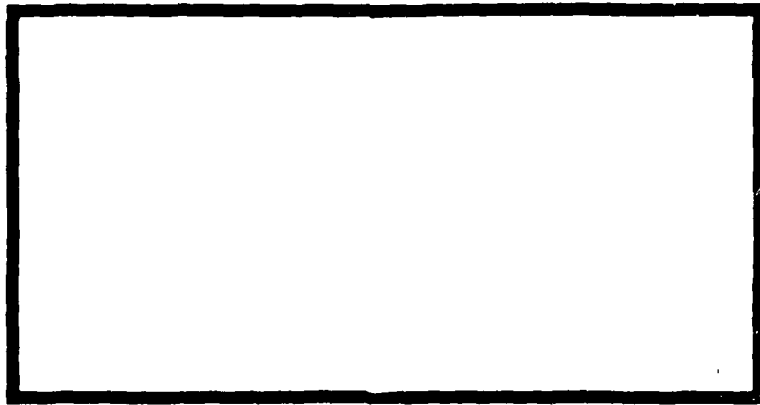


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Ⓢ A HIGHER-ORDER TRAPEZOIDAL VECTOR
VORTEX PANEL FOR SUBSONIC FLOW

Thesis

Ⓢ AFIT/GAE/AA/80D-14 Ronald E. Luther
Capt USAF

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AFIT/GAE/AA/80D-14

A HIGHER-ORDER TRAPEZOIDAL VECTOR
VORTEX PANEL FOR SUBSONIC FLOW

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

Ronald E. Luther, B.S.
Capt USAF

Graduate Aeronautical Engineering

December 1980

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Preface

I wish to thank my advisor, Major Stephen Koob, for his constant aid and guidance and also tolerance to my efforts. To my family, Freda and Amanda, I can only hope that in the future some reward for their tremendous sacrifice will be granted.

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List of Symbols

α	angle of attack
$\delta(x,y), \gamma(x,y)$	components of vorticity vector
δ^i, γ^i	nodal values of vortex vector
Δ	difference symbol
$\bar{\omega}$	vorticity vector
ϕ_j	set of planform nodal values of vorticity
$[A_{ij}]$	coefficient array
A, B, C, D, E, F	coefficients for δ distribution
G, H, I, J, K, L	coefficients for γ distribution
u, v, w,	perturbation velocities in x, y and z directions
\hat{i}, \hat{j}	unit vectors in x and y direction
$T_{L,T}^k$	integral of specific region
R	region of integration
U_∞	free stream velocity
M_∞	free stream Mach number
c	chordlength
C_p	pressure coefficient
C_L	local lift coefficient
C_m	local moment coefficient
k	ratio of specific heats
M	number of chordwise panels
N	number of spanwise panels
{ }	vector
[]	matrix

Subscripts and Superscripts

L,T Leading or trailing triangle

i Node number

Abstract

A higher-order trapezoidal vector vortex panel method is developed for application to linearized subsonic potential flow. Each panel is subdivided into two triangular subregions on which a quadratic vorticity strength distribution is prescribed for both the spanwise and chordwise components of the vorticity vector. The vorticity strength distribution is expressed as a function of the components of the vorticity vector at selected nodes on the boundary of each triangular subregion. Nodal values on the shared boundary of the subregions are made equal, assuring continuity of the vorticity distribution function throughout the trapezoidal panel. A lifting surface of no thickness is modeled with a network of the trapezoidal panels. Again, nodal values on the common panel boundaries are matched to achieve complete continuity of the vorticity distribution throughout the lifting surface. Aerodynamic data for several wing planforms is obtained with the flow model. Results from this method are compared to those from other computational and theoretical methods.

A HIGHER ORDER TRAPEZOIDAL VECTOR
VORTEX PANEL FOR SUBSONIC FLOW

I. Introduction

Background

The concept of modeling the flow over a lifting surface by replacing that surface with a distribution of vorticity began in the early part of this century and continues to be expanded and explored. The first quantitative results were achieved by Prandtl in 1919 with his lifting line theory (Ref 9: 112-123) in which the entire surface was represented by a single bound vortex and two infinitely long, free vortices. Despite the many simplifying hypotheses involved in Prandtl's theory, results obtained with it are sufficiently accurate for many purposes and the theory provided the foundation for many subsequent analytical analyses of the lifting surface problem. In 1925 Blenk (Ref 1) extended Prandtl's idea by representing the surface with not one but a distribution of bound vortices over the surface. Blenk's method was an improvement over the single lifting line approach, but still had limitations and the computations involved were very lengthy considering the absence of electronic computers at that time.

A more recent development has been the vortex-lattice theory (Ref 6). This method covers the surface with a grid

of horseshoe vortices and has produced very useful results. Yet another approach, and the one pursued in this report, is to subdivide the surface into a network of panels, each with a discrete vorticity distribution. Such an approach is termed a vortex paneling method.

An accurate flow model via the vortex panel technique is achieved as follows. The vorticity distribution is found such that, at as many points as possible on the surface, the normal component of velocity is zero. This is the so-called kinematic flow condition (Ref 9: 126). With the surface vorticity distribution known, the perturbation velocity at any point on the surface is easily determined. The velocity field and Bernoulli's theorem are then used to compute the pressure distribution and subsequently the aerodynamic lift and moment coefficients.

Ideally, the vorticity vector distribution used to model the lifting surface should be one that resembles the observed physical distribution of vorticity on a finite wing. That is, the vorticity vectors at the wing root lie predominantly in the spanwise direction while those in the region of the wing tip lie predominantly in the chordwise direction. Thus the chosen vorticity distribution function must permit the vorticity to vary in direction. Such a scheme has been proposed by Sparks (Ref 10). His results were flawed, however, until a computer program logic fault was detected by this author. Subsequent results have been encouraging. Sparks' vorticity distribution has two undesirable features. First,

continuity in vorticity distribution is not enforced throughout the surface (Ref 10: 9). This lack of continuity across panel boundaries permits the vorticity distribution at the boundaries to violate Helmholtz's second theorem regarding the continuity of vorticity (Ref 8: 168). Second, the vorticity distribution is linear. It has been shown (Ref 4) that increasing the number of terms in the polynomial expression representing the vorticity distribution has several advantages. The higher order representation reduces surface velocity errors and gives significantly improved accuracy as the number of panels used to model the surface is increased.

The present report develops a vortex panel having a quadratic vorticity distribution. The panel is derived specifically to provide a continuous vorticity distribution over the surface while identically satisfying the second Helmholtz condition at every point on the surface.

Approach

The basic problem is developed in Section II. First, the quadratic vorticity distribution function and the importance of the properties of the function are discussed. The panel geometry is presented and unknown values of the components of the vorticity vector are assigned to specified nodes on the panel boundary. This permits the vorticity distribution function to be expressed in terms of the unknown nodal values and the panel geometry. The method of joining panels to form a network to model a wing planform is then

described. This global network has a certain number of nodes and, consequently, a set number of global unknown nodal values to be determined.

Section III describes the solution process. The first step is the application of specific boundary conditions to the leading, tip, trailing and root edges of the planform. This process reduces the total number of unknown nodal values which must be explicitly solved for. The solution is obtained by generating an equal number of linear equations to be solved simultaneously.

Two conditions are satisfied on each panel to generate the required equations. First, the kinematic flow condition is enforced at two control points on each panel. To accomplish this, the normal velocity at each control point must be found. As the normal velocity at any point is affected by the vorticity distribution on the entire planform, each panel's individual contribution to the velocity at any point must be determined. The Biot-Savart Law is applied to the vorticity distribution function of each panel to calculate the induced velocity caused by that panel's vorticity on any desired point in the plane of the panel. The velocities induced by each panel on the desired control point are then summed. This summation yields one equation representing the total induced velocity at one control point. The second condition to be satisfied involves intra-panel continuity of the vorticity distribution. As will be explained in Section II, inter-panel continuity is achieved by the commonality of

nodal values at shared panel boundaries. Such commonality will be shown to not exist, however, on the intra-panel boundary, resulting in a discontinuity along that boundary. This problem is resolved by forcing commonality of sufficient nodal values along the boundary to ensure continuity. Satisfaction of the kinematic flow condition and intra-panel continuity condition results in a system of linear equations. The solution of these gives the nodal values for the vorticity distribution.

Section IV describes how the nodal values are used to calculate the components of the vorticity vector at any point on the planform. From the fully described vorticity distribution, the velocity distribution on the surface of the planform is readily determined and, subsequently, the pressure distribution and aerodynamic coefficients can be computed.

The computer code used to evaluate the theory is outlined in Section V. Section VI presents the results achieved. Comparisons are made to other theories for rectangular wings.

Section VII draws conclusions from the results of this method and offers recommendations for further improvement.

II. Basic Problem Development

Vorticity Distribution Function

The vorticity distribution is a vector function in the x-y plane:

$$\bar{\omega}(x,y) = \delta(x,y)\hat{i} + \gamma(x,y)\hat{j} \quad (2.1)$$

Each component of the vorticity vector is allowed to vary quadratically throughout the plane, thus expressions for the components are:

$$\delta(x,y) = A + Bx + Cy + Dxy + Ey^2 + Fx^2 \quad (2.2)$$

$$\gamma(x,y) = G + Hx + Iy + Jxy + Ky^2 + Lx^2 \quad (2.3)$$

It has been shown (Ref 8: 168) that vorticity is solenoidal and so:

$$\nabla \cdot \bar{\omega} = 0 \quad (2.4)$$

This statement of continuity is known as Helmholtz's second theorem (Ref 8: 168) and also as the condition of source-free vortex distribution (Ref 9: 124).

Equation (2.4) can be satisfied by replacing Eq (2.2) by

$$\delta(x,y) = A - Ix + Cy - 2Kxy + Ey^2 - \frac{1}{2}Jx^2 \quad (2.5)$$

Basic Panel Geometry

Figure 1 shows the geometry of the basic trapezoidal panel. This panel is subdivided into a leading triangle and a trailing triangle (hereafter denoted by the subscripts L and T respectively). The basic panel has nine nodes labeled as shown. Two features of the nodal coordinates should be noted. The x-coordinates of nodes 5-9 can be expressed in terms of the x-coordinates of nodes 1-4 (e.g. $x_5 = \frac{1}{2}(x_1+x_2)$). Only two y-coordinates need be specified in advance since $y_3 = \frac{1}{2}(y_1+y_2)$.

Interpolation Functions

It will be shown later that it is advantageous to represent the expressions for δ and γ as functions of nodal values and nodal coordinates. To accomplish this, each triangular subregion of the panel is treated separately. Figure 2 shows the panel split into its subregions with each subregion assigned nine nodal values. Since Eqs (2.5) and (2.3) involve nine coefficients, only nine nodal values are required to uniquely define δ and γ within the subregion. Substituting the nine nodal values of the leading triangle into Eqs (2.3) and (2.5) gives

$$\{\Gamma_L^j\} = [f_i(x_j, y_j)] \{F_L^i\} \quad (2.6)$$

where

$$\{\Gamma_L^j\} = [\delta_L^1 \ \delta_L^3 \ \delta_L^6 \ \delta_L^7 \ \gamma_L^1 \ \gamma_L^3 \ \gamma_L^4 \ \gamma_L^6 \ \gamma_L^7]^T \quad (2.7)$$

$$\{F_L^i\} = [A_L \ C_L \ E_L \ G_L \ H_L \ I_L \ J_L \ K_L \ L_L]^T \quad (2.8)$$

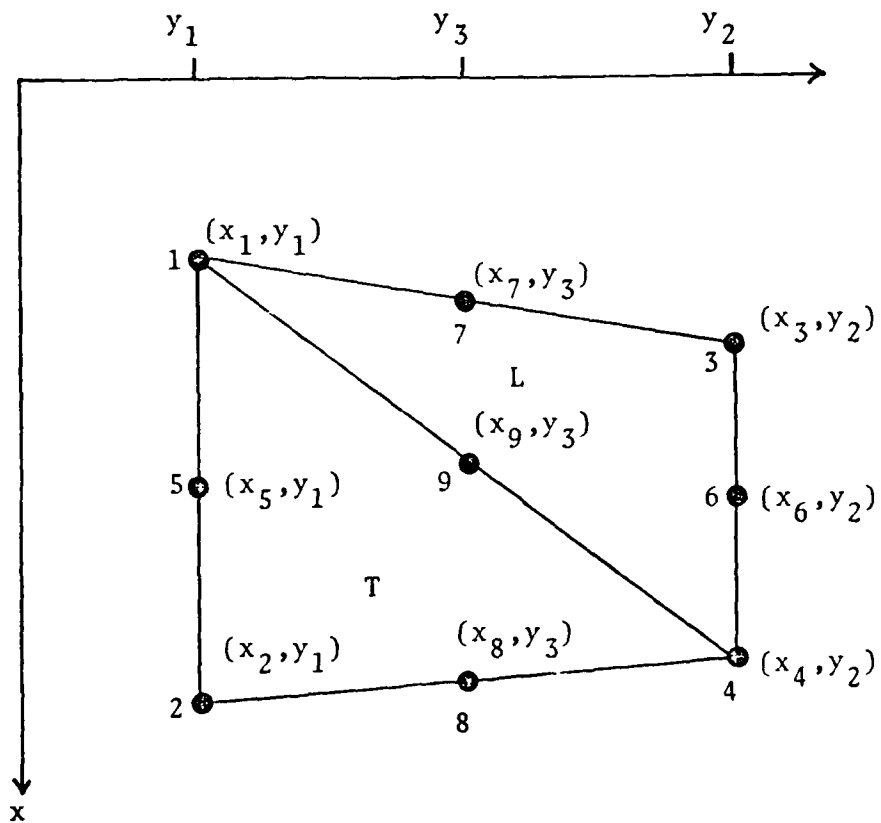


Fig 1. Basic Panel Geometry

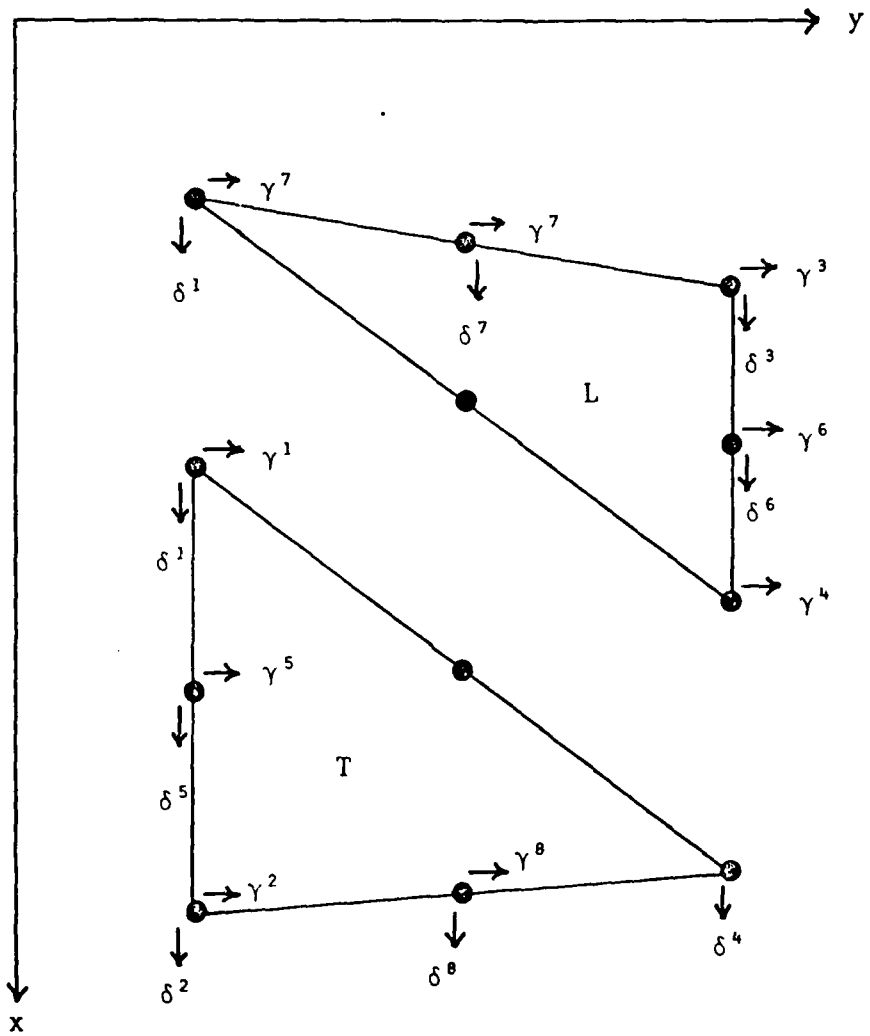


Fig 2. Panel Subregions With Assigned Nodal Values

Similarly for the trailing triangle:

$$\{\Gamma_T^j\} = [g_i(x_j, y_j)] \{F_T^i\} \quad (2.9)$$

where

$$\{\Gamma_T^j\} = [\delta_T^1 \ \delta_T^2 \ \delta_T^4 \ \delta_T^5 \ \delta_T^8 \ \gamma_T^1 \ \gamma_T^2 \ \gamma_T^5 \ \gamma_T^8]^T \quad (2.10)$$

$$F_T^i = [A_T \ C_T \ E_T \ G_T \ H_T \ I_T \ J_T \ K_T \ L_T]^T \quad (2.11)$$

Eqs (2.6) and (2.9) are solved for in the F_L^i and F_T^i , respectively. The method used for obtaining the solutions is explained in Appendix A.

The vorticity vector components can now be written in the following form:

$$\delta_{L,T}(x,y) = [f_{L,T}^i(x,y; x_j, y_j)] \{\Gamma_{L,T}^j\} \quad (2.12)$$

$$\gamma_{L,T}(x,y) = [g_{L,T}^i(x,y; x_j, y_j)] \{\Gamma_{L,T}^j\} \quad (2.13)$$

In Eqs (2.12) and (2.13) the functions $f_{L,T}^i$ and $g_{L,T}^i$ are the interpolating functions for the components $\delta_{L,T}$ and $\gamma_{L,T}$.

Network Assembly

Figure 3 shows how the panels are connected to model the semi-span of a wing planform. While the planform shown has straight leading and trailing edges, the method also permits analysis of cranked leading and/or trailing edges. The root edge, tip edge and all chordwise boundaries are parallel to the x-axis.

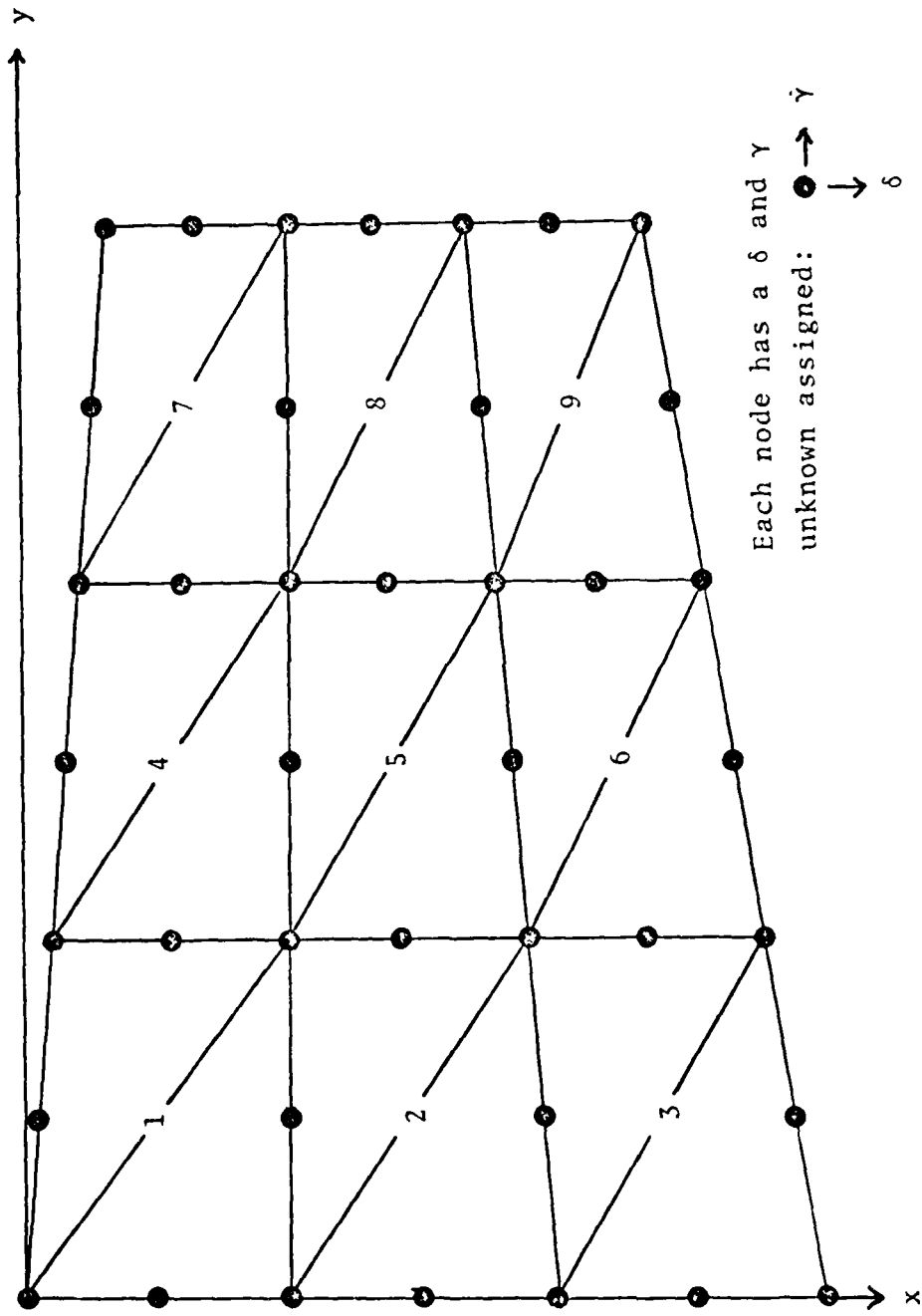


Fig 3. Typical Panel Network with Nodal Unknowns

Inter-Panel Continuity

Continuity of the vorticity function is assured across inter-panel boundaries for the following reason. The line forming a common boundary between two panels is described by

$$y = mx + b \quad (2.14)$$

Let A and B be the two panels whose common boundary is described by Eq (2.14) and examine the value of γ along the boundary for each panel. The value of γ for each panel along the boundary is given by substituting Eq (2.14) into Eq (2.13):

$$\gamma_A|_{\text{Boundary}} = C_1 + C_2x + C_3x^2 \quad (2.15)$$

$$\gamma_B|_{\text{Boundary}} = \bar{C}_1 + \bar{C}_2x + \bar{C}_3x^2 \quad (2.16)$$

The three constants in either Eq (2.15) or Eq (2.16) are determined if three values of γ are specified on the boundary. If the same three values and locations are specified for both Eqs (2.15) and (2.16), then the constants must be identical and γ is continuous along the boundary. The argument can be repeated for the δ function showing it too to be continuous along the boundary.

Intra-Panel Continuity

The same conditions that assure continuity between panels are applied to the shared boundary of the two triangular subregions of the basic panel. Figure 2 shows that continuity is not assured along this boundary since the only nodal values common to both subregions are the δ and γ

components at node 1. Two more common values for both components are required to establish continuity. This can be achieved by enforcing the following conditions. The δ component value at node 4 of the leading triangle is made equal to δ^4 . The γ component value at node 4 of the trailing triangle is made equal to γ^4 . Finally, the δ and γ component values at node 9 of the leading triangle are made equal to the δ and γ component values at the same node of the trailing triangle. The enforcement of the above conditions assures three common values and locations of both vorticity vector components along the boundary and guarantees intra-panel continuity. The method of incorporating these conditions into the solution process is explained in Section III.

III. Solution Process

Figure 3 shows the problem at hand. A quadratic vorticity vector distribution has been prescribed on the surface of a wing planform of no thickness. The vorticity distribution has been discretized by representing the wing as a network of vortex panels without sacrificing continuity of vorticity anywhere on the wing. By expressing the vorticity distribution in terms of interpolating functions and nodal values, the value of the vorticity vector is determined anywhere on the wing once the nodal values are obtained. This section details the method of solving for the nodal values.

Planform Edge Boundary Conditions

As stated in the introduction, the vorticity distribution has certain observed physical characteristics. These characteristics can be assigned to the model being developed and will serve to simplify the solution process by reducing the number of nodal values that must be determined in order to completely specify the wing's vorticity distribution.

The method used by Cohen (Ref 2) to develop vortex patterns on elliptic wings both with and without sweep is the basis for the boundary conditions that will be imposed here. These boundary conditions are not unique to Cohen's work, with Kuchemann (Ref 5: 140) suggesting similar vortex patterns.

Leading, Tip and Trailing Edges

Figures 4A and 4B illustrate Cohen's straightforward method for deriving the vortex pattern of a tapered wing in straight flight. Figure 4A shows an arbitrary distribution of lift assumed for the wing. Cohen shows the relationship that exists between the pressure distribution and the vortex pattern which yields Fig 4B. The contour lines in Fig 4B correspond to the vorticity pattern of a continuous vortex sheet. Based on these results, the boundary conditions for the leading, tip and trailing edges are formulated.

On the leading edge the vorticity vector is tangent to the leading edge. The slope of the leading edge is given by $\Delta y/\Delta x$, so the vorticity tangency requirement necessitates the δ and γ components at any point on the leading edge be related by the following

$$\frac{\gamma}{\delta} = \frac{\Delta y}{\Delta x}$$

or alternatively

$$\delta = \left(\frac{\Delta x}{\Delta y}\right)\gamma \quad (3.1)$$

In the solution process, therefore, the δ nodal values on the leading edge can be expressed as functions of the γ components and, consequently, need not be solved for simultaneously.

At the trailing edge, the Kutta condition that no pressure difference exists at the trailing edge requires that the γ nodal values on the trailing edge are identically zero.

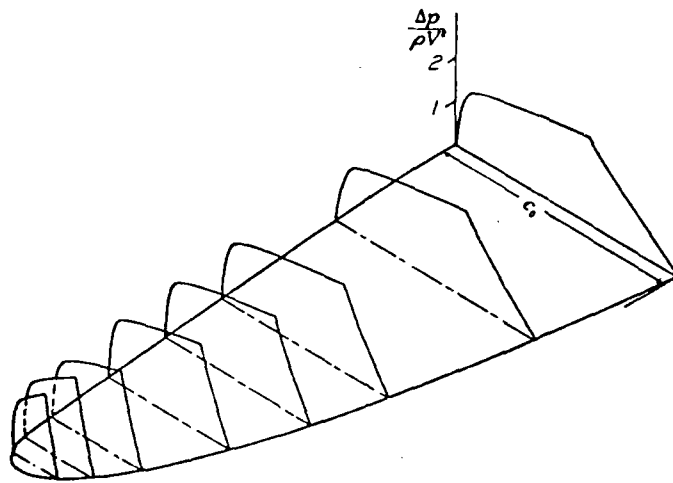


Fig 4A. Lift Distribution (Ref 2)

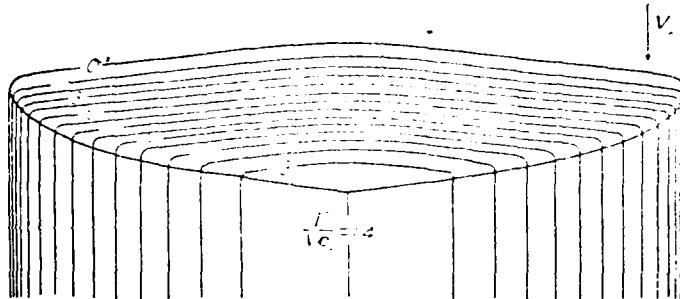


Fig 4B. Vortex Pattern - Straight Wing (Ref 2)

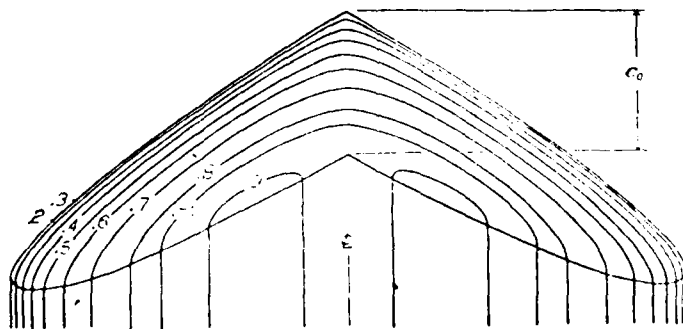


Fig 4C. Vortex Pattern - Swept Wing (Ref 2)

This is because the lift at any point on the surface is proportional to the cross-stream, or γ , component of the vorticity (Ref 2: 544), so requiring no load implies that γ be zero. Similarly, since the pressure differences between the upper and lower wing surfaces decrease to zero toward the wing tips, the γ component of vorticity must also decrease to zero at the tip. This results in the γ nodal values being zero along the tip edge.

Root Edge

Treatment of the root edge is not as straightforward as the leading, tip, and trailing edges. This is especially true for swept wings with pointed apices. Figure 4C shows Cohen's results for the vortex pattern on a sweptback wing. The vortex lines cross the root chord without a discontinuity in slope. It would, therefore, indicate that a boundary condition requiring the δ component of the vorticity vector to be zero along the root chord would be desirable. A conflict develops near the apex, however, where the leading edge boundary condition required that the δ component be non-zero to assure tangency at the leading edge. This dilemma is dealt with as follows. Figure 5A shows the leading edge, root chord, panel or a network modeling a swept wing (node 1 is the apex of the wing). The boundary condition of leading edge tangency is enforced at the apex. At node 2, however, the δ component is made equal to zero, permitting the vorticity vector to cross the root chord without discontinuity in slope. At

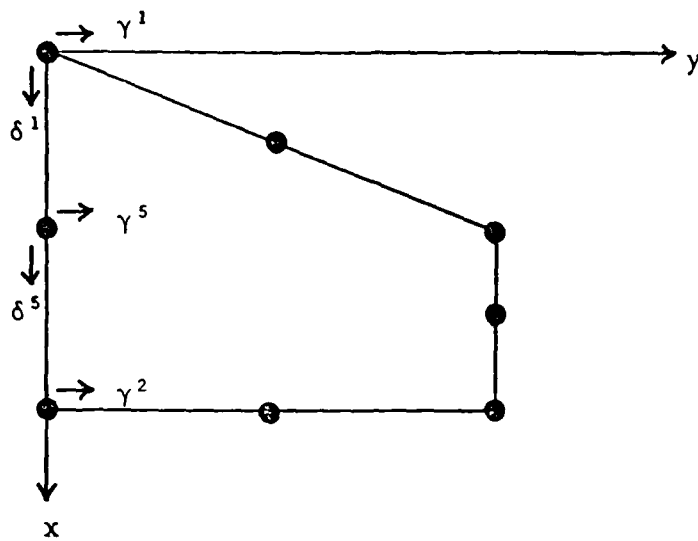


Fig 5A. Leading Edge Root Chord
Panel of Swept Wing

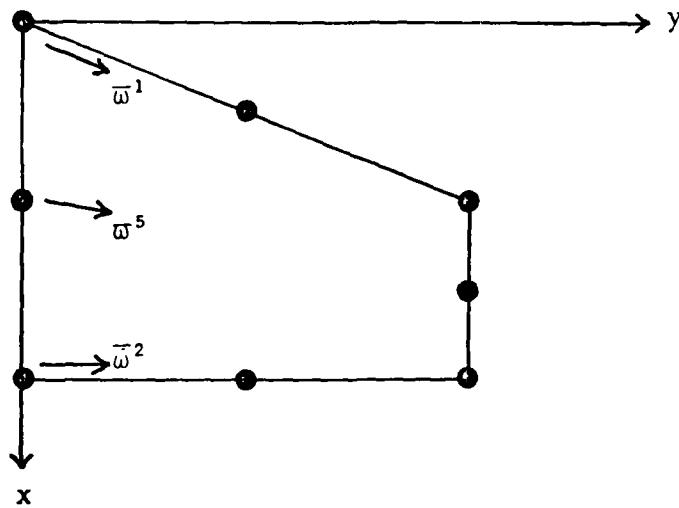


Fig 5A. Boundary Conditions Applied to
Leading Edge Root Chord Panel of Swept Wing

node 5, the δ component is permitted to be finite, but is restricted such that the slope, γ/δ , of the vorticity vector at node 5 is twice that of the vorticity vector at node 1. Figure 5B shows the nature of the vorticity vector, $\bar{\omega}$, on the leading, root chord panel. Because the δ component at node 5 is proportional to the γ component at that node, it does not have to be solved for explicitly. Any other nodes on the root chord due to other panels are treated similarly to node 2.

Applying all edge boundary conditions to the planform of Fig 3 reduces the number of nodal values which must be found to determine the vorticity at any point on the planform. Figure 6 shows the same planform with only the unspecified nodal values numbered. The number of nodal values which must be determined for any network arrangement is $6MN$, where M is the number of chordwise panels and N is the number of spanwise panels.

Kinematic Flow Condition

Ideally, the vortex distribution on the lifting surface would result in it being a stream surface such that the normal component of velocity were zero everywhere on that surface. The paneling method as developed here permits the enforcement of only a finite number of boundary conditions so the kinematic flow condition can be satisfied only at certain control points on the surface. In words, the kinematic flow condition states that when the induced velocity at a point on the

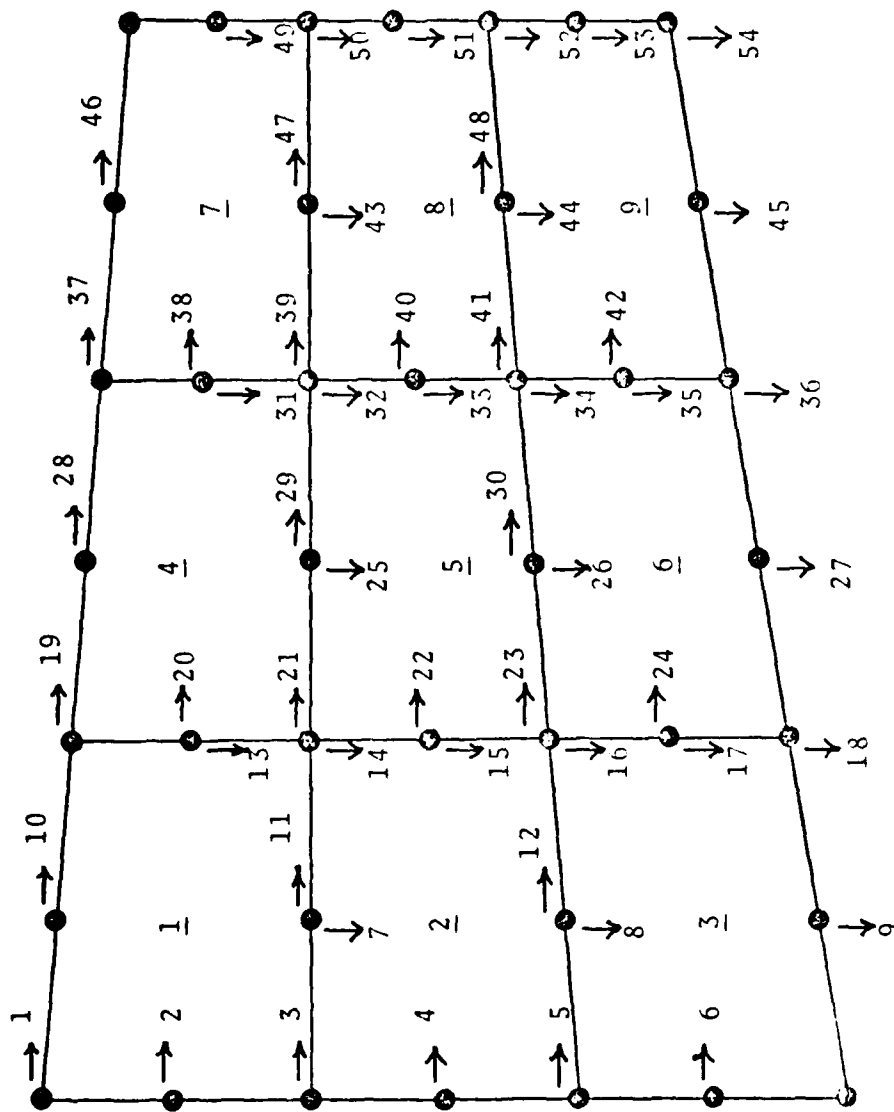


Fig 6. Nine Panel Network With Planform Boundary Conditions Applied

surface, $w(x,y)$, caused by a vortex distribution, $\bar{\omega}(x,y)$, on the surface is added to the normal component of velocity at the point caused by a free stream velocity U_∞ incident to the surface at some angle of attack, α , the resultant velocity is zero. Stated mathematically, the kinematic flow condition is

$$U_\infty \sin \alpha + w(x,y) = 0 \quad (3.2)$$

The Biot-Savart Law serves to uniquely define the induced velocity coexistent with a given vorticity field (Ref 8: 170). Sparks (Ref 10: 10) has used the Biot-Savart Law and shown that the normal velocity component induced at the origin of an x-y plane, when that plane has a vorticity distribution of the form (2.1), is given by

$$w(o,o) = \frac{1}{4\pi} \int_R \int (x\gamma - y\delta) / (x^2 + y^2)^{3/2} dR \quad (3.3)$$

Substituting Eqs (2.5) and (2.3) for δ and γ in (3.3) yields:

$$w = \frac{1}{4\pi} \int_R \int (Gx + Hx^2 + 2Ixy - Ay - Cy^2 + 3Kxy^2 - Ey^3 + Lx^3 + \frac{3}{2}Jx^2y) / (x^2 + y^2)^{3/2} dR \quad (3.4)$$

The induced velocity caused by one quadrilateral panel is the sum of the velocities induced by each of its two triangular subregions. The coefficients in Eq (3.4) are constants over a subregion, they being functions of the nodal values and panel geometry. Eq (3.4) is rewritten as:

$$\begin{aligned}
w = & (G_L T_L^1 + H_L T_L^2 + 2I_L T_L^3 - A_L T_L^4 - C_L T_L^5 + 3K_L T_L^6 \\
& - E_L T_L^7 + L_L T_L^8 + \frac{3}{2} J_L T_L^9 + G_T T_T^1 + H_T T_T^2 + 2I_T T_T^3 \quad (3.5) \\
& - A_T T_T^4 - C_T T_T^5 + 3K_T T_T^6 - E_T T_T^7 + L_T T_T^8 + \frac{3}{2} J_T T_T^9) / 4\pi
\end{aligned}$$

where

$$T_{L,T}^k = \int_R \int x^i y^j / (x^2 + y^2)^{3/2} dR \quad (3.6)$$

and

k =	1	2	3	4	5	6	7	8	9
i =	1	2	1	0	0	1	0	3	2
j =	0	0	1	1	2	2	3	0	1

The evaluation of the first five integrals given by Eq (3.6) is found in Sparks (Ref 10: 47-53). The last four were evaluated in a similar manner; the results are given in Appendix B. With the integrals evaluated in terms of the panel geometry, Eq (3.5) is expressed in the form:

$$w = \sum_{i=1}^8 (f_i \delta^i + g_i \gamma^i) \quad (3.7)$$

where the f_i and g_i are expressions involving only the panel geometry.

Eq (3.7) gives the normal velocity induced at the origin of the x-y plane by a vorticity vector distribution over a trapezoidal panel in the plane.

Intra-Panel Continuity Condition

Intra-panel continuity is achieved by enforcing the

four conditions set forth in Section II. The first of these is

$$\delta^4 = \delta_L(x_4, y_2) \quad (3.8)$$

Substituting Eq (2.12) for the right-hand side of Eq (3.8) gives an expression in terms of only nodal values and panel geometry. The second condition involves matching the γ values at node 4 and yields:

$$\gamma^4 = \gamma_T(x_4, y_2) \quad (3.9)$$

The right-hand side of Eq (3.9) is replaced by Eq (2.13), giving an expression in terms of nodal values and panel geometry.

The final two conditions match the δ and γ values at node 9:

$$\delta_T(x_9, y_3) = \delta_L(x_9, y_3) \quad (3.10)$$

$$\gamma_T(x_9, y_3) = \gamma_L(x_9, y_3) \quad (3.11)$$

Eqs (2.12) and (2.13) are substituted into the left- and right-hand sides, respectively, of Eqs (3.10) and (3.11).

Eqs (3.8) through (3.11) involve only nodal values and panel geometry, and satisfying these four expressions assures intra-panel continuity of the vorticity distribution function.

Eq (3.9) cannot be applied to the trailing edge root chord panel (panel 3 in Fig 6) under all conditions. If the trailing edge of this panel is parallel to the y -axis, Eq (3.9) is trivial. The reason for this can be seen by examining Figs 2 and 6. Suppose the panel in Fig 2 is the

trailing root chord panel. Then, by the planform edge boundary conditions, the following are all identically zero: δ^1 , δ^5 , δ^2 , γ^2 , γ^8 , and γ^4 . The value of γ on the trailing edge is

$$\gamma_T = C_1 + C_2\gamma + C_2\gamma^2 \quad (3.12)$$

since x is constant on the edge. Two values of γ , (γ^2 and γ^8), are specified. Helmholtz's second theorem, Eq (2.4), says that:

$$\frac{\partial f}{\partial x} + \frac{\partial \gamma}{\partial y} = 0 \quad (3.13)$$

Since δ^1 , δ^2 and δ^5 are all zero, $\frac{\partial \delta}{\partial x}$ at node 2 is zero, therefore $\frac{\partial \gamma}{\partial y}$ must also be zero at that node. Eq (3.12) is completely specified if the three conditions

$$\gamma^2 = \gamma^8 = \frac{\partial \gamma}{\partial y} = 0 \quad (3.14)$$

are dictated. The solution is that γ is zero along the line. Consequently, trying to apply Eq (3.9), recalling that γ^4 is also zero, is redundant. This problem is avoided by always insuring that the trailing edge of this panel (root-trailing edge) has a non-zero slope so that the edge is never parallel to the y -axis.

Matrix Formulation

Eq (3.7) is the induced velocity at a point caused by one panel. The total induced velocity at a point is found by summing the effects of all panels in the planform. The effect of panels on the planform to the left of the x -axis

is accounted for by reflecting the control point about the x-axis and applying Eq (3.7) to that point. Figure 7 illustrates this procedure. The induced velocity at point A caused by panels 1 and 2 is desired. The effect of panel 1 is found by straightforward application of Eq (3.7). From symmetry, the induced velocity at point A caused by panel 2 is identical to the effect of panel 1 on point A'. So, the induced velocity at point A due to panels 1 and 2 is the sum of the effect of panel 1 on A and the effect of panel 1 on A'. For a selected control point, the induced velocity is given by:

$$\sum_{j=1}^{6MN} A_{ij} \phi_j = w_i \quad (3.15)$$

In Eq (3.15) the coefficients A_{ij} are functions of panel geometry only, and the ϕ_j are the unknown nodal values of the vorticity vector distribution. The right-hand side of Eq (3.12) is the normal component of velocity such that Eq (3.2) is satisfied:

$$w_i = -U_\infty \sin \alpha_i \quad (3.16)$$

Two control points are selected per panel so Eq (3.15) gives 2MN equations for the 6MN unknown nodal values.

The remaining 4MN equations come from the four intra-panel continuity conditions being applied to each panel.

The resulting system of equations has the form:

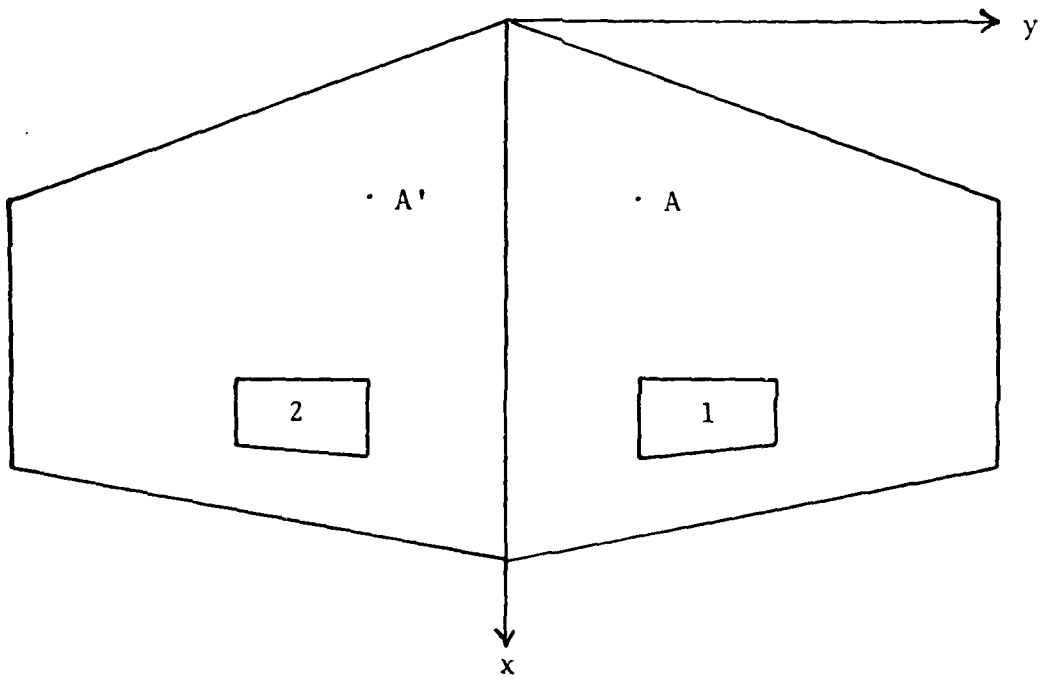


Fig 7. Control Point Reflection

$$[A_{ij}] \begin{Bmatrix} \phi_1 \\ \vdots \\ \phi_{6MN} \end{Bmatrix} = -U_\infty \begin{Bmatrix} \sin \alpha_1 \\ \vdots \\ \sin \alpha_{2MN} \\ 0 \end{Bmatrix} \quad (3.17)$$

The solution to this system is:

$$-\begin{Bmatrix} \phi_j \\ U_\infty \end{Bmatrix} = [A_{ij}]^{-1} \begin{Bmatrix} \sin \alpha_i \\ \vdots \\ 0 \end{Bmatrix} \quad (3.18)$$

Compressibility Correction

The Prandtl-Glauert Rule is used to account for the effect of compressibility as the Mach number is increased (Ref 8: 276). In practice, the adjustment for compressibility is made by multiplying all x-coordinates in the A matrix of Eq (3.18) by the factor

$$\beta = 1 / \sqrt{1 - M_\infty^2} \quad (3.19)$$

IV. Aerodynamic Data

The ϕ_j , as determined from Eq (3.15), are used in Eqs (2.12) and (2.13) to compute the components of the vorticity vector at any point on the planform. The perturbation velocities on the surface due to the vorticity distribution are:

$$\bar{u} = \pm \frac{1}{2}\gamma \hat{i} \quad (4.1)$$

$$\bar{v} = \mp \frac{1}{2}\delta \hat{j} \quad (4.2)$$

where the upper and lower signs correspond to the upper and lower sides of the surface (Ref 9: 124).

Following Sparks, the perturbation velocities are used to determine the pressure coefficient by one of two methods. The exact isentropic expression is (Ref 11: 433):

$$C_p = 2 \left[1 + \left(\frac{k-1}{2} M_\infty^2 \right) \left(1 - \frac{(U_\infty + u)^2 + v^2}{U_\infty^2} \right)^{k/(k-1)} \right] / k M_\infty^2 \quad (4.3)$$

while the second order approximation is (Ref 11: 435):

$$C_p = -2 \left[2u/U_\infty + (1 - M_\infty^2) u^2/U_\infty^2 + v^2/U_\infty^2 \right] \quad (4.4)$$

Let the difference in the pressure coefficients of the lower and upper surfaces be defined by:

$$\Delta C_p = C_{pL} - C_{pU} \quad (4.5)$$

The local coefficients of lift and moment are then given by
(Ref 9: 30):

$$C_L = \frac{1}{C} \int_0^C \Delta C_p \, dx \quad (4.6)$$

and

$$C_M = -\frac{1}{C^2} \int_0^C (\Delta C_p) x \, dx \quad (4.7)$$

V. Computer Code

The theory set forth in Sections II, III and IV has been incorporated into FORTRAN code. The program, WING2, takes its basic structure from the program WING, developed in Ref 10. WING2's mainline and two of its subroutines, MESH and LOADS2, are the result of only minor modifications to their counterparts in WING. Each subroutine is briefly described below and a complete listing of WING2 is in Appendix C.

WING2: Reads data describing the planform to be analyzed. Calls all subroutines.

MESH: Computes the x-y coordinates of the nodes and control points for the input planform.

BIOT: Generates the 2MN equations which result from enforcing the kinematic flow condition at each control point.

INT: Evaluates the nine integrals given by Eq (3.6).

STRIP: Is called by INT and contains expressions for evaluating the integrals.

CONT: Generated 4MN equations which result from assuring intra-panel continuity.

COEFF: Evaluates the coefficients required to define the interpolating functions.

LOADS2: Uses the nodal values of the vorticity distribution to determine the pressure distribution and aerodynamic coefficients.

VI. Results and Discussion

The theory was evaluated by obtaining aerodynamic data for a rectangular wing of aspect ratio 5 at an angle of attack of 5° . Two panel networks were used. First, the wing semispan was modeled with one panel. Second, four panels of equal size were used.

Figure 8A shows the distribution of the local lift coefficient, C_L , over the span using the one panel model and varying the control point location. Figure 8B shows the control point set locations on the semispan. The results shown for these locations illustrate the sensitivity of the solution to the control point placement. Analysis of results for the four control point locations shown and others indicated that control point set A was the most desirable.

Figure 9 shows the C_L distribution for control point set A compared to the distribution predicted by Truckenbrodt's lifting surface theory (Ref 9: 164). Although WING2 consistently underpredicted C_L values, the distribution is smooth and exhibits the desired rate of decay from a maximum at the root to zero at the tip.

Figure 10A illustrates the value of the vorticity vector at various points on the semispan, again for the single panel model. Also plotted on the figure are the two control point locations. The nature of the vortex pattern agrees

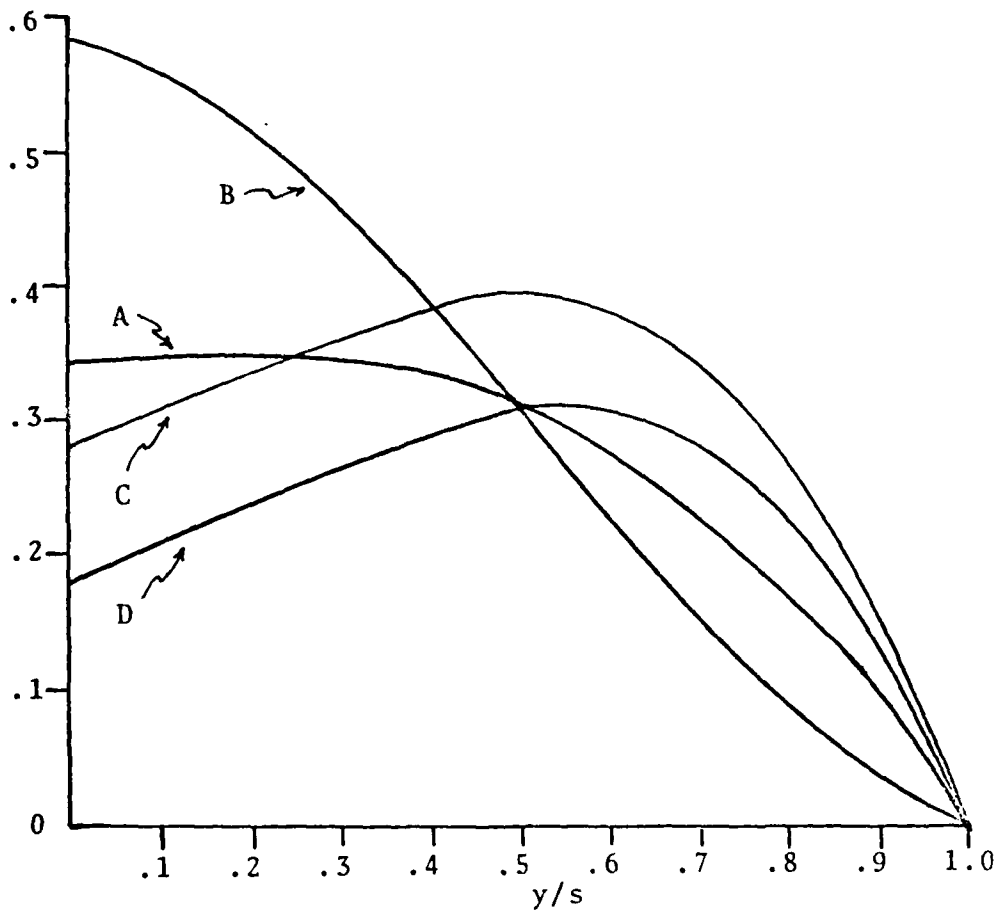


Fig 8A. Lift Distribution Versus Span Station for Various Control Point Locations

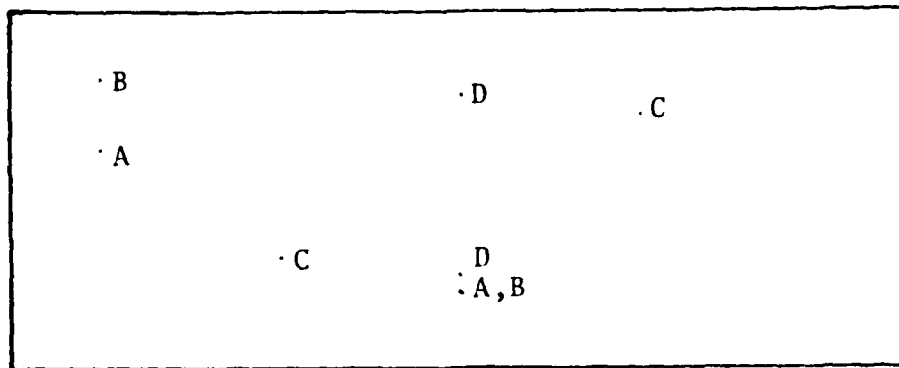


Fig 8B. Control Point Locations

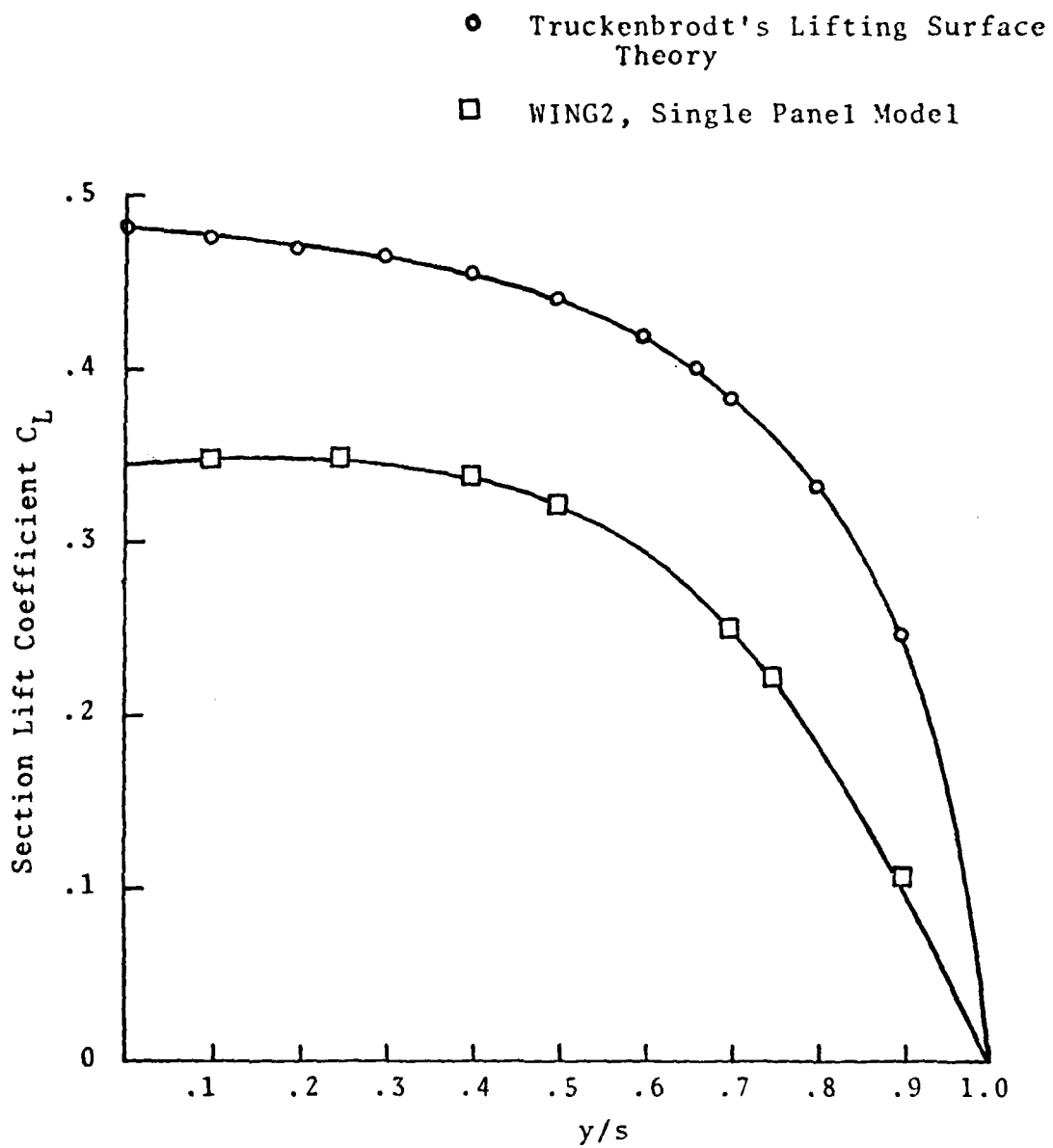


Fig 9. Lift Distribution Versus Span Station for Rectangular Wing; AR=5, $\alpha=5^\circ$

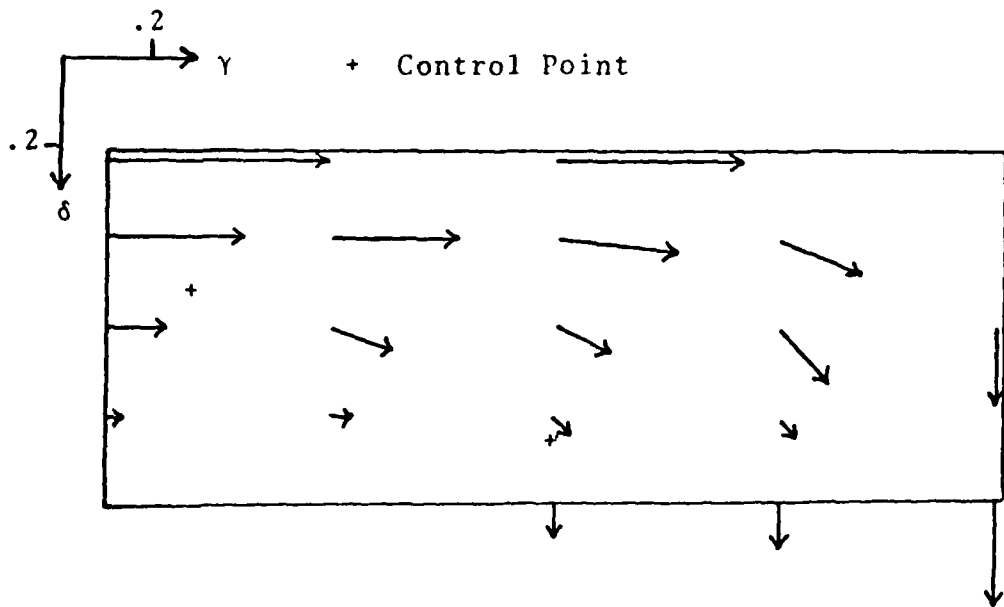


Fig 10A. Vortex Pattern - Single Panel Model

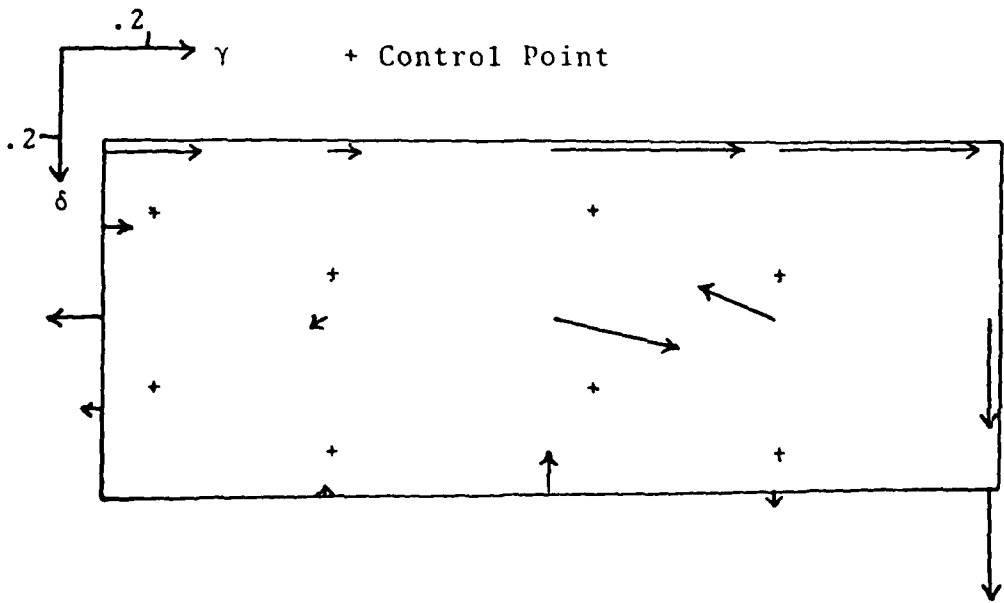


Fig 10B. Vortex Panel - Four Panel Model

with the desired pattern, Fig 4B.

Figure 11 plots the center of pressure versus span station. The X_{CP} shifts slightly forward towards the tip instead of slightly rearward as desired (Ref 9: 159).

The desirable features exhibited by the one panel model all but vanish when four panels are used. Figure 10B graphically shows the altered nature of the vortex pattern. The prevalence of negative values of both δ and γ is not physically reasonable for this low α case. A further illustration of the departure of the four panel solution is shown in Fig 12. Here the chordwise distribution of γ is given along the root chord. Three distributions are shown; the one panel model, four panel model, and the exact answer for two-dimensional flow about a flat plate (Ref 5: 62):

$$\gamma(x) = 2\alpha \left(\frac{x_L - x}{x - x_T} \right) \quad (6.1)$$

where x_L and x_T are the values of the leading and trailing edges respectively. For the single panel model, WING2 is more correct in the midchord region than in either the leading or trailing edge regions.

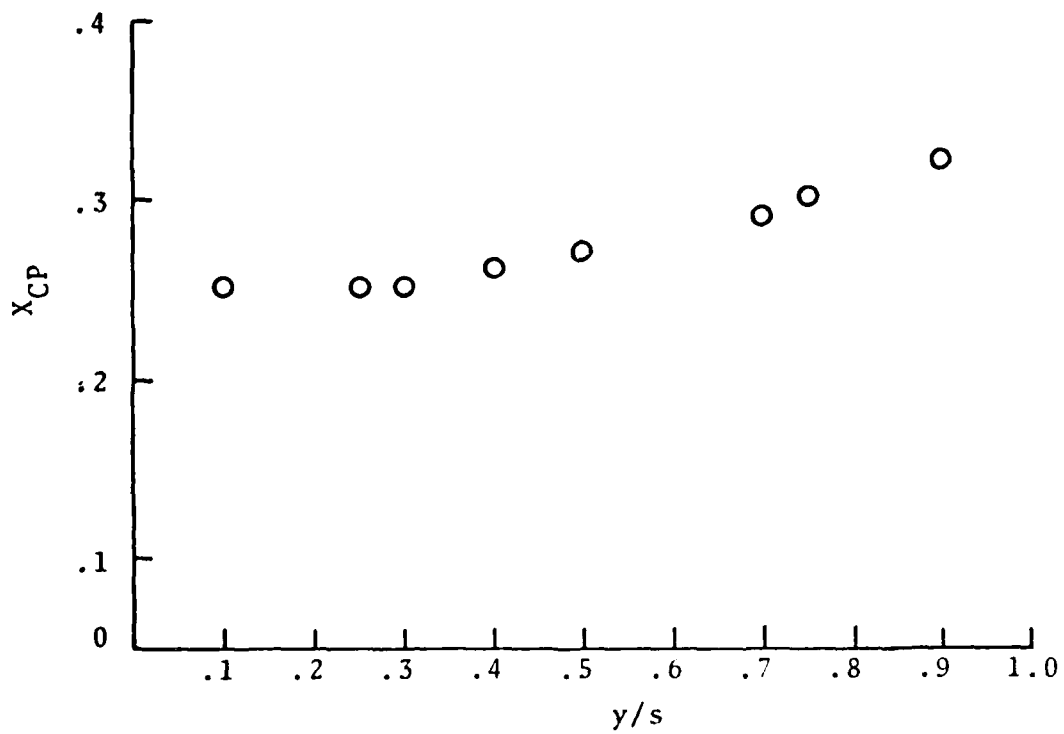


Fig 11. X_{CP} Versus Span Station for
Rectangular Wing; $AR=5$, $\alpha=5^\circ$

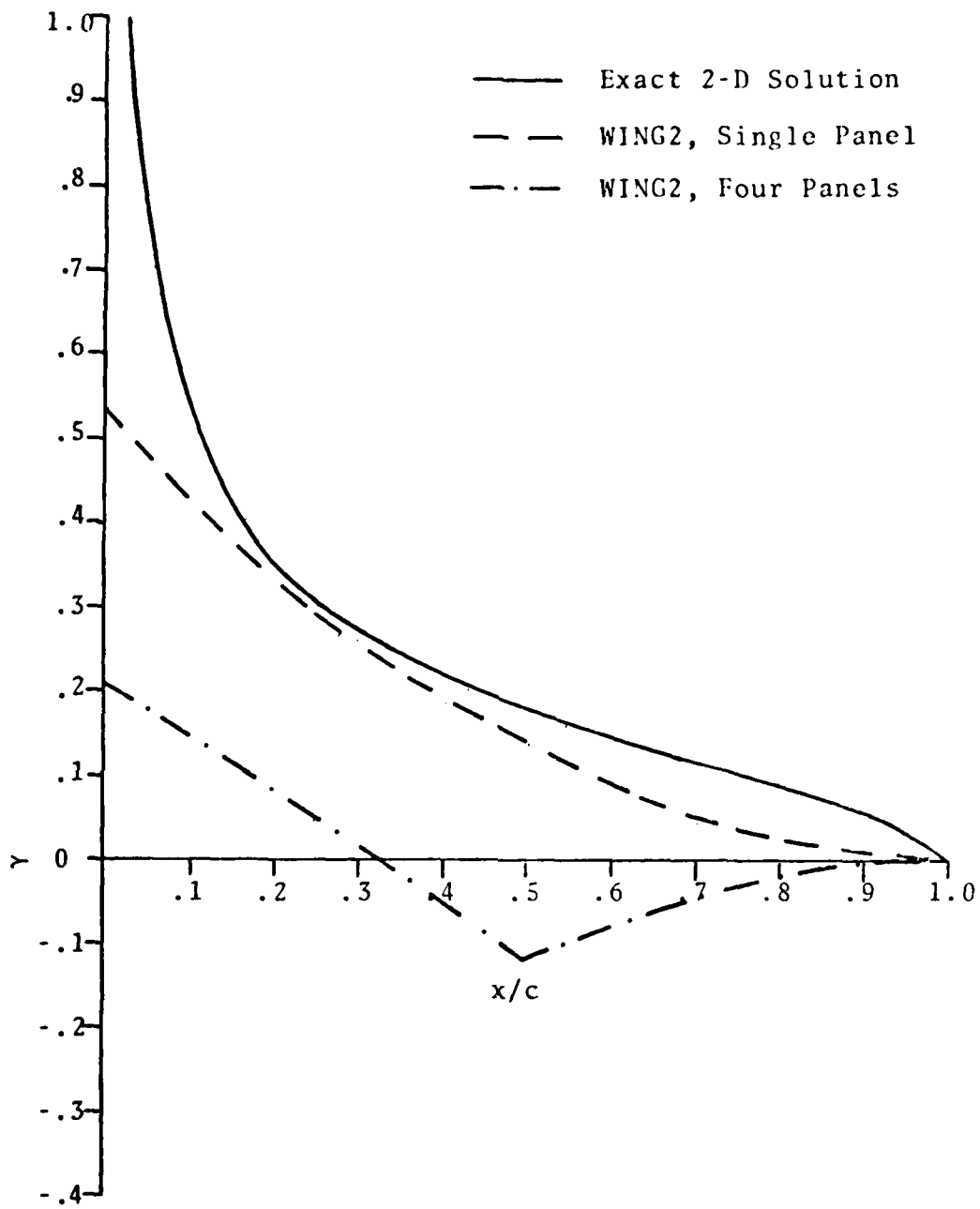


Fig 12. Comparison of γ Distribution Along Root Chord

VII. Conclusions and Recommendations

The linear discontinuous vorticity vector method of Sparks (Ref 10) has been extended to a quadratic, continuous vector vortex panel method. The desirable features of the higher-order, continuous vorticity distribution are apparent from the results of the single panel model. The spanwise and chordwise lift distributions, while consistently low in value, do exhibit the characteristics of the desired solutions. The method breaks down, however, if the planform is modeled with four panels.

No reason is readily apparent for the drastic change in the nature of the vortex pattern for the four panel solution. It is suspected that the vorticity distribution function must be further restricted so that the solution converges toward a physically reasonable result.

The following are recommendations which may improve the results of the method.

a. Sacrifice some control point equations to provide continuity in the first derivative of γ with respect to y across panel boundaries.

b. Incorporate analytical results into the solution process. Davies (Ref 3) has demonstrated an accurate closed form approximation to the behavior of the lift distribution near the wing apex. Using the analytical result will insure

a more correct result in the apex area and may then improve the overall lift distribution.

c. Adopt an iterative procedure. Starting with the single panel solution, use selected values of δ and γ as given quantities in the four panel solution process.

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APPENDIX A
Interpolating Function Equations

Interpolating Function Equations

Eqs (2.5) and (2.3) are transformed into functions of nodal values and nodal coordinates. The solutions to Eqs (2.7) and (2.10) are

$$\{F_L^i\} = [f_i(x_j, y_j)]^{-1} \{\Gamma_L^j\} \quad (A.1)$$

and

$$\{F_T^i\} = [g_i(x_j, y_j)]^{-1} \{\Gamma_T^j\} \quad (A.2)$$

The f_i and g_i matrices were symbolically inverted using the computer program MACSYMA (Ref 7). With the symbolic inversion the $F_{L,T}^i$ are readily expressed as linear polynomials in the nodal values. For example,

$$\begin{aligned} F_L^1 = A_L = & (ALD1)(\delta_L^1) + (ALD3)(\delta_L^3) + (ALD6)(\delta_L^6) \\ & + (ALD7)(\delta_L^7) + (ALG1)(\gamma_L^1) + (ALG3)(\gamma_L^3) \\ & + (ALG4)(\gamma_L^4) + (ALG6)(\gamma_L^6) + (ALG7)(\gamma_L^7) \end{aligned} \quad (A.3)$$

where the ALDX and ALGX represent the appropriate element of the inverted f_i matrix. The complete algebraic expressions of the inverted f_i and g_i matrices are found in the subroutine COEFF listed in Appendix C.

APPENDIX B
Evaluation of Integrals

Evaluation of Integrals

The procedure for evaluating the four integrals in Eqs (3.11) - (3.14) is given in detail in Sparks (Ref 10: 47-53). Only the results are given here.

Following Sparks' notation, the STRIP functions for the four additional integrals are:

$$\begin{aligned}
 S_6[(x_0, y_0), (x_1, y_1)] = & \left\{ \left(\frac{my_1}{2(m^2+1)} + \frac{3bm}{2(m^2+1)^2} \right) \right. \\
 & \cdot (m^2y_1^2 + m^2x_1^2)^{1/2} - \left(\frac{my_0}{2(m^2+1)} + \frac{3bm}{2(m^2+1)} \right) \\
 & \cdot (m^2y_0^2 + m^2x_0^2)^{1/2} + \frac{3bm^2 - m(m^2+1)b^2}{2(m^2+1)^{5/2}} \\
 & \left. \text{LN} \left(\frac{(m^2+1)^{1/2} (y_1^2+x_1^2)^{1/2} + my_1+x_1}{(m^2+1)^{1/2} (y_0^2+x_0^2)^{1/2} + my_0+x_0} \right) \right\} \quad (B.1)
 \end{aligned}$$

$$\begin{aligned}
 S_7[(x_0, y_0), (x_1, y_1)] = & \left\{ \left(\frac{2b-y_1}{2(m^2+1)} - \frac{3b}{2(m^2+1)^2} \right) \right. \\
 & \cdot (m^2y_1^2 + m^2x_1^2)^{1/2} + \left(\frac{2b+y_0}{2(m^2+1)} + \frac{3b}{2(m^2+1)^2} \right) \\
 & \cdot (m^2y_0^2 + m^2x_0^2) - \frac{3m^2b^2}{2(m^2+1)^{5/2}} \\
 & \left. \text{LN} \left(\frac{(m^2+1)^{1/2} (y_1^2+x_1^2)^{1/2} + my_1+x_1}{(m^2+1)^{1/2} (y_0^2+x_0^2)^{1/2} + my_0+x_0} \right) \right\} \quad (B.2)
 \end{aligned}$$

$$\begin{aligned}
S_8[(x_0, y_0), (x_1, y_1)] &= \left\{ \left(\frac{mx_1(m^2+1) - m^2b + 2b}{2(m^2+1)^2} - y_1 \right) \right. \\
&\cdot (x_1^2 + y_1^2)^{1/2} + \left(y_0 - \frac{mx_0(m^2+1) - m^2b + 2b}{2(m^2+1)^2} \right) \\
&\cdot (x_0^2 + y_0^2)^{1/2} + \frac{3mb^2}{2(m^2+1)^{5/2}} \\
&\left. \text{LN} \left(\frac{(m^2+1)^{1/2} (x_0^2 + y_0^2)^{1/2} + my_0 + x_0}{(m^2+1)^{1/2} (x_1^2 + y_1^2)^{1/2} + my_1 + x_1} \right) \right\} \quad (B.3)
\end{aligned}$$

$$\begin{aligned}
S_9[(x_0, y_0), (x_1, y_1)] &= \left\{ \left(\frac{3mb + m^4x_1 + m^2x_1}{2(m^2+1)^2} \right) \right. \\
&\cdot (y_1^2 + x_1^2)^{1/2} - \left(\frac{3mb + m^4x_0 + m^2x_0}{2(m^2+1)^2} \right) \\
&\cdot (y_0^2 + x_0^2)^{1/2} + \frac{y_0^2}{2} \text{LN}(x_0 + (x_0^2 + y_0^2)^{1/2}) \\
&- \frac{y_1^2}{2} \text{LN}(x_1 + (x_1^2 + y_1^2)^{1/2}) + \frac{2m^2b^2 - b^2}{2(m^2+1)^{5/2}} \\
&\left. \text{LN} \left(\frac{(m^2+1)^{1/2} (y_0^2 + x_0^2)^{1/2} + my_0 + x_0}{(m^2+1)^{1/2} (y_1^2 + x_1^2)^{1/2} + my_1 + x_1} \right) \right\} \quad (B.4)
\end{aligned}$$

The expressions for the four integrals are then

$$T_L^i = S_i[(x_1, y_1), (x_3, y_2)] - S_i[(x_1, y_1), (x_4, y_2)] \quad (B.5)$$

$$T_T^i = S_i[(x_1, y_1), (x_4, y_2)] - S_i[(x_2, y_1), (x_4, y_2)] \quad (B.6)$$

APPENDIX C
Computer Code


```
WRITE (5,*) 99) (A(I, J), J=1, 5.)  
579 FORMAT (5(2Y,=12.5))  
WRITE (5,*) 99)  
589 FORMAT (////)  
599 CONTINUE  
CALL LOADSC (NP, M, I, IP2)  
999 STOP  
END
```

```

SUBROUTINE INT(X1,X2,X3,X4,Y1,Y2)
3   THIS SUBROUTINE EVALUATES THE PANEL INTEGRALS. IT ALSO FUNCTIONS AS THE EXECUTIVE
3   CONTROL ROUTINE FOR SUBROUTINE STRIP.
COMMON/BLOCK1/IL(3),IT(3)
COMMON/BLOCK2/SPF(9,3),KILL
REAL IL,IT
3   CALCULATE STRIP FUNCTIONS FOR THE PANEL
3   LEADING EDGE WITH CORNER POINTS (X1,Y1)
3   AND (X3,Y2).
I=1
CALL STRIP(Y1,Y1,X3,Y2,I)
3   CALCULATE STRIP FUNCTIONS FOR THE PANEL
3   MAIN DIAGONAL WITH CORNER POINTS (X1,Y1)
3   AND (X3,Y2).
I=2
CALL STRIP(Y1,Y1,X1,Y2,I)
3   CALCULATE STRIP FUNCTIONS FOR THE PANEL
3   TRAILING EDGE WITH CORNER POINTS (X2,Y1)
3   AND (X4,Y2).
I=3
CALL STRIP(X2,Y1,X4,Y2,I)
3   EVALUATE PANEL INTEGRALS.
DO 1 I=1,3
IL(I)=SPF(1,1)-SPF(2,2)
IT(I)=SPF(1,2)-SPF(2,3)
1 CONTINUE
RETURN
END

```

```

SUBROUTINE STRIP(X,Y,X1,Y1,I)
SUBROUTINE TO CALCULATE STRIP FUNCTION.
THERE ARE 16 STRIP FUNCTION CALCULATIONS
PER BICENTRIC PANEL.
COMMON/PLDCKE/SPF(9,3),KILL
REAL M,MO,PO
USE STRIP FUNCTION EQUATIONS FOR 44 IN-
FINITE SLOPE IF X1=K.
IF (ABS(X1-X1).LT.1E-6) GO TO 1..
M=(Y1-Y1)/(X1-K)
PO=Y-Y1*M
DUM1=SQRT(X**2+Y1**2)
DUM2=SQRT(X**2+Y1**2)
DUM3=SQRT(1+M**2)
DUM4=(DUM1+(X**2+Y1**2)/DUM3)/(DUM2+(X1+M*Y1)/DUM3)
DUM5=ABS(DUM4)
DUM6=ALOG(DUM5)
DUM7=ABS(DUM1+X)
DUM8=ALOG(DUM7)
DUM9=ABS(DUM2+X1)
DUM10=ALOG(DUM9)
SPF(1,1)=-1+DUM6/DUM3
SPF(2,1)=Y*(DUM5-Y1/DUM4-1*(DUM1-DUM2)/DUM3**2-PO**2/DUM4**2)/DUM3**3
SPF(3,1)=M**2*(DUM1-DUM2)/DUM3**2-PO**2/DUM4**2
SPF(4,1)=DUM4/DUM3+DUM5-DUM6
SPF(5,1)=-DUM4**2/DUM3**2+DUM5**2/DUM3**2
K=1
SPF(1,2)=(MO*Y1)/2./DUM3**2+(3.*PO*MO)/2./DUM3**2+MO*DUM12-(MO*
1Y1)/2./DUM3**2+(3.*PO*MO)/2./DUM3**2+MO*DUM12-(MO*
1**2-PO**2+MO**2)/2./DUM3**2
SPF(2,2)=DUM2*(X1+DUM3)/2./DUM3**2-PO**2/DUM4**2+PO**2/DUM4
1*(Y1-PO)/2./DUM3**2+3.*PO/2./DUM3**2+PO**2/DUM4
1/DUM3**2
SPF(3,2)=(3.*PO*MO**2)/DUM3**2+DUM5*(MO*Y1+DUM12-
1-PO**2+MO**2)/DUM3**2+Y1*(DUM1-DUM2)/DUM3**2+MO**2/DUM3
1**2*(DUM1-DUM2)/DUM3**2
SPF(4,2)=(DUM1-DUM2)/DUM3**2+DUM5*(MO*Y1+DUM12-
1Y1+MO**2-PO**2)/DUM3**2+DUM12*(MO*Y1+DUM12-PO**2)/
2*(DUM1-DUM2)+DUM5*(Y1**2/2)-DUM6
WRITE(*,*)SPF(1,1),SPF(1,2)
RETURN
END
100 DUM1=SQRT(X**2+Y1**2)
DUM2=SQRT(X**2+Y1**2)
DUM3=ABS(DUM1+Y1)/(DUM2+Y1)
SPF(1,1)=-1+DUM3/DUM1
DUM4=ABS(DUM1+Y1)
DUM5=ALOG(DUM4)
DUM6=ABS(DUM1+X)
DUM7=ALOG(DUM6)
SPF(2,1)=Y*(DUM5-Y1/DUM3-1*(DUM1-DUM2)/DUM3**2-
SPF(3,1)=DUM3**2*(DUM1-DUM2)/DUM3**2-PO**2/DUM4**2
SPF(4,1)=DUM4/DUM3+DUM5-DUM7
DUM9=ABS(DUM1+X)
DUM10=ALOG(DUM9)
SPF(5,1)=-DUM4**2/DUM3**2+DUM5**2/DUM3**2
SPF(1,2)=(MO*Y1)/2./DUM3**2+(3.*PO*MO)/2./DUM3**2+MO*
1Y1)/2./DUM3**2+(3.*PO*MO)/2./DUM3**2+MO*DUM12-(MO*
1**2-PO**2+MO**2)/2./DUM3**2
SPF(2,2)=DUM2*(X1+DUM3)/2./DUM3**2-PO**2/DUM4**2+PO**2/DUM4
1*(Y1-PO)/2./DUM3**2+3.*PO/2./DUM3**2+PO**2/DUM4
1/DUM3**2
SPF(3,2)=(3.*PO*MO**2)/DUM3**2+DUM5*(MO*Y1+DUM12-
1-PO**2+MO**2)/DUM3**2+Y1*(DUM1-DUM2)/DUM3**2+MO**2/DUM3
1**2*(DUM1-DUM2)/DUM3**2
SPF(4,2)=(DUM1-DUM2)/DUM3**2+DUM5*(MO*Y1+DUM12-
1Y1+MO**2-PO**2)/DUM3**2+DUM12*(MO*Y1+DUM12-PO**2)/
2*(DUM1-DUM2)+DUM5*(Y1**2/2)-DUM7

```

```

DU47=DU5/DU4
DU47=ALOG(DU47)
SPF(6,I)=(Y1*(DU47)/2-(Y1*DU47)/2+((X1**2)/2)*DU47
WRITE(6,"SPF(6,I)=",SPF(6,I))
SPF(7,I)=X1*DU47-Y1*DU47
WRITE(7,"SPF(7,I)=",SPF(7,I))
SPF(8,I)=-Y1*DU47+Y1*DU47
WRITE(8,"SPF(8,I)=",SPF(8,I))
SPF(9,I)=(X1*(DU47)/2-((Y1**2)/2)*DU47-(X1*DU47)/2+((Y1**2)/2)*DU47
WRITE(9,"SPF(9,I)=",SPF(9,I))
RETURN
END

```



```

1** FORMAT (1X,12,12(1X,F7.2))
6  CONTINUE
  WRITE (6,1.0)
1** FORMAT (////1X,'SPAN=L',1X,'XCD',3X,'YCD',3X,'XCA',1X,'YCA',3X,
1'XCD',3X,'YCD')
  DO 51 I=1,N
  WRITE (6,1.0) I,XD(I),YD(I),XCA(I),YCA(I),XCD(I),YCD(I)
1** FORMAT (2X,12,'(1X,F7.2)')
6) CONTINUE
3)
      CALCULATE WING PLANFORM AREA
  AREA=0.
  DO 7 L=1,N-1
    R1=R(L,1)-R(L,1)
    R2=R(L+1,2)-R(L+1,1)
    A1=RL(L)
    A2=RL(L+1)
    A=(A1+A2)*R2-RL(L)*R2
    AREA=AREA+A*(A1+A2)
  CONTINUE
  WRITE (6,7) AREA
7) FORMAT (////2X,'TOTAL WING AREA =',F8.2)
3)
  STOP
END

```



```

SUBROUTINE COEFF (X1,X2,X3,Y1,Y2)
COMMON/PLUCKL/ALB1,ALB2,ALB3,ALB4,ALB5,ALB6,ALB7,ALB8,ALB9,
1 CLD1,CLD2,CLD3,CLD4,CLD5,CLD6,CLD7,CLD8,CLD9,
2 ELQ1,ELQ2,ELQ3,ELQ4,ELQ5,ELQ6,ELQ7,ELQ8,ELQ9,
3 ILQ1,ILQ2,ILQ3,ILQ4,ILQ5,ILQ6,ILQ7,ILQ8,ILQ9,
4 JLD1,JLD2,JLD3,JLD4,JLD5,JLD6,JLD7,JLD8,JLD9,
5 KLD1,KLD2,KLD3,KLD4,KLD5,KLD6,KLD7,KLD8,KLD9,
6 QLD1,QLD2,QLD3,QLD4,QLD5,QLD6,QLD7,QLD8,QLD9,
7 HLD1,HLD2,HLD3,HLD4,HLD5,HLD6,HLD7,HLD8,HLD9,
8 LLQ1,LLQ2,LLQ3,
9 ATQ1,ATQ2,ATQ3,ATQ4,ATQ5,ATQ6,ATQ7,ATQ8,ATQ9,
A CTQ1,CTQ2,CTQ3,CTQ4,CTQ5,CTQ6,CTQ7,CTQ8,CTQ9,
B ETQ1,ETQ2,ETQ3,ETQ4,ETQ5,ETQ6,ETQ7,ETQ8,ETQ9,
C ITQ1,ITQ2,ITQ3,ITQ4,ITQ5,ITQ6,ITQ7,ITQ8,ITQ9,
D JTQ1,JTQ2,JTQ3,
E KTD1,KTD2,KTD3,KTD4,KTD5,KTD6,KTD7,KTD8,KTD9,
F QTD1,QTD2,QTD3,QTD4,QTD5,QTD6,QTD7,QTD8,QTD9,
G HTD1,HTD2,HTD3,HTD4,HTD5,HTD6,HTD7,HTD8,HTD9,
H LTD1,LTD2,LTD3
REAL ILQ1,ILQ2,ILQ3,ILQ4,ILQ5,ILQ6,ILQ7,ILQ8,ILQ9,
1 QLD1,QLD2,QLD3,QLD4,QLD5,QLD6,QLD7,QLD8,QLD9,
2 KLD1,KLD2,KLD3,KLD4,KLD5,KLD6,KLD7,KLD8,KLD9,
3 LLQ1,LLQ2,LLQ3,
4 ITQ1,ITQ2,ITQ3,ITQ4,ITQ5,ITQ6,ITQ7,ITQ8,ITQ9,
5 JTQ1,JTQ2,JTQ3,
6 KTD1,KTD2,KTD3,KTD4,KTD5,KTD6,KTD7,KTD8,KTD9,
7 LTD1,LTD2,LTD3
ATQ1 = ((X1**2+Y1*Y2)*Y2**2+((-7*Y2-X1)*Y1*Y2**2+7*Y1*X2)*Y1**2)
1 2+ (5*Y1**2+(-X2-3*X1)*X1-Y1**2)/(Y2**2-2*X1*Y2+Y1**2)+Y2**2+
2 -2*Y2**2+4*Y1*Y2-2*Y1**2)*Y1*Y2+(X1**2-1*Y1*(X2+Y1**2)*Y1**2)
ATQ2 = ((Y1*Y2+X1**2)+Y2**2+((-3*X2-3*Y1)*X1-1*Y2**2+1*(X1*Y2+Y
1 1**2)+Y1*Y2+(X1**2+2*(Y2-3*Y1)*Y2)*Y1**2)/(X2**2-2*(X1*Y2+Y1
2 **2)+Y2**2+(-2*Y2**2+4*X1*Y2-2*X1**2)+Y1*Y2+(X2**2-2*(X1*Y2+X1
3 1**2)+Y1**2)
ATQ3 = (Y1*Y2+Y1**2)/(Y2**2-2*Y1*Y2+Y1**2)
ATQ4 = -(X1*Y2*Y2**2+(-1*Y2-4*Y1)*X1-1*(X2**2+1*Y1*X2)-Y1*
1 Y2+(12*X1**2+(X2-12*X1)*X1)*Y1**2)/(Y2**2-2*Y1*Y2+Y1**2)+Y2
2 **2+(-2*Y2**2+X1*Y2-2*Y1**2)*Y1*Y2+(Y2**2-2*Y1*X2+X1**2)*Y1**2
3 )
ATQ5 = -4*Y1*Y2/(Y2**2-2*Y1*Y2+Y1**2)
ATQ6 = ((X2*Y1**2+(-1*X2**2-1*(X1*Y2)*Y1+Y1*Y2**2)+Y1*Y2+(-
1 4*Y1**3+(X1*Y2+X1**2)*X1**2-X1*X2*X1)*Y1**2)/(Y2**2-2*X1*Y2+X1
2 **2)+Y2**3+(-3*Y2**2+Y1*Y2-3*Y1**2)*Y1*Y2**2+2*(X2**2-3*Y1*X2
3 +3*Y1**2)*Y1**2+Y2**2+(-X2**2+3*Y1*Y2-X1**2)-Y1**2)
ATQ7 = ((X2*Y1**2+(-1*Y2**2-1*(X1*Y2)*Y1-3*Y1*Y2**2+1*(X1**2+Y
1 1*Y1*Y2+(-1*Y2**2+1*(X2-12*X1)*X1)*Y1**2+(-1*Y1*(X2-3*Y1**2)*Y1
2 **2)/(X2**2-2*Y1*Y2+Y1**2)+Y2**3+(-3*Y2**2+Y1*Y2-3*Y1**2)*Y1
3 Y2**2+(3*X2**2-1*(X1*X2+Y1**2)*Y1**2)*Y2+(-X2**2+3*X1*X2-Y1**2)
4 *Y1**2)
ATQ8 = -(X1*Y2*Y1**2-1*(X1*Y2*X1-3*Y2**2+1*(Y1*Y2**2)+Y1*Y2+(-
1 9*Y1**3+15*Y1*Y1**2+(3*Y2**2-15*(X1*Y2)+X1)*Y1**2)/(Y2**2-2*Y1*
2 Y2+X1**2)*Y2**3+(-7*X2**2+Y1*Y2-3*Y1**2)*Y1*Y2**2+(7*X2**2-3
3 X1*Y2+3*Y1**2)*Y1**2+Y2**2+2*Y1*Y2-3*Y1**2)*Y1**2)
ATQ9 = -(X1*Y1*Y2-3*Y1*Y1**2)/(Y2**2-2*Y1*Y2+Y1**2)+Y1**2+Y2-3*
1 1**3)
CTQ1 = ((7*Y2+Y1)*Y2**2-7*Y2**2-3*Y1*Y2)*Y2+(-10*X2**2+(X1*Y2+Y
1 1)*X1-X2**2-3*X1*Y2)*Y1)/(X2**2-2*Y1*Y2+Y1**2)+Y2**2+(-2*Y2**2

```

$$2 \quad + \cdot X1^2 X2 - 2^2 Y1^2 + 2^2 Y1^2 Y2 + (X2^2 + 2 - 2^2 X1^2 Y2 + Y1^2 + 2) \cdot Y1^2 + 2$$

$$ET72 = ((1^2 X2 + 3^2 X1) \cdot X1 + 1^2 Y2 + 2^2 Y1^2 (X1 X2 - 7^2 Y1^2 + 2) \cdot Y2 + (-12^2 Y1^2 + 2$$

$$1 \quad + (21^2 X1 - 1^2 Y2) \cdot Y1 + 4^2 Y2^2 + 2 - 7^2 X1^2 X2 - X1^2 + 2) \cdot Y1) / ((Y2^2 - 1 - 2^2 X1^2 Y1^2 + Y1^2$$

$$2 \quad + 2) \cdot Y2^2 + 2 + (-2^2 X2 + 2 + 4^2 Y1^2 X2 - 2^2 X1^2 + 2) \cdot Y1^2 + Y2 + (X2^2 + 2 - 2^2 X1^2 X2 + Y1^2 + 2$$

$$3 \quad) \cdot Y1^2 + 2)$$

$$ET73 = -(Y2 + 2^2 Y1) / (Y2^2 + 2 - 2^2 Y1^2 Y2 + Y1^2 + 2)$$

$$ET75 = -(((12^2 X2 + 5^2 X1) \cdot X1 + 1^2 X2^2 + 2 - 2^2 X1^2 Y2) \cdot Y2 + (-2^2 X1^2 + 2 + 12^2 X$$

$$1 \quad 2 + 22^2 X1) \cdot X1 + 4^2 X2^2 + 2 - 12^2 X1^2 Y2 + Y1) / ((X1^2 + 2 - 2^2 X1^2 X2 + Y1^2 + 2) \cdot Y2 + 2 +$$

$$2 \quad (-2^2 Y2^2 + 2 + 1^2 X1^2 X2 - 2^2 X1^2 + 2) \cdot Y1^2 + 2 + (X2^2 + 2 - 2^2 Y1^2 Y2 + X1^2 + 2) \cdot Y1^2 + 2)$$

$$ET77 = (1^2 Y2 + 1^2 Y1) / (Y2^2 + 2 - 12^2 Y1^2 Y2 + Y1^2 + 2)$$

$$ET78 = -((1^2 X2^2 + X1^2 + 2 + (-1^2 Y2^2 + 2 - 1^2 X1^2 Y2) \cdot Y1 + 1^2 Y1^2 Y2 + 2) \cdot Y2 + (-1^2$$

$$1 \quad X^2 + 2 + (2^2 X2 + 3^2 X1) \cdot X1 + 1^2 Y2 + (-1^2 X2^2 + 2 - 12^2 Y1^2 Y2) \cdot Y1 + 1^2 Y1^2 X2 + 2) \cdot Y1)$$

$$2 \quad / ((X1^2 + 2 - 2^2 Y1^2 X2 + X1^2 + 2) \cdot Y1^2 + 2 + (-2^2 X2^2 + 2 - 1^2 X1^2 Y2 - 3^2 Y1^2 + 2) \cdot Y1^2 + 2 +$$

$$3 \quad + 2 + (3^2 X2^2 + 2 - 1^2 X1^2 Y2 + 3^2 X1^2 + 2) \cdot Y1^2 + 2 + Y2 + (-Y2^2 + 2 + 12^2 X1^2 Y2 + X1^2 + 2) \cdot X1$$

$$4 \quad + 2)$$

$$ET79 = -((1^2 Y2^2 + X1^2 + 2 + (1^2 Y2^2 + 2 - 12^2 Y1^2 X2) \cdot X1 + 1^2 Y1^2 Y2^2 + 2 + 1^2 Y1^2 + 2^2 X$$

$$1 \quad Y2) \cdot Y2 + (-1^2 Y1^2 + 2 + (2^2 X1 - 1^2 X2) \cdot X1 + 2 + (1^2 Y2^2 + 2 - 12^2 Y1^2 Y2 + 1^2 X1^2 + 2) \cdot X1)$$

$$2 \quad X^2 - 1^2 Y1^2 Y2^2 + 2 + 1^2 X1^2 + 2 + Y2) \cdot Y1) / ((X2^2 + 2 - 2^2 Y1^2 Y2 + Y1^2 + 2) \cdot Y2 + 2 + (-1^2$$

$$3 \quad X2^2 + 2 + 1^2 X1^2 X2 - 2^2 X1^2 + 2) \cdot Y1^2 + 2 + (3^2 X2^2 + 2 - 12^2 X1^2 X2 + 3^2 Y1^2 + 2) \cdot Y1^2 + 2$$

$$4 \quad + Y2 + (-X2^2 + 2 + 2^2 X1^2 X2 - X1^2 + 2) \cdot Y1^2 + 2)$$

$$ET85 = ((1^2 X2^2 + X1^2 + 2 - 12^2 X1^2 Y2^2 + X1^2 + 2 - 12^2 X1^2 Y2^2 + 1^2 X1^2 Y2^2 + 2) \cdot Y2 + (-1^2 X$$

$$1 \quad 4^2 X2 + 2 + 2^2 X1^2 Y2) \cdot X1 + 1^2 Y2^2 + 2 + 1^2 Y1^2 Y2^2 + 2 - 12^2 Y1^2 X2) \cdot X1 + 1^2 Y1^2 X2 + 2 + 1^2 Y1^2 Y2^2 + 2$$

$$2 \quad + 2) \cdot Y1) / ((Y2^2 + 2 - 2^2 Y1^2 X2 + Y1^2 + 2) \cdot Y2^2 + 2 + (-7^2 Y1^2 + 2 + 1^2 X2 - 3^2 Y1^2 + 2$$

$$3 \quad) \cdot Y1^2 Y2^2 + 2 + (2^2 X2^2 + 2 - 5^2 X1^2 X2 + 3^2 X1^2 + 2) \cdot Y1^2 + 2 + Y2 + (-Y2^2 + 2 + 12^2 X1^2 Y2 - X$$

$$4 \quad 1^2 + 2) \cdot Y1^2 + 2)$$

$$ET88 = (2^2 X2^2 + Y2 + (2^2 X2 - 12^2 X1) \cdot X1) / (Y2^2 + 2 - 3^2 Y1^2 Y2^2 + 2 + 7^2 Y1^2 + 2^2 Y2 -$$

$$1 \quad Y1^2 + 2)$$

$$ET91 = (1^2 Y2^2 + 2 + (-1^2 X2 - 4^2 Y1) \cdot X1 + 2^2 Y2^2 + 2 + 1^2 X1^2 Y2) / ((Y2^2 + 2 - 2^2 X1^2$$

$$1 \quad X2 + X1^2 + 2) \cdot Y2^2 + 2 + (-2^2 X2^2 + 2 + 1^2 X1^2 X2 - 2^2 X1^2 + 2) \cdot Y1^2 + 2 + (X2^2 + 2 - 2^2 X1^2 X2$$

$$2 \quad + X1^2 + 2) \cdot Y1^2 + 2)$$

$$ET92 = (1^2 Y2^2 + 2 - 12^2 Y1^2 X2 - 4^2 X2^2 + 2 + 2^2 X1^2 Y2 + 2^2 X1^2 + 2) / ((X2^2 + 2 - 12^2 X1$$

$$1 \quad X2 + X1^2 + 2) \cdot Y2^2 + 2 + (-2^2 X2^2 + 2 + 1^2 X1^2 X2 - 2^2 X1^2 + 2) \cdot Y1^2 + 2 + (X2^2 + 2 - 2^2 Y1^2 Y$$

$$2 \quad 2 + X1^2 + 2) \cdot Y1^2 + 2)$$

$$ET94 = 2 / (Y2^2 + 2 - 2^2 Y1^2 Y2 + Y1^2 + 2)$$

$$ET95 = -((12^2 Y2^2 + 2 + (-12^2 Y2 - 12^2 X1) \cdot X1 - 1^2 X2^2 + 2 + 12^2 X1^2 Y2) / ((X2^2 + 2 - 2$$

$$1 \quad X1^2 X2 + X1^2 + 2) \cdot Y2^2 + 2 + (-2^2 Y2^2 + 2 + 1^2 X1^2 Y2 - 2^2 X1^2 + 2) \cdot Y1^2 + 2 + (X2^2 + 2 - 2^2 Y$$

$$2 \quad 1^2 X2 + Y1^2 + 2) \cdot Y1^2 + 2)$$

$$ET98 = -1 / (Y2^2 + 2 - 2^2 Y1^2 Y2 + Y1^2 + 2)$$

$$ET99 = -((1^2 X2^2 + 2 + (-6^2 X2 - 12^2 Y1) \cdot X1 + 2 + (1^2 Y2^2 + 2 + 12^2 X1^2 Y2) \cdot X1 + 1^2 X1^2$$

$$1 \quad X2^2 + 2) / ((X2^2 + 2 - 12^2 Y1^2 X2 + X1^2 + 2) \cdot Y2^2 + 2 + (-3^2 X2^2 + 2 + 1^2 X1^2 X2 - 7^2 X1^2 + 2) \cdot X1$$

$$2 \quad Y1^2 Y2^2 + 2 + (3^2 X2^2 + 2 - 6^2 X1^2 X2 + 7^2 X1^2 + 2) \cdot Y1^2 + 2 + Y2 + (-X2^2 + 2 + 2^2 X1^2 Y2 - Y1^2$$

$$3 \quad + 2) \cdot Y1^2 + 2)$$

$$ET99 = -((1^2 X2^2 + 2 - 12^2 Y1^2 X2 + 1^2 X2 + (-1^2 Y2^2 + 2 + 1^2 X1^2 Y2 + 12^2 Y1^2 + 2) \cdot Y1^2 + 2 + Y$$

$$1 \quad 1^2 X2^2 + 2 - 2^2 Y1^2 + 2^2 Y2) / ((Y2^2 + 2 - 2^2 X1^2 X2 + Y1^2 + 2) \cdot Y2^2 + 2 + (-7^2 Y2^2 + 2 + 1^2 X1^2 X2$$

$$2 \quad + Y2 - 7^2 Y1^2 + 2) \cdot Y1^2 + 2 + (2^2 Y2^2 + 2 + (2^2 Y2^2 + 2 - 5^2 X1^2 X2 + 3^2 Y1^2 + 2) \cdot Y1^2 + 2 + Y2 + (-Y2^2 + 2$$

$$3 \quad 2 + 2^2 X1^2 X2 - X1^2 + 2) \cdot Y1^2 + 2)$$

$$ET98 = (2^2 Y2^2 + 2 + (-1^2 X2 - 1^2 Y1) \cdot X1 + 2 + (2^2 Y1^2 X2 - 12^2 Y2^2 + 2) \cdot X1 + 1^2 X2$$

$$1 \quad + 2 - 1^2 Y1^2 Y2 + 2) / ((Y2^2 + 2 - 2^2 Y1^2 Y2 + X1^2 + 2) \cdot Y2^2 + 2 + (-7^2 Y2^2 + 2 + 1^2 Y1^2 X2$$

$$2 \quad - 7^2 X1^2 + 2) \cdot Y1^2 + 2 + (3^2 X2^2 + 2 - 5^2 X1^2 X2 + 2^2 X1^2 + 2) \cdot Y1^2 + 2 + Y2 + (-X2^2 + 2 + 2$$

$$3 \quad + Y1^2 X2 - X1^2 + 2) \cdot Y1^2 + 2)$$

$$ET99 = (1^2 Y2 - 1^2 Y2) / (Y2^2 + 2 - 12^2 Y1^2 Y2 + 2 + 3^2 Y1^2 + 2^2 Y2 - Y1^2 + 2)$$

$$IT71 = ((3^2 X2 + X1) \cdot Y2 + (-1^2 X2 + X2 + 3^2 X1) \cdot Y1) / ((X2^2 + 2 - 2^2 X1^2 X2 + Y1^2 + 2$$

$$1 \quad) \cdot Y2 + (-Y2^2 + 2 + 1^2 Y1^2 Y2 - Y1^2 + 2) \cdot Y1)$$

$$IT72 = ((2^2 X2 + 3^2 X1) \cdot Y2 + (-1^2 X2 - 2^2 X2 + 3^2 Y1) \cdot Y1) / ((X2^2 + 2 - 2^2 X1^2 X2 + Y1^2 + 2$$

$$1 \quad + 2) \cdot Y2 + (-Y2^2 + 2 + 1^2 Y1^2 Y2 - Y1^2 + 2) \cdot Y1)$$

$$IT75 = -((1^2 Y2 + 1^2 Y1) \cdot Y2 + (-1^2 X2 - 4^2 X2 + 12^2 Y1) \cdot Y1) / ((X2^2 + 2 - 2^2 Y1^2 Y$$

$$1 \quad 2 + Y1^2 + 2) \cdot Y2 + (-Y2^2 + 2 + 2^2 X1^2 X2 - X1^2 + 2) \cdot Y1)$$


```

1 X3)*Y1*Y2+(Y4**2-2*Y3*Y1-3*Y3**2)*Y1**2)/((Y1**2-7*X3*(X1+Y7**2)
2 *Y2**2+(-2*Y4**2+X3*X1-2*Y3**2)*Y1*Y2+(Y4**2-2*Y7*Y4+X3**2)*Y
3 1**2)
ALG5 = -((3+Y1*Y3-12*X1**2)*Y2**2+(13*Y1*X1-1*X7**2)*Y1*Y2+X
1 3**2*Y1**2)/((Y1**2-2*Y7*Y4+X3**2)*Y2**2+(-2*Y4**2+X3*X1-2*X3
2 **2)*Y1*Y2+(X4**2-2*X3*Y4+Y7**2)*Y1**2)
ALG7 = -2*Y1*Y2/(Y2**2-2*Y1*Y2+Y1**2)
ALG1 = -((7*Y1*X1+Y1*Y7-7*Y1**2)*Y2**2+(Y1-3*Y3)*Y4-(3**2+7*X
1 1*X3)*Y1*Y2+(-Y3*X1-X3**2)*Y1**2)/((Y4-X3)*Y2**2+(7*X3-3*Y1)*Y1
2 *Y2**2+(3*Y1-3*X1)*Y1*Y2+Y3*(Y3-(1)-Y1**2)
ALG7 = -((X1*Y1**2+(X1*Y3-13*Y1**2)*Y1-7*Y1**2)*Y1+2*Y7*Y4*(X1**2)+
1 Y2**2+(3*Y1-Y3)*Y4**2+(-7*Y3**2+12*Y1*Y3-7*X1**2)*X4**2-(11*Y3**
2 2-17*X1**2*Y3)*Y1*Y2+(-7*Y7*(X1**2+(7*X1*X3-7*X7**2)*X4+3*Y1*Y3**
3 *2)*Y1**2)/((Y1**2-2*Y7*Y4+X3**2)*Y2**2+(3*Y1**2-4*Y7*Y4+3*X7**2)*Y1**2
4 *2)*Y2**2+(7*X1**2-2*X3*Y4+7*X3**2)*Y1**2+Y2+(-Y1**2+2*Y1*X1
5 -X3**2)*Y1**2)
ALG4 = ((Y1*Y3-Y1**2)*Y1-7*Y1*X7**2+3*Y1*X4**2+2*Y1*Y7*(Y1**2)*Y2**2+(
1 (X1**2-Y3**2)*X1+X7**2-7*X1*X3**2+3*Y1**2*X3)*Y1*Y2+((Y7**2-Y1**
2 Y3)*Y1+7*Y7**2-Y1*Y3**2)*Y1**2)/((Y1**2-2*Y7*Y4+Y3**2)*Y2**2+(-7*
3 X3**2+7*X3*X4-7*X1**2)*Y1*Y2**2+(3*X1**2-4*Y7*Y4+3*X7**2)*Y1**2
4 *Y2+(-Y1**2+2*Y1*X1-X3**2)*Y1**2)
ALG5 = -(((1+X1*X3-12*X1**2)*Y4-12*Y1*X3**2+2*Y1*Y7**2+Y1**2)*Y1**
1 3)*Y2**2+(11*Y1**2-7*Y3**2)*Y1+12*Y2**2+3-2*Y1*Y3**2+2*X1**2*X3
2 )*Y1*Y2+((1+X3**2-4*Y1*Y3)*X4+7*X3**2-7*Y1*Y7**2)*Y1**2)/((Y1**
3 2-2*X3*X1+Y7**2)*Y2**2+(3*Y1**2+4*Y7*Y4-3*Y3**2)*Y1*Y2**2+(3*Y
4 1**2-2*X3*X1+7*X3**2)*Y1**2+Y2**2+(-7*Y7**2+2*Y3*Y1-X7**2)*Y1**2)
ALG7 = ((3*X1*Y4+12*X1*Y7-2*X1**2)*Y2**2+(12*Y1-3*X3)*X4-12*X
1 3**2+2*X1*Y3)*Y1*Y2+(-7*Y7*X4-7*Y3**2)*Y1**2)/((Y1-X3)*Y2**2+(
2 3*X3-3*X1)*Y1*Y2**2+(3*Y1-3*Y3)*Y1**2+Y2+(Y1-Y3)*Y1**2)
ALG1 = -((7*Y2+Y1)/(Y2**2-2*Y1*Y2+Y1**2)
ALG3 = -((X4**2-2*Y3*X1-7*X3**2+7*Y1*Y3-2*Y3**2)*Y2+(3*Y1**2
1 -2*X3*X1-13*X7**2+11*X1*Y3)*Y1)/((Y1**2-2*Y3*Y1+Y7**2)*Y2**2+(-
2 *X4**2+7*Y3*Y1-2*Y7**2)*Y1*Y2+(X4**2-2*Y3*Y4+Y7**2)*Y1**2)
ALG5 = -((3*X3**2-32*X1*Y3+2*X1**2)*Y2+(11*Y7**2-11*X1*Y3)*Y1
1 1)/((Y1**2-2*X3*Y4+X3**2)*Y2**2+(2*Y1**2+4*Y7*Y4-2*X3**2)*Y1*Y2
2 +(Y4**2-2*Y7*Y4+X3**2)*Y1**2)
ALG7 = ((Y2+Y1)/(Y2**2-2*Y1*Y2+Y1**2)
ALG1 = -(((7*X3-7*X1)*Y4+Y1**2-12*X1*Y3+12*Y4-X2)*Y2+((11*Y3-Y1)
1 *Y4+7*X3**2-7*X1*X3)*Y1)/((X4-X3)*Y2**2+(3*Y3-3*Y4)*Y1*Y2**2+(7
2 *Y1-3*X3)*Y1**2+Y2+(Y3-Y4)*Y1**2)
ALG7 = -(((X7-1)*X1)*X4**2+(7*X3**2-7*Y1*Y3+7*X4**2)*X1-7*Y1**
1 X3**2+3*X1**2-X3-2)*Y1**2)*Y2+((7*Y7-7*X1)*X1**2+17*(3**2-2*X
2 X1*Y3+3*Y1**2)*X1-12*Y1*X3**2+11*Y1**2*Y3)*Y1)/((Y1**2-2*Y7*Y4
3 X3**2)*Y2**2+(3*X4**2+4*X7**2-3*Y7**2)*Y1*Y2**2+(7*Y4**2-2*Y7*
4 X4+3*X3**2)*Y1**2+Y2+(7*Y4**2+3*X3*X1-X3**2)*Y1**2)
ALG1 = ((Y3**2-2*Y1*Y7+X1**2)*Y4-7*Y7**2+6*Y1*Y7**2-6*X1**2*Y3+
1 4*X1**2)*Y2+((-X3**2+2*Y1*X1-X1**2)*X4-7*Y3**2+3*Y1*X3**2-7*X1**
2 *Y3)*Y1)/((Y1**2-2*X3*X1+X1**2)*Y2**2+(7*Y4**2+3*X3*X1-3*Y7*
3 *2)*Y1*Y2**2+(7*Y4**2-3*X3*X1+3*Y3**2)*Y1**2+Y2+(-Y4**2+2*Y7*X4
4 -Y7**2)*Y1**2)
ALG5 = -(((1+Y3**2-4*X1*Y7+Y1**2)*X4-12*X3**2+7*Y1*Y3**2-7*
1 *Y4**2+12*Y1*Y3)*Y2+((-7*X3**2+11*Y1*Y3-7*X1**2)*X4-7*Y7**2+3*
2 +7*Y1*Y3**2-2*X1**2*Y3)*Y1)/((X4-2*Y7*Y4+Y7**2)*Y2**2+(-2*Y4**2
3 +7*Y3*Y1-2*Y7**2)*Y1*Y2**2+(3*X1**2-2*Y7*Y4+3*X7**2)*Y1**2)*Y1**2
4 Y2+(-X4**2+2*X3*X1-X3**2)*Y1**2)
ALG7 = ((4*Y3-12*Y1)*Y4+12*X1*Y3**2-7*Y1*Y3**2+Y1**2)*Y2+((12*X
1 3-4*X1)*X4+2*(X3**2-2*Y1*Y3)*Y1)/((Y4-Y3)*Y2**2+(2*Y7-3*Y4)*Y1

```

$2 \quad *Y^2+2+(3*Y^2-3*Y^3)*Y1+4*Y^2*(Y^2-Y^3)*Y1^2$
 EL71 = $2/(Y^2+3-2*Y1-Y^2+Y1^2)$
 EL73 = $(2*Y^2+3-4*Y^3+Y^4-1-X^2+2+2*X1-Y^2-2*X1^2)/(X^2+2-2*$
 1 $X^2+X+X^2+2)*Y2+2+(-2*Y^2-2+2*Y^3+X^2-2*Y^3+2)*Y1^2+X^2+2-2*Y^2$
 2 $*X1+X^2+2)*Y1^2$
 EL75 = $(12*X^2+2-24*X1+Y^2+12*X1^2)/(Y^2+2-2*X^2+Y^2+Y^3+2)*Y2+$
 1 $*2+(-2*X^2+2+X^2*Y^2-2*Y^3+2)*Y1^2+X^2+2-2*Y^2+X^2+Y^2+Y^3+2)*Y1^2$
 2 $)$
 FL07 = $-1/(Y^2+2-2*Y1+Y2+Y1^2)$
 FL71 = $((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $+((3*Y^2-7*Y^3)-Y1+Y^2+2+(7*Y^2-3*Y^3)-Y1+2*Y2+(X^2-Y^2)*Y1^2)$
 FL73 = $((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $*Y^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 2 $*X^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 3 $*Y^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 FL74 = $(2*X^2+2-2*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $+Y^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 2 $+X^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 FL75 = $-(2+X^2+3-3*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $X^2+Y^2+Y^3+2)*Y2+2+(-2*Y^2-2+2*Y^3+X^2-2*Y^3+2)*Y1^2+X^2+2-2*Y^2$
 2 $-2*Y^3+X^2+3*Y^3+2)*Y1^2+2+2*Y^2+2+2*Y^3+X^2+X^2+2+2*Y^3+X^2+X^2+2)*Y1^2$
 EL77 = $-(2+X^2+3-3*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $2*Y^2+3+3*Y^3-7*Y^4)-Y1+Y^2+2+(2*X^2-7*X^3+Y^2+2*Y^3+Y^2+Y^3+Y^4)*Y1^2$
 IL78 = $((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $*Y^2+2+Y^3+Y^4+Y^2+2)*Y1^2$
 IL79 = $-(2+X^2+3-3*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $2*Y^2+2+Y^3+Y^4+Y^2+2)*Y1^2$
 IL81 = $-(2+X^2+3-3*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $2*Y^2+2+Y^3+Y^4+Y^2+2)*Y1^2$
 IL87 = $-(2+X^2+3-3*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $(-4*X^3-7*Y^3)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 2 $-2+4*X^3+X^2-2*X^2+2)*Y1^2+Y^2+Y^3+2+2*Y^3+X^2+X^2+2)*Y1^2$
 IL84 = $((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $*2+3*X1*Y^2)*Y1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 2 $-2*Y^3+2)*Y1^2+Y^2+Y^3+2+2*Y^3+X^2+X^2+2)*Y1^2$
 IL85 = $-(2+X^2+3-3*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $-4*X^3+X^2-12*Y^2+2+12*Y^3+Y^2+Y1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)$
 2 $-2*Y^2+2+4*Y^3+Y^2-2*Y^3+2)*Y1^2+X^2+2-2*Y^2+X^2+Y^2+Y^3+2)*Y1^2$
 IL87 = $((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $(2*X^3-2*X^2)*Y1^2+X^2+X^2+Y^2+2)$
 JL03 = $1/(Y^2+2-2*Y^2+Y^2+Y^3+2)$
 JL75 = $-1/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 JL91 = $-1/(Y^2+2-2*Y^2+Y^2+Y^3+2)$
 JL93 = $-(2*X^2+12*Y^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)*Y2+(-X^2+2+2*Y^3+Y^2+Y^3+2)*Y1^2$
 1 $(-Y^2+2)*Y1^2$
 JL95 = $-(2*X^2+12*Y^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)*Y2+(-X^2+2+2*Y^3+Y^2+Y^3+2)*Y1^2$
 1 $X^2+2+Y^3)$
 JL96 = $(2*X^2+12*Y^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)*Y2+(-Y^2+2+2*Y^3+Y^2+Y^3+2)*Y1^2$
 1 $-X^2+2)*Y1^2$
 JL97 = $1/(Y^2+2-2*Y^2+Y^2+Y^3+2)$
 KL73 = $-(2+X^2+3-3*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $X^2+Y^2+Y^3+2)*Y2+2+(-2*Y^2-2+2*Y^3+X^2-2*Y^3+2)*Y1^2+X^2+2-2*Y^2$
 2 $-2*Y^3+X^2+3*Y^3+2)*Y1^2+2+2*Y^2+2+2*Y^3+X^2+X^2+2+2*Y^3+X^2+X^2+2)*Y1^2$
 KL75 = $((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $*Y^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 2 $*X^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 KL76 = $(2*X^2+2-2*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $*Y^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 2 $*X^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 KL77 = $(2*X^2+2-2*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $*Y^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 2 $*X^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 KL78 = $(2*X^2+2-2*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $*Y^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 2 $*X^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 KL79 = $(2*X^2+2-2*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $*Y^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 2 $*X^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 KL80 = $(2*X^2+2-2*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $*Y^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 2 $*X^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 KL81 = $(2*X^2+2-2*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $*Y^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 2 $*X^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 KL82 = $(2*X^2+2-2*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $*Y^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 2 $*X^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 KL83 = $(2*X^2+2-2*Y^2+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 1 $*Y^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$
 2 $*X^2+2+((X^2+X-1)*Y+Y^2+Y^3+2+X1*Y^2+X^2+X1^2)/(X^2+2-2*Y^2+Y^2+Y^3+2)$

```

1  4*Y4+X3**2)*Y2**2+(-2*X1**2+X3*X4-2*X3**2)*Y1*Y2+(Y1**2-2*X3*Y
2  4+X3**2)*Y1**2)
  KLS4  = (2*X3**2-1+Y1*X3+2*Y1**2)/(Y1**2-2*X3*(X1+Y3**2)+Y2**2+
1  (-2*X1**2+X3*X4-2*X3**2)+Y1*Y2+(Y1**2-2*X3*Y1+Y3**2)+Y1**2)
  KLS5  = -(12*X3**2-2-2+X1*X3+12*X1**2)/(X1**2-2*X3*Y1+X3**2)*Y2
1  **2+(-2*X1**2+X3*X4-2*X3**2)*Y1*Y2+(X1**2-2*X3*Y1+X3**2)*Y1**
2  2)
  KLS7  = -(4*X1+12*X3-1+Y1)/(X1-X3)*Y2**2+(2*X3-2*Y4)*Y1*Y2+(X
1  4-Y3)*Y1**2)
  GLD3  = (1*Y1*Y2**2-2-X3*Y1*Y2)/(X1**2-2*X3*Y1+Y3**2)*Y2+(-Y1*
1  **2+2*X3*Y1-Y3**2)*Y1)
  GLD5  = -(1*X1*Y2**2+2-3*Y3-Y1*Y2)/(Y1**2-2*X3*Y1+X3**2)*Y2+(-X1
1  **2+2*X3*Y1-X3**2)*Y1)
  GLS1  = ((X1-X3+X1)*Y2**2+(X1+2*X3)*Y1*Y2)/(X1-X3)*Y2**2+(2*
1  Y3-2*Y4)*Y1*Y2+(X1-X3)*Y1**2)
  GLS7  = -(1*X1*Y1-Y1*Y3-1*(X1**2)*Y2**2+(-Y1**2+(2*X1-2*Y3)*Y1
1  **2+X1-X3)*Y1*Y2+(-Y1**2-Y3*Y1*Y1**2)/(X1**2-2*X3*Y1+X3**2)*Y2
2  **2+(-2*X1**2+X3*X4-2*X3**2)*Y1*Y2+(X1**2-2*X3*Y1+X3**2)*Y1**
3  2)
  GLS5  = ((Y1*Y4+1+Y3+2*Y1**2)*Y2**2+(-X3-Y1)*Y1+Y3**2-3*Y1*Y3
1  )*Y1*Y2+(X3*Y1+X3**2)*Y1**2)/((X1**2-2*X3*Y1+X3**2)+Y2**2+(-2*X
2  **2+Y3*Y1-2*X3**2)*Y1*Y2+(Y1**2-2*X3*Y1+X3**2)*Y1**2)
  GLS5  = -(1*X1*Y1-12*X1*X3+12*X1**2)/(Y1**2-2*X3*Y1+X3**2)*Y1+12
1  *X3**2-12*X1*X3)*Y1*Y2+X3*X4*Y1**2)/(Y1**2-2*X3*Y1+X3**2)*Y2
2  **2+(-2*X1**2+X3*X4-2*X3**2)*Y1*Y2+(X1**2-2*X3*Y1+X3**2)*Y1**
3  2)
  GLS7  = (1*X1*Y2**2+(-Y4-12*X3)*Y1*Y2)/(X1-X3)*Y2**2+(2*X3-
1  2*Y4)*Y1*Y2+(Y4-X3)*Y1**2)
  HLD3  = -5*Y2/(Y4**2-2*X3*Y1+X3**2)
  HLD5  = 6*Y2/(X1**2-2*X3*Y1+X3**2)
  HLS1  = 4*Y2/((Y1+Y3)-Y2*(X3-X1)*Y1)
  HLS7  = ((5*X1-X3-12*X1*Y1*Y2+(3*X4+X7)*Y1)/(Y4**2-2*X3*Y1+X3**2
1  )*Y2+(-X1**2+2*Y3*X1-X3**2)*Y1)
  HLS5  = -((X1-X3+Y1)*Y2+(-Y1-3*X3)*Y1)/(X1**2-2*X3*Y1+Y3**2)
1  *Y2+(-X1**2+2*X3*X4-X3**2)*Y1)
  HLS5  = ((X1-12*X3+12*Y1)*Y2+(-X1*Y4-4*Y3)*Y1)/(X1**2-2*Y3*Y4
1  +Y3**2)*Y2+(-X1**2+2*X3*Y1-(3**2)*Y1)
  HLS7  = -15*Y2/((X1-X3)*Y2+(X3-X1)*Y1)
  LLS7  = 2/(Y4**2-2*X3*Y1+X3**2)
  LLS4  = 2/(Y4**2-2*Y3*X1+X3**2)
  LLS5  = -1/(X1**2-2*X3*Y4+X3**2)
  RETURN
END

```


SUBROUTINE F12T (M,N,NP,NP2)

THIS SUBROUTINE COMPUTES THE INFLUENCE
COEFFICIENTS FOR 2NF EQUATIONS RESULTING
FROM FINDING THE HYDRAULIC FLOW
CONDITION AT 2NF CONTROL POINTS.

COMMON/BLUOK/AA(12,12),OR(12),OTR(12),SUM(6),ALPHA(1),I
COMMON/BLUOK/VEL(2),IT(2)

COMMON/BLUOK/AL(1,2,3,4,5,6,7,8,9,10,11,12),AL(2,3,4,5,6,7,8,9,10,11,12),AL(3,4,5,6,7,8,9,10,11,12),AL(4,5,6,7,8,9,10,11,12),AL(5,6,7,8,9,10,11,12),AL(6,7,8,9,10,11,12),AL(7,8,9,10,11,12),AL(8,9,10,11,12),AL(9,10,11,12),AL(10,11,12),AL(11,12),AL(12)

1 CLD1,CLD2,CLD3,CLD4,CLD5,CLD6,CLD7,CLD8,CLD9,CLD10,CLD11,CLD12,
2 CLD1,CLD2,CLD3,CLD4,CLD5,CLD6,CLD7,CLD8,CLD9,CLD10,CLD11,CLD12,
3 CLD1,CLD2,CLD3,CLD4,CLD5,CLD6,CLD7,CLD8,CLD9,CLD10,CLD11,CLD12,
4 CLD1,CLD2,CLD3,CLD4,CLD5,CLD6,CLD7,CLD8,CLD9,CLD10,CLD11,CLD12,
5 CLD1,CLD2,CLD3,CLD4,CLD5,CLD6,CLD7,CLD8,CLD9,CLD10,CLD11,CLD12,
6 CLD1,CLD2,CLD3,CLD4,CLD5,CLD6,CLD7,CLD8,CLD9,CLD10,CLD11,CLD12,
7 CLD1,CLD2,CLD3,CLD4,CLD5,CLD6,CLD7,CLD8,CLD9,CLD10,CLD11,CLD12,
8 LLD1,LLD2,LLD3,
9 LTD1,LTD2,LTD3,LTD4,LTD5,LTD6,LTD7,LTD8,LTD9,LTD10,LTD11,LTD12,
A LTD1,LTD2,LTD3,LTD4,LTD5,LTD6,LTD7,LTD8,LTD9,LTD10,LTD11,LTD12,
B LTD1,LTD2,LTD3,LTD4,LTD5,LTD6,LTD7,LTD8,LTD9,LTD10,LTD11,LTD12,
C LTD1,LTD2,LTD3,LTD4,LTD5,LTD6,LTD7,LTD8,LTD9,LTD10,LTD11,LTD12,
D LTD1,LTD2,LTD3,
E LTD1,LTD2,LTD3,LTD4,LTD5,LTD6,LTD7,LTD8,LTD9,LTD10,LTD11,LTD12,
F LTD1,LTD2,LTD3,LTD4,LTD5,LTD6,LTD7,LTD8,LTD9,LTD10,LTD11,LTD12,
G LTD1,LTD2,LTD3,LTD4,LTD5,LTD6,LTD7,LTD8,LTD9,LTD10,LTD11,LTD12,
H LTD1,LTD2,LTD3

REAL LLD1,LLD2,LLD3,LLD4,LLD5,LLD6,LLD7,
1 LLD1,LLD2,LLD3,LLD4,LLD5,LLD6,LLD7,
2 LLD1,LLD2,LLD3,LLD4,LLD5,LLD6,LLD7,
3 LLD1,LLD2,LLD3,
4 LTD1,LTD2,LTD3,LTD4,LTD5,LTD6,LTD7,LTD8,
5 LTD1,LTD2,LTD3,
6 LTD1,LTD2,LTD3,LTD4,LTD5,LTD6,LTD7,LTD8,
7 LTD1,LTD2,LTD3

REAL MACH,1,II
COMMON/BLUOK/XY(X),Y(X,Z),X(Z),Y(Z),R(1,2),CY,CX,XY1,CY2,
1CY1,CY2,YC(X),YD(X),YD(X),YD(X),YD(X)
COMMON/BLUOK/MS,MS2,C(12),C(12),C(12),C(12),C(12),C(12),C(12),C(12),C(12),C(12),C(12),C(12)
INTEGER D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12,D13,D14,D15,D16,D17,D18,D19,D20,D21
PI=3.141592
M2=1
MACH=SQRT(1-MACH**2)

WRITE OUT THE HYDRODYNAMIC INFLUENCE
AND CONTINUITY EQUATION COEFFICIENT ARRAY.

DO 10 I=1,NP2
DO 5 J=1,NP2
A(I,I)=1
5 CONTINUE
10 CONTINUE

START MASTER LOOP. THIS LOOP IS DONE TWICE,
ONCE FOR EACH SET OF CONTROL POINTS.

```

DO 10 I=1,2
WRITE*, "STARTING LOOP 10 ", I
    SELECT APPROPRIATE CONTROL POINT.
DO 11 J=1,4
WRITE*, "STARTING LOOP 11 ", J
    IF (II.EQ.0) GO TO 111
    L=J
    XC=YC1(J)
    YCJ=YC2(J)
    GO TO 112
111 CONTINUE
    L=2+J
    IF (L.EQ.4) GO TO 334
    YC1=YC2(J)
    YC2=YC3(J)
112 CONTINUE
    WITH THE CONTROL POINT ON PANEL J,
    COMPUTE INDUCED VELOCITY AT THAT POINT
    BY ALL OTHER PANELS.
DO 12 I=1,4
WRITE*, "STARTING LOOP 12 ", I
    APPLY A LINEAR TRANSFORMATION TO THE
    COORDINATES OF PANEL I WHERE THE NEW
    ORIGIN IS (XCJ,YCJ). APPLY PARABOLIC-GLAUCERT.
    X1=(Y (I,1)-YCJ)/AMCH
    X2=(Y (I,2)-YCJ)/AMCH
    X3=(Y (I,3)-XCJ)/AMCH
    X4=(X (I,1)-YCJ)/AMCH
    DETERMINE NOCAL UNKNOWN FOR PANEL I.
    CALL MODLUP (M,N,I)
    CALCULATE LEADING AND TRAILING EDGE CHECK
    PARAMETERS.
    N1=I-(I/2)+1
    J1=I-(I/2)+1
    FOLLOWING LOOP ACCOUNTS FOR INDUCED
    VELOCITY DUE TO PANEL I (K=1) AND THE
    IMAGE OF PANEL I (K=2).
DO 13 K=1,2
WRITE*, "STARTING LOOP 13 ", K
    J1M=(-1.)-K
    Y1=Y (I,1)+YCJ*J1M
    Y2=Y (I,2)+YCJ*J1M

```



```

A(L,03)=A(L,02)+(A(1,03)*IT(1)-A(2,03)*IT(2)-A(3,03)*IT(3))/2
A(L,04)=A(L,03)+(A(1,04)*IT(1)+A(2,04)*IT(2)+A(3,04)*IT(3)-A(4,04)*IT(4)-A(5,04)*IT(5))/2
1-A(1,04)*IT(1)+A(2,04)*IT(2)-A(3,04)*IT(3)+A(4,04)*IT(4)-A(5,04)*IT(5))/2
A(L,05)=A(L,04)+(A(1,05)*IT(1)+A(2,05)*IT(2)+A(3,05)*IT(3)-A(4,05)*IT(4)-A(5,05)*IT(5))/2
1-A(1,05)*IT(1)+A(2,05)*IT(2)-A(3,05)*IT(3)+A(4,05)*IT(4)-A(5,05)*IT(5))/2
WRITE(*,*)
IF(A(1,04,03) GO TO 129
A(L,06)=A(L,05)+(A(1,06)*IT(1)+A(2,06)*IT(2)+A(3,06)*IT(3)-A(4,06)*IT(4)-A(5,06)*IT(5))/2
1-A(1,06)*IT(1)+A(2,06)*IT(2)-A(3,06)*IT(3)+A(4,06)*IT(4)-A(5,06)*IT(5))/2
A(L,07)=A(L,06)+(A(1,07)*IT(1)+A(2,07)*IT(2)+A(3,07)*IT(3)-A(4,07)*IT(4)-A(5,07)*IT(5))/2
1-A(1,07)*IT(1)+A(2,07)*IT(2)-A(3,07)*IT(3)+A(4,07)*IT(4)-A(5,07)*IT(5))/2
WRITE(*,*)
129 CONTINUE
177 CONTINUE
178 CONTINUE
179 CONTINUE
180 CONTINUE
181 CONTINUE
182 CONTINUE
183 CONTINUE
I=2
L=3
X1=X(2,1)
X2=X(2,2)
X3=X(2,3)
Y1=X(3,1)
Y2=X(3,2)
CALL NORMUM (Y, I, I)
CALL COEFF (X1, X2, X3, Y1, Y2)
A(L,08)=0.75
A(L,09)=0.75
A(L,01)=0.75+2.0*K751*X2
A(L,02)=0.75+2.0*K752*X2
184 RETURN
END

```

```

SUBROUTINE COMI (M,N,MP)
COMMON/BL01/KC/A(12,12),S(12,1),D3R(5,1),SUM(1),ALPH(1),13
COMMON/BL02/KL/ALD1,ALD2,ALD3,ALD4,ALD5,ALD6,ALD7,ALD8,ALD9,ALD0,
1  ALD11,ALD12,ALD13,ALD14,ALD15,ALD16,ALD17,ALD18,ALD19,ALD20,
2  ALD21,ALD22,ALD23,ALD24,ALD25,ALD26,ALD27,ALD28,ALD29,ALD30,
3  ILD1,ILD2,ILD3,ILD4,ILD5,ILD6,ILD7,ILD8,
4  JLD1,JLD2,JLD3,JLD4,JLD5,JLD6,JLD7,
5  KLD1,KLD2,KLD3,KLD4,KLD5,KLD6,KLD7,
6  GLD1,GLD2,GLD3,GLD4,GLD5,GLD6,
7  HLD1,HLD2,HLD3,HLD4,HLD5,HLD6,HLD7,
8  LLD1,LLD2,LLD3,
9  ATD1,ATD2,ATD3,ATD4,ATD5,ATD6,ATD7,ATD8,ATD9,
A  STD1,STD2,STD3,STD4,STD5,STD6,STD7,STD8,STD9,STD10,
B  ETD1,ETD2,ETD3,ETD4,ETD5,ETD6,ETD7,ETD8,ETD9,ETD10,
C  ITD1,ITD2,ITD3,ITD4,ITD5,ITD6,ITD7,ITD8,
D  JTD1,JTD2,JTD3,
E  KTD1,KTD2,KTD3,KTD4,KTD5,KTD6,KTD7,
F  GTD1,GTD2,GTD3,GTD4,GTD5,GTD6,GTD7,
G  HTD1,HTD2,HTD3,HTD4,HTD5,HTD6,HTD7,
H  LTD1,LTD2,LTD3
REAL ILD3,ILD4,ILD5,ILD6,ILD7,ILD8,ILD9,ILD10,ILD11,ILD12,
1  JLD3,JLD4,JLD5,JLD6,JLD7,JLD8,JLD9,JLD10,JLD11,JLD12,
2  KLD3,KLD4,KLD5,KLD6,KLD7,KLD8,KLD9,KLD10,KLD11,KLD12,
3  LLD3,LLD4,LLD5,
4  ITD1,ITD2,ITD3,ITD4,ITD5,ITD6,ITD7,ITD8,
5  JTD1,JTD2,JTD3,
6  KTD1,KTD2,KTD3,KTD4,KTD5,KTD6,KTD7,
7  LTD1,LTD2,LTD3
COMMON/BL03/KX/1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18
INTEGER S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,S11,S12,S13,S14,S15,S16,S17,S18,S19,S20
REAL MACH
COMMON/BL04/KY/Y(1,1),Y(1,2),Y(1,3),Y(1,4),Y(1,5),Y(1,6),Y(1,7),Y(1,8),Y(1,9),Y(1,10),
10X Y(1,11),Y(1,12),Y(1,13),Y(1,14),Y(1,15),Y(1,16),Y(1,17),Y(1,18),Y(1,19),Y(1,20)
COMMON/BL05/KZ/S,S(1),S(2),S(3),S(4),S(5),S(6),S(7),S(8),S(9),S(10),S(11),S(12),S(13),S(14),S(15),S(16),S(17),S(18),S(19),S(20),MACH
MCH=
MACH=SQRT(2-MACH**2)
DO I=1,18
BASE EQUATION NUMBER
L=2*M*P+I-3
FOUR EQUATION NUMBERS.
L1=L
L2=L+1
L3=L+2
L4=L+3
EIGHT-EVEN COORDINATES.
X1=X(1,1)/MACH
Y1=Y(1,1)/MACH
Y2=Y(1,2)/MACH
Y3=X(1,3)/MACH
Y4=X(1,4)/MACH
Y5=X(1,5)/MACH
Y6=X(1,6)/MACH
Y7=X(1,7)/MACH
Y8=X(1,8)/MACH
Y9=X(1,9)/MACH
Y10=X(1,10)/MACH
Y11=X(1,11)/MACH
Y12=X(1,12)/MACH
Y13=X(1,13)/MACH
Y14=X(1,14)/MACH
Y15=X(1,15)/MACH
Y16=X(1,16)/MACH
Y17=X(1,17)/MACH
Y18=X(1,18)/MACH
Y19=X(1,19)/MACH
Y20=X(1,20)/MACH

```

```

Y2=Y(T,2)
Y3=Y(T,3)
3
3
3      COMPUTE APPLICABLE NOIAL VALUES.
3
3      CALL MODNUM (N,N,2)
3      COMPUTE GEOMETRIC COEFFICIENTS.
3
3      CALL COEFF (X1,X2,X3,Y1,Y2)
3
3      COMPUTE LEADING AND TRAILING EDGE CHECK PARAMETERS.
3
3      J3=I- (I/ )**2-1
3      J3=I- (I/ )**2
3
3      IF TIP PANEL GO TO TIP SECTION
3
3      IF (I,GT,(N-1)) GO TO 11
3      IF (I,LT,1) GO TO 11
3      Y=Y(T,I,2)
3      X1=Y (I+1,2)/ANOM
3
3      A(L1, G1)=(-GT1-ITD1*X4-ITD1*Y3-JTD1*X3+Y3-KTD1*Y3**2)-(X3-X1)
3      1/(Y2-Y1)
3      A(L1, G2)=(-LDS+LDS*X3+ILD3*Y3+JLD3*X3+Y3+KLD3*Y3**2)*(X1-X3)
3      1/(Y4-Y2)
3      A(L2, G1)=(-LDS1+ILD1*Y3+ELD1*Y3**2-ATD1+ITD1*X3-OTD1*Y3+2*KTD1*Y3
3      1**2/2)*(X3-Y1)/(Y2-Y1)
3      A(L2, G2)=(-LDS-ILD3*X3+JLD3*Y3-2*KLD3*X3*Y3+ELD3*Y3**2-JLD3*Y3**2
3      1**2/2)*(X1-Y3)/(Y2-Y1)
3      A(L2, G3)=(-LDS+JLD3*Y3+ELD3*Y3**2)-(Y3-X1)/(Y2-Y1)
3      A(L3, G1)=(-LDS+JLD3*Y3+ELD3*Y3**2)*(X1-X1)/(Y2-Y1)
3      A(L3, G2)=(-LDS-ILD3*X3+JLD3*Y3-2*KLD3*X3*Y3+ELD3*Y3**2-JLD3*Y3
3      1**2/2)*(Y1-Y3)/(Y2-Y1)
3      A(L3, G3)=(-LDS+JLD3*Y3+ELD3*Y3**2)-(Y3-X1)/(Y2-Y1)
3      A(L4, G1)=(-GT1+ITD1*X4+ITD1*Y3+JTD1*X3+Y3+KTD1*Y3**2)*(X3-X1)/
3      1/(Y2-Y1)
3      IF (I,GT,1) GO TO 12
3      A(L1, G1)=(-GT1-ITD1*X4-ITD1*Y3-JTD1*X3+Y3-KTD1*Y3**2)-(Y3-X1)
3      1/(Y2-Y1)/2
3      A(L2, G1)=(-LDS+JLD3*Y3+ELD3*Y3**2+KTD3*X3*Y3-ITD3*Y3**2+JTD3*X3
3      1**2/2)*(X3-Y1)/(Y2-Y1)/2
3      A(L3, G1)=(GT1+ITD1*Y3+ITD1*Y2+JTD3*X3+Y2+KTD3*Y3**2)*(Y2-X1)
3      1/(Y2-Y1)/2
3      GO TO 12.
11 CONTINUE
3      A(L1, G1)=(-LDS+LDS*X3+ILD3*Y3+JLD3*X3+Y3+KLD3*Y3**2)
3      A(L2, G1)=(-LDS-ILD3*X3+JLD3*Y3-2*KLD3*X3*Y3+ELD3*Y3**2-JLD3*Y3
3      1**2/2)
3      A(L2, G2)=(-LDS+JLD3*Y3+ELD3*Y3**2)
3      A(L3, G1)=(-LDS-ILD3*Y3+JLD3*Y3-2*KLD3*X3*Y3+ELD3*Y3**2-JLD3*Y3
3      1**2/2)
3      A(L3, G2)=(-LDS+JLD3*Y3+ELD3*Y3**2)
3      IF (I,GT,1) GO TO 12
3      A(L1, G1)=(-GT1-ITD1*X4-ITD1*Y3-JTD1*X3+Y3-KTD1*Y3**2)
3      A(L2, G1)=(-LDS+JLD3*Y3+ELD3*Y3**2-ATD1+ITD1*X3-OTD1*Y3+2*KTD1*Y3
3      1**2/2)

```

$A(L3, G1) = (ALD1 + CLD1 * Y2 + LD1 * Y2 * Z)$
 $A(L3, G2) = (GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z)$
100 00HTAME
 $A(L1, G1) = (GLD1 + HLD1 * X + ILD1 * Y2 + JLD1 * X * Y2 + KLD1 * Y2 * Z)$
 $A(L1, G2) = (-GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z - LT01 * X * Y2)$
 $A(L1, G3) = (GLG1 + HLG1 * X + ILG1 * Y2 + JLG1 * X * Y2 + KLG1 * Y2 * Z + LUG1 * X * Y2)$
 $A(L1, G4) = A(L1, G1) + (GLG1 + HLG1 * X + ILG1 * Y2 + JLG1 * X * Y2 + KLG1 * Y2 * Z + LUG1 * X * Y2)$
1-GLG1 + HTG1 * X - ITG1 * Y2 - JTG1 * X * Y2 - LUG1 * X * Y2
 $A(L1, G5) = A(L1, G3) + (GLG1 + HLG1 * X + ILG1 * Y2 + JLG1 * X * Y2 + KLG1 * Y2 * Z + LUG1 * X * Y2)$
1LLG1 * Y2 * Z
 $A(L1, G7) = (GLG7 + HLG7 * X + ILG7 * Y2 + JLG7 * X * Y2 + KLG7 * Y2 * Z)$
 $A(L2, G6) = (-GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z)$
 $A(L2, G8) = (GLD1 + HLD1 * X + ILD1 * Y2 + JLD1 * X * Y2 + KLD1 * Y2 * Z + LLD1 * Y2 * Z)$
1**2/2
 $A(L2, G1) = (-GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z)$
 $A(L2, G2) = A(L2, G1) + (-AT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z - LT01 * X * Y2)$
 $A(L2, G3) = (GLG1 + HLG1 * X + ILG1 * Y2 + JLG1 * X * Y2 + KLG1 * Y2 * Z + LUG1 * X * Y2)$
1**2/2
 $A(L2, G4) = A(L2, G1) + (GLG1 + HLG1 * X + ILG1 * Y2 + JLG1 * X * Y2 + KLG1 * X * Y2 + LUG1 * X * Y2)$
1- JLG1 * X * Y2 - LUG1 * X * Y2 + ITG1 * X * Y2 + JTG1 * X * Y2 + KTG1 * X * Y2 - LTG1 * X * Y2
 $A(L2, G5) = A(L2, G3) + (GLG1 + HLG1 * X + ILG1 * Y2 + JLG1 * X * Y2 + KLG1 * X * Y2 + LUG1 * X * Y2)$
1- JLG1 * X * Y2 + LUG1 * X * Y2
 $A(L2, G7) = A(L2, G2) + (GLG7 + HLG7 * X + ILG7 * Y2 + JLG7 * X * Y2 + KLG7 * X * Y2 + LUG7 * X * Y2)$
1- JLG7 * X * Y2 + LUG7 * X * Y2
 $A(L3, G6) = -1$
 $A(L3, G8) = (GLG1 + HLG1 * X + ILG1 * Y2 + JLG1 * X * Y2 + KLG1 * X * Y2 + LUG1 * X * Y2)$
1**2/2
 $A(L3, G4) = (GLG1 + HLG1 * X + ILG1 * Y2 + JLG1 * X * Y2 + KLG1 * X * Y2 + LUG1 * X * Y2)$
1**2/2
 $A(L3, G1) = A(L3, G4) + (GLG1 + HLG1 * X + ILG1 * Y2 + JLG1 * X * Y2 + KLG1 * X * Y2 + LUG1 * X * Y2)$
1- JLG1 * X * Y2 + LUG1 * X * Y2
 $A(L3, G3) = A(L3, G4) + (GLG1 + HLG1 * X + ILG1 * Y2 + JLG1 * X * Y2 + KLG1 * X * Y2 + LUG1 * X * Y2)$
1- JLG1 * X * Y2 + LUG1 * X * Y2
 $A(L3, G7) = A(L3, G2) + (GLG7 + HLG7 * X + ILG7 * Y2 + JLG7 * X * Y2 + KLG7 * X * Y2 + LUG7 * X * Y2)$
1- JLG7 * X * Y2 + LUG7 * X * Y2
 $A(L4, G1) = A(L4, G2) + (GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z + LT01 * X * Y2)$
 $A(L4, G3) = A(L4, G1) + (GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z + LT01 * X * Y2)$
Z(I, LT, (I+1), G, 100, 1)
 $A(L1, G2) = (-GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z)$
 $A(L1, G5) = (-GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z)$
 $A(L2, G2) = (-GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z - LT01 * X * Y2)$
1**2/2
 $A(L2, G7) = (-GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z + LT01 * X * Y2)$
1**2/2
 $A(L4, G2) = (GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z)$
 $A(L4, G3) = (GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z)$
100 00HTAME
 $A(L1, G1) = (GLD1 + HLD1 * X + ILD1 * Y2 + JLD1 * X * Y2 + KLD1 * Y2 * Z + LLD1 * X * Y2)$
 $A(L1, G2) = (-GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z - LT01 * X * Y2)$
 $A(L1, G4) = (-GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z)$
 $A(L2, G2) = (-GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z - LT01 * X * Y2)$
 $A(L2, G4) = (-GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z - LT01 * X * Y2)$
1**2/2
 $A(L2, G1) = (-GT01 + HT01 * X + IT01 * Y2 + JT01 * X * Y2 + KT01 * Y2 * Z)$
 $A(L3, G4) = (GLG1 + HLG1 * X + ILG1 * Y2 + JLG1 * X * Y2 + KLG1 * X * Y2 + LUG1 * X * Y2)$
1**2/2


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A(L4, G2) = (GTG2+HTG2*Y4+ITG2*Y2+KTG2*Y2**2+LTG2*X4**2)
A(L4, G1) = -1.
A(L4, G3) = (GTG3+HTG3*Y2+KTG3*Y2**2)
GO TO 15.
21 CONTINUE
IF(NF.NE.MO) GO TO 211
A(L1, G1) = (-GT11+HT11*Y1-IT11*Y3-J111*X1*Y3-K111*Y3**2)*(X3-X1)
1/(Y2-Y1)
A(L2, G1) = (-AL11+BL11*Y3+CL11*Y3**2-AT11+IT11*Y3-OT11*Y3**2+KT11*X1*Y3
1*Y3-IT11*Y3**2+JT11*X3**2/2)-(G3-Y1)/(Y2-Y1)
A(L2, G2) = (-AL11+BL11*Y3+CL11*Y3**2)*(Y2-Y1)/(Y2-Y1)
A(L3, G1) = (-AL11+BL11*Y3+CL11*Y3**2)*(Y3-Y1)/(Y2-Y1)
A(L3, G2) = (-AL11+BL11*Y3+CL11*Y3**2)*(X2-X1)/(X2-Y1)
A(L4, G1) = (-AT11+IT11*X1+JT11*Y3+KT11*X1*Y3**2+LT11*Y3**2)*(X3-X1)/
1*(Y2-Y1)
GO TO 22.
210 CONTINUE
A(L1, G1) = (-GT11+HT11*Y1-IT11*Y3-J111*X1*Y3-K111*Y3**2)
A(L1, G2) = (GL11+HL11*X1+IL11*Y3+JL11*X1*Y3**2+KL11*Y3**2)
A(L2, G1) = (-AL11+BL11*Y3+CL11*Y3**2-AT11+IT11*Y3-OT11*Y3**2+KT11*X1*Y3
1*Y3**2+JT11*X3**2/2)
A(L2, G2) = (-AL11+BL11*Y3+CL11*Y3**2-AT11+IT11*Y3-OT11*Y3**2+KT11*X1*Y3
1**2/2)
A(L3, G1) = (-AL11+BL11*Y3+CL11*Y3**2)
A(L3, G2) = (-AL11+BL11*Y3+CL11*Y3**2)
A(L3, G3) = (-AL11+BL11*Y3+CL11*Y3**2-AT11+IT11*Y3-OT11*Y3**2+KT11*X1*Y3
1**2/2)
A(L4, G1) = (GT11+HT11*Y1+IT11*Y3+JT11*X1*Y3**2+KT11*Y3**2)
211 CONTINUE
A(L1, G2) = (-GL11+HL11*X1+IL11*Y3+JL11*X1*Y3**2+KL11*Y3**2)
A(L1, G3) = (-GL11+HL11*X1+IL11*Y3+JL11*X1*Y3**2+KL11*Y3**2)
A(L1, G1) = A(L1, G1) + (GL11+HL11*X1+IL11*Y3+JL11*X1*Y3**2+KL11*Y3**2)
1-GT11-HT11*Y1-IT11*Y3-JT11*X1*Y3**2-LT11*Y3**2)
A(L1, G2) = (GL11+HL11*X1+IL11*Y3+JL11*X1*Y3**2+KL11*Y3**2)
A(L2, G1) = (-AT11+IT11*Y3-OT11*Y3**2)
A(L2, G2) = (-AL11+BL11*Y3+CL11*Y3**2-AT11+IT11*Y3-OT11*Y3**2+KT11*X1*Y3
1**2/2)
A(L2, G3) = (-AT11+IT11*Y3-OT11*Y3**2)
A(L2, G1) = A(L2, G1) + (AL11+BL11*Y3+CL11*Y3**2-AT11+IT11*Y3-OT11*Y3**2)
1-JL11*Y3**2/2)
A(L3, G1) = A(L2, G1) + (AL11+BL11*Y3+CL11*Y3**2-AT11+IT11*Y3-OT11*Y3**2)
1-JL11*Y3**2/2)
A(L3, G2) = A(L2, G2) + (AL11+BL11*Y3+CL11*Y3**2-AT11+IT11*Y3-OT11*Y3**2)
1-JL11*Y3**2/2)
A(L3, G3) = -1.
A(L3, G1) = (-AL11+BL11*Y3+CL11*Y3**2-AT11+IT11*Y3-OT11*Y3**2+KT11*X1*Y3
1**2/2)
A(L4, G1) = (GT11+HT11*Y1+IT11*Y3+JT11*X1*Y3**2+KT11*Y3**2)
A(L4, G2) = A(L4, G1) + (GT11+HT11*Y1+IT11*Y3+JT11*X1*Y3**2+KT11*Y3**2)
1-GL11*Y3**2/2-AT11+IT11*Y3-OT11*Y3**2+KT11*X1*Y3**2-LT11*Y3**2)
A(L1, G1) = (-GT11+HT11*Y1-IT11*Y3-JT11*X1*Y3**2+KT11*Y3**2)
A(L2, G2) = (-AT11+IT11*Y3-OT11*Y3**2+KT11*X1*Y3**2+JT11*Y3**2)

```

```

A(L2, G5) = (-ATG5 + ITG5 * X5 - OTG5 * Y5 + 2 * KTJ5 * X5 * Y5 - ETG5 * Y5 ** 2 + JTG5 * Y5
1 ** 2 / 2)
A(L4, G2) = (GTG2 + HTG2 * X4 + ITJ2 * Y2 + JTG2 * Y4 + Y2 + KTG2 * Y2 * 2)
A(L4, G3) = (GTG3 + HTG3 * X4 + ITJ3 * Y2 + JTG3 * X4 * Y2 + K(TG3 * Y2 * 2)
IF(N1, 20, 40) GO TO 20
A(L1, G2) = (-ATG2 - HTG2 * X2 - ITG2 * Y2 - KTG2 * Y2 * 2 - LTG2 * X2 * 2)
A(L1, G3) = (-OTG2 - ITG2 * Y2 - KTG2 * Y2 * 2)
A(L2, G2) = (-LTG2 + ITG2 * Y2 - OTG2 * Y2 + 2 * KTG2 * X2 * Y2 - ETG2 * Y2 * 2)
A(L2, G3) = (-ATG3 + LTG3 * X2 - OTG3 * Y2 + 2 * KTG3 * X2 * Y2 - ETG3 * Y2 * 2)
A(L4, G2) = (GTG2 + ITG2 * X4 + JTG2 * Y2 + KTG2 * Y2 * 2 + LTG2 * X4 * 2)
A(L4, G3) = (GTG3 + ITG3 * Y4 + JTG3 * Y2 + KTG3 * Y2 * 2)
27 CONTINUE
28 CONTINUE
17 CONTINUE
DO 500 I=1,500
WRITE (5, 150) (A(I, J), J=1, 4)
WRITE (5, 160) (A(I, J), J=1, 12)
WRITE (5, 170) (A(I, J), J=1, 2)
WRITE (5, 180) (A(I, J), J=2, 30)
WRITE (5, 190) (A(I, J), J=3, 45)
WRITE (5, 200) (A(I, J), J=4, 54)
619 FORMAT ('(2X, 112, 5)')
WRITE (5, 21)
627 FORMAT('///')
608 CONTINUE
RETURN
END

```



```

CALL SYMBOL (-.10, .25, .25, DHPLOYFORM, 3, .9)
CALL SYMBOL (-.10, .25, .25, DHPLOYFORM, 3, .9, 12)
CALL NUMBER (999, .5, .5, DHPLOY, 3, .1)
CALL SYMBOL (.25, -.10, .25, DHPLOY (DHPLOY, .9))
DOKI = 1. / DHPLOY

THIS LOOP DRAWS AND LABELS EACH OF THE PANELS.
DO 2 K=1, NP
  CALL PLOT (X(K,1)*DOKI, Y(K,1)*DOKI, 2)
  CALL PLOT (X(K,2)*DOKI, Y(K,1)*DOKI, 2)
  CALL PLOT (Y(K,1)*DOKI, Y(K,2)*DOKI, 2)
  CALL PLOT (Y(K,2)*DOKI, Y(K,2)*DOKI, 2)
  CALL PLOT (Y(K,1)*DOKI, Y(K,1)*DOKI, 2)
  CALL SYMBOL (Y(K,1)*DOKI, Y(K,1)*DOKI, .5, 3, .1, -1)
  CALL NUMBER (X(K,2)*DOKI+.7, Y(K,2)*DOKI-.15, .75,
1  PLOT (K), .5, -1)
2 CONTINUE
CONTINUE
3 SET DAMPER SLOPE EQUAL TO ZERO.
DO 1 I=1, NP
  CRAT = .
1 CONTINUE
3 READ NA SETS OF ANGLE OF ATTACK AND
  DAMPER SLOPE DISTRIBUTION DATA.
DO 31 L=1, NA
  READ (8,*) ALPHA(L), NCOG, NPF, CY, CX
31
31 NCOG IS THE DAMPER CHANGE PARAMETER.
  ENTER 0 TO READ A NEW DAMPER SLOPE DISTRIBUTION OR ENTER 1 TO RETAIN THE PREVIOUS
  DISTRIBUTION.
31 NPF IS THE PRESSURE OPTION PARAMETER.
  ENTER 0 TO USE THE EXACT IDENTIFIED
  EXPRESSION OR ENTER 1 TO USE THE LINEAR-
  IZED FORM.
31 COMPUTE PRESSURES AT PANEL LOCATION CY
  AND CX. NOTE - PRESSURES MAY BE COMPUTED
  AT POINTS OTHER THAN THE CONTROL POINTS.
31 CALCULATE PRESSURE EVALUATION POINTS.
DO 17 I=1, NP
  YC(I) = (1-CY)*Y(I,1) + CY*Y(I,2)
17 YC(I) = CY*X(I,1) + (1-CY)*Y(I,1) + CY*(CY*(X(I,1)-X(I,2)) + (1-CY)*
  (X(I,2)-Y(I,1)))
  IF (NCOG.EQ.1) GO TO 2
  READ(9,*) (CB(I), I=1, NP)
2 CONTINUE
3 FORMULATE LINEARIZED FORM OF THE FLOW
  TANGENCY BOUNDARY CONDITION. SUM(I)
  REPRESENTS THE LINEARIZED 7 COMPONENT OF
  VELOCITY FOR PANEL I.
DO 3 I=1, NP
  DFCGRAD = 7.075 / 795
  SUM(I) = 576.0 * (ATAN(CB(I) / 795.0) - ALPHA(L) / 57.6)
3 CONTINUE
DO 31 I=1, NP

```


3
3

ARE FOR ALL NON TIP PANELS.

```
IF(NP,NE,NO) GO TO 111  
Y1=Y(I+1,2)  
X1=Y(I+1,3)/A-CH  
DEL1=(X3-Y1)/(Y2-Y1)*SG(G1)  
WRITE *, "DEL1", DEL1  
DEL3=(X1-Y3)/(Y1-Y2)*SG(G3)  
WRITE *, "DEL3", DEL3  
DEL7=(X3-Y1)/(Y2-Y1)*SG(G7)  
WRITE *, "DEL7", DEL7  
IF(I,GT,1) GO TO 12  
DEL5=(X3-Y1)/(Y2-Y1)/2*SG(G5)  
GO TO 12  
111 CONTINUE  
DEL3=SG(G3)  
WRITE *, "DEL3", DEL3  
DEL7=SG(G7)  
WRITE *, "DEL7", DEL7  
IF(I,LT,(N+1)) GO TO 121  
DEL1=SG(G1)  
WRITE *, "DEL1", DEL1  
112 CONTINUE  
DEL4=SG(G4)  
WRITE *, "DEL4", DEL4  
DEL5=SG(G5)  
DEL8=SG(G8)  
WRITE *, "DEL8", DEL8  
GAM1=SG(G1)  
WRITE *, "GAM1", GAM1  
GAM3=SG(G3)  
WRITE *, "GAM3", GAM3  
GAM5=SG(G5)  
WRITE *, "GAM5", GAM5  
GAM6=SG(G6)  
WRITE *, "GAM6", GAM6  
GAM7=SG(G7)  
WRITE *, "GAM7", GAM7  
IF(I,LT,(N+1)) GO TO 121  
DEL2=SG(G2)  
WRITE *, "DEL2", DEL2  
DEL6=SG(G6)  
113 CONTINUE  
IF(NP,EO,NO) GO TO 310  
GAM2=SG(G2)  
WRITE *, "GAM2", GAM2  
GAM4=SG(G4)  
WRITE *, "GAM4", GAM4  
GAM8=SG(G8)  
WRITE *, "GAM8", GAM8  
GO TO 310  
311 CONTINUE
```

3
3
3
3

LOCAL VALUES DETERMINED IN THIS
ARE FOR NING TIP PANELS.

IF(NP,NE,NO) GO TO 310

```

DEL1=(X3-X1)/(Y2-Y1)*SG(G1)
WRITE*, "DEL1", DEL1
DEL7=(X3-X1)/(Y2-Y1)*SG(G7)
WRITE*, "DEL7", DEL7
GO TO 72
3) CONTINUE
DEL1=SG(G1)
WRITE*, "DEL1", DEL1
DEL7=SG(G7)
WRITE*, "DEL7", DEL7
DEL7=SG(G7)
WRITE*, "DEL7", DEL7
3) CONTINUE
GA41=SG(G1)
WRITE*, "GA41", GA41
GA47=SG(G7)
WRITE*, "GA47", GA47
GA45=SG(G5)
WRITE*, "GA45", GA45
DEL4=SG(G4)
WRITE*, "DEL4", DEL4
DEL5=SG(G5)
WRITE*, "DEL5", DEL5
DEL9=SG(G9)
WRITE*, "DEL9", DEL9
DEL2=SG(G2)
WRITE*, "DEL2", DEL2
DEL3=SG(G3)
IF(NR, EQ, NC) GO TO 410
GA42=SG(G2)
WRITE*, "GA42", GA42
GA44=SG(G4)
WRITE*, "GA44", GA44
4) CONTINUE

      COMPUTE NODAL GEOMETRIC COEFFICIENTS.
      CALL COEFF (X1, X2, X3, X4, Y1, Y2)

      COMPUTE COEFFICIENTS REQUIRED FOR GAMMA.

      GL=GL03*DEL3+GL06*DEL6+GL01*GA41+GL03*GA43+GL04*GA44+GL05*GA45
      1+GL07*GA47

      HL=HL03*DEL3+HL06*DEL6+HL01*GA41+HL03*GA43+HL04*GA44+HL05*GA45
      1+HL07*GA47

      LI=LI03*DEL3+LI06*DEL6+LI01*GA41+LI03*GA43+LI04*GA44+LI05*GA45
      1+LI07*GA47

      JL=JL03*DEL3+JL06*DEL6+JL01*GA41+JL03*GA43+JL04*GA44+JL05*GA45
      1+JL07*GA47

      KL=KL03*DEL3+KL06*DEL6+KL01*GA41+KL03*GA43+KL04*GA44+KL05*GA45
      1+KL07*GA47

      LL=LL03*GA43+LL04*GA44+LL05*GA45

```



```

      CM1=CM1+CM(N)
77  CONTINUE
      XC=(X1/CL1)*CHORD(N)
      CLC(N)=CL1/CLC(N)
      CM=CM1/(CHORD(N)**2)
77  WRITE(7,7) YC(N),XCP,THROD(N),CLC(N),CM,NS
79  FORMAT(10X,F8.2,FX,F8.2,FX,F7.2,2X,F8.3,FX,F8.3,10)
79  CONTINUE
79  CONTINUE
      WING TIP CONDITIONS
      NOTE CHORD LENGTH AT WING TIP IS AN ARBITRARY
      NO., SINCE MULTIPLIED BY ZERO CLC
      YC(NS)=1.
      CLC(NS)=.
      CHORD(NS)=1.
      WRITE(8,8) YC(NS),CLC(NS),NS
79  FORMAT(10X,F8.2,10X,F7.3,1X,1)
80
80  THIS SECTION FINDS THE WING TOTAL LIFT
80  COEFFICIENT BY NUMERICALLY INTEGRATING
80  THE LOCAL LIFT COEFF. TIMES LOCAL CHORD
80  (=LOCAL LIFT/RY AND PRESSURE) VS. SPANWISE
80  DISTANCE CURVE. THE INTEGRATION IS
80  ACCOMPLISHED USING TRAPEZOIDAL RULES.
80  TOTAL CL IS THEN THIS VALUE DIVIDED BY WING
80  AREA.
      ARI=.
      NS1=NS-1
      DO 797 I=1,NS1
      LIFT(I)=CLC(I)*CHORD(I)
      IF (7.FO.NT) GO TO 799
      DELY(I)=(YS(I+1)-YC(I))*SSPN
79  CONTINUE
      DO 797 I=1,NS1
      ARI=ARI+.5*(LIFT(I)+LIFT(I+1))*DELY(I)
79  CONTINUE
      CALCULATE AREA UNDER CURVE FROM FOOT CHORD
      TO FIRST CONTROL POINT CHORD, ASSUMING A
      LINEAR CURVE FROM FOOT CHORD TO CL(2)*CHORD(2)
      CL77=CLD(1)*CHORD(1)-((CLD(2)*CHORD(2)-CLD(1)*CHORD(1))/
      1(YS(2)-YS(1)))*YC(1)
      AR2=.5*YS(1)*SSPN*(CL77+CLD(1)*CHORD(1))
      TOTAL LIFT COEFF.
      CLTOT(L)=2*ARI/AREA
      WRITE(9,9) CLTOT(L)
90  FORMAT (//21X,10L,TOTAL =,F7.2)
80
80  CALCULATE WING BENDING MOMENTS AS A FUNCTION
80  OF SPANWISE STATION
      WRITE(9,10)
81  FORMAT (//7X, WING BENDING, //21X,1Y,5F,1X,1SHAP//,
      1Y,10CONT//)
      DO 347 J=1,NS1
      SHEAR(J)=.
      MOMENT(J)=.
      DO 347 J=1,NS1
      AZ=(YS(J)-YC(I))*SSPN

```

```

      THIS SECTION PLOTS CL,TOTAL VS ALPHA, IF APPLICABLE
IF (N.PLOT.EC,0) GO TO 91.
CALL PLOT (15,,,-3)
CALL FACTOR (,75)
ALPHA (NA+1)=-1.
ALPHA (NA+2)=-.75.
CALL SCALE (CLTOT,1,,NA,1)
CALL AXIS (1,,12(ALPHA (000)),-11,8,,ALPHA (NA+1),ALP 12(NA+2))
CALL AXIS (4,,100,100,,CLTOT (NA+1),CLTOT (NA+3))
CALL SYMBOL (1.5,1.5,,20,0 LEFT COEFF. ANGLE,.,2)
CALL SYMBOL (3.7,3.25,,15,0,1,,2)
CALL SYMBOL (2.1,2.075,,20,0 OF STACK,.,0)
CALL SYMBOL (1,,2,12,10,,-1)
CALL SYMBOL (103,105,,12,14(CL,TOTAL)/,.,11)
CALL SYMBOL (109,109,,12,12,,-1)
CALL SYMBOL (103,110,,12,14(ALPHA) = ,.,1)
CALL NUMBER (913,105,,12,CLALPH,.,3)
CALL SYMBOL (109,105,,12,14/300,.,4)
CALL PLINE (ALPHA,CLTOT,-1,NA,1,1,1)
      PLOTTER TERMINATION ROUTINE.
CALL PLOT(1)
CONTINUE
RETURN
END

```

Vita

Ronald E. Luther was born in Waukegan, Illinois on 11 June 1948. He was graduated from Chillicothe High School, Chillicothe, Ohio in 1966. He graduated from Purdue University with a Bachelor of Science degree (Aeronautical Engineering) in June 1970 and was commissioned into the Air Force at that same time. After completing Undergraduate Pilot Training he was assigned to the F-111 weapons system and was stationed at Nellis AFB, Nevada, Taklihi Royal Thai AFB, Thailand, and RAF Upper Heyford, England. In 1979 he entered the Air Force Institute of Technology.

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Block 20:

expressed as a function of the components of the vorticity vector at selected nodes on the boundary of each triangular subregion. Nodal values on the shared boundary of the subregions are made equal, assuring continuity of the vorticity distribution function throughout the trapezoidal panel. A lifting surface of no thickness is modeled with a network of the trapezoidal panels. Again, nodal values on the common panel boundaries are matched to achieve complete continuity of the vorticity distribution throughout the lifting surface. Aerodynamic data for several wing planforms is obtained with the flow model. Results from this method are compared to those from other computational and theoretical methods.

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