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SYNTHESIS OF ANTENNA PATTERNS WITH NULL CONSTRAINTS

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The Gaussian antenna pattern synthesis method is extended to include constraints such as pattern nulls or pattern derivative nulls at a given set of angles. The problem is formulated as a constrained approximation problem which is solved exactly, and a clear geometrical interpretation of the solution is given in a multidimensional vector space. The relation of the present method to those of constrained gain-maximization and signal-noise ratio maximization is discussed and conditions for their equivalence stated. For a

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20. Abstract (Continued)

linear uniform N-element array, it is shown that when M nulls are imposed on a given "quiescent" pattern, the optimum solution for the constrained pattern is the initial pattern and a set of M weighted (sin Nx)/sin x-beams. Each beam is centered exactly at the corresponding pattern null, irrespective of its relative location. In addition, simple quantitative expressions are derived for the pattern change and gain cost associated with the forced pattern nulls.

Preface

The contribution of Dr. Robert Mailloux, who initially suggested the problem, is gratefully acknowledged.

Contents

Illustrations

Synthesis of Antenna Patterns With Null Constraints

I. INTRODUCTION

The problem of forming nulls in the radiation pattern of an antenna, in order to suppress interference from certain directions, presently receives must attention. Most work is in the area of adaptive nulling systems, as discussed by Applebaum,[1] where a performance index such as the signal-noise ratio is maximized. In the case where jammers are the dominant noise source, this process automatically places pattern nulls in the directions of the jammers. A seemingly different approach is that of Drane-McIlvenna,[2] where another index, antenna gain, is maximized, subject to a set of null constraints on the pattern. In both methods the performance index is the quantity of prime interest, whereas the role of the antenna pattern is not sufficiently clear, which to an antenna engineer is unsatisfactory.

The purpose of this paper is to show that the problem can be formulated as a direct antenna pattern synthesis problem and that exact expressions for the effects of the forced pattern nulls, in terms of pattern change and gain cost, can be

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(Received for publication 5 January 1981)

1. Applebaum, S. (1976) Adaptive arrays, IEEE Trans. Antennas Propagation AP-24:585-598.

2. Drane, C. and McIlvenna, J. (1970) Gain maximization and controlled null placement simultaneously achieved in aerial array patterns, The Radio and Electronic Eng. 39(No. 1):49-57.

derived. The synthesis method is based on Gaussian or least-mean-square approximation[3] which allows a simple and very attractive geometrical interpretation in a unitary space.

The relation of the present approach to those of signal-noise ratio and gain optimization, which has been indicated[4, 5] is also discussed.

2. FORMULATION OF THE PROBLEM

We consider a situation where an antenna is being illuminated by desired signals and also by infinitely strong interference signals from certain discrete directions. The optimum antenna pattern for this case is reasonably defined as the desired pattern in the absence of the jammers, the so-called quiescent pattern, suitably modified so as to form pattern nulls in the interference directions. The degrees of freedom available in the antenna pattern are thus used in a first-hand way to form the pattern nulls, with remaining degrees of freedom being used for approximation of the quiescent pattern.

The corresponding antenna pattern synthesis problem consists of determining the closest approximation p_a to a given quiescent pattern p_o, subject to a set of null constraints. The solution of this problem requires a definition of "distance" between two patterns; this will be defined in Gauss' sense as the mean square difference between the patterns. This particular metric provides an overall measure of approximation and, in contrast to (for instance) Chebyshev approximation, gives no bound on the maximum deviation from the desired function at any particular point. However, it is the only metric that allows the approximation problem to be solved with any sense of generality.

The general antenna with N degrees of freedom is discussed in Appendix A. Here for simplicity we consider a linear array of N isotropic antenna elements with uniform half-wavelength spacing. Setting $u = \sin\theta$ (see Figure 1), the antenna far-field pattern is described by the array factor

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3. Schell, A. and Ishimaru, A. (1969) Antenna pattern synthesis, in Antenna Theory, Part I, ed. Collins and Zucker, McGraw-Hill, NY.

4. Mayhan, J., Simmons, A., and Cummings, W. (1980) Wide-band nulling with adaptive arrays using tapped delay lines, 1980 Int. IEEE AP-S Symposium Digest, Quebec, Canada, pp 110-113.

5. Fujita, M. and Takas, K. (1978) Asymptotic feature of an adaptive array and its application to array pattern synthesis, Transactions IECE of Japan E61(No. 8):599-603.

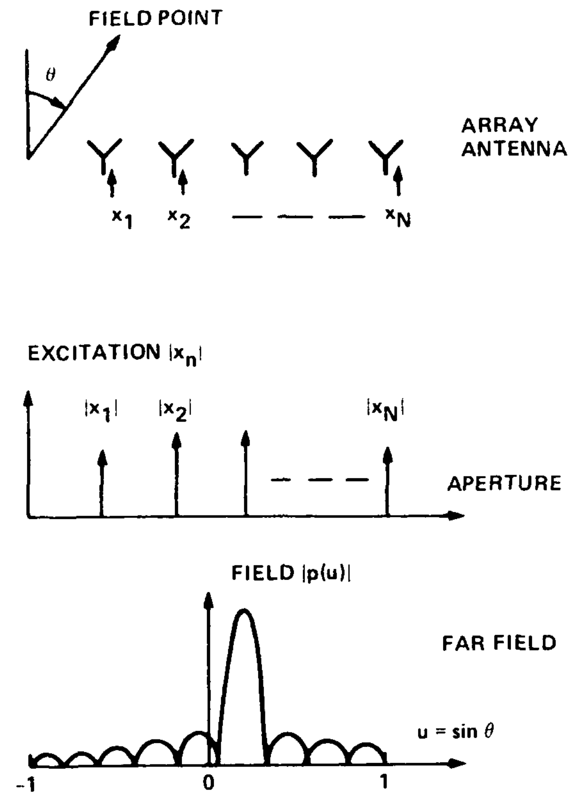

Figure 1. The Array Antenna, Its Aperture and Far Field

$$p(u) = \sum_{1}^{N} x_n e^{-i\pi n u} \tag{1}$$

where x_n denotes the complex excitation of the nth array element. The synthesis problem can now be stated mathematically: Find the pattern $p_a(u)$, such that the mean-square difference

$$\epsilon(p_a) = \frac{1}{2} \int_{-1}^{1} |p_o(u) - p_a(u)|^2 \, du = \text{minimum} \tag{2a}$$

subject to the constraints

$$p_a(u_m) = 0 \qquad m = 1, \ldots, M \tag{2b}$$

where $\{u_m\}_1^M$ denotes the angular location of the M interference sources.

We assume that the desired quiescent pattern is given as a sum of N harmonics, as represented by Eq. (1). For the general case, where $p_o(u)$ has any functional form, p_o may be simply approximated by the first N terms of its Fourier-series expansion. Although the synthesis procedure then involves two subsequent approximations, it is easily shown to lead to the correct least-mean-square approximation of the initial pattern.[6] We also assume that the null constraints in Eq. (2b) are linearly independent, as defined in the next section.

3. METHOD OF SOLUTION

The synthesis problem posed above is most conveniently described in a unitary space, which allows for a clear interpretation of the approximations involved.

To describe the array pattern we introduce an N-dimensional complex pattern space P, spanned by basis functions $\{e^{-i\pi nu}\}_1^N$. *The inner product of two patterns* p and q we define as $(p, q)_p = (1/2)\int p(u)q(u)^*du$, with the asterisk denoting complex conjugate, and for the norm of p we use $\|p\|_p = (p, p)_p^{1/2}$. Under this inner product the basis functions are orthonormal.

Similarly, we introduce an N-dimensional excitation space X in which an array excitation with coefficients $\{x_n\}_1^N$ is represented by a corresponding excitation vector $\overline{x} = (x_1, x_2, \ldots, x_N)$. The inner product we define as $(\overline{x}, \overline{y}) = \Sigma x_n y_n^*$ and the norm $\|\overline{x}\| = (\overline{x}, \overline{x})^{1/2}$.

The spaces P and X are related by a one-to-one mapping, the explicit form of which is given by Eq. (1). Furthermore, as a consequence of our choice for the inner products

$$(p, q)_p = (\overline{x}_p, \overline{x}_q) \tag{3}$$

with $\overline{x}_p$ and $\overline{x}_q$ denoting the excitations of p and q respectively, and hence

$$\|p - q\|_p = \|\overline{x}_p - \overline{x}_q\| \quad . \tag{4}$$

6. Steyskal, H. (1970) On antenna power pattern synthesis, IEEE Trans. Antennas Propagation AP-18:123-124.

Equation (4) is an important relation expressing the fact that, with our metric, the distance measured between two points in pattern space is the same as the distance measured between the corresponding points in excitation space. Thus a least-mean-square match of a desired quiescent pattern p_o is tantamount to a least-mean-square match of the quiescent pattern p_o is tantamount to a least-mean-square match of the quiescent array excitation $\overline{x}_o$.

Finally we define in excitation space a set of constraint vectors $\{\overline{y}_m\}_1^M$ by

$$\overline{y}_m = \left(e^{i\pi u_m}, e^{i2\pi u_m}, \ldots, e^{iN\pi u_m}\right) \tag{5}$$

which we assume to be linearly independent in accordance with the similar assumption about the pattern nulls earlier. In pattern space each constraint vector corresponds to a pattern

$$q_m(u) = e^{i\pi u_m} e^{-i\pi u} + e^{i2\pi u_m} e^{-i2\pi u} + \ldots + e^{iN\pi u_m} e^{-iN\pi u}$$

$$= \frac{\sin [N\pi(u - u_m)/2]}{\sin [\pi(u - u_m)/2]} e^{-i\pi(N+1)(u-u_m)/2} \quad . \tag{6}$$

For large N the amplitudes of these beams have the familiar behavior of a sinc-function centered at $u = u_m$, resulting from the constant amplitude-linear phase excitation represented by the constraint vectors.

Applying these definitions to Eqs. (2a) and (2b) results in the former equation being formulated in pattern space and the latter being formulated in excitation space, but by Eq. (4) they are readily translated from one space to the other. Choosing to work in excitation space we obtain

$$\begin{cases} \epsilon = \|\overline{x}_o - \overline{x}_a\|^2 = \text{min.} & \text{(7a)} \\ (\overline{x}_a, \overline{y}_m) = 0 \qquad m = 1, \ldots, M & \text{(7b)} \end{cases}$$

where $\overline{x}_o$ and $\overline{x}_a$ denote the excitation of the quiescent and the constrained pattern, respectively.

Here we note that in the same way as the condition of a pattern null, the condition of a null in the pattern derivative (of any order) is readily expressed by a constraint vector and included in Eq. (7). Setting the first derivative equal to zero locates a sidelobe at that particular point; setting the pattern and its derivative equal to zero at the same point broadens the pattern null.[7]

Equation (7) shows that the desired solution $\bar{x}_a$ is orthogonal to the constraint vectors $\{\bar{v}_m\}_1^M$. A geometrical interpretation of this relation is obtained if the space X is divided into an M-dimensional subspace Y, spanned by the vectors $\{\bar{v}_m\}_1^M$ and its (N-M)-dimensional orthogonal complement Z. Any vector $\bar{x}$ now has a unique decomposition[8]

$$\bar{x} = \bar{y} + \bar{z} \tag{8}$$

where $\bar{y} \in Y$, $\bar{z} \in Z$, $z \perp Y$, and due to this orthogonality

$$\|\bar{x}\|^2 = \|\bar{y}\|^2 + \|\bar{z}\|^2 \quad . \tag{9}$$

Using this decomposition for $\bar{x}_o$ and $\bar{x}_a$ we get from Eqs. (7), (8), and (9)

$$\begin{cases} \epsilon = \|\bar{y}_o - \bar{y}_a\|^2 + \|\bar{z}_o - \bar{z}_a\|^2 \to \min \quad . & \text{(10a)} \\ (\bar{x}_a, \bar{v}_m) = (\bar{y}_a, \bar{v}_m) = 0 \qquad m = 1, \ldots, M \quad . & \text{(10b)} \end{cases}$$

Equation (10b) yields

$$\bar{y}_a \equiv 0 \tag{11}$$

and therefore ϵ in Eq. (10a) is minimized by setting $\bar{z}_a = \bar{z}_o$ leading to

$$\bar{x}_a = \bar{x}_o - \bar{y}_o \tag{12}$$

and

$$\epsilon_{min} = \|\bar{x}_o - \bar{x}_a\|^2 = \|\bar{y}_o\|^2 \quad . \tag{13}$$

Equations (12) and (13) constitute the solution to the posed problem.

The method of solution is illustrated in Figure 2. The vector $\bar{x}_o$, which is to be approximated, has the projections $\bar{y}_o$ and $\bar{z}_o$ in subspaces Y and Z.

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7. Kwok, P. and Brandon, P. (1980) Maximization of signal noise ratio in array with broadened zero, Electronics Letters 16(No. 2):60-72.

8. Gantmacher, F. (1959) The Theory of Matrices, Vol. 1, Chelsea Publication Co., NY.

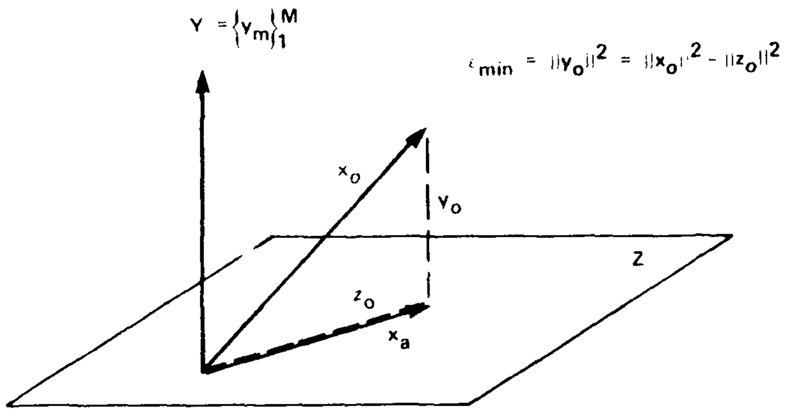

Figure 2. Geometrical Illustration of the Approximation Problem: Desired Point = x_o, Closest Approximation Confined to Subspace Z is $\overline{x}_a = \overline{z}_o$

Equation (10b) implies that the approximation $\overline{x}_a$ is *orthogonal* to the vector set $\{y_m\}_1^M$ which spans Y, and therefore $\overline{x}_a$ is confined to the subspace Z. Under these circumstances the best approximation to $\overline{x}_o$ is obtained by setting $\overline{x}_a = \overline{z}_o$, since of all elements $\overline{z} \in Z$, this point is closest to $\overline{x}_o$.

The antenna pattern corresponding to this solution will be discussed in Sections 5 and 6.

4. COMPARISON WITH OTHER METHODS

In this section we intend to show that the present method of pattern synthesis is related to the methods of constrained gain maximization by Drane-McIlvenna[2] and signal-noise ratio maximization by Applebaum.[1]

Following Drane-McIlvenna[2] we seek the maximum of the antenna directivity G in the "look" direction u_ℓ

$$G(u_\ell) = \frac{2\,|p(u_\ell)|^2}{\int_{-1}^{1} |p(u)|^2\, du} \rightarrow \max. \tag{14a}$$

subject to the constraints

$$p(u_m) = 0 \qquad m = 1, \ldots, M \quad . \tag{14b}$$

When these equations are transformed into excitation space they become

$$G(u_\ell) = \frac{|(\overline{x}, \overline{x}_\ell)|^2}{\|\overline{x}\|^2} = \max. \tag{15a}$$

and, compare with Eqs. (6) and (7b),

$$(\overline{x}, \overline{y}_1) = \ldots = (\overline{x}, \overline{y}_M) = 0 \tag{15b}$$

where the vector

$$\overline{x}_\ell = (e^{i\pi u_\ell}, e^{i2\pi u_\ell}, \ldots, e^{iN\pi u_\ell}) \quad . \tag{16}$$

As before we divide excitation space X into subspace Y, spanned by the constraint vectors $\{\overline{y}_m\}_1^M$ and its orthogonal complement Z, and then decompose the vector $\overline{x}$ into its projections, leading to

$$\overline{x} = \overline{y} + \overline{z} \quad , \quad \overline{y} \in Y, \ \overline{z} \in Z \quad . \tag{17a), (17b}$$

In view of Eq. (17b) we have $\overline{y} \equiv 0$, which upon substitution in Eq. (17a) gives

$$G(u_\ell) = \frac{|(\overline{z}, \overline{y}_\ell + \overline{z}_\ell)|^2}{\|\overline{z}\|^2} = \left|\left(\frac{\overline{z}}{\|\overline{z}\|}, z_\ell\right)\right|^2 = \max. \tag{18}$$

Since $\overline{z}/\|z\|$ is a vector of unit magnitude, clearly G attains its maximum when $\overline{z}$ is parallel to $\overline{z}_\ell$. The sought excitation vector thus is

$$\overline{x} = \lambda \overline{z}_\ell = \lambda(\overline{x}_\ell - \overline{y}_\ell) \tag{19}$$

where λ is a proportionality constant.

In order to compare the solution for maximum gain [Eq. (19)] with the solution of pattern synthesis [Eq. (12)], we must specify the excitation for the desired quiescent pattern $\overline{p}_o$. As is well known for the unconstrained case, the desired excitation $\overline{x}_o$ for maximum gain in the direction u_ℓ is given by

$$\overline{x}_o = (e^{i\pi u_f}, e^{i2\pi u_f}, \ldots, e^{iN\pi u_f}) \quad . \tag{20}$$

Comparing this with Eq. (18) we find that indeed $\overline{x}_o = \overline{x}_f$ and the identity of the two solutions is established.

We note that the pattern synthesis method and the gain maximization method lead to the same solution if in Eq. (15a) we generalize $\overline{x}_f$ to be any desired vector $\overline{x}_o$ – which may or may not coincide with $\overline{x}_f$ as given by Eq. (16). In the latter case, however, strictly speaking we are no longer optimizing gain.

Next we consider Applebaum's[1] approach in which we seek a set of array element weights, denoted by the row vector $\overline{w} = (w_1, \ldots, w_N)$ that maximizes the generalized signal-noise ratio

$$\left(\frac{S}{N}\right) = \frac{|(\overline{w}, \overline{t}^*)|^2}{(\overline{w}^* M, w^*)} \quad . \tag{21}$$

Here $\overline{t}$ is the desired quiescent signal vector and M the noise covariance matrix. The latter consists of two parts:

$$M = M_o + M_j \tag{22}$$

where M_o represents the quiescent environment (receiver noise only) and M_j represents the statistically independent, external interference sources (jammers). For the present comparison with pattern synthesis we can assume all array elements to contribute uncorrelated noise of equal power $|V_o|^2$, which leads to

$$M_o = |V_o|^2 \, I \tag{23}$$

where I is the identity matrix.

The matrix M_j is derived as per Applebaum.[1] Assuming M jammers located in the directions $\{u_m\}_1^M$, the total interference signal at the kth element is

$$v_k = \sum_1^M V_m \, e^{ik\pi u_m} \tag{24}$$

where V_m denotes the complex amplitude of the individual jammer. The terms $u_{k\ell}$ of the matrix M_j are given by

$$u_{k\ell} = E(v_k^* v_\ell) \tag{25}$$

where E denotes "expected value" and, in view of the [illegible] statistically independent,

$$u_{k\ell} = \sum_{m=1}^{M} |V_m|^2 e^{-i\pi(k-\ell)u_m} \quad . \tag{26}$$

A pleasant consequence of Eq. (26) is that M_j can be written as the sum of the covariance matrices of the individual jammers and therefore

$$M_j = \sum_{1}^{M} |V_m|^2 M_{jm} \tag{27}$$

with

$$M_{jm} = \begin{pmatrix} 1 & e^{i\pi u_m} & - - & e^{i\pi(N-1)u_m} \\ e^{-i\pi u_m} & 1 & & \\ - - - & - - - & - - - & - - - \\ e^{-i\pi(N-1)u_m} & & & 1 \end{pmatrix} \tag{28}$$

Substitution of Eqs. (23) and (27) in Eq. (21) yields

$$\left(\frac{S}{N}\right) = \frac{|(w, t*)|^2}{(\overline{w}*[|V_o|^2 + |V_1|^2 M_{j1} + \ldots + |V_m|^2 M_{jm}], \overline{w}*)} \quad . \tag{29}$$

In the limit of infinitely strong jammers, a necessary condition for a non-trivial result is

$$(\overline{w}*M_{jm}, \overline{w}*) = 0 \qquad m = 1, \ldots, M \quad . \tag{30}$$

Noting that M_{jm} can be rewritten as the outer product of the vectors $\overline{y}_m^\dagger$ and $\overline{y}_m$, where $\overline{y}_m$ is given by Eq. (6) and † denotes the complex conjugate transpose, it is easily shown that Eq. (30) is equivalent to

$$(\overline{w}^*, \overline{v}_m) = 0 \qquad m = 1, \ldots, M \quad . \tag{31}$$

Summarizing Eqs. (29), (30), and (31) we thus arrive at the problem to seek

$$\left| \left(\frac{\overline{w}^*}{|\overline{w}^*|}, \overline{t} \right) \right|^2 = \text{maximum} \tag{32a}$$

subject to the constraints

$$(\overline{w}^*, \overline{v}_m) = 0 \qquad m = 1, \ldots, M \quad . \tag{32b}$$

Equation (32) is identical to Drane-McIlvenna's[2] generalized problem discussed above, since $\overline{t} = \overline{x}_0$, the desired, quiescent array excitation, and $\overline{w}^* = \overline{x}$ in view of the phase conjugacy between a weight distribution in the receive mode and an aperture distribution in the transmit mode. Thus the solution is the same as that obtained by the constrained pattern synthesis method.

A more recent approach[9] considers a generalized form of our initial Eq. (2) where the right-hand side of Eq. (2b) may have any desired value (not necessarily zero). The pattern p(u), which minimizes received noise power and simultaneously maintains a fixed value at a desired look direction u_ℓ, is determined as a solution; that is, p(u) is solved from

$$\begin{cases} \int N_0(u) |p(u)|^2 \, du = \text{minimum} \\ p(u_\ell) = 1 \end{cases} \tag{33}$$

where $N_0(u)$ is the known angular noise power distribution. Neither in this case are the properties of the resulting antenna pattern known.

We have thus shown that the published methods[1, 2, 9] can be manipulated to yield the same result as our pattern synthesis method. Conceptionally, however, the latter method may be more appealing since it is a more direct approach and provides valuable insight.

9. Mucci, R.A., Tufts, D., and Lewis, J. (1976) Constrained least-squares synthesis of coefficients for arrays of sensors and FIR digital filters, IEEE Trans. Aerospace and Electronic Syst. AES-12:195-201.

5. THE SYNTHESIZED PATTERN

Returning to the solution $\overline{x}_a$ for the constrained excitation, as given by Eq. (12), we note that the vector $\overline{y}_i$ is a linear combination of the vectors $\overline{y}_m$, and therefore $\overline{x}_a$ may be written as

$$\overline{x}_a = \overline{x}_o - \sum_{1}^{M} \alpha_m \overline{y}_m \quad . \tag{34}$$

The unknown coefficients α_m may be determined from Eq. (10b), which leads to the following system of equations:

$$\begin{pmatrix} (\overline{y}_1, \overline{x}_o) \\ \cdot \\ \cdot \\ \cdot \\ (\overline{y}_M, \overline{x}_o) \end{pmatrix} = \begin{pmatrix} (\overline{y}_1, \overline{y}_1) & (\overline{y}_1, \overline{y}_2) & \cdots & (\overline{y}_1, \overline{y}_M) \\ (\overline{y}_2, \overline{y}_1) & (\overline{y}_2, \overline{y}_2) & & \\ \cdots & \cdots & \cdots & \cdots \\ (\overline{y}_M, \overline{y}_1) & (\overline{y}_M, \overline{y}_2) & & (\overline{y}_M, \overline{y}_M) \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_M \end{pmatrix} \quad . \tag{35}$$

Applying Cramer's rule to solve for α_m and substituting back in Eq. (34) yields

$$\overline{x}_a = \overline{x}_o - \frac{1}{G} \sum_{1}^{M} D_m \overline{y}_m \tag{36}$$

where the Gram determinant $G = G(\overline{y}_1, \ldots, \overline{y}_M)$ is the determinant of the coefficient matrix in Eq. (35) (see Gantmacher[8]) and D_m is the determinant of the same coefficient matrix with the mth column replaced by the column vector $((\overline{y}_1, \overline{x}_o), (\overline{y}_2, \overline{x}_o), \ldots, (\overline{y}_M, \overline{x}_o))$.

Equation (36) expresses the optimum excitation $\overline{x}_a$ as the quiescent excitation $\overline{x}_o$ minus a weighted sum of the constraint vectors $\overline{y}_m$ (constant amplitude-linear phase excitations). The pattern $p_a(u)$ corresponding to the excitation $\overline{x}_a$ is immediately obtained, using Eqs. (1) and (6), as

$$p_a(u) = p_o(u) - \frac{1}{G} \sum_{1}^{M} D_m q_m(u) \tag{37}$$

and thus the solution to our initial problem Eq. (2) is seen to consist of the desired quiescent pattern and M superimposed sinc-beams. This result agrees with the known case of a single interference source.[1]

The cancellation beams $\{q_m(u)\}_1^M$ represent M degrees of freedom and clearly it must be possible to realize M pattern nulls with these. However, it is noteworthy that each of these beams is determined solely by the direction of the corresponding interference source. Thus with the appearance of a new jammer only the amplitudes but not the directions of the beams need to be changed (apart from the requirement of a new beam). For an adaptive nulling system this may possibly lead to a faster convergence rate in pattern space than in excitation space.

6. THE EFFECTS OF NULL CONSTRAINTS ON THE PATTERN

It is clear that forcing the antenna pattern to zero at certain directions does affect the pattern over the entire angular region, and the extent of these effects is a matter of practical as well as theoretical interest. The following two measures for the difference between the quiescent and the constrained pattern seem natural:

1. Pattern change $\epsilon = \frac{1}{2}\int |p_o - p_a|^2 \, du$. This overall measure is relevant for shaped beam patterns or for patterns described over the entire angular sector.

2. Gain cost $\epsilon_g = G_o(u_f) - G_a(u_f)$, which is the reduction in directivity in the look direction u_f and is of interest, particularly for pencil beams.

By definition, the pattern change ϵ is minimized by the constrained solution p_a. However, it is a happy coincidence that the gain cost ϵ_g is minimized simultaneously, as discussed in Section 4.

The pattern change may be written using Eq. (13) together with Eqs. (4) and (9)

$$\epsilon_{min} = \|p_o - p_a\|^2 = \|\overline{x}_o\|^2 - \|\overline{z}_o\|^2 \quad . \tag{38}$$

The last term in Eq. (38) can be concisely expressed[8] as the ratio of two Gram determinants, leading to

$$\epsilon_{min} = \|\overline{x}_o\|^2 - \frac{G(\overline{x}_1, \ldots, \overline{x}_M, \overline{x}_o)}{G(\overline{x}_1, \ldots, \overline{x}_M)} \quad . \tag{39}$$

The advantage of this form is primarily that the explicit calculation of the coefficients $\{\alpha_i\}$ is avoided and also determinants are readily evaluated by computer.

A simple estimate for ϵ_{min} is obtained when the constraint vectors $\bar{y}_m$ can be considered to be orthogonal. This is true whenever the jammers are spread an odd multiple of π/N apart and is approximately true as soon as the jammers are more than a beamwidth apart. Stated mathematically, the condition is

$$(\bar{y}_m, \bar{y}_n) \ll \|\bar{y}_m\|\|\bar{y}_n\| \qquad m \neq n \tag{40}$$

In this case the vectors $\{\bar{y}_m\}_1^M$ after normalization by $\|\bar{y}_m\| = \sqrt{N}$ form an orthonormal basis for the subspace Y, and thus we find

$$\epsilon_{min} = \|\bar{y}_o\|^2 \simeq \frac{1}{N} \sum_1^M |(\bar{x}_o, \bar{y}_m)|^2 = \frac{1}{N} \sum_1^M |p_o(u_m)|^2 \quad . \tag{41}$$

Equation (41) for ϵ_{min} seems reasonable since (1) the cancellation beams, which are superimposed on p_o to produce the nulls, are proportional to $p_o(u_m)$, and (2) the beamwidth of the cancellation beams and therefore their power content is inversely proportional to N.

For the gain cost ϵ_g we obtain, using Eqs. (15) and (12), the alternative expressions

$$\epsilon_g = 2\left|\frac{|p_o(u_\ell)|^2}{\|p_o\|^2} - \frac{|p_a(u_\ell)|^2}{\|p_a\|^2}\right| = G_o(u_\ell)\left|1 - \frac{\|\bar{x}_o\|^2}{\|\bar{x}_a\|^2}\,\frac{|(\bar{x}_o, \bar{x}_\ell) - (\bar{y}_o, \bar{x}_\ell)|^2}{|(\bar{x}_o, \bar{x}_\ell)|^2}\right| \quad . \tag{42}$$

For the particular case where the quiescent excitation $\bar{x}_o = \bar{x}_\ell$, corresponding to a quiescent sinc-pattern pointed at u_ℓ, Eq. (42) simplifies to $\epsilon_g = \|\bar{y}_o\|^2$ which is seen to be equivalent to the pattern change ϵ as given by Eq. (38). In this case, therefore, minimizing the pattern change simultaneously minimizes the gain cost.

An estimate for ϵ_g can be obtained when the constraint vectors are approximately orthogonal again. Assuming a highly directive quiescent pattern and setting for simplicity $u_\ell = 0$, it is easily shown from Eq. (42) that

$$\epsilon_g < 2\, G_o(o) \frac{|(\bar{y}_o, \bar{x}_\ell)|}{|(\bar{x}_o, \bar{x}_\ell)|} \simeq 2 \frac{G_o(o)}{N p_o(o)} \left| \sum_1^M (\bar{x}_o, \bar{y}_m)(\bar{y}_m, \bar{x}_\ell) \right| -$$

$$2 \frac{G_o(o)}{N\, p_o(o)} \left| \sum_1^M p_o(u_m)\, q_m(o) \right| \leq$$

$$2\, G_o(o) \sum_1^M \left| \frac{p_o(u_m)}{p_o(o)} \; \frac{\sin(N\pi u_m/2)}{N \sin(\pi u_m/2)} \right| . \tag{43}$$

7. SUMMARY AND CONCLUSION

We have extended the general method of Gaussian antenna pattern synthesis[3] to include null constraints on the pattern and its derivatives. The problem has been posed as a constrained approximation problem and an exact solution has been obtained. The relation to other known methods to achieve pattern nulls under mathematically well-defined conditions has been discussed.

For a linear uniform array we have shown that, with M interference sources, the constrained pattern is the sum of the quiescent pattern and M weighted sinc-beams. Each beam points exactly at the corresponding interference source, irrespective of its relative location. In addition, we have derived simple quantitative expressions for the pattern change and the gain cost associated with the forced pattern nulls.

Finally, it is worth noting that we have formulated the constrained pattern synthesis method in quite general terms, involving only the excitation at the antenna input ports and the radiated beam patterns. The antenna therefore may include any desired linear passive beamforming network. It is hoped that this approach can further contribute to an understanding of the role played by the antenna in an adaptive nulling system.

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Appendix A

The General Antenna

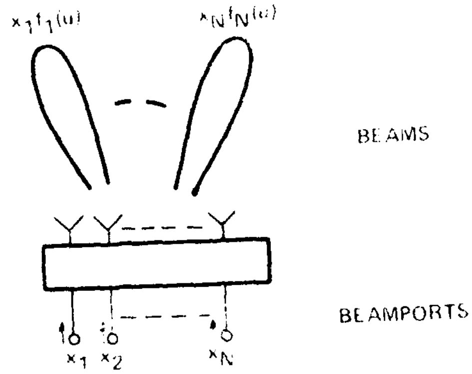
The synthesis method presented above can be generalized to an antenna with an arbitrary feed network, such as shown in Figure A1. Applying a complex excitation x_n at the nth beamport gives rise to a radiated field distribution $x_n f_n(u)$, where $f_n(u)$ is the far-field pattern and u the angular variable. The general pattern thus is

$$p(u) = \sum_{1}^{N} x_n f_n(u) \tag{A1}$$

and our synthesis problem consists in finding a pattern $p_a(u)$ such that

$$\begin{cases} \epsilon = \dfrac{1}{2} \displaystyle\int_{-1}^{1} |p_o(u) - p_a(u)|^2 \, w(u) \, du = \text{minimum} & \text{(A2a)} \\[2ex] p_a(u_m) = 0 \qquad m = 1, \ldots, M & \text{(A2b)} \end{cases}$$

where the quiescent pattern $p_o(u)$ and the weighting function w(u) are given. The latter function can be used to assign more relative importance to a specific angular sector by weighting it more heavily, for instance.



[illegible] A1. [illegible] Antenna [illegible] N Input Ports

To reduce the present problem to the [illegible] form as [illegible] the inner product $(p, q)_p$ [illegible] $(1/2)\int pq^* \, du$ in pattern space [illegible] basis $\{e_n(u)\}_1^N$ from the set $\{f_n(u)\}_1^N$. A pattern thus has the two alternative representations

$$p(u) = \sum_1^N x_n f_n = \sum_1^N [illegible]$$

The relation between the coefficients [illegible] inner product of p with e_m, leading to

$$[illegible]_m = \sum_{n=1}^N x_n (f_n, e_m)_p$$

or

$$\bar{[illegible]} = \bar{x} A$$

[illegible] $(\ldots)$ and $\bar{x} = (x_1, \ldots$ [illegible]

$$[illegible] = (f_n, e_m)_p$$

Defining generalized constraint vectors

$$\bar{v}_m = (f_1(u_m), \ldots, f_N(u_m))^* \qquad \text{(A6)}$$

we can finally write Eq. (A2) as

$$\int \epsilon \, |\bar{g}_o - \bar{g}_a|^2 = \text{minimum} \qquad \text{(A7a)}$$

$$(\bar{g}_a, \bar{g}_m) = 0 \qquad m = 1, \ldots, M \qquad \text{(A7b)}$$

where

$$\bar{g} = \bar{v}_m (A^{-1})^{\dagger} \quad . \qquad \text{(A8)}$$

The synthesis problem for the general antenna now has the same form as for the uniform array antenna and therefore may be solved in an identical manner.

Considerable simplifications result for a lossless antenna with matched isolated input ports and a weighting function w(u) = 1. Using [illegible] the initial patterns $f_\nu(u)$ form an orthonormal set in this case.[A2] Therefore $v_\nu = f_\nu$, the matrix A = I, and Eq. (7) reduces to

$$\int \epsilon \, |\bar{x} - \bar{x}_o|^2 = \text{minimum} \qquad \text{(A9a)}$$

$$(\bar{x}_m, \bar{x}) = 0 \qquad m = 1, \ldots, M \qquad \text{(A9b)}$$

with the solution, cf. Eqs. (34) and (35),

$$\bar{x} = \bar{x}_o - \sum_{m=1}^{M} \alpha_m \bar{x}_m \qquad \text{(A10a)}$$

$$(\bar{x}_m, \bar{x}_o) = \sum [illegible] (\bar{x}_{m'}, \bar{x}_m) \qquad [illegible] = 1, \ldots, M \quad . \qquad \text{(A10b)}$$

A2. [illegible] IEEE Trans. [illegible]

These equations are the same as for the uniform array, but now the constraint vectors Eq. (A6) are expressed in terms of the general beam patterns $f_n(u)$ instead of the functions $e^{-in\pi u}$ used earlier.

The cancellation beams thus become

$$q_m(u) = \sum_{n=1}^{N} f_n(u_m)^* f_n(u) \qquad m = 1, \ldots, M \tag{A11}$$

and the pattern

$$p_a(u) = p_o(u) - \frac{1}{G} \sum_{1}^{M} D_m^* q_m(u) \quad . \tag{A12}$$

We note that the shape of the cancellation beams no longer is independent of the jammer direction u_m, as it was for the simple array antenna.

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