

REPORT DOCUMENT/ PAGE	READ INSTRUCTIONS
REPORT NUMBER ,2 GOVT ACCESSION NO	3. REC PIENT'S CATALOG NUMBER
N00014-75-C-0428 Vib-A099	045
TITLE (and Subilile)	5. TYPE OF REPORT & PERIOD COVERE
	1/1/75 - 3/31/81
RAINDUM TIME PRUCESSES	6. PERFORMING ORG. REPORT NUMBER
AUTHOR(a)	8. CONTRACT OR GRANT NUMBER(*)
Murrav Rosenblatt, Ph.D.	N00014-75-C-0428
Professor of Mathematics	
PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
THE REGENTS OF THE UNIVERSITY OF CALIFORNIA University of California, San Diego	
La Jolla, California 92093	1
CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
800 North Quincy Street	1 FIAY 1981
Arlington, Virginia 22217	5
4. MONITORING AGENCY NAME & ADDRESS(11 dillorent from Controlling Office) Office of Naval Research Resident Representative	15. SECURITY CLASS. (of this report)
Mail Code Q-043	Unclassified
University of California, San Diego	154. DECLASSIFICATION/DOWNGRADING SCHEDULE
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- ⁷ The behavior of k-dimensional kernel density (probability) estimates was examined.
- The asymptotic behavior of partial sums of dependent random variables was also examined.

The relationship between long-range dependence and non-Gaussian structure of the limiting distributions was determined in some cases.

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REPORT N00014-75-C-0428

RANDOM TIME PROCESSES.

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- Murray/Rosenblatt, Ph.D. Professor of Mathematics University of California, San Diego Mail Code C-012 La Jolla, California 92093
- 1 May 1981

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9 Final Report, for Period 1 January 1975 - 31 March 1981,

Prepared for:

OFFICE OF NAVAL RESEARCH 800 North Quincy Street Arlington, Virginia 22217

OFFICE OF NAVAL RESEARCH RESIDENT REPRESENTATIVE ATTN: Mr. Robert L. Bachman Administrative Contracting Officer Mail Code Q-043 University of California, San Diego La Jolla, California 92093 Introduction. The duration of the contract N00014-75-C-0428 was from January 1, 1975 to march 31, 1981. The principal investigator was Murray Rosenblatt of the Department of Mathematics at U.C.S.D. The following list contains the names of other people who were supported at some time.

> Richard Bradley Richard Davis Larry Goldstein Dr. Kenneth Helland Dr. Keh-Shin Lii Y. P. Mack Dr. John Rice Daniel Sorenson Stewart Strait

<u>List of doctoral theses</u>. The following doctoral theses were completed by students working with the principal investigator. These students were supported in part by funds from the ONR contract.

Keh-Shin Lii "On a global measure of the deviation of a spline estimate of a density function" 1975

Yue-Pok Mack "k-nearest neighbor estimation" 1978

Richard Bradley, Jr. "Measure of dependence on stationary sequences of random variables" 1978

Richard Davis "Extremes of stationary processes" 1979

Stewart Strait "A quadratic measure of deviation of spectral estimates" 1979

List of research papers published. Below a list is given of papers supported in part by the contract. The papers with no attribution are authored by

M. Rosenblatt.

- 1. (with Keh-shin Lii) "Asymptotic behavior of a spline estimate of a density function" Comp. and Maths. with Appl. 1, 223-235, 1975.
- 2. "Multiply schemes and shuffling" Math. of Comput. 29, 929-934, 1975.
- 3. "The local behavior of the derivative of a cubic spline interpolator" J. Approx. Th. 15, 382-387, 1975.
- 4. "Asymptotics and representation of cubic splines" J. Approx. Th., 17, 332-343, 1976.

- 5. (with Keh-Shin Lii and C. Van Atta) "Bispectral measurements in turbulence" J. Fluid Mech. 77, 45-62, 1976
- 6. "On the maximal deviation of k-dimensional density estimates" Ann. Prob., 4, 1009-1015, 1976
- 7. "Fractional integrals of stationary processes and the central limit theorem" J. Appl. Prob., 13, 723-732, 1976
- (with P.A. W. Lewis, L. H. Liu and D. W. Robinson) "Empirical sampling study of a goodness of fit statistic for density function estimation" Multivariate Analysis IV, 159-174, 1977
- 9. (with John Rice) "Estimation of the log survivor function and hazard function" Sankhya, 38 Series A, Pt. 1, 60-78, 1976
- 10. (with K. N. Helland and K. S. Lii) "Bispectra of atmospheric and wind tunnel turbulence" Appl. of Statistics, 223-248, 1977
- 11. "Energy transfer for the Burgers equation" Phys. Fluids 21, 1694-1697, 1978
- (with K.N. Helland) "Spectral variance estimation and the analysis of turbulence", Phys. Fluids, 22, 819-823, 1979
- (with K.N. Helland and K.S. Lii) "Bispectra and energy transfer in grid-generated turbulence" Develop. in Statist. Vol. 2, 123-155, 1979
- 14. "Some remarks on a mixing condition" Ann. Prob. 7, 170-172, 1979
- 15. (with Y.P. Mack) "Multivariate k-nearest neighbor density estimates" J. Multiv. Anal., 9, 1-15, 1979
- Richard A. Davis "Maxima and minima of stationary sequences" Ann. Prob., 7, 453-460, 1979
- 17. Richard C. Bradley "A remark on the central limit theorem for dependent random variables" J. Appl. Prob., 17, 94-101, 1980
- Richard C. Bradley "On the strong mixing and weak Bernoulli conditions" Zeit. f. Warschein. 51, 49-54, 1980
- 19. Richard C. Bradley "A note on a mixing condition Ann. Prob., 8, 636-638, 1980
- 20. Y. P. Mack "Asymptotic normality of multivariate k-nn density estimates" Sankhya, 42, Series A, 1980
- 21. "Some limit theorems for partial sums of stationary sequences" Multivariate Analysis V 239-248, 1980
- 22. "Some limit theorems for partial sums of quadratic forms in stationary Gaussian variables, Zeit. f. Warschein. 49, 125-132, 1979
- 23. "Linear processes and bispectra" J. Appl. Prob. 17, 265-270, 1980

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Discussion of the research

In the discussion of research, papers will be referred to by numbers as they are enumerated in the list of research papers published.

The paper [1] considered spline estimates of probability density functions as suggested by Boneva, Kendall and Stepanov and determined the asymptotic bias and variance of these estimates. They were also shown to be asymptotically normai under appropriate conditions. The results of [1] were based in part on [3] where the error terms in cubic spline interpolation were precisely estimated asymptotically. In [4], the effects of the boundary in the case of a cubic free spline interpolator are shown to be the major contribution to the integrated square error. The results of [2] suggest the type of improvement shuffling makes in the generation of pseudo-random numbers.

Bispectral estimates are employed in the multiple author paper [5] as a means of getting information about aspects of the nonlinear transfer of energy in turbulence. It is applied in the analysis of atmospheric data. These ideas are employed in a more extensive analysis of atmospheric and wind tunnel turbulence in [10] and [13]. A discussion of energy transfer for the model Burgers equation is to be found in [11].

The behavior of k-dimensional kernel density (probability) estimates is examined in [6]. An empirical Monte Carlo study of the behavior of a quadratic goodness of fit statistic for a density function estimate is carried out in [8]. The asymptotic behavior of multivariate k-nearest neighbor density estimates is determined in [15]. Related ideas are used to compare and evaluate the behavior of a number of estimates of the log survivor function and hazard function when the data is dependent [9]. Such questions are of interest in the study of reliability.

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The asymptotic behavior of partial sums of dependent random variables is examined in a number of papers. The dependent random variables are assumed to be stationary. In [7] conditions are specified under which one still gets asymptotic normality though the limiting normal process may not be the classical Brownian motion but rather a fractional Brownian motion. Related questions are taken up in [14] and [21]. In [22] cases are taken up in which there is sufficient long-range dependence and non-Gaussian structure so that the limiting distribution of partial sums is non-Gaussian and even nonstable. The normalization in these cases also turns out to be quite different from the standard normalization. Related questions of scale renormalization have been brought up in statistical physics and the study of turbulence.

In [16] the joint asymptotic distribution of maxima and minima of stationary sequences are determined under rather weak mixing conditions. In paper [17], [13], and [19] a number of mixing conditions for stationary sequences are examined. Some remarks are also made about the boundary of the domain within which the central limit theorem (asymptotic normality) is still valid for partial sums of dependent sequences.

Paper [23] considers linear non-Gaussian schemes and the problem of parameter estimation for such schemes. The sense in which Gaussian linear schemes are atypical is remarked on.

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