ADA097162

Technical Note ⊛CEL

TN no. N-1590

title: SHOCK SPECTRUM CALCULATION FROM
ACCELERATION TIME HISTORIES

author: H. A. Gaberson

date: September 1980

sponsor: Naval Facilities Engineering Command

program nos: YF53.534.006.01.017

CIVIL ENGINEERING LABORATORY

NAVAL CONSTRUCTION BATTALION CENTER
Port Hueneme, California 93043

Approved for public release; distribution unlimited.

81 3 31 027

(14) CEL-TN-1590

(9) Rept ... 6-May 78

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER TN-1590 ✓	2. GOVT ACCESSION NO. DN04406	3. RECIPIENT'S CATALOG NUMBER 4D-A097162
4. TITLE (and Subtitle) SHOCK SPECTRUM CALCULATION FROM ACCELERATION TIME HISTORIES		5. TYPE OF REPORT & PERIOD COVERED Not final; July 1976-May 1978
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) H. A. Gaberson		8. CONTRACT OR GRANT NUMBER(s) (16) F...5341
9. PERFORMING ORGANIZATION NAME AND ADDRESS CIVIL ENGINEERING LABORATORY Naval Construction Battalion Center Port Hueneme, California 93043		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62755N; (17) YF53.534.006 01.017
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Facilities Engineering Command Alexandria, Virginia 22332		12. REPORT DATE September 1980
		13. NUMBER OF PAGES 66 (12) 67
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Shock spectrum, response spectrum, data analysis, shock design, shock (mechanics), hardening, earthquakes.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The report mainly discusses, compares, and derives one new and improved and two popular shock spectrum computation methods. The new one is a single recursive equation method that approximates the acceleration as a straight line between the digitized values. The new method is easily derived without recourse to Z Transform theory and, thus, should contribute to improved understanding of the computations. A new source of low frequency

(continued)

DD FORM 1473 EDITION OF 1 NOV 65 IS OBSOLETE
1 JAN 73

391 111

20. Continued

error common to all of the calculation methods was found, and empirical testing of the
coefficients was used to establish digitizing rules to avoid the error. A FCRTRAN listing
of a program using the new method is given in an appendix.

Library Card

CONTENTS

INTRODUCTION

The Naval Facilities Engineering Command seeks to expand the technology base upon which future shore facilities are founded. An important area in which a new design approach is being developed is in the management and understanding of those characteristics of earthquake and explosively generated foundation motions that damage or destroy Naval equipment. This report is a part of the design method development. Specifically, it documents and offers a new method of shock and response spectrum computation. The new method is more accurate than any known to be in use, it runs on a computer with less computer time than other procedures with similar sophistication, and the theory is more easily derivable and understood. The shock spectrum is the concept used in the new design method for quantifying the destructive capacity of explosively and earthquake generated violent equipment foundation motions.

The report begins with an overview of the shock and response spectrum concept, and then describes the computations that are required to transform time histories into shock spectra. It then discusses current computation methods and compares them to the new computation method. The new method of computing the "during" values of the spectrum is presented here for the first time; the theoretical detail is given separately in Appendix A. The procedure for the computation of the residual spectrum values has been presented previously in an interim report (Ref 1) but is repeated here for completeness. A FORTRAN IV listing of the programs is given in Appendix B, as it is now being used on a time-sharing computer.

NTIS GRA&I	☑
DTIC TAB	☐
Unannounced	☐
Justification	

By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A	

BACKGROUND

The term "shock spectrum" has been in use for about 40 years, and so one might reasonably assume the technology to be complete. Respected firms now sell preprogramed minicomputers that calculate and plot a shock spectrum at the touch of a button. Thus, it is felt that some explanation for yet another report on the subject must be offered.

A precise definition of the term "shock spectrum" has not yet been accepted. The term is used by DOD and its contractors, while the earthquake community synonomously use the term "response spectrum." The term is used mostly to describe a short-duration violent motion, but it also is used to describe a force transient. For the case where it is used to describe a motion, it is invariably "defined" to be the peak response of single-degree-of-freedom systems to that motion plotted versus the natural frequency of the single-degree-of-freedom system. Within that definition there are at least 108* different plots, even all with the same damping, which one could call the shock spectrum of that motion. This state breeds confusion, and prevents one from acquiring experience in the appearance of severe shock spectra.

Thus, a precise definition is required for use in any design procedure, and one was given along with reasons for it in the preliminary design method (Ref 2). Specifically, the shock spectrum is taken to be a plot of the peak relative displacement of a single-degree-of-freedom system exposed to the motion being analyzed, as a function of undamped natural frequency, and plotted on four-coordinate paper. The shock spectrum, therefore, is treated as a precise technical property of a transient motion. A property is defined by how it is measured and, particularly in the case of shock spectra, by how it is actually computed. This report in that sense defines the shock spectrum.

---

*Plot it log, linear, or semi-log, or four-coordinate; plot absolute or relative; plot peak acceleration, velocity, or displacement; give plus and minus values, or the overall value; plot the during, residual, or the total peak. Four-coordinate is the same as log but the paper has more lines.

Although defined and specified for purposes of this work, interest
and additional study of the shock spectrum will continue. Analysis of
dynamic data in terms of its effect on single-degree-of-freedom systems
is a basically fundamental approach that has not yet been fully exploited.
The references for this work are current; work on explaining the calcu-
lation and improving accuracy continues. Appendix A of this report is a
new development. The comments at the end of Appendix A indicate that
even this effort can be continued. However, the current state of shock
spectrum understanding is adequate for a useful design method. This is
explained in the preliminary design manual, Reference 2. For those who
have to compute their own spectra, or use preprogramed machines and
desire an appreciation for the computations, this report should help.

The shock spectrum is the key to an upgraded design method for
installation of shock-resistant equipment. Pursuance of the concept can
make facilities much safer for Navy personnel. For example, a common
problem from earthquakes is falling fluorescent light fixtures. Attention
to the shock spectrum can cause one to use a fixture, not necessarily
more expensive, that is not sensitive to motions with earthquake fre-
quencies. Indeed all appurtenances to the structure and all installed
equipment have estimatable natural frequencies. The shock spectrum
gives the maximum values of oscillatory motion to expect from equipment
with various natural frequencies. Dynamic design is considerably sim-
plified when one can predict the peak acceleration and displacement.
Thus, the mere understanding of, and attention to, the information in
the predicted shock spectrum permits an evaluation of installed equipment,
especially as regards the equipment breaking free and injuring personnel.

DESCRIPTION OF THE SHOCK AND RESPONSE SPECTRUM

The explosively generated mechanical shock motion or the earthquake-
generated motion of equipment foundations is often recorded as a signal
from an accelerometer on magnetic tape. These records are converted or

3

digitized into a list of closely spaced numerical values of the acceleration with known equal time intervals separating the values. In this form the information is not readily useful for design. The shock spectrum transforms this motion time listing into a set of the peak responses the motion is able to cause in a set of simple mass-spring-dashpot vibratory systems. Thus, the shock spectrum describes the shock motion in terms of its capacity to excite simple vibratory systems. This transformation of the data is now routinely accomplished on digital computers; several programs are available in the literature (Ref 3-7).

However, the algebra required to derive the calculating equations is lengthy and seldom published. For example, Lane's algorithm (Ref 8), which was published in 1964, is extremely efficient, requires unbelievably few calculations, and is touted as being accurate. It was adopted by virtually all of the aerospace industry and is still in use. It was derived by using Z Transform Theory (Ref 9) which few understood. In 1973, Cronin (Ref 4) published a method of deriving Lane's result by manipulating a Duhamel's integral. Cronin's work showed the severity of the approximation required to obtain the Lane result, which raised doubts about the accuracy of the method. O'Hara (Ref 10), Nigam and Jennings (Ref 3), and Vernon (Ref 5) all published two-equation forms for calculatng equations. Their assumptions were clearly laid out and satisfactory, but computing with those equations was comparatively time consuming. A comparison of Cronin's work with these two-equation calculating schemes made possible the development of a new single-calculating equation. This new theory is presented here for the first time. The new method is more simply derived and should contribute to a wider understanding of the calculations, and thereby make them more believable and useful.

COMPUTATIONS

To understand the computing to be accomplished, the stage must be set. The machine will be given a list of perhaps 1,000 acceleration values in sequence; they are equally spaced values separated by a known time interval sampled from the acceleration versus time graph of the

4

motion to be analyzed. The resulting spectrum to be calculated will be a plot versus frequency of the peak responses of simple vibratory systems of a given damping ratio when exposed to that input motion. Figure 1 contains two computed shock spectra, one for zero damping and one for 5% damping. It is plotted on four-coordinate paper, as discussed in Reference 2. As can be read from the figure, this is the shock spectrum of a foundation motion that would cause an undamped 10-Hertz oscillator to attain a peak deflection of 3 inches, or a 100-Hertz oscillator to attain a peak deflection of 0.26 inch and a peak acceleration of about 250g. Each curve on Figure 1 consists of 180 equally spaced points connected by straight lines; on that sized paper this makes a very smooth plot. Now the computation can be visualized; one must numerically calculate the response, one at a time, of 180 different natural frequency vibratory systems and pick out and save the peak value for each frequency. A new set of 180 values is computed for each damping ratio. The computer does it quickly and inexpensively; even the plotting is done by machine.

SINGLE-DEGREE-OF-FREEDOM SYSTEM RESPONSE TO A FOUNDATION MOTION

The single-degree-of-freedom vibratory system to be conceptually used for this data analysis is shown in Figure 2. The absolute position of the foundation is y, and the absolute position of the mass, m, is given by x. The spring stretch, or relative displacement of the mass with respect to the foundation, is z, or

$$z = x - y \tag{1}$$

Dots are used to indicate differentiation with respect to time; thus, a free body diagram of the mass with the reversed inertia force is as shown in Figure 3. Summing the forces yields

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0 \tag{2}$$

5

Writing Equation 2 in terms of the relative displacement, z, defined in Equation 1 yields

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} \qquad (3)$$

By dividing by m, and using the traditional definitions and symbols in Reference 11 given below

$$\omega = +\sqrt{k/m} \qquad (4a)$$

$$c_c = 2m\omega \qquad (4b)$$

$$\zeta = c/c_c \qquad (4c)$$

one obtains

$$\ddot{z} + 2\zeta\omega\dot{z} + \omega^2 z = -\ddot{y} \qquad (5)$$

Equation 5 gives the dynamics of the simple system in terms of its more interpretable characteristics, i.e., its undamped natural frequency, ω, and its damping ratio, ζ. It non-sizes the equation; all single-degree-of-freedom systems with the same natural frequency and damping ratio must respond identically to the same transient acceleration. All shocks or earthquakes are transient motions, which can be described in terms of their transient accelerations. Thus, their effect on single-degree-of-freedom systems is an excellent way of organizing or classifying their capability to damage equipment.

The right hand side of Equation 5, the $\ddot{y}$, or input to the equation, will be a list of values. Solutions for Equation 5 will be developed that generate a list of z's that result from the input. The theory will also give us equations for $\ddot{x}$, $\ddot{z}$, and $\dot{z}$ once the list of z's has been found.

Several approaches exist for deducing the solution of Equation 5 for a transient input, which is what is needed here. A good form, the Duhamel integral form, is developed in most vibration texts; Thomson

6

(Ref 11) develops it for foundation excited motions, and O'Hara (Ref 10) gives it explicitly with initial conditions but without derivation as follows:

$$z = z_o \, e^{-\zeta \omega t} \left(\cos \omega_d t + \frac{\zeta}{\eta} \sin \omega_d t \right) + \frac{\dot{z}_o \, e^{-\zeta \omega t}}{\omega_d} \sin \omega_d t$$

$$- \frac{1}{\omega_d} \int_o^t \ddot{y}(\tau) \, e^{-\zeta \omega(t-\tau)} \sin \omega_d(t-\tau) \, d\tau \qquad (6)$$

where $\quad z_o, \dot{z}_o \;=\;$ initial values of z and $\dot{z}$

$\omega_d \;=\;$ damped natural frequency, $\eta \omega$

$\eta \;=\; \sqrt{1 - \zeta^2}$

$\tau \;=\;$ a dummy time variable of integration

One should not be intimidated by the complicated appearance of Equation 6. No one uses it in this form except to derive simpler relations. Equation 6 gives the homogeneous solution of Equation 5 in its first two terms; the motion described by these terms is caused by the initial velocity, $\dot{z}_o$, and displacement, z_o, existing at time equal to zero. The third term, the integral, is a formula for finding the particular solution (or the part of the motion) being caused by the excitation, $\ddot{y}$, occurring during the time interval in which Equation 6 is being applied. The contribution to shock spectrum computing technology that has been made by this research is the way in which Equation 6 was used. In the next few paragraphs, two very popular computing equations are reviewed. All of the results presented below are derived and further discussed in Appendix A.

DISPLACEMENT SINGLE RECURSIVE EQUATION, TRAPEZOIDAL RULE INTEGRAL APPROXIMATION

The simplest of the calculating solutions will be discussed first, and will serve to illustrate the use of all the calculating solutions.

7

It was first presented by Lane (Ref 8) and is in current use by most of the aerospace community (Ref 4,12,13). The result is given below

$$z_i = \alpha_{15} z_{i-2} + \alpha_{16} z_{i-1} + \gamma_1 \ddot{y}_{i-1} \tag{7}$$

where the constants are given by

$$\alpha_{15} = - e^{-2\zeta \omega h} \tag{7a}$$

$$\alpha_{16} = 2 e^{-\zeta \omega h} \cos \omega_d h \tag{7b}$$

$$\gamma_1 = \frac{- h e^{-\zeta \omega h} \sin \omega_d h}{\omega_d} \tag{7c}$$

and where h is the time interval between samples of $\ddot{y}$. This is derived in Appendix A as Equation A-11a.

The use of Equation 7 can be envisioned as follows. Consider that for a given damping value, one wants to compute the peak z for some frequency ω_1. The constants α_{15}, α_{16}, and γ_1 are first calculated. Then $\ddot{y}_1$ is substituted into Equation 7 with z_1 and z_0 equaling zero. This yields a value for z_2. Next $\ddot{y}_2$ and z_2 are used to compute z_3 with z_1 again taken equal to zero. The value z_4 is calculated from Equation 7 by using $\ddot{y}_3$, z_3, and z_2; and, thus, the process of calculating the z's from the $\ddot{y}$'s continues. At each step of the computation, each newly computed value of z is tested to see if it should replace what up until that time has been the most positive and negative values. The absolute largest of the two is the maximum. When the processing of the list of $\ddot{y}$'s has been completed and the maximum z found, one has found the maximum "during" values of z for that frequency. This process is repeated for each frequency at which a shock spectrum value is desired.

One can refer to Equation 7 as a displacement recursive equation for use with equally spaced digitized values of acceleration. The list of displacements computed therefrom completely defines the resulting motion. In Appendix A it is shown that if the velocity at any time is desired, it can be computed from these displacement values as follows

8

$$\dot{z}_i = \alpha_{11} z_{i-1} + \alpha_{12} z_i + \gamma_2 \ddot{y}_i \tag{7d}$$

where the constants are given by

$$\alpha_{11} = -(\eta \omega e^{-\zeta \omega h})/(\sin \eta \omega h)$$

$$\alpha_{12} = \omega(\eta/\tan \eta \omega h - \zeta)$$

$$\gamma_2 = - h/2$$

If the relative acceleration, $\ddot{z}_i$, is of interest, it is obtained by using the values z_i and $\ddot{y}_i$, along with the computed value of $\dot{z}_i$ in Equation 5, which gives

$$\ddot{z}_i = - 2 \zeta \omega \dot{z}_i - \omega^2 z_i - \ddot{y}_i \tag{7e}$$

If the absolute acceleration is desired, one notes from Equation 1 that

$$\ddot{x}_i = \ddot{z}_i + \ddot{y}_i \tag{7f}$$

and, thus, from Equation 7e one finds

$$\ddot{x}_i = - 2 \zeta \omega \dot{z}_i - \omega^2 z_i \tag{7g}$$

Equations 7, the aerospace industry equations, are a complete set. References 4 and 13 give Fortran program lists for their use. As mentioned in Appendix A, the apparent crudeness of the approximate integration method was not made clear until Reference 4 was published in 1973. Nothing has been written criticizing these equations, including Reference 4, except that they are completely ignored by the respected earthquake analysis community (Ref 3 and 7). Reed (Ref 13) compares several computing approaches and finds these equations inaccurate for coarse sampling rates, but otherwise acceptable. The computational approach advocated

here resulted from trying to reconcile this computing approach with that
of References 3, 7, and 10. A suitably simple theory was found for
transforming the conventional equations (Ref 3 and 10) into a single
displacement recursive equation so that simplicity of a single equation
could be retained along with the accuracy of the more conventional
integration approximations.

DISPLACEMENT AND VELOCITY PAIR OF RECURSIVE EQUATIONS, ACCELERATION APPROXIMATED AS A STRAIGHT LINE IN INTEGRAL

The more conventional equations referred to above were first pre-
sented by O'Hara (Ref 10) in 1962 and then independently again by Nigam
and Jennings (Ref 3) in 1968. They approximate the acceleration by a
straight line between the sample points and then integrate the integral
of Equation 6. This yields a pair of equations for processing the data.
They are derived in Appendix A as Equations A-18c and A-18d and are as
follows

$$z_i = \alpha_1 z_{i-1} + \alpha_2 \dot{z}_{i-1} + \alpha_{29} \ddot{y}_{i-1} + \alpha_{30} \ddot{y}_i \tag{8a}$$

$$\dot{z}_i = \alpha_4 z_{i-1} + \alpha_5 \dot{z}_{i-1} + \alpha_{31} \ddot{y}_{i-1} + \alpha_{32} \ddot{y}_i \tag{8b}$$

The constants, which are a complicated expression, but of the same
variables as in Equation 7, are given in Appendix A as Equations A-4c,
A-4d, A-4f, A-4g, A-18i, A-18j, A-18k, A-18l. In this computing method,
two outputs, both z and $\dot{z}$, must be computed for each step, and then used
in the next. Since both z and $\dot{z}$ are being constantly computed, it is a
simple matter to use Equation 7g to compute the acceleration of the
mass, $\ddot{X}$, at each point if desired. This is the procedure currently in
use for most of the U.S. earthquake data (Ref 7), and hence is a highly
respected approach. Note that for each step, or for each point of input
data, eight multiplications and six additions are required. What was
found during the course of this work was that the above pair could be
exactly reduced to a single equation; this results in each step only
requiring five multiplications and four additions.

10

DISPLACEMENT SINGLE RECURSIVE EQUATION, ACCELERATION APPROXIMATED AS A STRAIGHT LINE IN INTEGRAL

In Appendix A it is shown that with some shifting of the indices, one can combine Equations 8a and 8b into the single equation given below

$$z_i = \alpha_{15} z_{i-2} + \alpha_{16} z_{i-1} + \alpha_{24} \ddot{y}_{i-2} + \alpha_{25} \ddot{y}_{i-1} + \alpha_{26} \ddot{y}_i \qquad (9a)$$

The constants again are complicated, but easily computed and are given in Appendix A as Equations A-10e, A-10f, A-19b, A-19c, and A-19d. Note that the constants applied to the z's are the same as those of Equation 7, Lane's result (Ref 8). No matter how sophisticated an approximate integration is, these same two constants appear. It does seem more reasonable that three input values are required for each step, including the value, $\ddot{y}_i$, for the time at which the output, z_i, is being computed.

As was the case previously, the data or list of $\ddot{y}_i$'s is marched through Equation 9a, which yields a z_i for each $\ddot{y}_i$. If the velocity at any point is desired, it can be computed as shown in Appendix A, Equation A-20a, from the z's as follows

$$\dot{z}_i = \alpha_{11} z_{i-1} + \alpha_{12} z_i + \alpha_{27} \ddot{y}_{i-1} + \alpha_{28} \ddot{y}_i \qquad (9b)$$

The constants are given in Appendix A as Equations A-10a, A-10c, A-20b, and A-20c. If the absolute acceleration of the mass, $\ddot{x}_i$, is desired, $\dot{z}_i$ is computed and used with z_i in Equation 7$_6$.

RESIDUAL SPECTRUM CALCULATION

As has been discussed, the spectrum computation consists of exciting the theoretical model of Figure 1, which is described by the differential Equation 5, by means of a calculation algorithm (such as Equations 7, 8, or 9) with the excitation given as a sequence of equally spaced values.

11

After the excitation is over, the oscillator continues to ring, and the low frequency oscillators attain their greatest values after the excitation has passed. In the previously discussed algorithms, the calculation of these "residual values" is accomplished by continuing the algorithm with zero excitation values for one period of the oscillator. However, this is not the only way of doing it, or necessarily the best way. Often one computes spectral values for oscillators with periods far longer than the excitation duration, and, in this case, the residual integration region is necessarily longer than the original pulse. Another approach is to compute the final velocity and displacement, $\dot{z}$ and z, at the end of the excitation, and then calculate the positive maximum and negative minimum from the free vibration solution. This is the procedure used in the new shock spectrum computing method and, thus, will be derived here with some of the detail given in Appendix C.

The solution for a damped oscillator undergoing free (or decaying) vibrations can be taken from the first two terms of Equation 6 (since $\ddot{y}$ is then zero), or

$$z = z_o e^{-\zeta \omega t}\left(\cos \omega_d t + \frac{\zeta}{\eta} \sin \omega_d t\right)$$

$$+ \frac{z_o e^{-\zeta \omega t}}{\omega_d} \sin \omega_d t \qquad (10)$$

By rearranging, this can be put in a form more convenient for manipulating as follows:

$$z = e^{-\zeta \omega t} (A \sin \omega_d t + B \cos \omega_d t) \qquad (11)$$

$$\text{where} \quad A = \frac{z_o \zeta}{\eta} + \frac{\dot{z}_o}{\omega_d} \qquad (11a)$$

$$B = z_o \qquad (11b)$$

It can be shown by substitution that a convenient form for the derivative of Equation 11 or $\dot{z}$ is

12

$$\dot{z} = \omega \, e^{-\zeta \omega t} \, [A \cos(\omega_d t + \delta) - B \sin(\omega_d t + \delta)] \tag{12}$$

where

$$\sin \delta = \zeta \tag{12a}$$

$$\cos \delta = \eta \tag{12b}$$

$$\eta = \sqrt{1 - \zeta^2} \tag{12c}$$

$$\omega_d = \eta \, \omega \tag{12d}$$

Since Equation 11 is a damped vibration or at the most with $\zeta = 0$, a constant vibration, both the most positive and the most negative value must occur in the first cycle or at $t = 0$. Thus, z_{max} and z_{min} must be pursued in the first cycle. If not, when $t = 0$, these occur when $\dot{z}$ equals zero or when

$$A \cos(\omega_d t + \delta) = B \sin(\omega_d t + \delta) \tag{13}$$

If

$$\beta = \omega_d t + \delta \tag{14}$$

one seeks β such that

$$A \cos \beta = B \sin \beta \tag{15}$$

or

$$\tan \beta = A/B \tag{15a}$$

and also seeks two values β_1 and β_2 such that the results $(\omega_d t)_1$ and $(\omega_d t)_2$ are between zero and 2π. Clearly, β_1 and β_2 are consecutive angles at which the velocity goes to zero; thus (since the tangent goes through zero every π radians),

13

$$\beta_2 = \beta_1 + \pi \tag{16}$$

Now one must go through an amount of detail to assure the finding of the very first time after time equals zero that $\dot{z}$ equals zero. Nine cases must be considered as indicated:

	A > 0	A = 0	A < 0
B > 0	I	II	III
B = 0	IV	V	VI
B < 0	VII	VIII	IX

This is done carefully in Appendix C. The results are given below as the set of "if" statements used in programming the computation.

1. If both A and B are zero, no residual response results.

2. If not, and A = 0, then $\beta_1 = 0$. <div style="float:right">(17a)</div>

3. If not, and B = 0, then $\beta_1 = \frac{\pi}{2}$. <div style="float:right">(17b)</div>

4. If not, and A and B have the same sign,

$$\beta_1 = \text{Tan}^{-1} (A/B) \tag{17c}$$

5. If not,

$$\beta_1 = \pi - \text{Tan}^{-1} (-A/B) \tag{17d}$$

Now with this value of β_1, one final test is required. If, and only if

$$\beta - \dot{\varepsilon} < 0 \tag{17e}$$

one must add π to β_1, or

$$\beta_1 = (\beta_1)_{old} + \pi \tag{17f}$$

Now with the acceptable value of β_1, $(\omega_d t)_1$ can be obtained with Equation 14 as follows

$$(\omega_d t)_1 = \beta_1 - \delta \tag{17g}$$

The second value of $(\omega_d t)$ is obtained from

$$(\omega_d t)_2 = (\omega_d t)_1 + \pi \tag{17h}$$

These values of $(\omega_d t)_1$ and $(\omega_d t)_2$ are substituted successively into Equation 11, which will yield values for z_{max} and z_{min}. These values are compared with z_o to make sure it is not greater than z_{max} or less than z_{min}, and, thus, the residual spectrum values are computed. Subroutine RESID listed at the end of Program SPCTRM in Appendix B uses this procedure to compute the residual spectrum values.

Some additional explanation of the need for the residual spectrum computation procedure is given here, since the previously mentioned programs do not go to this trouble. To get the residual response, one must first use a numerical method (such as Equations 7, 8, or 9) to find the displacement and velocity (z_o and $\dot{z}_o$) at the end of the "during" portion of the response. The other computations continue the numerical procedure to find the residual maximum values. This procedure uses a theoretically exact method to compute the values; it is faster, more accurate, but a little clumsier to program.

THE COMPUTER PROGRAMS

Appendix B gives the Fortran listings for the two computer programs used in the preparation of shock and response spectra by this new method. The first, SPCTRM, computes the shock spectrum and lists as output the values of the maximum and minimum displacement and pseudovelocity for each frequency considered. The second, PLTVLF, is the program used to plot the maximum, positive or negative shock spectrum on the four-coordinate paper. Note that the precise frequencies used for calculation

15

are selected by the program so that they will be equally spaced when
plotted logarithmically. Appendix D documents the logic for this section
of program SPCTRM. Otherwise the steps and symbols of the program
coincide with the report body and appendices, and should be completely
readable. The programs are in current use on a CDC time-sharing terminal;
the plotting is done on an interconnected Hewlett Packard Model 7202A
Graphic Platten. The format statements show the form of the input data.
The programs interrogate the user and, thus, supply the needed documen-
tation.

COMMENTS

The Accuracy of the Algorithms

Equation 9a is more accurate than any others thus far published.
Because these recursive calculating equations use previously computed
results for each succeeding step, they propagate any errors introduced.
Thus, if fewer multiplications and additions are required for each step,
there is less opportunity for error accumulation due to roundoff and
subtraction of nearly equal values.

Of the algorithms reviewed, Equation 7, the "trapezoidal" approach,
has the fewest multiplications and additions. However, it assumes each
factor of the integral is constant during the integration. If one
graphs the value of the factors over the integration interval and then
compares an estimate of the integral of their product with that assumed
by the algorithm, the crudeness of the approximation becomes apparent.
Thus, the "trapezoidal" rule approach was discarded because of these
initial assumptions.

Conversely, if one graphs the values of the factors similarly with
the straight line integration approach, one can infer a convergence to
the "true" value as the steps become smaller. Such an examination even
leads one to assume the straight line approach is exact; such a comment
was even made in Reference 3. That is why most of the algorithms use

16

the straight line approach. Of these the new single recursive equation
developed here has by far the fewest multiplications and additions and,
thus, is the most accurate.

A rigorous study of the accuracy of the approaches has not been
undertaken, and would certainly be welcomed. However, perfunctory
accuracy statements that pretend to bound the error by referring to
higher derivatives of the input are not applicable to digitized data.
Nor is there any way in which one can construe the data to be composed
of low order polynomials. As is discussed further in Appendix A, one
does not want to sample any coarser than about 10 points per cycle of
the highest frequency present. At this high a sampling rate, the straight-
line-between-the-points assumption makes an accurate appearing plot of
the data, which leaves one to believe the integration has been adequately
approximated.

Allowable Frequency Ranges

This new program, as well as all other shock spectrum computer
programs, has limitations on the frequencies for which it can compute
accurate shock spectrum values. The allowable frequencies are dependent
upon the sampling rate of the data used as input and upon the number of
digits the particular computer uses in performing its arithmetic opera-
tions. The high frequency limit for the programs discussed in this
report is one tenth of the sampling rate. This assures a 5% accuracy
and is discussed by most authors (Ref 3).

That a low frequency limit exists has not been published up to now.
The constants that apply to the input data in Equations 7, 8, and 9a
contain sines and cosines with arguments that become very small at low
frequencies and lead to the subtraction of nearly equal numbers (e.g.,
equal to the 12th or 13th digit in a 14-digit number). At very low
frequencies this can lead to the computer generating ridiculous values
off by a factor of ten or more. Time did not permit a thorough investi-
gation of this effect, but an empirical testing of the constants did
indicate that for the computer used, which calculates to 14-digit accuracy
in single precision, accuracy could be maintained for frequencies as low
as one eight-thousandth of the sampling rate.

17

These frequency limitations are more completely discussed in
Appendix A. In symbols, the range of allowable frequencies for the
programs can be expressed as

$$10 \leq \frac{f_s}{f} \leq 8000 \tag{18}$$

Thus, for data that have been sampled at 10,000 points per second, one
could compute shock spectrum values for frequencies as high as 1,000 Hertz
and as low as 1.25 Hertz. The current program does not impose this
restriction; it is the responsibility of the user.

SUMMARY

The body of the report has presented a background in shock spectrum
calculation methods applicable to digital computation that has not been
available. In general terms in the text and in detail in Appendix A,
the two most popular methods and a new method are described, defined,
and completely derived. This new method decreases the numbers of addi-
tions and multiplications required by 40% and, thereby, increases accuracy
while decreasing cost. The main purpose of the report is to document
this new method and make it available. Appendix B gives computer program
lists for this new method.

There are still unresolved questions and certainly room for improve-
ment; these matters are discussed. In particular, a new potential
source of inaccuracy was discovered in trying to extend to earthquake
frequencies data that had been sampled for explosion frequencies. The
only way one can compute spectra for the very low earthquake frequencies
is to have the data digitized at an appropriate sampling rate.

The author believes that a greater understanding of the shock
spectrum is valuable for evaluating equipment installation in dynamic
environments. Mere plotting of the shock spectrum on four-coordinate
paper gives a vastly improved appreciation of the damaging capacity of a
given environment. Required rattlespace stands out and, to the extent

that one can approximate the gross natural frequency of the equipment on its support, the peak breakaway forces can be estimated. A considerable increase in safety and facility hardness from the effects of explosion and earthquake is available inexpensively from increased understanding and use of the shock spectrum.

Finally, this study of the shock spectrum analysis of sampled acceleration time histories not only reduced and clarified the required calculations, it also provoked several ideas for even further reducing the program size. It must be assumed that soon small calculators will be doing the analysis. Inexpensive digitizers and plotters will follow, and all will have access to many new tools for intelligent use of the data that now is only plotted.

ACKNOWLEDGMENT

REFERENCES

1. Civil Engineering Laboratory. Technical Memorandum M-63-76-12: Shock spectrum calculation from sampled acceleration time histories, by H. A. Gaberson and D. Pal. Port Hueneme, Calif., Sep 1976.

2. Civil Engineering Laboratory. Interim Report No. YF53.534.006.01.017: A design method for shock resisting equipment installation, by H. A. Gaberson. Port Hueneme, Calif., Apr 1975.

3. California Institute of Technology, Earthquake Engineering Research Laboratory. Digital calculation of response spectra from strong-motion earthquake records, by N. C. Nigam and P. C. Jennings. Pasadena, Calif., Jun 1968.

4. D. L. Cronin. "Numerical integration of uncoupled equations of motion using recussive digital filtering," International Journal for Numerical Methods in Engineering, vol 6, 1973, pp 137-152.

5. J. B. Vernon. Linear vibration and control system theory. New York, N.Y., John Wiley and Sons, Inc., 1967, pp 251-252.

6. Naval Ordnance Laboratory. Report No. NOLTR-68-37: Shock spectra and an application to antillery projectile shock, by P. S. Hughes. White Oak, Md., Mar 1968.

7. National Information Service. Earthquake engineering computer applications, Programs 06-573, SPECTR, and C8-573 SPECEQ/SPECUQ, Earthquake Engineering/Computer Applications, Davis Hall, University of California. Berkeley, Calif., Listing of Apr 1978.

8. Naval Research Laboratory, Shock and Vibration Information Center. Digital shock spectrum analysis by recussive filtering, Shock and Vibration Bulletin V33, Section 2, by D. W. Lane. Washington, D.C., 1964, pp 173-181.

9. E. I. Jury. Theory and applications of the Z-Transform Method. New York, N.Y., John Wiley and Sons, Inc., 1964.

10. Naval Research Laboratory. Report No. 5772: A numerical procedure for shock and Fourier analysis, by G. J. O'Hara. Washington, D.C., Jun 1962.

11. W. T. Thomson. Vibration theory and applications. Englewood Cliffs, N.J., Prentice-Hall, Inc., 1965, pp 56 and 104.

12. Naval Research Laboratory, Shock and Vibration Information Center. SVM-5: Principles and techniques of shock data analysis, by R. D. Kelly and G. Richman. Washington, D.C., 1969.

13. Naval Ordnance Laboratory. NOLTR 70-243: Comparison of methods used for shock and Fourier spectra computations, by R. S. Reed, Jr. Silver Spring, Md., Nov 1970.

14. L. A. Pipes. Applied mathematics for engineers and scientists, 2nd ed. New York, N.Y., McGraw-Hill Book Company, 1958.

15. Sandia Laboratories. Report No. SAND-74-0392: Shock spectra, by R. Rodeman. Albuquerque, N.M., Oct 1974.

16. R. S. Burrington. Handbook of mathematical tables and formulas, 5th ed. New York, N.Y. McGraw-Hill Book Company, 1973.
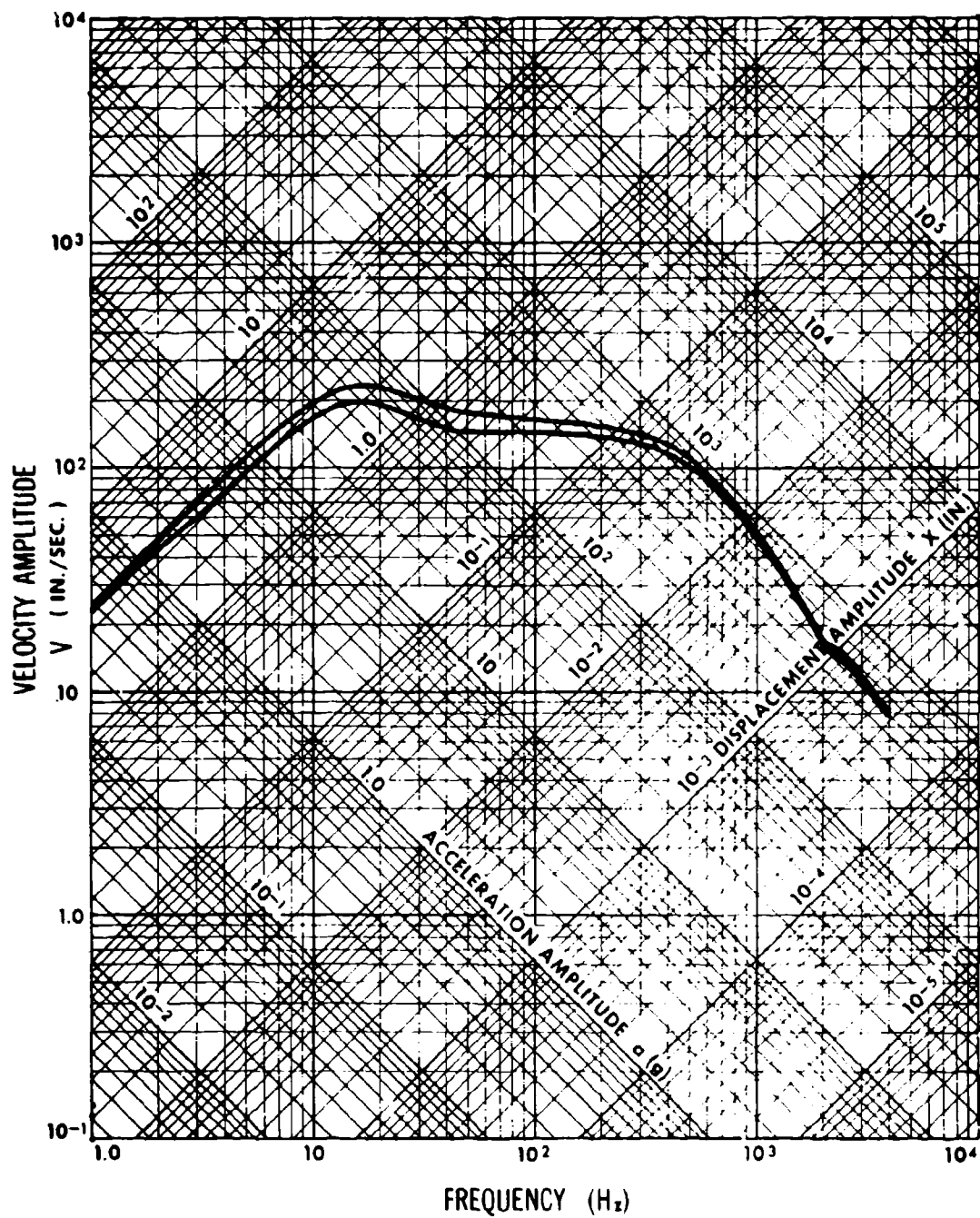
Figure 1. Relative displacement shock spectrum plotted on four-coordinate paper. Upper curve is undamped, the lower curve is for 5% damping.
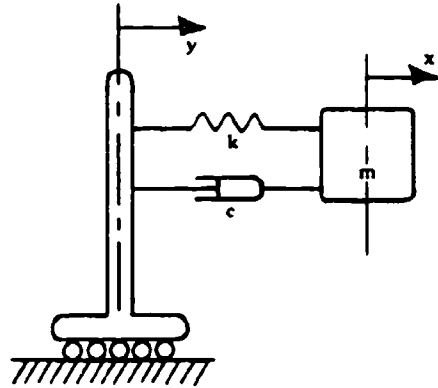
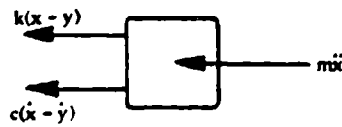Figure 2. Single-degree-of-freedom vibratory system.

Figure 3. Free body diagram.

Appendix A

DERIVATION OF RECURSIVE EQUATIONS

DERIVATION APPROACH AND NEW SYMBOLS

As has been discussed in the text, one seeks a solution of Equation 5,

$$\ddot{z} + 2 \zeta \omega \dot{z} + \omega^2 z = -\ddot{y} \tag{5}$$

applicable to a list of closely and equally spaced values of the input foundation acceleration, $\ddot{y}$. Hence, three such solutions of Equation 5 will be developed that apply from one excitation point to the next. They will be recursive calculating equations, through which the input data are successively passed, thereby yielding a list of output values, z. The text equations and definitions 1 through 6 will be used. The approach of this Appendix is to give a general beginning leading to two known results and the new result.

The <u>Duhamel</u> integral solution, Equation 6, is the point of departure. The derivative of this solution with respect to time is needed, and this must be done carefully since t is a limit of, and occurs within, the integral. Pipes (Ref 14) explains this carefully, and following his instruction one obtains

$$\dot{z} = -\frac{z_0 \omega}{\eta} e^{-\zeta \omega t} \sin \omega_d t + \frac{\dot{z}_0}{\eta} e^{-\zeta \omega t}(-\zeta \sin \omega_d t$$

$$+ \eta \cos \omega_d t) - \frac{1}{\eta} \int_0^t \ddot{y}(\tau) e^{-\zeta \omega(t-\tau)}[-\zeta \sin \omega_d(t-\tau)$$

$$+ \eta \cos \omega_d(t-\tau)] d\tau \tag{A-1}$$

25

Consider applying Equations 6 and A-1 from one value of $\ddot{y}$ to the next; i.e., from $\ddot{y}_0$ to $\ddot{y}_1$. As can be read, the equations presume z_o and $\dot{z}_o$ are known. Equation 6 would give z_1, and Equation A-1 would give $\dot{z}_1$.

To clarify what follows, some new symbols must be defined. The time interval between the equally spaced values will be h. Equations 6 and A-1 are to be applied when t equals h. Therefore, define

$$\psi \equiv e^{-\zeta\omega h} \sin \omega_d h \tag{A-2a}$$

$$\equiv e^{-\zeta\omega h} \cos \omega_d h \tag{A-2b}$$

$$_{01} \equiv \int_0^h \ddot{y}(\tau) \, e^{-\zeta\omega(h-\tau)} \sin \omega_d(h-\tau) \, d\tau \tag{A-2c}$$

$$_{01} \equiv \int_0^h \ddot{y}(\tau) \, e^{-\zeta\omega(h-\tau)} \cos \omega_d(h-\tau) \, d\tau \tag{A-2d}$$

Using these new symbols, Equations 6 and A-1 become

$$z_1 = \left(X + \frac{\zeta}{\eta} \psi \right) z_o + \frac{\psi}{\eta \, \omega} \dot{z}_o - \frac{1}{\eta \, \omega} S_{01} \tag{A-3a}$$

$$\dot{z}_1 = -\frac{\omega \, \psi}{\eta} z_o + \left(-\frac{\zeta}{\eta} \psi + X \right) \dot{z}_o + \frac{\zeta}{\eta} S_{01} - C_{01} \tag{A-3b}$$

Note that as these equations are applied to the input list of $\ddot{y}$'s, h, ω, ζ, ω_d, η, X, and ψ are constants. If subscripted α's are to represent constants, Equations A-3 become

$$z_1 = \alpha_1 z_o + \alpha_2 \dot{z}_o + \alpha_3 S_{01} \tag{A-4a}$$

$$\dot{z}_1 = \alpha_4 z_o + \alpha_5 \dot{z}_o + \alpha_6 S_{01} + \alpha_7 C_{01} \tag{A-4b}$$

where $\quad \alpha_1 \equiv X + \frac{\zeta}{\eta} \psi$ $\tag{A-4c}$

26

$$\alpha_2 \equiv \frac{\psi}{\eta \, \omega} \tag{A-4d}$$

$$\alpha_3 \equiv \frac{-1}{\eta \, \omega} \tag{A-4e}$$

$$\alpha_4 \equiv -\frac{\omega \, \psi}{\eta} \tag{A-4f}$$

$$\alpha_5 \equiv -\frac{\zeta \, \psi}{\eta} + X \tag{A-4g}$$

$$\alpha_6 \equiv \frac{\zeta}{\eta} \tag{A-4h}$$

$$\alpha_7 \equiv -1 \tag{A-4i}$$

Consider the integrals S and C. The way in which they are evaluated constitutes the differences between the various results.

TRAPEZOIDAL APPROXIMATION

The crudest method in popular use is the trapezoidal rule (Ref 4), which states that the integrand is taken to be constant over the interval and equal to the average of the values of the integrand at the beginning and end of the interval.

From Equation A-2c,

$$S_{01} = \int_0^h \ddot{y}(\tau) \, e^{-\zeta \omega (h-\tau)} \sin \omega_d (h-\tau) \, d\tau \tag{A-2c}$$

The average of the end point values is

$$A_1 = \frac{1}{2} \left(\ddot{y}_0 \, e^{-\zeta \omega h} \sin \omega_d h + 0 \right)$$

or

$$A_1 = \frac{\psi}{2} \ddot{y}_0 \tag{A-5a}$$

27

Using this as the integrand in A-2c yields

$$S_{01} = \frac{\psi h}{2} \ddot{y}_0 \qquad \text{(A-5b)}$$

for the value of S_{01} by the trapezoidal rule. To get the value of C_{01}, consider

$$C_{01} = \int_0^h \ddot{y}(\tau) \, e^{-\zeta\omega(h-\tau)} \cos \omega_d(h-\tau) \, d\tau \qquad \text{(A-2d)}$$

The average of the end point values is

$$A_2 = \frac{1}{2} \left(\ddot{y}_0 \, e^{-\zeta\omega h} \cos \omega_d h + \ddot{y}_1 \right) \qquad \text{(A-5c)}$$

Using this as the integrand in A-2d yields

$$C_{01} = \frac{h}{2} \left(X \ddot{y}_0 + \ddot{y}_1 \right) \qquad \text{(A-5d)}$$

The point to note here is that in this case, and in the other cases to be considered, the integrals, S_{01} and C_{01}, are composed of constants and input data values; no z's or output values. This can be seen more clearly as follows.

By substituting these values of S_{01} and C_{01}, Equations A-5b and A-5d into Equations A-4a and A-4b, one obtains

$$z_1 = \alpha_1 z_0 + \alpha_2 \dot{z}_0 + \alpha'_3 \ddot{y}_0 \qquad \text{(A-6a)}$$

and

$$\dot{z}_1 = \alpha_4 z_0 + \alpha_5 \dot{z}_0 + \alpha'_6 \ddot{y}_0 + \alpha'_7 \ddot{y}_1 \qquad \text{(A-6b)}$$

where these new α's are defined to be

$$\alpha_3' \equiv \alpha_3 \psi h/2 \tag{A-6c}$$

$$\alpha_6' \equiv \frac{h}{2}\left(\alpha_6 \psi + \alpha_7\right) \tag{A-6d}$$

$$\alpha_7' \equiv \alpha_7 h/2 \tag{A-6e}$$

Thus, it is seen that, upon evaluation of S_{01} and C_{01}, which become products of constants and the input $\ddot{y}$'s, Equations A-4a and A-4b become Equations A-6a and A-6b. These form a pair that are used together to generate a list of both z's and $\dot{z}$'s, from the $\ddot{y}$'s. Assuming the α's have been computed and the full list of the $\ddot{y}$'s is on hand, one would start the calculating at $\ddot{y}_0$, taking z_0 and $\dot{z}_0$ equal to zero, and proceed to z_i. Knowing z_i and $\dot{z}_i$, the pair of equations will yield z_{i+1} and $\dot{z}_{i+1}$ and so on.

With this in mind it is now possible to go back and combine the two equations (A-4a and A-4b) into a single equation for z_i alone, thereby eliminating the need to keep computing both z and $\dot{z}$. This development does not require Z Transform Theory (Ref 8 and 9) and is more straight-forward than the approach of Cronin (Ref 4).

DEVELOPMENT OF A SINGLE RECURSIVE EQUATION

One begins by solving Equation A-4a for $\dot{z}_0$ and defining new α's. This yields

$$\dot{z}_0 = \alpha_8 z_0 + \alpha_9 z_1 + \alpha_{10} S_{01} \tag{A-7a}$$

$$\text{where} \quad \alpha_8 \equiv -\frac{\alpha_1}{\alpha_2} \tag{A-7b}$$

$$\alpha_9 \equiv \frac{1}{\alpha_2} \tag{A-7c}$$

$$\alpha_{10} \equiv -\frac{\alpha_3}{\alpha_2} \tag{A-7d}$$

Equation A-7a is next substituted into A-4b, which then becomes

$$\dot{z}_1 = \alpha_4 z_o + \alpha_5(\alpha_8 z_o + \alpha_9 z_1 + \alpha_{10} S_{01})$$
$$+ \alpha_6 S_{01} + \alpha_7 C_{01} \tag{A-8a}$$

By defining new α's, this can be written as

$$\dot{z}_1 = \alpha_{11} z_o + \alpha_{12} z_1 + \alpha_{13} S_{01} + \alpha_{14} C_{01} \tag{A-8b}$$

where $\quad \alpha_{11} \equiv \alpha_4 + \alpha_5 \sigma_8 \tag{A-8c}$

$$\alpha_{12} \equiv \alpha_5 \alpha_9 \tag{A-8d}$$

$$\alpha_{13} \equiv \alpha_5 \alpha_{10} + \alpha_6 \tag{A-8e}$$

$$\alpha_{14} \equiv \alpha_7 \tag{A-8f}$$

This is a very interesting result. It states that the velocity at point 1 is exactly given by z_o and z_1, and the values of the two integrals. Note that it does not require z_o.

The next step is to realize that Equation A-4a also applies between points 1 and 2 equally as well as between points 0 and 1, and, thus, the following can be written

$$z_2 = \alpha_1 z_1 + \alpha_2 \dot{z}_1 + \alpha_3 S_{12} \tag{A-9a}$$

The value of $\dot{z}_1$ from Equation A-8b is substituted into this form, which yields

$$z_2 = \alpha_1 z_1 + \alpha_2(\alpha_{11} z_o + \alpha_{12} z_1 + \alpha_{13} S_{01} + \alpha_{14} C_{01}) \tag{A-9b}$$
$$+ \alpha_3 S_{12}$$

And finally by defining more α's, this becomes

$$z_2 = \alpha_{15} z_0 + \alpha_{16} z_1 + \alpha_{17} S_{01} + \alpha_{18} S_{12} + \alpha_{19} C_{01} \tag{A-9c}$$

where $\quad \alpha_{15} \equiv \alpha_2 \alpha_{11}$ \hfill (A-9d)

$$\alpha_{16} \equiv \alpha_1 + \alpha_2 \alpha_{12} \tag{A-9e}$$

$$\alpha_{17} \equiv \alpha_2 \alpha_{13} \tag{A-9f}$$

$$\alpha_{18} \equiv \alpha_3 \tag{A-9g}$$

$$\alpha_{19} \equiv \alpha_2 \alpha_{14} \tag{A-9h}$$

Equation A-9c is the new result. Note that it is quite different
from Equation A-4a in that it requires no input velocity; thus,
Equation A-4b does not have to be computed along with it. It reduces
the number of computations and, thus, the accumulation of error. It is
therefore more accurate and economical. If the velocity is desired, it
can be calculated with Equation A-8b. As it stands it is exact. Approx-
imation will be introduced when the integrals are evaluated and by
round-off error accumulated in the calculations.

Before using Equation A-9c with various approximations for the
values of the integrals, S_{01} and C_{01}, many of the α values will be
required and so the most important ones are listed below.

$$\alpha_{11} = -\frac{\eta \omega}{\psi} e^{-2\zeta\omega h} = -\frac{\eta \omega e^{-\zeta\omega h}}{\sin \omega_d h} \tag{A-10a}$$

$$\alpha_{12} = \omega \frac{\eta X}{\psi} - \zeta = \omega \frac{\eta}{\tan \omega_d h} - \zeta \tag{A-10b}$$

$$\alpha_{13} = \frac{X}{\psi} = \frac{1}{\tan \omega_d h} \tag{A-10c}$$

$$\alpha_{14} = -1 \tag{A-10d}$$

$$\alpha_{15} = -e^{-2\zeta\omega h} \tag{A-10e}$$

$$\alpha_{16} = 2X \tag{A-10f}$$

$$\alpha_{17} = \frac{X}{\eta\,\omega} \tag{A-10g}$$

$$\alpha_{13} = \frac{-1}{\eta\,\omega} \tag{A-10h}$$

$$\alpha_{19} = \frac{-\Psi}{\eta\,\omega} \tag{A-10i}$$

TRAPEZOIDAL RULE RECURSIVE EQUATIONS

The calculating equations that result from approximating the integrals by the trapezoidal rule are obtained by substituting Equations A-5b and A-5d into Equation A-9c. This yields

$$z_2 = \alpha_{15}\, z_0 + \alpha_{16}\, z_1 + \gamma_1\, \ddot{y}_1 \tag{A-11a}$$

where

$$\gamma_1 \equiv -(h\,\psi)/(\eta\,\omega) \tag{A-11b}$$

This is the result given in References 4 and 8. Lane (Ref 8) derived it in 1964 by Z Transform Theory, which did not make clear the approximation used in evaluating the integral. Cronin (Ref 4) in 1973 was the first to derive this result by conventional means; his work showed that it could be derived with the trapezoidal rule. It is indeed simple: from only one input value, and not even the value of the input corresponding to the time at which the output is being computed, and two previous output values, the equation yields a new output value.

The velocity equation is given by substituting Equations A-5b and A-5d into Equation A-8b. This yields

$$\dot{z}_1 = \alpha_{11}\, z_0 + \alpha_{12}\, z_1 + \gamma_2\, \ddot{y}_1 \tag{A-11c}$$

where

$$\gamma_2 \equiv -h/2 \tag{A-11d}$$

This equation is used to compute a velocity if required; e.g., the final velocity is required as input to the residual response calculation. Reiterating, Equation A-11a is used to compute a list of z's from the list of $\ddot{y}$'s; i.e., if $\ddot{y}_1$ is the first non-zero point, Equation A-11a yields z_2 as the first non-zero output value. Next $\ddot{y}_2$ and z_2 are used to compute z_3; next $\ddot{y}_3$, z_3, and z_2 are used to compute z_4, etc. Equation A-11a written for the general (n+1) point is

$$z_{n+1} = \alpha_{15} z_{n-1} + \alpha_{16} z_n + \gamma_1 \ddot{y}_n \qquad \text{(A-11e)}$$

which is a more common way of writing a recursive equation.

STRAIGHT LINE ACCELERATION APPROXIMATION IN INTEGRAL

There are several other approaches one could take in evaluating the integrals S and C. Cronin (Ref 4) suggested considering the values of the integrand at three consecutive points as defining a parabola and then integrating the parabola so defined. Rather than approximate the whole integrand by an approximating function, O'Hara (Ref 10) in 1962, and Nigam and Jennings (Ref 3) in 1968, both derived the pair of computing equations that result from Equations A-4a and A-4b when the acceleration input is approximated as a straight line between two adjacent values. They then integrated the product of the approximate acceleration function and the remainder of the integrand. Vernon (Ref 5) in 1967 published an unusual pair, similar to Equations A-4a and A-4b, again with the straight line acceleration approximation, except he chose to compute $\ddot{x}$ and x as output values (as opposed to z and $\dot{z}$). Rodeman (Ref 15) in 1974 used Z Transform Theory more accurately than Lane (Ref 8), approximating only the acceleration as a straight line, and deriving a single recursive equation with acceleration, $\ddot{x}$, as the output, as opposed to z. Thus, the straight line approximation has a host of advocates. Except for Cronin (Ref 4), none found the single recursive equation without Z Transform Theory.

As to the accuracy of the straight line approximation, Reference 3 calls the method "exact". O'Hara (Ref 10) gives formulas for approximating the acceleration by a parabola, which clearly has to be better than a straight line. The philosophy taken here is that, if the sequence of digits when plotted with straight lines connecting adjacent points is a good visual approximation to the analog data, then the digitizing is adequate for approximating the acceleration function with straight lines.

To evaluate the integrals S_{01} and C_{01} with the acceleration taken as a straight line between adjacent values, or $\ddot{y}_0$ and $\ddot{y}_1$, let

$$\ddot{y}(\tau) = a + b\,\tau \tag{A-12a}$$

where

$$a \equiv \ddot{y}_0 \tag{A-12b}$$

$$b \equiv (\ddot{y}_1 - \ddot{y}_0)/h \tag{A-12c}$$

Equation A-12a is used in the definition of S_{01}, Equation A-2c, to yield

$$S_{01} = \int_0^h (a + b\,\tau)\, e^{-\zeta\omega(h-\tau)}\, \sin \omega_d(h-\tau)\, d\tau \tag{A-13a}$$

To integrate this expression, change the variable

$$u = h - \tau \tag{A-13b}$$

Thus,

$$du = - d\tau \tag{A-13c}$$

and

$$\tau = h - u \tag{A-13d}$$

For the limits, note from Equation A-13b that when

$$\tau = 0, \qquad u = h \tag{A-13e}$$

34

and when

$$\tau = h, \qquad u = 0 \tag{A-13f}$$

By using Equations A-13c through f in A-13a one obtains

$$S_{01} = -\int_h^0 (a + b\,h - b\,u)\, e^{-\zeta \omega u} \sin \omega_d u\, du \tag{A-14a}$$

which can be written as

$$S_{01} = (a + b\,h) \int_0^h e^{-\zeta \omega u} \sin \omega_d u\, du \tag{A-14b}$$

$$- b \int_0^h u\, e^{-\zeta \omega u} \sin \omega_d u\, du$$

These are tabulated integrals,* and their evaluation is straightforward, but lengthy. After considerable simplification, one obtains

$$S_{01} = \alpha_{20}\, y_0 + \alpha_{21}\, y_1 \tag{A-15a}$$

where

$$\alpha_{20} \equiv \frac{-1}{\omega} \left\{ \zeta\, \psi + \eta\, X + \frac{1}{\omega h} \left[(2\, \zeta^2 - 1)\psi + 2\, \zeta\, \eta (X - 1) \right] \right\} \tag{A-15b}$$

$$\alpha_{21} \equiv \frac{1}{\omega} \left\{ \eta + \frac{1}{\omega h} \left[(2\, \zeta^2 - 1)\psi + 2\, \zeta\, \eta (X - 1) \right] \right\} \tag{A-15c}$$

It should be noticed from Equations A-12b and A-12c that the same result would be found if the integration were from $\ddot{y}_1$ to $\ddot{y}_2$; thus, S_{12} is given by

---

*See Reference 16, pp 99 and 101.

35

$$S_{12} = \alpha_{20} y_1 + \alpha_{21} y_2 \qquad \text{(A-15d)}$$

Next, Equations A-12 are used in the definition of C_{01}, Equation A-2d, which becomes

$$C_{01} = \int_0^h (a + b \tau) e^{-\zeta\omega(h-\tau)} \cos \omega_d(h-\tau) \, d\tau \qquad \text{(A-16a)}$$

By using the same change of variable indicated in Equations A-13, one obtains similarly

$$C_{01} = (a + b h) \int_0^h e^{-\zeta\omega u} \cos \omega_d u \, du \qquad \text{(A-16b)}$$

$$- b \int_0^h u \, e^{-\zeta\omega u} \cos \omega_d u \, du$$

which as before contains integrals that are tabulated in Burrington (Ref 16). Again, after considerable simplification, the result can be written as

$$C_{01} = \alpha_{22} y_0 + \alpha_{23} y_1 \qquad \text{(A-17a)}$$

where

$$\alpha_{22} \equiv \frac{1}{\omega} \left\{ \eta \psi - \zeta X + \frac{1}{\omega h} [2 \zeta \eta \psi - (2 \zeta^2 - 1)(X - 1)] \right\} \qquad \text{(A-17b)}$$

$$\alpha_{23} \equiv \frac{1}{\omega} \left\{ \zeta - \frac{1}{\omega h} [2 \zeta \eta \psi - (2 \zeta^2 - 1)(X - 1)] \right\} \qquad \text{(A-17c)}$$

36

TWO-EQUATION COMPUTING PAIR FOR STRAIGHT LINE ACCELERATION

To permit verification of the results, substitute Equations A-15a and A-17a into Equations A-4a and A-4b to show that correct values of the constants were derived. This yields

$$z_1 = \alpha_1 z_o + \alpha_2 \dot{z}_o + \alpha_3(\alpha_{20} \ddot{y}_0 + \alpha_{21} \ddot{y}_1) \qquad \text{(A-18a)}$$

and

$$\dot{z}_1 = \alpha_4 z_o + \alpha_5 \dot{z}_o + \alpha_6(\alpha_{20} \ddot{y}_0 + \alpha_{21} \ddot{y}_1) \qquad \text{(A-18b)}$$
$$+ \alpha_7(\alpha_{22} \ddot{y}_0 + \alpha_{23} \ddot{y}_1)$$

Defining new constants, these can be written as

$$z_1 = \alpha_1 z_o + \alpha_2 \dot{z}_o + \alpha_{29} \ddot{y}_0 + \alpha_{30} \ddot{y}_1 \qquad \text{(A-18c)}$$

$$\dot{z}_1 = \alpha_4 z_o + \alpha_5 \dot{z}_o + \alpha_{31} \ddot{y}_0 + \alpha_{32} \ddot{y}_1 \qquad \text{(A-18d)}$$

where
$$\alpha_{29} \equiv \alpha_3 \alpha_{20} \qquad \text{(A-18e)}$$

$$\alpha_{30} \equiv \alpha_3 \alpha_{21} \qquad \text{(A-18f)}$$

$$\alpha_{31} \equiv \alpha_6 \alpha_{20} + \alpha_7 \alpha_{22} \qquad \text{(A-18g)}$$

$$\alpha_{32} \equiv \alpha_6 \alpha_{21} + \alpha_7 \alpha_{23} \qquad \text{(A-18h)}$$

Using the previously obtained values of the indicated α's, these are found to be

$$\alpha_{29} = \frac{1}{\omega^3 h} \left[(2 \zeta^2 - 1 + \zeta \omega h) \frac{\psi}{\eta} + (2 \zeta + \omega h) X - 2 \zeta \right] \qquad \text{(A-18i)}$$

$$\alpha_{30} = \frac{1}{\omega^3 h} \left[(1 - 2 \zeta^2) \frac{\psi}{\eta} - 2 \zeta(X - 1) - \omega h \right] \qquad \text{(A-18j)}$$

$$\alpha_{31} = \frac{1}{\omega^2 h} \left[1 - (\zeta + \omega h) \frac{\psi}{\eta} - X \right] \qquad (A\text{-}18k)$$

$$\alpha_{32} = \frac{1}{\omega^2 h} \left(\frac{\zeta \psi}{\eta} + X - 1 \right) \qquad (A\text{-}18l)$$

These are the equations used in Reference 3, and the constants agree, except for two typographical misprints, which are corrected in the Fortran program listing. One might at first think these expressions adequate if both the relative velocity and displacement are desired. However, such is not the case because the recursive process would then involve more calculations and, thus, a greater accumulation of error. The equations are only given here for completeness.

NEW SINGLE RECURSIVE EQUATION FOR A STRAIGHT LINE ACCELERATION APPROXIMATION

Now the results of evaluating the integrals S_{01}, S_{12}, and C_{01}, with the acceleration taken as a straight line between the digitized values, are obtained by merely substituting Equations A-15 and A-17 into Equation A-9c. The result can be arranged as follows

$$z_2 = \alpha_{15} z_0 + \alpha_{16} z_1 + \alpha_{24} \ddot{y}_0 + \alpha_{25} \ddot{y}_1 + \alpha_{26} \ddot{y}_2 \qquad (A\text{-}19a)$$

where the new constants are found to be

$$\alpha_{24} \equiv \frac{1}{\omega^3 h} \left[2 \zeta X - e^{-2\zeta\omega h}(2 \zeta + \omega h) + (1 - 2 \zeta^2)\frac{\psi}{\eta} \right] \qquad (A\text{-}19b)$$

$$\alpha_{25} \equiv \frac{2}{\omega^3 h} \left[\omega h X - (1 - 2 \zeta^2) \frac{\psi}{\eta} - \zeta(1 - e^{-2\zeta\omega h}) \right] \qquad (A\text{-}19c)$$

$$\alpha_{26} \equiv \frac{1}{\omega^3 h} \left[(1 - 2 \zeta^2) \frac{\psi}{\eta} + 2 \zeta(1 - X) - \omega h \right] \qquad (A\text{-}19d)$$

38

and the first two constants are repeated here for convenience

$$\alpha_{15} = -e^{-2\zeta\omega h} \tag{A-10h}$$

$$\alpha_{16} = 2X \tag{A-10i}$$

It is hoped that the reader has observed that the indices 0, 1, and 2 on the z and $\ddot{y}$ values do not restrict this result to the first three points of the data list, i.e., they are perfectly general; in fact, to better conform to other writings on recursive calculating equations, Equation A-19a can be written for calculating the ith value as

$$z_i = \alpha_{15} z_{i-2} + \alpha_{16} z_{i-1} + \alpha_{24} \ddot{y}_{i-2} + \alpha_{25} \ddot{y}_{i-1} + \alpha_{26} \ddot{y}_i \tag{A-19e}$$

Though it is a trivial observation, especially after going through the algebra three times, the reader will note on his first time through that

$$\psi^2 + X^2 = e^{-2\zeta\omega h} \tag{A-19f}$$

is a group that recurs.

The velocity equation for calculating velocities from the computed displacements is similarly obtained by substituting Equations A-15 and A-17 into A-8b which again, after simplification, can be written as

$$\dot{z}_i = \alpha_{11} z_{i-1} + \alpha_{12} z_i + \alpha_{27} \ddot{y}_{i-1} + \alpha_{28} \ddot{y}_i \tag{A-20a}$$

where

$$\alpha_{27} \equiv \frac{1}{\omega^2 h}\left[\frac{2\zeta\eta X}{\psi} - \frac{\eta\, e^{-2\zeta\omega h}}{\psi}(2\zeta + \omega h) - 2\zeta^2 + 1\right] \tag{A-20b}$$

$$\alpha_{28} \equiv \frac{1}{\omega^2 h}\left[\frac{1}{\psi} 2\zeta\eta\, e^{-2\zeta\omega h} + (\omega h - 2\zeta)\frac{\eta X}{\psi} - \zeta - 1\right] \tag{A-20c}$$

and from the previous results

$$\alpha_{11} = -\frac{1}{\psi} \eta \omega e^{-2\zeta\omega h} \qquad (A-10a)$$

$$\alpha_{12} = \omega\left(\frac{\eta X}{\psi} - \zeta\right) \qquad (A-10c)$$

Equations A-19e and A-20a are the new result. Equation A-19e is the equation that marches through the data yielding relative displacements, z_i's, from only previously computed z_i's and excitation values. Note also that each step requires only five multiplications and four additions. It is theoretically identical to using both Equations A-18c and A-18d, but avoids relative velocity as an intermediate result. Equation A-20a is the velocity calculation; it exactly calculates the velocity at any desired point from computed displacements and input values.

Incidentally, it is more straightforward to deduce Equations A-18c and A-18d directly from the original differential Equation 5 by sta.:ting with the straight line acceleration approximaticn in the right-hand side and obtaining the needed particular solution by undetermined coefficients. This is the technique of Reference 3. Then Equations A-18c and A-18d are transformed to Equation A-19e by going through the steps of Equations A-7 to A-9 for this specific case rather than for the general case.

FURTHER COMMENTS ON ACCURACY AND REMAINING PROBLEMS

Some comments were made in the report body on accuracy, and they are applicable here. Additionally, one might say that if both the relative displacement and the relative velocity are desired, one might just as well use the two-equation approach of References 3 and 10. That would be less accurate because the propagation of the solution through the data would require eight as opposed to five multiplications per step and six as opposed to four additions per step. Accuracy would suffer.

One can certainly envision parabolic or cubic approximation of the
acceleration input during the integration, and one would certainly
expect this to be an improved approximation. It would permit one to
sample more coarsely, perhaps at only five points per cycle of highest
frequency present. But in a sense this becomes defeating and introduces
complications. In the method presented here, the results are only
evaluated at each point. As long as the points occur at least 10 points
per cycle, the maximum error can only be 5%. Making the sampling coarser
would then require an additional procedure of evaluating the result in
between points. This is suggested in Reference 10 and implemented in
Reference 5. At this point it appears to be a substantial effort to
implement and test; the value of the outcome is at best uncertain.

Finally, what is most disappointing and complicating is the fact
that all of the constants in all of the methods have insidious low
frequency problems. An examination of all the α's reveals that crucial
ones in all of the methods approach indeterminate forms for very small
values of ωh. Thus, one cannot without extreme caution evaluate for
frequencies outside the range

$$10 \leq f_s/f \leq 8,000 \qquad\qquad (A-21)$$

This range was determined empirically for a computer that carries about
14 significant figures. Double precision improves the situation, but
not considerably. Undoubtedly this is being resolved by those producing
specialized shock spectrum computing machinery; however, their algorithms
are invariably proprietary and, thus, not subject to technical review.
What will have to be done is a careful series expansion of the α's, and
then the programs will have to shift to the expanded α's when frequencies
are low enough to permit their use. Until such time as this can be
done, the above limits must be imposed. It is also presumed that since
none of the authors point this out, they are unaware of the problem. It
is insidious because there is no way you can realize it is happening.
All of a sudden spurious peaks and valleys occur in the spectrum with no
continuity from one frequency to the next, and one finds that the computer
is generating incorrect values for the α's.

Appendix B

COMPUTER PROGRAMS

LIST

PROGRAM   SPCTRM

```
00100 PROGRAM SPCTRM(INPUT,OUTPUT,TAPE1,TAPE10)
00110 DIMENSION Y(2050),Z(2050),OUT(5,500)
00120 PRINT,*INPUT ACCELS IN G'S*
00130 PRINT,*NO.OF SAMPLES OF INPUT DATA (I)*
00140 READ,N1
00150 PRINT,*APPROX LOW FREQ, (HZ), (F)*
00160 READ,FLOW
00170 PRINT,*APPROX HIGH FREQ, (F)*
00180 READ,FHIGH
00190 PRINT,*SAMPLING RATE (SAMPLES/SEC.)    (F)*
00200 READ,S
00210 PRINT,*FREQS PER DECADE, (ABT 25), (F)*
00220 READ,FPD
00230 PRINT,*SCALING FACTOR (G.S/DATA UNIT)   (F)*
00240 READ,SF
00250 PRINT,*ZERO OFFSET   (DATA UNITS)   (F)*
00260 READ,ZO
00270 FLOWLOG=ALOG10(FLOW)
00280 C2=FLOAT(INT(FLOWLOG)-1)
00290 AJLOW=FPD*(FLOWLOG-C2)
00300 JSTART=INT(AJLOW)
00310 IF(AJLOW.NE.FLOAT(JSTART))JSTART=JSTART+1
00320 FSTART=10.**(FLOAT(JSTART)/FPD+C2)
00330 JSTOP=INT(FPD*(ALOG10(FHIGH)-C2))
00340 FSTOP=10.**(FLOAT(JSTOP)/FPD+C2)
00350 NFREQS=JSTOP-JSTART+1
00360 PRINT,*LOWEST AND HIGHEST FREQUENCIES ARE*,FSTART,FSTOP
00370 PRINT,*NO. OF FREQS IS*,NFREQS
00380 TPI=6.28318531
00390 READ(1,69)(Y(I),I=1,N1)
00400 69 FORMAT(5X,4(1X,I5))
00410 GSF=SF*386.088
00420 DO 33 I=1,N1
00430 Y(I)=GSF*((FLOAT(Y(I)))-ZO)
00440 33  CONTINUE
00450 PRINT,*DAMPING RATIO?   (F)*
00460 READ,ZETA
00470 H=1./S
00480 ETA=SQRT(1.-ZETA*ZETA)
00490 G1=2.*ZETA
00500 G2=1.-G1*ZETA
00510 DO 111 J=JSTART,JSTOP
00520 F=10.**(FLOAT(J)/FPD+C2)
00530 WOM=TPI*F
00540 G3=WOM*H
00550 G4=EXP(-ZETA*G3)
00560 A15=-G4*G4
00570 G5=ETA*G3
```

```
00580 G6=G4/ETA*SIN(G5)
00590 CHI=G4*COS(G5)
00600 G7=CHI/G6
00610 G8=-A15/G6
00620 A16=2.*CHI
00630 A11=-WOM*G8
00640 A12=WOM*(G7-ZETA)
00650 G9=G3*WOM
00660 G10=G1+G3
00670 G11=G2*G6
00680 G12=G9*WOM
00690 A24=(G1*CHI+A15*G10+G11)/G12
00700 A25=2./G12*(G3*CHI-G11-ZETA*(1.+A15))
00710 A26=(G11+G1*(1.-CHI)-G3)/G12
00720 A27=(G1*G7-G8*G10+G2)/G9
00730 A28=(G1*G8+(G3-G1)*(G7-ZETA)-1.)/G9
00740 ZIMIN=0.
00750 ZIMAX=0.
00760 Z(1)=A26*Y(1)
00770 Z(2)=A16*Z(1)+A25*Y(1)+A26*Y(2)
00780 DO 13 I=3,NI
00790 Z(I)=A15*Z(I-2)+A16*Z(I-1)+A24*Y(I-2)+A25*Y(I-1)+A26*Y(I)
00800 IF(Z(I).GT.ZIMAX)ZIMAX=Z(I)
00810 IF(Z(I).LT.ZIMIN)ZIMIN=Z(I)
00820 13 CONTINUE
00830 ZO=Z(NI).
00840 ZDO=A11*Z(NI-1)+A12*Z(NI)+A27*Y(NI-1)+A28*Y(NI)
00850 CALL RESID(WOM,ZETA,ETA,ZO,ZDO,ZRMAX,ZRMIN)
00860 ZMIN=AMIN1(ZIMIN,ZRMIN)
00870 ZMAX=AMAX1(ZIMAX,ZRMAX)
00880 SVMIN=WOM*ZMIN
00890 SVMAX=WOM*ZMAX
00900 OUT(1,J)=ZMIN
00910 OUT(2,J)=ZMAX
00920 OUT(3,J)=SVMIN
00930 OUT(4,J)=SVMAX
00940 OUT(5,J)=F
00950 111 CONTINUE
00960 DO 410 I=JSTART,JSTOP
00970 410 WRITE(10,425)OUT(5,I),(OUT(KK,I),KK=1,4)
00980 PRINT,/* PLEASE NOTE*/
00990 PRINT,*ON THE OUTPUT FILE THE DATA COLUMNS READ AS*
01000 PRINT,*FREQUENCY(HZ), ZMIN(IN), ZMAX, SVMIN(IPS), SVMAX*
01010 PRINT,*TYPE "SAVE,TAPE10=____"*
01020 PRINT,*TYPE"RETURN,TAPE10"*
01030 425 FORMAT(F12.5,4E15.6)
01040 END FILE 10
01050 STOP
01060 END
01070 SUBROUTINE RESID(WOM,ZETA,ETA,ZO,ZDO,ZRMAX,ZRMIN)
01080C  THIS SUBROUTINE COMPUTES RESIDUAL RESPONSE
01090 B=ZO
01100 DELT=ASIN(ZETA)
01110 A=(ZO*ZETA+ZDO/WOM)/ETA
```

45

```
01120 PI=3.141592654
01130 IF(A.EQ.0..AND.B.EQ.0.)GO TO 24
01140 IF(A.EQ.0.) GO TO 26
01150 IF(B.EQ.0.) GO TO 28
01160 IF(A.AND.B.GT.0. .OR.A.AND.B.LT.0.) GO TO 30
01170 BETAI=PI-ATAN(-A/B)
01180 GO TO 80
01190 30 BETAI=ATAN(A/B)
01200 GO TO 80
01210 28 BETAI=PI/2.
01220 GO TO 80
01230 24 ZRMIN=0.
01240 ZRMAX=0.
01250 GO TO 100
01260 26 BETAI=0.
01270 80 IF(BETAI.LT.DELT)BETAI=BETAI+PI
01280 WDT1=BETAI-DELT
01290 WDT2=WDT1+PI
01300 Z1=EXP(-ZETA*WDT1/ETA)*(A*SIN(WDT1)+B*COS(WDT1))
01310 Z2=EXP(-ZETA*WDT2/ETA)*(A*SIN(WDT2)+B*COS(WDT2))
01320 ZRMAX=AMAX1(Z1,Z2)
01330 ZRMIN=AMIN1(Z1,Z2)
01340 100 RETURN
01350 END
```

```
00100 PROGRAM PLTVELF(INPUT,OUTPUT,TAPE1,TAPE10)
00110 DIMENSION FREQ(250),VEL(250),IFREQ(250),IVEL(250),VELN(250)
00120 PRINT,*TYPE#FTV#FOR A FT PLOT OR TYPE #SS# FOR A SS PLOT*
00130 READ73,FT
00140 73 FORMAT(A3)
00150 IF(FT.EQ.3HFTV)GOTO74
00160 PRINT,*TYPE#+1#FOR POS.SPECT.OR#-1#FOR NEG.SPECT.OR#+0#FOR 0.ASS*
00170 READ 99,IPLOT
00180 99 FORMAT(I2)
00190 74 PRINT,*NO.OF AMPLITUDES TO BE PLOTTED*
00200 READ,N
00210 PRINT,*TYPE#PLTL#FOR A LINE PLOT OR#PLTP# FOR A PT.PLOT*
00220 READ 71,PLOT
00230 71 FORMAT(A5)
00240 IF(FT.EQ.3HFTV)GOTO75
00250 IF(IPLOT)4,5,6
00260 75 READ(1,20)(FREQ(J),VEL(J),J=1,N)
00270 20 FORMAT(2E15.6)
00280 GO TO 7
00290 4 READ(1,18)(FREQ(J),VEL(J),J=1,N)
00300 18 FORMAT(F12.5,30X,E15.6)
00310 GO TO 7
00320 17 FORMAT(F12.5,45X,E15.6)
00330 6 READ(1,17)(FREQ(J),VEL(J),J=1,N)
00340 7 K=0
00350 IF(FREQ(1).LT.1.) K=2500
00360 IF(K.EQ.0)PRINT,*THE ORIGIN IS AT A FREQUENCY OF 1(HZ)*
00370 IF(K.NE.0)PRINT,*THE ORIGIN IS AT A FREQUENCY OF 0.1 HZ.*
00380 DO 88 I=1,N
00390 IFREQ(I)=INT(ALOG10(FREQ(I))*9999./4.)
00400 IFREQ(I)=IFREQ(I)+K
00410 VEL(I)=ABS(VEL(I))
00420 88 IVEL(I)=INT((ALOG10(VEL(I))+1.)*9999./5.)
00430 PRINT,*ALL THE VALUES ARE IN*
00440 PRINT,*SET UP PLOTTER TO PLOT; TURN ON PUNCH; HIT CR*
00450 PAUSE
00460 PRINT,*HERE IS SOME LEADER*
00470 PRINT71,PLOT
00480 DO 30 I=1,N
00490 30 PRINT 40,IFREQ(I),IVEL(I)
00500 40 FORMAT(2I5)
00510 PRINT,*PLTT*
00520 PRINT,*SHUT OFF PLOTTER; TURN OFF PUNCH*
00530 GO TO 10
00540 19 FORMAT(F12.5,30X,2E15.6)
00550 5 READ(1,19)(FREQ(J),VELN(J),VEL(J),J=1,N)
00560 K=0
00570 IF(FREQ(1).LT.1.) K=2500
00580 IF(K.EQ.0)PRINT,*THE ORIGIN IS AT A FREQ.OF 1.0 (HZ)*
00590 IF(K.EQ.2500)PRINT,*THE ORIGIN IS AT A FREQ.OF 0.1 (HZ)*
00600 DO 21 I=1,N
00610 IFREQ(I)=INT(ALOG10(FREQ(I))*9999./4.)
```

47

```
00620 IFREQ(I)=IFREQ(I)+K
00630 VELN(I)=ABS(VELN(I))
00640 VEL(I)=ABS(VEL(I))
00650 VEL(I)=AMAX1(VELN(I),VEL(I))
00660 21 IVEL(I)=INT((ALOG10(VEL(I))+1.)*9999./5.)
00670 PRINT,*ALL THE VALUES ARE IN*
00680 PRINT,*SET UP PLOTTER TO PLOT; TURN ON PUNCH; HIT CR*
00690 PAUSE
00700 PRINT,*HERE IS SOME LEADER*
00710 PRINT71,PLOT
00720 DO 31 I=1,N
00730 31 PRINT 40,IFREQ(I),IVEL(I)
00740 PRINT,*PLTT*
00750 PRINT,*SHUT OFF PLOTTER; TURN OFF PUNCH*
00760 10 STOP
00770 END
```

Appendix C

DETAILS OF THE RESIDUAL SPECTRUM CALCULATION

·

The table given in the main text is repeated below to organize the examination of the nine cases

	A > 0	A = 0	A < 0
B > 0	I	II	III
B = 0	IV	V	VI
B < 0	VII	VIII	IX

Because many values of β will satisfy Equation 15a, one imagines the signs of A and B in Equation 15 to deduce the proper value of β for β_1.

Case I, A > 0, B > 0

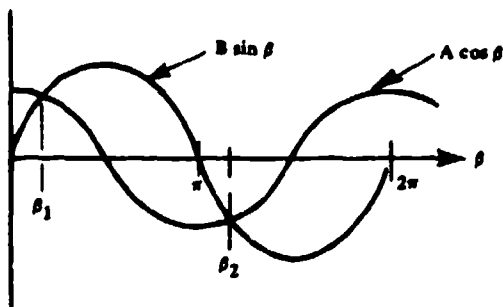Consider a sketch of A cos β and B sin β as shown in Figure C-1.

Figure C-1.

As can be seen, β_1 is an acute angle, with tangent defined by Equation 15a, and using principal angle notation

$$\beta_1 = \text{Tan}^{-1} (A/B) \tag{C-1}$$

Recall that $(\omega_d t)$ must be greater than zero; thus, from Equation 14, the following check is made

$$\beta_1 - \delta \geq 0 \tag{C-2}$$

If Condition C-2 is not fulfilled, the value of β_1 must be increased by π. The value of β_2 is given by Equation 16 in all cases.

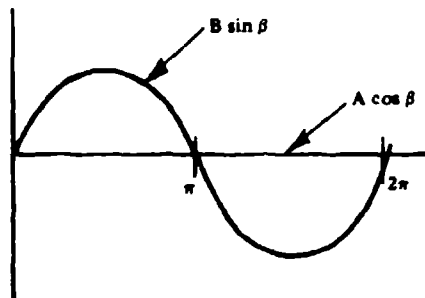## Case II, A = 0, B > 0

Consider Figure C-2.

Figure C-2.

In every case

$$\beta_1 \approx 0 \tag{C-3}$$

However, in every case with damping, this value of β_1 will not satisfy Condition C-2; thus,

$$\beta_1 = \pi, \quad \text{if } \zeta = 0 \tag{C-4}$$

Again β_2 will be given by Equation 16.

50

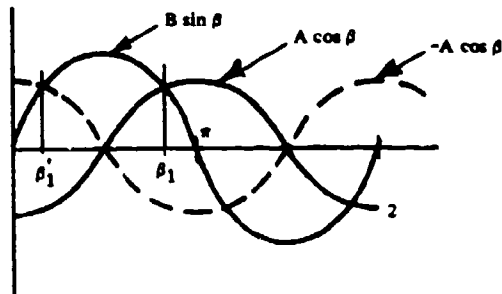## Case III, A < 0, B > 0

Consider the sketch of Figure C-3.

Figure C-3.

Clearly β_1 is between $\pi/2$ and π, but note that if the negative of A cos β is drawn dotted, an angle β_1' can be defined as

$$\beta_1' = \text{Tan}^{-1} (-A/B) \tag{C-5}$$

The angle β_1 is then given by $(\pi - \beta_1')$ or

$$\beta_1 = \pi - \text{Tan}^{-1} (-A/B) \tag{C-6}$$

Condition C-2 would again be used to see if β_1 should be increased by π, and Equation 16 would give β_2.

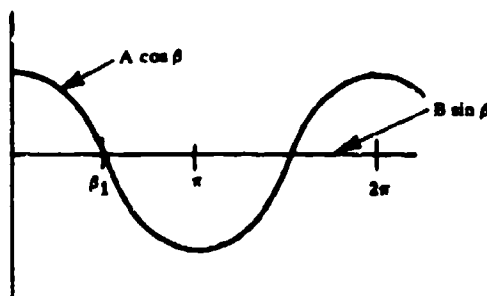## Case IV, A > 0, B = 0

Consider the sketch of Figure C-4.

Figure C-4.

51

Clearly $\beta_1 = \pi/2$. Condition C-2 would be checked, and Equation 16 used for β_2.

Case V, A = 0, B = 0

Going back to the definition of A and B in Equations 11a and 11b, one sees that both z_o and $\dot{z}_o$ are zero. Hence, no residual response results, and both z_{min} and z_{max} are zero.

Case VI, A < 0, B = 0

Consider the sketch of Figure C-5.

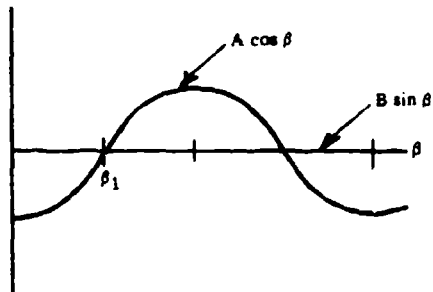

Figure C-5.

Clearly $\beta_1 = \pi/2$. Condition C-2 would be checked, and Equation 16 used for β_2.

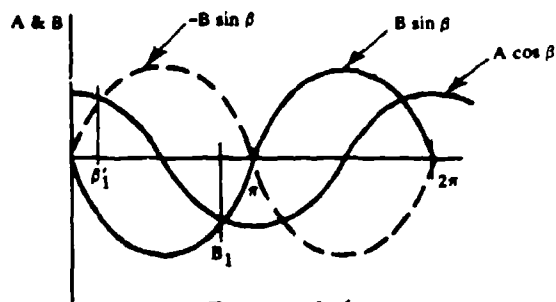## Case VII, B < 0, A > 0

Consider the sketch of Figure C-6.

Figure C-6.

In this case β_1 lies between $\pi/2$ and π. If the negative of B is used, the acute angle β' can be obtained and subtracted from π as indicated below.

$$\beta' = \operatorname{Tau}^{-1} (A/-B) \qquad \text{(C-7a)}$$

$$\beta_1 = \pi - \beta' \qquad \text{(C-7b)}$$

Case VIII, $A = 0$, $B < 0$
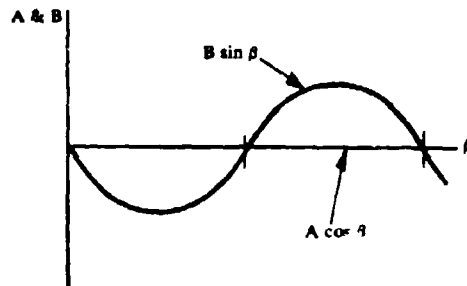
Consider this situation drawn in Figure C-7.

Figure C-7.

As in Case II,

$$\beta_1 = 0 \qquad \text{(C-8a)}$$

but again in every case with damping, this value of β_1 will not satisfy Condition C-2; thus,

$$\beta_1 = \pi, \qquad \text{if } \zeta = 0 \qquad \text{(C-8b)}$$

As in all, cases β_2 is given by Equation 16.

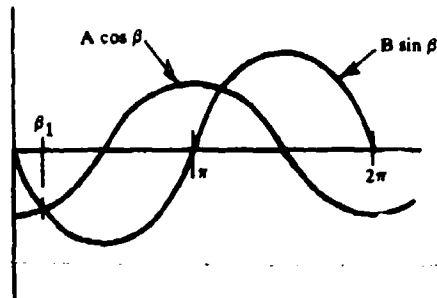Case IX, A < 0, B < 0

Consider the sketch drawn in Figure C-8.

Figure C-8.

This use is the inverse of Case I, and β_1 will always be acute and given simply by

$$\beta_1 = \text{Tan}^{-1} (A/B) \qquad\qquad (C-9)$$

If Condition C-2 is not fulfilled, the value of β_1 from Equation C-9 must be increased by π; the value of β_2 is obtained from Equation 16.

That completes the examination of the nine possible cases identified in Table 1. By going through them, one can organize the results as follows.

Case V: A = B = 0,

   no residual response occurs

Cases I, IX: A > 0, B > 0; or A < 0, B < 0,

$$\beta_1 = \text{Tan}^{-1} (A/B)$$

Cases III, VII: A < 0, B > 0; or B < 0, A > 0,

$$\beta_1 = \pi - \text{Tan}^{-1} (-A/B)$$

Cases II, VIII: A = 0, B > 0; or A = 0, B < 0,

$$\beta_1 = 0$$

54

Cases IV, VI: $A > 0$, $B = 0$; or $A < 0$, $B = 0$,

$$\beta_1 = \frac{\pi}{2}$$

Now this summary can be organized into a more concise set of rules for β_1 determination as follows:

1. If both A and B are zero, no residual response results.

2. If not, and $A = 0$, $\beta_1 = 0$.

3. If not, and $B = 0$, $\beta_1 = \pi/2$.

4. If not, and A and B have the same sign,

    $$\beta_1 = \text{Tan}^{-1} (A/B)$$

5. If not,

    $$\beta_1 = \pi - \text{Tan}^{-1} (-A/B)$$

Now given a value of β_1, one applies Condition C-2, or if

$$\beta_1 - \delta < 0$$

$$\beta_1 = (\beta_1)_{old} + \pi$$

which yields acceptable values of β_1 from which $(w_d t)_1$ can be obtained with Equation 14 as follows

$$(w_d t)_1 = \beta_1 - \delta \qquad\qquad (C-10)$$

The second value of $(w_d t)$ is obtained from Equation 16 and is

$$(w_d t)_2 = (w_d t)_1 + \pi$$

Appendix D

FREQUENCY GENERATION FOR EQUALLY SPACED VALUES
WHEN PLOTTED ON LOGARITHMIC PAPER

For a number of reasons it is advantageous to plot shock spectra on four-coordinate paper, which is a logarithmic paper, as discussed in Reference 2. To do this requires the frequencies to be selected in the following manner. For equally spaced values of frequency on logarithmic paper one wants the logarithm of the frequency to have equally spaced values. One can observe that, if J is the computer "do-loop" index, that increments by unity for each step, the logs will be equally spaced if

$$\log_{10}F = C_1 J + C_2 \tag{D-1}$$

Corresponding to this, the frequency will be given by

$$F = 10.^{(C_1 J + C_2)} \tag{D-2}$$

The user will select a lowest frequency of interest, F_{low}, a highest frequency of interest, F_{high}, and the number of frequencies per decade at which he wants shock spectrum values calculated, F_{pd}. The remainder of this argument is more simply written in FORTRAN because integer logic is necessary. Thus, Equations D-1 and D-2 become

$$ALOG10(F) = C1*J + C2 \tag{D-3}$$

$$F = 10.**(C1*J + C2) \tag{D-4}$$

Each time the log of the frequency increases by unity, the frequency
will have increased by a factor of 10, or gone through a decade; thus,
C1 is the reciprocal of F_{pd}, and Equations D-3 and D-4 may be written

$$ALOG10(F) = J/FPD + C2 \qquad (D-5)$$

$$F = 10.**(J/FPD + C2) \qquad (D-6)$$

Now, rather than start and stop at the precise low and high frequencies
selected by the user, one will start at FSTART and stop at FSTOP, defined
to be the frequencies computed by Equation D-6 from the values of J
given as JSTART and JSTOP. Using functional notation this can be indi-
cated by saying

$$FSTOP = F(JSTOP) \qquad (D-7)$$

$$FSTART = F(JSTART) \qquad (D-8)$$

The variables JSTOP and JSTART are defined such that

$$F(JSTART - 1) .LT. FLOW .LE. F(JSTART) \qquad (D-9)$$

and

$$F(JSTOP) .LE. FHIGH .LT. F(JSTOP + 1) \qquad (D-10)$$

This is convenient because the user will in general select an FPD, an
FLOW, and an FHIGH that are not mutually consistent. In this way, if
the user selects integer values of FPD, it is convenient to arrange C2
so that frequency values divisible by 10 are always included in the list
selected.

In Equation D-5, C2 is an integer used so that J does not have to
assume zero or negative values. Whenever the starting frequency is less
than unity, the log of F will be negative. If C2 is one less than the
characteristic of the log of FSTART, J will always have to be greater
than zero. Thus, define:

$$C2 = INT(ALOG10(FLOW)) - 1 \qquad \text{(D-11)}$$

To formulate a procedure to find JSTART, proceed as follows. Substitute Equation D-6 into Condition D-9; take the log of all three terms, which yields

$$(JSTART - 1)/FPD + C2 \text{ .LT. } JLOW/FPD + C2 \text{ .LE.}$$

$$(JSTART/FPD + C2) \qquad \text{(D-12a)}$$

Subtract C2 from each group, and then multiply through by the positive number FPD which will yield

$$JSTART - 1 \text{ .LT. } JLOW \text{ .LE. } JSTART \qquad \text{(D-12b)}$$

where JSTART is an integer and JLOW is floating point and formed from Equation D-5 with F equal to FLOW or

$$JLOW = FPD*(ALOG10(FLOW) - C2) \qquad \text{(D-13)}$$

In general, JLOW will not have integral value; for the case where JLOW is not integral

$$JSTART = INT(JLOW) + 1 \qquad \text{(D-14)}$$

but when JLOW does have integral value

$$JSTART = INT(JLOW) \qquad \text{(D-15)}$$

This can be programed as follows

$$FLOWLOG = ALOG10(FLOW)$$

$$C2 = FLOAT(INT(FLOWLOG) - 1)$$

$$AJLOW = FPD*(FLOWLOG - C2)$$

```
JSTART  =  INT(AJLOW)

IF(AJLOW .NE. FLOAT(JSTART)) JSTART  =  JSTART + 1

FSTART  =  10.**(FLOAT(JSTART)/FPD + C2)
```

A few values are hand-computed to document that this does deliver
Condition D-9. The results are shown in Table D-1.

To derive formulas for JSTOP and FSTOP, substitute Equation D-6 in
Condition D-10 and take the log of all three terms, which yields

$$JSTOP/FPD + C2 .LE. JHIGH/FPD + C2 .LT. (JSTOP + 1)/FPD + C2$$

Subtracting C2 from each term, and then multiplying each by the positive
number FPD, yields

$$JSTOP .LE. JHIGH .LT. JSTOP + 1 \qquad\qquad (D-16)$$

where JSTOP is an integer, and JHIGH is found from Equation D-5 with F
equal to FHIGH, or

$$JHIGH = FPD*(ALOG10(FHIGH) - C2) \qquad\qquad (D-17)$$

In general, JHIGH will not be integral, but even when it is, Condition
D-16 will be satisfied if we take

$$JSTOP = INT(JHIGH) \qquad\qquad (D-18)$$

Therefore, to get JSTOP and FSTOP, one adds the following steps to the
program given after Equation D-15

```
JSTOP  =  INT(FPD*(ALOG10(FHIGH) - C2))

FSTOP  =  10.**(FLOAT(JSTOP)/FPD + C2)          (D-19)
```

Table D-1

FPD	FLOW	FLOWLOG	C2	AFLOW	JSTART	FSTART	F(JSTART - 1)
20	0.001	-3.	-4.	20.	20	0.001	0.000891
20	0.0011	-2.9586	-3.	0.8728	1	0.001122	0.001
20	0.0099	-2.0043	-3.	19.913	20	0.01	0.00891
20	1.	0.	-1.	20	20	1.	0.89125
20	12.	1.07918	0.	21.583	22	12.589	11.72
20	57.	1.756	0.	35.1174	36	63.095	56.234
20	101.	2.00432	1.	20.08642	21	112.202	100.

DISTRIBUTION LIST

AF HQ PREES Washington DC (R P Reid)

AFB AF Tech Office (Mgt & Ops), Tyndall, FL; AFCEC/XR,Tyndall FL; CESCH, Wright-Patterson; HQ
Tactical Air Cmd (R. E. Fisher), Langley AFB VA; HQAFESC/DEMM, Tyndall AFB, FL; MAC/DET
(Col. P. Thompson) Scott, IL; SAMSO/MNND, Norton AFB CA; Stinfo Library, Offutt NE; WPNS Safety
Div, Norton, CA

AMMO HAZARDS REVIEW BD Chairman, NAS, Norfolk, VA

AMTRAK President, Wash DC

ARMY ARRADCOM, Dover, NJ; ARRCOM, Rock Island, IL; BMDSC-RE (H. McClellan) Huntsville AL;
DAEN-CWE-M (LT C D Binning), Washington DC; DAEN-FEU-E (J. Ronan), Washington DC;
DAEN-MCE-D (R L Wight) Washington DC; DAEN-MPE-D Washington DC; DAEN-MPU, Washington
DC; DARCOM; ERADCOM Tech Supp Dir. (DELSD-L) Ft. Monmouth, NJ; Savanna Army Depot
(Bynd), IL; Savanna Army Depot (Ernst), IL; Tech. Ref. Div., Fort Huachuca, AZ

ARMY - CERL Library, Champaign IL

ARMY AMMUNITION PLANT SARHW-FEM Hawthorne, NV

ARMY COE Philadelphia Dist. (LIBRARY) Philadelphia, PA

ARMY CORPS OF ENGINEERS MRD-Eng. Div., Omaha NE; Seattle Dist. Library, Seattle WA

ARMY ENG DIV ED-CS (S.Bolin) Huntsville, AL; HNDED-CS, Huntsville AL; HNDED-SR, Huntsville, AL;
Huntsville, AL

ARMY ENG WATERWAYS EXP STA (S Kiger) Vicksburg, MS; Library, Vicksburg MS

ARMY ENVIRON. HYGIENE AGCY B620, Edgewood Arsenal MD; Water Qual Div (Doner), Aberdeen
Prov Ground, MD

ARMY MATERIALS & MECHANICS RESEARCH CENTER Dr. Lenoe, Watertown MA

ARMY MISSILE R&D CMD Redstone Arsenal AL Sci. Info. Cen (Documents)

ARMY MTMC Trans Engr Agency MTT-CE, Newport News, VA

ASST SECRETARY OF THE NAVY Spec. Assist Energy (Leonard), Washington, DC

BUMED Field Med Svc School, Camp Pendleton, CA

BUREAU OF RECLAMATION Code 1512 (C. Selander) Denver CO

CINCLANT Civil Engr. Supp. Plans. Ofr Norfolk, VA

CINCPAC Fac Engrng Div (J44) Makalapa, HI

CNO Code NOP-964, Washington DC; OP-411F, Wash DC; OP987J (J. Boosman), Pentagon

COMFLEACT, OKINAWA PWO, Kadena, Okinawa

COMNAVMARIANAS Code N4, Guam

COMOCEANSYSPAC SCE, Pearl Harbor HI

DEFENSE CIVIL PREPAREDNESS AGENCY J.O. Buchanan, Washington DC

DEFENSE INTELLIGENCE AGENCY Dir., Washington DC

DNA STTL, Washington DC

DOD Explos Safety Board, Wash DC; Explosives Safety Board (Library), Washington DC

DOE Dr. Cohen

DOT Office of Hazardous Matl Ops, Wash DC

DTIC Defense Technical Info Ctr/Alexandria, VA

DTNSRDC Code 1706, Bethesda MD; Code 4111 (R. Gierich), Bethesda MD

DTNSRDC Code 4121 (R. Rivers), Annapolis, MD

DTNSRDC Code 42, Bethesda MD

FED RAILROAD AMIN Administrator, Wash DC

FLTCOMBATTRACENLANT PWO, Virginia Bch VA

FMFLANT CEC Offr, Norfolk VA

KWAJALEIN MISRAN BMDSC-RKL-C

LIBRARY OF CONGRESS Washington, DC (Sciences & Tech Div)

MARINE CORPS BASE Camp Pendleton CA 92055; Code 43-260, Camp Lejeune NC; M & R Division, Camp
Lejeune NC; PWO Camp Lejeune NC; PWO, Camp S. D. Butler, Kawasaki Japan

MARINE CORPS HQS Code LFF-2, Washington DC

MCAS Facil. Engr. Div. Cherry Point NC; CO, Kaneohe Bay HI; Code PWE, Kaneohe Bay HI; Code S4,
Quantico VA; PWD, Dir. Maint. Control Div., Iwakuni Japan; PWO Kaneohe Bay HI; PWO, Yuma AZ

MCRD PWO, San Diego Ca

NAD Engr. Dir. Hawthorne, NV

NAF PWD - Engr Div, Atsugi, Japan; PWO Sigonella Sicily; PWO, Atsugi Japan

NALF OINC, San Diego, CA

NARF Code 100, Cherry Point, NC; Code 640, Pensacola FL

NAS CO, Guantanamo Bay Cuba; Code 114, Alameda CA; Code 183 (Fac. Plan BR MGR); Code 187, Jacksonville FL; Code 18700, Brunswick ME; Code 18U (ENS P.J. Hickey), Corpus Christi TX; Code 6234 (G. Trask), Point Mugu CA; Code 70, Atlanta, Marietta GA; Dir. Util. Div., Bermuda; ENS Buchholz, Pensacola, FL; Lakehurst, NJ; PW (J. Maguire), Corpus Christi TX; PWD Maint. Div., New Orleans, Belle Chasse LA; PWD, Maintenance Control Dir., Bermuda; PWD, Willow Grove PA; PWO Belle Chasse, LA; PWO Chase Field Beeville, TX; PWO Key West FL; PWO Whiting Fld, Milton FL; PWO, Dallas TX; PWO, Glenview IL; PWO, Kingsville TX; PWO, Millington TN; PWO, Miramar, San Diego CA; PWO,, Moffett Field CA; SCE Lant Fleet Norfolk, VA; SCE Norfolk, VA; Security Offr, Alameda CA

NATL RESEARCH COUNCIL Naval Studies Board, Washington DC

NATNAVMEDCEN PWO Bethesda, MD

NAVACT PWO, London UK

NAVAEROSPREGMEDCEN SCE, Pensacola FL

NAVAIRSTA Code 500, San Diego, CA

NAVAIRSYSCOM PMA 258 Washington, DC

NAVAVIONICFAC PWD Deputy Dir. D/701, Indianapolis, IN

NAVCOASTSYSTCTR Code 423 (D. Good), Panama City FL; Code 713 (J. Quirk) Panama City, FL; Library Panama City, FL

NAVCOMMAREAMSTRSTA Code W-602, Honolulu, Wahiawa HI; PWO, Norfolk VA; PWO, Wahiawa HI; SCE Unit 1 Naples Italy

NAVCOMMSTA CO (61E) Puerto Rico; CO, San Miguel, R.P.; Code 401 Nea Makri, Greece; PWO, Exmouth, Australia; PWO, Fort Amador Canal Zone

NAVEDTRAPRODEVCEN Tech. Library

NAVEDUTRACEN Engr Dept (Code 42) Newport, RI

NAVEODFAC Code 605, Indian Head MD

NAVFAC PWO, Brawdy Wales UK; PWO, Cape Hatteras, Buxton NC; PWO, Centerville Bch. Ferndale CA; PWO, Guam

NAVFAC PWO, Lewes DE

NAVFACENGCOM Code 043 Alexandria, VA; Code 044 Alexandria, VA; Code 0451 Alexandria, VA; Code 0454B Alexandria, Va; Code 04B3 Alexandria, VA; Code 04B5 Alexandria, VA; Code 100 Alexandria, VA; Code 1113 (M. Carr) Alexandria, VA; Code 1113 (T. Stevens) Alexandria, VA; Code 1113 Alexandria, VA; Morrison Yap, Caroline Is.; P W Brewer Alexandria, VA

NAVFACENGCOM - CHES DIV. Code 101 Wash. DC; Code 403 (H. DeVoe) Wash, DC; Code 405 Wash, DC; Code FPO-1 Wash, DC

NAVFACENGCOM - LANT DIV. CDR E. Peltier; Code 111, Norfolk, VA; Eur. BR Deputy Dir, Naples Italy; European Branch, New York; RDT&ELO 102, Norfolk VA

NAVFACENGCOM - NORTH DIV. CO; Code 1028, RDT&ELO, Philadelphia PA; Code 111 (Castranovo) Philadelphia, PA; Code 114 (A. Rhoads); Design Div. (R. Masino), Philadelphia PA; ROICC, Contracts, Crane IN

NAVFACENGCOM - PAC DIV. (Kyi) Code 101, Pearl Harbor, HI; Code 2011 Pearl Harbor, HI; Code 402, RDT&E, Pearl Harbor HI; Commander, Pearl Harbor, HI

NAVFACENGCOM - SOUTH DIV. Code 09 (Watts) Charleston, SC; Code 90, RDT&ELO, Charleston SC

NAVFACENGCOM - WEST DIV. 102; 112; AROICC, Contracts, Twentynine Palms CA; Code 04B San Bruno, CA; Code 402 San Bruno, CA; O9P/20 San Bruno, CA; RDT&ELO Code 2011 San Bruno, CA

NAVFACENGCOM CONTRACT AROICC, Point Mugu CA; AROICC, Quantico, VA; Code 05, TRIDENT, Bremerton WA; Dir. Eng. Div., Exmouth, Australia; Eng Div dir, Southwest Pac, Manila, PI; OICC, Southwest Pac, Manila, PI; OICC/ROICC, Balboa Canal Zone; ROICC AF Guam; ROICC LANT DIV., Norfolk VA; ROICC, Keflavik, Iceland; ROICC, Pacific, San Bruno CA

NAVMAG SCE, Guam

NAVNUPWRU MUSE DET Code NPU-30 Port Hueneme, CA

NAVOCEANSYSCEN Code 41, San Diego, CA; Code 6700, San Diego, CA; Research Lib., San Diego CA; Tech. Library, Code 447

NAVORDSTA PWO, Louisville KY

NAVPETOFF Code 30, Alexandria VA

NAVPGSCOL Code 61WL (O. Wilson) Monterey CA

NAVPHIBASE CO, ACB 2 Norfolk, VA; Code S3T, Norfolk VA

NAVRADRECFAC PWO, Kami Seya Japan

NAVREGMEDCEN Chief of Police, Camp Pendleton CA; Code 29, Env. Health Serv, (Al Bryson) San Diego, CA; PWO Newport RI; PWO Portsmouth, VA; SCE (D. Kaye); SCE San Diego, CA; SCE, Camp Pendleton CA; SCE, Guam; SCE, Oakland CA

NAVSCOLCECOFF C35 Port Hueneme, CA; CO, Code C44A Port Hueneme, CA

NAVSEASYSCOM Code 05M13 (Newhouse) Wash DC; Code 62T2, Wash DC; SEA-04H, Wash DC

NAVSEC Code 6034 (Library), Washington DC

NAVSECGRUACT PWO, Adak AK; PWO, Edzell Scotland; PWO, Puerto Rico; PWO, Torri Sta, Okinawa

NAVSHIPREPFAC SCE Subic Bay

NAVSHIPYD; Code 202.4, Long Beach CA; Code 202.5 (Library) Puget Sound, Bremerton WA; Code 380, (Woodroff) Norfolk, Portsmouth, VA; Code 400, Puget Sound; Code 400.03 Long Beach, CA; Code 404 (LT J. Riccio), Norfolk, Portsmouth VA; Code 410, Mare Is., Vailejo CA; Code 440 Portsmouth NH; Code 440, Norfolk; Code 440, Puget Sound, Bremerton WA; Code 450, Charleston SC; Code 453 (Util. Supr), Vallejo CA; L.D. Vivian; Library, Portsmouth NH; PWD (Code 400), Philadelphia PA; PWO, Mare Is.; PWO, Puget Sound; SCE, Pearl Harbor HI; Tech Library, Vallejo, CA

NAVSTA CO Naval Station, Mayport FL; CO Roosevelt Roads P.R. Puerto Rico; Dir Mech Engr, Gtmo; Engr. Dir., Rota Spain; Long Beach, CA; Maint. Cont. Div., Guantanamo Bay Cuba; Maint. Div. Dir/Code 531, Rodman Canal Zone; PWD (LTJG.P.M. Motolenich), Puerto Rico; PWO Midway Island; PWO, Keflavik Iceland; PWO, Mayport FL; ROICC, Rota Spain; SCE, Guam; SCE, San Diego CA; SCE, Subic Bay, R.P.; Utilities Engr Off. (A.S. Ritchie), Rota Spain

NAVSUBASE ENS S. Dove, Groton, CT; SCE, Pearl Harbor HI

NAVSUPPACT CO, Seattle WA; Code 4, 12 Marine Corps Dist. Treasure Is., San Francisco CA; Code 413, Seattle WA; LTJG McGarrah, SEC, Vallejo, CA; Plan/Engr Div., Naples Italy

NAVSURFWPNCEN PWO, White Oak, Silver Spring, MD

NAVTECHTRACEN SCE, Pensacola FL

NAVWPNCEN Code 2636 (W. Bonner), China Lake CA; Code 3276 China Lake, CA; PWO (Code 26), China Lake CA; ROICC (Code 702), China Lake CA

NAVWPNSTA (Clebak) Colts Neck, NJ; Code 09, Concord, CA; Code 092, Colts Neck NJ; Code 092A (C. Fredericks) Seal Beach CA; Maint. Control Dir., Yorktown VA; Naval Weapons Handling Center, Earle, NJ

NAVWPNSTA PW Office (Code 09C1) Yorktown, VA

NAVWPNSTA Safety Br, Concord, CA; Security Offr, Colts Neck NJ

NAVWPNSUPPCEN Code 09 Crane IN

NCBU 405 OIC, San Diego, CA

NCBC Code 10 Davisville, RI; Code 155, Port Hueneme CA; Code 156, Port Hueneme, CA; Code 400, Gulfport MS; PW Engrg, Gulfport MS; PWO (Code 80) Port Hueneme, CA; PWO, Davisville RI

NCBU 411 OIC, Norfolk VA

NCR 20, Commander

NCSO BAHRAIN Security Offr, Bahrain

NMCB 5, Operations Dept.; 74, CO; Forty, CO; THREE, Operations Off.

NORDA Code 440 (Ocean Rsch Off) Bay St Louis MS

NRL Code 8400 Washington, DC; Code 8441 (R.A. Skop), Washington DC

NSC Code 54.1 (Wynne), Norfolk VA

NSD SCE, Subic Bay, R.P.

NTC Commander Orlando, FL

NTSB Chairman, Wash DC

NUSC Code 131 New London, CT; Code EA123 (R.S. Munn), New London CT

OCEANSYSLANT LT A.R. Giancola, Norfolk VA

OFFICE SECRETARY OF DEFENSE OASD (MRA&L) Pentagon (T. Casberg), Washington, DC

ONR Code 700F Arlington VA

PHIBCB 1 P&E, Coronado, CA

PMTC Pat. Counsel, Point Mugu CA

PWC (Lt E.S. Agonoy) Pensacola, FL; ACE Office (LTJG St. Germain) Norfolk VA; CO Norfolk, VA; CO, (Code 10), Oakland, CA; CO, Great Lakes IL; Code 10, Great Lakes, IL; Code 120, Oakland CA; Code 120C, (Library) San Diego, CA; Code 128, Guam; Code 154, Great Lakes, IL; Code 200, Great Lakes IL; Code 220 Oakland, CA; Code 220.1, Norfolk VA; Code 30C, San Diego, CA; Code 400, Great Lakes, IL; Code 400, Oakland, CA; Code 400, Pearl Harbor, HI; Code 400, San Diego, CA; Code 420, Great Lakes, IL; Code 420, Oakland, CA; Code 42B (R. Pascua), Pearl Harbor HI; Code 505A (H. Wheeler); Code 600,

Great Lakes, IL; Code 601, Oakland, CA; Code 610, San Diego Ca; Code 700, Great Lakes, IL; Code 700, San Diego, CA; LTJG J.L. McClaine, Yokosuka, Japan; Utilities Officer, Guam; XO (Code 20) Oakland, CA

SPCC PWO (Code 120) Mechanicsburg PA

NAF PWO (Code 30) El Centro, CA

U.S. MERCHANT MARINE ACADEMY Kings Point, NY (Reprint Custodian)

USCG (Smith), Washington, DC; G-EOE-4/61 (T. Dowd), Washington DC

USEUCOM (ECJ4/L-LO), Wright, Stuttgart, GE

USNA Ch. Mech. Engr. Dept Annapolis MD; PWD Engr. Div. (C. Bradford) Annapolis MD

CALIFORNIA STATE UNIVERSITY LONG BEACH, CA (CHELAPATI)

CORNELL UNIVERSITY Ithaca NY (Serials Dept, Engr Lib.)

DAMES & MOORE LIBRARY LOS ANGELES, CA

ILLINOIS STATE GEO. SURVEY Urbana IL

LEHIGH UNIVERSITY Bethlehem PA (Fritz Engr. Lab No. 13, Beedle); Bethlehem PA (Linderman Lib. No.30, Flecksteiner)

MICHIGAN TECHNOLOGICAL UNIVERSITY Houghton, MI (Haas)

MIT Cambridge MA; Cambridge MA (Rm 10-500, Tech. Reports, Engr. Lib.); Cambridge MA (Whitman)

NEW MEXICO SOLAR ENERGY INST. Dr. Zwibel Las Cruces NM

NY CITY COMMUNITY COLLEGE BROOKLYN, NY (LIBRARY)

PURDUE UNIVERSITY Lafayette, IN (CE Engr. Lib)

CONNECTICUT Hartford CT (Dept of Plan. & Energy Policy)

SEATTLE U Prof Schwaegler Seattle WA

SOUTHWEST RSCH INST R. DeHart, San Antonio TX

STANFORD UNIVERSITY Engr Lib, Stanford CA

STATE UNIV. OF NEW YORK Buffalo, NY

TEXAS A&M UNIVERSITY W.B. Ledbetter College Station, TX

UNIVERSITY OF CALIFORNIA BERKELEY, CA (CE DEPT, GERWICK); Berkeley CA (E. Pearson); DAVIS, CA (CE DEPT, TAYLOR); LIVERMORE, CA (LAWRENCE LIVERMORE LAB, TOKARZ)

UNIVERSITY OF DELAWARE Newark, DE (Dept of Civil Engineering, Chesson)

UNIVERSITY OF HAWAII Honolulu HI (Dr. Szilard)

UNIVERSITY OF ILLINOIS Metz Ref Rm, Urbana IL; URBANA, IL (LIBRARY); URBANA, IL (NEWMARK); Urbana IL (CE Dept, W. Gamble)

UNIVERSITY OF MASSACHUSETTS (Heronemus), Amherst MA CE Dept

UNIVERSITY OF MICHIGAN Ann Arbor MI (Richart)

UNIVERSITY OF NEBRASKA-LINCOLN Lincoln, NE (Ross Ice Shelf Proj.)

UNIVERSITY OF NOTRE DAME Katona, Notre Dame, IN

UNIVERSITY OF TEXAS Inst. Marine Sci (Library), Port Arkansas TX

UNIVERSITY OF TEXAS AT AUSTIN AUSTIN, TX (THOMPSON); Austin, TX (Breen)

UNIVERSITY OF WASHINGTON Dept of Civil Engr (Dr. Mattock), Seattle WA; SEATTLE, WA (MERCHANT)

UNIVERSITY OF WISCONSIN Milwaukee WI (Ctr of Great Lakes Studies)

AMMAN & WHITNEY CONSULT ENGRS N Dobbs New York, NY

ARVID GRANT OLYMPIA, WA

ASSOC AMER RR Bureau of Explosive, Chicago, IL

ATLANTIC RICHFIELD CO. DALLAS, TX (SMITH)

AUSTRALIA Dept of Const, Canberra; Dept of Hsng & Const, Sydney

BECHTEL CORP. SAN FRANCISCO, CA (PHELPS)

BROWN & CALDWELL E M Saunders Walnut Creek, CA

CANADA Mem Univ Newfoundland (Chari), St Johns; Trans-Mnt Oil Pipe Lone Corp, Vancouver, BC Canada

COLUMBIA GULF TRANSMISSION CO. HOUSTON, TX (ENG. LIB.)

HERCULES INC. D Richardson, Magna, UT

DURLACH, O'NEAL, JENKINS & ASSOC. Columbia SC

FORD, BACON & DAVIS, INC. New York (Library)

FRANCE Dr. Dutertre, Boulogne

GLIDDEN CO. STRONGSVILLE, OH (RSCH LIB)

HUGHES AIRCRAFT Culver City CA (Tech. Doc. Ctr)

ITALY M. Caironi, Milan

KAMAN SCIENCES CORP T Cook Colorado Springs, CO

LOCKHEED MISSILES & SPACE CO. INC. Sunnyvale, CA (K.L. Krug)

MCDONNEL AIRCRAFT CO. Dept 501 (R.H. Fayman), St Louis MO; Navy Tech Rep St Louis, MO; R Carson, St. Louis, MO

NEWPORT NEWS SHIPBLDG & DRYDOCK CO. Newport News VA (Tech. Lib.)

NORWAY DET NORSKE VERITAS (Library), Oslo; DET NORSKE VERITAS (Roren) Oslo; I. Foss, Oslo; J. Creed, Ski; Norwegian Tech Univ (Brandtzaeg), Trondheim

PORTLAND CEMENT ASSOC. Skokie IL (Rsch & Dev Lab, Lib.)

RAND CORP. Santa Monica CA (A. Laupa)

RAYMOND INTERNATIONAL INC. E Colle Soil Tech Dept, Pennsauken, NJ

SANDIA LABORATORIES Library Div., Livermore CA; Seabed Progress Div 4536 (D. Talbert) Albuquerque NM

SCHUPACK ASSOC SO. NORWALK, CT (SCHUPACK)

SHELL OIL CO. HOUSTON, TX (MARSHALL)

SWEDEN Cement & Concrete Research Inst., Stockholm; GeoTech Inst

SWITZERLAND T Schneider, Zurich

THE NETHERLANDS Director Tech Rsch Prins Maurits Lab, Rijswijk

TRW SYSTEMS REDONDO BEACH, CA (DAI)

UNITED KINGDOM Atomic Energy Auth (F R Farmer) London; Cement & Concrete Assoc Wexham Springs, Slough Bucks; D. Lee, London; D. New, G. Maunsell & Partners, London; R. Browne, Southall, Middlesex; Taylor, Woodrow Constr (014P), Southall, Middlesex

WESTINGHOUSE ELECTRIC CORP. Annapolis MD (Oceanic Div Lib, Bryan); Library, Pittsburgh PA

WESTINTRUCORP Egerton, Oxnard, CA

WISS, JANNEY, ELSTNER, & ASSOC Northbrook, IL (D.W. Pfeifer)

WOODWARD-CLYDE CONSULTANTS (A. Harrigan) San Francisco

BRAHTZ La Jolla, CA

BROWN, ROBERT University, AL

BRYANT ROSE Johnson Div. UOP, Glendora CA

BULLOCK La Canada

F. HEUZE Alamo, CA

CAPT MURPHY Sunnyvale, CA

R.F. BESIER Old Saybrook CT

T.W. MERMEL Washington DC