability and independence as primitive concepts. Along with the development of fuzzy set theory, De Luca and Termini (55) as well as Okuda, Tanaka, and Asai (58) have studied information measures in connection with the fuzzy set theory. Although they have defined fuzzy entropy measures, it is hard to be convinced that any one of them has a sound basis.

4.3.1 Definition

The usual definition of entropy is based on probability concepts, and does not pertain to individual values, but rather to random values, i.e., to probability distributions within a given group of values. We find it advantageous to treat the total information in the system (input, network topology, output) as being comprised of two parts; namely that due to the randomness and vagueness. Thus in addition to the information due to the randomness described by probability theory we recognize the fuzzy information contained in the imprecision of the system described by fuzzy set theory. Consider a functional defined on the class of generalized characteristic functions (fuzzy sets). We denote this as "fuzzy entropy". Thus we obtain a global measure of the "uncertainty" related to the situations described by the fuzzy sets. This "fuzzy entropy" may be regarded as a measure of a quantity which is related to the randomness of the experiments and the impreciseness in the system.

Classical probability theory is based on properties such as $P(\Omega)=1$ (exhaustivity) and countable additivity. It would be useful to have a new measure and calculus which might eliminate the inherent need to be exhaustive, while restricting every sample point of the structure to a well-defined set (51).

Definition 4.1: Let B be a Borel field (σ-algebra) of subsets of the real line Ω. A set function $\chi(.)$ defined on B is called a fuzzy measure if it hass the following properties:

1) $\chi(\Phi) = 0$ (Φ is the empty set of Ω) (4.3.1)

2) $\chi(\Omega) = 1$ (4.3.2)

3) If α, $\beta \varepsilon$ B with $\alpha \subset \beta$, then $\chi(\alpha) \leq \chi(\beta)$. (4.3.3)

4) If $\{\alpha | 1 \leq j < \infty\}$ is a monotone sequence, then (4.3.4)

$$\lim_{j\to\infty} (\chi(\alpha_j)) = \chi(\lim_{j\to\infty}(\alpha_j)).$$

Definition 4.2: Let $\mu : \Omega \to [0,1]$ and $\chi : \{y| \ y \geq x\} \to [0,1]$. The fuzzy expected value (FEV) of μ over a set A, with respect to the measure $\chi(.)$ is defined as

$$FEV(\mu) = \text{s}\!\cdot\!\cdot \{ \min \{ \mu, \chi(\xi_x)\}\}, \qquad (4.3.5)$$

where $\xi x = \{y | y \geq x\} \subset A$.

Definition 4.3: The fuzzy entropy $H(\mu)$ of μ over a set A with respect to the measure $\chi(.)$ is defined as

$$H(\mu) = -FEV(\mu) \log_2 FEV(\mu). \qquad (4.3.6)$$

We may also use the logarithmic fuzzy entropy defined by DeLuca (55,56).

Definition 4.4: The fuzzy entropy $(H(\mu | x))$ of μ over a set A with respect to the measure $\chi(.)$ given x is defined as

$$H(\mu|x) = -\lambda\mu(x) \log_2 \mu(x) - (1- \lambda) \chi(\xi_x) \log_2 \chi(\xi x) \qquad (4.3.7)$$

where $\lambda = \dfrac{FEV(\mu) - \chi(\xi x)}{\mu(x) - \chi(\xi x)}$

Theorem 4.1) For any x in a set A, $H(\mu)$ is larger than or equal to $H(\mu \mid x)$.

Proof: Suppose supremum of $H(\mu)$ occurs at x^* in A, then $\mu(x^*) = \chi(\xi x^*)$.

$$H(\mu) = -\mu(x^*)\log_2\mu(x^*)$$

$$H(\mu) - H(\mu|x) = -\mu(x^*)\log_2\mu(x^*) + \lambda\mu(x)\log_2\mu(x) + (1-\lambda)\chi(\xi_x)\log_2\chi(\xi_x)$$

where $\lambda = \dfrac{\mu(x^*)-\chi(\xi x)}{\mu(x)-\chi(\xi x)}$, $\mu(x^*) = \lambda\mu(x) + (1-\lambda)\chi(\xi x)$.

$\mu(x^*)$ is in between $\mu(x)$ and $\chi(\xi x)$ and $-t\log_2 t$ is a concave function and $\mu(x^*)$ is a linear combination of $\mu(x)$ and $\chi(\xi x)$. Therefore, $H(\mu)-H(\mu \mid x)>0$. Equality holds when $\lambda = 0$ or 1.

Definition 4.5: The fuzzy mutual information $H(\mu, \mu|x)$ is defined by $H(\mu)-H(\mu \mid x)$.

Corollary 4.1: The fuzzy mutual information $H(\mu, \mu \mid x)$ is a positive number.

Proof: It is a direct consequence of Theorem 4.1.

4.3.2 Measurability of Faults in Analog Networks

The nature of symptoms in a faulty analog system is usually not as clearcut as is the case for a faulty digital system. Rather, the output responses are given in functional form and the set of deviations of the output responses are potentially the symptoms of the several component faults. We are restricting our interest only to such deviations as the possible fault symptoms.

The response deviation function $C_{ij}(.)$ of the ith faulty component and the jth port together with fuzzy membership function $f_{C_{ij}}(.)$ will form a fuzzy set representing the degree of fault response due to the jth port response deviation and the ith component. The set of response deviations can be represented by a semi-closed interval, whose lower and upper bound $(a_{ij}, b_{ij}]$ may be approximated by using a worst case analysis (69). It is assumed that the response deviation interval X_j due to the C_i can be predetermined and denoted by a fuzzy set X_{ij} whose characteristic function is as follows.

$$\mu_{ij}(x) = \begin{cases} 1, & x \varepsilon (a_{ij}, b_{ij}] \\ 0, & \text{otherwise.} \end{cases} \tag{4.3.8}$$

It is also assumed that the fuzzy membership function $f_{C_{ij}}(.)$ is continuous and that the values of $f_{C_{ij}}(.)$ are assigned so that if the reponse deviation increases then the corresponding fuzzy membership values are nondecreasing. Therefore the membership function $f_{X_{ij}}(.)$ of the response deviations in $X_{ij}(.)$ with respect to C_{ij} can be represented by

$$f_{X_{ij}}(x) = \mu_{ij}(x) \wedge f_{C_{ij}}(x). \tag{4.3.9}$$

We denote a fuzzy set $\{x, f_{X_{ij}} > 0\}$ as $\underset{\sim}{X}_{ij}$. Next we defined the fuzzy measure $\chi_{ij}(.)$ for the response deviation X_{ij} as a normed weighted length denoted by

$$\chi_{ij}(x) = \frac{\int_x^b f_{X_{ij}}(t)\, dt}{\int_a^b f_{X_{ij}}(t)\, dt} \qquad (4.3.10)$$

where $X_{ij} = \{t | \quad a < t < b\}$.

We can subjectively interpret $\chi_{ij}(x)$ as the degree of belief in the existence of a fault when the measurements are larger than x. When fuzzy membership function $f_{X_{ij}}(.)$ is constant, $\chi_{ij}(.)$ behaves similar to a uniform distribution function in probability theory.

Fuzzy expected value for the responses deviation X_{ij} can be expressed as follows.

$$FEV(f_{X_{ij}}(x)) = \sup\{ \min f_{X_{ij}}(x), \chi_{ij}(x) \} \qquad (4.3.11)$$

We can easily show that for any nondecreasing continuous fuzzy membership $f_{X_{ij}}(x)$ over a fuzzy set X_{ij}, there exists a unique fuzzy expected value of $f_{X_{ij}}(x)$ with respect to the fuzzy measure $\chi_{ij}(x)$.

In our case, FEV's are used as measures of central tendency for the response deviation sets X_{ij}'s.

4.3.3 Fault Properties, Fault Averages, and Fuzzy Expected Values

Even though the set of membership functions $f_{X_{ij}}(.)$'s for each ith component and jth response are obtained, the problem of formulating a fault isolation criterion still remains. The main reason is that the set of membership functions $f_{X_{ij}}(.)$'s have two properties; the occurence of the ith component fault and the subjective observation at the jth response. On the one hand, when we focus our attention on the occurence of a faulty component, we seek some averaging method to determine a typical fault. On the other hand, we may focus our attention on the subjective interpretation of the faults at the jth response to seek the one which exhibits maximum deviation properties. Therefore the weights of individual properties and collective behavior seem to have essential roles in determining the likely faults. The concepts of property set were introduced by A. D. Allen (88). It seems reasonable to use FEV's as a set of values considering both properties and collective tendencies. It's interpretation is such that the FEV as a typical value of corresponding fuzzy component membership function and the measurement whose fuzzy membership is equal to the FEV as the corresponding typical measurement. The criterion we are proposing for fault isolation is to select the minimum of the sum of the difference between the observed values and the FEV's at each point.

The $FEV_{ij}(x)$ for the ith faulty component at jth port are given as follows:

$$FEV_{ij}(x) = \begin{cases} FEV(f_{X_{ij}}(x)), & x \in X_{ij}, \\ c > 1 & , x \notin X_{ij}. \end{cases} \tag{4.3.12}$$

where c is a constant determined by the designer.

The proposed criterion can be written as

$$\min_i \sum_j |FEV_{ij}(x) - \chi_{ij}(x)|.$$

Application of this criterion mainly depends on the accuracy in determination of the fuzzy membership functions. Therefore it is vital to develop a strategy to upgrade the accuracy of the response deviation membership function.

4.3.4 An Algorithm of Fuzzy Measure to Analog Fault Isolation

Our objective now is to minimize errors in fault isolation as well as to construct a structure for an effective fault isolation scheme. We start by being aware of the possibly overlapped response regions. Recall our subjective interpretation of a response deviation fuzzy membership value for a particular response deviation. It indicates the degree of fault due to the component deviation while all other components are subject to drift within tolerances and under noisy mesurements. We assume only a single component fault occurs at each set of observations.

The proposed algorithm is a follows:

1. Set up the set of fuzzy membership functions $f_{X_{ij}}$ for each ith component and jth port.

2. According to the fuzzy membership functions $f_{X_{ij}}$, set of fuzzy measure χ_{ij}'s are calculated.

3. Fuzzy expected values (FEV's) are calculated using eq. 4.3.11.

4. Observed values are compared and ordered according to the criterion.

$$\min_{i} \sum_{j} |FEV_{ij}(x) - \chi_{ij}(x)|$$

The ith component having the minimum indicates the most likely fault.

5. Confirmation of correct isolation directs the update of $f_{X_{ij}}$'s. Suppose x_0 is the observed set of values.

$$f_{X_{ij}}(x) \leftarrow \alpha f_{X_{ij}}(x) + (1-\alpha)\phi(x), \quad 0 < \alpha < 1$$

$$\phi(x) = \begin{cases} \chi(x-x_0) & \begin{cases} 1 \text{ if } x \geq x_0 \\ 0 \text{ otherwise} \end{cases} \\ f_{X_{ij}}(x) & \end{cases}$$

where $x, x_0 \in X$.

When x_0 is correctly classified this information reinforces the correct decision for the next similar fault. When x_0 is incorrectly classified we do not use it to update

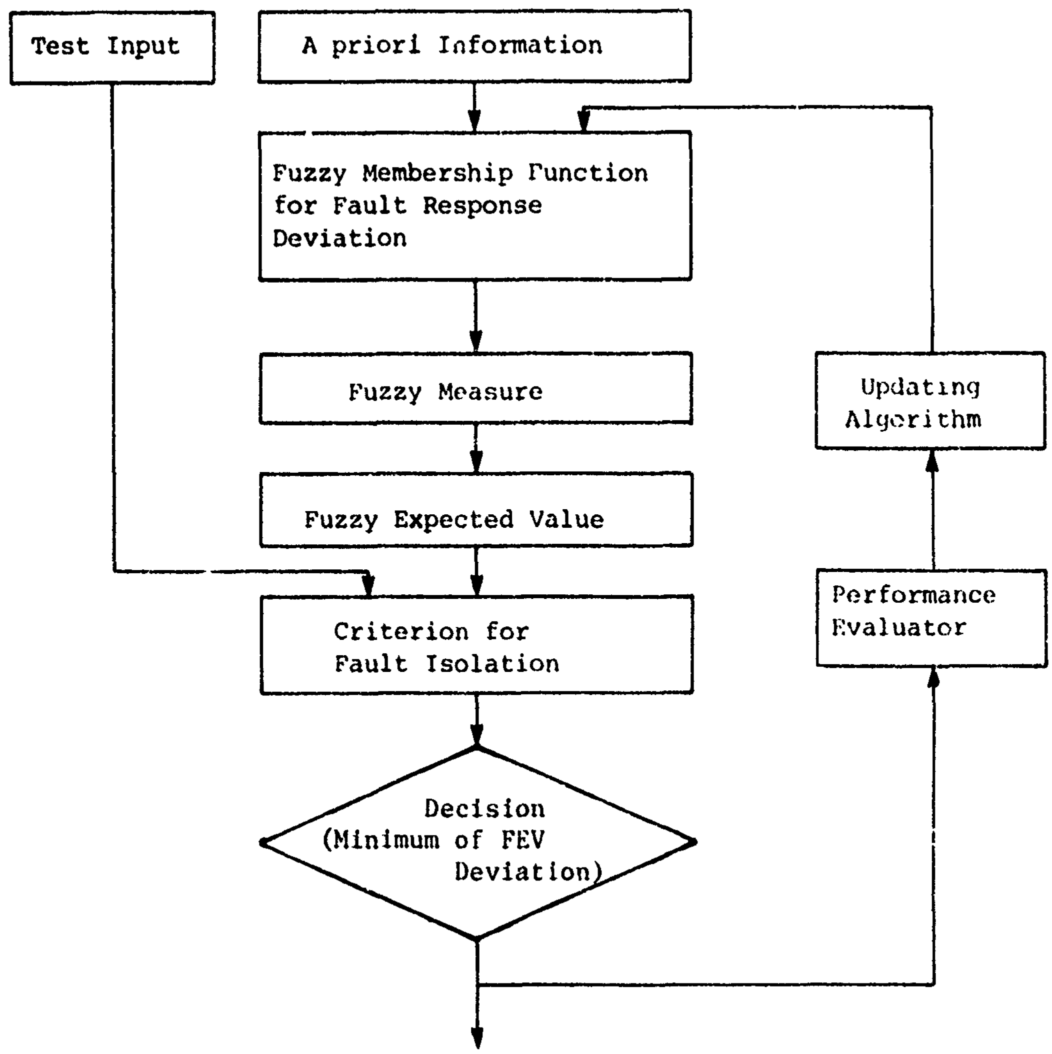


Fig. 4.1 Fault Isolation Using A Fuzzy Measure

the $f_{X_{ij}}$.

6. When we have new observations for a different unit under test go to step 2.

A macro flow diagram of this algorithm is shown in Fig. 4.1. In this way we may able to start fault isolation even though there exist overlapped responses and the probability distribution of the fault response deviation are not precisely specified.

4.4 A Fault Isolation Method in Nonlinear Analog Networks Using Fuzzy Distance Measure

As a possible means of representing fault patterns, we can use power measurements. We utilize a special form of Tellegen's theorem to get the necessary values of port currents and voltages for diagnostic purposes. Furthermore, we present an algorithm that makes use of the available measured data on port responses to isolate the faulty components using fuzzy distance measure detailed in section 4.2. An illustrative example using the NAP2 Network Analysis Program is included. The results are compared with other criteria.

4.4.1 Introduction

In analog electronic networks which are designed to perform certain analog functions, probability distributions for the value of each component are often available. It is, however, still difficult to calculate the port responses from such network component data. Moreover, the nominal port response which is the subset of all the possible port responses could be rather imprecise because it is only given by the actual designer's experiences or by calculated values assuming some specific preset normal conditions. Whenever actual measurements fall in the predetermined fault conditions, due to the port responses, impreciseness in the criteria of the predetermined fault conditions, and the inadequacy of available ports, the faulty nonlinear analog network can be interpreted as a fuzzy system.

Herein we present an algorithm to make use of all the available data, namely data on port responses to isolate the faulty components using fuzzy set concepts. Also, a possible way of selecting a set of features leading toward better fault isolation is briefly studied.

Using a network analysis computer program, we can get the port responses for both nominal and faulty conditions. A special form of Tellegen's theorem is applied to get the necessary values of port currents and voltages for diagnostic purposes.

4.4.2 Reference Feature Generation of Faulty Networks using Tellegen's Theorem

Throughout this section, we assume that only a single component fault occurs. This assumption is mainly for notational convenience. Recall that a "soft failure" entails a change of component value to such a degree that the network response is just outside the specification while all the non-faulty components are subject to drift within their tolerance ranges. It has been discussed (12) that such failures are more troublesome to isolate than "hard failures" such as an open or short circuit of the component. We restrict our discussion only to "soft" failures. If desired, a "hard" failure can be approximated by the extreme case of "soft" failure.

One of the most powerful tools available to solve the network is Tellegen's theorem. One general statement of Tellegen's theorem is stated as follows:

$$\sum_p \Lambda' i_p \Lambda'' v_p = \sum_\alpha \Lambda' i_\alpha \Lambda'' v_\alpha \tag{4.4.1}$$

where i_p, v_p are port currents and voltages; and i_α, v_α are branch currents and voltages. Λ' and Λ'' are any Kirchhoff operators. The above theorem can be applied to networks including non-linear elements as long as the network topology is not changed. We proceed to apportion the currents and voltages at the ports and branches respectively to be consistent with their nominal values and their faulty values as follows:

$$i_p(t) = i_p^{(0)}(t) + i_p^{(1)}(t) \tag{4.4.2}$$

$$v_p(t) = v_p^{(0)}(t) + v_p^{(1)}(t) \tag{4.4.3}$$

$$i_\alpha(t) = i_\alpha^{(0)}(t) + i_\alpha^{(1)}(t) \tag{4.4.4}$$

$$v_\alpha(t) = v_\alpha^{(0)}(t) + v_\alpha^{(1)}(t) \tag{4.4.5}$$

where $^{(0)}$ stands for nominal portion and $^{(1)}$ stands for faulty portion. Then we find that Tellegen's theorem is expressible as four separate expressions:

$$\sum_p i_p^{(0)} v_p^{(0)} = \sum_\alpha i_\alpha^{(0)} v_\alpha^{(0)} \qquad (4.4.6)$$

$$\sum_p i_p^{(0)} v_p^{(1)} = \sum_\alpha i_\alpha^{(0)} v_\alpha^{(1)} \qquad (4.4.7)$$

$$\sum_p i_p^{(1)} v_p^{(0)} = \sum_\alpha i_\alpha^{(1)} v_\alpha^{(0)} \qquad (4.4.8)$$

$$\sum_p i_p^{(1)} v_p^{(1)} = \sum_\alpha i_\alpha^{(1)} v_\alpha^{(1)} \qquad (4.4.9)$$

In contrast with the power equation (4.4.6), we can call eqs. (4.4.7), (4.4.8), (4.4.9) pseudo-power equations. Port responses under the nominal faulty conditions can be calculated easily using a network analysis program. We assume that the port currents and voltages of the nominal network which have n components and m ports are represented

as $i_{p1}^{(0)} \ldots i_{pm}^{(0)},\ v_{p1}^{(0)} \ldots v_{pm}^{(0)}$ (nominal case)

and ${}_k i_{p1} \ldots {}_k i_{pm},\ {}_k v_{p1} \ldots {}_k v_{pm}$ (k component faulty case).

The corresponding faulty network port currents and voltages are represented by

$$({}_k i_{p1}^{(1)}, \ldots, {}_k i_{pm}^{(1)}, {}_k v_{p1}^{(1)}, \ldots, {}_k v_{pm}^{(1)}) \qquad (4.4.10)$$

$$= ({}_k i_{p1} - i_{p1}^{(0)}, \ldots, {}_k i_{pm} - i_{pm}^{(0)}, {}_k i_{p1} - v_{p1}^{(0)}, \ldots, {}_k v_{pm} - v_{pm}^{(0)}).$$

We then have three possibly independent equations for each k:

$$\sum_p i_p^{(0)} {}_k v^{(1)} = \sum_\alpha i_\alpha^{(0)} {}_k v_\alpha^{(1)} \qquad (4.4.11)$$

$$\sum_{p} {}_{k}i_{p}^{(1)} v_{p}^{(0)} = \sum_{\alpha} {}_{k}i_{\alpha}^{(1)} v_{\alpha}^{(0)} \tag{4.4.12}$$

$$\sum_{p} {}_{k}i_{p}^{(1)} {}_{k}v_{p}^{(1)} = \sum_{\alpha} {}_{k}i_{\alpha}^{(1)} {}_{k}v_{\alpha}^{(1)} \tag{4.4.13}$$

Thus it is clear that we have the port measurements for the nominal and typical single fault cases. The problem, however, is how to isolate the faulty component from given measurements. We can select port voltages and currents independent of each other by forming the appropriate spanning tree of the network. From eq. (4.4.10), we can generate the 2^m possible equations by choosing ports. Then, we have

$$i_{p\ell}^{(0)} {}_{k}v_{p\ell}^{(1)} = - \sum_{\ell = \ell'} i_{p\ell'}^{(0)} {}_{k}v_{p\ell'}^{(1)} + \sum i_{\alpha}^{(0)} {}_{k}v_{\alpha}^{(1)} \tag{4.4.14}$$

where $\ell = 1...m$.

From eq. (4.4.14), we get m independent equations where the lefthand sides are known and the corresponding righthand sides are unknown. These equations show that the pseudo-power disturbance due to the one faulty component is revealed by the port responses. We will use the lefthand side quantity to isolate the faulty component. Likewise eq. (4.4.12) and (4.4.13) yield 2m more possibly independent equations. By suitable extension of this approach, multiple component fault cases can be handled.

4.4.3 Fuzzy Distance Measure in Fault Isolation

We assume that the system has n single fault cases. We further assume that every fault that occurs is near the typical single fault calculated by the present conditions. Herein we are only interested in isolating faulty units under test to the given fault cases.

Then the problem of fault isolation reduces to determining the maximum equivalence between measurement y_j of unit under test Y and preset values x_{ij}'s of typical fault X_i, where j stands for number of measurements made and i stands for different fault cases. Since independency of the measurement data are not given precisely, we use fault' equivalence $\mu(Y \equiv X_i)$ as a simple averaging of the port fault equivalence $\mu(y_i \equiv x_{ij})$.

4.4.4 A Fault Isolating Algorithm Using the Fuzzy Set Concept

A description of a proposed algorithm is as follows:

1. Calculate port voltages and currents under nominal and faulty conditions using a network analysis program.

2. Calculate pseudo-powers

$$i_p^{(0)}\ {}_kv_p^{(1)},\ {}_ki_p^{(1)},\ {}_ki_p^{(1)}\ v_p^{(0)},\ {}_ki_p^{(1)}\ {}_kv_p^{(1)}$$

for all k's and p's. It is assumed that the m(≤n) independent ports are available, and we have ℓ faulty cases and n components.

3. Form a pseudo-power matrix X' which has columns and m' (≤3m) rows as follows:

$$X = \begin{pmatrix} i_{p1}^{(0)} & {}_1v_{p1}^{(1)} & \cdots & i_{p1}^{(0)} & {}_\ell v_{p1}^{(1)} \\ \vdots & & & & \\ i_{pm}^{(0)} & {}_1v_{pm}^{(1)} & \cdots & i_{pm}^{(0)} & {}_\ell v_{pm}^{(1)} \\ i_{p1}^{(1)} & {}_1v_{p1}^{(0)} & \cdots & i_{p1}^{(1)} & {}_\ell v_{p1}^{(0)} \\ \vdots & & & & \\ i_{pm}^{(1)} & {}_1v_{pm}^{(0)} & \cdots & i_{pm}^{(1)} & {}_\ell v_{pm}^{(0)} \\ i_{p1}^{(1)} & {}_1v_{p1}^{(1)} & \cdots & i_{p1}^{(1)} & {}_\ell v_{p1}^{(1)} \\ \vdots & & & & \\ i_{pm}^{(1)} & {}_1v_{pm}^{(1)} & \cdots & i_{pm}^{(1)} & {}_\ell v_{pm}^{(1)} \end{pmatrix} = \begin{pmatrix} x_{11} \cdots x_{1\ell} \\ x_{ij} \\ x_{3m1} \cdots x_{3m\ell} \end{pmatrix}.$$

$$X' = \begin{pmatrix} x_{11} \cdots x_{1\ell} \\ x_{m'1} \cdots x_{m'\ell} \end{pmatrix}$$

X' is formed by reducing X. If jth row of X is all zeroes, then jth row is deleted.

4. A measure of separability S_j is formed to assess the diagnostic worth of jth set of measurements. When we order x_{ij} for every i given j(= 1,...,m'), we get new matrix Y, where its element y_{ij} is ordered. We set the measure S_j, as

$$S_j = \sum_{q=1}^{\ell-1} \frac{Y_{q+1,j} - Y_{q,j}}{Y_{\ell,j} - Y_{1,j}} \log \frac{Y_{q+1,j} - Y_{q,j}}{Y_{\ell,j} - Y_{1,j}}$$

If the measurements are evenly spaced, then

$$S_j = -n*\frac{1}{n}\log\frac{1}{n} = -\log\frac{1}{n} = \log n .$$

If the measurements are spaced unevenly, then $S_j < \log n$. Therefore S_j indicates the diagnostic worth of jth set of measurements.

5. Measure the output port voltages and currents of unit under test simultaneously. The voltages and currents representing faults are indicated by

$$i_{p'1} \ldots i_{p'm'} , v_{p'1} \ldots v_{p'm'} .$$

6. Calculate the rank order for each jth row.

a. If there are i' equal values at {i,...(i+i'-1)}th order then the rank would be

$$[i \times \ldots \times (i+i'-1)]^{1/i'}.$$

b. Remember the zero crossing rank order.

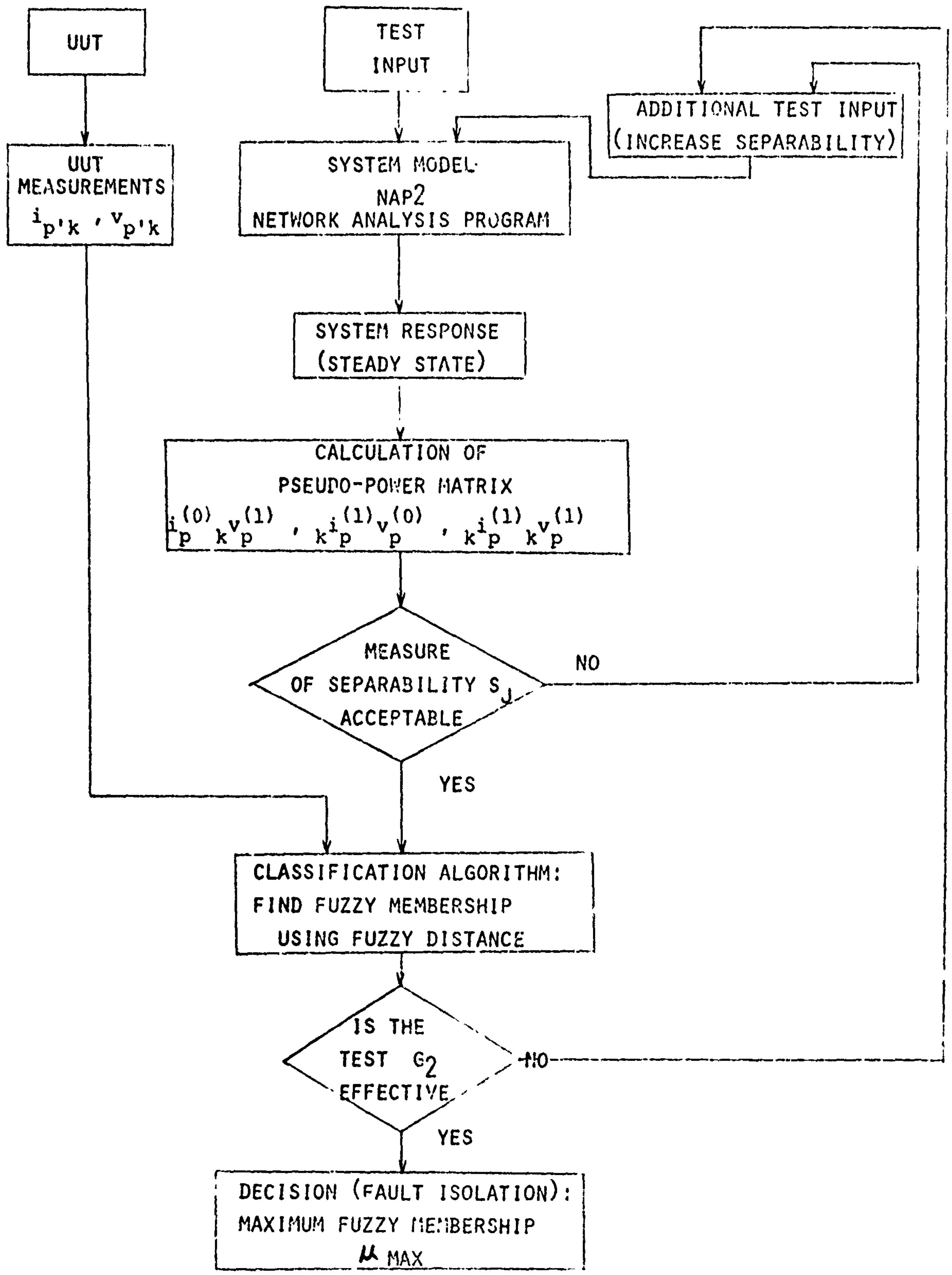
If zero crossing occurs between {i", i"+1}th order, then the zero crossing rank is $((i'' \times (i''+1))^{\frac{1}{2}}$.

7. An attribute to the rth fuzzy membership is given by

$$P_{rj} = 1 - \log_{\ell} \text{(rank order of rth component)}$$

where $0 \le P_{rj} \le 1$.

Fig 4.2 Simplified Diagram of Fault Isolation Method Using Fuzzy Distance



8. The fuzzy membership is given by the average of P_{rj}

$$\mu_r = \frac{1}{m'} \sum_{j=1}^{m'} P_{rj} .$$

9. Order the fuzzy membership function. The maximum of the fuzzy membership function indicates the most likely fault. The relative valus of fuzzy membership function will also have some meaning.

10. Validity of the Test: Noting that fuzzy membership $0 \leq \mu \leq 1$, we define test criteria as:

a. $\mu_{max} \rightarrow 1$ indicates high confidence in the decision.

b. Let $G1 = \frac{\mu_{max} - \mu_{nom}}{\log_\ell 2}$. A large G1 indicates an effective test.

c. Let $G2 = \frac{\mu_{max} - \mu_{sec\ max}}{\log_\ell 2}$. A large G2 also indicates an effective test.

11. If the resulting decision is not satisfactory, return to step 4 and select new port measurements with good local separability S_{qj} near troublesome components.

$$S_{qj} = -\sum_{r=1}^{2} \frac{Y_{q+2r-3,j} - Y_{q,j}}{Y_{\ell,j} - Y_{1,j}} \log \frac{Y_{q+2r-3,j} - Y_{q,j}}{Y_{\ell,j} - Y_{1,j}}$$

where q is the component, j is the new port measurement. Simplified flow diagram is shown in Fig. 4.2.

Table 4.1 List of Selected Faults

FAULTS INSERTED	FAULT DESCRIPTION
1. Open -12V source	R3 and R5 open
2. Open 5V source	Base of Q1 open
3. Open X_1	R_1 open
4. Open 12V source	R2, R6 and emitter of Q2 open
5. Q1 short emitter to collector	Replace 0.1 to emitter to collector
6. Open R2	R2←$10^9\Omega$
7. Short R2	R2←0.1Ω
8. R3 increase	R3←2 x R3
9 CR1 short	CR1←0.1Ω
10. R5 increase	R5←2 x R5
11. Q4 short emitter to collector	Replace 0.1Ω to emitter to collector
12. Q3 short base to emitter	Replace 0.1Ω to base to emitter
13. R6 open	R6←$10^9\Omega$
14. Short CR3	CR3←0.1Ω

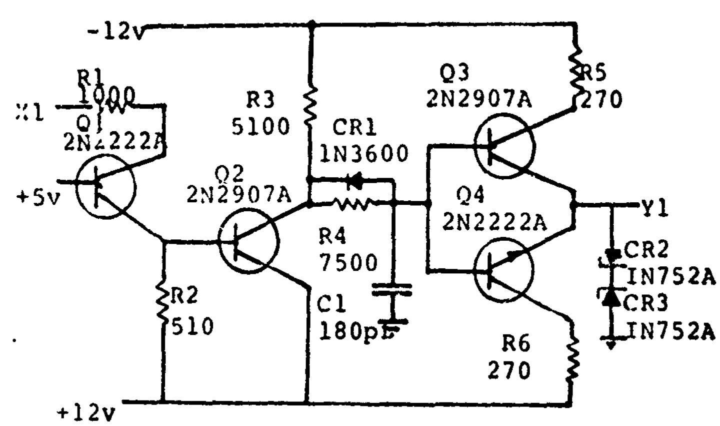


Fig. 4.3 Partial Diagram of the Communication I/O Circuit

FAULT TYPE	FUZZY DISTANCE	NEAREST NEIGHBOR RULE
1	3	3
2	2	3
3	3	0
4	3	0
5	3	3
6	2	1
7	0	0
8	3	2
9	3	3
10	3	2
11	3	3
12	3	3
13	2	3
14	3	2
TOTAL	36	28

Table 4.2 The Number of Correct Diagnoses from total of 42 Measurements (3 samples each)

FAULT TYPE	μ_{MAX}		$\mu_{SEC\ MAX}$		G_2
1	1	0.729	10	0.645	0.320
2	8	0.526	2	0.522	0.015
3	3	0.614	8	0.517	0.369
4	4	0.614	7	0.505	0.415
5	5	1.000	11	0.469	2.022
6	6	0.887	11	0.433	1.729
7	7	0.522	2	0.521	0.004
8	8	0.680	9	0.600	0.304
9	9	0.861	8	0.460	1.603
10	10	0.773	14	0.534	0.910
11	11	0.908	1	0.608	1.142
12	12	0.908	1	0.548	1.371
13	13	0.633	8	0.605	0.107
14	14	0.908	10	0.589	1.184

Table 4.3 Effectiveness of Test G_2

4.4.5 An Illustative Example

We use the Line Receiver Circuit of the communication I/O circuit card shown in Fig. 4.3 which has 4 transistors, 3 diodes, 6 resistors, 1 capacitor, 3 power sources, 1 input port, and 1 output port. For convenience we assume the 14 fault cases shown in table 4.1. The NAP2 Nonlinear Analysis Program is used to simulate the circuit and calculate the port measurements. We measure the output port voltages, currents and the input port currents with 0 and 5.5V at the input port. Out of 12 sets of pseudo-power measurements, 5 sets of pseudo-power measurements are retained. We have 3 samples for each fault type. Each fault type is recognized by a specific fault condition with all the other components subject to drift with normal distribution within 5 percent of their nominal values. Table 4.2 shows the number of correct diagnoses for each fault type based on a fuzzy distance criterion and the nearest neighbor rule. Using the algorithm with a fuzzy distance criterion, we find that 36 out of 42 samples are correctly classified. Under the same conditions, the nearest neighbor rule classifies only 28 samples correctly. Table 4.3 illustrates the effectiveness of test G2, defined in the previous section, for 14 samles. In this table, fault type 2 is classified incorrectly. This is not suprising since this fault yields a very low effectiveness value.

4.4.6 Discussion and Futher Remarks

A key point in this approach is that such fault isolation is in fact converted to a simplified form of pattern recognition. We feel that because of availability for very limited number of samples of fault measurements, the nearest neighbor rule might be inadequate as a decision criterion. At the same time, the voting technique is avoided, because it requires an optimum threshold level which is not easy to obtain. Although the number of samples is very limited, results appear to show rather easy and effective diagnosis possible based on a fuzzy distance measure.

4.5 An Application of Fuzzy Entropy Measure for Analog Fault Isolation

4.5.1 Introduction

The deviation from the normal responses in analog networks can be viewed as functions of the variability of the faulty components. However several ambiguities arise: due to nonlinearity, component drift, and noise. These along with changing ambiguity in different stages are interpreted in the context of a fuzzy system. A fuzzy measure is introduced to facilitate analog fault diagnosis under these circumstances.

For problems of automatic analog fault isolation, one can as already indicated adopt the viewpoint of pattern classification. In the pattern classification, many of the theoretical problems have been resolved by using statistical methods. However, in practical analog fault dignosis, almost all of the available statistical methods encounter rather unrelistic assumptions such as the availability of very large sample sizes and known probability distribution of the systems. On the one hand for the specific case of analog fault isolation, we usually have avaialable a large amount of information in the form of a circuit description. On the other hand we often have only limited sample sizes for characterizing the type of faults. The probability density is unknown or at best only partially known. Also under the framework of existing statistical distance measures, it is very difficult to take the contextual information of the fault pattern into consideration. Furthermore the increased computational efforts required for effectiveness of certain existing statistical analog fault isolation methods must be traded off against the classification error reduction obtained. Therefore we must balance the need for computational simplicity and the level of exactness. It seems both appealing and useful to adopt the fuzzy set idea. In this way we enhance fault isolation and maintain the level of ambiguities in the fault isolation procedure while achieving the necessary output requirements. Because of the ambiguities in the procedure not only the most likely fault, but also the ordering of the possible faults retains some significance.

With this in mind, we formulate the fault isolation problem utilizing the fuzzy set concept which will enhance the ATPG effort. A fuzzy entropy measure was developed in section 4.3 utilizing vague measurements for isolating faulty components in analog systems. An applicable fault isolation algorithm is given next based on the fuzzy entropy measure.

4.5.2 A model of Fault Membership Function of the Response

When there is a fault in an analog electronic network, it usually means the observed response is out of the tolerance limit for that particular nominal response. Therefore, the fault is a function of the deviation from its nominal response. It is often true that the deviation from its nominal response tends to increase when the component deviation from its nominal value increases. We shall distinguish three cases of faults. One can logically assume that when the response deviation departs from its tolerance limit the network is considered faulty. Initially, fuzzy membership 0 is arbitrarily assigned. By the decision maker's choice, when the deviation reaches a certain point β the network is definitely faulty. Fuzzy membership 1 is assigned at this particular point. Beyond this point, no matter what the observations on the network, the assignment of the fuzzy membership values for the deviation remains 1. There is a gradual transition region between the fuzzy membership assignment of 0 and 1. For simplicity, in this region we assume that the degree of fault increases linearly due to the increment of the deviations. Fig. 4.4 shows the simple model of the network fault membership function R_j at jth response deviation as a fuzzy membership function of response deviations described above.

Now let us turn our attention to the network fault due to a specific faulty component. We are restricting our

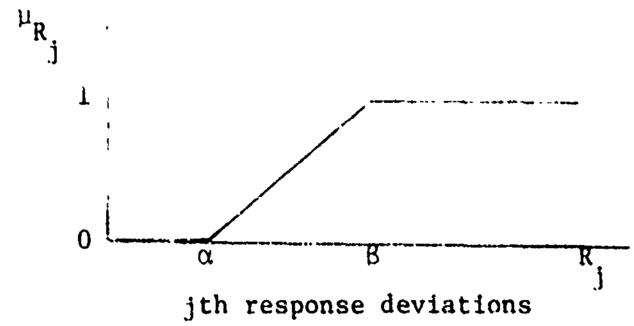


Fig. 4.4   A Network fault membership function

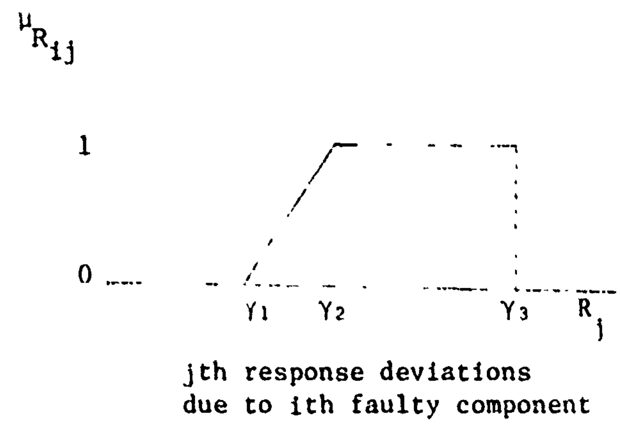


Fig. 4.5   A Component fault membership function

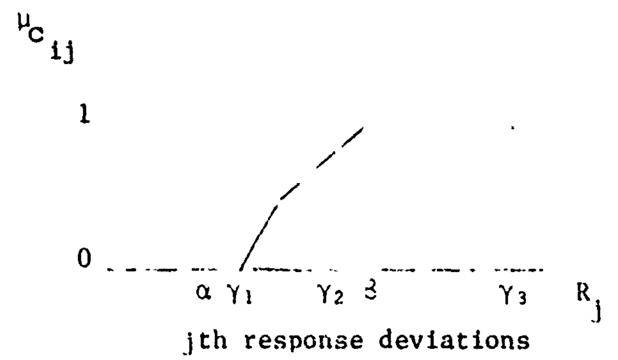


Fig. 4.6   The network fault membership function due to ith component fault

intrest to isolating the single fault case only. By a similar reasoning as network fault, for each component deviation, we consider three response deviation points γ_1, γ_2, and γ_3 as the response deviations when the component is on the verge of its tolerance limit, when the component is its usual fault state chosen by the decision maker, and when the component is at its extreme value such as an open or short. Fig. 4.5 shows this model of the component fault membership function $\mu_{R_{ij}}$ as a function of ith response deviation due to a specific fault component.

A network fault membership function due to the jth faulty component $\mu_{c_{ij}}$ is defined to be the minimum of the network fault membership function and ith component fault membership function. Fig. 4.6 shows the network fault membership function due to the ith faulty component.

The initial measure $\mu(\xi_{x_{ij}})$ is calculated as

$$\mu(\xi_{x_{ij}}) = \frac{\int_x^\infty \mu_{c_{ij}}(t)\, dt}{\int_0^\infty \mu_{c_{ij}}(t)\, dt} . \qquad (4.5.1)$$

The FEV of c_{ij} is calculated as

$$\mathrm{FEV}(c_{ij}) = \sup_{x \in R_j} (\min(\mu_{c_{ij}}(x), \mu(\xi_{x_{ij}})). \qquad (4.5.2)$$

We may also update the initial fuzzy measure $\mu(\xi_{c_{ij}})$ by adding new information from "correct data". A set of "correct data" is obtained by the success of actual faulty component replacement as determined by the criterion discussed in the previous chapter. One method using linear reinforcement scheme is as follows.

$$\mu(\xi_{c_{ij}}) = \frac{\int_x^{\infty}\mu_{c_{ij}}(t) + \alpha\delta(t-t^*)\, dt}{\int_0^{\infty}\mu_{c_{ij}}(t)\, dt + \alpha} \qquad (4.5.3)$$

where $\delta(t)$ is a unit impulse function, $0 \le \alpha \le 1$, and t^* denotes a correct observation.

4.5.3 A fault Isolation Algorithm using Fuzzy Entropy Criterion

It is again assumed that we have n single fault cases ($i = 1,\ldots,n$) and m different response setups ($j = 1,\ldots,m$) available. From the fuzzy membership function $\mu(c_{ij}(x))$ and the fuzzy measure $\mu(\xi_{x_{ij}})$ described in the previous section, we can calculate FEV(c_{ij}) for such jth response and ith component fault. When there is a set of observations $(x_1,\ldots,x_n)$ for the response deviations at the response setup $(R_1,\ldots,R_m)$ from UUT, the observations are adjusted by an amount of the difference between the response deviation at the FEV and the response deviation of γ_2. For new adjusted observations $(x_1,\ldots,x_n)$, $\mu(c_{ij}(x))$ and $\mu(\xi_{x_{ij}})$ for each jth response and ith component is calculated.

Our objective is to determine i while minimizing the

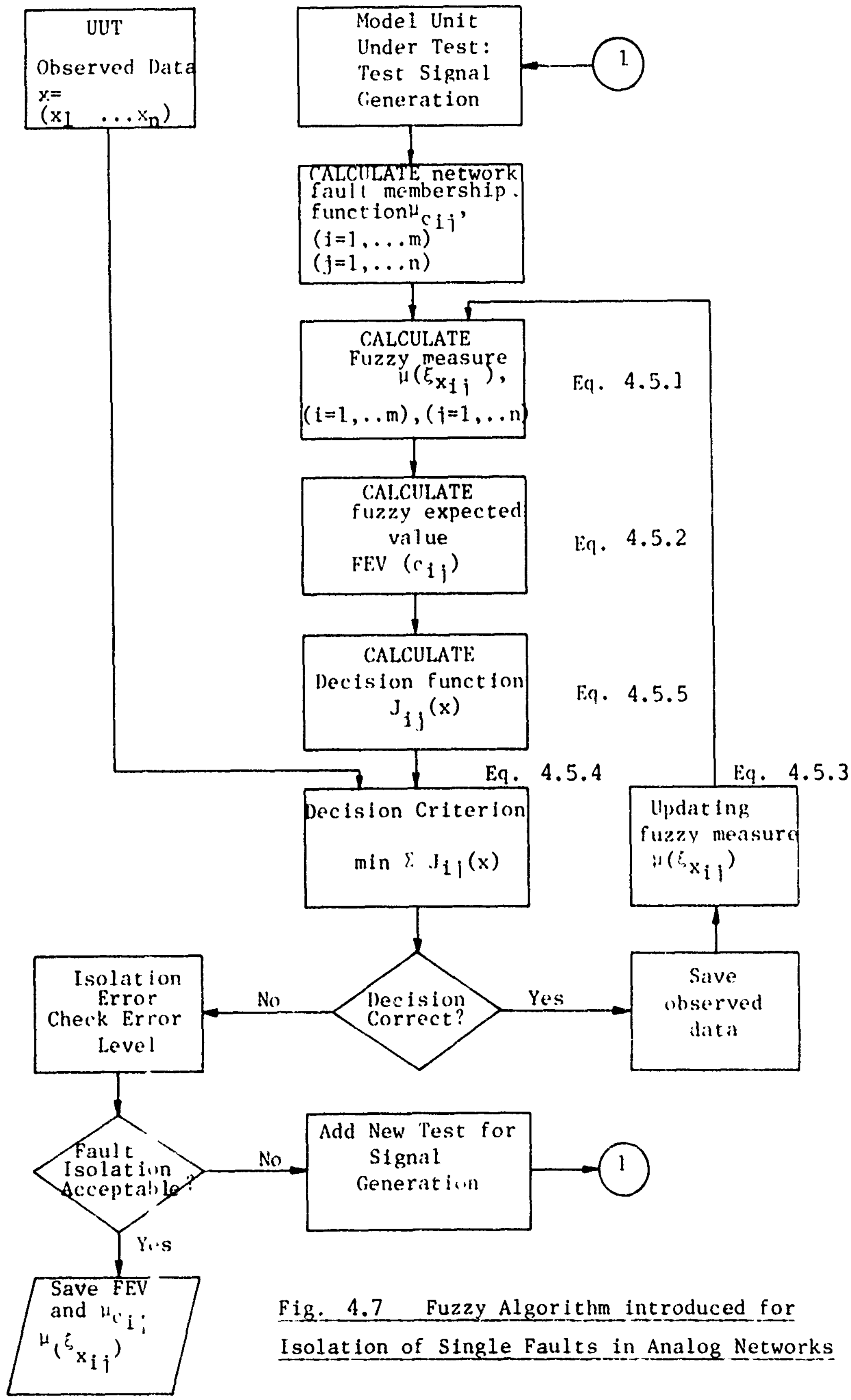


Fig. 4.7 Fuzzy Algorithm introduced for Isolation of Single Faults in Analog Networks

possible number of incorrect decisions. As a possible criterion, we are proposing that we find i such that

$$\min_{i} \sum_{j} J_{ij}(x) \tag{4.5.4}$$

where

$$J_{ij}(x) = H(FEV(c_{ij}))-\lambda H(\mu(c_{ij}(x))-(1-\lambda)H(\mu(\xi_{x_{ij}})), \tag{4.5.5}$$

$$H(t) = -t \log_2(t) \text{ and } \lambda = \frac{FEV(c_{ij})-\mu(\xi_{x_{ij}})}{\mu(c_{ij}(x))-\mu(\xi_{x_{ij}})}$$

After gathering some correct data, we can update the fuzzy measure $\mu(\xi_x)$. A flow diagram of the described algorithms is given in Fig. 4.7.

4.5.4 An Illustrative Example

To facilitate comparisons we use the same circuit and the same 14 faults as that given in section 4.4. We measure the output voltages, currents and the input port currents with 0- and 5.5-V excitation at the input port. We obtained three samples for each fault type using the NAP2 Nonlinear Analysis Program. Each fault type is recognized by a specific fault condition with all other components in this circuit subject to drift within 5% normal distribution. For simplicity, we assume that the network starts being assigned faulty membership at 5% response deviations and the network is definitely classified as faulty after 10% response

Criterion / Fault Type	"Fuzzy Distance"	"Fuzzy Entropy"	"Nearest Neighbor Rule"
1	3	3	3
2	2	0	3
3	3	3	0
4	3	3	0
5	3	2	3
6	2	1	1
7	0	3	0
8	3	0	2
9	3	3	3
10	3	3	2
11	3	3	3
12	3	3	3
13	2	2	3
14	3	3	2
Total	36	32	28

Table 4.4 Comparisons of Different Criteria

deviations. As an initial guess, we also assume that $\gamma_1, \gamma_2, \gamma_3$ as the 80, 100, and 110 percent of the typical response deviations of the given faulty component. Table 4.4 shows the number of correct diagnoses obtained for each fault type based on the fuzzy entropy criterion with the results obtained in section 4.4. We find that 32 out of 42 samples are correctly classified using the fuzzy entropy criterion while the nearest neighbor rule classified only 28 samples correctly.

4.5.5 Discussion and further remarks

A fault isolation method for analog circuits is discussed using fuzzy set concepts. Even though the information derived from the circuit under test is usaually inadequate to apply statistical methods, application of our fuzzy measure can provide adequate fault isolation capability. The reason seems to be that the proposed algorithm includes an experienced designer's notion of "fault" which tends to enhance decision making. Also the upper and lower bounds of the response deviations contribute to improve the decision through the proposed simulation model.

4.6 Chapter Summary

An evaluation of analog fault isolation techniques is attempted through a fuzzy distance measure and a fuzzy entropy measure. A metric on the lattice giving a measure of the "distance" apart of two propositions under a valuation is defined as the logarithmic value of the rank order of the two propositions. And its quasimetric properties are discussed.

The "fuzzy entropy" is introduced as a measure of a quantity which is related to the randomness of the experiments and the impreciseness in the system. The fuzzy entropy is defined as the information contained in the fuzzy expected value.

Fault properties in connection with the fuzzy measure are discussed. These fuzzy measures are the bases for fault isolation algorithms. The results of simulation study based on these decision criteria yield improved fault isolation.

CHAPTER 5 CONCLUSION AND SUGGESTIONS FOR FUTURE RESEARCH

5.1 Summary and Conclusion

To attack the long standing fault isolation problem in analog electronic circuits, we have focused on two of the major problems. One is the presence of uncertainties such as indeterminacy, vagueness, randomness, and so on that naturally arise during the solution procedure of analog fault isolation. The other is the presence of topological restrictions inherent in specific circuit configurations.

Our main attention was focused on dealing with the fault isolation problem involving various kinds of uncertainties such as indeterminacy or vagueness. We show that such problems lend themselves very well to and in fact can be solved by adopting fuzzy set concepts. In particular this line of research has produced a modified fuzzy set technique applicable to automatic fault isolation. Topological aspects utilizing graph theory may be used effectively to assist in preanalysis of faulty analog electronic circuits. As a spin off of a consideration of these problems, we developed some new theorems for element value solvability. It should be made clear however that effective fault isolation can be accomplished with or without this preanalysis to assist in resolving the more fundamental problem incurred by uncertainty.

As a consequence, this research yields the following specific results:

1. A base line automatic isolation system which can be used to deal with various kinds of uncertainties. A fuzzy automaton model served as a point of departure for the base line system. Various fuzzy relations are used to select and update the parameters and structures of the system.
2. Set of algorithms and new decision criteria which can be implemented easily and used for effective fault isolation. A fuzzy distance measure and a fuzzy entropy measure are used for decision making in the fault isolation algorithms. The results are shown to be generally more effective than existing techniques.
3. Ample illustrative examples and simulation studies are included to back up these new methods. Several examples such as low pass filter, band pass filter, and communication I/O circuits are used to illustrate the simulation studies. The results of simulation studies demonstrate the applicability of a fuzzy set technique.

5.2 Suggestions for Future Studies

The following topics are suggested for future investigation:

1. Future investigation on the properties of fuzzy sets is needed to adapt and facilitate its applicability to multiple fault cases.
2. Further investigation on refining the proposed single fault techniques to achieve potential improvements in the order of 50%.
3. Based on extension of the analog fault diagnosis algorithm, fault diagnosis of hybrid electronic systems needs consideration and further intensive efforts.

Appendix A

Graph Theoretic Aspects of Analog Fault Isolation

The state of art in fault analysis of analog networks using graph theory is reviewed briefly and some possible extensions are explored. In particular, the topological interpretation including network solvability and the key subgraph concept are reviewed and extended. A new algorithm to determine solvability based on network topology is given. This graph theoretical approach is useful to determine the suffiency of the available access points. Therefore this graph theoretical approach can be used as a preanalysis for the application of fuzzy set technique.

A.1 Introduction

In the body of the dissertation, we have been applying the fuzzy set concepts to alleviate some of the difficulties of handling the fault isolation of analog networks in the single fault case. Graph theory is used to investigate a specific network structures. Furthermore enumeration of the possible number of measurements on linear networks with N-accessible terminals, we gain insight into the behavior of faulty analog networks.

Conventional graph theoretical aspects previously applied for analog fault dignosis will be briefly discussed. Especially the network solvability and key subgraph concepts will be reviewed and extended. The connection of graph theory with fuzzy sets for application to analog fault diagnosis is also indicated.

A.2 Existing Graph Theoretic Aspects of Analog Fault Isolation

A graph representation for network dates back to Kirchhoff (1847). Hence the applications of graph theory to the analysis and design of electrical network is not new. But applications of graph theory to network analysis did not prove to be advantageous until the advent of the high speed digital computer. More recently applications of graph theory for the fault analysis of combinatorial networks or digital networks have been made quite successful (81). Yet, applications of graph theory to analog fault analysis have been relatively infrequent. Berkowitz (74) developed the concept of element value solvability for passive linear lumped network. Bedrosian introduced the key subgraph concept to provide some insight into solutions of active as well as passive lumped networks. Identification of the key subgraph leads directly to a set of equations which in the case of a single element kind network is an homogeneous multilinear algebraic form. Very recently Navid and Willson (14) presented some sufficient conditions for the element value solvability for linear elements. Here we present a method of determining necessary conditions to the network solvability which is tighter than that given by Berkowitz.

A.3 Element Value Solvability

A.3.1 New Theorems

We start with a few definitions:

Definition A.1) Available nodes (A) : External network nodes at which voltages and currents can be applied and/or measured. In other words, available node can be opened or shorted.

Definition A.2) Partly available nodes (P) : External network nodes at which voltages can be applied and/or measured but currents can not be applied or measured. In other words, a partly available node can be shorted but can not be opened.

Definition A.3) Nonavailable nodes (I) : Nodes internal to network at which neither voltages nor currents can be measured.

Definition A.4) Key subgraph (K) : The subgraph of network N which consists of the subset Bk of all branches incident on all the nonavailable nodes.

Definition A.5) Core graph (C) : The subgraph of network N which consists of the subset Bc of all (concealed) branches incident only on all the nonavailable nodes.

Theorem A.1) For any star graph whose core node is only one nonavailable node and at least two of the outside nodes are A , then the network is solvable.

Proof. Consider Fig. A.1 as a model of step 4.

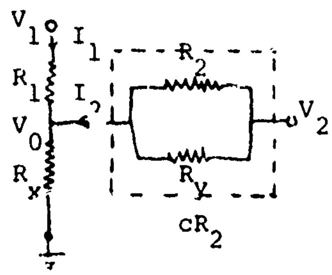


Fig. A.1 A Model for the Step 4

We set two available nodes as A_1 and A_2. We also set the rest of the nodes connected to nonavailable node I as P_1, P_2, ..., P_r. Node voltages and currents to the I node are represented as V_{A1}, V_{A2}, V_{P1}, V_{P2}, ...,V_{Pr} and I_{A1}, I_{A2}, I_{P1}, I_{P2}, ...,I_{Pr}. The node voltage of I is represented as V_o. The unknown admittance of each branch is represented as Y_{A1}, Y_{A2}, Y_{P1},Y_{P2}, ...,Y_{Pr}.

Step 1) Set $V_{A1} = V_1$, $V_{A2} = V_{P1} = V_{P2} = \ldots = V_{Pr} = 0$.

Set $Y = Y_{P1} + Y_{P2} + \ldots + Y_{Pr}$.

Measure $I_{A1} = I_1$, then

$$\frac{1}{Y_{A1}} + \frac{1}{Y_{A2} + Y} = \frac{V_{A1}}{I_{A1}} = \frac{V_1}{I_1} = \alpha \qquad (A.1)$$

Step 2) Set $V_{A1} = V_2$, V_{A2}: open, $I_{A2} = 0$,

$V_{P1} = V_{P2} = \ldots = V_{Pr} = 0.$

Measure $I_{Ak} = I_2$, then

$$\frac{1}{Y_{Ak}} = \frac{1}{Y} = \frac{V_{A1}}{I_{A1}} = \frac{V_2}{I_2} = \beta \tag{A.2}$$

Step 3) Set V_{A1}: open, $V_{A2} = V_3$, $I_{A1} = 0$,

$V_{P1} = V_{P2} = \ldots = V_{Pr} = 0.$

Measure $I_{A2} = I_3$, then

$$\frac{1}{Y_{A2}} + \frac{1}{Y} = \frac{V_{A2}}{I_{A2}} = \frac{V_3}{I_3} = \gamma \tag{A.3}$$

From Eq's (A.1), (A.2), and (A.3), Y_{A1}, Y_{A2}, Y is solvable.

Step 4) As is in Fig. A.1 choose $V_{Pk} = 0$, anb $V_{A2} = V_{P1} = \ldots = V_{P(k-1)} = V_{P(k+1)} = \ldots V_{Pr} = V_2$, $V_{A1} = V_1$.

$$\frac{1}{R} = \frac{1}{R_x} + \frac{1}{R_y} \tag{A.4}$$

where R is the total impedance of branches, R_x is the impedance of P_k branch, and R_y is the total impedance excluding P_x branch. From Eq. (A.4) and Fig. A.1

$$\frac{1}{R_2} + \frac{1}{R} - \frac{1}{R_X} = \frac{1}{cR_2} \tag{A.5}$$

where c is a constant.

$$V_1 - R_1 I_1 = (I_1 + I_2)\, R_x \tag{A.6}$$

$$V_1 - R_1 I_1 = V_2 - cR_2 I_2 \tag{A.7}$$

From Eq. (A.6),

$$R_x = \frac{V_1 - R_1 I_1}{I_k + I_2} \tag{A.8}$$

$$V_1 - R_1 I_1 = V_2 - \frac{1}{\frac{1}{R_2} + \frac{1}{R} - \frac{1}{R_x}}\, I_2 \tag{A.9}$$

From Eq. (A.9)

$$I_2 = (V_2 - V_1 + R_1 I_1)\left(\frac{1}{R_2} + \frac{1}{R} - \frac{1}{R_x} \right) \tag{A.10}$$

Substitute Eq. (A.10) for Eq. (A.8), then

$$R_x = \frac{V_1 - R_1 I_1}{I_1 + (V_2 - V_1 + R_1 I_1)\dfrac{R_x(R + R_2) - RR_2}{RR_2 R_x}} \tag{A.11}$$

Solving Eq. (A.11), we get

$$R_x = \frac{RR_2 V_2}{I_1 R R_2 + (V_2 - V_1 + R_1 I_1)(R + R_2)} .$$

Likewise, we can determine all the $Y_{p1}, \ldots, Y_{pr}$.

qed.

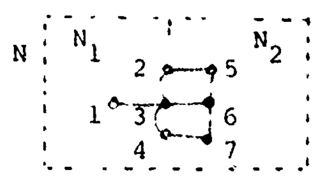
Consider a given network N partitioned into two subnets N_1 and N_2 along with some common nodes N_3. We assume all the branches among N_3 are included in the subnet N_1. We denote N_2' as N_2 with replacing N_3 to all available nodes.

<u>Theorem A.2</u>) If N_1 and N_2' are solvable, then N is also solvable.

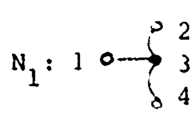
Proof. Since N_1 is solvable, we can get the element values of subnet N_1. Therefore we can get te voltages and currents of N_3 always. That is we can consider N_3 as available nodes. Now we can divide the network N into two subnets along with the available nodes. We know N_1 and N_2' are solvable. Therefore the network N is solvable.

qed.

Example A.1

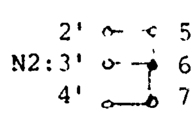


$N_3 = \{2, 3, 4\}$



: solvable

N is solvable.



: solvable

Consider the network N which meets the following conditions.

1) All the nodes in the network N are tied to one nonavailable node.

2) The network N has at least three separate subnets when we eliminate all the branches to the nonavailable node.

3) Each newly generated subnet is solvable.

4) The network N has at least one isolated available node when we eliminate all the branches to the nonavailable node.

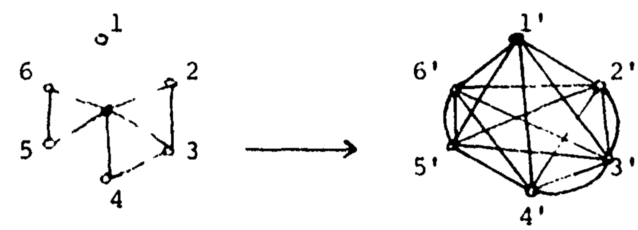
Theorem A.3) The above network N is element value solvable.

Proof. Suppose the network N is divided into subnets N_1, and N_2, and N_3. N consists of only one available node. Number of nodes for N_2 and N_3 are n_2 and n_3. That means we have $n = 1+n_2+n_3$ available or partly available nodes. When we apply star-mesh transformation, the network will be a complete graph K_n. We can measure all the branch voltages and currents. And also the transformed portion from star network of the mesh network should satisfy Shen's (79) condition. We can generate all the element values only using the measurements between the N_1 and N_2, N_1 and N_3, and N_2 and N_3.

Suppose we want to calculate for any element values between n_{3i} and n_{3j} in N_3. We measure the element value of the branch $n_1 n_{3j}$, $n_{2k} n_{3j}$, and $n_1 n_{2k}$. n_{2k} is an available node of N_2. Therefore we can get all the values in K_n. The difference between theoretical measurement and actual measurement represented as an admittance value is the element value before star-mesh transformation. Therefore we can solve the element values of the original network by mesh-star transformation.

qed.

Example A.2



$$Y_{23} = Y_{2'3'} - \frac{Y_{1'6'}}{Y_{1'3'}Y_{2'6'}}$$

$$Y_{34} = Y_{3'4'} - \frac{Y_{1'6'}}{Y_{1'4'}Y_{3'6'}}$$

$$Y_{56} = Y_{5'6'} - \frac{Y_{1'2'}}{Y_{1'5'}Y_{2'6'}}$$

Theorem A.4) If a key subgraph is tree graph and the core nodes of tree graph satisfy following conditions then the key subgraph is also solvable.

1) Each core node which incidents only one other core node is connected with at least 1 A and 1 A or P.

2) Core nodes which incident with more than two other core

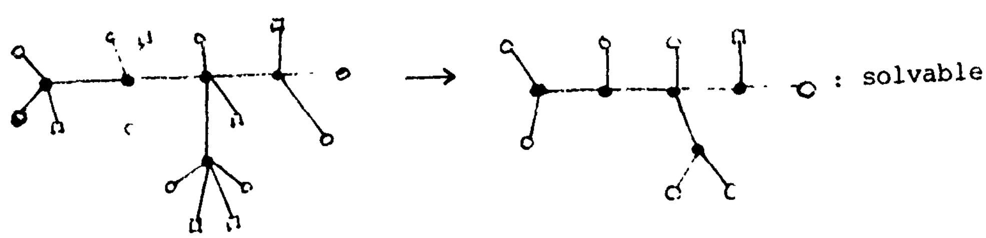
nodes are connected with at least 1 A.

Proof. It sufficies to show that the condition given in this theorem agrees with the condition given in the Theorem A.3. We can eliminate the core node by star-mesh transformation until only one nonavailable node is left. And the conditions given in Theorem A.4 guarantees the unique solvability.

qed.

Theorem A.5) If for any nonavailable node I, at least two nodes are connected to free A and all others are connected to A or P and only one other nonavailable node is connected, then you can reduced the network for network-solvable-purpose.

Example A.3



Original graph

Reduced graph

If the reduced network is solvable, then the original network is also solvable.

Col A.1) A network N is solvable if following conditions are all satisfied.

1. Its key subgraph is solvable.

2. The network remaining after all branches adjacent to the core graph C of key subgraph K are deleted is solvable.

3. The subgraph, which is comprised of key subgraph K enlarged by including all the branches both ends of which are on the key subgraph K is also solvable.

We note in passing that our Theorem A.3 appears to be related to the theorem given recently by Mayeda (82).

A.3.2 Algorithm for Checking Network Solvability of the Key Subgraph K

1. Find the longest length of core graph (tree).

2. Disrupt connectivity (To leave the most favorable branch set).

3. The reduced graph K of K generated by following rule is solvable.

a) Every node of branch connectivity one in the core graph can be changed to the available terminal A if the node has at least two branch connectivity with available nodes.

b) Every node of branch connectivity one in the core graph can be changed to the partly available nodes if the node has at least two partly available nodes.

4. If the reduced graph K of K is solvable graph then the key subgraph K is solvable, otherwise go to 5.

5. If we can reduce further go to step 3 and repeat, otherwise stop and K is not solvable.

A.3.3 Two Element Kind Network

Theorem A.6) For a two element kind network if the following reduced subgraphs are solvable then the network itself is solvable.

1. Exclude all the Y elements, then we have the graph Gxo composed of X element only. Find the subgraph of Gxo which is solvable.

2. Short circuit all the Y elements, we have the graph Gxs composed of X element only. New subgraph will have m nodes less than the original graph. Find the subgraph of Gxs which is solvable.

3. If the union of the branches of the solvable subgraph in the step 1 and 2 is X, then the elements of X are solvable.

4. Exclude all the X elements, then we have the graph Gyo composed of Y elements only. Find the subgraph of Gyo which is solvable.

5. Short circuit all the X elements, we have the graph Gys composed of Y elements only. New subgraph will have n nodes less than the original graph. Find the subgraph of Gys which is solvable.

6. If the union of the branches of the solvable subgraph in the step 4 and 5 is Y, then the elements of Y are

solvable.

7. X and Y elements are all solvable, then the network is solvable.

A.4 Summary

The main results of this chapter are :

1. Introduction of the core graph as an important key to the solvability of the networks.

2. A refined method of determining the element value solvability of the system.

   ---- Necessary and sufficient conditions for single and two element kind network solvability.

3. Algorithm of reducing the system for solvability purpose.

   ---- A systematic procedure has been developed for determining the network solvability given three kinds of terminals.

4. Illusrative examples including previous results of Bedrosian (78).

Appendix B

NAP2 Nonlinear Analysis Program (85,86)

B.1. Introduction

For convenience we summarize the main features of the computer program utilized to simulate the electronic circuits used to develop and illustrate the analog fault isolation techniques.

NAP2 is a Nonlinear Analysis Program for lumped electronic circuit simulations. The program covers DC, transient, and frequency domain analysis. The input language is format free and allows the user to build his own models that can be stored in a library for later use. The solution is based upon a hybrid formulation of network equations and sparce matrix technique. For nonlinear circuits the Newton-Raphson method is followed and in transient analysis a implicit, variable-order, variable-step integration scheme is used. Sensitivities are computed from the adjoint network in the DC analysis, while the time dependent sensitivities are calculated directly from the difference equations produced by the integration formula.

NAP2 has the following features. The program is coded in FORTRAN IV for an IBM 370/165 system. The storage requirements of the present version are 104 K bytes. With this region size the program limitations are:

Nodes

+number of primary current variables	$\leq$	50
Circuit description statements		
+number of diodes		
+6 * number of bipolar transistors		
+6 * number of field effect transistors		
+number of output options	$\leq$	195
Subparameters		
+2 * number of functional values		
+4 * number of nonlinear couplings		
+4 * number of bipolar transistors		
+8 * number of field effect transistors		
+2 * number of diodes	$\leq$	276

B.2. Model Library

NAP2 provides an arbitrary collection of statements to be stored in a library for later use. Six libraries are available under the names: LIB1, LIB2,..., LIB6. Although the program offers diodes, transistors, and field effect transistors as built-in models, we might enrich the program by using the libraries.

During the modeling of the circuit, we have generated and used several transistor and diode models.

B.3. Model of a Transistor and two diodes

```
*LIB3 NEW
*LIB3 IN649+
QEXP/EXP/ A -0.251E-9 B 0.251E-9 D 3.831E-2 L -0.5 U 2.
QCJ1/ABS/ B 0.108E-10 C 0.864 D -0.577
RB 3 2 0.532
RS 1 3 0.123E12
CJ 1 3 1*QCJ(VID)
ID 1 3 1*QEXP1(VID)
>

*LIB3 IN752+
QEXP2/EXP/ A -1.25E-11 B 1.25E-11 D 3.247E-2 E 1. L -.5 U 2.
QCD2//     B 3.1E-6 C 1.25E-11
QCJ2/ABS/  B 3.31E-10 C .75 D -.5
RB 3 2 11
RS 1 3 1.E6
CD 1 3 1*QCD2(IID)
CJ 1 3 1*QCJ2(VID)
ID 1 3 1*QEXP2(VID)
>

*LIB3 N2222A+
QJE4/EXP/ A -3.02E-11 B 3.02E-11 D 40. L -0.5 U 2.
QJC4/EXP/ A -1.19E-10 B 1.19E-10 D 38. L -0.5 U 2.
QCE4/ABS/ B 2.2E-11 C 0.9 D 0.4
QCC4/ABS/ B 1.3E-11 C 0.9 D 0.35
RB 9 1 0.05
RC 7 2 2.8E-3
CE 9 8 1*QCE4(VCE)
CC 1 2 1*QCC4(VCC)
IE 1 8 1*QJE4(VCE)
IC 1 2 1*QJC4(VCC)
IN 2 1 0.9927*QJE4(VCE)
II 8 1 0.697*QJC4(VCC)
>
```

References

A. Fault Analysis Algorithm and Pattern Recognition

1. L. Phung and S.G. Chan, "On Fault Isolation." Proc. Fourth Asilomar Confr. on Circuits and Systems, Univ. of Ill., pp. 41-49, 1970.
2. G.O. Martens and J.D. Dyck, "Fault Identification in Electronic Circuits with the Aid of Bilinear Transformations," IEEE Trans. on Reliability, Vol. R-21, pp. 99-104, May 1972.
3. H. Sriyananda, D.R. Towill, "Fault diagnosis using Time Domain Measurements," The Radio and Electronic Engineer, Vol. 43, No. 9, September 1973.
4. S.D. Bedrosian, D.W.C. Shen, "Adaptive Learning Model for Optimization of Analog Tests," Proc. 20th Midwest Symposium on Circuits and Systems, Texas Tech., pp. 206-210, August 1977.
5. H. Sriyananda, D.R. Towill, "Voting Techniques for Fault Diagnosis from Freuquency-Domain Test-Data," IEEE Trans. on Reliability, Vol. R-24, No. 4, pp. 260-267, October 1975.
6. S.D.Bedrosian and J.H. Lee, "Automatic Fault Test Generation for Nonlinear Analog Circuits Using the Fuzzy Concept," Proc. 21st Midwest Symp. on Circuits and Systems, Iowa State Univ., pp. 399-403, August 1978.
7. C. Tinaztepe, Automated Test Design, Ph.D. Dissertation, University of Pennsylvania, 1977.
8. Duda, Hart, Pattern Classification and Scene Analysis, Wiley- Interscience, 1973.
9. K. Fu, Synthetic Methods in Pattern Recognition, Academic Press, 1974.
10. L.F. Pau, "Diagnosis of Equipment Failures by Pattern Recognition," IEEE Trans. Reliability, R-23, No. 3, pp. 202-208, August 1974.
11. R. Gayer, "Fault Isolation in Solid State Circuits," M.S.E. Thesis, Moore School, Univ. of Pennsylvania, 1963.
12. S.D. Bedrosian, J.H. Lee, and D.W.C. Shen, "Learning Techniques Applied to Analog Fault Testing," Autotestcon '77, pp. - , Hyanis, MA, November 1977.
13. S.D. Bedrosian and J.H. Lee, "A Fault Isolation Method in Nonlinear Analog Networks Using Fuzzy Set Concepts," Proc. 1978 Asilomar Conf. on Circuits and Systems, Pacific Grove CA, pp.478-482.
14. N. Navid and A.N. Willson,Jr., "A Theory and an Algorithm for Analog Circuit Fault Diagnosis," IEEE Trans. Circuits and Systems, Vol. CAS-26, No. 7 pp. 440-456, July 1979.
15. P.Duhamel and J.C. Rault, "Automatic Test Generation Techniques for Analog Circuits and Systems : A Review," IEEE Trans. Circuits and Systems, Vol. CAS-26, No. 7, pp. 411-439, July 1979.

16. L. Buchsbaum, M. Dunning, T.J.B. Hannon, and L. Math, "Investigation of Fault Diagnosis by Computational Method," Remington Rand UNIVAC, AD-601 204, May 1964.
17. R.S. Berkowitz and R.L. Wexelblat, "Statistical Considerations In Element Value Solutions," IRE Trans. Mil. Electron., Vol., Mil-6, pp.282-288, July 1962.
18. W.J. Hankley and H.M. Merrill, "A Pattern Recognition Technique for System Error Analysis," IEEE Trans. Rel., Vol. R-20, pp.148-153, Aug. 1971.
19. H.M. Merrill, "Failure Diagnosis using Quadratic Programing," IEEE Trans. Rel., Vol. R-22, pp.207-213, Oct. 1973.
20. E.C. Neu, "Combinatorial Analysis For Checkout Techniques," Joint National Meeting of the American Astronautical Society, pp.17-20, June 1969.
21. L.J. Buchsbaum, "Fault Diagnosis by Computational Methods," M.S.E. Thesis, The Moore School of Electrical Engineering of the Univ. of Pennsylvania, Dec. 1963.
22. T.S. Bush, "A Hybrid Circuit Test System," presented at IEEE INTERCON 1975, New York, NY, April 1975.
23. R.F. Barry, R.S. Fisher, and R.R. Mattison, "Programmed Algorithm for Test Point Selection and Fault Isolation," RCA Aerospace Systems Division, Burlington, MA, Report Contract AF 33(615)-3033, Vol. 1, 2, Aug. 1966.
24. A.L. Brown and G.J. Sussman, "Localization of Failures in Radio Circuits, A Study of Causal and Teleological Reasoning," MIT Artificial Intelligence Laboratory, Memo 319, Dec. 1974.
25. J.J. Narrary, "Accessibility in Systems," the Automation of Testing, in IEE Conf. No. 91, pp.1-6, Sept., 1972.
26. P.E.K. Chow and A.C. Davies, "The Synthesis of Cyclic Code Generators," Electron. Eng., pp.253-259, April, 1964.
27. W.D.T. Davies, "Generation and Properties of Maximum-length Sequencies," Control, Vol. 10, No. 96, pp.302-304, June 1966.
28. A.D.G. Hazlerigg and A.R.M. Norton, "Application of Cross- Correlating Equipment to Linear System Identification," Proc. IEE, Vol. 112, No. 12, pp.2385-2400, Dec., 1965.
29. D.R. Towill, "Dynamic Test Techniques utilized in ATE," REME Journal, No. 22, pp.35-43, April 1972.
30 J.D. Lamb, "System Frequency Response using P-N Binary Waveforms," IEEE Trans. AC-15, No. 4, pp.478-480, Aug., 1970.
31. R.F. Garzia, "Fault Isolation Testing," Proc. IEEE Conf. on Advanced Maintainability, Philadelphia, Nov., 1972.
32. H. Sriyananda, D.R. Towill and J.H. Williams, "Selection of Test Frequencies for System Fault

Diagnosis," AIAA Paper No. 170-864, AIAA Guidance and Control Conf., Key Biscayne, Florida, Aug., 1973.

33. W.A. Plice, "Techniques for the Automatic Generation of Fault Isolation Tests for Analog Circuits," ASSC Conference Record, pp.127-130, Oct., 1975.

34. Y.C. Ho and R.L. Kashyap, "An Algorithm for Linear Inequalities and its Applications," IEEE Trans. Electronic Computers Vol. EC-14, No. 5, pp.683-688, Oct., 1965.

B. Fuzzy Set Theory

35. L.A. Zadeh, "Fuzzy Sets," Information and Control, Vol. 8, pp. 338-353, June 1965.
36. L.A. Zadeh, Fu, Tanaka, Simura, Fuzzy Sets and their Applications to Cognitive and Decision Process, Academic Press, 1975.
37. C.V. Negoita, Applications of Fuzzy Sets to Systems Analysis, Halsted Press, 1975.
38. L.A. Zadeh, "Probability Measures of Fuzzy Events," J. Math. Anal. Appl. 23, pp.482-505, 1963.
39. MacLane, Birkhoff, Algebra, MacMillan Book Co., 1967.
40. B.R.Gains, "Fuzzy and Probability Uncertainty Logics," Information and Control 38, pp. 154-169, 1978.
41. S. Watanabe, "A Generalized Fuzzy-Set Theory," IEEE Trans. on Systems, Man, and Cybernetics, Vol. SMC-8, No. 10, Oct. 1978, pp. 756-760.
42. B. R. Gaines and L.J.Kahout, "The Fuzzy Decade; a Bibliography of Fuzzy Systems and Closely Related Topics," International Journal of Man-Machine Studies 9, pp. 1-68, 1977.
43. B.R. Gaines, "Fuzzy and Probability Uncertainty Logics," Information and Control 38, pp. 154-169, 1978.
44. J.Knopfmacher, "On Measures of Fuzziness," J. Math. Anal. Appl. 49, pp. 529-534, 1975.
45. L.A. Zadeh, "Fuzzy Languages and their Relation to Human and Machine Intelligence," Man and Computer, Proc. Int. Conf., Bordeaux, pp.136-165, 1972.
46. L.A. Zadeh, "A Fuzzy-Set-Theoretic Interpretation of Linguistic Hedges," Memorandum No. ERL-M355, Univ. of California, Berkely, April 1972.
47. L.A. Zadeh, "Fuzzy Sets and Systems," Proc. 1965 Symp. on System Theory, Brooklyn, New York.
48. R. Bellman, R. Kalaba and L.A. Zadeh, "Abstraction and Pattern Classification," Journal of Mathematical Analysis and Applications, Vol. 13, No. 1, pp.1-7, Jan., 1966.
49. A. Kaufmann, Introduction to the Theory of Fuzzy Subsets, Vol. 1, Academic Press, New York, 1975.
50. E. Sanchez, "Inverse of Fuzzy Relations. Application to Possibility Distribution and Medical Diagnosis," IEEE Control and Decision Conf., pp.1384-1389, 1977.

C. Information Theory and Fuzzy Information

51. T.Terano and M. Sugeno, "Conditional Fuzzy Measures and Their Applications," Fuzzy Sets and Their Applications to Cognitive and Decision Processes, Academic Press, 1975.

52. R.J. Solomonoff, "A Formal Theory of Inducive Inference," Information and Control 7, pp. 1-23, 1964.

53. M. Lof, "The Definition of Random Sequences," Information and Control 9, pp. 602-619, 1966.

54. A.N. Kolmogorov, "Logical Basis for Information Theory and Probability Theory," IEEE Trans. Inform. Theory, IT-14, pp. 662-664, 1968.

55. De Luca and S. Termini, "A Definition of Nonprobabilistic Entropy in the Setting of Fuzzy Sets Theory," Information and Control 20, pp. 301-322, 1972.

56. R. Capocelli and De Luca, "Fuzzy Sets and Decision Theory," Information and Control 23, pp. 446-473, 1973.

57. J. Cerney and P. brunovsky, "A Note on Information Without Probability," Information and Control 25, pp. 134-144, 1974.

58. T. Okuda, H. Tanaka, and K.Asai, "A Formulation of Fuzzy Decision Problems with Fuzzy Information Using Probability Measures of Fuzzy Events," Information and Control 38, pp. 135-147, 1978.

D. Automata Theory

59. Shannon and McCarthy, Automata Studies, Princeton University Press, 1956.

60. B. Chandrasekaran, Contribution to the Theory of Learning Automata, Ph.D. Dissertation, Moore School, University of Pennsylvania, 1967.

61. W.Wee, On Generalizations of Adaptive Algorithms and Applications of the Fuzzy Sets Concept to Pattern Classification, Ph.D. dissertation, Purdue University, 1967.

62. G.A. Wong, and D.C. Shen, "On the Learning Behavior of Fuzzy Automata," Proc. Int. Cong. on Cybernetics and Systems, Oxford University, Oxford, UK, 1972.

63. E.S. Santos, "Maxmin Automata," Information and Control 13, pp.366-377, 1968.

64. E.S. Santos, "Realizations of Fuzzy Languages by Probabilstic, Max-product, and Maxmin Automata," Information Sciences 8, pp.39-53, 1975.

65. M.G. Thomason, "Finite Fuzzy Automata, Regular Fuzzy Languages, and Pattern Recognition," Pattern Recognition, Vol. 5, pp. 383-390, Pergamon Press, 1973.

66. E. Tazaki and M. Amagasa, "Heuristic Structure Synthesis in a Class of Systems using a Fuzzy Automaton," IEEE Conference on Decision and Control, pp. 1414-1418, December 1977.

67. S.Tamura, S. Higuchi, and K.Tanaka, "Pattern Classification Based on Fuzzy Relations," IEEE Trans. Systems, Man, Cybernetics, Vol. SMC-1, pp. 61-66, January 1971.

68. D.A. Callahan, Computer Aided Network Design, McGraw-Hill Book Co., 1968.

69. W.E. Heinlein, W.H. Holmes, Active Filters for Integrated Circuits, Prentice Hall International, 1974.

70. P.A. Payne, "An Improved Technique of Transfer Function Synthesis from Frequency Response Data," IEEE Trans. Automatic Control, Vol. Ac-15, pp. 480-483, August 1970.

71. R. Kalman, "A new Approach to Linear Filtering and Prediction Problems," Trans. Amer. Soc. Mech. Engrs., J. Basic Engng., 82D, pp. 35-45, March 1960.

72. F.T. Boesch, ed., Large-Scale Networks: Theory and Design, IEEE Press, 1975.

73. S.W. Director, "The Generalized Adjoint Network and Network Sensitivities," IEEE Trans. on Circuit Theory, Vol. CT-16, No. 3. pp. 318-323, August 1969.

74. R.S. Berkowitz, "Conditions for Network-Element-Value Solvability," IRE Trans. on Circuit Theory, pp.24-29, March 1962.

75. G.L. Berry and S.D. Bedrosian, "Normalized Element Value Representation of Filter Networks," Journal of the Franklin Institute, Vol. 280, No. 4, pp. 334-342, October 1965.

76. J. H. Lee and S. D. Bedrosian, "Fault Isolation Algorithm for Analog Electronic Systems Using the Fuzzy Concept," Special Issue of IEEE Trans. on Circuits and Systems for Analog Fault Testing, Vol. CAS 26, pp. 518-522, July 1979.

77. R. Saeks and S. Liberty, Rational Fault Analysis, Marcel Dekker, 1977.

78. S.D. Bedrosian, On Element Value Solution of Single-Element-Kind Networks, Ph.D. Dissertation, University of Pennsylvania, 1961.

79. D.W.C. Shen, "Generalized Star and Mesh Transformations," Philosophical Magazine, Ser. 7, Vol. 38, pp.267-275, April, 1947.

80. S.D. Bedrosian and J.H. Lee, "Graph Theoretical Aspects for Analog Fault Diagnosis," 17th Annual Allerton Conference on Communication, Control, and Computing, pp.164-171, Univ. of Ill., Oct. 1979.

81. C.L. Liu, Introduction to Combinatorial Mathematics, McGraw-Hill, 1968.

82. W. Mayeda, Graph Theory, New York, Wiley Interscience, 1972.

83. P. Penfield, et al, Tellegen's Theorem and Electrical Networks, the MIT Press, 1970.

84. W. Silvert, "Symmetric Summation: A Class of Operations on Fuzzy Sets," Trans. on Systems, Man, and Cybernetics, Vol. SMC-9, No. 10, pp.657-659, Oct., 1979.

85. T. Rubner-Petersen, "NAP2, Application Examples," Report 30/8 73, Institute of Circuit Theory and

Telecommunication, Technical University of Denmark.

86. T. Rubner-Petersen, "NAP2, Users Manual," Report 16/5 73, Institute of Circuit Theory and Telecommunication, Technical University of Denmark.

87. J.T. Tou, Applied Automata Theory, Academic Press, 1968.

88. A.D. Allen, "Measuring the Empirical Properties of Sets," IEEE Trans. on Systems, Man and Cybernetics, Vol. SMC-4, No. 1, pp. 66-73, Jan. 1974.

89. D. Becker, "Extended SCEPTRE User's Manual, AD-A009-594," GTE Slvania, Inc., MA, Dec. 1974.

90. S.D. Bedrosian, J.H. Lee, "Analog Fault Isolation in Nonlinear Electronic Circuits with Fuzzy Learning Scheme," to be presented ECTCD'80, Warsaw, Poland. Sept. 1980.

INDEX

DISTRIBUTION LIST FOR ONR ELECTRONIC

AND SOLID STATE SCIENCES

Director Advanced Research Projects Agency Attn: Technical Library 1400 Wilson Boulevard Arlington, VA 22209	1 copy
Office of Naval Research Electronics Program Office (Code 427) 800 North Quincy Street Arlington, VA 22217	1 copy
Office of Naval Research Code 105 800 North Quincy Street Arlington, VA 22217	6 copies
Director Naval Research Laboratory 4555 Overlook Ave., S.W. Washington, D. C. 20375	
attn: Technical Library	6 copies
Code 5200	1 copy (Abstract)
Code 5210	1 copy (Abstract)
Code 5270	1 copy (Abstract)
Code 6400	1 copy (Abstract)
Office of the Director of Defense Research and Engineering Information Office Library Branch The Pentagon Washington, D. C. 20301	1 copy
Defense Documentation Center Cameron Station Alexandria, VA 22314	12 copies
Commanding Officer Office of Naval Research Branch Office 536 South Clark Street Chicago, IL 60605	1 copy (Abstract)
San Francisco Area Office Office of Naval Research 50 Fell Street San Francisco, CA 94102	1 copy (Abstract)

Commanding Officer Office of Naval Research Branch Office 1030 East Green Street Pasadena, CA 91101	1 copy (Abstract)
Commanding Officer Office of Naval Research Branch Office 495 Summer Street Boston, MA 02210	1 copy (Abstract)
New York Area Office Office of Naval Research 207 West 24th Street New York, New York 10011	1 copy (Abstract)
ODDR&E Advisory Group on Electron Devices 201 Varick Street New York, New York 10014	1 copy (Abstract)
Naval Air Development Center Attn: Technical Library Johnsville Warminster, PA 18974	1 copy
Naval Weapons Center China Lake, California 93555 attn: Technical Library	1 copy
Code 6010	1 copy (Abstract)
Naval Research Laboratory Underwater Sound Reference Division Technical Library P.O. Box 8337 Orlando, Fla. 32806	1 copy
Navy Underwater Sound Laboratory Technical Library Fort Trumbull New London, Conn. 06320	1 copy
Commandant, Marine Corps. Scientific Advisor (Code AX) Washington, DC 20380	1 copy (Abstract)
Naval Ordanance Station Technical Library Indian Head, MD 20640	1 copy
Naval Postgraduate School Monterey, CA 93940 attn: Technical Library	1 copy
Electrical Engineering Dept.	1 copy (Abstract)

Naval Missile Center Technical Library (Code 5632.2) Point Mugu, CA 93010	1 copy	
Naval Electronics Laboratory Center San Diego, CA		
Attn: Technical Library	1 copy	
Code 2300	1 copy	(Abstract)
Code 2600	1 copy	(Abstract)
Code 4800	1 Copy	(Abstract)
Naval Undersea Warfare Center Technical Library 3202 East Foothill Boulevard Pasadena, CA 91107	1 Copy	
Naval We pons Laboratory Technical Libra Dahlgren, VA 22448	1 copy	
Naval Ship Research & Development Center Central Library (Code L42 and L43) Washington, DC 20007	1 copy	
Naval Surface Weapons Center White Oak Laboratory Silver Spring, MD 20910		
Attn: Technical Library	1 copy	
Code 200	1 copy	(Abstract)
Code 212	1 copy	(Abstract)
Deputy Chief of Naval Operations (Development) Technical Analysis & Advisory Group (Code NOP-077D) Washington, DC 20350	1 copy	(Abstract)
Commander Naval Air Systems Command Washington, DC		
Attn: Code 310	1 copy	(Abstract)
Code 360	1 copy	(Abstract)
Commander Naval Electronics Systems Command Washington, DC 20360		
Attn: Code 304	1 copy	(Abstract)
Attn: Code 310	1 copy	(Abstract)
Commander Naval Sea Systems Command Washington, DC 20360	1 copy	(Abstract)
Naval Surface Weapons Center Attn: Library Dahlgren, Virginia 22448	1 copy	

Air Force Office of Scientific Research 1 copy
Attn: Electronic and Solid State Sciences Division
Department of the Air Force
Washington, DC 20333

Air Force Weapon Laboratory 1 copy
Technical Library
Kirtland Air Force Base
Albuquerque, NM 87117

Air Force Avionics Laboratory 1 copy
Air Force Systems Command
Technical Library
Wright-Patterson Air Force Base
Dayton, Ohio 45433

Air Force Cambridge Research Laboratory 1 copy
L.G. Hanscom Field
Technical Library
Cambridge, Mass. 02138

Harry Diamond Laboratories 1 copy
Technical Library
Connecticut Ave. at Van Ness, N.W.
Washington, DC 20438

U.S. Army Research Office 1 copy
Box CM, Duke Station
Durham, NC 27706

Director 1 copy
U.S. Army Engineering Research and Development Laboratories
Fort Belvoir, VA 22060
Attn: Technical Documents Center

Director National Bureau of Standards 1 copy
Attn: Technical Library
Washington, DC 20234