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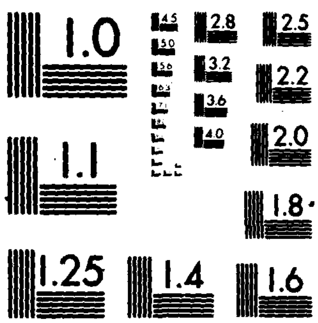

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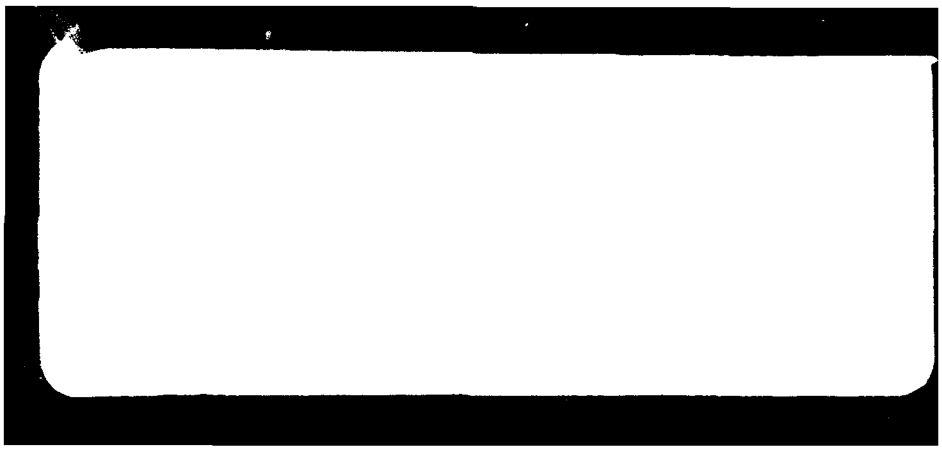

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LEVEL II

A nonlinear effect in Mode II crack problems

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Summary

A recent result of Stephenson [1] shows that, when finite deformations are taken into account, a crack under Mode II loading conditions in plane strain will open, at least near the crack-tip and at least for certain elastic materials. In this note, the matter is investigated further, and it is shown that, in general, nonlinear effects — even at small loads — lead either to crack-opening or apparent interpenetration of the crack-faces when the loading is of Mode II-type.

Introduction

When treated on the basis of the linearized theory of elasticity, the plane strain equilibrium problem of an infinite — homogeneous and isotropic — solid containing a crack of finite length and subject to Mode II shear loading at infinity leads to a solution in which the component of displacement normal to the crack vanishes at all points of each crack-face. Particles on opposing faces of the crack therefore undergo oppositely directed sliding displacements parallel to the crack itself. Thus, according to this description, the crack remains straight after deformation, and its faces —

[1] The results reported in this paper were obtained in the course of an investigation supported in part by Contract N00014-75-C-0196 with the Office of Naval Research.

although traction free — remain contiguous.

Stephenson [1] has recently studied the local structure near a crack-tip of solutions of plane strain problems in finite elastostatics in enough generality to encompass both Mode I and Mode II loadings at infinity. He treats a rather broad class of homogeneous, isotropic, incompressible materials. For some of the materials considered, he finds that, under Mode II conditions, the nonlinear theory predicts that the crack will open, at least near the tips. For the remaining materials in his class, Stephenson's analysis — which is essentially local — leaves undecided the question of whether the crack opens in the Mode II case.

Stephenson's result suggests that, in the finite elasticity problem, there may be a nonvanishing normal component of displacement along the crack-faces which is of second — or higher — order in the loading parameter and therefore undectectable by linearized theory. Should such a displacement occur in the solution of the nonlinear Mode II problem, then interpenetration of the crack-faces — as an alternative to crack-opening — might be predicted. If this is the case, the Mode II problem has no physically acceptable solution as posed.

In the present note, we consider the Mode II "small-scale nonlinear crack problem" for a fully general homogeneous, isotropic compressible elastic material within the framework of finite elastostatics in order to study further the question of crack-opening at small loads. In this small-scale problem, one replaces the finite crack by a semi-infinite one, and one seeks a solution of the field equations of finite elasticity which — far from the crack-tip — coincides asymptotically with the field predicted by linearized theory near the crack-tip in the original Mode II problem.

We find that, for certain materials, the crack will indeed open at points "moderately near" the crack-tip, but for other materials, the small-scale problem results in an apparent interpenetration of the crack-faces, in violation of the required one-to-one character of the deformation. We give a restriction on the second-order elastic constants which delineates these two classes of materials. Special choices of the strain energy density characteristic of the material supply examples illustrating the various possibilities.

1. The Mode II crack problem

The cross-section of the undeformed solid containing a crack of length 2b, as well as the cartesian material coordinates, are shown in Fig.1. The Mode II problem which underlies our considerations consists in finding displacements u_α and nominal — or Piola — stresses[1] $\sigma_{\alpha\beta}$ (forces per unit undeformed area) which satisfy the field equations of finite elastostatic plane strain away from the crack-tips, leave the deformed faces of the crack free of traction, and satisfy the conditions

$$u_1 \sim kx_2, \quad u_2 \to 0 \quad \text{as} \quad x_1^2 + x_2^2 \to \infty \quad , \tag{1.1}$$

corresponding to shear of the Mode II type at infinity. Here $k > 0$ is the given amount of shear. A particle at the point (x_1, x_2) in the undeformed state is carried by the deformation to the point (y_1, y_2), where

$$y_\alpha = x_\alpha + u_\alpha(x_1, x_2) \quad . \tag{1.2}$$

It is required that the displacements furnish a one-to-one mapping

[1]All components of vectors and tensors are taken with respect to the x_1, x_2-frame. Greek subscripts have the range 1,2; repeated subscripts are summed, and a subscript preceded by a comma indicates partial differentiation with respect to the corresponding x-coordinate.

$(x_1, x_2) \leftrightarrow (y_1, y_2)$ between the exteriors of the undeformed and deformed cracks. Finally, the displacements must be bounded near the crack-tips.

The differential equations of equilibrium in terms of nominal stresses are

$$\sigma_{\alpha\beta,\beta} = 0 \quad . \tag{1.3}$$

The constitutive law which relates $\sigma_{\alpha\beta}$ to u_α is given by

$$\sigma_{\alpha\beta} = (2W_I + W_J)\delta_{\alpha\beta} + 2W_I u_{\alpha,\beta} + W_J \varepsilon_{\alpha\rho}\varepsilon_{\beta\gamma} u_{\rho,\gamma} \quad , \tag{1.4}$$

corresponding to a homogeneous, isotropic, compressible material with plane strain elastic potential $W(I,J)$ (strain energy per unit undeformed volume). In (1.4), $\delta_{\alpha\beta}$ is the Kronecker delta, $\varepsilon_{\alpha\beta}$ is the two-dimensional alternator $(\varepsilon_{11} = \varepsilon_{22} = 0,\ \varepsilon_{12} = -\varepsilon_{21} = 1)$, and the two deformation invariants I and J are given by

$$I = 2 + 2u_{\alpha,\alpha} + u_{\alpha,\beta} u_{\alpha,\beta} \quad , \tag{1.5}$$

$$J = 1 + u_{\alpha,\alpha} + \frac{1}{2}\varepsilon_{\alpha\lambda}\varepsilon_{\beta\gamma} u_{\alpha,\beta} u_{\lambda,\gamma} \quad ; \tag{1.6}$$

in particular, J is the Jacobian determinant of the deformation and must be positive. W_I and W_J are the derivatives of W.

The condition of vanishing <u>true</u> traction on the deformed faces of the crack can be shown to be equivalent to the vanishing of the nominal traction on the undeformed crack-faces.[1] The latter condition in turn is

$$\sigma_{12} = \sigma_{22} = 0 \quad \text{at} \quad x_2 = 0\pm, \quad -b < x_1 < b \quad . \tag{1.7}$$

[1] For a fuller discussion of this point, as well as of the field equations of plane finite elastostatics, see [2].

Once u_α and $\sigma_{\alpha\beta}$ have been found, the true (or Cauchy) stresses $\tau_{\alpha\beta}$ are determined from the relation

$$\tau_{\alpha\beta} = \frac{1}{J}\,\sigma_{\alpha\gamma}(\delta_{\gamma\beta} + u_{\beta,\gamma}) \quad . \tag{1.8}$$

From (1.4), (1.8), it follows that

$$\tau_{\alpha\beta} = (\frac{2}{J}W_I + W_J)\delta_{\alpha\beta} + \frac{2}{J}W_I(u_{\beta,\alpha} + u_{\alpha,\beta} + u_{\alpha,\gamma}u_{\beta,\gamma}) \quad . \tag{1.9}$$

To obtain the formulation of the Mode II problem according to linear theory,[1] one need only linearize (1.4)-(1.6) with respect to $u_{\alpha,\beta}$; the remaining equations — (1.1), (1.3), (1.7) — are retained without change. The relation (1.8) between true and nominal stresses reduces to $\tau_{\alpha\beta} \sim \sigma_{\alpha\beta}$ upon linearization. In carrying out the linearization process, one must make use of the fact that, in the undeformed state, one has, by (1.5), (1.6), $I = 2$, $J = 1$; moreover,

$$2W_I(2,1) + W_J(2,1) = 0 \tag{1.10}$$

if the undeformed state is to be unstressed. One further needs the relations which determine the shear modulus μ and Poisson's ratio ν of the linearized theory in terms of W:

$$\mu = 2W_I(2,1), \quad \frac{\mu}{1-2\nu} = (4W_{II} + 4W_{IJ} + W_{JJ})_{I=2,\ J=1} \quad . \tag{1.11}$$

The global solution of the linearized version of the Mode II problem can be found exactly by making use of the work of Inglis [3]. From the solution one can show (see [4]) that the displacements $\overset{\circ}{u}_\alpha$ furnished by the linear theory are given asymptotically near the right crack-tip by

[1] We do not spell out this formulation here.

$$\overset{\circ}{u}_\alpha \sim r^{1/2} U_\alpha(\theta) \quad \text{as} \quad r \to 0 \ , \tag{1.12}$$

where

$$U_1(\theta) = c\left[\left(\frac{9}{2} - 4\nu\right)\sin\frac{\theta}{2} + \frac{1}{2}\sin\frac{3\theta}{2}\right],\ U_2(\theta) = c\left[\left(-\frac{3}{2} + 4\nu\right)\cos\frac{\theta}{2} - \frac{1}{2}\cos\frac{3\theta}{2}\right] \ , \tag{1.13}$$

$$c = \frac{k}{2}\sqrt{\frac{b}{2}} \ , \tag{1.14}$$ [1]

and (r,θ) are polar coordinates at the crack-tip (Fig.1). According to the second of (1.13), $U_2(\pm\pi) = 0$, reflecting the fact that $u_2 = 0$ on the crack-faces in linear theory. We note that U_2 is an even function of θ.

2. The small-scale nonlinear crack problem

If the amount of shear k prescribed at infinity is small, one would expect that the field near a crack-tip — say the right one — could be determined on the basis of an asymptotic scheme in which the crack of finite length is replaced by a semi-infinite one, and the far field is required to match the elastostatic field near the crack-tip as predicted by the solution of the original problem according to linearized theory. One thus takes the view that, for small loads, there is an inner region in the immediate vicinity of each crack-tip in which nonlinear effects are dominant; surrounding this, there is an intermediate region in which the field is described by the near-tip approximation supplied by linearized theory. At points which lie exterior to the intermediate region (points in the outer region), the full solution of the linearized problem is required to describe the field.

In this small-scale nonlinear crack problem, we thus seek displacements

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[1] In linear theory, the given shear strain k is related to the applied shear stress σ by $\sigma = \mu k$; in addition, $c = (2\mu\sqrt{2\pi})^{-1}K_{II}$, where K_{II} is the Mode II stress intensity factor.

u_α and nominal stresses $\sigma_{\alpha\beta}$ which satisfy the field equations (1.3)-(1.6) away from the crack-tip, the free surface conditions

$$\sigma_{12} = \sigma_{22} = 0, \quad x_2 = 0\pm, \quad -\infty < x_1 < 0 \quad , \tag{2.1}$$ [1]

and the matching conditions (see (1.12))

$$u_\alpha \sim r^{1/2} U_\alpha(\theta) \quad \text{as} \quad r \to \infty \quad , \tag{2.2}$$

where the U_α's are given by (1.13). Again, the displacements are to be bounded near the crack-tip, and the mapping $(x_1, x_2) \leftrightarrow (y_1, y_2)$ is to be one-to-one between the exteriors of the crack before and after deformation.

In order to study the issue of crack-opening, it is fortunately not necessary to solve the small-scale problem globally. It turns out that determining the second terms in the large r expansions of the displacements u_α — the first terms are prescribed in (2.2) — furnishes useful information. By investigating the solution of the small-scale problem for large r, we are in effect studying the behavior of the solution of the original Mode II problem in the transition zone between the inner and intermediate regions. We shall speak of points which lie in this transition zone as moderately near the crack.

We write

$$u_\alpha \sim r^{1/2} U_\alpha(\theta) + V_\alpha(\theta), \quad r \to \infty \quad , \tag{2.3}$$ [2]

and we seek functions V_α such that (2.3) is asymptotically consistent as $r \to \infty$ with the field equations (1.3)-(1.6) and the boundary conditions

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[1] The origin has been moved to the right crack-tip.

[2] If, instead of (2.3), one sets $u_\alpha \sim r^{1/2} U_\alpha + r^{m_\alpha} V_\alpha$ (no sum), where nothing is assumed about the exponents m_α except that $m_\alpha < 1/2$, then one can show that $m_1 = m_2 = 0$, as is assumed in (2.3).

(2.1). By setting $\theta = \pi$ in (2.3) and making use of the fact that $U_2(\pi) = 0$ (see (1.13)), one finds that the curve representing the deformation image of the upper crack-face is asymptotically tangent as $r \to \infty$ to the horizontal line $y_2 = V_2(\pi)$. Moreover, it is easy to show that particles which, in the undeformed state, lie immediately above the upper crack-face are carried to points above this curve. Similarly, the undeformed lower crack-face is deformed to a curve which is asymptotically tangent to the line $y_2 = V_2(-\pi)$, and the adjacent particles lie below this curve in the deformed state. If follows that, according to the small-scale problem, the deformed crack is open as $r \to \infty$ if $V_2(\pi) - V_2(-\pi) > 0$, while interpenetration of the crack-faces is predicted if $V_2(\pi) - V_2(-\pi) < 0$. In the latter case, the one-to-one requirement imposed on the deformation is violated, and the problem as posed has no solution. Our interest thus centers on the difference $V_2(\pi) - V_2(-\pi)$.

The calculation which determines V_α is routine in principle but lengthy and elaborate in detail; we omit it here. One substitutes (2.3) into (1.5), (1.6) to obtain the expansions as $r \to \infty$ of I and J. These expansions, which show that I is near 2 and J near 1, must be inserted into (1.4) to produce the corresponding expansions for the nominal stresses $\sigma_{\alpha\beta}$. In order to carry out this step, it is first necessary to expand $W_I(I,J)$ and $W_J(I,J)$ in powers of $I-2$ and $J-1$ up to and including quadratic terms. Once the expansions for $\sigma_{\alpha\beta}$ have been found, they are substituted into the equilibrium equations (1.3) and the boundary conditions (2.1). To leading order in r, the $\sigma_{\alpha\beta}$'s are of course precisely the near-tip stresses of linear theory, so that, to leading order, (1.3) and (2.1) are satisfied automatically. When (1.3) and (2.1) are enforced to second order, they result in differential equations and boundary conditions for the determination of the V_α's. After much algebra, one then finds in particular that

$$V_2(\theta) = a_2 - \frac{c^2}{2(1-\nu)}\left\{\left[(2-3\nu)\alpha - \beta - \frac{11}{4} + \frac{13}{2}\nu - 4\nu^2\right]\theta \right.$$
$$+\left[\left(-\frac{9}{8}+4\nu\right)\alpha + 2\beta + \frac{65}{16} - 12\nu - 4\nu^2\right]\sin\theta$$
$$\left. +\left(\frac{\nu}{2}\alpha + \frac{1}{2}\beta - \frac{9}{8} + \frac{5}{4}\nu\right)\sin 2\theta + \left(\frac{3}{8}\alpha - \frac{3}{16}\right)\sin 3\theta\right\} . \qquad (2.4)$$

Here a_2 is an arbitrary constant associated with a rigid body translation which is left undetermined by the small-scale problem, c is given in terms of crack half-length b and loading parameter k by (1.14), ν is Poisson's ratio, and α and β are second-order elastic constants given by

$$\alpha = \frac{2(1-2\nu)}{\mu}\left(2W_{II} + W_{IJ}\right)_{I=2,J=1} , \qquad (2.5)$$

$$\beta = \frac{(1-2\nu)^3}{\mu}\left(4W_{III} + 6W_{IIJ} + 3W_{IJJ} + \frac{1}{2}W_{JJJ}\right)_{I=2,J=1} . \qquad (2.6)$$

It may be noted that $V_2(\theta)$ does not have the same parity in θ as does $U_2(\theta)$; except for the additive constant, $V_2(\theta)$ is <u>odd</u> in θ, while $U_2(\theta)$ is <u>even</u>. This reflects the fact that, in contrast to the Mode I case, the solution of the nonlinear Mode II problem does not have the same parity in x_2 as that of its counterpart in linearized theory.[1]

From (2.4), we may define the <u>crack-face separation at large</u> r — call it δ — as

$$\delta \equiv V_2(\pi) - V_2(-\pi) = \frac{\pi c^2}{1-\nu}\left[\beta - (2-3\nu)\alpha + \frac{11}{4} - \frac{13}{2}\nu + 4\nu^2\right] ; \qquad (2.7)$$

δ need not be positive.

[1] For an extensive discussion of this point, see [1].

3. Discussion

The small-scale problem predicts crack-opening or interpenetration according as δ is positive or negative. The sign of δ, in turn, depends only on the material, being determined by Poisson's ratio ν and the second-order elastic constants α and β.

When $\delta > 0$, the amount of crack-opening is of second-order in the loading parameter k, as is clear from (2.7), (1.14).

In the degenerate case for which $\delta = 0$, the issue of crack-opening versus interpenetration cannot be decided without consideration of terms beyond the second in the large-r expansions (2.3) for the displacements.

In order to study further the question of the sign of δ, it is convenient to recall first some properties of two special homogeneous plane finite deformations: simple shear and uniaxial stress. In simple shear with amount of shear k, the displacements are given by

$$u_1 = kx_2, \quad u_2 = 0 \quad . \tag{3.1}$$

From (1.5), (1.6), it follows that $I = 2 + k^2$, $J = 1$; the true stresses can then be computed from (1.9) as

$$\tau_{12} = 2W_I(2 + k^2, 1)k \quad , \tag{3.2}$$

$$\tau_{11} = 2W_I(2+k^2,1)(1+k^2)+W_J(2+k^2,1) \quad , \quad \tau_{22} = 2W_I(2+k^2,1)+W_J(2+k^2,1) \quad . \tag{3.3}$$

If k is small, then (3.2), (3.3) reduce approximately to (see (1.11), (2.5))

$$\tau_{12} \sim \mu k, \quad \tau_{11} \sim \left[1 + \frac{\alpha}{2(1-2\nu)}\right]\mu k^2, \quad \tau_{22} \sim \frac{\alpha}{2(1-2\nu)}\mu k^2 \quad . \tag{3.4}$$

The presence of the normal stresses τ_{11}, τ_{22} in simple shear, sometimes called a <u>Poynting</u> effect, arises from the nonlinear character of the theory.

Since such normal stresses must be present at infinity in the finite elastostatic version of the Mode II problem, one might conjecture that crack-opening or interpenetration is associated with the sign of the normal stress τ_{22} at infinity. According to (3.4), this sign is determined by the sign of α when k is small. We explore this conjecture further below.

In a plane deformation which corresponds to uniaxial stress in the x_1-direction, one has

$$u_1 = (\lambda - 1)x_1, \quad u_2 = (\bar{\lambda} - 1)x_2 \quad , \tag{3.5}$$

where the constants λ and $\bar{\lambda}$ are the direct and transverse stretch ratios, respectively. The invariants I and J are given by $I = \lambda^2 + \bar{\lambda}^2$, $J = \lambda\bar{\lambda}$, and the requirement that $\tau_{22} = 0$ leads through (1.9) and (3.5) to

$$\left(2\frac{\bar{\lambda}}{\lambda}W_I + W_J\right)_{I_1 = \lambda^2 + \bar{\lambda}^2,\ J = \lambda\bar{\lambda}} = 0 \quad . \tag{3.6}$$

Assuming that (3.6) determines $\bar{\lambda}$ as a function of λ, the stress-stretch relation then follows from (1.9) with $\alpha = \beta = 1$ as

$$\tau(\lambda) \equiv \tau_{11} = \left(2\frac{\lambda}{\bar{\lambda}}W_I + W_J\right)_{I = \lambda^2 + \bar{\lambda}^2,\ J = \lambda\bar{\lambda}} . \tag{3.7}$$

When λ is near unity,

$$\tau(\lambda) \sim \tau'(1)(\lambda - 1) + \frac{1}{2}\tau''(1)(\lambda - 1)^2 \quad ; \tag{3.8}$$

a direct calculation based on (3.7), (3.6) shows that

$$\tau'(1) = \frac{2\mu}{1-\nu} \ , \quad \tau''(1) = \frac{2\mu}{(1-2\nu)^3}\left[\beta + \frac{3}{2}\alpha - \nu(1 - \nu)\right] \quad . \tag{3.9}$$

The sign of the second derivative $\tau''(1)$ may be positive for some materials, negative for others. We shall say that a material is hardening (at the un-undeformed state) in uniaxial stress if $\tau''(1) > 0$, softening if $\tau''(1) < 0$.

With the help of these results pertaining to simple shear and uniaxial stress in plane strain, we now consider the crack separation δ of (2.7) for some special choices of the strain energy density W. First, we take

$$W(I,J) = \frac{\mu}{2}\left(\frac{I}{J^2} + 2J - 4\right) \quad , \tag{3.10}$$

corresponding to the elastic potential proposed by Blatz and Ko [5] in connection with experiments on foam rubber.[1] From (3.10), (1.11), one finds that $\nu = 1/4$ for this material, while (3.10), (2.5), (2.6) give

$$\alpha = -1, \quad \beta = -3/8 \quad . \tag{3.11}$$

The crack-face separation δ is then found from (2.7), (3.11) to be

$$\delta = 3\pi c^2 > 0 \quad , \tag{3.12}$$

so the crack-opening is predicted for the Blatz-Ko material. To test the conjecture that crack-opening is associated with a tensile normal stress τ_{22} at infinity, we specialize the Poynting effect formula in (3.4), obtaining

$$\tau_{22} \sim -\mu k^2 < 0 \quad . \tag{3.13}$$

The normal stress τ_{22} at infinity is thus compressive, upsetting the conjecture. (The normal stress τ_{11} in simple shear vanishes identically for the Blatz-Ko material, as may be verified from (3.10) and the first of (3.3).) Finally we note that, in plane strain uniaxial stress, (3.9), (3.11) give $\tau''(1) = -88\mu/9 < 0$, so that the Blatz-Ko material softens at the undeformed

[1] The theoretical properties of a material characterized by the Blatz-Ko strain energy density have been analyzed in detail in [6].

state.

Next we consider the case of the harmonic materials introduced by John [7]. For these,

$$W(I,J) = 2\mu[H(R) - J] \ , \quad R = \sqrt{I + 2J} \ , \tag{3.14}$$

where H is a given function characteristic of the particular harmonic material under consideration. In order to assure appropriate behavior at infinitesimal deformations, it is required that H satisfy the conditions

$$H(2) = 1, \quad H'(2) = 1, \quad H''(2) = \frac{1-\nu}{1-2\nu} \ , \tag{3.15}$$

where ν is Poisson's ratio.[1] For these materials, (3.14), (3.15), (2.5), (2.6) furnish the second order elastic constants as

$$\alpha = 1/2, \quad \beta = (1 - 2\nu)^3\left[H'''(2) - \frac{3}{4}\ \frac{1}{1-2\nu}\right] \ , \tag{3.16}$$

so that, from (2.7)

$$\delta = \frac{\pi c^2}{1-\nu}\left[(1 - 2\nu)^3 H'''(2) + (1 - \nu)^2\right] \ . \tag{3.17}$$

In plane strain uniaxial stress, the general formula in (3.9) for $\tau''(1)$ presently specializes to

$$\tau''(1) = \frac{2\mu}{(1-\nu)^3}\ \left[(1 - 2\nu)^3 H'''(2) + 2\nu(1 - \nu)\right] \ . \tag{3.18}$$

Eliminating $H'''(2)$ between (3.17) and (3.18) yields

$$\delta = \frac{1}{2}\pi c^2\left[\frac{\tau''(1)}{\mu} + \frac{1-3\nu}{(1-\nu)^2}\right] \ . \tag{3.19}$$

[1] For a fuller discussion of the characteristics of harmonic materials in plane strain, see [7] and [8].

It follows that the small-scale nonlinear crack problem predicts interpenetration of the crack-faces if

$$\tau''(1) < -\frac{1-3\nu}{(1-\nu)^2}\mu \quad . \tag{3.20}$$

Thus, for all harmonic materials with common, fixed values of μ, ν, (3.20) will hold for those which soften rapidly enough (or, if $\nu > 1/3$, for those which do not harden too rapidly) in uniaxial stress at the undeformed state. For those materials which harden rapidly enough, the crack will open.

From (3.4), (3.16), one finds that both normal stresses τ_{11}, τ_{22} are tensile in simple shear for any harmonic material.

These examples serve to show that crack-opening may occur despite the presence of a compressive normal stress τ_{22} at infinity, and that interpenetration of the crack-faces can be predicted even though the normal stress is tensile.

In general, for given values of ν and α, the crack-face separation δ of (2.7) will be negative — corresponding to interpenetration — if β is sufficiently negative. In view of (3.9), this means that for given ν and α, interpenetration will be predicted for materials which soften rapidly enough.

Since only first- and second-order elastic constants are involved in (2.7), it is not necessary to know the full strain energy density W to determine the sign of the crack-face separation δ. Hughes and Kelly [9] have experimentally determined the elastic constants up through second-order for polystyrene, Armco iron and pyrex glass. When their results are used to determine μ, ν, α and β for these materials, it is found that crack-opening occurs for polystyrene and iron, but interpenetration is predicted for glass.

It must be emphasized that the present analysis pertains only to the behavior of the crack-faces at points moderately near the crack-tips. Whether crack-opening or interpenetration is predicted at other points cannot be inferred from the results given here. To deal with this question, one would need an analysis of the original nonlinear Mode II problem accurate to second-order in the loading parameter and uniformly valid with respect to position along the crack. Nevertheless, to the extent that the small-scale problem faithfully describes the small-load behavior of the solution of the original Mode II problem, the present results furnish a condition $(\delta < 0)$ on the material which is sufficient to assure the violation — through apparent crack-face interpenetration — of the one-to-one requirement imposed on the deformation.

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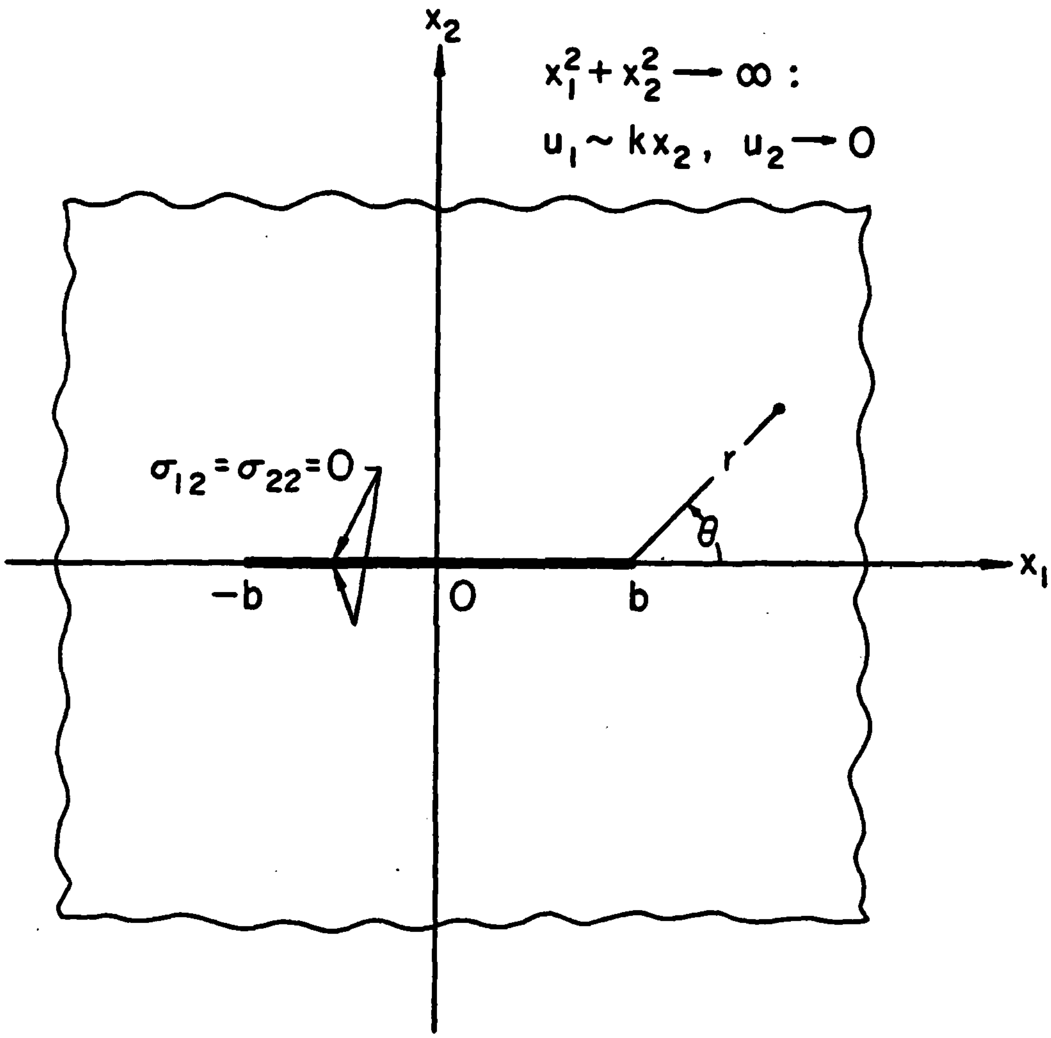

FIGURE 1. CROSS-SECTION OF UNDEFORMED BODY WITH CRACK AND COORDINATES; MODE II LOADING.

(9) Technical rept.

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A recent result of Stephenson shows that, when finite deformations are taken into account, a crack under Mode II loading conditions in plane strain will open, at least near the crack-tip and at least for certain elastic materials. In this note, the matter is investigated further, and it is shown that, in general, nonlinear effects—even at small loads—lead either to crack-opening or apparent interpenetration of the crack-faces when the loading is of Mode II-type.

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