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JAN 81 H ARSHAM, A R BALANA, D GROSS   N00014-75-C-0729

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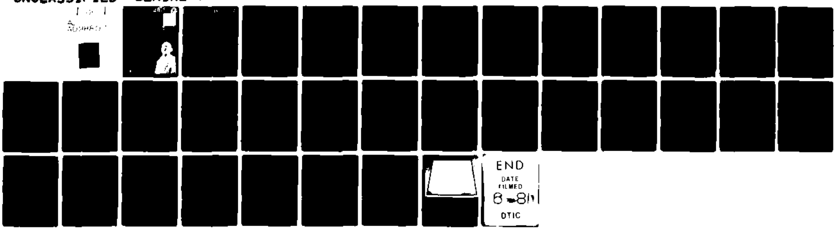

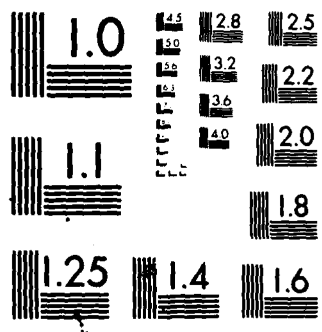

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NUMERICAL METHODS FOR TRANSIENT SOLUTIONS OF MACHINE REPAIR PROBLEMS

by

Hossein Arsham
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1. Introduction

The classic machine repair problem with spares is a typical example of a finite state space queueing problem and consists of a fixed number of identical machines of which initially M are operating and Y are spares. The M machines are in parallel and are independent. When one fails, it is instantaneously replaced by a spare if a spare is available; if not, less than M machines will operate until a repaired machine becomes available. Simultaneously, the failed machine goes instantaneously into a repair facility.

1.1 Assumptions and Problem Statement

The following assumptions are made concerning the machine repair example.

(a) The system failure rate is proportional to the number of operating machines.

(b) Each machine has exponential failure time with mean $1/\lambda$.

(c) There are c parallel servers in the repair facility.

(d) Each server has exponential service time with mean $1/\mu$.

Thus, the machine repair problem is Markovian and the states of this Markov process can be described by a single number i , where i represents the number of machines in the repair station. The intensity matrix of this process is given by

$$Q \equiv \begin{bmatrix} -\lambda_0 & \lambda_0 & 0 & \cdots & & 0 \\ \mu_1 & -\lambda_1-\mu_1 & \lambda_1 & 0 & & 0 \\ 0 & \mu_2 & -\lambda_2-\mu_2 & \lambda_2 & & 0 \\ \vdots & & & & & \vdots \\ 0 & & & & \mu_{M+Y} & -\mu_{M+Y} \end{bmatrix}$$

where

$$\lambda_i = \begin{cases} M\lambda , & (0 \leqq i < Y) \\ (M-i+Y)\lambda , & (Y \leqq i \leqq Y+M) \\ 0 , & (i \geqq Y+M) \end{cases}$$

$$\mu_i = \begin{cases} i\mu , & (0 \leqq i \leqq c) \\ c\mu , & (i \geqq c) \end{cases} .$$

For this problem, steady state solutions in closed form are readily attainable [see, for example, Gross and Harris (1974)].

1.2 Transient Solutions

It is desired to find the transient solutions for the machine repair problem. If the problem has N states $(N = M+Y+1)$, the intensity matrix provides N equations

$$\Pi'(t) = \Pi(t) \cdot Q , \tag{1}$$

where $\Pi(t)$ is a $N = M+Y+1$ component vector whose elements are $\pi_i(t)$, the unconditional probability that the system is in state i at time t , and $\Pi'(t)$ a vector of derivatives of $\pi_i(t)$. Finding solutions in closed form [see, for example, Marlow (1978)] is extremely difficult and in most cases impossible (unless $M+Y$ is very small). However, a variety of procedures is available which can yield numerical solutions to the differential equations. In this paper we will discuss several of these numerical procedures and attempt to apply each procedure to a machine repair problem with the following parameters:

$M=4$, number of machines initially operating
$Y=1$, number of spares
$c=2$, number of service channels
$\lambda=.15$, machine failure rate
$\mu=.5$, service rate.

Under the assumptions mentioned in the preceding section one can obtain the following initialized first order system of differential equations. Assuming at $t=0$ all machines are up:

$$\begin{bmatrix} \pi_0'(t) \\ \pi_1'(t) \\ \pi_2'(t) \\ \pi_3'(t) \\ \pi_4'(t) \\ \pi_5'(t) \end{bmatrix} = \begin{bmatrix} \pi_0(t) \\ \pi_1(t) \\ \pi_2(t) \\ \pi_3(t) \\ \pi_4(t) \\ \pi_5(t) \end{bmatrix}^T \begin{bmatrix} -.6 & .6 & 0 & 0 & 0 & 0 \\ .5 & -1.1 & .6 & 0 & 0 & 0 \\ 0 & 1 & -1.45 & .45 & 0 & 0 \\ 0 & 0 & 1 & -1.3 & .3 & 0 \\ 0 & 0 & 0 & 1 & -1.15 & .15 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

with initial value $\Pi(0) = [1,0,0,0,0,0]$.

2. Randomization Techniques

The randomization procedures give a method of calculating the transition probability matrix $P(t)$, i.e., the order $(M+Y+1)$ square matrix whose elements are $p_{ij}(t)$, the probability that the system is in state j at time t given that it started in state i $(0 \le i,j \le M+Y)$. From $P(t)$, by using the initial probability vector $\Pi(0)$,

$\Pi(t)$ can be calculated from $\Pi(t) = \Pi(0)P(t)$. We denote the $(i,j)th$ element of Q by q_{ij} , and define a scalar β as

$$\beta = \max_i |q_{ii}| . \tag{2}$$

Now define a matrix P with elements p_{ij} as

$$P = \{p_{ij}\} \equiv I + \frac{1}{\beta} Q ,$$

where I is an identity matrix having the same order as Q .

The matrix P is stochastic, and it has been shown [see Cohen (1969)] that the elements of $P(t)$ can be obtained by

$$p_{ij}(t) = e^{-\beta t} \sum_{n=0}^{\infty} \frac{\beta^n t^n}{n!} p_{ij}^{(n)} , \tag{3}$$

where $p_{ij}^{(n)}$ is the $(i,j)th$ element of matrix P raised to the nth power. This method is called randomization because it can be interpreted as a discrete time Markov chain with transition probabilities p_{ij} and transition time generated by a Poisson process at rate β .

To compute $p_{ij}(t)$ involves raising the matrix P to the nth power for $n=0,1,2,\ldots$. The numerical procedure truncates n at some appropriate value, say m , so

$$p_{ij}(t) = e^{-\beta t} \sum_{n=0}^{m-1} \frac{\beta^n t^n}{n!} p_{ij}^{(n)} + R_m , \tag{4}$$

where R_m is the error due to truncation.

2.1 Barzily and Gross Method

Barzily and Gross (1979) proposed a criterion to truncate n such that for a given $\varepsilon > 0$, the smallest m is chosen such that the error R_m obeys

$$R_m \leq 1 - e^{-\beta t} \sum_{n=0}^{m-1} \frac{(\beta t)^n}{n!} \leq \varepsilon . \qquad (5)$$

Their paper shows that such an m exists. For a machine repair problem with parameters $M = Y = C = 1$ and $\varepsilon = 10^{-3}$, the result of applying this criterion has been shown to be quite satisfactory, in fact accurate up to the second decimal place. A complete description of the Barzily-Gross method together with the study of the transient effects and the speed of convergence to steady state for machine repair problems can be found in the above cited reference. The algorithm has been coded in FORTRAN to run on the IBM 3031 computer, under the program name WONG. The following section is a brief description of program WONG.

2.2 Program WONG

WONG, a FORTRAN code originally designed to find the spares inventory level and number of repair channels necessary to guarantee a prespecified service level for a machine repair problem, included in it a program to provide transient solutions for the system state probabilities. This portion of the program was separated from the original and updated to stand alone as a provider of transient solutions to machine repair problems. Input requirements for program WONG are given in Appendix 1. Results of applying program WONG to the sample problem of Section 1.2 are given in Section 5.

3. The QUE Package

Grassmann and Servranckx (1979) developed a FORTRAN based package for finding transient solutions for moderate sized queueing networks (up to ten state variables). The method adopted in this package is in fact based on the randomization procedure discussed in the preceding sections. The truncation criteria are somewhat different and are described in the following section. We have adopted the sample problem of Section 1.2 to the specifications of this package and the results are presented, along with those of program WONG, in Section 5.

Grassmann (1977) has also shown that the truncation error, R_m, can be bounded with reasonable accuracy; that is,

$$R_m \leq 1 - \sum_{n=0}^{m-1} \frac{(\beta t)^n e^{-\beta t}}{n!} . \tag{6}$$

For small βt, the sum in the RHS of (6) can easily be evaluated for fixed m, and thus m can be determined such that R_m will be below a prescribed value $\varepsilon > 0$, as was done by Barzily and Gross (1979). For large βt one can approximate the Poisson distribution by the normal distribution. In the QUE package, m is set equal to

$$m = \beta t + 4\sqrt{\beta t} + 5 . \tag{7}$$

Using Poisson tables for $\beta t \leq 20$, or normal tables otherwise this procedure guarantees that such an m yields an R_m less than 10^{-4}.

3.1 Formulation of the Sample Problem for the QUE Package

To solve the machine repair problem by using the QUE package, one must formulate the problem to fit the network structure input requirements. The output statistics are then obtained from the program. One way of formulating the problem to fit the QUE package requirements is shown in Figure 1.

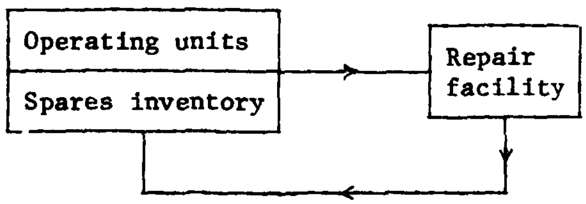

Figure 1.--Formulation of the sample machine repair problem for the QUE package.

It is necessary to define state variables and event descriptions together with type, conditions, net effect, and rate parameters. These are shown in Table 1.

TABLE 1

THE EVENT DESCRIPTION OF THE PROBLEM FOR THE QUE PROGRAM

Events	Type	Rate	Condition	Net Effect
1. Arrival into repair station				
a. when no machine is in repair station	1	.6	X1=0	(+1)
b. when there are some machines in the repair station	1	.75 - .15X1	$1 \leq X1 \leq 4$	(+1)
2. Service: when only one repairman is busy	1	.5	X1=1	(-1)
3. Service: when both repairmen are busy	1	1	$2 \leq X1 \leq 5$	(-1)

The state variable is described by X1 ≡ number of machines in repair station, $0 \leq X1 \leq 5$.

Each event has a rate function which associates the rate of each transition with the starting state. The rate function may be constant or a function of the state variable. Types of events are classified as type one, having finite rate event, or as type two, having infinite rate event. The state space of the system is defined by general conditions represented by linear inequalities involving the state variables. The net effect is the value of the state variable after the event occurs and is determined by incrementing or decrementing the value by a constant prior to the event's occurrence. A more detailed explanation can be found in Grassmann and Servanckx (1979). Appendix 2 shows the QUE program input requirements for the sample problem.

4. Numerical Integration Methods

Numerical integration methods can be employed to solve a system of ordinary differential equations described by

$$Y'(t) \equiv \begin{bmatrix} y_1'(t) \\ y_2'(t) \\ \vdots \\ y_p'(t) \end{bmatrix} = \begin{bmatrix} f_1(y_1,\ldots,y_p;t) \\ f_2(y_1,\ldots,y_p;t) \\ \vdots \\ f_p(y_1,\ldots,y_p;t) \end{bmatrix} \equiv f(Y,t) \qquad (8)$$

with known initial value $Y(t_0)$. The standard techniques are generally variations of either Runge-Kutta (R-K) or predictor-corrector (p-c) methods.

Runge-Kutta methods are based on formulas that approximate the Taylor series solutions

$$y_i(t+h) = y_i(t) + h \cdot y_i'(t) + \frac{h^2}{2} y_i''(t) + \ldots + \frac{h^k}{k!} y_i^{(k)}(t) + R_k \ ,$$

$i=1,\ldots p$. These methods use approximations for the second and higher-order derivatives. Euler's method is a special R-K method, with $k=1$. These methods have been used by several authors [e.g., Bookbinder and Martell (1979), Grassmann (1977), Liitschwager and Ames (1975) and Neuts (1975)] to find transient solutions in queueing systems.

The predictor-corrector methods require information about several previous points in order to evaluate the next point. These methods involve using one formula to predict the next $Y(t)$ value, followed by the application of a more accurate corrector formula. Unlike the R-K methods, p-c methods are not self-starting; hence, they use the R-K method to obtain the first $Y(t)$ value.

Predictor-corrector methods can provide an estimate of the local truncation error at each step in the calculations, in contrast to the R-K methods, which cannot obtain such an estimate.

Predictor-corrector methods include Milne's method, Hamming's method, and Adams' methods. Methods based on the Adams formulas have performed very well in test problems [see Hull, Enright, Fellen, and Sedgwick (1972)] even for nonstiff[1] systems or when function evaluations are relatively expensive. Hull *et al.* also concluded that R-K methods are not competitive, although fourth or fifth order methods are best for problems in which function evaluations are not very expensive and accuracy requirements are not very stringent.

Predictor-corrector methods have been used by Ashour and Jha (1973) for queueing problems.

A variety of routines is available for solving a system of ordinary differential equations. They include RKGS, DRKGS (fourth order R-K formulas), HPCG, DHPCG, HPCL, DHPCL (Hamming's Methods), all from the IBM Scientific Subroutine Package [IBM (1968)]; and DVERK (Verner's fifth and sixth order R-K formulas) and DVOGER (Gear's Method) in the International Mathematical and Statistical Libraries (IMSL) package [IMSL (Ed. 6)]. One routine based on extrapolation methods is DREBS, also in the IMSL package, which uses the Bulvisch-Stoer method.

4.1 Gear's Algorithm

C. W. Gear (1971a, 1971b) proposed a variable-order integration method based on Adams' predictor-corrector formulas of orders one through seven. It uses an order one formula to start and, for this reason, must start with very small step size when the error tolerance is stringent.

Gear's algorithm includes a special approach for dealing with stiff differential equations.

---

[1] A stiff system of ordinary differential equations is characterized by the property that the ratio of the largest to the smallest eigenvalue is much greater than one.

The Adams' formulas fall under two general categories--open and closed formulas. The Adams-Bashforth *pth* order formulas (open formula) can be written as

$$y_n = y_{n-1} + h \sum_{K=1}^{p} \beta_K y'_{n-K} , \quad (9)$$

where $y_i = y(t_i)$, $t_i = ih$, $y'_i = f(y_i, t_i)$. The order of the method is one less than the order of the truncation error per step. The Adams-Moulton *pth* order formulas (closed formulas) can be written as

$$y_{n,m} = y_{n-1} + \beta_0^* h f\left(y_{n,m-1}, t_n\right) + h \sum_{K=1}^{p-1} \beta_K^* y'_{n-K} . \quad (10)$$

The coefficients β_K and β_K^* are given by Henrici (1962). Equation (9) is used as the first approximation in Equation (10). Thus (9) is used as the predictor equation and (10) as the corrector equation. Whenever (10) converges (as is true when h is small and f is smooth), the truncation error introduced at the *nth* integration step is $C_{p+1}^A h^{p+1} y^{(p+1)}(t_n) + 0(h^{p+2})$, where $y^{(K)}$ is the *kth* derivative of y , and C_{p+1}^A are constants [see Henrici (1962)].

The predictor equation (9) is equivalent to fitting a *pth* degree polynomial through the known quantitites y_{n-1}, hy'_{n-1}, ..., hy'_{n-p} . For more details of the algorithm, see Gear (1971a, 1971b).

4.2 DVOGER Subroutine

DVOGER is a FORTRAN routine based on Gear's algorithm designed to solve a set of first order ordinary differential equations. The algorithm chooses the order of approximation such that the step size is increased, thereby decreasing the solution time. The option of using a particular method is done through a switch variable (MTH). Results of using DVOGER on the sample problem are also given in section 5. Appendix 3 shows the input and programming requirements for exercising DVOGER.

5. Conclusions and Numerical Results

We have presented three numerical methods in computing the transient probabilities for finite Markovian queues. Transient probabilities often provide a realistic picture of actual queueing systems, and sometimes it is desirable to know how fast they converge to steady state [Barzily and Gross (1979)].

The methods considered in this paper fall into two categories, namely, randomization and predictor-corrector numerical integration. In general these methods give reasonably accurate results for a moderate sized problem. Table 2 shows the output of these programs for t=1,3,5,7,9,12. The QUE package and DVOGER show almost equal results, while program WONG deviates from the other two by at most 3×10^{-4}, which is reasonably compatible.

In terms of set-up effort, program WONG gives the least degree of difficulty since it was written primarily for machine repair problem. The biggest concern with respect to the QUE program was the huge core storage requirement, which exceeds the current daytime capacity of 384K bytes of the IBM 3031 at the GWU Center for Academic and Administrative Computing. Future modification by redimensioning is suggested. In using DVOGER, one must carefully choose applicable parameter values, as in the step size. Total running times of the programs are 2.35 seconds for program QUE, 6.13 seconds for program WONG, and 162.62 seconds for DVOGER. The reason for the length of the latter is that with step size fixed at 1×10^{-4}, DVOGER must be called 120,000 times to integrate for each time point from $t=0$ to $t=12$. There is a need to explore further the best options of DVOGER to find those which might reduce running time considerably.

TABLE 2

PROGRAM OUTPUT OF WONG, QUE, DVOGER FOR THE TRANSIENT SOLUTION OF A MACHINE REPAIR PROBLEM WITH PARAMETERS M=4, Y=1, C=2, λ=.15, μ=.5 and $\Pi(0)$=(1,0,0,0,0,0) AT t=1,3,5,7,9,12

Time	Program	$\pi_0(t)$	$\pi_1(t)$	$\pi_2(t)$	$\pi_3(t)$	$\pi_4(t)$	$\pi_5(t)$
1	WONG	.6235	.2946	.0708	.0096	.0007	.0000
	QUE	.6237	.2949	.0709	.0097	.0007	.0000
	DVOGER	.6237	.2949	.0709	.0097	.0007	.0000
3	WONG	.3944	.3686	.1722	.0536	.0099	.0008
	QUE	.3945	.3688	.1723	.0536	.0099	.0008
	DVOGER	.3945	.3688	.1723	.0536	.0099	.0008
5	WONG	.3331	.3663	.2005	.0781	.0192	.0022
	QUE	.3333	.3665	.2006	.0782	.0192	.0022
	DVOGER	.3333	.3665	.2006	.0782	.0192	.0022
7	WONG	.3119	.3618	.2091	.0886	.0244	.0033
	QUE	.3122	.3620	.2093	.0887	.0244	.0033
	DVOGER	.3122	.3621	.2093	.0887	.0244	.0033
9	WONG	.3038	.3595	.2123	.0931	.0269	.0038
	QUE	.3039	.3597	.2124	.0932	.0269	.0038
	DVOGER	.3039	.3597	.2124	.0932	.0269	.0038
12	WONG	.2995	.3580	.2138	.0955	.0283	.0042
	QUE	.2997	.3582	.2140	.0956	.0284	.0042
	DVOGER	.2997	.3582	.2140	.0956	.0284	.0042

6. Acknowledgments

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APPENDIX 1: PROGRAM WONG

A. Input Requirements

TABLE A1.1

PARAMETER CARD INPUT

Columns	Format	Input Name	Explanation
1-5	I5	M	Number of machines initially operating
6-10	I5	IC	Number of service channels
11-15	I5	IY	Number of spares
16-27	F12.7	RLAM	Machine failure rate (Poisson mean)
28-39	F12.7	RMU	Service rate (Poisson mean)
40-49	F10.6	EPS	Tolerance value

TABLE A1.2

TIME INPUT

Columns	Format	Input Name	Explanation*
1-7	F7.3	TDEL	Time T

*Time at which transient probability is required is format free, but it must be coded starting from column one, and a separate card is required for every time desired.

B. Numerical Example Input

We shall illustrate the use of program WONG on the sample problem given in Section 1.2. The cards for the sample problem, with $\varepsilon = 10^{-3}$

and $t = 3,5$ are shown in Figure A1.1. The output obtained for this problem is tabulated in Table A1.3.

Card Type	Card Image
Job	// STANDARD JOB CARD
JCL	// EXEC FØRT2
JCL	//FØRT.SYSIN DD *
Program Deck	[Program WONG deck]
JCL	//GØ.SYSIN DD *
Parameter	4 2 1 .15 .5 .001
Time	3
Time	5
JCL	//

Figure A1.1--Card input program WONG for sample problem.

TABLE A1.3

THE OUTPUT OF PROGRAM WONG
FOR THE SAMPLE PROBLEM*

t	$\pi(t)$					
	$\pi_0(t)$	$\pi_1(t)$	$\pi_2(t)$	$\pi_3(t)$	$\pi_4(t)$	$\pi_5(t)$
3	.3944	.3686	.1722	.0536	.0099	.0008
5	.3331	.3663	.2005	.0781	.0192	.0022

*Note: the initial distribution of the system is assumed to be $\pi_0(0)=1$, $\pi_i(0)=0$, $i \geq 1$.

APPENDIX 2: PROGRAM QUE

A. Input Deck Format Specification

TABLE A2.1

INPUT DECK FORMAT FOR PROGRAM QUE

1. Problem title card:
   Function: for documentation only

Columns	Format	Field description
1-80	20A4	Problem title

2. Problem specification card:
   Function: describes the number of state variables, number of events, and the number of general system conditions

Columns	Format	Field description
1-2	I2	Number of state variables
3-4	I2	Number of events
5-6	I2	Number of state space restrictions

3. Maximum vector card (one card for each state variable--for machine repair problems only one is required)
   Function: to describe the highest possible value of each state variable (a maximum of ten state variables)

Columns	Format	Field description
1-2	I2	Maximum value of state variable 1

4. Event title cards (one for each event)
   Function: for documentation purposes only

Columns	Format	Field description
1-80	20A4	Event title

TABLE A2.1--*continued*

5. Event specification card (one for each event)
Function: indicates type of event and the rate of this event

Columns	Format	Field description
1	I1	Print flag for transitions (1 = Yes)
2	I1	Event type (1 for machine repair problem)
3	I1	Number of specific conditions (0 for machine repair problem)
4	I1	Flag for new minima and maxima of the state variable (1 = Yes)
5-10	F6.2	Rate of this event
11-12	I2	State variable on which rate depends (if zero, rate is a constant)
13-18	F6.2	Increase of rate

6. New maxima and minima vector card (one for each event)
Function: resets the maximum and minimum values for the state variable

Columns	Format	Field description
1-2	I2	New maximum for state variable X1
3-4	I2	New minimum for state variable X1

7. Net effect card
Function: defines the function f(x) which converts the starting state into the target state

Columns	Format	Field description
1-3	I3	Net effect for state variable X1

8. Trailer card
Function: delimiter card used to indicate the end of events section of the system's input. The first four bytes of the record must contain the string "END"

Columns	Format	Field description
1-4	A4	Control field (value is "END")
5-80	19A4	Ignored

TABLE A2.1--*continued*

9. Probability specification card
Function: gives the number of nonzero initial probabilities (only the nonzero ones need to be entered)

Columns	Format	Field description
1-2	I2	Number of nonzero initial probabilities
3-8	F6.2	The starting time of the system

10. Initial probability card
Function: specified initial probability and the state to which it pertains (one for each state variable--only one required for machine repair problem)

Columns	Format	Field description
1-6	F6.5	Initial probability
7-8	I2	State variable X1

11. Time specification card
Function: to indicate the time for which transient solutions are required, to indicate what measures are to be printed, and to give a criterion whether or not to continue calculating the times on the following card

Columns	Format	Field description
1	I1	Number of times for which solutions are required (5 or less)
2	I1	Print flag; if value is 1, joint distributions are printed
3	I1	Print flag; if value is 1, marginal distributions are printed
4	I1	Print flag; if value is 1, expectations are printed
5	I1	If value is 1, other cards follow (having the same format as this one)
6-11	F6.5	Stopping criterion (accuracy desired)

12. Time card
Function: gives each time (a maximum of five) for which results are desired

TABLE A2.1--*continued*

Columns	Format	Field description
1-5	F5.2	First time t_1
6-10	F5.2	Second time t_2
⋮	⋮	⋮

the times $t_1, t_2, \ldots$ must be input in increasing order of magnitude

B. Numerical Example Input

Input data for the sample machine repair problem is shown in Table A2.2.

TABLE A2.2

INPUT CARDS FOR PROGRAM QUE

Card Type	Card Input
1	TRANSIENT SOLUTION FOR MACHINE REPAIR PROBLEM
2	010400
3	05
4	ARRIVAL INTO REPAIR STN: (A) NO MACHINE
5	1101000.60
6	0000
7	001
4	ARRIVAL INTO REPAIR STN: (B) 1 MACHINE
5	1101000.7501-00.15
6	0401
7	001
4	SERVICE WHEN ONE REPAIRMAN IS IDLE
5	1101000.50
6	0301
7	-01

TABLE A2.2--*continued*

Card Type	Card Input
4	SERVICE WHEN BOTH REPAIRMEN ARE BUSY
5	1101001.00
6	0502
7	-01
8	END OF EVENT SECTION
9	01000.00
10	1.000000
11	41111.0001
12	00.2500.5000.7501.00
11	41111.0001
12	01.2501.5001.7502.00
11	41111.0001
12	02.2502.5002.7503.00
11	41111.0001
12	03.2503.5003.7504.00
11	41111.0001
12	04.2504.5004.7505.00
11	41111.0001
12	05.2505.5005.7506.00
11	41111.0001
12	06.2506.5006.7507.00
11	41111.0001
12	07.2507.5007.7508.00
11	41111.0001
12	08.2508.5008.7509.00
11	41111.0001
12	09.2509.5009.7510.00
11	41111.0001
12	10.2510.5010.7511.00
11	41111.0001
12	11.2511.5011.7512.00
11	00000

APPENDIX 3: DVOGER SUBROUTINE

A. Input and Options

This section discusses the input requirements for DVOGER. Since DVOGER is a library subroutine, one must write a computer program in order to use it. The advantage is in the flexibility of inputting the parameter values, as well as in the choice of output variables, frequency of printing the solutions, and so forth.

The input structure is as follows.

1. Job and JCL cards. See Section B for the standard and job control language cards.

2. Main program. The main or the calling program is to be written in FORTRAN. The proper dimensioning of arrays, the input mode of parameter values, the number of calls to DVOGER, and the frequency of printing the solution must be determined by the user. Moreover, an external subroutine DFUN is to be written by the user to compute functional values F(y,t) or the Jacobian of F(y,t) .

   The parameters needed for the main program include:

   N = number of first order differential equations

   M = order of Jacobian (M=N)

   T = initial value of independent variable (e.g., time)

   MTH = method indicator

   = { 0, predictor-corrector (Adams) method
   1, variable-order method, suitable for stiff systems (partial derivatives provided by user)
   2, variable-order method (partial derivatives computed by numerical differencing)

Y(1,N) = an input array of initial solutions at T ; array Y is to be declared 8×N

YMAX(N) = an input array of maximum absolute value of solution

HMIN = smallest step size allowable

HMAX = maximum step size allowable

H = step size to be attempted on the next step; this is to be used if it does not cause a larger error than requested

JSTART = { -1, repeat the last step with a new H
 0, initialize the integration (for first call to DVOGER)
 1, take a new step continuing from the last

EPS = maximum error criterion such that the single step error estimates divided by YMAX(I) are less than EPS in norm.

The call to DVOGER is done by the statement,

CALL DVOGER (DFUN, Y, T, N, MTH, MAXDER, JSTART, H, HMIN, HMAX, EPS, YMAX, ERROR, WK, IER).

3. Subroutine DFUN. DFUN is user-supplied and is to be declared by an EXTERNAL DFUN statement in the main or calling program. DFUN specifies the problem for DVOGER. It provides the system of equations and the Jacobian. The parameters include:

YP(1,N) = vector of solution TP

TP = present time

M = order of Jacobian

IND = { 0, DFUN computes F(YP,TP)
 1, DFUN computes Jacobian of F evaluated at (YP,TP)

YP is to be declared as an 8×N array.

B. Numerical Example Input

We shall illustrate the solution of the sample problem using the DVOGER subroutine. The structure of a single job run is as follows.

(a) Job and JCL cards

```
// STANDARD JOB CARD
//           EXEC FØRG2
//FØRT.SYSIN DD *
[Main Program and Subroutine]
//GØ.SYSLIB  DD
//           DD DSN=GWU.IMSL6.LMØD.D,DISP=SHR
//           DD DSN=GWU.IMSL6.LMØD.S,DISP=SHR
//GØ.SYSIN DD *
```

(b) Figure A3.1 shows the main program and subroutine DFUN, and the input and our options, where the transient solutions are required for times T=1,2, and 3 .

(c) Program Output. The step size h was fixed at 0.0001 by specifying HMAX=HMIN=H=0.0001 . The method used was the predictor-corrector method based on the Adams formulas (MTH=0) . The output of transient solutions was printed out at every time increment of .01 starting at t=0 to t=3 . The error tolerance was set at 10^{-7} .

The program below was written for this specific problem, although a more general program can be written to handle any problem with arbitrary parameter values.

The authors have not tested this subroutine for options which give minimum execution time, as this was not the purpose here.

```
      DIMENSION YP(8,10),PW(10,10),Y(8,10),YMAX(10),DY(10)
      DIMENSION WK(140,20),ERROR(10),C(10)
      DOUBLE PRECISION YP
      DOUBLE PRECISION C,D,E,H,P,T,Y,R1,BND,EPS,EUP,EDWN,ENQ1,D1,D2,
     *                 ENQ2,ENQ3,HMAX,HMIN,HNEW,HOLD,TOLD,YMAX,
     *                 ERROR,RACUM,WK,XK,ZERO,HALF,ONE,ONEP,PW
      EXTERNAL DFUN
      KK=0
      N=6
      M=6
      T=0.0D0
      Y(1,6)=0.0D0
      Y(1,5)=0.0D0
      Y(1,4)=0.0D0
      Y(1,3)=0.0D0
      Y(1,2)=0.0D0
      Y(1,1)=1.0D0
      YMAX(1)=1.0D0
      YMAX(2)=1.0D0
      YMAX(3)=1.0D0
      YMAX(4)=1.0D0
      YMAX(5)=1.0D0
      YMAX(6)=1.0D0
      JSTART=0
      IND=0
      MTH=0
      HMAX=1.0D-4
      HMIN=1.0D-4
      H=1.0D-4
      EPS=1.0D-7
      WRITE(6,100)
  100 FORMAT('0',9X,'T',12X,'P0',12X,'P1',12X,'P2',12X,'P3',
     * 12X,'P4',12X,'P5')
      WRITE(6,200) T,Y(1,1),Y(1,2),Y(1,3),Y(1,4),Y(1,5),Y(1,6)
      DO 10 K=1,30000
      CALL DVOGER(DFUN,Y,T,N,MTH,MAXDER,JSTART,H,HMIN,HMAX,EPS,
     * YMAX,ERROR,WK,IER)
      KK=KK+1
      IF(KK.NE.100) GOTO 250
      WRITE(6,200) T,Y(1,1),Y(1,2),Y(1,3),Y(1,4),Y(1,5),Y(1,6)
  200 FORMAT('0',7(2X,F10.8))
      KK=0
  250 CONTINUE
      H=1.0D-2
      IND=0
      MTH=0
      IF(MTH.NE.1) GOTO 10
      PW(1,1)=-.6
      PW(1,2)=.5
      PW(1,3)=0.0
```

Figure A3.1--Program listing to call DVOGER for the machine repair problem, t=1,2 and 3 .

```
      PW(1,4)=0.0
      PW(1,5)=0.0
      PW(1,6)=0.0
      PW(2,1)=.6
      PW(2,2)=-1.1
      PW(2,3)=1.0
      PW(2,4)=0.0
      PW(2,5)=0.0
      PW(2,6)=0.0
      PW(3,1)=0.0
      PW(3,2)=.6
      PW(3,3)=-1.45
      PW(3,4)=1.0
      PW(3,5)=0.0
      PW(3,6)=0.0
      PW(4,1)=0.0
      PW(4,2)=0.0
      PW(4,3)=.45
      PW(4,4)=-1.3
      PW(4,5)=1.0
      PW(4,6)=0.0
      PW(5,1)=0.0
      PW(5,2)=0.0
      PW(5,3)=0.0
      PW(5,4)=.3
      PW(5,5)=-1.15
      PW(5,6)=1.0
      PW(6,1)=0.0
      PW(6,2)=0.0
      PW(6,3)=0.0
      PW(6,4)=0.0
      PW(6,5)=.15
      PW(6,6)=-1.0
   10 CONTINUE
      STOP
      END
      SUBROUTINE DFUN(YP,TP,M,DY,PW,IND)
      DIMENSION PW(10,10),YP(8,10),DY(10)
      DOUBLE PRECISION YP,TP,DY,PW
      IF(IND.EQ.0) GOTO 5
      PW(1,1)=-.6
      PW(1,2)=.5
      PW(1,3)=0.0
      PW(1,4)=0.0
      PW(1,5)=0.0
      PW(1,6)=0.0
      PW(2,1)=.6
      PW(2,2)=-1.1
      PW(2,3)=1.0
      PW(2,4)=0.0
      PW(2,5)=0.0
      PW(2,6)=0.0
      PW(3,1)=0.0
      PW(3,2)=.6
      PW(3,3)=-1.45
```

Figure A3.1--*continued*

```
      PW(3,4)=1.0
      PW(3,5)=0.0
      PW(3,6)=0.0
      PW(4,1)=0.0
      PW(4,2)=0.0
      PW(4,3)=.45
      PW(4,4)=-1.3
      PW(4,5)=1.0
      PW(4,6)=0.0
      PW(5,1)=0.0
      PW(5,2)=0.0
      PW(5,3)=0.0
      PW(5,4)=0.3
      PW(5,5)=-1.15
      PW(5,6)=1.0
      PW(6,1)=0.0
      PW(6,2)=0.0
      PW(6,3)=0.0
      PW(6,4)=0.0
      PW(6,5)=.15
      PW(6,6)=-1.0
      GOTO 10
    5 CONTINUE
      DY(1)=-.6*YP(1,1)+.5*YP(1,2)
      DY(2)=.6*YP(1,1)-1.1*YP(1,2)+YP(1,3)
      DY(3)=.6*YP(1,2)-1.45*YP(1,3)+YP(1,4)
      DY(4)=.45*YP(1,3)-1.3*YP(1,4)+YP(1,5)
      DY(5)=.3*YP(1,4)-1.15*YP(1,5)+YP(1,6)
      DY(6)=.15*YP(1,5)-YP(1,6)
   10 RETURN
      END
```

Figure A3.1--*continued*

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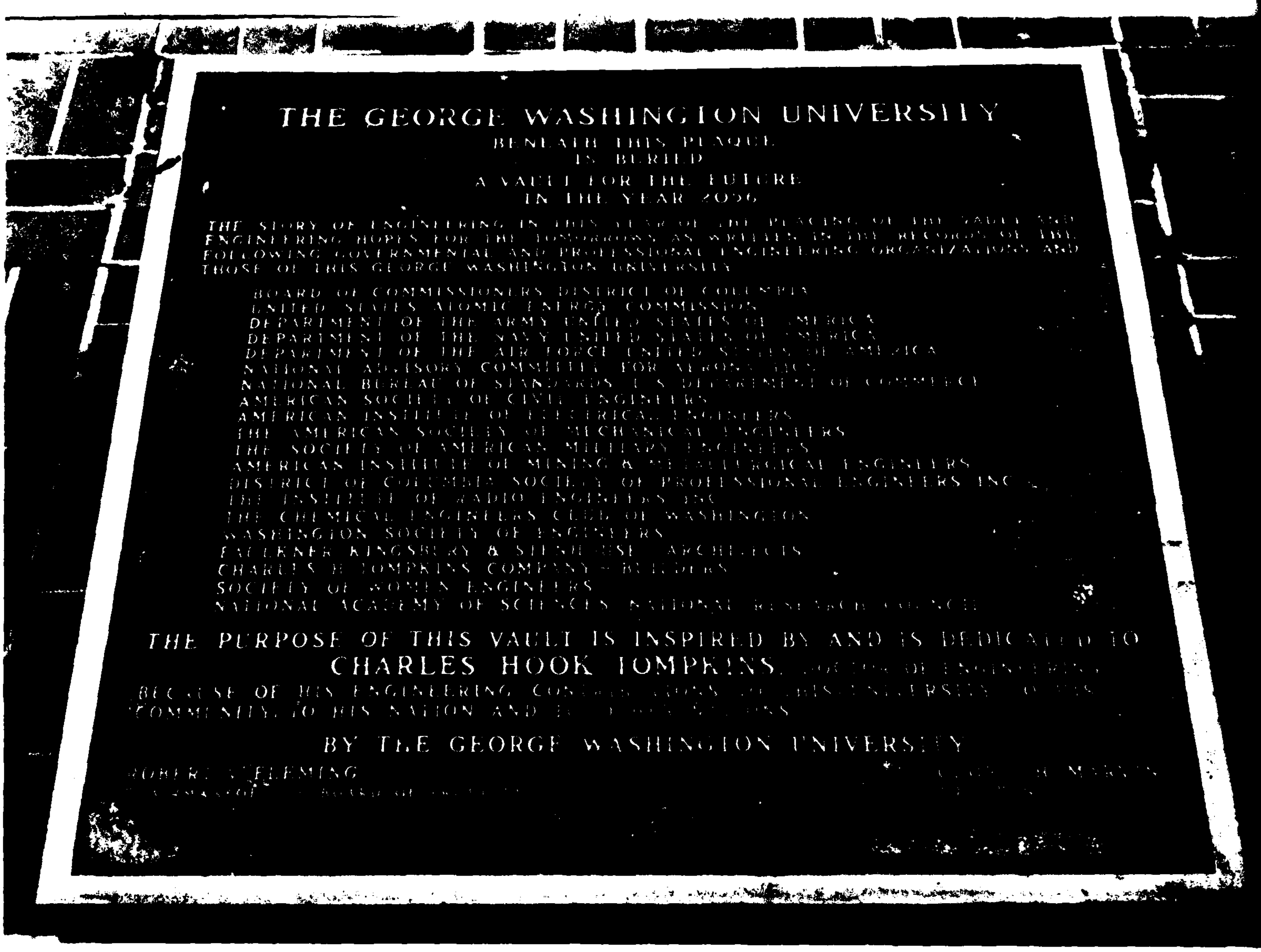

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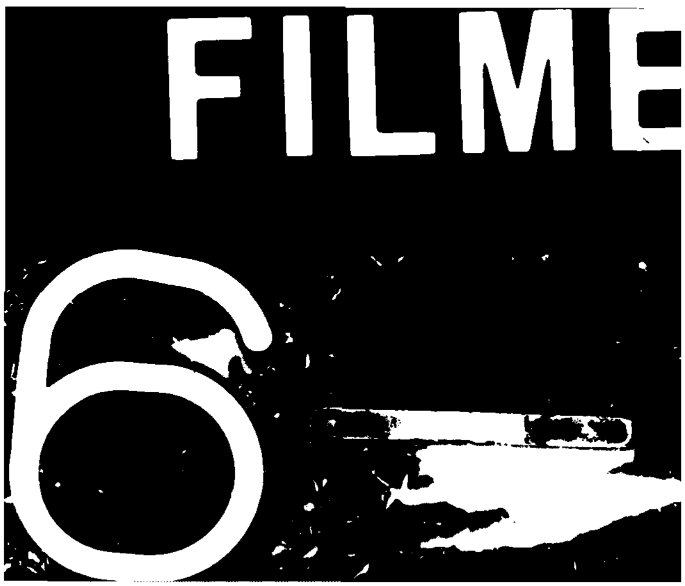
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