A COGNITIVE THEORY OF INTERACTIVE TEACHING. (U)

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Allan Collins and Albert L. Stevens

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Abstract

The theory presented is derived from analysis of a variety of teachers who use the case, inquiry, or Socratic method. This paper attempts to show how techniques that different inquiry teachers use can be applied in diverse domains, such as arithmetic, moral education, and geography. The techniques are illustrated by excerpts from transcripts of some of the different teachers we have been analyzing. The techniques include strategies for constructing cases to give students, for getting students to formulate and test hypotheses, and for teaching students to recognize misconceptions and question authority.
INTRODUCTION

We have been studying transcripts of a variety of interactive teachers. The teachers we have studied all use some form of the case, inquiry, discovery, or Socratic method (Anderson and Faust, 1974; Davis, 1966; Sigel and Saunders, 1979). The topics they are teaching range over different domains: mathematics, geography, moral education, law, medicine, and computer science. But we think it is possible to abstract common elements of their teaching strategies, and show how these can be extended to different domains. In this way we think it is possible to identify the most effective techniques that each of these teachers has discovered, so that they can be made available to anyone who wants to apply these techniques in their own teaching (Collins, 1978).

In a related paper (Collins & Stevens, 1981) we have attempted to specify a formal theory to describe the goals and strategies of the teachers we have been analyzing. In this paper we instead want to pick the most striking techniques they are using, and show how these can be applied across widely disparate domains.

The theory of instruction we are developing in these two papers is at base a descriptive theory in the terms of Reigeluth and Merrill (this volume). We are trying to describe expert
performance, in the current tradition of cognitive science (e.g., Chase & Simon, 1973; Larkin, 1979; Simon & Simon, 1979). By focussing on experts, the descriptive theory becomes a prescriptive theory as well. That is to say, a descriptive theory of expert performance is in fact a prescriptive theory for the non-expert performer.

Our theory of inquiry teaching is domain independent (see Block, this volume). This is not to say that this is the only useful kind of analysis of expert teaching. There is much to be gained from careful examination of the kind of misconceptions students have in different domains (e.g., Brown & Burton, 1978; Stevens, Collins, & Goldin, 1979) and of the specific methods suited to teaching a particular domain (e.g., VanLehn & Brown, 1980). But at the same time task analysis can be used to abstract the significant generalizations about teaching that cut across domains. Comparison across diverse domains makes it possible to see what teachers are doing in a more general way, and forces insights into teaching that might not otherwise be noticed.

The theory is cast in a framework similar to that used by Newell and Simon (1972) to describe human problem solving. It contains three parts:
1. The goals and subgoals of teachers.
2. The strategies used to realize different goals and subgoals.
3. The control structure for selecting and pursuing different goals and subgoals.

Teachers typically pursue several goals simultaneously. Each goal has associated with it a set of strategies for selecting cases, asking questions, and giving comments. In pursuing goals simultaneously, teachers maintain an agenda which allows them to allocate their time among the various goals efficiently (Collins, Warnock, & Passafiume, 1975b; Stevens & Collins, 1977). The theory therefore encompasses goals, strategies, and control structure.

**Terminology Used in the Theory**

Many of the teaching strategies we describe serve to communicate the teacher's understanding of the causal structure of a domain to a student. Thus we need a way to notate a causal structure. One way of representing causal dependencies is in terms of an **and/or graph** (Stevens and Collins, 1980). Figure 1 shows such a graph for the causal dependencies derived by a student in a dialogue on growing grain in different places (Collins, Warnock, Aiello, & Miller 1975a). Each place that was discussed functioned as a case in the terminology of the theory. In the figure, rice growing is the dependent variable and is
treated as a function having two possible values: either you can grow rice or you can't. In other sections of the dialogue, wheat growing and corn growing were discussed as alternative dependent variables. Unlike grain growing, which the student treated as a threshold function, many dependent variables are treated as continuous functions (e.g. a place is colder or warmer), where there is a continuous range of values.

During the course of the dialogue, the student identified four principal factors affecting rice growing: fresh water, a flat area, fertile soil, and warm temperature. These were configured as shown in the diagram. These factors (or independent variables) are linked to rice growing through chains with various intermediate steps. In fact any step in a chain can be considered as a factor.

Given a set of factors and a dependent variable, a rule is any function that relates values of one or more factors to values of the dependent variable. A rule can be more or less complete depending on how well it takes into account all the relevant factors and the entire range of values of the dependent variable. For example, a rule about rice growing might assert that growing rice depends on heavy rainfall and fertile soil. Such a rule is obviously incomplete with respect to the mini-theory shown in Figure 1. A theory specifies the causal structure interrelating different rules. In complex domains like rice growing and medicine, no theory is ever complete.
Figure 1. A student’s analysis of the causal factors affecting rice growing.
Given the dependencies in the diagram, it is apparent that a factor like heavy rainfall is neither necessary nor sufficient for rice growing. It is not necessary because obtaining a supply of fresh water (which is a necessary factor) can also be satisfied by irrigation from a river or lake. It is not sufficient because other factors, such as a warm temperature, are required. When prior steps are connected into a step by an "or", any of the prior steps is sufficient and none is necessary. For example, either heavy rainfall or a river or a lake is a sufficient source for fresh water, but neither heavy rainfall nor a river nor a lake are necessary. In contrast, when prior steps are connected into a step by an "and", all of the prior nodes are necessary and none is sufficient. For example, fresh water is necessary to flood a flat area, but is not sufficient. Any variable not included as a factor in the diagram is effectively treated as irrelevant to the theory.
Independent and Dependent Variables in Different Domains

Table 1 illustrates how the terminology applies to teaching strategies in different domains. We believe that these teaching techniques can be applied to virtually any domain. In Table 1 we are not trying to list all possible independent and dependent variables, nor are we ruling out other possible assignments; these are merely meant to indicate the most common assignments that teachers make.

Let us briefly explain these examples:

1. In arithmetic, a student solves problems in order to learn how to handle different operations, numbers, variables, etc. Because of the procedural emphasis in arithmetic, it is the domain that fits our terminology least well.

2. In art history, the teacher attempts to teach students how different techniques, uses of texture or color, structural interrelationships, etc., create certain effects on the viewer.

3. In law, historical cases are used to teach students how different variables (historical precedents,
<table>
<thead>
<tr>
<th>Discipline</th>
<th>Cases</th>
<th>Independent Variables</th>
<th>Dependent Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic</td>
<td>Problems</td>
<td>Numbers, Operators, Variable Assignments</td>
<td>Answers</td>
</tr>
<tr>
<td>Art History</td>
<td>Pictures, Sculptures</td>
<td>Techniques, Relation of Parts</td>
<td>Effects on Viewer</td>
</tr>
<tr>
<td>Law</td>
<td>Legal Cases</td>
<td>Laws, Past Rulings, Facts of Case</td>
<td>Court Decisions</td>
</tr>
<tr>
<td>Medicine</td>
<td>Medical Cases</td>
<td>Symptoms, History, Course of Symptoms</td>
<td>Diseases</td>
</tr>
<tr>
<td>Geography</td>
<td>Places</td>
<td>e.g., Latitude, Altitude, Currents</td>
<td>e.g., Climate</td>
</tr>
<tr>
<td>Moral Education</td>
<td>Situations, Events</td>
<td>Actions, Rules of Behavior</td>
<td>Fairness</td>
</tr>
<tr>
<td>Botany</td>
<td>Particular Plants</td>
<td>Shapes, Leaf and Branch Structure</td>
<td>Type of Plant</td>
</tr>
</tbody>
</table>
laws, aspects of the particular case, etc.) affect legal outcomes.

4. In medicine, the goal is to teach students how to diagnose different diseases, given patterns of symptoms, their course of development, and the patient's history and appearance.

5. In geography, most variables are treated both as independent and dependent variables on different occasions. For example, average temperature is a dependent variable with respect to the first-order factors, latitude and altitude, and general second-order factors, distance from the sea, wind and sea currents, tree and cloud cover, etc. But, in turn, temperature is a factor affecting dependent variables such as population density, products, land types, etc.

6. In moral education, teachers try to teach rules of moral behavior by considering different situations with respect to the actions and motives of the participants.

7. In botany, one learns what configurations of the shape, branches, leaves, etc., go with what plant names.
Whether a variable is treated as a dependent or independent variable depends on what the teacher is trying to teach. It does not depend on the direction of causality. What functions as a dependent variable is merely what one tries to make predictions about in the real world.

Data Analyzed

The dialogues we have analyzed range over a variety of subject matter domains and take place in a variety of situations. Some are with individual students and some with groups of students. The students range in age from preschoolers to adults. In some cases the teacher has a well-worked out plan as to where the dialogue will go; in others, the teacher does not. We can illustrate the variety by describing briefly each of the dialogues we have analyzed, which we have listed in Table 2.

1. In arithmetic Professor Richard Anderson of the University of Illinois systematically varied different variables in problems of the form $7 \times 4 + 3 \times 4 = ?$ until the student discovered the shortcut to solving them based on the distributive law (i.e., $(7 + 3) \times 4$).
Table 2

Teachers, Students, Domains, and Topics

in the Analyzed Transcripts

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Student</th>
<th>Domain</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anderson, R.C.</td>
<td>Junior high girl</td>
<td>Arithmetic</td>
<td>Distributive Law</td>
</tr>
<tr>
<td>Anderson, R.C.</td>
<td>Hypothetical student</td>
<td>Geography</td>
<td>Factors affecting temperature</td>
</tr>
<tr>
<td>Anderson, R.C.</td>
<td>Hypothetical student</td>
<td>Moral Education</td>
<td>Morality of American revolutionaries</td>
</tr>
<tr>
<td>Beberman, M.</td>
<td>Junior high students</td>
<td>Arithmetic</td>
<td>Numbers vs. Numerals</td>
</tr>
<tr>
<td>Beberman, M.</td>
<td>Junior high students</td>
<td>Arithmetic</td>
<td>Addition of real numbers</td>
</tr>
<tr>
<td>Collins, A.</td>
<td>Adults</td>
<td>Geography</td>
<td>Grain growing</td>
</tr>
<tr>
<td>Collins, A.</td>
<td>Adults</td>
<td>Geography</td>
<td>Population density</td>
</tr>
<tr>
<td>Teacher</td>
<td>Student</td>
<td>Domain</td>
<td>Topic</td>
</tr>
<tr>
<td>------------------</td>
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<td>----------------------------</td>
</tr>
<tr>
<td>Mentor</td>
<td>Hypothetical student</td>
<td>Medicine</td>
<td>Diagnosis of disease</td>
</tr>
<tr>
<td>Miller, A.</td>
<td>Adults</td>
<td>Law</td>
<td>Fairness of sentences</td>
</tr>
<tr>
<td>Schank, R.</td>
<td>Graduate students</td>
<td>Computer Science</td>
<td>Types of plans</td>
</tr>
<tr>
<td>Socrates (Plato)</td>
<td>Slave boy</td>
<td>Arithmetic</td>
<td>Area of squares</td>
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<tr>
<td>Stevens &amp; Collins</td>
<td>Adults</td>
<td>Geography</td>
<td>Causes of rainfall</td>
</tr>
<tr>
<td>Warman, E.</td>
<td>Preschoolers</td>
<td>Moral Education</td>
<td>Who can play with blocks</td>
</tr>
<tr>
<td>Warman, E.</td>
<td>Preschoolers</td>
<td>Moral Education</td>
<td>Character's morality in</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Peter Pan</td>
</tr>
</tbody>
</table>
2. In geography Anderson compared different places to get the student to see that average winter temperature depends on distance-from-the-ocean as well as latitude.

3. In moral education Anderson compared the American revolutionaries to draft resisters to force the student to consider what factors make rebellion right or wrong.

4. In one dialogue Professor Max Beberman of the University of Illinois had junior high students figure out the pattern underlying the wrong answers in an arithmetic test \((5 + 7 = 57, \frac{1}{2} \text{ of } 8 = 3)\), where the answers were derived by manipulating the symbols (i.e., numerals) rather than number concepts. His goal was to teach the difference between numbers and their symbols.

5. In the other Beberman dialogue he got students to abstract the rules for addition of real numbers. He gave them problems to work on graph paper by drawing lines to the right for positive numbers and lines to the left for negative numbers.

6. In two dialogues on grain growing, Collins (the first author) questioned adults about whether
different places grow rice, wheat, and corn in order to extract the factors that determine which grains are grown.

7. In two dialogues on population density, Collins asked why different places have more or fewer people to determine what factors affect population density.

8. Mentor is a computer system developed by Feurzeig, Munter, Swets & Breen (1964). In its medical dialogues the student tries to identify a particular disease by asking the system about symptoms and test results. In turn, the system interrogates the student about his hypotheses.

9. In a television series Professor Arthur Miller of Harvard Law School conducted a dialogue with his audience on whether or not there should be mandatory sentencing, by considering what would be fair sentences for various hypothetical crimes.

10. In his computer science class Professor Roger Schank of Yale asked students first to define a plan, then to form a taxonomy of different types of plans, and finally to analyze a real plan in terms of the taxonomy.
11. In the Meno dialogue Socrates (Plato, 1924) uses systematic questioning to get a slave boy to figure out that the area of a square can be doubled by multiplying each side by 2.

12. In the Stevens and Collins (1977) dialogues, several adults were questioned about the factors leading to heavy rainfall or little rainfall in different places.

13. In a dialogue with a class of preschoolers, Eloise Warman tried to get the students to solve the problem that arose because the boys were always playing with the blocks, thus preventing the girls from playing with them.

14. In another dialogue, Warman questioned the children about the morality of different characters after the children had seen a film of Peter Pan.

Excerpts from many of these dialogues will be shown as we discuss the various strategies that inquiry teachers use to get their students to solve different problems.
THE THEORY

Our theory of interactive teaching has three parts: (1) the goals of teachers, (2) the strategies teachers use, and (3) the control structure governing their teaching. Each of these is discussed below.

Goals of Teachers

There are two top-level goals that teachers in inquiry dialogues pursue: (1) teaching students particular rules or theories, and (2) teaching students how to derive rules or theories. There are several subgoals associated with each of these top-level goals. The top-level goals and subgoals that we have identified are shown in Table 3.

The most frequent goal is for the student to derive a specific rule or theory that the teacher has in mind. For example, in arithmetic Beberman tried to get students to derive the rule for addition of real numbers, and Anderson the distributive law. In geography Anderson tried to get the student to understand how distance-from-ocean affected temperature, and
Table 3
Goals and Subgoals of Teachers

1. Learn a general rule or theory. (e.g., Beberman, Anderson, Collins)
   a. Debug incorrect hypotheses. (e.g., Beberman on numbers and numerals, Socrates, Stevens & Collins, Feurzeig, et al., Anderson on moral education and geography)
   b. Learn how to make predictions in novel cases. (e.g., Beberman, Anderson in arithmetic, Warman, Collins, Feurzeig, et al.)

2. Learn how to derive a general rule or theory (e.g., Schank, Warman)
   a. Learn what questions to ask. (e.g., Schank, Warman)
   b. Learn what is the nature of a theory. (e.g., Schank, Beberman, Stevens & Collins)
   c. Learn how to test a rule or theory. (e.g., Anderson in geography, Schank)
   d. Learn to verbalize and defend rules or theories. (e.g., Warman, Miller, Schank)
Stevens and Collins tried to get students to build a first-order theory of the factors affecting rainfall.

Along with trying to teach a particular rule or theory, teachers often try to elicit and "debug" incorrect rules or theories. The teachers want the student to confront incorrect hypotheses during learning, so that they won't fall into the same traps later. This kind of goal is evident in Beberman's dialogue where he tries to teach the difference between numbers and numerals, in Socrates' dialogues where he traces the consequences of his student's hypothesis down to a contradiction, and in Anderson's dialogues on geography and moral education where he entraps students into revealing their misconceptions.

Another goal that frequently pairs with teaching a given rule or theory is learning how to make novel predictions based on the rule or theory. Simply knowing the structure of a theory is not enough; one must be able to operate on that structure to deal with new problems. For example, Anderson in mathematics gives harder and harder problems for the student to predict the answer. Collins and Stevens in geography start with cases that exemplify first-order factors and gradually move to more difficult cases to predict. Peurzeig et al. are trying to get students to diagnose novel cases. This goal emphasizes the ability to use the theory one has learned.
The other top-level goal of inquiry teachers is to teach students how to derive a new rule or theory. For example, Schank tried to get his students to formulate a new theory of planning, and Warman tried to get her preschoolers to devise a new rule for allocating blocks. Many of the dialogues had a similar aim.

One related ability is knowing what questions to ask in order to derive a new rule or theory on your own. For example, Warman teaches her preschoolers to evaluate any rule by how fair it is. Schank tries to get students to construct a theory by asking taxonomic kinds of questions. Feurzeig et al. emphasize considering different diagnoses before reaching a conclusion.

A goal that underlies many of the dialogues is to teach students what form a rule or theory should take. In Schank's case, the structure of a theory is a set of primitive elements as in chemistry. In one of Beberman's dialogues he taught students the form of arithmetic rules, where variables replace numbers in order to be general. Stevens' and Collins' (1977) notion of a theory of rainfall was a hierarchically-organized, process theory. The principal method for achieving this goal seems to be to construct rules or theories of the idealized type.

Occasionally in the dialogues the teachers pursue a goal of teaching students how to evaluate a rule or theory that has been constructed. For example, Anderson in teaching about what
affects temperature tried to get the student to learn how to control one factor while testing for another. Schank, after his students had specified a set of primitive plan types, tried to get them to test out their theory by applying it to a real world plan (i.e., becoming president). The strategies teachers use are specific to the kind of evaluation methods being taught.

Finally, it was a clear goal of both Warman and Schank to get their students to verbalize and defend their rules or theories. For example, it is clear why Warman's children were always interrupting to give their ideas: she was constantly encouraging and rewarding them for joining in. Similarly, Schank tried to get each student in the class to either offer his ideas, adopt one of the other's ideas, criticize one of the other's ideas, etc. Both stressed the questioning of authority in their dialogues as a means to push students to formulate their own ideas.

These are the top-level goals we have been able to identify so far. In pursuing these goals, teachers adopt subgoals of identifying particular omissions or misconceptions and debugging them (Stevens & Collins, 1977). Thus these top-level goals spawn subgoals that drive the dialogue more locally. This will be discussed more fully in the section on control structure.
Strategies for Inquiry Teaching

We have decided to focus on ten of the most important strategies that inquiry teachers use. The ten strategies are listed in Table 4 together with the teachers who used them. Our plan is to show excerpts of the teachers illustrating each of these techniques, and then show how the technique can be extended to two other domains.

The domains we will use to illustrate the techniques are mathematics, geography, moral education, medicine and law. These domains cover radically different kinds of education: mathematics exemplifies a highly-precise, procedural domain; moral education and law exemplify domains where loosely-structured belief systems are paramount (Abelson, 1979), and geography and medicine exemplify domains where open-ended, causal knowledge systems are paramount (Collins, et al., 1975a).

Selecting positive and negative exemplars

Teachers often choose positive or negative paradigm cases in order to highlight the relevant factors. Paradigm cases are cases where the relevant factors are all consistent with a particular value of the dependent variable. This strategy was
Table 4
Different Instructional Techniques
and their Practitioners

1. Selecting positive and negative exemplars (Anderson; Miller; Stevens & Collins)
2. Varying cases systematically (Anderson; Stevens & Collins)
3. Selecting counterexamples (Collins; Anderson)
4. Generating hypothetical cases (Warman; Miller)
5. Forming hypotheses (Warman; Schank; Anderson; Beberman)
6. Testing hypotheses (Anderson; Schank)
7. Considering alternative predictions (Feurzeig et al.; Warman)
8. Entrapping students (Anderson; Collins; Feurzeig et al.)
9. Tracing consequences to a contradiction (Socrates; Anderson)
10. Questioning authority (Schank; Warman)
most evident in the geographical dialogues of Stevens and Collins (1977), but it is also apparent in Anderson's arithmetic dialogue, and Miller's law dialogues.

We can illustrate this strategy for geography in terms of selecting paradigm cases for rainfall. In the beginning of their teaching Stevens and Collins chose positive exemplars such as the Amazon, Oregon, and Ireland where all the relevant factors had values that lead to heavy rainfall. They also chose negative exemplars like southern California, northern Africa, and northern Chile where all the relevant factors have values that lead to little rainfall. Only later would they take up cases like the eastern United States or China where the factors affecting rainfall have a more complicated pattern.

The method that Anderson used to select cases to illustrate the distributive law in arithmetic was based on the strategy of selecting positive exemplars. For example, the first problem he presented was $7 \times 5 + 3 \times 5 = ?$ He wanted the student to see that because the 5 entered the equation twice, the problem could be easily solved by adding 7 and 3 and multiplying by 5. There are a number of aspects of this particular problem (and the subsequent problems he gave) that make it a paradigm case: (1) because 7 and 3 add up to 10, the 5 appears as the only significant digit in the answer, (2) the 5 appears in the same position in both parts of the equation, (3) the 5 is distinct
from the other digits in the equation. All these serve to highlight the digit the student must factor out.

In his work on discovery learning Davis (1966) advocated a similar strategy for selecting cases. In getting students to discover how to solve quadratic equations by graphing them, he would give problems of the form: \( x^2 - 5x + 6 = 0 \), where the roots are 3 and 2, or \( x^2 - 12x + 35 = 0 \), where the roots are 5 and 7. The fact that both roots had the same sign was essential to getting the students to make the correct discovery; only when there are roots of the same sign is it readily apparent that the \( x \) coefficient is the sum of the two roots.

This same attempt to pick paradigm cases is apparent in Miller’s law dialogues. In considering what should be a mandatory sentence for a crime, he considers worst cases, where all the relevant factors (e.g., tough guy, repeat offender, no dependents) would lead a judge to give a heavy sentence, and best cases, where all the relevant factors (e.g., mother with dependents, first offender) would lead to a light sentence. This exactly parallels the Stevens and Collins strategy in geography.

There are also other strategies for picking positive and negative exemplars that we have named "near hits" and "near misses" after Winston (1973). Near misses are cases where all the necessary factors but one hold. For example, Florida is a
near miss for rice growing, since rice could be grown there except for the poor soil. Near misses highlight a particular factor that is necessary. Near hits are their counterparts for sufficient factors: cases which would not have a particular value on the dependent variable, except for the occurrence of a particular sufficient factor. For example, it is possible to grow rice in Egypt despite little rainfall, because of irrigation from the Nile. Near misses and near hits are important strategies for highlighting particular necessary or sufficient factors.

Varying cases systematically

Teachers often choose cases in systematic sequences to emphasize particular factors that they want the student to notice. This is most evident in the dialogue where Anderson got a junior high school girl to derive the distributive law in arithmetic. He started out giving her problems to work, like $7 \times 5 + 3 \times 5$ and $7 \times 12 + 3 \times 12$, where the only factor that changed was the multiplier, which shows up in the answer (50 or 120) as the significant digits. He then gave problems where he varied the addends systematically, $70 \times 8 + 30 \times 8$ and $6 \times 4 + 4 \times 4$, but preserved the fact that the multiplier formed the significant digits. Then he relaxed that constraint to examples like $11 \times 6 + 9 \times 6$, $110 \times 4 + 90 \times 4$, and finally $4 \times 3 + 8 \times 3$, so that the student would formulate the
distributive law in its most general form. Anderson was systematically varying one factor after another in the problems he gave the student, so that the student could see how each factor in turn affected the answer.

We can illustrate this technique in geography by showing how teachers can systematically choose cases to vary the different factors affecting average temperature. First, the teacher might systematically vary latitude while holding other variables constant (e.g., the Amazon jungle, the Pampas, Antarctica), then vary altitude while holding other variables constant (e.g., the Amazon jungle, the city of Quito, the top of Kilimanjaro), then other factors such as distance-from-the-ocean, sea and wind currents, cloud and tree cover, etc. The separation of individual factors in this way is precisely what Anderson was doing in arithmetic.

In moral education it is possible to consider what punishment is appropriate by considering cases where the punishable behavior is systematically varied in different respects. For example, the teacher could systematically vary the malice of the intention, the severity of the act, and the damage of the consequences one at a time while holding each of the other factors constant.
In Collins and Stevens (1981) we point out four different ways this kind of systematic variation can occur. The cases cited above involve differentiation; in differentiation a set of non-focused factors is held constant, while the teacher shows how variation of one factor affects the dependent variable. Its inverse, generalization, occurs when the teacher holds the focused factor and the dependent variable constant, while varying the non-focused factors. The two other strategies highlight the range of variability of either the focused factor or the dependent variable: in one strategy the teacher holds the focused factor constant while showing how widely the value of the dependent variable may vary (because of variation in non-focused factors); in the other strategy the teacher holds the dependent variable constant and shows how widely the value of the focused factor may vary. These four strategies allow teachers to stress various interactions between different factors and the dependent variable.

Selecting counterexamples

A third method of choosing cases that teachers use in the dialogues we have analyzed is selecting counterexamples. We can illustrate two different kinds of counterexamples in the following short dialogue from Collins (1977) on growing rice:
AC: Where in North America do you think rice might be grown?
S: Louisiana.
AC: Why there?
S: Places where there is a lot of water. I think rice requires the ability to selectively flood fields.
AC: OK. Do you think there's a lot of rice in, say, Washington and Oregon? (Counterexample for an insufficient factor)
S: Aha, I don't think so.
AC: Why?
S: There's a lot of water up there too, but there's two reasons. First the climate isn't conducive, and second, I don't think the land is flat enough. You've got to have flat land so you can flood a lot of it, unless you terrace it.
AC: What about Japan? (Counterexample for an unnecessary factor)
S: Yeah, well they have this elaborate technology I suppose for terracing land so they can flood it selectively even though it's tilted overall.

The first counterexample (for an insufficient factor) was chosen because the student gave rainfall as a sufficient cause of rice growing. So a place was chosen that had a lot of rainfall, but
no rice. When the student mentioned mountains as a reason why no rice is grown in Oregon, Japan was chosen as a counterexample (for an unnecessary factor), because it is mountainous but produces rice. As can be seen in the dialogue, counterexamples like these force the student to pay attention to different factors affecting the dependent variable.

One can see this same strategy for choosing a counterexample applied to moral education in the following excerpt from Anderson (in Collins, 1977):

RA: If you'd been alive during the American Revolution, which side would you have been on?
S: The American side.
RA: Why?
S: They were fighting for their rights.
RA: You admire people who fight for their rights. Is that true?
S: Yes.
RA: How about the young men who broke into the draft office and burned the records? Do you admire them? (Counterexample for insufficient factors)
S: No, what they did was wrong.

What Anderson had done is to pick a counterexample for an insufficient factor. He knows the student does not admire
everyone who fights for their rights, so there must be other factors involved. This line of questioning forces the student to think about some of the different factors that determine the morality of an action.

We can illustrate the use of counterexample in mathematics with an example from analytic geometry. Suppose a student hypothesizes on the basis of the graph for \( x^2 + y^2 = 1 \) (which yields a circle of radius 1) that the term on the right of the equation is the radius of the circle. Then the teacher might ask the student to plot the graph of \( x^2 + y^2 = 4 \). The student will find that this yields a circle of radius 2 rather than radius 4, and may infer that the radius is the square root of the term on the right. Learning to construct counterexamples is particularly useful in mathematics where many proofs and intuitions rest upon this skill.

Two kinds of counterexamples were seen in the first excerpt from geography: counterexample for insufficient factors, and for unnecessary factors. There can also be counterexamples for irrelevant factors and incorrect values of factors (Collins & Stevens, 1981).

Generating hypothetical cases

In the dialogues of Eloise Warman on moral education and Arthur Miller on fairness of sentencing, they often generate
hypothetical cases to challenge their students' reasoning. Warman's use of the strategy was most apparent in a class discussion about a problem that arose because the boys (B) in the class were always playing with the blocks, thus preventing the girls (G) from playing with them. Two examples of Warman's use of the strategy occur in the excerpt below:

B: How about no girls play with anything and boys play with everything.
EW: OK. Let's take a vote. Boys, how about if you don't play with any toys here in school? Would you like that? (Hypothetical case)
B: No
G: Yea.
EW: OK. David said something. What did you say?
B: I would stay home.
EW: He would stay home. OK. How about if we had boys could play with everything but blocks? (Hypothetical Case)
B: No. Rats.

What Warman does systematically is to illustrate the unfairness of the current or a proposed situation by reversing the roles as to who gets the advantage. Thus in the first hypothetical case above she reverses the boy's proposed rule by substituting boys for girls. In the second hypothetical case she reverses the
current situation where girls don't get to play with the blocks. She reverses the polarity of some factor in the situation to force the students to see what factors will make things fairer.

A somewhat different version of this strategy is used by Miller in his television show Miller's Court. In a show on sentencing, for example, he carried on a dialogue along the following lines with one man (M).

AM: You believe that there should be mandatory sentences? What do you think should be the sentence for armed robbery?
M: 10 years.
AM: So if a hardened criminal robs a bank of $1000, he should get 10 years in prison with no possibility of parole? (Hypothetical Case)
M: Yes that seems fair.
AM: What if a poor young woman with children, who needs money to feed her kids, holds up a grocery store with an unloaded gun. Should she get 10 years too? (Hypothetical Case)

What Miller does is entrap the man into a confirmation of a harsh rule of sentencing with one hypothetical case. His second case faces the man with the opposite extreme (as did Warman) where the man's rule is satisfied (armed robbery), but where other factors
override the man's evaluation of the fairness of the rule. Both Mille: and Warman use hypothetical case construction to force their respondents to take into account other factors in forming a general rule of behavior.

We can illustrate how this technique can be extended to geography with an example. Suppose a student thinks rice is grown in Louisiana because it rains a lot. The teacher might ask "Suppose it didn't rain a lot in Louisiana, could they still grow rice?" In fact, irrigation could be used to grow rice. In the Collins and Stevens (1981) paper, we outline four different kinds of hypothetical cases the teacher can construct, which parallel the four kinds of counterexamples.

Forming hypotheses

The most prevalent strategy that teachers use is to get students to formulate general rules relating different factors to values of the dependent variable. We can illustrate these attempts in all three domains by excerpts from Beberman, Anderson, and Warman.

In one dialogue Beberman was trying to get students to formulate a general rule for addition of real numbers. To this end he gave students a procedure to work through on graph paper to add a set of real numbers, by going right for positive numbers and going left for negative numbers. After a while students
found a shortcut for doing this: they would add together the
positive numbers, then the negative numbers, and take the
difference. He subsequently tried to get them to formulate this
shortcut procedure into a few general rules for adding real
numbers, which can be seen in the dialogue excerpt below:

MB: I want to state a rule here which would tell
somebody how to add negative numbers if they didn’t
know how to do it before. Christine?

S: The absolute value -- well -- a plus b equals uh --
negative --

MB: Yes, what do we do when we try to do a problem like
that? Christine is on the right track. What do
you actually do? Go ahead, Christine.

S: You add the numbers of arithmetic 5 and 7, and then
you --

MB: I add the numbers of arithmetic 5 and 7; but how do
I get the numbers of arithmetic when I’m talking
with pronumerals like this?

We can illustrate the attempt to get students to formulate
rules in geography with an excerpt from Anderson (in Collins,
1977) on the factors affecting average temperature. In an
earlier part of the dialogue the student had been forced to the
realization that there were places in the northern hemisphere
that were warmer on the average than places to the south of them.

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The following excerpt shows Anderson's emphasis on hypothesis formation:

S: Some other factor besides north-south distance must also affect temperature.
RA: Yes! Right! What could this factor be?
S: I don't have any idea.
RA: Why don't you look at your map of North America. Do you see any differences between Montana and Newfoundland?
S: Montana is in the center of the country. Newfoundland is on the ocean.
RA: What do you mean by "in the center of the country?"
S: It's a long way from the ocean.
RA: Do you suppose that distance from the ocean affects temperature?
S: I'm not sure. It would just be a guess.
RA: True! The name for such a guess is a hypothesis. Supposing the hypothesis were correct, what exactly would you predict?
S: The further a place is from the ocean, the lower the temperature will be in the winter.

Warman in her dialogue on who could play with blocks never explicitly asked the children to formulate a new rule, but she stated the problem and encouraged them strongly whenever anyone
offered a new rule for allocating the blocks. This can be seen in the two short excerpts below; in the first she rejects a proposed rule because it is the same as the current rule, and in the second she accepts the rule as the solution to the problem.

G: I've got a good idea. Everybody play with blocks.

EW: What do you think about that?

B: Rats.

EW: Isn't that the rule we have right now? That everyone can play with blocks. But what's the problem?

----

B: I've got one idea.

EW: Oh, Greg's got a good idea. (Reward rule formulation.)

B: The girls can play with the big blocks only on 2 days.

EW: Hey, listen we come to school 4 days a week. If the girls play with the big blocks on 2 days that gives the boys 2 other days to play with blocks. Does that sound fair? (Restate rule. Ask for rule evaluation.)

G: Yea! Yea!
There are a variety of strategies for prodding students to formulate hypotheses about what factors are involved and how they affect the dependent variable. These are enumerated in Collins & Stevens (1981) as strategies for identifying different elements in a rule or theory.

Evaluating hypotheses

Sometimes teachers follow up the hypothesis formation stage by trying to get students to systematically test out their hypotheses. This strategy is seen most clearly in the Anderson and the Schank dialogues. Anderson tries to get the student to test his hypothesis by comparing temperature in different places in the real world. Schank tries to get his students to test out their notions about what are the basic elements in planning by applying their taxonomy to a real world problem, such as running for president. We will show how testing hypotheses can be applied to the three examples shown above of hypothesis formation.

We will start with the Anderson example above, where the student's hypothesis was that distance from the ocean affects average temperature. The dialogue continued as follows:

RA: How could you test your hypothesis?
S: By comparing temperatures of places different distances from the ocean.

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RA: Very good. Let's do that. Suppose we take St. Louis, Missouri. Which would be best to compare, Atlanta, Georgia or Washington, D.C.?
S: I'm not sure.
RA: Why don't you look at your map? Maybe that will help you decide.
S: I would pick Washington.
RA: Why?
S: Because it's at the same latitude as St. Louis.
RA: Why is that important?
S: Well, if Atlanta were warmer, I wouldn't know whether it was because it was nearer the ocean or further south.

What Anderson is doing here is teaching the student how to hold other variables constant when testing out a hypothesis. This is also one of the strategies used by teachers in the systematic variation of cases described earlier.

After Beberman got the students to formulate several rules for the addition of real numbers, he could have had students test their rules by generating widely different examples to see if the rules as formulated could handle them. For example one rule the class formulated was "If both a and b are negative, add the absolute value of a and the absolute value of b and give the sum a negative sign." There were such rules to handle different
cases. To test out the rules he could get students to generate different pairs of numbers to see if the rules produce the same answers as the line drawing procedure. In this case it is particularly important to make sure the rules work for special cases, such as when a or b equal zero.

In the Warman excerpt, where Greg formulates a rule that boys get to play with the blocks on two days and girls on two days, she explicitly asks students to evaluate the rule for fairness. This in fact led later to one amendment, that the girls get to go first since they have been deprived previously. She could have gone further in evaluating Greg's rule by asking the students to consider its fairness for all the people involved: boys, girls, teachers, particular children, etc. If they had done this they might have amended the rule further to let the child (or children) who was playing with the blocks to invite one member of the opposite sex to play, since one of the boys had expressed a desire to play with one of the girls. They could have even tested the rule farther in this situation by trying it out for a day where the boys got the blocks half the time and the girls half the time, to see whether the new rule worked.

There are different aspects to hypothesis evaluation, such as controlling variables or testing out special cases, that are important for students to learn. These can be brought out by getting students to systematically evaluate their hypotheses.
Considering alternative predictions

Hypothesis formation is concerned with identifying different factors and how they relate to values of the dependent variable. Thus Anderson was trying to get students to consider different factors that affect temperature and to specify a rule relating the factors to temperature. Sometimes teachers, particularly in the Feurzeig, et al. and Warman dialogues, try to get the students to consider different alternative values for the dependent variable.

We can see the teacher trying to get the student to consider alternative predictions in the following dialogue on medical diagnosis (Feurzeig, et al., 1964):

T: We’ve considered one possibility (i.e., pulmonary infarction). Do you have another diagnosis in mind?
S: No.
T: In that case I’d like to talk about viral pneumonia. The tachycardia, high WBC, elevated respiratory rate, shaking chills, bloody sputum, and severe pleural pain all lend weight to that diagnosis -- right?

What the teacher is doing here is trying to get the student to consider how the values of the known factors fit with different
possible values of the dependent variable. This forces the student to weigh different alternatives in making any predictions or judgments. This same strategy was applied by Collins in his dialogues on the factors affecting grain growing when he would ask students to consider whether wheat or rice or corn could be grown in the same place.

An excerpt from Warman illustrates the same strategy applied to moral education. The excerpt is from a dialogue discussing the morality of the different characters in Peter Pan, which the children had just seen:

EW: Are the Indians good in Peter Pan?
S: Good.
EW: Why are the Indians good?
S: No. It's the Chief, because he caught all of the boys.
EW: So the Chief catches all the boys; so is the Chief good?
S: Nope. He's bad.
EW: He's bad? Is he always bad? Or is he good sometimes, or what do you think? That's a tough question. Is the Indian Chief always bad, or is he sometimes bad? What would you say?
Here she tries to get the children to consider different points on the morality continuum, as to where the actions of the Indian Chief fall on that continuum. Sometimes dependent variables have a discrete set of values as in medicine, and sometimes they are continuous variables, as in moral education, but in either case it is possible to get students to consider different possible values.

This strategy can be illustrated in mathematics with an example from geometry. Suppose the teacher wants the student to figure out what the regular polygon is with the most number of sides that can cover a plane surface. The student might have decided that 4 must be the answer, because you can cover a surface with triangles and squares but not with pentagons. The teacher might press the student to consider 6, 8, and 12 as possible answers. Part of a mathematician's skill depends on being able systematically to generate other plausible solutions and to prove they can not hold.

Encouraging students to consider other values of the dependent variable forces them into the more powerful methods of differential diagnosis or comparative hypothesis testing as opposed to the more natural tendency to consider only one alternative at a time. This is particularly important to prevent people from jumping to a conclusion without considering the best alternative.
Entrapping students

The teachers we have analyzed often use entrapment strategies to get the students to reveal their underlying misconceptions. This is most apparent in the dialogues of Anderson, Collins, and Feurzeig, et al. We can illustrate the use of entrapment in the different domains by excerpts from three of the dialogues.

Anderson frequently uses a kind of entrapment strategy where he takes the student's reasons and turns them into a general rule. One example of this occurred in the excerpt on page 25, where he formulated the general rule "You admire people who fight for their rights", and then suggested a counterexample. This strategy can be seen later in the same dialogue when the student defended the American revolutionaries:

S: They were in the right. They didn't have any voice in the government. There was taxation without representation.

RA: So you would say that people do have a right to disobey laws if they don't have a voice in the government? (Formulate a general rule for an insufficient factor.)

Anderson's formulation of general rules can be applied not only to reasons based on insufficient factors, as in these two
examples, but also to unnecessary factors, irrelevant factors and incorrect values of factors (Collins & Stevens, 1981).

Another somewhat different kind of entrapment can be seen in the following dialogue excerpt from Collins (1977):

AC: Is it very hot along the coast here? (points to Peruvian coast near the equator) (Entrapment into a prediction based on insufficient factors)
S: I don't remember.
AC: It turns out there's a very cold current coming up along the coast; and it bumps against Peru, and tends to make the coastal area cooler, although it's near the equator.

Here the teacher tries to entrap the student into a wrong prediction based on the equatorial latitude, which is overridden in this case by an ocean current. Anderson (in Collins, 1977) also uses this kind of entrapment in his geographical dialogue when he asks "Which is likely to have the coldest winter days, Newfoundland or Montana?" The student is likely to guess Newfoundland because it is further north. Entrapment into incorrect predictions can also occur in different forms (Collins & Stevens, 1981).

Another kind of entrapment occurs in the medical dialogues of Feurzeig et al. (1964). This can be seen in the excerpt on
page 36 where the teacher suggests that several symptoms lend weight to a diagnosis of viral pneumonia. In fact all the symptoms mentioned either have incorrect values or are irrelevant to a diagnosis of viral pneumonia. Here the entrapment takes the form of a suggestion that particular factors lead to a given value of the dependent variable.

We can illustrate how entrapment might be used in mathematics by considering Socrates' dialogue with the slave boy in the Meno dialogue (Plato, 1924), where he tried to get the boy to figure out the area of a square:

Soc: So the space is twice two feet?
Boy: Yes
Soc: Then how many are twice two feet? Count and tell me.
Boy: Four, Socrates.
Soc: Well could there be another such space, twice as big; but of the same shape, with all the lines equal like this one?
Boy: Yes.
Soc: How many feet will there be in that, then?
Boy: Eight
Soc: Very well, now try to tell me how long will be each line of that one. The line of this one is two feet; how long would the line of the double one be?
Boy: The line would be double, Socrates, that is clear.

Here the boy is entrapped into a wrong hypothesis, that double the area is produced by a side double in length, in a manner similar to the geographical example above. The entrapment would have been even stronger if Socrates had suggested, "Would the line of the double square be twice as long?" This is entrapment into an incorrect prediction, but other forms of entrapment are equally applicable with respect to mathematical rules or factors.

Entrapment is used to force the student to face difficulties that may arise later in other circumstances. By getting the student to reveal and correct his misconceptions during learning, the teacher assures that the student has a deeper understanding of the subject matter.

Tracing consequences to a contradiction

One of the ways teachers try to get students to correct their misconceptions is to trace the consequences of the misconceptions to some conclusion that the student will agree cannot be correct. This kind of approach is most evident in Socrates' Meno dialogue and Anderson's moral education dialogue.

We can illustrate Socrates' use of this technique by picking up just after the slave boy had predicted that to double the area of a square, you must double the length of the side (the line segments are shown in Figure 2):
Soc: Then this line (ac) is double this (ab), if we add as much (bc) to it on this side.
Boy: Of course.
Soc: Then if we put four like this (ac), you say we shall get this eight-foot space.
Boy: Yes.
Soc: Then let us draw these four equal lines (ac, cd, de, ea). Is that the space which you say will be eight feet?
Boy: Of course.
Soc: Can’t you see in it these four spaces here (A, B, C, D), each of them equal to the one we began with, the four-foot space?
Boy: Yes.
Soc: Well how big is this new one? Is it not four times the old one?
Boy: Surely it is!
Soc: Is four times the old one, double?
Boy: Why no, upon my word!
Soc: How big then?
Boy: Four times as big!
Soc: Then, my boy, from a double line we get a space four times as big, not double.
Boy: That’s true.
What Socrates has done is follow the chain of consequences until the slave boy recognizes the contradiction.

Anderson applied this same strategy in his dialogue comparing the Vietnam draft resisters to the American revolutionaries. In the segment shown there is a series of four questions where he traces out several different consequences of the student's previous statements until the student finally finds a distinction that differentiates the two cases for him:

S: I don't think Viet Nam is such a good thing, but you just can't have individuals deciding which laws they are going to obey.
RA: So, you would say the American revolutionaries should have followed the law.
S: Yes, I guess so.
RA: If they had obediently followed all the laws we might not have had the American Revolution. Is that right?
S: Yes.
RA: They should have obeyed the laws even if they believed they were unjust. Is that right?
S: I'm not sure. I suppose I have to say yes.
Figure 2. Diagram referred to by Socrates in the Meno dialogue.
RA: In other words what the American revolutionaries did was wrong. That's true isn't it?

S: No, damn it. They were in the right. They were fighting for their liberty. They didn't have any voice in the government. There was taxation without representation.

We can illustrate how this same technique can be extended to geography with an example from one of the Stevens' and Collins' dialogues on the causes of rainfall in the Amazon. When asked where the moisture evaporated from that caused the heavy rainfall in the Amazon jungle, the student incorrectly answered the Amazon River. The implications of this could be traced with a series of questions such as: (1) Does most of the water in the river evaporate or flow into the ocean? (2) If most of the water flows into the ocean, won't the process soon dry up? The student will quickly be forced by this line of reasoning to see that most evaporation must occur from the ocean rather than from the river.

Tracing consequences in this way forces students to actively debug their own theories. This may prevent students from making similar mistakes in the future, and it teaches them to evaluate a theory by testing out its consequences.
Questioning authority

A striking aspect of both the Schank and Warman dialogues is the effort they make to get the students not to look to the teacher or the book for the correct answers, but rather to construct their own theories. This is a particularly important strategy to Schank and Warman's goal of teaching students how to develop their own rules or theories.

We can illustrate how Schank and Warman apply the strategy with short excerpts from their class sessions. The segment from Schank shows him trying to get students to form a taxonomy of basic types of plans. He complains when he recognizes that they are just repeating what they read in the book:

RS: Give me some categories of plans.
S2: Bargain object. (laughter)
RS: Give me a better one than that. Anyway it's not a category, that's a plan.
S1: Plans to obtain objects. (Schank writes it down)
S4: Is there a reason why we want that category?
RS: No, I'm just looking for gross categories.
S2: to establish social control over something.
RS: The of you are agreeing that everything from the book is gospel. It's all right. Give me something new -- I wrote those -- invent something.
Warman comes to the same problem in her dialogue when she is trying to get her preschoolers to develop a new rule to decide who is allowed to play with the toy blocks in the classroom. The current rule is that anybody can play with anything, but the boys are dominating the use of the blocks, thus keeping the girls from them. The following excerpt shows her argument against deciding by authority:

EW: Do you think that it should be all right for only one person should get to make all the choices (sic) for who gets to play with blocks. Or do you think it should be something we all decide on?

G: I think it should be the teachers.

EW: But why just the teachers? It doesn’t seem to work. We had an idea. We’ve been trying.

The questioning of authority is an important strategy for getting students to think like scientists, to get them to try out theory construction on their own, and to get them to question those things that may appear to be givens.

Dialogue Control Structure

The control structure that the teacher uses to allocate time between different goals and subgoals may be the most crucial aspect for effective teaching. An earlier attempt at a theory of the control structure was developed in Stevens and Collins.
(1977), based on tutors' comments about what they thought a student knew after each answer and about why they asked each question. The four basic parts of the control-structure theory are: (1) a set of strategies for selecting cases with respect to the top-level goals, (2) a student model, (3) an agenda, and (4) a set of priority rules for adding goals and subgoals to the agenda.

Given a set of top-level goals, the teacher selects cases that optimize the ability of the student to master those goals. There appear to be several overall strategies that teachers apply in selecting cases:

1. **Select cases that illustrate more important factors before less important factors.** For example, in teaching about rainfall, Stevens and Collins move from cases like the Amazon and Ireland that exemplify a first-order theory to cases like Eastern America or Patagonia where the factors are more complex.

2. **Select cases to move from concrete to abstract factors.** Teachers tend to select cases that emphasize concrete factors initially, in order to make contact with the student's experience and move to cases that emphasize more abstract factors.
3. **Select more important or more frequent cases before less important or less frequent cases.** Other things being equal, a geography teacher will select cases like the United States, Europe and China that are more important. A medical professor will select the most frequent diseases and the ones that are most important to diagnose.

When a case is selected, the teacher begins questioning students about the values of the dependent and independent variables, and the rules interrelating them. The answers reveal what the student does and does not know with respect to the teacher's theory (Stevens & Collins, 1977). As the teacher gains information about the student's understanding, factors in the teacher's theory are tagged as known, in error, not known, etc. This is the basic student model.

The teacher's model of the student also includes a priori expectations of how likely any student is to know a given piece of information in the theory (Collins, et al., 1975b). As a particular student reveals what he knows, his level of sophistication with respect to the teacher's theory can be gauged. From this an estimate can be made as to the likelihood that the student will know any given factor in the theory. This enables the teacher to focus on adding information near the edge of what the student knows a priori. The details of how this operates are given in Collins, et al., (1975b).
As specific bugs (i.e., errors and omissions) in the student's theory or reasoning processes are identified, they create subgoals to diagnose the underlying causes of the bug and to correct them. Often the questions reveal multiple bugs. In such cases the teacher can only pursue one bug at a time. Thus there has to be an agenda, which orders the subgoals according to which will be pursued first, second, third, etc.

In adding subgoals to the agenda, there must be a set of priority rules. The priorities we found in the earlier work (Stevens & Collins, 1977) were:

1. Errors before omissions.
2. Prior steps before later steps.
3. Shorter fixes before longer fixes.
4. More important factors before less important factors.

Errors take priority over omissions because they have more devastating consequences. Prior steps take priority because the teacher wants to take things up in a rational order, to the degree the order is not determined by the student's responses. Shorter fixes, like telling the student the right answer, take priority because they are easier to complete. More important factors take priority because of the order implied by the overall goals.
When more than one bug has been diagnosed, the teacher holds all but the one pursued on the agenda, in order of their priority. When he has fixed one bug, he takes up the next highest priority bug, and attempts to fix that. Sometimes when he is trying to fix one bug, he diagnoses another bug. If the new bug is of a higher priority, he sometimes interrupts the goal he is pursuing to fix the higher priority bug. Thus in the dialogues there is a pattern of diagnosing bugs at different times and holding them there until there is time to correct them.

CONCLUSION

These techniques of inquiry teaching are designed to teach students to construct rules and theories by dealing with specific cases, and to apply these rules and theories to new cases. In this process the student is learning two kinds of things: (1) specific theories about the knowledge domain, and (2) a variety of reasoning skills. In some sense the inquiry method models for the student the process of being a scientist.

The kinds of reasoning skills we think the student learns from this process are: forming hypotheses, testing hypotheses, making predictions, selecting optimal cases to test a theory, generating counterexamples and hypothetical cases, distinguishing between necessary and sufficient conditions, considering alternative hypotheses, knowing the forms that rules and theories
can take, knowing what questions to ask, etc. In short, all the reasoning skills that scientists need arise in inquiry teaching.

Furthermore the technique is exceptionally motivating for students. They become involved in the process of creating new theories or recreating theories that have been developed over centuries. It can be an exhilarating experience for the students.

In summary, by turning learning into problem-solving, by carefully selecting cases that optimize the abilities the teacher is trying to teach, by making students grapple with counterexamples and entrapments, the students are challenged more than by any other teaching method. They come out of the experience able to attack novel problems by applying these strategies themselves.
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