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RADAR ECHO ANALYSIS BY THE SINGULARITY EXPANSION METHOD. (U)  
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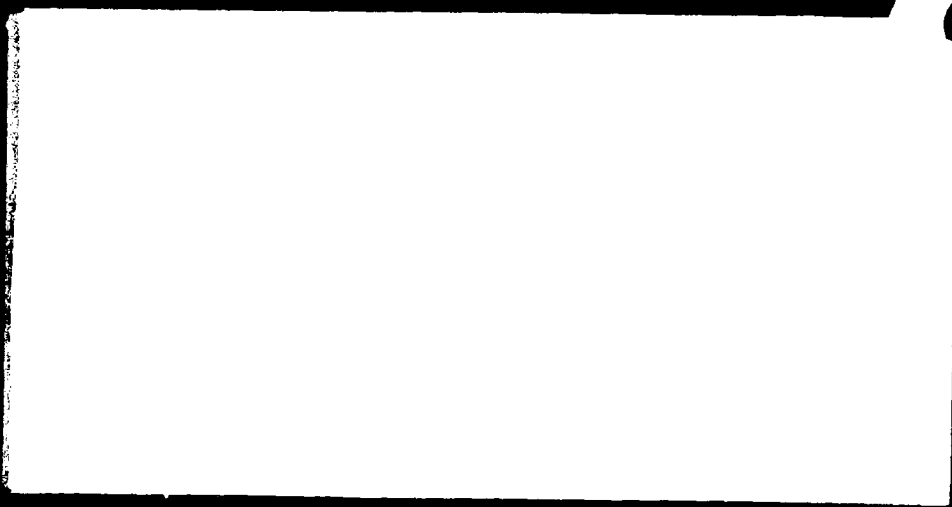
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RADAR ECHO ANALYSIS

BY THE

SINGULARITY EXPANSION METHOD

(10) Calvin H. Wilcox

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ABSTRACT.

Pulse mode radar operation is analyzed under the assumption that the scattering object  $\Gamma$  lies in the far field of both the transmitter and the receiver. It is shown that, in this approximation, the radar signal is a plane wave  $s(x \cdot \theta_0 - t, \theta_0)$  near  $\Gamma$ , where  $\theta_0$  is a unit vector directed from the transmitter toward  $\Gamma$ , and similarly the echo is a plane wave  $e(x \cdot \theta - t, \theta, \theta_0)$  near the receiver, where  $\theta$  is a unit vector directed from  $\Gamma$  toward the receiver. Moreover, it is shown that

$$e(\tau, \theta, \theta_0) = \text{Re} \left\{ \int_0^{\infty} e^{i\tau\omega} T(\omega, \theta, \theta_0) \hat{s}(\omega, \theta_0) d\omega \right\}$$

where  $\hat{s}(\omega, \theta_0)$  is the Fourier transform of  $s(\tau, \theta_0)$  and  $T(\omega, \theta, \theta_0)$  is the scattering amplitude in the direction  $\theta$  due to the scattering by  $\Gamma$  of a CW mode plane wave with frequency  $\omega$  and propagation direction  $\theta_0$ .

Finally the singularity expansion method is used to show that

$$e(\tau, \theta, \theta_0) \sim \sum e^{i\tau\omega_n} T_n(\theta, \theta_0) \hat{s}(\omega_n, \theta_0), \text{Im } \omega_n < 0.$$

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## 1. INTRODUCTION - RADAR ECHO PREDICTION.

➤ This paper presents an application of C. E. Baum's singularity expansion method (SEM) [2] and the author's method of asymptotic wave functions [11, 12, 13] to the prediction of pulse mode radar echoes from bounded scatterers. The results presented here are generalizations of corresponding results for sonar echoes [15]. Only a summary of the principal concepts and results is presented here. A complete exposition of the theory is planned for a separate publication.

1.1 Physical Assumptions. Radar echo structure is analyzed under the following assumptions.

- The radar system (transmitter and receiver) operates in a stationary homogeneous isotropic unlimited medium,
- The system is stationary with respect to the medium,
- The scatterers are bounded perfectly conducting objects,
- The scatterers are stationary with respect to the medium.
- The transmitter and receiver are in the far field of the scattering objects.

In addition it is assumed that secondary echoes due to the radar system components are negligible.

1.2 Mathematical Formulation. A fixed Cartesian coordinate system is used throughout the paper.  $x = (x_1, x_2, x_3) \in \mathbb{R}^3$  denotes a coordinate triple of this system and  $t \in \mathbb{R}$  denotes a time coordinate.  $\Gamma$  denotes a closed bounded subset of  $\mathbb{R}^3$  that represents the scatterers and  $\Omega = \mathbb{R}^3 - \Gamma$  denotes the domain exterior to  $\Gamma$ . The common frontier of  $\Gamma$  and  $\Omega$ , which

represents the surface of the scatterers, is denoted by  $\partial\Omega$ . The medium filling  $\Omega$  is characterized by a dielectric constant  $\epsilon$  and a magnetic permeability  $\mu$ . It will be assumed that  $\epsilon = 1$  and  $\mu = 1$  since this can be achieved by a suitable choice of units.

The electric and magnetic fields will be described by their components,  $(E_1, E_2, E_3)$  and  $(H_1, H_2, H_3)$  respectively, relative to the fixed Cartesian system. It will be convenient to use the notation and conventions of matrix algebra and to characterize the electromagnetic field by the  $6 \times 1$  column matrix

$$(1.1) \quad u = u(t, x) = (E_1 \ E_2 \ E_3 \ H_1 \ H_2 \ H_3)^T$$

where  $M^T$  denotes the transpose of matrix  $M$ . Similarly, if the electric and magnetic current densities that generate the field are described by their components,  $(J_1, J_2, J_3)$  and  $(J'_1, J'_2, J'_3)$  respectively, then

$$(1.2) \quad f = f(t, x) = (J_1 \ J_2 \ J_3 \ J'_1 \ J'_2 \ J'_3)^T$$

characterizes the field sources. With these conventions Maxwell's field equations can be written

$$(1.3) \quad D_t u + \sum_{j=1}^3 A_j D_j u + f = 0 \text{ for } t \in \mathbb{R}, x \in \Omega$$

where  $D_t = \partial/\partial t$ ,  $D_j = \partial/\partial x_j$  ( $j = 1, 2, 3$ ) and  $A_1, A_2, A_3$  are the three symmetric  $6 \times 6$  matrices defined by

$$(1.4) \quad \sum_{j=1}^3 A_j P_j = \begin{pmatrix} 0 & M(p) \\ -M(p) & 0 \end{pmatrix}, \quad M(p) = \begin{pmatrix} 0 & p_3 & -p_2 \\ -p_3 & 0 & p_1 \\ p_2 & -p_1 & 0 \end{pmatrix}.$$

The field equations (1.3) will be supplemented by the boundary condition for a perfect electrical conductor. It can be written

$$(1.5) \quad M(n) E = 0 \text{ on } \partial\Omega$$

where  $n = (n_1, n_2, n_3)$  is a unit vector on  $\partial\Omega$  and  $E = (E_1, E_2, E_3)^T$  is the electric part of  $u$ .

A theory of solutions with finite energy of (1.3), (1.5) was given in [10]. The total field energy at time  $t$  is given by

$$(1.6) \quad E = \frac{1}{2} \int_{\Omega} u(t, x)^T u(t, x) dx$$

where  $dx = dx_1 dx_2 dx_3$ . The theory of [10] makes use of the energy norm

$$(1.7) \quad \|u\|_{\Omega} = \left( \frac{1}{2} \int_{\Omega} u(x)^T u(x) dx \right)^{1/2}$$

and corresponding Hilbert space  $\mathcal{K}$ . The pulse mode radar echoes constructed below are in  $\mathcal{K}$ .

## 2. PULSE MODE RADAR SIGNAL STRUCTURE.

The transmitter will be assumed to be localized in the ball  $B(x_0, \delta_0) = \{x : |x - x_0| \leq \delta_0\}$  and to act during an interval  $0 \leq t \leq t_0$ . Thus the source distribution  $f$  in (1.3) will have support

$$(2.1) \quad \text{supp } f \subset \{(t, x) : 0 \leq t \leq t_0 \text{ and } |x - x_0| \leq \delta_0\}.$$

The corresponding pulse mode radar signal is the electromagnetic field  $u_0(t, x)$  that is generated by  $f$  when no scatterers are present. Thus  $u_0$  is characterized by the conditions

$$(2.2) \quad D_t u_0 + \sum_{j=1}^3 A_j D_j u_0 + f = 0 \text{ for } t \in \mathbb{R}, x \in \mathbb{R}^3,$$

$$(2.3) \quad u_0(t, x) = 0 \text{ for } t < 0, x \in \mathbb{R}^3.$$

The field  $u_0$  can be constructed by Fourier analysis or by the method of retarded potentials [11, 13, 15] but these constructions will not be recorded here.

**2.1 Asymptotic Wave Fields.** For definiteness the scatterers are assumed to be localized in the ball  $B(0, \delta)$  centered on the origin:  $\Gamma \subset B(0, \delta)$ . With this convention the assumption that the transmitter lies in the far field of  $\Gamma$  can be formulated as  $|x_0| \gg 1$ . The signal, propagating at the speed  $c = (\epsilon\mu)^{-1/2} = 1$ , will arrive at  $\Gamma$  at a time  $t$  of the same magnitude as  $|x_0|$ , whence  $t \gg 1$ .

It was shown in [13] that each signal  $u_0$  with finite energy has an asymptotic wave field  $u_0^\infty$  of the form



$$(2.4) \quad u_0^\infty(t, x) = |x - x_0|^{-1} s(|x - x_0| - t, \theta), \quad \theta = (x - x_0)/|x - x_0|$$

such that

$$(2.5) \quad \lim_{t \rightarrow +\infty} \|u_0(t, \cdot) - u_0^\infty(t, \cdot)\|_{R^3} = 0.$$

The wave profile  $s(\tau, \theta)$  is defined for all  $(\tau, \theta) \in R \times S^2$  where  $S^2$  is the unit sphere in  $R^3$ . Moreover, by specializing the results of [13] it can be shown that  $s(\tau, \theta)$  has the properties

$$(2.6) \quad \int_R \int_{S^2} s(\tau, \theta)^T s(\tau, \theta) d\theta d\tau < \infty,$$

where  $d\theta$  is the element of area on  $S^2$  (solid angle), and

$$(2.7) \quad P(\theta) s(\tau, \theta) = s(\tau, \theta)$$

where

$$(2.8) \quad P(\theta) = \frac{1}{2} \begin{pmatrix} 1 - \theta\theta & M(\theta) \\ -M(\theta) & 1 - \theta\theta \end{pmatrix} \text{ for all } \theta \in S^2.$$

In (2.8),  $\theta\theta$  denotes the dyadic, or tensor, product of  $\theta$  with itself with components  $\theta_j \theta_k$ . Property (2.7), (2.8) characterizes the polarization properties of the asymptotic wave fields  $u_0^\infty$ .

The function  $s(\tau, \theta)$  will be called the pulse mode transmitter radiation pattern. It can be constructed from the source function  $f$ ; see [15]. However, it will be assumed here that  $s$ , rather than  $f$ , is given since  $s$  is the important function in pulse mode transmitter design. The construction of a transmitter with a prescribed radiation pattern is the task of the transmitter design engineer.

2.2 The Plane Wave Approximation. Define  $\theta_0 \in S^2$  by  $x_0 = -|x_0| \theta_0$ . Then  $\theta_0$  is directed from the transmitter toward the scatterers and for  $x$  near  $\Gamma$  one has

$$(2.9) \quad |x - x_0| = |x_0| + \theta_0 \cdot x + O(|x_0|^{-1}) \text{ for } |x_0| \gg 1.$$

Hence, by (2.4),

$$(2.10) \quad u_0^\infty(t) = |x_0|^{-1} s(\theta_0 \cdot x - t + |x_0|, \theta_0) + O(|x_0|^{-2})$$

near  $\Gamma$ . If the error term is dropped one has a pulse mode plane wave signal. This approximation is made in the remainder of the paper.

### 3. PULSE MODE PLANE WAVE SCATTERING.

A plane wave signal

$$(3.1) \quad u_0(t, x) = s(x \cdot \theta_0 - t, \theta_0), \quad \text{supp } s(\cdot, \theta_0) \subset [a, b],$$

is assumed where the wave profile  $s(\tau, \theta_0)$  satisfies

$$(3.2) \quad P(\theta_0) s(\tau, \theta_0) = s(\tau, \theta_0).$$

Such a field is a solution of Maxwell's equations (2.2) with  $f = 0$ . The total field  $u(t, x)$  resulting from the interaction of  $u_0(t, x)$  with the scatterers is characterized by the properties

$$(3.3) \quad D_t u + \sum_{j=1}^3 A_j D_j u = 0 \quad \text{for } t \in \mathbb{R}, \quad x \in \Omega,$$

$$(3.4) \quad M(n) E = 0 \quad \text{for } t \in \mathbb{R}, \quad x \in \partial\Omega,$$

$$(3.5) \quad u(t, x) = u_0(t, x) \quad \text{for } t + b + \delta < 0, \quad x \in \Omega$$

where  $E = (u_1 \ u_2 \ u_3)^T$  is the electric part of  $u$ . The scattered field, or echo, is defined by

$$(3.6) \quad u_e(t, x) = u(t, x) - u_0(t, x) \quad \text{for } t \in \mathbb{R}, \quad x \in \Omega.$$

The author has shown, by the method of [11, 15], that  $u_e$  has an asymptotic wave field

$$(3.7) \quad u_e^\infty(t, x) = |x|^{-1} e(|x| - t, \theta, \theta_0), \quad x = |x| \theta$$

that converges to  $u_e(t, x)$  in energy when  $t \rightarrow \infty$ :

$$(3.8) \quad \lim_{t \rightarrow \infty} \|u_e(t, \cdot) - u_e^\infty(t, \cdot)\|_\Omega = 0.$$

The proof follows that for the scalar case of [15].

Points  $x$  in the far field of  $\Gamma$  satisfy  $|x| \gg 1$ . The echo  $u_e$  will arrive at a receiver at such a point when  $t \gg 1$ . Hence the echo may be approximated in the far field by the asymptotic field (3.7). For this reason  $e(\tau, \theta, \theta_0)$  will be called the echo waveform. It depends on the direction of incidence of the plane wave (3.1) and the direction of observation  $\theta$ . In this approximation, the echo prediction problem is the problem of constructing  $e(\tau, \theta, \theta_0)$  when the transmitter radiation pattern  $s(\tau, \theta_0)$  and the scatterers  $\Gamma$  are given. The solution to this problem given below is based on the theory of CW mode radar echoes outlined in the next two sections.

#### 4. CW MODE SIGNAL STRUCTURE.

The CW mode electromagnetic fields are solutions of the field equations (1.3) of the form

$$(4.1) \quad u(t,x) = e^{-i\omega t} v(x), \quad f(t,x) = e^{-i\omega t} \rho(x)$$

whence

$$(4.2) \quad \sum_{j=1}^3 A_j D_j v - i\omega v = \rho.$$

CW mode signals in  $R^3$  are generated by the Green's matrix [7]

$$(4.3) \quad G(x,x',\omega) = \begin{pmatrix} \nabla\nabla + \omega^2 1_3 & -i\omega M(\nabla) \\ i\omega M(\nabla) & \nabla\nabla + \omega^2 1_3 \end{pmatrix} \frac{e^{i\omega|x-x'|}}{4\pi\omega|x-x'|} 1_6.$$

where  $1_n$  denotes the  $n \times n$  unit matrix.  $G$  is the outgoing solution of the equation

$$(4.4) \quad \left( \sum_{j=1}^3 A_j D_j - i\omega \right) G(x,x',\omega) = \delta(x-x') 1.$$

The outgoing solution in  $R^3$  of (4.2) is

$$(4.5) \quad v(x) = \int_{R^3} G(x,x',\omega) \rho(x') dx'$$

Asymptotic evaluation of  $v(x)$  for large  $|x|$  using (4.3) and (4.5) gives the far field form

$$(4.6) \quad v(x) = (2\pi)^{1/2} \omega \frac{e^{i\omega|x|}}{|x|} P(\theta) \hat{\rho}(-\omega\theta) + O(|x|^{-2})$$

where  $x = |x|\theta$ ,  $P(\theta)$  is defined by (2.8) and

$$(4.7) \quad \hat{\rho}(p) = \frac{1}{(2\pi)^{3/2}} \int_{\mathbb{R}^3} e^{-ip \cdot x} \rho(x) dx$$

is the Fourier transform of  $\rho(x)$ . In particular, noting that  $P(-\theta)P(\theta) = 0$ , it is seen that the Silver-Müller radiation condition for  $v(x)$  can be written

$$(4.8) \quad P(-\theta) v(|x|\theta) = O(|x|^{-2}), \quad |x| \rightarrow \infty.$$

4.1 CW Mode Plane Waves.  $G(x, x', \omega)$  represents a CW spherical wave from a point source at the point  $x'$ . On putting  $x' = -|x'|\eta$  in (4.3) and making  $|x'| \rightarrow \infty$  with  $x$  fixed one finds after a short calculation

$$(4.9) \quad G(x, x', \omega) = (2\pi|x'|)^{-1} \omega e^{i\omega|x'|} e^{i\omega\eta \cdot x} P(\eta) + O(|x'|^{-2}).$$

Dropping the error term gives a matrix CW mode plane wave electromagnetic field. The general CW mode plane wave field is obtained by applying (4.9) to a constant vector and dropping the error term. It has the form

$$(4.10) \quad v(x) = e^{ip \cdot x} P(\eta)c, \quad p = |p|\eta$$

where  $c$  is an arbitrary 6-component vector. This may also be derived from (3.1), (3.2) by taking  $s(\tau, \eta) = e^{i\omega\tau} P(\eta)c$ . (4.10) is equivalent to the familiar formulas

$$(4.11) \quad E(x) = e^{ip \cdot x} \alpha, \quad H(x) = e^{ip \cdot x} (\eta \times \alpha), \quad \alpha \cdot \eta = 0$$

where  $v = \begin{pmatrix} E \\ H \end{pmatrix}$  and  $p = |p|\eta$ .

5. CW MODE ECHO STRUCTURE.

The columns of the  $6 \times 6$  matrix-valued function

$$(5.1) \quad \Psi^0(x, p) = (2\pi)^{-3/2} e^{ip \cdot x} p(\eta), \quad p = |p|\eta,$$

are CW mode plane waves of the form (4.10). The scattering of the CW mode matrix plane wave (5.1) by  $\Gamma$  produces a CW mode matrix-valued field

$$(5.2) \quad \Psi(x, p) = \Psi^0(x, p) + \Psi^{sc}(x, p), \quad x \in \Omega, \quad p \in \mathbb{R}^3 - \{0\}$$

that is characterized by the properties

$$(5.3) \quad \left[ \sum_{j=1}^3 A_j D_j - i|p| \right] \Psi(x, p) = 0, \quad x \in \Omega,$$

$$(5.4) \quad M(n) \Psi_E(x, p) = 0, \quad x \in \partial\Omega$$

$$(5.5) \quad p(-\theta) \Psi^{sc}(|x|\theta, p) = O(|x|^{-2}), \quad |x| \rightarrow \infty,$$

where  $\Psi_E$  is the electric part of  $\Psi$  (a  $3 \times 6$  matrix). The author has shown the existence and uniqueness of  $\Psi(x, p)$  for a large class of domains  $\Omega$ , including the "cone domains" of N. Weck [9] and domains having S. Agmon's "restricted cone property" [1]. The proofs, which generalize the results of [11] to Maxwell's equations, are based on compactness results of N. Weck [9] and C. Weber [8], respectively. In the special case that  $\partial\Omega$  is a smooth surface  $\Psi(x, p)$  can be constructed by the integral equation method described below.

5.1 Far Field Form of CW Mode Echoes.  $\Psi^{sc}(x, p)$  is the CW mode echo produced by the scattering of  $\Psi^0(x, p)$  by  $\Gamma$ . An integral representation



of  $\Psi^{\text{sc}}$  by the Green's matrix (4.3) can be used to derive the far field form

$$(5.6) \quad \Psi^{\text{sc}}(\mathbf{x}, \mathbf{p}) = \frac{e^{i|\mathbf{p}||\mathbf{x}|}}{4\pi|\mathbf{x}|} T(|\mathbf{p}|\theta, \mathbf{p}) + O(|\mathbf{x}|^{-2}), \quad \mathbf{x} = |\mathbf{x}|\theta,$$

where  $T(\mathbf{p}, \mathbf{p}')$  is a  $6 \times 6$  matrix-valued scattering amplitude. The polarization of the echo in the far field is characterized by the property

$$(5.7) \quad P(\eta) T(|\mathbf{p}|\eta, |\mathbf{p}|\eta') = 0.$$

5.2 Construction of  $T(\mathbf{p}, \mathbf{p}')$ . Define

$$(5.8) \quad \mathbf{J}(\mathbf{x}, \mathbf{p}) = \mathbf{n}(\mathbf{x}) \times \Psi_{\mathbf{H}}(\mathbf{x}, \mathbf{p}), \quad \mathbf{x} \in \partial\Omega,$$

where  $\Psi_{\mathbf{H}}$  is the magnetic part of  $\Psi$ .  $\mathbf{J}(\mathbf{x}, \mathbf{p})$  is the matrix electric current density on  $\partial\Omega$  induced by the plane wave  $\Psi^0$ . The divergence theorem and the jump relations of potential theory can be used to show that

$$(5.9) \quad \mathbf{J}(\mathbf{x}, \mathbf{p}) = 2(\mathbf{n} \times \Psi_{\mathbf{H}}^0(\mathbf{x}, \mathbf{p})) + \int_{\partial\Omega} \mathbf{K}(\mathbf{x}, \mathbf{x}', |\mathbf{p}|) \mathbf{J}(\mathbf{x}', \mathbf{p}) \, dS'$$

where  $\mathbf{K}$  is the  $3 \times 3$  matrix-valued kernel

$$(5.10) \quad \mathbf{K}(\mathbf{x}, \mathbf{x}', \omega) = \frac{1}{2\pi} \left\{ \nabla \frac{e^{i\omega|\mathbf{x}-\mathbf{x}'|}}{|\mathbf{x}-\mathbf{x}'|} \mathbf{n}(\mathbf{x}) \cdot - \frac{\partial}{\partial n} \frac{e^{i\omega|\mathbf{x}-\mathbf{x}'|}}{|\mathbf{x}-\mathbf{x}'|} \mathbf{1}_3 \right\}.$$

If  $\partial\Omega$  is smooth then (5.9) is a Fredholm equation and can be used to construct  $\mathbf{J}(\mathbf{x}, \mathbf{p})$  and  $\Psi(\mathbf{x}, \mathbf{p})$ ; cf. L. Marin and R. W. Latham [4] and

L. Marin [5]. The scattering amplitude can be calculated from  $J(x,p)$  and the relation

$$(5.11) \quad T(p,p') = (2\pi)^{3/2} 2i|p| \int_{\partial\Omega} \psi^0(x,p)^* \begin{pmatrix} J(x,p') \\ 0 \end{pmatrix} dS, \quad |p| = |p'|.$$

## 6. PULSE MODE RADAR ECHO STRUCTURE.

The solution of the pulse mode radar echo prediction problem formulated in §3 is given by the relation

$$(6.1) \quad e(\tau, \theta, \theta_0) = \text{Re} \left\{ \int_0^{\infty} e^{i\tau\omega} T(\omega\theta, \omega\theta_0) \hat{s}(\omega, \theta_0) d\omega \right\}$$

where

$$(6.2) \quad \hat{s}(\omega, \theta_0) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} e^{-i\omega\tau} s(\tau, \theta_0) d\tau$$

is the Fourier transform of  $s(\tau, \theta_0)$ . Thus under the far field assumptions of §1 the echo waveform is determined by the transmitter waveform and the matrix scattering amplitude  $T(\omega\theta, \omega\theta_0)$ . The latter can be calculated by solving the integral equation (5.9) and using relation (5.11).

Equation (6.1) is the generalization to electromagnetic fields of the analogous result for acoustic scattering that was derived in [15]. A proof of (6.1) may be given by the method of [15]. The key item in the proof is the theorem that the CW mode fields  $\Psi(x, p)$  are a complete family of generalized eigenfunctions for the Maxwell system. A proof along the lines of [11] may be based on the results of Weck [9] or Weber [8].

## 7. SEM EXPANSION OF PULSE MODE RADAR ECHOES.

If the scatterers  $\Gamma$  are bounded by smooth surfaces the integral equation (5.9) can be solved for  $J(x, \omega\theta)$  by the Fredholm determinant method [14]. Note that  $\Psi^0(x, \omega\theta)$  and  $K(x, x', \omega)$  are entire functions of  $\omega$ . It follows from the Fredholm theory that

$$(7.1) \quad J(x, \omega\theta) = \frac{M(x, \omega\theta)}{D(\omega)}$$

and hence

$$(7.2) \quad T(\omega\theta, \omega\theta_0) = \frac{N(\omega\theta, \omega\theta_0)}{D(\omega)}$$

where  $D(\omega)$ ,  $M(x, \omega\theta)$  and  $N(\omega\theta, \omega\theta_0)$  are entire functions of  $\omega$ . Moreover, the poles of  $T(\omega\theta, \omega\theta_0)$  can be shown to lie in the lower half-plane. These facts can be used to develop an SEM expansion of the echo waveform (6.1).

The reality of  $s(\tau, \theta_0)$  and symmetry properties of  $T(p, p')$  imply that (6.1) can be rewritten

$$(7.3) \quad e(\tau, \theta, \theta_0) = \frac{1}{2} \int_{-\infty}^{\infty} e^{i\tau\omega} T(\omega\theta, \omega\theta_0) \hat{s}(\omega, \theta_0) d\omega.$$

It is natural to regard this integral as a contour integral in the  $\omega$ -plane and to shift the contour to a line  $\text{Im } \omega = -b < 0$ . Assume that the poles  $\omega_n$  of  $T(\omega\theta, \omega\theta_0)$  satisfy

$$(7.4) \quad D'(\omega_n) \neq 0, \quad n = 1, 2, 3, \dots$$

$$(7.5) \quad \{n : -b \leq \text{Im } \omega_n < 0\} \text{ is finite}$$

$$(7.6) \quad |N(\omega\theta, \omega\theta_0)| \leq C|\omega|^m \text{ for } -b \leq \text{Im } \omega \leq 0$$

where  $C$  and  $m$  are constants. Then (7.3) implies

$$(7.7) \quad e(\tau, \theta, \theta_0) = \sum_{\text{Im } \omega_n > -b} e^{i\tau\omega_n} T_n(\theta, \theta_0) \hat{s}(\omega_n, \theta_0) + O(e^{b\tau})$$

where

$$(7.8) \quad T_n(\theta, \theta_0) = -\pi i \text{Res}_{\omega_n} T(\omega\theta, \omega\theta_0).$$

Hypothesis (7.4) is inessential. If  $T(\omega\theta, \omega\theta_0)$  has a higher order pole then in (7.7)  $e^{i\tau\omega_n}$  will be multiplied by a polynomial in  $\tau$ . Hypotheses (7.5) and (7.6) are closely connected with the geometry of  $\Gamma$  and the associated question of the exponential decay on bounded sets of the scattered fields. For acoustic scattering there is a considerable literature on these questions: see [3] and [6] and the literature cited there. The electromagnetic case awaits further analysis.

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13. ABSTRACT  <p style="text-align: center;"><u>ABSTRACT.</u></p> <p>Pulse mode radar operation is analyzed under the assumption that the scattering object <math>\Gamma</math> lies in the far field of both the transmitter and the receiver. It is shown that, in this approximation, the radar signal is a plane wave <math>s(x \cdot \theta_0 - t, \theta_0)</math> near <math>\Gamma</math>, where <math>\theta_0</math> is a unit vector directed from the transmitter toward <math>\Gamma</math>, and similarly the echo is a plane wave <math>e(x \cdot \theta - t, \theta, \theta_0)</math> near the receiver, where <math>\theta</math> is a unit vector directed from <math>\Gamma</math> toward the receiver. Moreover, it is shown that</p> $e(\tau, \theta, \theta_0) = \text{Re} \left\{ \int_0^{\infty} e^{i\tau\omega} T(\omega, \theta, \theta_0) \hat{s}(\omega, \theta_0) d\omega \right\}$ <p>where <math>\hat{s}(\omega, \theta_0)</math> is the Fourier transform of <math>s(\tau, \theta_0)</math> and <math>T(\omega, \theta, \theta_0)</math> is the scattering amplitude in the direction <math>\theta</math> due to the scattering by <math>\Gamma</math> of a CW mode plane wave with frequency <math>\omega</math> and propagation direction <math>\theta_0</math>. Finally the singularity expansion method is used to show that</p> $e(\tau, \theta, \theta_0) \sim \int e^{i\tau\omega_n} T_n(\theta, \theta_0) \hat{s}(\omega_n, \theta_0) d\omega_n, \text{ as } \omega_n < 0.$			

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