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THE SIGNIFICANCE OF DATA COLLECTION PARAMETERS WHEN UTILIZING R--ETC(U)
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THE SIGNIFICANCE OF DATA COLLECTION PARAMETERS WHEN UTILIZING RADAR AND DIGITAL TERRAIN MODELS TO LOCATE TERRAIN FEATURES

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The combination of cartographic and hypsographic data with radar data can generate the location of terrain features in the ground reference system. The significance of selected data collection parameters used in this task is investigated in three phases:

1. The development of deterministic radar models from given digital terrain models and varying radar parameters.

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2) The perturbation of the terrain feature coordinates which result from errors in radar position, radar orientation, range, and resolution.

3) A factorial analysis of the terrain feature errors to establish the significance of the main fixed factors and their interactions. The main factors are azimuth from radar to terrain feature, range, resolution, terrain height and coordinate.

The results indicate that the order of greatest to least importance for the factors is coordinate, range, resolution, terrain height and azimuth.

THE SIGNIFICANCE OF DATA COLLECTION PARAMETERS
WHEN UTILIZING RADAR AND DIGITAL TERRAIN MODELS
TO LOCATE TERRAIN FEATURES

ABSTRACT

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1) The development of deterministic radar models from given digital terrain models and varying radar parameters.
2) The perturbation of the terrain feature coordinates which result from errors in radar position, radar orientation, range, and resolution.
3) A factorial analysis of the terrain feature errors to establish the significance of the main fixed factors and their interactions. The main factors are azimuth from radar to terrain feature, range, resolution, terrain height and coordinate.

The results indicate that the order of greatest to least significance for the factors studied is coordinate, range, resolution, terrain height and azimuth. Additional results are given in fourteen figures and eleven tables. An appendix is included on the projection equations for real-aperture radar.

THE SIGNIFICANCE OF DATA COLLECTION PARAMETERS WHEN UTILIZING RADAR AND DIGITAL TERRAIN MODELS TO LOCATE TERRAIN FEATURES

1.0 INTRODUCTION

1.1 BACKGROUND

The continuing improvement in radar systems coupled with their unique capabilities of information collection indicate that they will play an increasing role in future mapping systems. The ultimate role or path to that role has yet to be determined.

Considerable experimentation has taken place during the past decade with the goal of determining the value of radar as a mapping or terrain feature positioning tool. The geometry applicable to radar mapping has been well documented, and the mathematical models have been developed with the thought of combining the radar sensor with other types of remote sensors. The most significant combinations of sensors have yet to be determined.

The digital terrain model (DTM) is an emerging information source which will have an impact on the use of radar as a mapping tool. Ground truth is an important part of any mapping task and the availability of DTM data can be used in many ways to provide a geometric framework for the radar data. The optimum combinations of these two data sources will not be easy to ascertain and this topic will need considerable study.

1.2 CONCEPT AND PURPOSE OF THE INVESTIGATION

The overall concept of the mapping task considered in this study is to combine the capability of the radar with the DTM to produce the location of terrain features in the X, Y, and Z DTM coordinate system. The assumption is that a DTM is available for the area of interest. A radar sensor is to be flown in the vicinity of the area of concern in order to collect imagery.

Assuming a given sensor, whose general capability is known, the object is to determine the most efficient operational use of the system. The determination of target positions is a function of several parameters whose values are known within certain limits. If the values of the data collection parameters are changed, will the results also be changed? Some of the parameters can be varied *mathematically* for stronger functional relationships, however, some depend solely on the original data collection procedures. Accordingly, the purpose of this study was to select several important data collection parameters and to study their significance in the location of terrain features when radar and DTM data are combined. The determination of the relative importance of data collection parameters should aid in the design of successful data collection efforts. For example, Figure 1 schematically demonstrates the effect one can expect from the variation of range, azimuth, and terrain height during the data collection process. The standard deviations are for the conditions imposed in this study.

The general procedure employed was to develop deterministic models, to perturb the selected parameters in a random fashion, to establish errors for specific parameter combinations and to evaluate the results with a factorial analysis. Section 2 elaborates on the details of the procedure and section 3 states and discusses the results. Section 4 gives conclusions and recommendations. Appendices A and B provide details on the computational and statistical procedures.

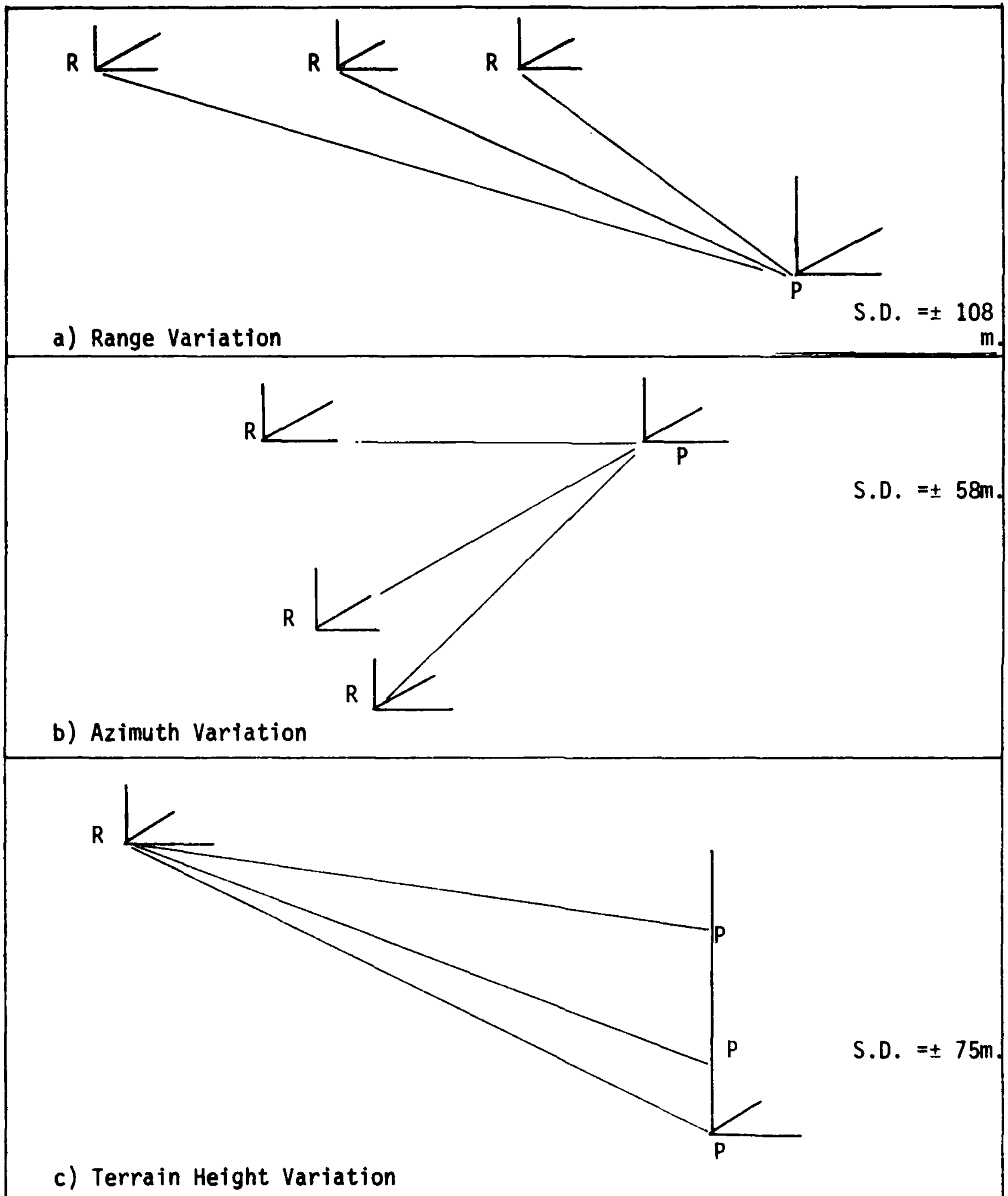


FIGURE 1 - FLUCUATION OF COMPUTED POSITION STANDARD DEVIATIONS AS A RESULT OF VARYING DIFFERENT DATA COLLECTION PARAMETERS

R-Radar P-Terrain Feature

2.0 EXPERIMENTAL PROCEDURE

2.1 GENERAL

This section outlines the procedure employed during the study. It consists of several sub-sections, namely; the definition of the deterministic model which served as the basis of the study, the error propagation phase of the effort which perturbed the deterministic model in a random fashion, and the factorial analysis of the errors introduced by perturbing the selected parameters.

2.2 DETERMINISTIC MODEL

Two coordinate systems are utilized in setting up the model. The first is the ground coordinate reference system which is the basis for the DTM. In this instance the RADC DME Navigation Range in the vicinity of Rome, N.Y. was selected for the study. Cartographic and hypsographic data have been collected for this area and a DTM exists. A local radar coordinate system also has to be established and an expression adopted for the transformation of one system into the second and vice versa. Figure 2 shows an example of the relative position of the two reference systems. Equation 1 adopted from Franz Leberl states the mathematical model which is the basis for the study. (Leberl, 1976).

$$\begin{bmatrix}\text{Point Coordinates}\\ \text{In DTM}\\ \text{Reference System}\end{bmatrix} = \begin{bmatrix}\text{Radar Location}\\ \text{In DTM Reference}\\ \text{System}\end{bmatrix} + \begin{bmatrix}\text{Radar}\\ \text{Orientation}\end{bmatrix} \begin{bmatrix}\text{Point Coordi-}\\ \text{nates in Radar}\\ \text{Reference System}\end{bmatrix}$$

$$\begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix} = \begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} + \begin{bmatrix} A(k,\phi,w) \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} \quad (1)$$

$$\text{where: } x_p = (\sin \tau)\, r \quad (2)$$

$$y_p = (\sin^2 \Omega - \sin^2 \tau)^{1/2}\, r$$

$$z_p = (-\cos \Omega)\, r$$

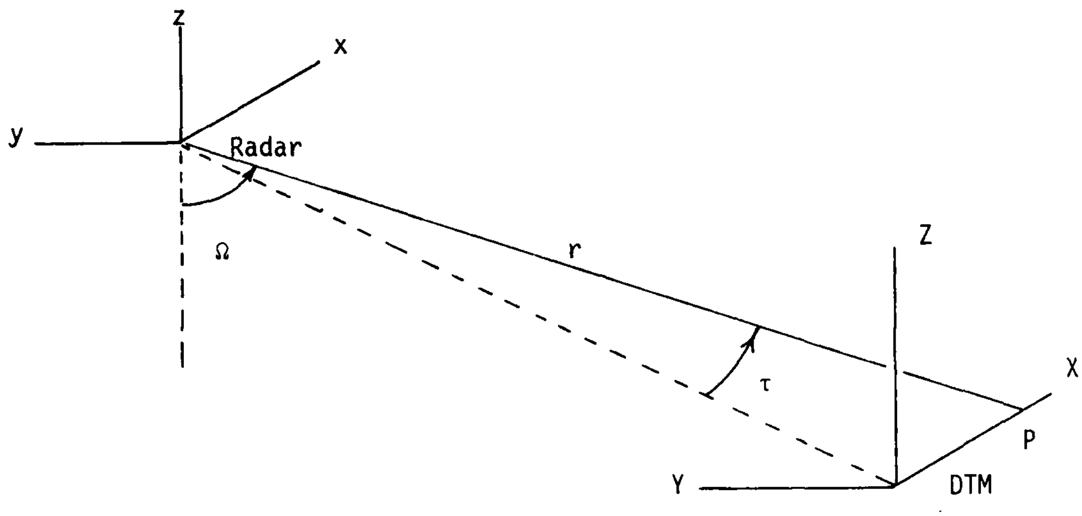


FIGURE 2 - RADAR COORDINATE SYSTEM AND DTM REFERENCE SYSTEM

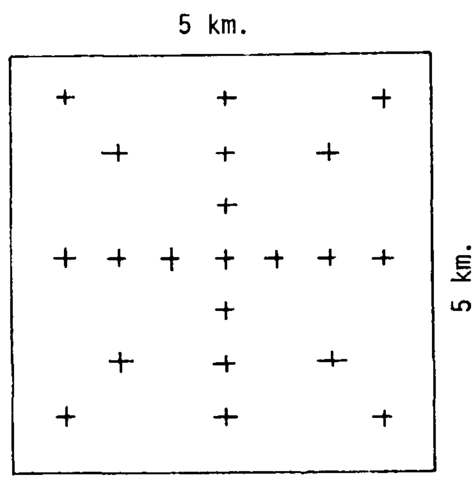


FIGURE 3 - NOMINAL PLAN POSITION OF CONTROL POINTS IN THE DTM

A = Orientation matrix in terms of k,¢,w
as normally adopted in photogrammetry.

r = Range from radar to DTM point P.

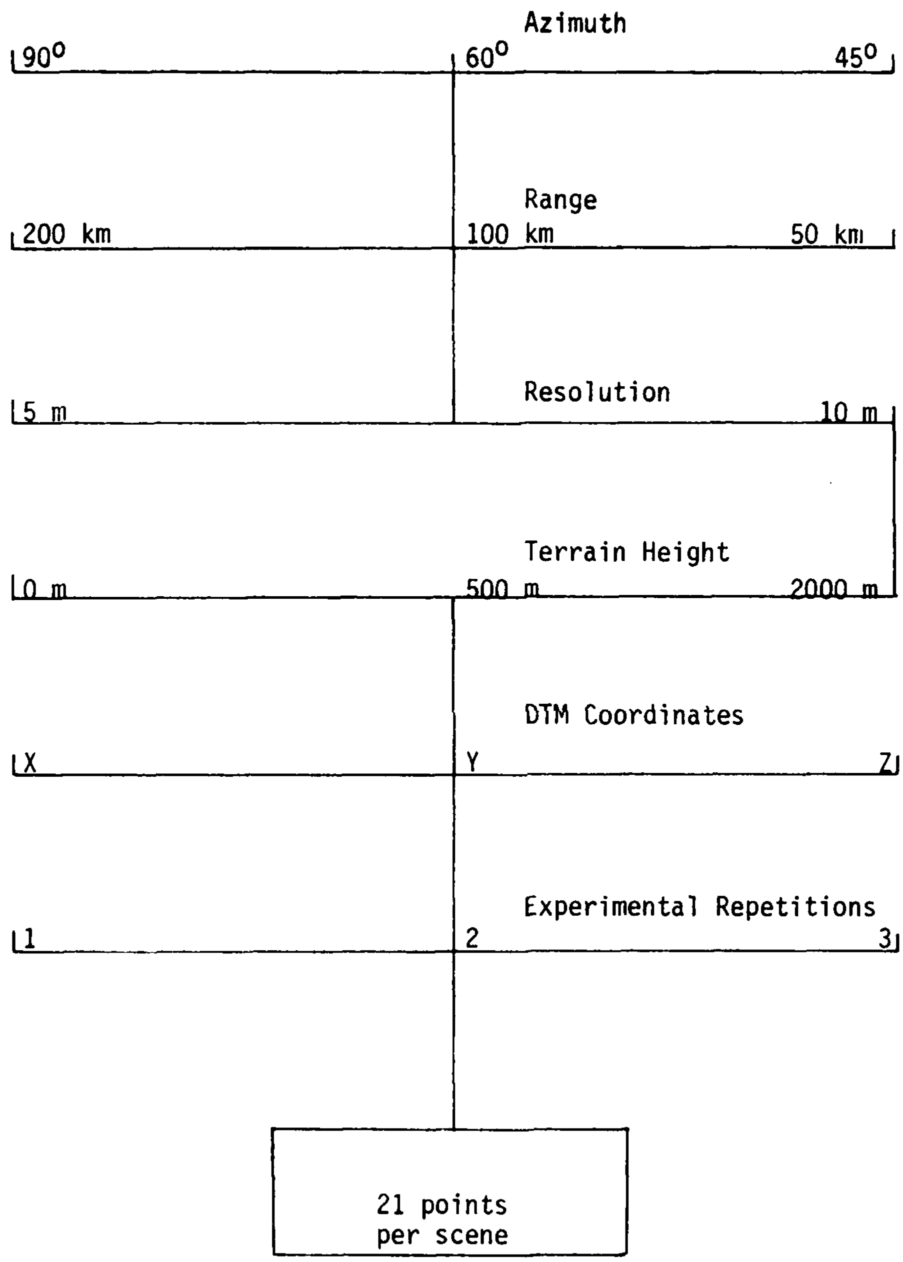
τ = Squint angle

Ω = Elevation angle

A total of 27 unique radar scenes were generated with equations (1) and (2). X, Y, and Z coordinates for twenty-one points were generated in each scene which nominally covered an area in the DTM of 5 km, by 5 km. This experiment was repeated three times for each of the 27 scenes. Figure 3 shows the positions of the selected 21 control points in the DTM.

Recalling that the purpose of the study is to determine the relative importance of radar data collection parameters, the next task is to select the main parameters or factors for study. The parameters studied in this effort were range, azimuth or radar direction which was angle kappa in this application, the resolution of the radar, the height of the DTM point, and the particular coordinate for the point of interest. When locating random points in a radar scene and transforming the radar location into a meaningful position in the DTM, any or all of these parameters may be important. The task is to determine how the relative values of these parameters affect the accuracy of the final point position in the DTM. Are their particular combinations to either avoid or encourage for the sake of accurate mapping or position determination?

The total experiment is summarized in Figure 4. For each of the computations noted in Figure 4, the values of range (r), squint angle (τ) and elevation angle (Ω) were computed when the values X_p, Y_p, Z_p, X_R, Y_R, Z_R, k, ϕ, and w were provided.



(3 Az.)(3 Ra.)(2 Res.)(3 Ht.)(3 Coord.)(3 Exp.)(21 pt.)

FIGURE 4 - SUMMARY OF EXPERIMENT SHOWING THE MAIN FACTORS AND LEVELS OF EACH

Formulas used for the computation follow:

$$\text{RANGE } (r) = \left[(X_P - X_R)^2 + (Y_P - Y_R)^2 + (Z_P - Z_R)^2 \right]^{\frac{1}{2}}$$

$$\text{SQUINT ANGLE } (\tau) = \text{SIN}^{-1} \left[\frac{A_{11} \ (X_p - X_R) + A_{12} \ (Y_P - Y_R) + A_{31} \ (Z_P - Z_R)}{r} \right]$$

$$\text{ELEVATION ANGLE } (\Omega) = \text{COS}^{-1} \left[\frac{A_{31} \ (X_P - X_R) + A_{32} \ (Y_P - Y_R) + A_{33} \ (Z_P - Z_R)}{-r} \right]$$

2.3 ERROR MODELS

The computed DTM coordinates (X_p, Y_p, Z_p) are a function of X_R, Y_R, Z_R, k, ϕ, w, r, τ, and Ω. Under operational conditions these values must be provided either directly or indirectly. Each value will have, at least, a random error associated with it. Previous studies and experimental measurement in this area by the authors and others provide us with reasonable estimates for the appropriate random error to be applied for each parameter. (Brock, Zulch 1979).

The position of the radar is provided by DME navigation. It is estimated that $X_R = \pm 9$ m., $Y_R = \pm 3$ m., and $Z_R = \pm 20$ m. Stabalized platforms are estimated to provide k, ϕ, and w to $\pm 0.1^{\circ}$. The error in range is set at ±5 m. One of the additional factors was radar resolution. Two cases were assumed. The first assumed that the minimum element of information had a diameter of 5 m. and the second case assumed that the minimum element size was 10 m. For a range of 200 km, for instance, this would then place the random error for τ and Ω at $\pm 0.0015^{\circ}$ for the 5 m. case and $\pm 0.003^{\circ}$ for the 10 m. case.

Utilizing the above standard deviations and a random number generator, random errors were applied to each of the 9 parameters utilized in the computation of the X_p,Y_p, and Z_p coordinates. This provided perturbed values for X_p,Y_p, and Z_p. The perturbed values were subtracted from the true values previously assigned. The resulting errors provided the basis for the factorial analysis portion of the study. See Table 1 for an example of true and perturbed values.

2.4 FACTORIAL ANALYSIS

The main objective of the effort was to demonstrate the importance of azimuth, range, resolution, terrain height and coordinates in the utilization of radar and DTM data to provide DTM positions of random points of interest.

A factorial analysis was selected since it not only allows one to examine the effects of each parameter but the effects of the interactions of the factors as well. In addition all values may be used to investigate each of the factors. The possibility of ascribing variation to the wrong factor is reduced since all factors are contained in the model. A summary of the main experimental factors and the levels of each is given in Figure 4.

TABLE 1 - SAMPLE OF TRUE AND PERTURBED VALUES FOR PARAMETERS AND A TERRAIN FEATURE OF INTEREST

PARAMETER	TRUE VALUE	PERTURBED VALUE
X_R	302,000 M.	302,010 M.
Y_R	523,000 M.	522,997 M.
Z_R	12,192 M.	12,187 M.
k	1.570796 Rad.	1.569871 Rad.
ϕ	0.0 Rad.	0.002028 Rad.
w	0.0 Rad	0.001062 Rad
r	198,944 M.	198,936 M.
τ	0.000000 Rad.	-0.000024 Rad.
Ω	1.512017 Rad.	1.512037 Rad.
TERRAIN FEATURE		
X_p	500,600 M.	500,590 M.
Y_p	523,000 M.	523,162 M.
Z_p	505 M.	294 M.

3.0 RESULTS

3.1 GENERAL

The results will be presented in three parts. First, sample results will be given for the generation of deterministic models. Second, the results of several error models will be listed and last the factorial analysis results will be stated. Where appropriate, graphical results will also be given. The last portion of this section will deal with a discussion of the results as found for the conditions of this study.

3.2 DETERMINISTIC MODEL RESULTS

As previously stated a total of 27 unique radar scenes were mathematically created. First, the radar was directed along three different azimuths with each azimuth being used for three distinct ranges. Figure 5 shows the nine radar positions created by these two variables. For each of the nine positions, scenes having three different average heights were created thus making 27 scenes. A set of 21 ficticious rays were created for each of the 27 scenes. The range, squint angle and elevation angle were calculated for each ray. Figure 6 shows partial results for one of the 27 scenes.

3.3 RESULTS OF ERROR MODELS

As discussed in Section 2.3 the 21 ficticious rays created for each scene were randomly perturbed by introducing random errors into the radar position and orientation and the ray distance and direction. For a given set of parameters a set of X, Y, and Z error components were calculated. Tables 2 through 6 are five sets of errors selected from the 162 sets which were computed. These error sets provide examples of noteworthy results which are discussed in Section 3.5.

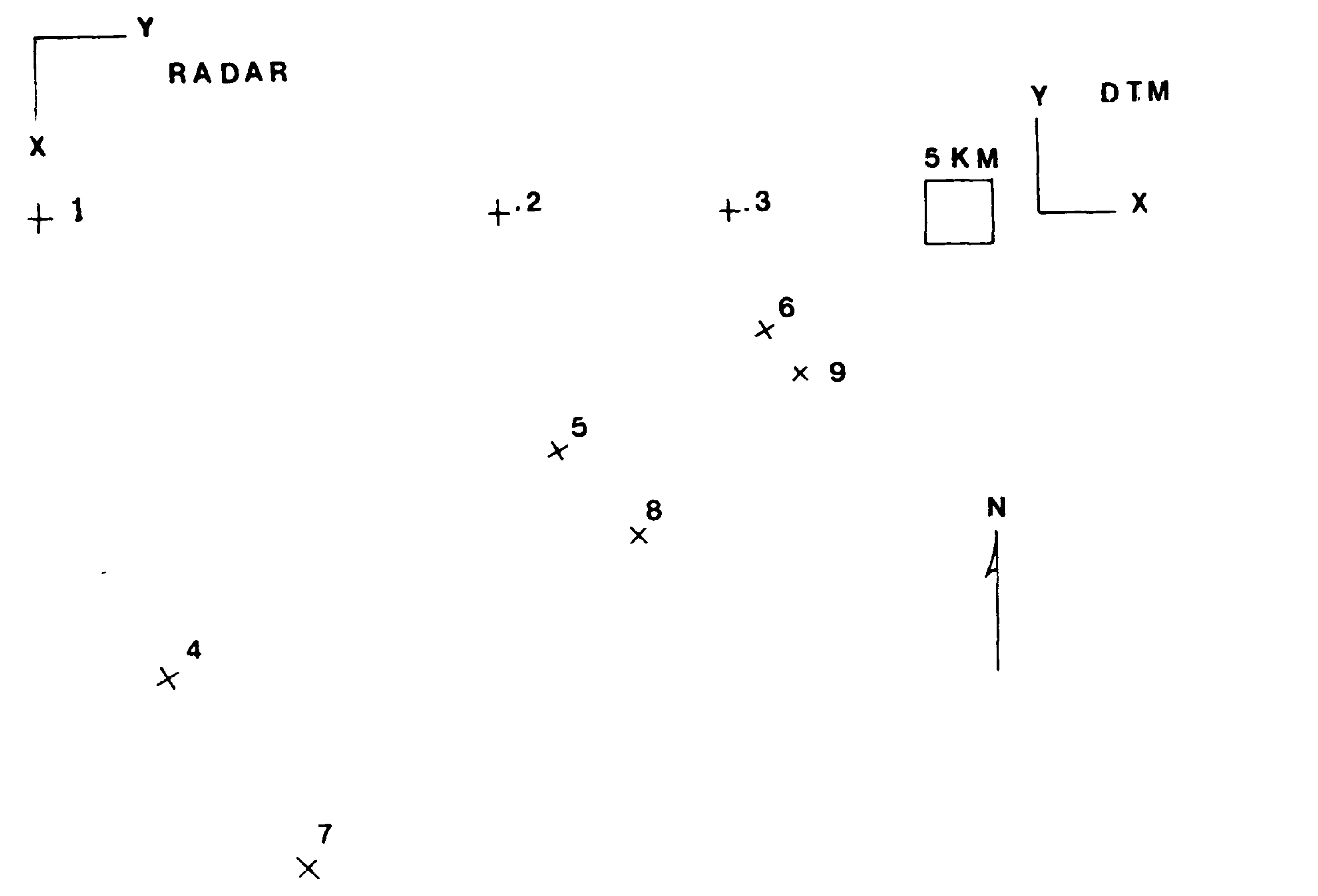
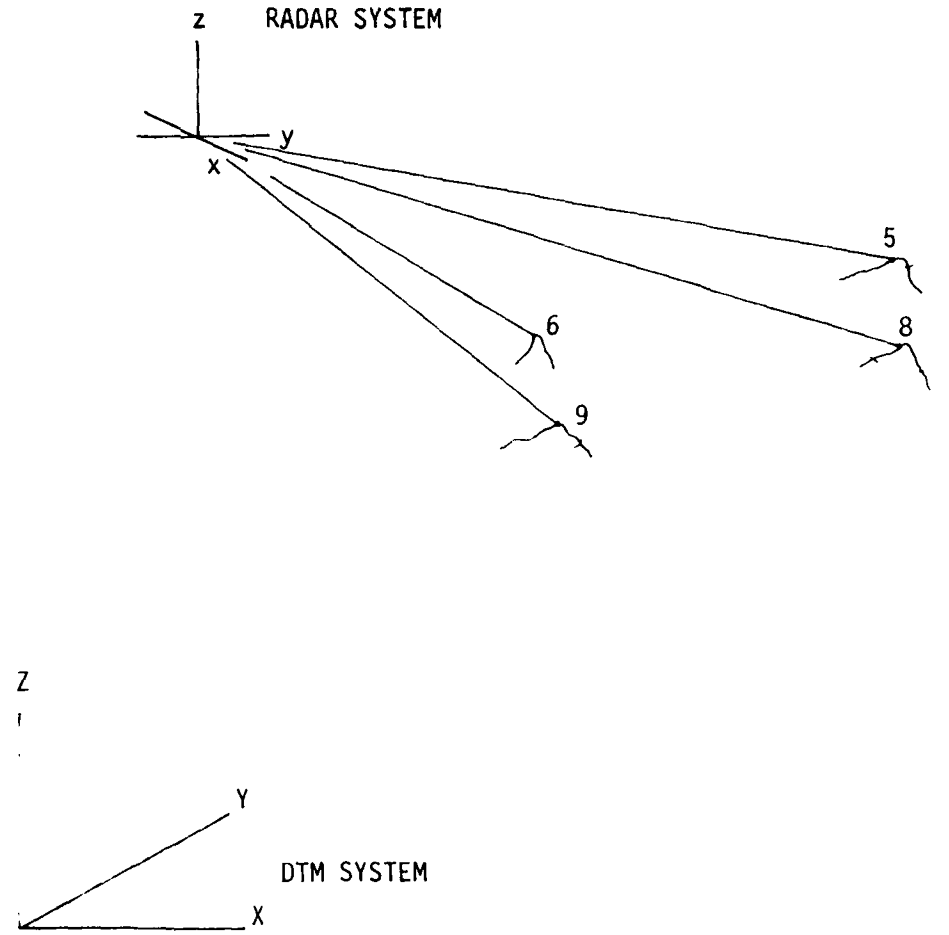


FIGURE 5. RADAR POSITIONS USED IN THE STUDY



DTM POINT	RANGE (r) M.	SQUINT ANGLE (τ)°	ELEVATION ANGLE (Ω)°
5	102,138	-0.79	83.15
6	99,332	-0.82	82.95
8	102,138	0.79	83.15
9	99,331	0.82	82.95

FIGURE 6. PARTIAL RESULTS FOR RADAR POSITION 8 AS SHOWN IN FIGURE 5.

Table 2 - PROPAGATED ERRORS FOR CONTROL POINT COORDINATES-M.
Azimuth - 45^{o}, Range 200 km., Resolution - 5 m.
Terrain Height - 0 m.

Control Point No.	Coordinates X	Y	Z	Resultant $(X^2 + Y^2 + Z^2)^{1/2}$
1	251.86	-361.70	-963.87	1059.89
2	-399.83	324.90	-480.73	704.64
3	-611.52	658.00	782.70	1191.44
4	-458.83	530.01	391.82	803.08
5	468.66	-501.44	-772.95	1033.70
6	25.09	- 12.61	534.84	535.58
7	-238.15	276.26	-200.15	416.05
8	563.18	-509.07	828.64	1123.82
9	-473.49	463.31	-417.57	783.08
10	488.53	-356.36	974.50	1146.87
11	-500.96	477.97	-421.11	810.40
12	-568.16	625.99	746.33	1127.69
13	-451.65	382.80	-609.50	849.71
14	-558.00	583.47	- 23.99	807.70
15	707.39	-684.52	135.20	993.60
16	2.05	- 37.60	-459.08	460.62
17	193.21	-172.96	421.29	494.70
18	-148.85	205.69	696.21	741.06
19	-164.81	101.04	-796.52	819.64
20	695.21	-753.83	-354.92	1085.15
21	689.66	-587.81	896.70	1274.84
Average	-23.30	31.03	43.23	58.09
Std. Deviation	462.32	459.41	624.25	902.49

Table 3 - PROPAGATED ERRORS FOR CONTROL POINT COORDINATES-M.
Azimuth - 45°, Range - 50 km., Resolution - 5 m.
Terrain Height - 0 m.

Control Point No.	Coordinates X	Y	Z	Resultant $(X^2 + Y^2 + Z^2)^{1/2}$
1	61.68	- 2.74	227.35	235.58
2	41.57	- 83.88	-104.46	140.27
3	78.54	-147.57	-181.23	246.56
4	-95.74	34.73	-246.95	249.38
5	-42.64	36.83	114.57	127.67
6	-27.56	- 45.13	-173.29	181.18
7	107.27	-220.22	-138.95	281.62
8	44.21	- 39.50	81.89	101.10
9	-73.99	86.87	5.29	114.23
10	77.91	10.71	184.49	200.55
11	-110.47	42.34	-105.30	158.38
12	128.95	- 92.98	197.62	253.63
13	- 40.40	129.48	192.84	235.76
14	- 14.27	- 67.76	-186.96	199.37
15	-109.22	42.54	-260.00	285.20
16	- 37.55	54.78	- 43.93	79.63
17	19.78	- 9.09	70.77	74.04
18	- 3.76	15.15	125.09	126.06
19	134.49	- 70.51	205.62	255.61
20	- 36.62	21.63	-133.00	139.63
21	- 65.05	87.16	64.40	126.40
Average	1.77	- 10.34	- 4.91	11.58
Std. Deviation	74.47	81.31	159.76	194.11

Table 4 - PROPAGATED ERRORS FOR CONTROL POINT COORDINATES-M.
Azimuth - 90^o, Range - 50 km., Resolution - 5 m.
Terrain Height - 0 m.

Control Point No.	Coordinates X	Y	Z	Resultant $(X^2 + Y^2 + Z^2)^{1/2}$
1	75.67	- 19.71	272.29	283.30
2	- 26.76	-161.52	- 79.13	181.84
3	53.82	186.73	35.14	197.48
4	- 39.09	108.55	-190.51	222.72
5	- 11.04	-251.48	29.42	253.44
6	- 10.97	96.68	-110.03	146.88
7	16.10	86.07	100.21	133.08
8	82.05	- 75.98	241.17	265.83
9	- 51.22	24.95	-147.83	158.43
10	- 14.10	98.65	70.22	121.91
11	- 1.39	- 61.22	- 40.72	73.54
12	23.88	84.53	143.57	168.31
13	31.51	36.44	229.79	234.79
14	- 59.16	30.44	-178.75	190.73
15	31.29	170.94	117.80	209.94
16	- 55.89	181.36	-190.76	269.08
17	41.50	159.50	273.09	318.97
18	24.49	- 29.48	208.80	212.29
19	29.79	-120.26	146.84	192.12
20	6.75	-195.79	32.87	198.64
21	- 7.36	115.39	- 41.03	122.69
Average	6.66	22.13	43.93	49.64
Std. Deviation	39.32	124.72	152.00	200.51

Table 5 - PROPAGATED ERRORS FOR CONTROL POINT COORDINATES-M.
Azimuth - 45^o, Range - 50 km., Resolution - 5 m.
Terrain Height - 2000 m.

Control Point No.	Coordinates X	Y	Z	Resultant $(X^2 + Y^2 + Z^2)^{1/2}$
1	175.31	-117.24	184.55	280.25
2	-119.38	158.96	130.09	237.58
3	9.93	53.22	258.75	264.35
4	219.38	-196.15	140.38	326.05
5	- 27.31	77.12	236.37	250.13
6	-160.57	57.03	-274.87	323.40
7	- 73.52	47.75	-224.67	241.17
8	- 28.81	- 19.45	-109.71	115.08
9	84.48	-204.11	-178.65	284.10
10	-123.69	103.50	- 86.44	182.98
11	- 15.55	78.26	276.94	288.21
12	26.16	18.63	25.69	41.13
13	72.70	-102.01	- 76.65	146.86
14	58.26	- 75.34	- 59.56	112.33
15	156.55	-134.50	50.44	212.47
16	192.61	-145.81	60.39	249.01
17	10.72	- 58.44	-248.97	255.96
18	-146.65	167.19	75.49	234.86
19	77.90	- 46.74	137.00	164.38
20	79.29	-155.06	-198.27	263.90
21	100.16	- 82.92	80.32	152.84
Average	27.05	- 27.41	9.46	39.65
Std. Deviation	108.73	110.25	168.22	228.64

Table 6 - PROPAGATED ERRORS FOR CONTROL POINT COORDINATES-M.
Azimuth - 45^o, Range - 50 km., Resolution - 10 m.
Terrain Height - 0 m.

Control Point No.	Coordinates X	Y	Z	Resultant $(X^2 + Y^2 + Z^2)^{½}$
1	-132.38	190.30	256.18	345.49
2	-170.43	154.91	-115.42	257.61
3	162.97	-131.66	120.20	241.54
4	-112.00	115.17	- 55.15	169.85
5	- 28.60	- 7.06	-161.37	164.04
6	6.11	- 7.05	20.43	22.46
7	29.34	- 42.27	- 82.77	97.46
8	- 22.00	14.56	81.51	85.67
9	169.34	-151.47	48.74	232.37
10	22.43	-103.11	-225.66	249.11
11	62.65	- 96.92	-271.02	294.57
12	145.87	-115.61	140.22	233.04
13	38.87	41.58	224.91	232.00
14	35.68	11.53	137.85	142.86
15	227.89	-178.53	52.40	294.20
16	135.80	- 64.43	186.40	239.45
17	- 6.90	- 58.08	-123.94	137.05
18	67.37	- 96.59	- 32.70	122.22
19	-198.92	151.72	-222.39	334.73
20	84.06	-145.40	-168.32	237.78
21	- 32.08	34.52	-122.15	130.93
Average	23.10	- 23.04	- 14.86	35.85
Std. Deviation	110.51	105.57	153.43	216.56

The computations in Table 3, for example, were repeated three times, each with a different set of random errors. These three computations were referred to as experiments. Table 7 shows the average error results of these three computations for the parameter values given in Table 3.

TABLE 7. AVERAGE ERROR RESULTS FOR THREE EXPERIMENTS (Table 3 parameters used)

Experiment	Coordinate - m.		
	X	Y	Z
1	1.76	-10.34	- 4.92
2	44.87	-43.37	23.36
3	-46.88	43.40	- 7.33
Average	- 0.08	- 3.44	3.70
Std. Deviation	45.90	43.79	17.07

This is an important point of the study since the test for significance is against the experimental error. If one cannot repeat a coordinate determination to better than ±45 m., then a parameter cannot be judged to be significant unless the error produced by the variation of the parameter is statistically larger than ±45 m.

3.4 RESULTS OF ANALYSIS OF VARIANCE

The basic null hypothesis (H_0) for the analysis of variance (ANOVA) model in this study is one of no significance between the factors or interaction of the factors at the 10% level. Thus, if the analysis shows significance, it is indicating that one could expect a Type I error 10 times out of 100 in the long run.

The results of the Factorial Analysis are given in Table 8, where the fixed factors of concern are as follows:

A - Azimuth
B - Range
C - Resolution
D - Terrain Height
E - DTM Coordinate

The F test for significance depends upon the variable introducing a variance which is significantly larger than that generated by completing the experiment independently three times. It is seen that only the DTM coordinate is significant. The order of importance for the main factors from most to least is DTM coordinate, range, resolution, terrain height and azimuth.

The ANOVA computation also provides the average errors for all levels of the five main factors. See Table 9 for these values.

Referring to the coordinate results it is seen that the significance indication can be attributed to the large variations in the Y coordinate.

Considering the interaction of any two of the factors it is seen that the combination of range and terrain height provide significance. Most of the variance can be traced to the case of 200 KM range and 2000 M. terrain height. (Table 10). A look at three factor interactions show significance for range, terrain height and coordinate combinations. The greatest variance is provided at the 200 KM range and in particular the 200 KM range, 2000 M. terrain height and Z coordinate. (Table 11). Four factor interactions have one significant combination, namely; azimuth, resolution, terrain height and coordinate. The intermediate level of 10 M. resolution, 500 M. terrain height, Z coordinate and 90° azimuth gives the greatest variance.

TABLE 8 - ANALYSIS OF VARIANCE RESULTS

Source	Degrees Of Freedom	Sum of Squares $(m.)^2$	Mean Square $(m.)^2$	F. Test
A - Azimuth	2	6702	3551	0.59
B - Range	2	23477	11738	2.06
C - Resolution	1	6342	6343	1.12
D - Terrain Ht.	2	11168	5584	0.98
E - Coordinate	2	29162	14581	2.56*
AB	4	5618	1405	0.25
AC	2	7361	3680	0.65
AD	4	23147	5787	1.02
BC	2	2741	1371	0.24
BD	4	48529	12132	2.13*
CD	2	1452	726	0.13
AE	4	23263	5816	1.02
DE	4	33176	8294	1.46
BE	4	34947	8737	1.54
CE	2	524	262	0.05
ABC	4	3556	889	0.16
ABD	8	22820	2853	0.50
BCD	4	6423	1606	0.28
ACD	4	23088	5772	1.02
CDE	4	21207	5302	0.93
DEA	8	26420	3303	0.58
ABE	8	30044	3755	0.66
BDE	8	86500	10813	1.90*
ACE	4	18628	4657	0.82
BCE	4	13116	3279	0.58
ABCD	8	10640	1330	0.23
BCDE	8	33907	4238	0.75
CDEA	8	85927	10741	1.89*
ABDE	16	40329	2521	0.44
ABCE	8	27373	3422	0.60
ABCDE	16	92878	5805	1.02
Experimental Error	324	1842264	5686	
Total	485	2642728		

* Significant at 10% level.
All others not significant.

TABLE 9 - AVERAGE ERRORS IN METERS FOR MAIN FACTORS

(MAIN FACTORS ARE LISTED IN ORDER OF SIGNIFICANCE)

Order of results	Factor Values	$(EX_i)^{2*}$ ($m.^2$)	Ave. X_i (± m.)
		Coordinate Results	
1	Z	63,360	1.6
2	X	191,784	2.7
3	Y	6,255,196	15.4
		Range Results	
1	100 km	27,626	1.1
2	50 km	40,453	1.2
3	200 km	5,521,278	14.5
		Resolution Results	
1	5 m.	78,144	1.2
2	10 m.	4,142,361	8.4
		Terrain Height Results	
1	0 m.	21,734	0.9
2	500 m.	504,628	4.4
3	2000 m.	3,069,059	10.8
		Azimuth Results	
1	45^o	20,021	0.9
2	60^o	349,957	3.6
3	90^o	2,501,950	9.8

* X_i = Average error in meters for 21 point scene.

TABLE 10 - AVERAGE ERRORS FOR INTERACTIONS BETWEEN

TWO OF THE MAIN FACTORS.

(SELECTED COMBINATIONS ARE GIVEN IN ORDER OF MOST SIGNIFICANT TO LEAST SIGNIFICANT).

Order of Result	Combination Values		$(EX_i)^2$ ($m.^2$)	Ave. X_i ($\pm$ m.)
	Range and Terrain Height Results			
	Range	Terrain Ht.		
1	50 km	500 m.	3	0.1
2	100 km	2000 m.	11,137	2.0
3	50 km	0 m.	11,437	2.0
4	100 km	0 m.	14,755	2.2
5	200 km	0 m.	17,662	2.5
6	50 km	2000 m.	93,817	5.7
7	200 km	500 m.	101,721	5.9
8	100 km	500 m.	154,618	7.3
9	200 km	2000 m.	4,681,606	40.0
	Range and Coordinate Results			
	Range	Coordinate		
1	50 km	X	378	0.4
2	50 km	Y	598	0.5
3	100 km	X	4,603	1.3
4	100 km	Z	6,850	1.5
5	50 km	Z	60,035	4.5
6	100 km	Y	100,374	5.9
7	200 km	X	151,727	7.2
8	200 km	Z	335,815	10.7
9	200 km	Y	4,664,588	40.0

TABLE 10 - (CONT'D)

Terrain Height and Coordinate Results

	Terrain Ht.	Coordinate		
1	500 m.	Z	3,348	1.1
2	2000 m.	X	36,290	3.5
3	500 m.	Y	81,175	5.3
4	500 m.	X	135,128	6.8
5	0 m.	X	378,261	11.4
6	0 m.	Z	561,545	13.9
7	2000 m.	Z	889,652	17.5
8	2000 m.	Y	998,319	18.5
9	0 m.	Y	1,481,013	22.5

Azimuth and Terrain Height

	Azimuth	Terrain Height		
1	45°	0 m.	5080	1.3
2	45°	500 m.	48,293	4.1
3	60°	500 m.	63,594	4.7
4	60°	0 m.	83,260	5.3
5	90°	0 m.	133,005	6.8
6	45°	2000 m.	187,077	8.0
7	60°	2000 m.	308,249	10.3
8	90°	2000 m.	583,922	14.2
9	90°	500 m.	1,397,848	21.9

Azimuth and Coordinate Results

	Azimuth	Coordinate		
1	90°	X	4	0.1
2	60°	X	555	0.4
3	45°	Z	10,838	1.9
4	90°	Z	39,780	3.7
5	60°	Y	71,866	5.0
6	60°	Z	120,449	6.4
7	45°	X	173,386	7.7
8	45°	Y	205,921	8.4
9	90°	Y	3,165,470	33.0

TABLE 10 - (CONT'D)

Azimuth and Resolution Results

	Azimuth	Resolution		
1	45^o	10 m.	974	0.4
2	90^o	5 m.	5576	0.9
3	60^o	5 m.	8948	1.2
4	45^o	5 m.	12159	1.4
5	60^o	10 m.	246,983	6.1
6	90^o	10 m.	2,271,297	18.6

Azimuth and Range Results

	Azimuth	Range		
1	90^o	50 km	412	0.4
2	45^o	100 km	1,153	0.6
3	60^o	100 km	2,203	0.9
4	60^o	50 km	6,767	1.5
5	45^o	50 km	19,367	2.6
6	90^o	100 km	23,482	2.8
7	45^o	200 km	98,991	5.8
8	60^o	200 km	392,999	11.6
9	90^o	200 km	1,983,070	26.1

Range and Resolution Results

	Range	Resolution		
1	100 km	5 m.	93,358	3.8
2	50 km	10 m.	132,844	4.5
3	100 km	10 m.	222,553	5.8
4	50 km	5 m.	319,910	7.0
5	200 km	5 m.	1,324,096	14.2
6	200 km	10 m.	1,437,712	14.8

TABLE 11 - AVERAGE ERRORS FOR INTERACTIONS BETWEEN

THREE OF THE MAIN FACTORS

Order of Results	Combination Values			Ave. Error (± m.)
	Range (km)	Terrain Ht (m)	Coord.	
1	50	2000	X	0.4
2	50	500	X	1.1
3	50	0	X	1.8
4	50	0	Y	2.0
5	100	500	X	2.0
6	200	500	Y	2.1
7	50	0	Z	2.1
8	100	2000	Y	2.2
9	100	2000	X	2.5
10	50	500	Z	2.5
11	50	500	Y	3.5
12	200	500	Z	4.0
13	50	2000	Y	4.1
14	100	2000	Z	6.1
15	100	0	Z	8.1
16	100	0	X	8.2
17	100	0	Y	9.6
18	100	500	Z	9.6
19	100	500	Y	10.2
20	50	2000	Z	13.3
21	200	2000	X	13.4
22	200	500	X	19.6
23	200	0	X	27.8
24	200	0	Z	35.6
25	200	0	Y	56.0
26	200	2000	Y	61.8
27	200	2000	Z	71.8

3.5 DISCUSSION OF RESULTS

The purpose of the study was to determine the relative significance of several mapping variables used in locating terrain features with radar and DTM data. Assumed standard errors were employed which may or may not be applicable for specific cases. In the long run these specific error magnitudes should not affect the relative significance of the factors involved. In addition the relative sizes of the errors in Tables 9, 10 and 11 should not be greatly changed provided the errors generated are randomly distributed.

Considering the five main factors it was shown that the coordinate factor was significant. The actual average errors for the 21-point scenes provided the input for the ANOVA computation. Based on this computation there is a standard deviation of 121 m. which can be attributed to the variation between X, Y and Z coordinates. The standard deviation attributed to the experimental error (repeating the computation three times with different random errors) was 75 m. The weakness in consistency for coordinate computation is largely attributed to poor geometry and weak functional relationships during the computations. A comparison of Tables 3 and 4 shows an example where an azimuth of 45^{0} (Table 3) provides approximately equal standard deviations for X and Y but, where the azimuth changes to 90^{0} (Table 4), the standard deviation for Y is approximately three times that for X. The values for average error are also greater for the Y coordinate as seen in Table 9.

Although range shows similar tendencies the range standard deviation of 108 m. was not significant at the 10% level. Comparing Tables 2 and 3 it can be seen that the standard deviations for X, Y and Z coordinates are several times larger at a range of 200 km. as opposed to 50 km. In addition, Table 9 shows the overall average error to be much greater for the 200 km. range.

The standard deviation for resolution is 80 m. which was not significant when compared to the experimental error of 75 m. The effect of resolution can be seen by comparing Tables 3 and 6. The average errors and standard deviations are generally larger for the 10 m. resolution.

Terrain height variations yield a standard deviation of 75 m. which is not significant. Tables 3 and 5 show that a change from 0 to 2000 m. for average terrain height will increase the average errors and standard deviations for all coordinates.

The least significant of the main factors was azimuth whose variation accounted for a standard deviation of 60 m. Tables 3 and 4 show the error pattern is more consistent and generally smaller for an azimuth of 45^{o} as compared to the 90^{o} azimuth.

The five main factors can be combined into 10 different pairs. Table 8 indicates that the pairing of range and terrain height is significant with a standard deviation of 110 m. Some of the 9 combinations of range and terrain height covered in this study are to be avoided. Table 10 indicates that the combination of 200 km. range and 2000 m. terrain height is the main contributer to the significance of this pairing. Refer to Figure 7 for an overview of the average errors produced by the range-terrain height combinations. Each of the 9 plotted points is an average error compiled from 54 values representing 1134 rays.

When considering the combination of range and coordinate, the standard deviation due to the 9 possible combinations was 93 m. This

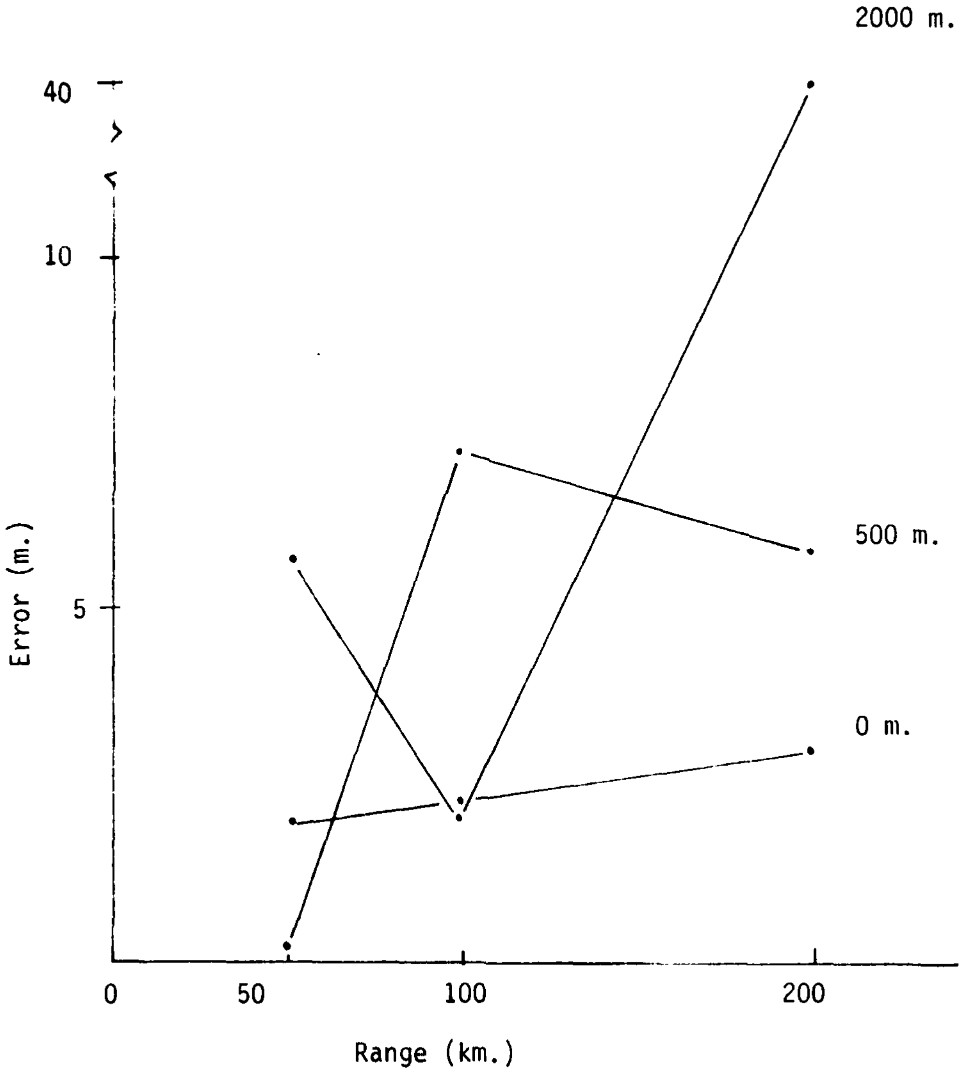


FIGURE 7 - AVERAGE ERROR VALUES FOR RANGE AND TERRAIN TERRAIN HEIGHT COMBINATIONS.

combination was not significant. Table 10 reveals that most of the variation is attributed to the Y coordinate at 200 km. Figure 8 displays the coordinate trends at the three ranges. Once again 54 values are represented by each plotted point. The trends in X and Y are fairly consistent but Z displays some randomness.

The 9 pairings of terrain height and coordinates were again not significant with a standard deviation of 91 m. Table 10 gives the results of the pairings which are plotted in Figure 9. The most consistent results for all coordinates were obtained at the 500 m. terrain height. The X coordinate was the most consistent.

The variations ascribed to azimuth and terrain height combinations were basically the same magnitude as those introduced by the experimental error. Table 10 lists the average errors which are plotted in Figure 10. Fairly consistent trends are exhibited and low terrain heights and 45° to 60° azimuths are favored.

The results for three factor combinations showed that range, terrain height and coordinates were a significant combination which produced a standard deviation of 104 m. between the 27 combinations possible. Table 11 lists the average errors which each represent 18 values. It can be seen that range values behave fairly consistently with best results at 50 km. and poorest results at 200 km. The terrain heights results are fairly random throughout the list of 27 and the coordinate listings generally favor X, Y and Z in that order. Most other three factor combinations produce less variation than the experimental error and consequently are not at all significant.

One of the four factor combinations was significant, namely;

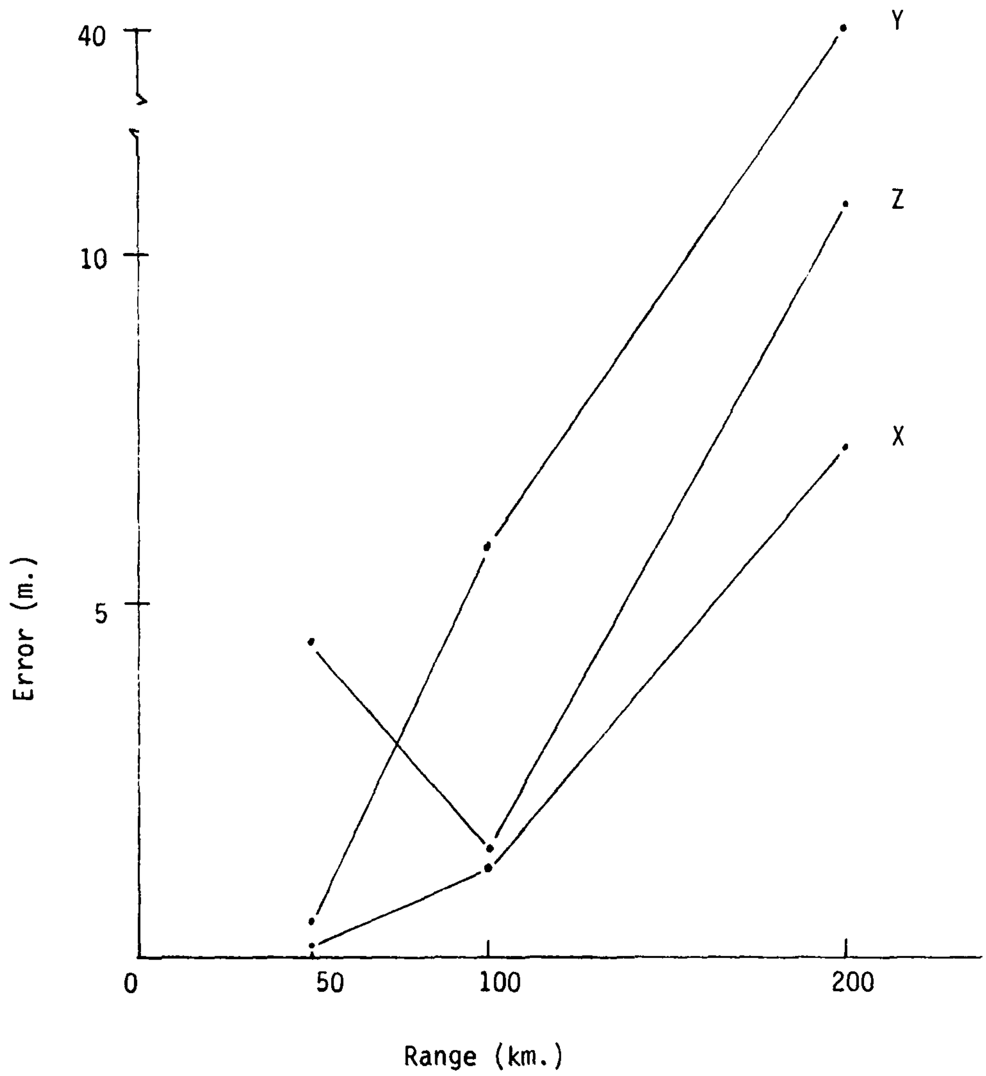


FIGURE 8 - AVERAGE ERROR VALUES FOR RANGE AND COORDINATE COMBINATIONS.

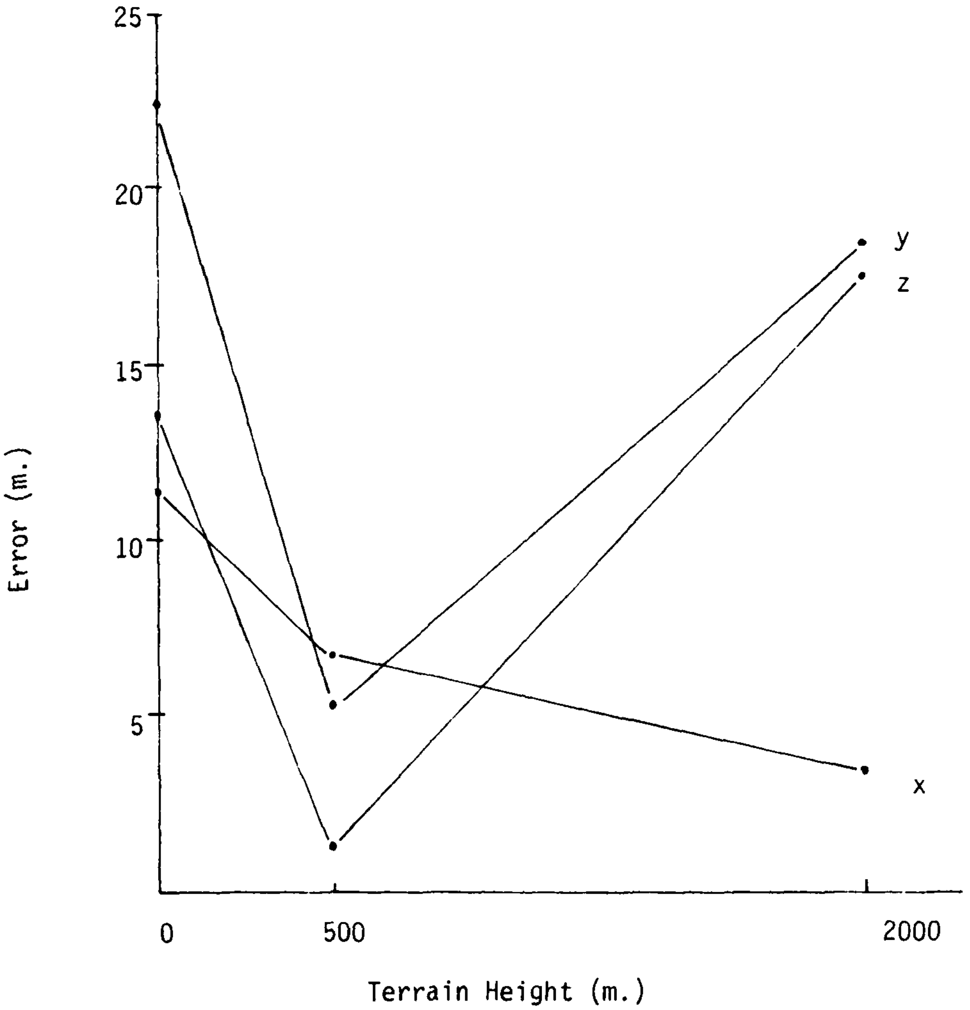


FIGURE 9 - AVERAGE ERROR VALUES FOR TERRAIN HEIGHT AND COORDINATE COMBINATIONS.

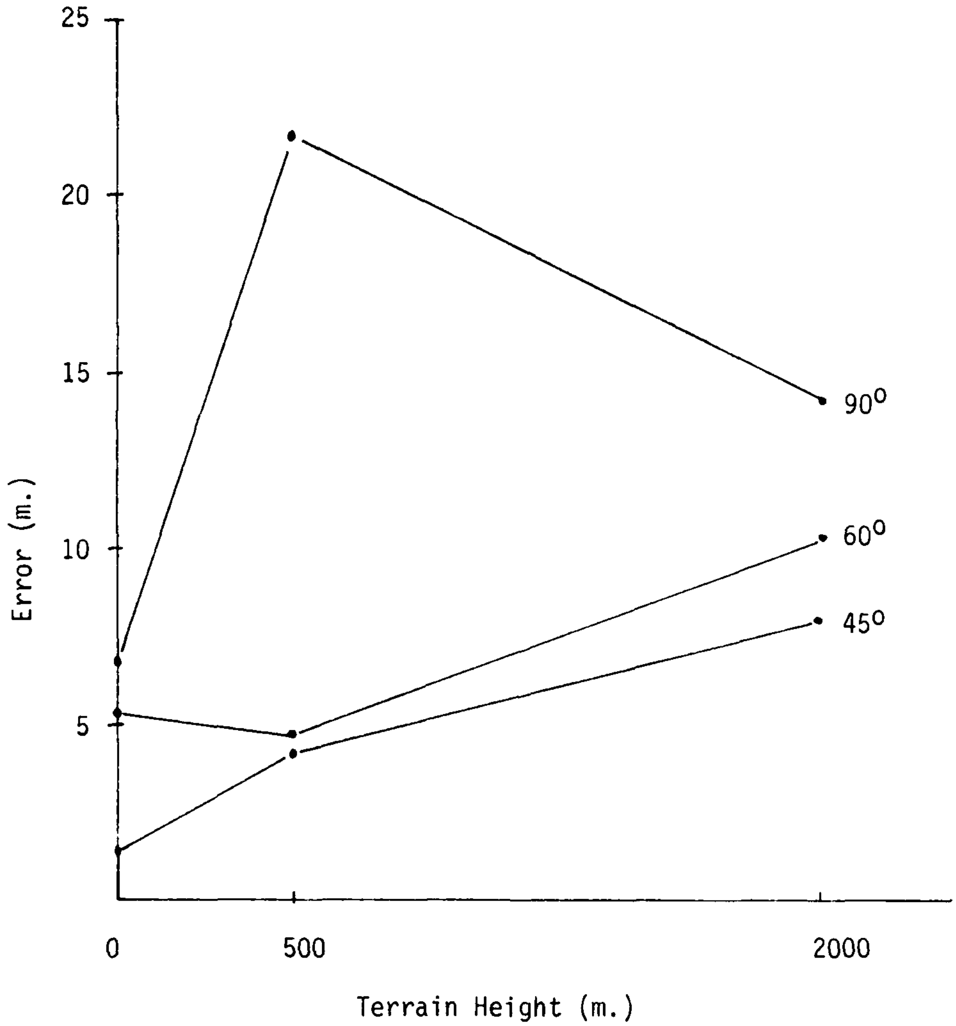


FIGURE 10 - AVERAGE ERROR VALUES FOR TERRAIN HEIGHT AND AZIMUTH.

resolution, terrain height, coordinate and azimuth (CDFA). All other four factor combinations did not approach significance. Fifty-four values are represented in the CDEA combination. Detailed results show the greatest variation is caused by 10 m. resolution, 500 m. terrain height, Z coordinate and 90^{o} azimuth; however, only nine values or 189 individual rays are represented by this result.

The particular significance of the results will depend on the application, however, specific data collection designs are necessary to fine tune the mapping system. For this particular set of data the results indicate that one should collect radar data at an azimuth of 45^{o}, a range of 100-50 km, a resolution of 5 m. and over low level terrain for the best results.

4.0 CONCLUSIONS AND RECOMMENDATION

4.1 CONCLUSIONS

The following conclusions are based on the fixed factors of this experiment.

1. Avoid collecting radar data when the axes of the radar coordinate system are parallel with axses of the DTM coordinate system. DTM coordinates along track (Y) contained large errors because of this. If unavoidable, an intermediate axes rotation may prove helpful.
2. Radar operating at maximum range generally yields less reliable measurement results.
3. A small increase in resolution greatly improves the geometric integrity of the mapping effort.

4. Increased terrain height leads to decreased accuracy and precision in position determination.
5. Variable geometry results in considerable variation in the reliability of X, Y, and Z coordinate determination. The conditions in this experiment were most unfavorable for the Y coordinate.
6. The proper design of radar flights and data reduction is essential if the data is to be used to favorably compliment mapping efforts.

4.2 RECOMMENDATION

The experiment identified important parameter variations and dependencies. More extensive analysis of data collection parameters would prove productive; particularly when sensors are combined for a common cause. Computer simulations are relatively inexpensive yet often pin point the not so obvious. It is recognized that simulations are far removed from real world data collection, analysis and mapping; however, experimental data is available for establishing variance on measurement systems and this data can be used to fortify simulation results.

The combination of simulation and factorial analysis has been applied successfully, and it is recommended that this data reduction combination be used more frequently in the preliminary design of remote sensing systems.

5.0 REFERENCES

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APPENDIX A

BASIC PROJECTION EQUATIONS FOR REAL-APERTURE RADAR

The purpose of this appendix is to explicitly set down the basic projection equations for real aperture radar as used in this error propagation study. Although these equations have been published in several forms (Leberl, 1972 and Leberl, 1976 particularly) their availability is still limited and it is felt that a development of the versions used in this report would be helpful.

Real-aperture radar imagery can be expressed in terms of a general projection equation as given in A-1, A-2, and A-3 below. (See Figure A-1).

$$X_P = X_R + A^T \left[B^T \cdot x_P + \Delta\bar{X} \right] \qquad \text{(A-1)}$$

[Point Coordinates in DTM System] = [Radar Location in DTM System] + [Exterior Orientation of Radar Mount] [[Rotates Sensor System into Sensors Mounting System] [x_P - Vector of model point sensors coord. system] + [Vector between origin of sensor and origin of mount.]] (A-2)

$$\begin{bmatrix} X_P \\ Y_P \\ Z_P \end{bmatrix} = \begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} + A^T \left[B^T \begin{bmatrix} x_P \\ y_P \\ z_P \end{bmatrix} + \begin{bmatrix} \Delta\bar{X} \\ \Delta\bar{Y} \\ \Delta\bar{Z} \end{bmatrix} \right] \qquad \text{(A-3)}$$

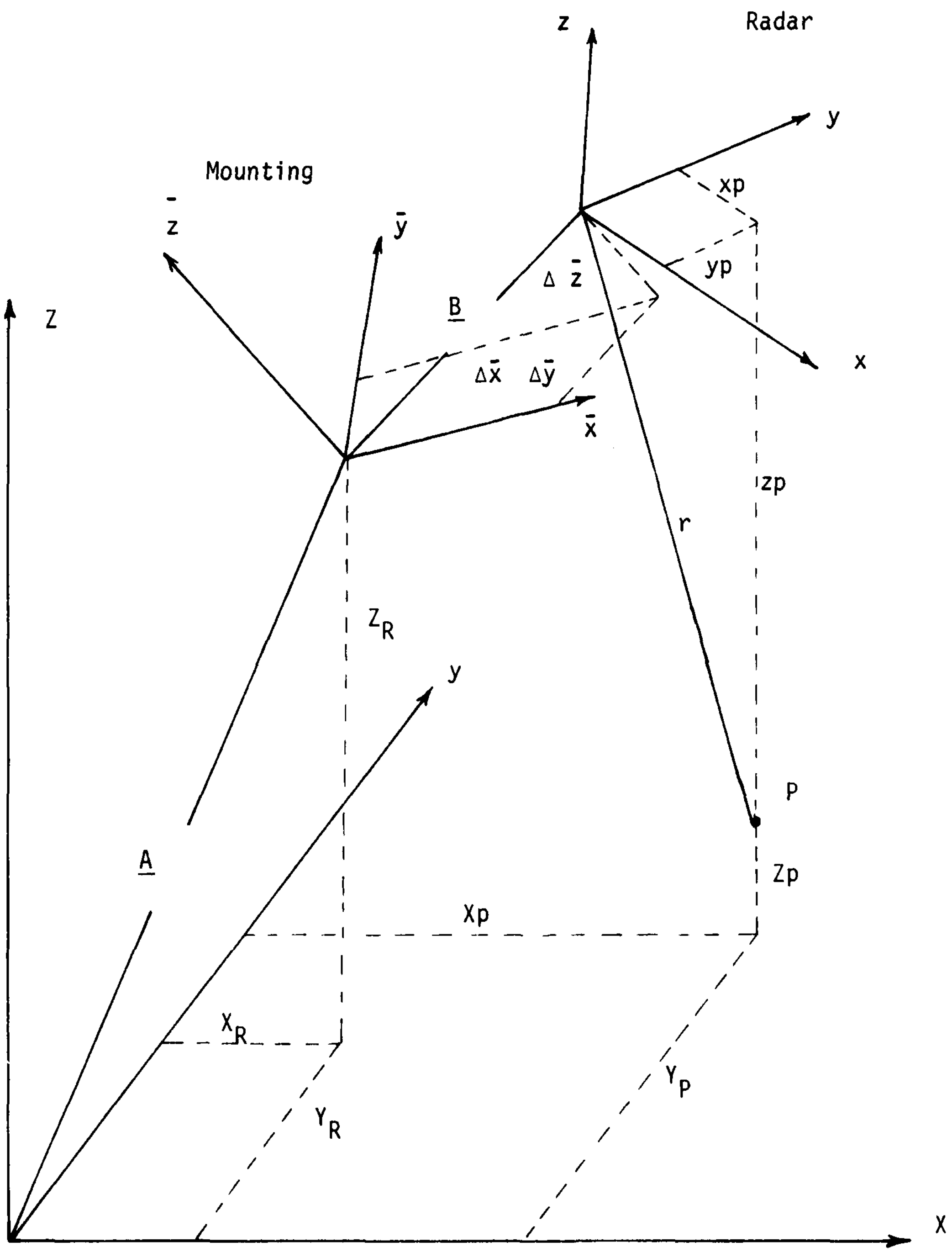

FIGURE A-1 COORDINATE SYSTEM FOR RADAR PROJECTION EQUATIONS

A simplification of equation (A-3) will now be made by assuming that the sensor is parallel and coincident with the mount. In this case B becomes a unit matrix and $\Delta\bar{X}$ is zero.

Thus:

$$\begin{bmatrix} X_P \\ Y_P \\ Z_P \end{bmatrix} = \begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} + \begin{bmatrix} A^T\ (k,\ \phi,\ w) \end{bmatrix} \begin{bmatrix} x_P \\ y_P \\ z_P \end{bmatrix} \qquad \text{(A-4)}$$

where:

$x_p = (\sin \tau)\ r$

$y_p = (\sin^2 \Omega - \sin^2 \tau)^{\frac{1}{2}}\ r$

$z_p = (-\cos \Omega)\ r$

r = Range from radar to DTM point P.

τ = Squint angle

Ω = Elevation angle

The A matrix utilized in this study in (A-4) is defined below:

$$A = \begin{bmatrix} \cos\phi \cos k & \cos w \sin k + \sin w \sin\phi \cos k & \sin w \sin k - \cos w \sin\phi \cos k \\ -\cos\phi \sin k & \cos w \cos k - \sin w \sin\phi \sin k & \sin w \cos k + \cos w \sin\phi \sin k \\ \sin\phi & -\sin w \cos\phi & \cos\phi \cos w \end{bmatrix} \qquad \text{(A-5)}$$

where:

+w = clockwise rotation about x axis - primary rotation

$+\phi$ = " " " y axis - secondary rotation

+k = " " " z axis - tertiary rotation

(Figure A-2)

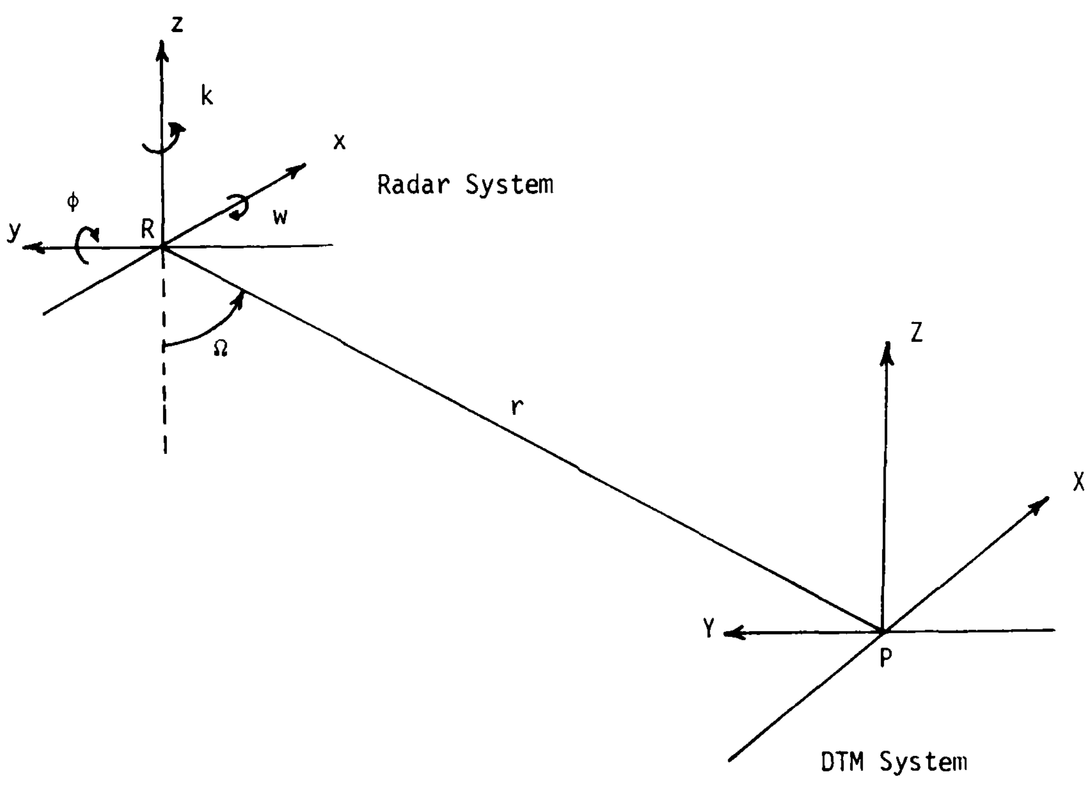

FIGURE A-2 - EXTERIOR ORIENTATION OF RADAR

In order to provide a complete sample computation such as that carried out in the study it will be necessary to refer to a sketch of the ficticious locations of radar and ground position. (See Figure A-2).

R represents the radar location in an aircraft. The coordinates of the radar in the DTM system are:

X_R = 0 m.; Y_R = + 99,254 m.; Z_R = + 12,192 m.

P represents the target or point of interest in the DTM system. The coordinates of P are:

X_P = 0 m.; Y_P = 0 m.; Z_P = 0 m.

Utilizing these coordinates range is computed to be 100,000 m.

The exterior orientation of the radar is:

K = 0 degrees; ϕ = 0 degrees; w = 0 degrees

(A-4) can be rearranged as (A-6) and utilized to compute τ (squint angle) and Ω (the elevation angle).

$$\begin{bmatrix} x_P \\ y_P \\ z_P \end{bmatrix} = \begin{bmatrix} (\sin \tau)\, r \\ (\sin^2 \Omega - \sin^2 \tau)^{\frac{1}{2}}\, r \\ (-\cos \Omega)\, r \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} X_P - X_R \\ Y_P - Y_R \\ Z_P - Z_R \end{bmatrix} \qquad \text{(A-6)}$$

$$\tau = \sin^{-1} \left[\frac{a_{11}(X_P - X_R) + a_{12}(Y_P - Y_R) + a_{13}(Z_P - Z_R)}{r} \right] \qquad \text{(A-7)}$$

$$\Omega = \cos^{-1} \left[\frac{a_{31}(X_P - X_R) + a_{32}(Y_P - Y_R) + a_{33}(Z_P - Z_R)}{-r} \right] \qquad \text{(A-8)}$$

Substituting the values for our example yields:

$$\begin{bmatrix} (\sin\tau)\ 100{,}000 \\ (\sin^2\Omega - \sin^2\tau)^{\frac{1}{2}}\ 100{,}000 \\ (-\cos\Omega)\ 100{,}000 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & - & 0 \\ 0 & - & 99{,}254 \\ 0 & - & 12{,}192 \end{bmatrix} = \begin{bmatrix} 0 \\ -99{,}254 \\ -12{,}192 \end{bmatrix} = \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix}$$

Solving for τ and Ω:

$$\tau = \sin^{-1}\left[\frac{0}{100{,}000}\right] = 0 \text{ degrees}$$

$$\Omega = \cos^{-1}\left[\frac{-12{,}192}{-100{,}000}\right] = 83 \text{ degrees}$$

This completes the computation of a single ficticious ray from radar to ground point. Each ray was then perturbed with random errors. Errors were introduced for radar position (X_R, Y_R, Z_R), radar orientation (k, ϕ, w), and ray length and direction (r, τ, Ω).

For example assume the following random errors occurred in the above 9 parameters:

X_R = + 9 m.	k = +0.1 deg.	r = + 5 m.
Y_R = - 3 m.	ϕ = -0.1 deg.	τ = + 0.003 deg.
Z_R = +20 m.	w = -0.1 deg.	Ω = - 0.003 deg.

These error magnitudes were considered reasonable values from previous studies. The adjusted parameter values are now:

X_R = 9 m. | k = + 0.1 deg. | r = 100,005 m.

Y_R = 99,251 m. | ϕ = - 0.1 deg. | τ = +0.003 deg.

Z_R = 12,212 m. | w = - 0.1 deg. | Ω = 82.997 deg.

Utilizing equation (A-4) the coordinates of P now are:

X_P = - 137.72 m.; Y_P = -28.91 m.; Z_P = 192.85 m.

Recall the correct value for each coordinate was zero.

The total experiment entailed the computation and perturbation of ficticious rays or 162 scenes each having 21 rays or points of interest.

These coordinate errors were the bases of the input for the analysis of the variance computation.

APPENDIX B

ANALYSIS OF VARIANCE MODEL

The factors studied in this experiment were azimuth, range, resolution, terrain height, and coordinates. The purpose of the analysis of variance procedure was to aid in selecting the best combinations of the above variables when using radar to locate points of interest on the terrain. Assuming that all factors in the error propagations are constant except those above, there is a need to analyze the propagated errors in such a way that one can determine not only how each individual factor affects the results but in addition, how the interactions of all factors affect the results. A factorial experiment was used to accomplish this. The levels of each factor were selected to introduce ranges in the variables which were of particular interest in this study. This produced a fixed-effects model which is shown schematically in Figure B-1.

The mathematical model which represents the factorial experiment is:

$$Y_{ijklmn} = \mu + \alpha_i^{A} + \alpha_j^{B} + \alpha_k^{C} + \alpha_l^{D} + \alpha_m^{E} + \alpha_{ij}^{AB} + \alpha_{ik}^{AC}$$

$$+ \alpha_{il}^{AD} + \alpha_{jk}^{BC} + \alpha_{jl}^{BD} + \alpha_{kl}^{CD} + \alpha_{im}^{AE} + \alpha_{lm}^{DE}$$

$$+ \alpha_{jm}^{BE} + \alpha_{km}^{CE} + \alpha_{ijk}^{ABC} + \alpha_{ijl}^{ABD} + \alpha_{jkl}^{BCD}$$

$$+ \alpha_{ikl}^{ACD} + \alpha_{klm}^{CDE} + \alpha_{lmi}^{DEA} + \alpha_{ijm}^{ABC} + \alpha_{jlm}^{BDE}$$

$$+ \alpha_{ikm}^{ACE} + \alpha_{jkm}^{BCE} + \alpha_{ijkl}^{ABCD} + \alpha_{jklm}^{BCDE}$$

$$+ \alpha_{klmi}^{CDEA} + \alpha_{ijlm}^{ABDE} + \alpha_{ijkm}^{ABCE} + \alpha_{ijklm}^{ABCDE}$$

$$+ \delta_{ijklmn}$$

where: $Y_{ijklmno}$ = propagated error

A	= azimuth-fixed	D	= terrain height-fixed
B	= range-fixed	E	= coordinate-fixed
C	= resolution-fixed	δ	= experimental error

α_{ij}^{AB} is the interaction effect between the ith azimuth and the jth range, etc. The number of levels for each of the fixed factors are:

I = 3, J = 3, K = 2, L = 3, and M = 3.

It is assumed that the linear additive model as written above is applicable for this study. The α's are fixed effects and $\Sigma\alpha_i^A$ = $\Sigma\alpha_j^B$ = ... = $\Sigma\alpha_{ijklm}^{ABCDE}$ = 0 is assumed true. The $\delta_{ijklmno}$'s are the experimental unit and they are assumed to be normally and independently distributed with zero mean and variance σ_s^2.

For each item in the model equation, the following type of hypothesis is tested.

$$H_0 : \hat{\alpha}_1 = \hat{\alpha}_2 = \hat{\alpha}_3 = 0$$

$$H_1 : \text{at least one } \hat{\alpha}_i \neq 0$$

FIGURE B-1 ANALYSIS OF VARIANCE TABLE FOR MAIN EXPERIMENT
EACH CELL CONTAINS AVERAGE ERROR FOR 21 TERRAIN POINTS.

AZIMUTH Ai	RANGE Bi	RES. Ci	TERRAIN HEIGHT Di 1									2									3								
			COORDINATE Ei X			Y			Z			COORDINATE X			Y			Z			COORDINATE X			Y			Z		
			REPETITION 1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1	1	1																											
1	1	2																											
1	2	1																											
1	2	2																											
1	3	1																											
1	3	2																											
2	1	1																											
2	1	2																											
2	2	1																											
2	2	2																											
2	3	1																											
2	3	2																											
3	1	1																											
3	1	2																											
3	2	1																											
3	2	2																											
3	3	1																											
3	3	2																											

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