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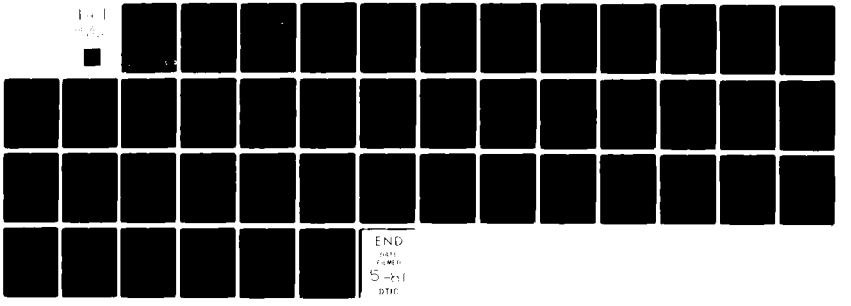

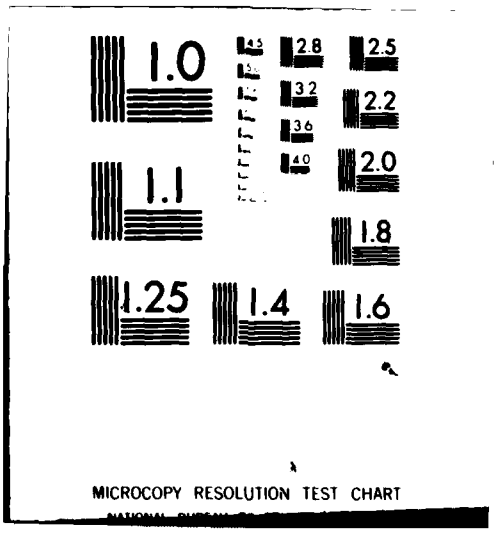
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THE UNIVERSITY OF IOWA

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On the Use of a Cumulative Distribution as a Utility Function in Educational or Employment Selection

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The University of Iowa

Abstract

Formal decision theory can make important contributions to educational or employment decision-making, provided one can quantify the utilities of different possible outcomes such as test scores, grade-point averages or other common outcome variables. Utility is usually a monotonic increasing function of true ability or performance score. A cumulative probability function is then very convenient for describing one's utilities. Moreover, calculations of expected utility of a decision is greatly simplified when the utility and the probability function have the same functional form, e.g. both normal. A least-squares procedure for fitting a utility function is described and applied to truncated normal and beta distribution functions.

Introduction

Bayesian approaches to the problems of selection or certification have been discussed by several authors. Gross and Su (1975), Petersen (1976), and Huynh (1976, 1977) use threshold utility or a constant loss function to derive the cut-off scores in various models. Van der Linden and Mellenbergh (1977) extend the utility (loss) function to a linear form. Novick and Lindley (1978) suggest using a parametric utility function. They discuss several advantages of using a cumulative distribution as a utility function and recommend using the cumulative normal utility function. However, one restrictive property of the cumulative normal utility is its symmetry about the mean. This can be avoided by using only a portion of a symmetric function, e.g. a truncated normal function, or using a non-symmetric function, e.g. a beta function.

In this paper we consider using a truncated normal or a generalized beta cumulative distribution function (cdf) as a utility function. We will refer to them as a truncated normal utility (TNU) and a generalized beta utility (GBU) respectively. Applications of the two utility functions will be discussed specifically. A TNU and a GBU both have the advantage of being cumulative distribution functions and at the same time they provide increased flexibility in applications. For example, it can be symmetrical between risk-aversion and risk-proneness or it can be risk-averse or risk-prone throughout its range. Moreover, if the utility and the posterior distribution have the same functional form, both normal or both beta, then the computations of the expected utility become simple.

The major problem in utilizing utility theory is the assessment of the utility function. Procedures for obtaining a utility function have been

given by Mosteller and Nogee (1951), Pratt, Raiffa and Schlaifer (1965), Schlaifer (1969), and Keeney and Raiffa (1976). One of the difficulties in utility assessment is resolving the incoherence of subjects. In resolving subject incoherence, one can require a subject to give more than the minimum number of judgments in fitting a utility function. In this paper we discuss a procedure for fitting a cumulative utility function based on fixed state least-squares utility assessment as given by Novick and Lindley (1979).

A Truncated Normal Utility

Consider the problem of educational or personnel selection. Let θ denote the ability (or a measure of performance) of an applicant and $U(\theta)$ be the utility for selecting an applicant with ability θ. Assume the range of θ is $[\theta_o, \theta_n]$, then a truncated normal utility (TNU) function is defined by

$$U(\theta) = \frac{\Phi\left[\frac{\theta - \mu}{\sigma}\right] - \Phi\left[\frac{\theta_o - \mu}{\sigma}\right]}{\Phi\left[\frac{\theta_n - \mu}{\sigma}\right] - \Phi\left[\frac{\theta_o - \mu}{\sigma}\right]} \qquad \theta_o \leq \theta \leq \theta_n \quad (1)$$

where μ and σ are respectively the mean and standard deviation and $\Phi(\cdot)$ is the standardized normal distribution function. Hence, a TNU uses only a portion of a normal cumulative function. The utilities at the two end points are 0 and 1.

To investigate the behaviors of a person from the utility function, Pratt (1964) develops the risk aversion function $r(\theta)$ for a measure of risk aversion of a utility function, where

$$r(\theta) = -\frac{d^2U(\theta)}{d\theta^2} \Big/ \frac{dU(\theta)}{d\theta}$$

If $r(\theta)$ is positive at $\theta = \theta'$ then $U(\theta)$ is risk-averse at θ'; if $r(\theta)$ is negative at θ' then $U(\theta)$ is risk-prone at θ'; and if $r(\theta)$ is zero at θ' then $U(\theta)$ is risk neutral at θ'. The risk aversion function for the TNU defined in (1) is

$$r(\theta) = \frac{\theta - \mu}{\sigma^2} \qquad \theta_o \leq \theta \leq \theta_n$$

Thus, the sign of $r(\theta)$ depends on the position of μ. If $\theta_o < \mu$, then $r(\theta)$ is positive for all θ, $U(\theta)$ is risk-averse through its range. If $\theta_n > \mu$, then $r(\theta)$ is negative for all θ, $U(\theta)$ is risk-prone through its range. If $\theta_o < \mu < \theta_n$ then $r(\theta)$ is negative for $\theta < \mu$ and positive for $\theta > \mu$, $U(\theta)$ is risk-prone for low θ values and risk-averse for high θ values. This property is true for most unimodal distribution functions. Since the derivative of the risk aversion function in the normal case is

$$r'(\theta) = \frac{1}{\sigma^2} .$$

This implies that $U(\theta)$ has strictly increasing risk aversion. This characteristic is particularly useful in educational selection. For $\theta_o < \mu < \theta_n$, a TNU reflects a decision maker who is risk-taking for low ability θ, and risk-avoid for high ability θ; and his willingness to take the risk decreases as θ increases. This seems reasonable and common in educational selection.

Finally, for $\theta_o < \mu < \theta_n$ the limiting forms of the TNU become very simple. If $\sigma \to o$ then

$$U(\theta) = \begin{cases} 1 & \text{for } \theta \geq \mu \\ 0 & \text{for } \theta < \mu . \end{cases}$$

This is a threshold utility with threshold point at $\theta = \mu$. And if $\sigma \to \infty$ then

$$U(\theta) = \frac{\theta - \theta_o}{\theta_n - \theta_o} , \qquad \theta_o \leq \theta \leq \theta_n$$

The utility becomes a linear function with slope $(\theta_n - \theta_o)^{-1}$. (This result also holds for $\mu > \theta_n$ or $\mu < \theta_o$). Thus, for $0 < \sigma < \infty$ the function results in a smooth curve so that the lower portion ($\theta < \mu$) is convex and the upper portion ($\theta > \mu$) is concave with the point of inflection at $\theta = \mu$.

with observed score x is

$$E\ U(\theta) = \int U(\theta)\ d\ F(\theta \mid x) \qquad (2)$$

where $F(\theta \mid x)$ is the posterior distribution of θ given observed score x. The Bayes rule is to select a candidate with the highest expected utility. Therefore, the computations of the expected utility become important in the selection analysis.

As Lindley (1977) and Novick and Lindley (1978) indicate, if one can match the function form of utility $U(\theta)$ with the (posterior) distribution $F(\theta \mid x)$ then the computations of expected utility become simple. In most applications $F(\theta \mid x)$ is or can be approximated by a normal distribution Huynh (1979). Assume $F(\theta \mid x)$ is a normal distribution with mean μ_o and variance σ_o^2, on $[\theta_o, \theta_n]$ then the expected utility of the TNU in (1) is

$$E\ U(\theta) = \int_{\theta_o}^{\theta_n} \frac{\Phi\left(\frac{\theta - \mu}{\sigma}\right) - a}{(b - a)(b_o - a_o)}\ d\ \Phi\left(\frac{\theta - \mu_o}{\sigma_o}\right) \quad , \quad \theta_o \le \theta \le \theta_n$$

$$= \frac{-\int_{-\infty}^{\infty} \Phi\left(\frac{\theta - \mu}{\sigma}\right)\ d\ \Phi\left(\frac{\theta - \mu_o}{\sigma_o}\right)}{(b - a)(b_o - a_o)} - \frac{a}{b - a} \quad , \quad \theta_o \le \theta \le \theta_n$$

$$= \frac{Pr\ (\xi - \theta \le 0)}{(b-a)(b_o - a_o)} - \frac{a}{b - a} \quad , \quad \theta_o \le \theta \le \theta_n$$

$$= \frac{Pr(\xi - \theta \le 0,\ \theta_o \le \theta \le \theta_n)}{(b - a)(b_o - a_o)} - \frac{a}{b - a} \qquad (3)$$

where $a = \Phi\left(\frac{\theta_o - \mu}{\sigma}\right)$, $b = \Phi\left(\frac{\theta_n - \mu}{\sigma}\right)$, $a_o = \Phi\left(\frac{\theta_o - \mu_o}{\sigma_o}\right)$,

$b_o = \Phi\left(\frac{\theta_n - \mu_o}{\theta_o}\right)$ and ξ and θ are two independent random variables such that $\xi \sim N(\mu,\sigma)$ and $\theta \sim N(\mu_o,\sigma_o)$. Since ξ and θ are both normal, $(\xi - \theta, \theta)$ has a bivariate normal distribution with mean $(\mu - \mu_o, \mu_o)$ and covariance matrix

$$\begin{pmatrix} \sigma^2 + \sigma_o^2 & -\sigma_o^2 \\ -\sigma_o^2 & \sigma_o^2 \end{pmatrix}.$$

The expected utility can be obtained by computing the probability of the bivariate normal variable over the region $\xi - \theta \leq 0$ and $\theta_o \leq \theta \leq \theta_n$. Tables for bivariate normal probabilities are compiled by the (U.S.) National Bureau of Standards (1959). If $b_o - a_o \cong 1$, e.g. $\theta_o \leq \mu_o - 4\sigma_o < \mu_o + 4\sigma_o \leq \theta_n$, then the expected utility in (3) is

$$E\,U(\theta) = \frac{Pr(\xi - \theta \leq 0)}{b - a} - \frac{a}{b - a}$$

$$= \frac{c - a}{b - a}$$

where

$$c = \Phi\left(\frac{\mu_o - \mu}{(\sigma_o^2 + \sigma^2)^{1/2}}\right),$$

and a, b, c may be obtained from a univariate normal distribution table. A computer algorithm for computing bivariate normal probabilities is given by Divgi (1979).

In the previous discussions θ is considered to be on the interval $[\theta_o, \theta_n]$, in some cases θ may be defined beyond $[\theta_o, \theta_n]$. For example, in latent trait models the range of θ is $(-\infty, \infty)$. In this case, we can assign $U(\theta) = 1$ for $\theta > \theta_n$ and $U(0) = 0$ for $\theta < \theta_o$. Then the expected utility in (3) with $F(0 \mid x)$ defined over $(-\infty, \infty)$ becomes

$$E\,U(\theta) = \frac{Pr(\xi - \theta \leq 0,\ \theta_o \leq \theta \leq \theta_n) - a}{(b - a)} + (1 - b_o). \qquad (3)'$$

where ξ, θ, a, b and b_o are as previously defined.

In the problem of setting cut-off (passing) scores, it is necessary to require monotonic expected utility. More precisely, for a given increasing utility function $U(\theta)$ the expected utility should be a monotonic function of x, the observed score. A sufficient condition for monotonic expected utility is that the posterior distribution is stochastically increasing in x, i.e. if $x < x'$ implies $F(\theta \mid x) > F(\theta \mid x')$ for all θ (Chuang, Chen and Novick, 1981). It is easy to show that if the mean μ_o of the normal distribution $F(\theta \mid x)$ is an increasing function of x and the variance σ_o is independent of x then $F(\theta \mid x)$ is stochastically increasing in x. So the expected utility in (3) or (3)′ will increase as x increases.

We now give an example of applying the TNU to the regression model (Petersen and Novick 1976) $y = \alpha + \beta (x - \bar{x}) + e$ where $e \sim N(0, \sigma^2)$, in the selection of applicants. The example is based on the data used in Petersen's (1976) model for selection under restrictions. The criterion variable Y is the first semester college grade-point average. The ACT composite score is used for the predictor variable X. The applicants can be divided into disadvantaged and advantaged groups. The range of Y is from 0 to 4 and the range of X is from 0 to 36. Table 1 gives the sample size of N, mean $\bar{x}$ and $\bar{y}$, standard deviation s_x and s_y, correlation r_{xy} and regression coefficient $\hat{\beta}$ for each group.

Table 1 near here

Assume indifference prior distributions of α, β and σ for each group, then the posterior predictive distribution for y given an applicant with score x_o is

$$\frac{y - [\bar{y} + \hat{\beta}(x_o - \bar{x})]}{s\left[\frac{N+1}{N} + \frac{(x_o - \bar{x})^2}{\Sigma(x - \bar{x})^2}\right]^{\frac{1}{2}}} \sim t_{N-2} \qquad (4)$$

where $s^2 = N s_y^2 (1 - r_{xy}^2)/(N - 2)$ is the mean square error of the regression equation. If N is large then the t distribution in (4) is approximated by a normal variable and

$$\frac{N+1}{N} + \frac{(x_o - \bar{x})^2}{\Sigma(x-\bar{x})^2} \longrightarrow 1 .$$

Therefore, the posterior predictive distribution for y for a person with score x_o approximates to a normal distribution with mean $\bar{y} + \hat{\beta}\ (x_o - \bar{x})$ and variance s^2. For a positive value of $\hat{\beta}$, the mean is an increasing function of x_o, and the variance is independent of x_o, so the expected utility is increasing in x_o.

Suppose the utility for disadvantaged group has mean 1.5 and variance 2 and the utility for advantaged group has mean 2 and variance 1.5, the expected utilities computed from (3) of each group are given in Table 2. It is easy to see that of two applicants from different groups with the same test score, the applicant from the disadvantaged group is preferred. For the given data set and utility functions, Table 2 can be used for the selection of applicants.

Table 2 near here

A Generalized Beta Utility

We now consider the use of a generalized beta cumulative distribution as a utility function. Again, assume the range of θ is $[\theta_o, \theta_n]$. A generalized beta utility is defined as

$$U(\theta) = \int_{\theta_o}^{\theta} \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \quad \frac{(x-\theta_o)^{a-1}(\theta_n - x)^{b-1}}{(\theta_n - \theta_o)^{a+b-1}} \, dx \qquad \theta_o \leq \theta \leq \theta_n \quad (5)$$

The standard beta distribution has $\theta_o = 0$ and $\theta_n = 1$. The risk aversion function for a GBU is

$$r(\theta) = \frac{b-1}{(\theta_n - \theta)} - \frac{a-1}{(\theta - \theta_o)} \qquad \theta_o \leq \theta \leq \theta_n$$

Taking the derivative to find where the GBU has constant, decreasing or increasing risk-aversion, we have

$$r'(\theta) = \frac{b-1}{(\theta_n - \theta)^2} + \frac{a-1}{(\theta - \theta_o)^2} \qquad \theta_o \leq \theta \leq \theta_n$$

With the choices of a and b, beta utility provides a wide variety of utility functions. For example, for $a < 1$ and $b < 1$ the GBU is risk averse when θ is less than θ_a, and is risk prone when θ is greater than θ_a, where $\theta_a = [(a-1)\,\theta_n - (b-1)\,\theta_o]/\,(a+b-2)$ is the antimode of the generalized beta function. It has strictly decreasing risk aversion. For $a < 1$ and $b = 1$ the beta utility is risk averse through its range and it has strictly decreasing risk aversion. For $a < 1$ and $b > 1$ the beta utility is risk averse through its range and it has decreasing risk aversion when $\theta < \theta_b$ and it has increasing risk-aversion when $\theta > \theta_b$ where $\theta_b = \left(\theta_n - \theta_o\sqrt{\frac{-(b-1)}{(a-1)}}\right) / \left(1 + \sqrt{-\frac{(b-1)}{(a-1)}}\right)$. A decreasing risk-averse utility function is useful in economic applications.

In the previous section, we discussed some properties of the TNU function. A GBU with $a \geq 1$ and $b \geq 1$ has similar characteristics. It is increasing risk aversion. For $a > 1$ and $b > 1$ the beta utility is risk prone when $\theta > \theta_a$ and is risk averse when $\theta < \theta_a$, θ_a is the mode of the generalized beta function. For $a > 1$ and $b = 1$ the beta utility is risk prone through its range. For $a = 1$ and $b > 1$ the beta utility is risk averse through its range. The last two types represent the lower half curve and the upper half curve of a normal utility respectively. Finally, for $a = 1$ and $b = 1$ the beta utility is equivalent to a TNU with $\sigma = \infty$. We summarize the above results in Table 3.

Table 3 near here

The expected utility of a GBU can be computed by simple numerical integration. If a and b are integers, the beta utility in (5) becomes a polynomial utility with degree $a + b - 1$. Then expected utility is the linear combinations of the moments of the posterior distribution $F(\theta|x)$. The moments for many common probability distributions are tabulated and could be used in the calculations of the expectation of a polynomial utility.

If $F(\theta \mid x)$ has a beta distribution then the expected utility becomes one beta variable less than or equal to another beta variable. Assume ξ and θ are two independent beta random variables with distributions $U(\theta)$ and $F(\theta \mid x)$ respectively. Then the expected utility of a GBU in (5) is

$$E\,U(\theta) = Pr\,(\xi \leq \theta)$$

Note that the beta variable θ is also defined on $[\theta_o, \theta_n]$. Let $\xi' = \frac{\theta - \theta_o}{\theta_n - \theta_o}$ and $\theta' = \frac{\theta - \theta_o}{\theta_n - \theta_o}$, then ξ' and θ' are usual beta random variables defined on $[0;1]$ and

$$Pr(\xi \leq \theta) = Pr\,(\xi' \leq \theta').$$

Hence

$$E\ U\ (\theta) = Pr\ (\xi' \leq \theta'). \qquad (6)$$

The probability on the right hand side does not depend on θ_o or θ_n. A formula for computing this probability is given in Altham (1969) and Weisberg (1972).

We now provide an example of the use of the beta utility when the test scores follow a beta-binomial model. This model has been used extensively in the theory of criterion-referenced testing by Huynh and others. For a test of n items, given an applicant with ability θ the probability that he answers x items correctly is a binomial distribution. Assume a person's ability has a beta distribution with parameters p and q. Then the posterior distribution of θ given x is a beta (p + x, q + n - x). If a, b, p, q are integers, Altham (1969) provided the following formula for computing the probability in equation (6):

$$E\ U(\theta) = \sum_{s = \max\{p+x-b,0\}}^{p+x-1} \frac{\binom{a+p+x-1}{s}\binom{b+q+n-x-1}{p+q+n-s-1}}{\binom{a+b+p+q+n-2}{p+q+n-1}}$$

Again, consider the selection on the restricted model. Suppose the utility for the disadvantaged group has beta cumulative function with parameters 2 and 4 and the utility for the advantaged group has beta cumulative function with parameters 2 and 2. Assume indifference prior (Novick and Jackson 1974, p. 156) for each group, i.e. p = q = 0. For a test of 16 items the expected utility of each group is given in Table 4. It is easy to see that the expected utility is an increasing function of test score. In fact, the expected utility preserves the shape of the utility function. For example, for the advantaged group the expected utility is symmetric at the score of 8.

Table 4 near here

Fitting a Utility Function

We have seen above that there are some advantages in using the cumulative normal or beta distribution function as a utility function. In this section we consider a least-squares procedure given in Novick and Lindley (1979) to fit the parameters $(\tau_1\tau_2)$ of a utility, where (τ_1,τ_2) are (μ,σ) for a TNU and (a,b) for a GBU.

Consider (n+1) points with $\theta_o < \theta_1 < \ldots < \theta_n$ and let $U(\theta_i) = U_i$, $i = 0,1, \ldots, n$. For a given triplet $(\theta_i, \theta_j, \theta_k)$, a subject is asked to compare θ_j for sure with a gamble on θ_i and θ_k where $i < j < k$. Specifically, the subject is required to state a probability, p_{ijk}, such that he is indifferent between θ_j for sure and the gamble: chance p_{ijk} of θ_k and $1 - p_{ijk}$ of θ_i. Then

$$U_j = p_{ijk} U_k + (1 - p_{ijk}) U_i$$

or

$$\frac{p_{ijk}}{1-p_{ijk}} = \frac{U_j - U_i}{U_k - U_j}$$

(i) If $U(\theta)$ is a TNU function, then

$$\frac{p_{ijk}}{1 - p_{ijk}} = \frac{\Phi\left(\frac{\theta_j - \mu}{\sigma}\right) - \Phi\left(\frac{\theta_i - \mu}{\sigma}\right)}{\Phi\left(\frac{\theta_k - \mu}{\sigma}\right) - \Phi\left(\frac{\theta_j - \mu}{\sigma}\right)} \qquad (7)$$

(ii) If $U(\theta)$ is a GBU function, then

$$\frac{p_{ijk}}{1 - p_{ijk}} = \frac{U'\left(\frac{\theta_i - \theta_o}{\theta_n - \theta_o}\right) - U'\left(\frac{\theta_i - \theta_o}{\theta_n - \theta_o}\right)}{U'\left(\frac{\theta_k - \theta_o}{\theta_n - \theta_o}\right) - U'\left(\frac{\theta_j - \theta_o}{\theta_n - \theta_o}\right)}$$

where $U'(\cdot)$ is a standard beta distribution function, i.e.

$$U'(\theta') = \frac{\Gamma(a + b)}{\Gamma(a)\,\Gamma(b)} \int_0^{\theta'} x^{a-1} (1-x)^{b-1}\, dx, \qquad 0 \leq \theta' \leq 1.$$

Two values of p_{ijk} are sufficient to determine τ_1 and τ_2. For checking the coherence subject is required to assess more than two indifference probabilities p_{ijk}. Making the same assumption as in Novick and Lindley (1979) about the use of log-odds, a utility can be fitted by minimizing the following sum of squares with respect to τ_1 and τ_2

$$\sum_{ijk} \left(\log \frac{p_{ijk}}{1 - p_{ijk}} - \log \frac{U_j - U_i}{U_k - U_j}\right)^2 .$$

An example of using the least-squares procedure to fit a TNU and a GBU for a state university administrator on grade point average (GPA) is given below. The range of GPA is from 0 to 4, 14 different gambles are formed from the 9 points 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4. The list of the gambles is shown in column 1 to 3 of Table 5. (There are 84 possible gambles). For each gamble the assessed indifference probabilities are given in column 4. The Gauss-Newton method was used to minimize sum of squares. The algorithm is given in the appendix. The parameters of the fitted normal and beta utility are (1.44, 0.84) and (1.72, 2.99), respectively. The fitted utilities for the nine points are given in column 5 and 6. Two utilities are reasonably close to each other. The fitted beta has a mean of 1.46 and a mode of 1.06. The utilities rise very rapidly for GPA below 2 and very slowly for GPA above 3. Perhaps, this is the state university regulation that requires a continuing average of 1.0 to remain in college on probation, an average of 2.0 to maintain regular status and an average of 3.0 to take honor courses. Finally, for the comparison of the goodness of fits, the utilities from nine points least-squares fit are given in column 7. It also shows that the utilities at GPA = 0.5, 1, 1.5, 2 increase most rapidly, and the three fits are very close for GPA above 1.5.

Table 5 near here

Note that equation (7) is independent of $a = \Phi\left(\frac{\theta_o - \mu}{\sigma}\right)$ and $b = \Phi\left(\frac{\theta_n - \mu}{\sigma}\right)$. This procedure will fit the best normal curve over the interval $[\theta_o, \theta_n]$. However, if μ is far from the midpoint of θ_o and θ_n or/and σ is large then the performance of the fitted procedure will decrease. We briefly examine this difficulty by using the Monte Carlo methods for the model

$$\log \frac{p_{ijk}}{1-p_{ijk}} = \log \frac{\Phi\left(\frac{\theta_j - \mu}{\sigma}\right) - \Phi\left(\frac{\theta_i - \mu}{\sigma}\right)}{\Phi\left(\frac{\theta_k - \mu}{\sigma}\right) - \Phi\left(\frac{\theta_j - \mu}{\sigma}\right)} + \nu .$$

14 gambles are used for each fit, the (i, j, k) are generated uniformly from the 84 gambles and the error ν from independent $N(0, \frac{1}{2})$ population. Table 4 gives the average of 25 fitted mean and standard deviations for each combination of μ=2, 2.5, 3, 4 and σ=0.5, 1, 1.5, 2. The standard error of the averages are also provided. They are indicated in parenthesis. We consider the biases (|fitted-true|) and standard errors of the fitted values as a measure of the performance of the procedure. Table 4 indicates that an increase of σ or μ worsens the fit as one would expect. For $\mu = 4$ and $\sigma = 0.5$, the utilities for $\theta \leq 3$ are so close together (close to 0) that the procedure does not converge. This also suggests that θ_i should be chosen so that the utilities of $U(\theta_i)$ increase significantly at each θ_i.

Table 6 near here

Fitting a Utility in Two Groups

The problem of using probability distribution function as a utility function arises when the utilities are specified for more than one group. In the previous section, both utility bounds for advantaged and disadvantaged groups are assumed to be 1 and 0. This may not be true in some situations. For example, of two applicants with the same GPA of 4, one may prefer the disadvantaged group applicant. Therefore, it is important to check coherence between groups on multiple groups assessment. We now give a procedure to exploit coherence between groups. We will use GPA as an example to describe the procedure.

Let $\bar{U}_D(\theta)$ and $\bar{U}_A(\theta)$ be the utility function for disadvantaged and advantaged groups respectively. Suppose selecting a disadvantaged group applicant with GPA of 4 has the highest utility of 1 and selecting an advantaged group applicant with GPA of 0 has the lowest utility of 0. Hence, $\bar{U}_D(4)=1$ and $\bar{U}_A(0)=0$. Consider the utility for disadvantaged groups, for a given GPA of θ_i , the subject is asked to compare the following two options:

For sure	disadvantaged group applicant with GPA=θ_i

and

p	disadvantaged group applicant with GPA=4
1- p	advantaged group applicant with GPA=0

Let p_i be the probability for which the subject is indifferent between the two options. Then

$$\bar{U}_D(\theta_i) = (1 - p_i)\ \bar{U}_A(0) + p_i\ \bar{U}_D(4)$$
$$= p_i \qquad (8)$$

for any i. Assume $U_D(\theta)$ and $U_A(\theta)$ are two cumulative utility functions elicited independently from the procedure described in the last section. Because $U_D(\theta)$ and $\bar{U}_D(\theta)$ are two utility functions for the same population, we have

$$\bar{U}_D(\theta) = \alpha_D + \beta_D\ U_D(\theta)$$
$$= (1 - \beta_D) + \beta_D\ U_D(\theta) \qquad (9)$$

The last equation follows from $\bar{U}_D(4) = U_D(4) = 1$. Substituting (8) into (9) and simplifying gives

$$(1 - p_i) = \beta_D\ (1 - U_D(\theta_i)) \qquad (10)$$

β_D can be solved immediately from P_i and $U_D(\theta_i)$. Note that for coherence, β_D should be less than or equal to 1 (or $U(\theta_i)$ should be less than or equal to p_i). Equation (10) is a linear regression equation without intercept. By assessing at different θ_i points, the least squares procedure for estimating β_D can be used for checking coherence. Similarily, the utility for advantaged group is

$$\bar{U}_A(\theta) = \beta_A\ U_A(\theta) \qquad 0 \leq \beta_A \leq 1$$

And β_A can be estimated from the regression line

$$p_i = \beta_A\ U_A(\theta_i). \qquad (11)$$

where p_i is the indifference probability between the two options:

For sure advantaged group applicant with GPA=θ_i

and

p disadvantaged group applicant with GPA=4

$1-p$ advantaged group applicant with GPA=0 .

If $\hat{\beta}_D$ and $\hat{\beta}_A$ are the least squares solutions from equation (10) and (11) then the assessed utility for the two groups are

$$\bar{U}_D(\theta) = (1 - \hat{\beta}_D) + \hat{\beta}_D \, U_D(\theta)$$

and

$$\bar{U}_A(\theta) = \hat{\beta}_A \, U_A(\theta) \ .$$

Since the two bounds of $\bar{U}_D(\theta)$ and $\bar{U}_A(\theta)$ are not 0 and 1, they cannot be considered as a cumulative distribution function; however, they retain all the properties of a TNU described above. The expected utilities can be computed directly from $E(U_D(\theta))$ and $E(U_A(\theta))$ respectively, i.e.

$$E \ \bar{U}_D(\theta) = (1 - \hat{\beta}_D) + \hat{\beta}_D \, E \, U_D(\theta)$$

and

$$E \, \bar{U}_A(\theta) = \hat{\beta}_A \, E \, U_A(\theta).$$

Note we assume that the highest utility and the lowest utility are in two different groups, i.e. $\bar{U}_D(4) = 1$ and $\bar{U}_A(0) = 1$. Suppose the two extreme utilities 0 and 1 occur in the same group. Without loss of generality, it could alternatively be assumed that $\bar{U}_A(4) = 1$ and $\bar{U}_A(0) = 0$, then only the utility for the disadvantaged group has to be rescaled. Following the same arguments described above we have

$$p_i = \alpha_D + \beta_D U_D \qquad 0 < \alpha_D , 0 < \beta_D < 1 \qquad (12)$$

where p_i is the indifference probability for the two options

For sure disadvantaged group applicant with GPA= θ_i

and

p advantaged group applicant with GPA=4

1-p advantaged group applicant with GPA=0.

α_D and β_D in (12) can be estimated by fitting the linear regression line. If $\hat{\alpha}_D$ and $\hat{\beta}_D$ are two fitted values, then the utility for the two groups are

$$\bar{U}_D(\theta) = \hat{\alpha}_D + \hat{\beta}_D U_D(\theta)$$

and

$$\bar{U}_A(\theta) = U_A(\theta).$$

Summary

The use of some simple utility or loss functions in educational or employment evaluation has recently been studied by Petersen (1976) Huynh (1976, 1977), Van der Linden and Mellenbergh (1977). Novick and Lindley (1978) demonstrated that more realistic utility functions can be easily used without increasing the complexity and may be preferable in some applications.

This paper illustrates the use of a truncated normal or a generalized beta cumulative distribution function as a utility function. They are more flexible than threshold or linear functions. If a person's utility function can be fitted to a distribution function the analysis of utility is simple and easy. For example, its derivative is the familiar density function. It provides the rate of the increase of utility. However, one should not force a person's utility function into this form if it does not fit.

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Table 1 Sample Data for the Use of Truncated Normal Utility for Educational Selection

Group	N_i	$\bar{x}$	S_x	$\bar{Y}$	S_y	r_{xy}	β
Disadvantaged	305	13.47	4.787	1.68	1.088	.2772	.063
Advantaged	2182	19.03	5.276	2.07	1.015	.3732	.072

Source of Data - 1971 ACT Research Services used in Petersen (1976).

Table 2 The Expected Values of Two Truncated Normal Utilities

	Disadvantaged $\mu_o = 1.5$ $\sigma_o = 2$	Advantaged $\mu_o = 2$ $\sigma_o = 1.5$
2	0.298	0.197
4	0.324	0.222
6	0.353	0.250
8	0.382	0.280
10	0.413	0.312
12	0.444	0.347
14	0.476	0.383
16	0.509	0.421
18	0.542	0.460
20	0.575	0.499
22	0.607	0.538
24	0.639	0.577
26	0.670	0.615
28	0.700	0.651
30	0.729	0.686
32	0.756	0.718
34	0.782	0.749
36	0.806	0.776

Table 3 Risk Aversion Properties of Beta Utility Functions

	$r(\theta)$	$r'(\theta)$
$a < 1$ $b < 1$	$+$ if $\theta < \theta_a$ 0 if $\theta = \theta_a$ $-$ if $\theta > \theta_a$	$-$
$a < 1$ $b = 1$	$+$	$-$
$a < 1$ $b > 1$	$+$	$-$ if $\theta < \theta_b$ 0 if $\theta = \theta_b$ $+$ if $\theta > \theta_b$
$a = 1$ $b < 1$	$-$	$-$
$a = 1$ $b = 1$	0	0
$a = 1$ $b > 1$	$+$	$+$
$a > 1$ $b < 1$	$-$	$+$ if $\theta < \theta_b$ 0 if $\theta = \theta_b$ $-$ if $\theta > \theta_b$
$a > 1$ $b = 1$	$-$	$+$
$a > 1$ $b > 1$	$-$ if $\theta < \theta_a$ 0 if $\theta = \theta_a$ $+$ if $\theta > \theta_a$	$+$
	$\theta_a = \dfrac{(a-1)\theta_n - (b-1)\theta_o}{(a+b-2)}$	$\theta_b = \dfrac{\left(\theta_n - \theta_o\sqrt{-\frac{b-1}{a-1}}\right)}{\left(1+\sqrt{-\frac{b-1}{a-1}}\right)}$

Table 4 The Expected Values of Two Generalized Beta Utilities

Score	Disadvantaged a = 2 b = 4	Advantaged a = 2 b = 2
1	0.053	0.020
2	0.140	0.056
3	0.249	0.108
4	0.366	0.172
5	0.483	0.245
6	0.594	0.326
7	0.693	0.412
8	0.779	0.500
9	0.848	0.588
10	0.902	0.674
11	0.942	0.755
12	0.969	0.828
13	0.986	0.892
14	0.995	0.944
15	0.999	0.980

Table 5 Fitted Least Squares Utilities for a University Administrator

	Gamble			Normal Fit	Beta Fit	9 point fit
For sure	1-P	P	IP			
0.0	--	--	--	0	0	0
0.5	0.0	1.0	.68	.09	.12	.19
1.0	0.5	1.5	.30	.27	.33	.28
1.5	1.0	2.0	.45	.51	.55	.48
2.0	1.5	2.5	.59	.74	.74	.72
2.5	2.0	3.0	.66	.89	.88	.89
3.0	2.5	3.5	.78	.97	.96	.97
3.5	3.0	4.0	.80	.99	.99	.99
4.0	--	--	--	1.00	1.00	1.00
1.0	0.0	2.0	.38			
1.5	0.5	2.5	.41			
2.0	1.0	3.0	.63			
2.5	1.5	3.5	.79			
3.0	2.0	4.0	.92			
1.5	0.0	3.0	.50			
2.0	0.0	4.0	.72			

Table 6 Simulated Studies for the Performance of normal utility fits for Various μ and σ .

σ \ μ	2	2.5	3	4
0.5	2.00 (0.038) 0.50 (0.015)	2.51 (0.053) 0.50 (0.017)	2.97 (0.042) 0.49 (0.009)	X X
1	1.97 (0.123) 0.99 (0.110)	2.48 (0.131) 1.01 (0.117)	2.85 (0.172) 0.95 (0.090)	3.54 (0.240) 0.88 (0.067)
1.5	2.02 (0.305) 1.59 (0.555)	2.44 (0.340) 1.46 (0.304)	2.77 (0.262) 1.35 (0.254)	3.24 (0.362) 1.18 (0.187)
2	1.88 (0.560) 1.93 (0.600)	2.40 (0.396) 2.11 (0.717)	2.84 (0.477) 2.02 (0.475)	3.28 (0.500) 1.62 (0.317)

Least-squares procedure to fit a normal or a beta cdf utility

Appendix

Let p_{ijk} be the indifference probability for the triplet $(\theta_i, \theta_j, \theta_k)$ then

$$U(\theta_j) = p_{ijk}\, U(\theta_k) + (1-p_{ijk})\, U(\theta_i) \ .$$

We will minimize the following sum of squares with respect to the parameter $\underset{\sim}{\tau} = (\tau_1, \tau_2)$ of $U(\theta)$

$$Q = \sum_{ijk} \left(\log \frac{p_{ijk}}{1 - p_{ijk}} - \log \frac{U(\theta_j) - U(\theta_i)}{U(\theta_k) - U(\theta_j)} \right)^2 \ . \qquad \text{(A1)}$$

Let $\log \frac{p_{ijk}}{1-p_{ijk}} = y_{ijk}$ and $\log \frac{U(\theta_j) - U(\theta_i)}{U(\theta_k) - U(\theta_j)} = f_{ijk}(\underset{\sim}{\tau})$.

Consider the function $y - f(\underset{\sim}{\tau})$ the first-order Taylor series expansion for the function about $\underset{\sim}{\tau}^{(0)}$ is

$$y - f(\tau^{(0)}) = \frac{\partial}{\partial \tau_1} f(\tau^{(0)}) \ (\tau_1 - \tau_1^{(0)}) + \frac{\partial}{\partial \tau_2} f(\underset{\sim}{\tau}^{(0)}) \ (\tau_2 - \tau_1^{(0)}) + R$$

where R is the remainder. The least-squares solution for the linear model

$$y_{ijk} - f_{ijk} = b_1 \ \frac{\partial}{\partial \tau_1} \ f_{ijk} + b_2 \ \frac{\partial}{\partial \tau_2} \ f_{ijk} + R_{ijk}$$

can be used to approximate the solution in (A1).

If we write

$$X^{(0)} = \left[\begin{array}{cc} \frac{\partial}{\partial \tau_1} f_{ijk}, & \frac{\partial}{\partial \tau_2} f_{ijk} \\ \vdots & \vdots \end{array} \right]_{\underset{\sim}{\tau} = \underset{\sim}{\tau}^{(0)}} \quad \text{and} \quad \underset{\sim}{y} - \underset{\sim}{f}^{(0)} = \left[\begin{array}{c} y_{ijk} - f_{ijk} \\ \vdots \end{array} \right]_{\underset{\sim}{\tau} = \underset{\sim}{\tau}^{(0)}}$$

then the least-square estimate of b is

$$\underset{\sim}{b}^{(0)} = (X'^{(0)} X^{(0)})^{-1} X'^{(0)} (\underset{\sim}{y} - \underset{\sim}{f}^{(0)})$$

$\underset{\sim}{\tau}$ can be estimated by the interactive process from the above equation, that is

$$\begin{aligned} \underset{\sim}{\tau}^{(n+1)} &= \underset{\sim}{\tau}^{(n)} + \underset{\sim}{b}^{(n)} \\ &= \underset{\sim}{\tau}^{(n)} + (X'^{(n)} X^{(n)})^{-1} X'^{(n)} (\underset{\sim}{y} - \underset{\sim}{f}^{(n)}). \end{aligned}$$

We now give the derivate for two useful parametric utility functions:

(1) $U(\theta)$ is a normal or truncated normal cdf with mean μ and variance σ^2, i.e. $U(\theta) = \phi(\frac{\theta - \mu}{\sigma})$.

Then

$$\frac{\partial}{\partial \mu} f_{ijk}(\mu,\sigma) = \frac{\phi_j - \phi_i}{\sigma(\phi_j - \phi_i)} - \frac{\phi_k - \phi_j}{\sigma(\phi_k - \phi_j)}$$

$$\frac{\partial}{\partial \sigma} f_{ijk}(\mu,\sigma) = \frac{(\theta_j - \mu)\phi_j - (\theta_i - \mu)\phi_i}{\sigma^2(\phi_j - \phi_i)} - \frac{(\theta_k - \mu)\phi_k - (\theta_j - \mu)\phi_j}{\sigma^2(\phi_k - \phi_j)}$$

where $\phi_i = \phi(\frac{\theta_i - \mu}{\sigma})$, $\phi_i = \phi(\frac{\theta_i - \mu}{\sigma})$ and $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$.

(2) $U(\theta)$ is a beta cdf with parameters a and b, i.e.

$$U(\theta) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^{\theta} x^{a-1} (1-x)^{b-1} \, dx \quad . \quad \text{Then}$$

$$\frac{\partial}{\partial a} f_{ijk}(a,b) = \frac{v_{ij}}{u_{ij}} - \frac{v_{jk}}{u_{jk}}$$

$$\frac{\partial}{\partial b} f_{ijk}(a,b) = \frac{w_{ij}}{u_{ij}} - \frac{w_{jk}}{u_{jk}}$$

where $u_{ij} = \int_{\theta_i}^{\theta_j} x^{a-1} (1-x)^{b-1} \, dx$, $v_{ij} = \int_{\theta_i}^{\theta_j} \log x \, x^{a-1}(1-x)^{b-1} \, dx$

and $w_{ij} = \int_{\theta_i}^{\theta_j} \log (1-x) \, x^{a-1} (1-x)^{b-1} \, dx.$

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