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AIR UNIVERSITY (ATC)
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Wright-Patterson Air Force Base, Ohio

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THREE DIMENSIONAL ORBITAL STABILITY ABOUT THE EARTH-MOON EQUILATERAL LIBRATION POINTS.

THESIS

AFIT/GA/AA/80D-4 Howard A. Tilton
Captain USAF

AFIT/GA/AA/80D-3

THREE DIMENSIONAL ORBITAL STABILITY ABOUT THE EARTH-MOON EQUILATERAL LIBRATION POINTS

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University (ATC)
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

Howard A. Tilton, B.S.
Captain USAF

Graduate Astronautics

December 1980

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List of Symbols

$r, \dot{r}$	position; velocity magnitude
m	mass
[illegible]	[illegible]
d	declination
e	obliquity of the ecliptic
MD	mean moon distance
TU	absolute time unit
dv	small increment

Vectors

$\bar{r}$	position
$\ddot{\bar{r}}$	acceleration
$\bar{p}, d\bar{p}, d\bar{r}$	ephemeris position vectors
A	work matrix

Subscripts

i,j,k	summation indices
e	earth
s	sun
m	moon
c	satellite
J	Jupiter
em	earth-moon
es	earth-sun
ec	earth-satellite
eJ	earth-Jupiter
ms	moon-sun

eM	moon-satellite
eS	sun-satellite
eJ	Jupiter-satellite

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Abstract

A search [illegible] satellite about L4 is performed. A proposed two dimensional very restricted orbit is used to supply the initial conditions required for the search. An ephemeris of high accuracy is generated from a specific date and time using actual positions for the sun and moon. The generated sun and moon position and velocity vectors are used in the integration of the system's equations of motion. A stable orbit is found and is tested for its length of stability. The orbit is found to have a stable lifetime in excess of six hundred lunar synodic months. The sensitivity of the orbit to the sun's and moon's position is tested and found to be only slightly sensitive for an error in position of one quarter day. Finally, a predicted 180° out of phase orbit is found and is determined to be only marginally stable.

[illegible]

ABOUT THE EARTH-MOON EQUILATERAL

[illegible]

I. INTRODUCTION

Background

In recent years, there has arisen a great deal of interest in the orbital analysis of the earth-moon Lagrangian points. Both the civilian and military space programs have produced studies and proposals about the use of the Lagrangian points as orbital areas for satellites now under consideration for future missions. The areas of common interest in both the military and civilian space programs, lie in both manned and unmanned vehicles. Current proposals have included platforms for the industrialization and colonization of space. Such platforms have included Solar Power Satellites for beaming power back to the Earth, large space manufacturing facilities that are able to use the "zero gravity" environment for the production of materials that are currently extremely difficult or impossible to produce on Earth. A side effect of these manufactories would be the colonization of the moon and cislunar space. The military

Interest in space applications [illegible] [illegible] [illegible] satellites is a problem of increasing concern. If the [illegible] about L4 or L5, then [illegible] [illegible] ineffective. Such weapons would include "killer satellites" and Earth-based laser weapons.

Studies undertaken so far have largely dealt with two dimensional analysis of the very restricted four body problem. Some of the restictions have been circular orbits for both the moon and sun, an unrealistic mass for the sun, and planar motion for all bodies. Some analysis has been undertaken in the past to study the three dimensional problem, but these have always been restricted in their scope and lead to a vague conclusion about the existence of three dimensional orbital stability in the Lagrangian vicinities. This report will attempt to show actual stability about L4 for a period of at least fifty years, and to set conditions for further studies that follow this report.

Most recently, at the Air Force Institute of Technology, where this report was written, two studies were completed, one by Major William Beekman (Ref 3), and the other by Captain John Wheeler (Ref 7). Each did a study of two dimensions of orbital stability about L4. Both also lacked an analysis of the moon and sun's actual positions and [illegible] therefore only able to report on [illegible] with their various [illegible].

Capt. [illegible]'s report [illegible] sun in a [illegible] to construct a proposed orbit. His orbit was limited by the various constraints that were imposed on the problem. The major limitation came about by assuming the moon and sun to be in circular orbits about their respective system barycenters. Capt. Wheeler's conclusions at the end of his study indicated linear stability exists for his system.

Major Beekman's study was based on three reports, one of them being Capt. Wheeler's. The other reports were by Kolenkiewicz and Carpenter, in 1968, (Ref 6), and by Barkham, Modi, and Soudack in 1975, (Ref 2). In his investigation, Major Beekman confirmed the Wheeler model's stability, for a period of at least twenty years, by removing the restriction of circular orbits. He also showed the other orbits studied were marginally stable in the same time frame, even though the planar restriction was still in force.

Problem and Scope

The search for a three dimensional stable orbit about L4 and L5 is a problem of increasing importance in the evolution of space exploration. For any serious long term "[illegible] [illegible]" colonization [illegible] place in the earth-moon [illegible]

[illegible] without the [illegible] of [illegible] this study should be valid for the [illegible]tment of hardware to [illegible] mission p[illegible] [illegible], the problem must address realistic [illegible] in the four body problem. Consideration must al[illegible] be given to the effect of the remainder of the solar system on the motion of the colony and the Earth. The sun and [illegible]'s position in this study should be known at all times to allow planners a reasonable starting point for their mission. In the final analysis, the orbit described must have the restrictions of the previous analyses removed, specifically two dimensionality and the unperturbed circular orbits of the moon and sun.

II. [illegible]

[illegible]

In the analysis of the problem, the equations of motion of the moon and the colony are those given by T. A. Heppenheimer, in an article published in 1978, (Ref 5). His equations are given in rectangular nonrotating Earth-centered inertial coordinates for two dimensions. These equations are then simply expanded to three dimensions. The sun's motion is then described in the same coordinate system as a two body problem. Since the equations, are given in only two dimensions the expansion to three dimensions follows the derivation of the two dimensional case.

The equations of motion for a general "n-body" problem, in rectangular coordinates is given by:

$$\ddot{\bar{r}}_{ij} = -\sum_{j \neq i}^{n} Gm_j \frac{(\bar{r}_i - \bar{r}_j)}{r_{ij}^3} \qquad \text{EQ II-1}$$

where $r_{ij} = |r_j - r_i|$. Letting i=1 be the reference body, the Earth, i=2 the body whose motion we wish to study, and i=3 and i=4 be the indices of the two other bodies in the system. For the Earth-Sun system:

[illegible]:

$$\ddot{\bar{r}}_{12} = \sum_{j \neq 1}^{n} \frac{Gm_j(\bar{r}_1 - \bar{r}_j)}{r_{1j}^3} - \sum_{k \neq 2}^{n} \frac{Gm_k(\bar{r}_2 - \bar{r}_k)}{r_{2k}^3}$$

Expanding and combining terms:

$$\ddot{\bar{r}}_{12} = -\frac{G(m_1+m_2)\bar{r}_{12}}{r_{12}^3} - \sum_{j=3}^{n} Gm_j\left(\frac{\bar{r}_2-\bar{r}_j}{r_{2j}^3} - \frac{\bar{r}_1-\bar{r}_j}{r_{1j}^3}\right) \quad \text{EQ II-3}$$

Realizing the moon and the satellite have only a negligible effect on the motion of the sun, EQ II-3 reduces to:

$$\ddot{\bar{r}}_{12} = -\frac{G(m_1+m_2)\bar{r}_{12}}{r_{12}^3} \quad \text{or}$$

$$\ddot{\bar{r}}_{es} = -\frac{Gm_s\bar{r}_{es}}{r_{es}^3} \quad \text{EQ II-4}$$

From EQ II-3, the equations of the motion of the moon are written, also realizing the satellite has no effect on the moon's motion:

$$\ddot{\bar{r}}_{ec} = -Gm_e\frac{\bar{r}_{ec}}{r_{ec}^3} - Gm_s\left(\frac{\bar{r}_c - \bar{r}_s}{r_{cs}^3} - \frac{\bar{r}_e - \bar{r}_s}{r_{es}^3}\right) - Gm_m\left(\frac{\bar{r}_c - \bar{r}_m}{r_{cm}^3} - \frac{\bar{r}_e - \bar{r}_m}{r_{em}^3}\right)$$

EQ II-6

Using the Earth as a convenient reference frame, the equations finally become:

$$\ddot{\bar{r}}_s = -Gm_s\frac{\bar{r}_s}{r_s^3}$$ EQ II-7

$$\ddot{\bar{r}} = -Gm_e\frac{\bar{r}_m}{r_m^3} - Gm_s\left(\frac{\bar{r}_m - \bar{r}_s}{r_{ms}^3} + \frac{\bar{r}_s}{r_s^3}\right)$$ EQ II-8

$$\ddot{\bar{r}}_c = -Gm_e\frac{\bar{r}_c}{r_c^3} - Gm_s\left(\frac{\bar{r}_c - \bar{r}_s}{r_{cs}^3} + \frac{\bar{r}_s}{r_s^3}\right) - Gm_m\left(\frac{\bar{r}_c - \bar{r}_m}{r_{cm}^3} + \frac{\bar{r}_m}{r_m^3}\right)$$

EQ II-9

The equations of motion for analysis purposes are then represented in state vector form for ease in handling. See Appendix A for the subroutine pertaining to the equations of motion.

Coordinate System

As mentioned previously, the coordinate system used in the study is a nonrotating, rectangular coordinate system centered with the Earth. The position vectors associated with the moon and the sun in this frame are developed from Ref 8. Since the moon vector given by Ref 8 is described in right ascension, a, declination, d, and Earth radii, r, it must be transformed to the frame we wish to use. The rectangular coordinates are given by:

$$
\begin{aligned}
x &= r*\cos(a)*\cos(d) \\
y &= r*\sin(a)*\cos(d) \qquad \text{EQ II-10} \\
z &= r*\sin(d)
\end{aligned}
$$

and the velocity vector elements are:

$$
\begin{aligned}
\dot{x} &= \dot{r}*\cos(a)\cos(d)-r*\dot{a}*\sin(a)\cos(d)-r*\dot{d}*\cos(a)\sin(d) \\
\dot{y} &= \dot{r}*\sin(a)\cos(d)-r*\dot{a}*\cos(a)\cos(d)-r*\dot{d}*\sin(a)\sin(d) \\
\dot{z} &= \dot{r}*\sin(d)+r*\dot{d}*\cos(d)
\end{aligned}
$$

EQ II-11

The vectors are still not in the proper frame and need to be rotated to the ecliptic. If e is the obliquity of the ecliptic, then the transformation matrix for this is:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(e) & \sin(e) \\ 0 & -\sin(e) & \cos(e) \end{bmatrix} \qquad \text{EQ II-12}$$

The sun's position vector is already in Earth-centered rectangular coordinates and only needs to be rotated to the proper frame by the use of the above transformation matrix.

The frame for the analysis of the problem will be an Earth-centered ecliptic nonrotating rectangular system. The X-axis will point toward the vernal equinox and the Z-axis will be perpendicular to the ecliptic having the XY-plane coincident with the ecliptic plane. The frame for the presentation of the output of the analysis will be a rotating frame with the x-axis through the center of mass of the moon. It also will be an Earth-centered rectangular ecliptic frame. See Fig 1 and Fig 2 for a pictorial representation of each coordinate system.

Ephemeris Generation

Ref 8 provides the position vector for both the moon and sun, but does not provide a velocity vector for each appropriate time step. In order to integrate the equations of motion, the velocities must be known at any given time. Since Ref 8 provides position at a known time, a central difference velocity can be determined. This velocity is

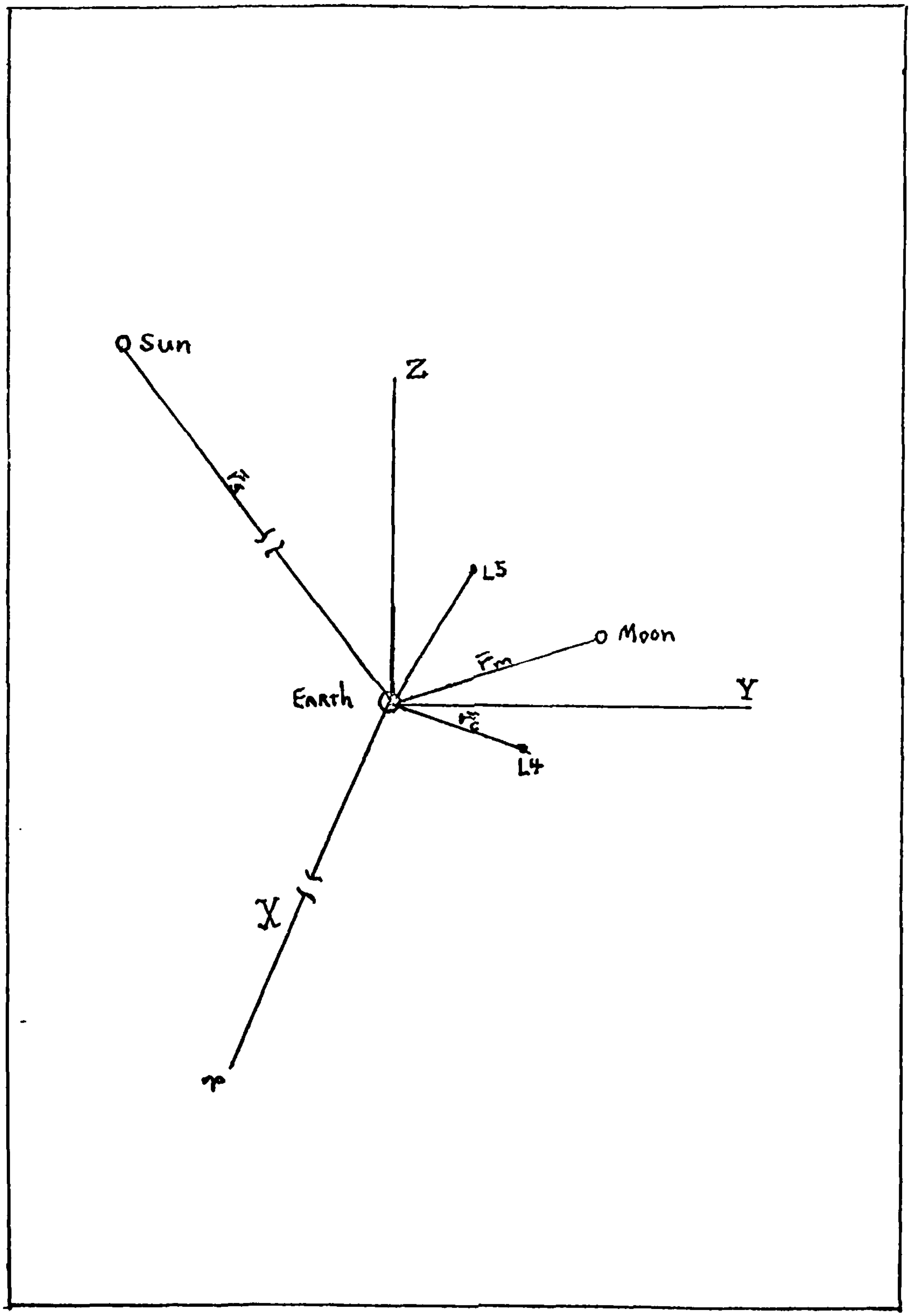

Fig 1. Earth-Centered Nonrotating Frame

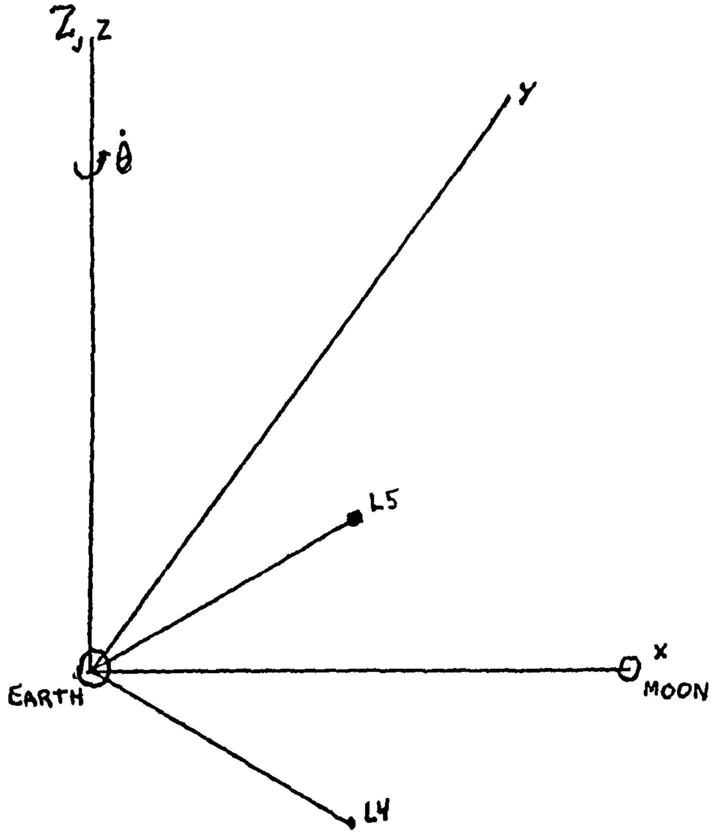

Fig 2. Earth-Centered Rotating Frame

crude but effective for relatively short time spans, but will [illegible] [illegible]

percent of positional error over a time span of about one year for this particular study. In order to accomplish this accuracy, a dynamic differential corrector routine was used to modify the velocities obtained from Ref 8.

Using the position and velocity vectors as initial conditions, the equations of motion are integrated forward to a reference time, t_o, where the position vector, $\overline{P}_o$, was known. The reference time selected was a function of the time step available from Ref 8, ten days for the sun and one half hour for the moon. The corrector was first applied only to the sun's velocity and, after repetition at longer reference periods, the corrector was applied to the moon's velocity. The outcome of the corrector, the velocity vectors of the moon and sun at the initial time, was substituted for the crude velocities of Ref 8.

The start date for the ephemeris generation was chosen as 5 Jan 1979, of the Equinox of 1950. This particular date was chosen because the moon was relatively near the equatorial plane of the Earth. Using a time when the $\dot{Z}$-component of the moon's velocity vector approaches zero will give highly inaccurate velocities when computed by central differences and will require more repetitions of the velocity corrector to achieve the velocity desired.

The first step in computing the velocity corrector is to integrate the equations of motion forward to the appropriate reference time, t_o. The integrated position, $\bar{p}_1$, is compared to the reference position, $\bar{p}_o$, and this vector, $d\bar{r}=\bar{p}_1-\bar{p}_o$, is stored for later use. The equations of motion are then integrated forward three more times from the initial conditions with the initial position vectors and the following velocity vectors in turn:

$$\bar{v}= \begin{bmatrix} x+dv \\ y \\ z \end{bmatrix}, \begin{bmatrix} x \\ y+dv \\ z \end{bmatrix}, \begin{bmatrix} x \\ y \\ z+dv \end{bmatrix} \qquad \text{EQ II-13}$$

where dv is a small velocity increment. It should be noted that two of the velocity components are the original initial velocity conditions for each integration. Each time the equations are integrated forward a new columnar position vector, $\bar{p}_i$, is formed. Subtracting $\bar{p}_o$ from $\bar{p}_i$, one obtains the vector $d\bar{p}_i=\bar{p}_i-\bar{p}_o$, where $i=1,2,3$. After the three integrations these position vectors may be combined into matrix form, $A=[dp_1:dp_2:dp_3]$. Dividing this matrix by the delta velocity that was added to the initial velocity vectors, a differential matrix is obtained of the form:

[illegible]

Noting that $d\bar{r}=Ad\bar{v}$, and since [illegible] the [illegible] vector $d\bar{r}$ are known, the velocity vector [illegible] is obtained by inverting A and postmultiplying [illegible] dr, $d\bar{v}=A^{-1}d\bar{r}$. This velocity corrector vector is then added to the initial conditions velocity vector. This procedure is then repeated using a new reference time, until the velocity vector yields a postion vector at the end of the ephemeris span to within the accuracy desired. See Appendix B for a subroutine pertaining to these calculations.

Constants

The primary constants used for the problem analysis were obtained from Ref [8:529]. All secondary constants were derived from these values. The accuracy of the constants in Ref 8 is on the order of six digits or less for masses and distances but this will be shown to be adequate for the problem analysis. The use of constants other than those obtained in Ref 8 are used only in the duplication of the Wheeler model and are obtained from Ref's 3 and 7.

[illegible] the unit of mass will be a function of the mass ratio of the respective bodies using the sum of the Earth's and moon's masses. The constants in both systems are modified to reflect these standard units.

The Wheeler Model

The Wheeler orbit should be obtainable from the method of analysis. In order to achieve this, the equations of motion are modified to place the moon and sun in circular orbits about the Earth. Wheeler's constants are then corrected to the unit constants described in the preceeding section. The initial conditions are then transformed to the frame coordinates being used, and the equations of motion are then integrated forward for the appropriate time span, one lunar synodic month. See Fig 3 for a representation of the Wheeler system and Fig 4 for the predicted Wheeler orbit.

The initial conditions are selected from the restrictions of the Wheeler model. The sun and moon are in circular orbits and they are also initially on the frame's negative X-axis. The sun is placed at one A. U. with a circular velocity at that point solely in the negative Y-direction. The sun's X-component is transformed into the correct units and the same with the velocity. The moon's position is also on the negative X-axis at one moon distance

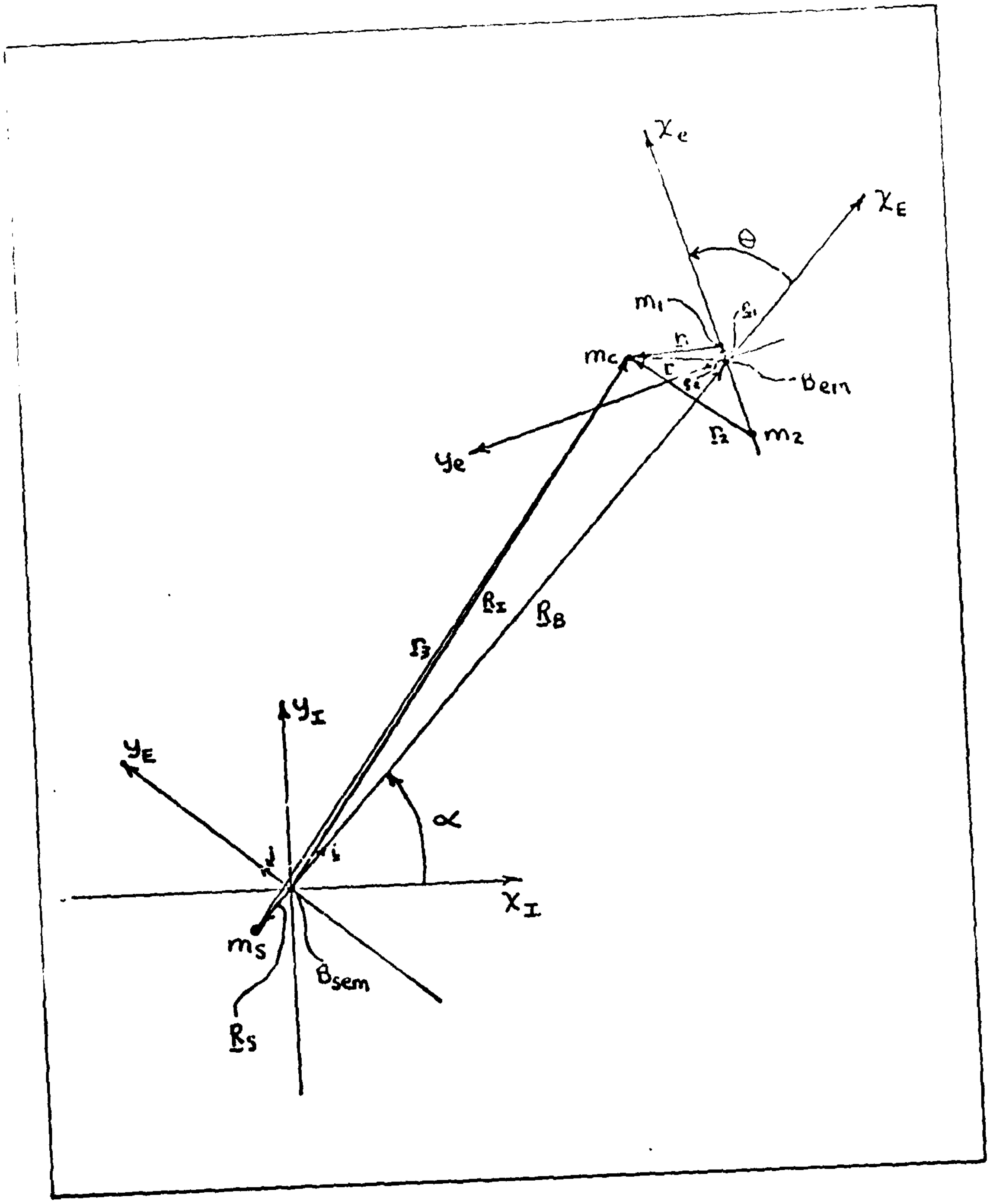

Fig 3. The Wheeler Frame

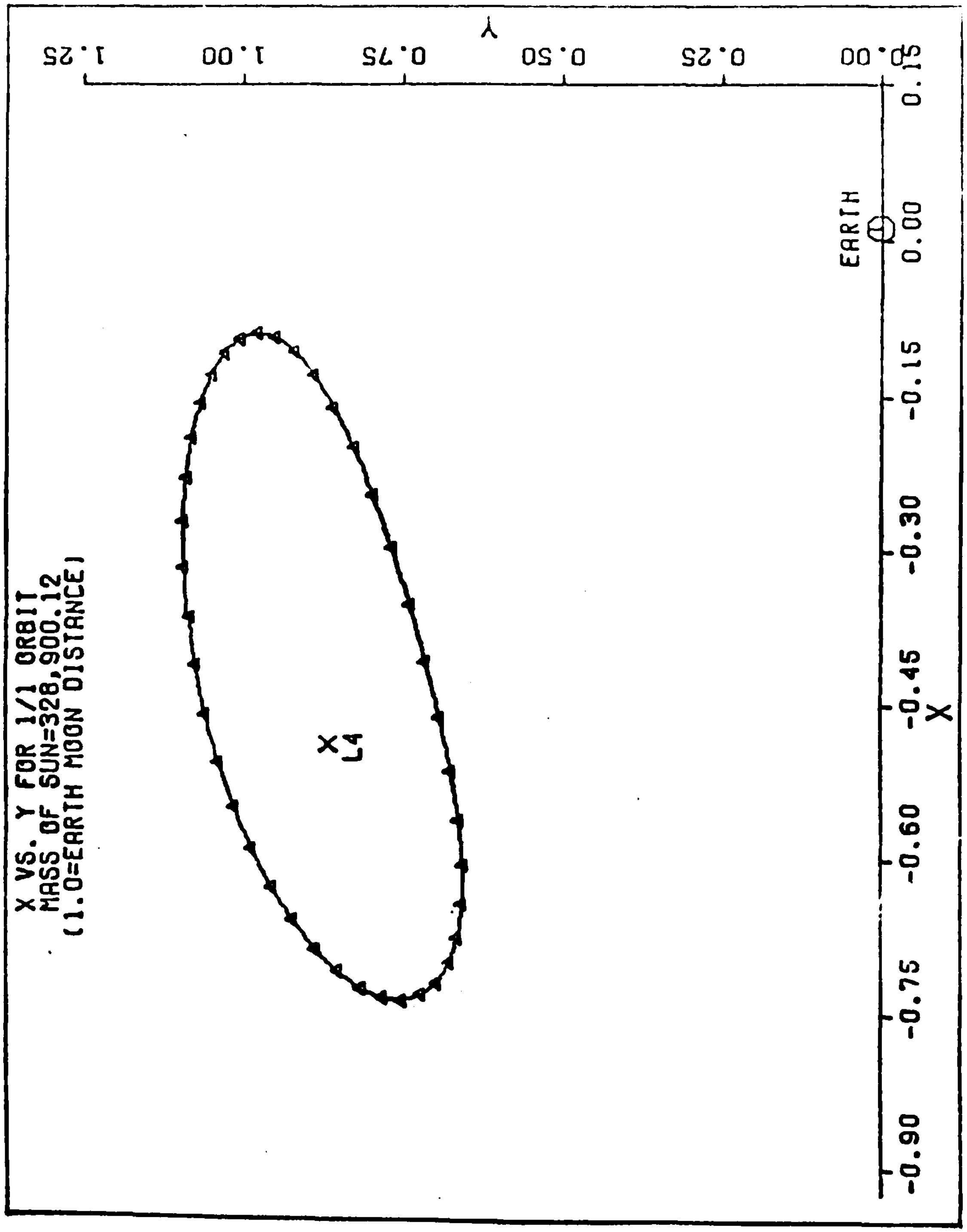

Fig 4. The Predicted Wheeler Orbit

with the velocity component also totally in the negative Y direction. The sun's position and velocity vector is:

$$\bar{r}_s = \begin{vmatrix} -388.83244 \\ 0. \\ 0. \end{vmatrix} \qquad \dot{\bar{r}} = \begin{vmatrix} 0. \\ -6.688534 \\ . \end{vmatrix} \qquad \text{EQ II-15}$$

The moon's position and velocity vector are also given by:

$$\bar{r}_m = \begin{vmatrix} -1. \\ 0. \\ 0. \end{vmatrix} \qquad \dot{\bar{r}}_m = \begin{vmatrix} 0. \\ -.229971 \\ 0. \end{vmatrix} \qquad \text{EQ II-16}$$

The colony's initial position and velocity vectors are given by Ref [7:62]. However, Ref [3:31] provides the vectors in an earth-centered frame. The correction that Beekman made is sufficient for the position vector, but the velocity vector has the wrong units. The units used are in MD/TU, mean moon distance per absolute time unit, while the units required are in MD/DAY, mean moon distance per day. Ref [3:22] provides the TU relationship and this is used in the correction. Finally the position and velocity vectors are given by:

$$\bar{r}_c = \begin{vmatrix} \text{[illegible]} \\ \text{[illegible]} \\ 0. \end{vmatrix} \qquad \dot{\bar{r}}_c = \begin{vmatrix} -.\text{[illegible]} \\ -.\text{[illegible]} \\ 0. \end{vmatrix} \quad \text{[illegible]}$$

Solar System Effects

The effect of the remainder of the solar system on the satellite's motion needs to be investigated. Since the motion of the sun and moon are known, the only terms that need to be considered in the "n-body" equation are the motion of the Earth and that of the satellite. Even though this is an "n-body" problem most of the mass of the solar system is at such a distance that the effects are going to be minimal. With that in mind, the problem can be reduced to the effect of Jupiter's tidal acceleration on the Earth's and satellite's motion. Jupiter is the closest of the massive planets and would reflect the largest effect on their motion. As a comparison, the tidal acceleration of Jupiter can be related to the sun's tidal acceleration. The tidal acceleration equation can be written as:

$$\ddot{\bar{r}}_{cJ} = -Gm_J(\bar{r}_{eJ} r_{eJ}^{-3} - \bar{r}_{cJ} r_{cJ}^{-3}) \qquad \text{EQ II-18}$$

where $\bar{r}_{cJ} = \bar{r}_{eJ} + \bar{r}_c$. Expanding the cubic term of the colony:

$$r^{-3} = \left[\bar{r} \bullet \bar{r} \quad \ldots \bullet \bar{r} + \bar{r} \bullet \bar{r}\right]^{-3/2}$$

where $\bar{r}_c \cdot \bar{r}_c$ can be regarded as zero. The remaining terms can then be binomially expanded to form:

$$\bar{r}_{cJ}^{-3} = r_{eJ}^{-3} - 3r_{eJ}^{-5}(\bar{r}_{eJ} \bullet \bar{r}_c) + O_2 \qquad \text{EQ II-20}$$

Substituting this back into EQ II-18 to form:

$$\ddot{\bar{r}}_{cJ} = -Gm_J\left[\bar{r}_{eJ} r_{eJ}^{-3} - (\bar{r}_{eJ}+\bar{r}_c)(r_{eJ}^{-3} - 3r_{eJ}^{-5}(\bar{r}_{eJ} \cdot \bar{r}_c))\right]$$

Collecting terms finally gives:

$$\ddot{\bar{r}}_{cJ} = -Gm_J\left[\frac{3(\bar{r}_{eJ} \cdot \bar{r}_c)(\bar{r}_{eJ}+\bar{r}_c) - \bar{r}_c}{r_{eJ}^5}\right] \qquad \text{EQ II-21}$$

Ref [2:360] provides a relative value of Jupiter's mass and distance from the selected frame. These can be converted into values relative to the sun for the comparison. Replacing m_J with its equivalent, $.001m_S$, and $\bar{r}_{eJ}$ with $4.20\bar{r}_s$, into EQ II-21 will allow a numerical evaluation of the acceleration. Noting that $\bar{r}_{eJ}+\bar{r}_c \cong \bar{r}_{eJ} = 4.20\,\bar{r}_s$, EQ II-21 can be reduced to:

$$\ddot{\bar{r}}_{cJ} = \text{[illegible]}$$

Evaluating this expression leads to:

$$\ddot{\bar{r}}_{cJ} = 1.2*10^{-7} \frac{G m_s \bar{r}_s}{r_s^3}$$

which demonstrates the tidal acceleration due to Jupiter or the remainder of the solar system is approximately 10^{-7} less than that of the sun's tidal acceleration. Therefore, we need not concern ourselves with any solar system perturbations other than the sun's.

The Four Body Problem

The initial conditions of the four body problem are the basis on which the whole stability question lies. If the wrong initial conditions are chosen the search for stability will be long and tedious. Capt Wheeler's model however, demonstrates linear stability in his frame. Therefore the initial conditions selected should have a greater chance of three dimensional stability than any selected by random. Using the position and velocity vector determined in the analysis of the Wheeler model will provide the needed vectors in two dimensions. Since the velocity and position vectors are only in two dimensions we need to add the third dimension component to each. The Moon's X and $\dot{X}$ vector components are [illegible], [illegible] [illegible] components of the colony's position and velocity vector.

As mentioned in the [illegible], the initial conditions now selected are only an initial selection. Stability should not exist in the Lagrangian vicinities for these starting conditions. Therefore it will become necessary in the analysis to modify these conditions in the search for stability. The manner in which the modification is made does not matter and therefore can be done by any method available. Once the modification is made the equations of motion can then be integrated forward for a relatively short period of time. If the colony is still in the vicinity of L4, they can be integrated forward for a greater length of time. This can be repeated until the period of stability reaches that as desired. Using short integration time spans in the initial search for stability will greatly reduce the amount of execution time required for the integration of the eighteen equations of motion.

III. [illegible]

Selected Ephemeris

The span of the ephemeris was selected to be approximately one year. This particular length was chosen due to its amount of integration necessary for the generation. Longer time spans would be nice but the execution time increases linearly for the integrations required. The span does allow a sufficient number of starting points, new moons, for the analysis. The addition of the dynamic differential corrector for the velocity vectors of the moon and sun produced a positional error related to time of approximately six hours for both the moon and sun for the slightly less than one year generation span. This error is slightly less than .07 percent, or less than the error required at the outset of the problem. If the error were cumulative throughout the study, then the total error for a fifty year period would approach 3.5 percent, or 12.5 days. This total error would have little, if any, effect on the question of stability.

The sun's velocity was corrected three times, the first time for a period of one day, then ten days, twenty days, and finally fifty days. The moon on the other hand, required four corrections of the velocity, probably due to [illegible] for a [illegible], [illegible] by twenty and then fifty as in the case of the sun. For times longer than fifty [illegible], and for times less than fifty [illegible] error increased in the overall search for accuracy. The reason for this was not investigated because the fifty day reference period produced the accuracy required and such an investigation is beyond the scope of this report. The error in the resultant ephemeris is linear throughout the entire span. The major cause for this would be a minor error still existant in the initial conditions used in its generation. The use of a tabulated ephemeris that contained the entire span needed would be of some help, provided that the accuracy is greater than that achieved by this method. However, as this analysis indicates, such an ephemeris is not needed. Table I provides selected portions of the moon and sun ephemeris in ephemeris coordinates which can easily be compared to Ref 8 to note their accuracy. Table II is the state vector of the sun-moon conditions on 5.0 Jan 1979.

The Wheeler Orbit

The initial conditions derived from Ref [3:31] were initially integrated forward for a period of one lunar synodic month. The data obtained in the nonrotating frame was transformed into a rotating frame in which the moon

Table I. Generated Moon and Sun Ephemeris

```
TIME ELAPSED= [illegible].

SUN COORDINATES IN A.U.

 X= .2[illegible]12[illegible] Y= -.82[illegible]  Z= -.[illegible]177[illegible]

MOON POSITION IN EPHEMERIS COORDINATES

GEOCENTRIC DISTANCE= 5[illegible].[illegible]
LONGITUDE= 8 DEG 1[illegible] MIN 32.[illegible]1223 SEC
LATITUDE= -[illegible] DEG -[illegible] MIN -[illegible] SEC
```

```
TIME ELAPSED= [illegible].

SUN COORDINATES IN A.U.

 X= .8922[illegible]25[illegible] Y= .[illegible]3[illegible]13[illegible]39[illegible]3 Z= -.[illegible]212[illegible]15[illegible]

MOON POSITION IN EPHEMERIS COORDINATES

GEOCENTRIC DISTANCE= 5[illegible].652362784436
LONGITUDE= 296 DEG 45 MIN 21.[illegible]72 SEC
LATITUDE= 4 DEG 5 MIN [illegible]9.99158[illegible]4234 SEC
```

```
TIME ELAPSED= 180.

SUN COORDINATES IN A.U.

 X= -.19[illegible]26242568 Y= .95342[illegible]459175[illegible] Z= .2[illegible]274[illegible]23[illegible]429

MOON POSITION IN EPHEMERIS COORDINATES

GEOCENTRIC DISTANCE= 6[illegible].3[illegible]9290716
LONGITUDE= 2[illegible] DEG 59 MIN [illegible].2613419[illegible]492 SEC
LATITUDE= 3 DEG 35 MIN 46.[illegible]239[illegible]298 SEC
```

```
TIME ELAPSED= 3[illegible].

SUN COORDINATES IN A.U

 X= .1415573457[illegible]3 Y= -.872692135[illegible]892 Z= -.[illegible]2494[illegible]81985[illegible]35

MOON POSITION IN EPHEMERIS COORDINATES

GEOCENTRIC DISTANCE= 5[illegible].699[illegible]51[illegible]52[illegible]1
LONGITUDE= [illegible] DEG 54 MIN [illegible]2.16479[illegible]86326 SEC
LATITUDE= -[illegible] DEG -6 MIN -7.25[illegible]26161[illegible]2 SEC
```

Table II. State Vector of 5.0 Jan 1979

INITIAL CONDITIONS VECTOR

X(1)= 90.262722

X(2)= -371.8793511

X(3)= -.0177276746

X(4)= 6.61597474739

X(5)= 1.60285058302

X(6)= .0000744396864064

X(7)= .9690805

X(8)= .1392251

X(9)= -.0272228

X(10)= -.02055542857908

X(11)= .2301949796881

X(12)= -.0198502010047

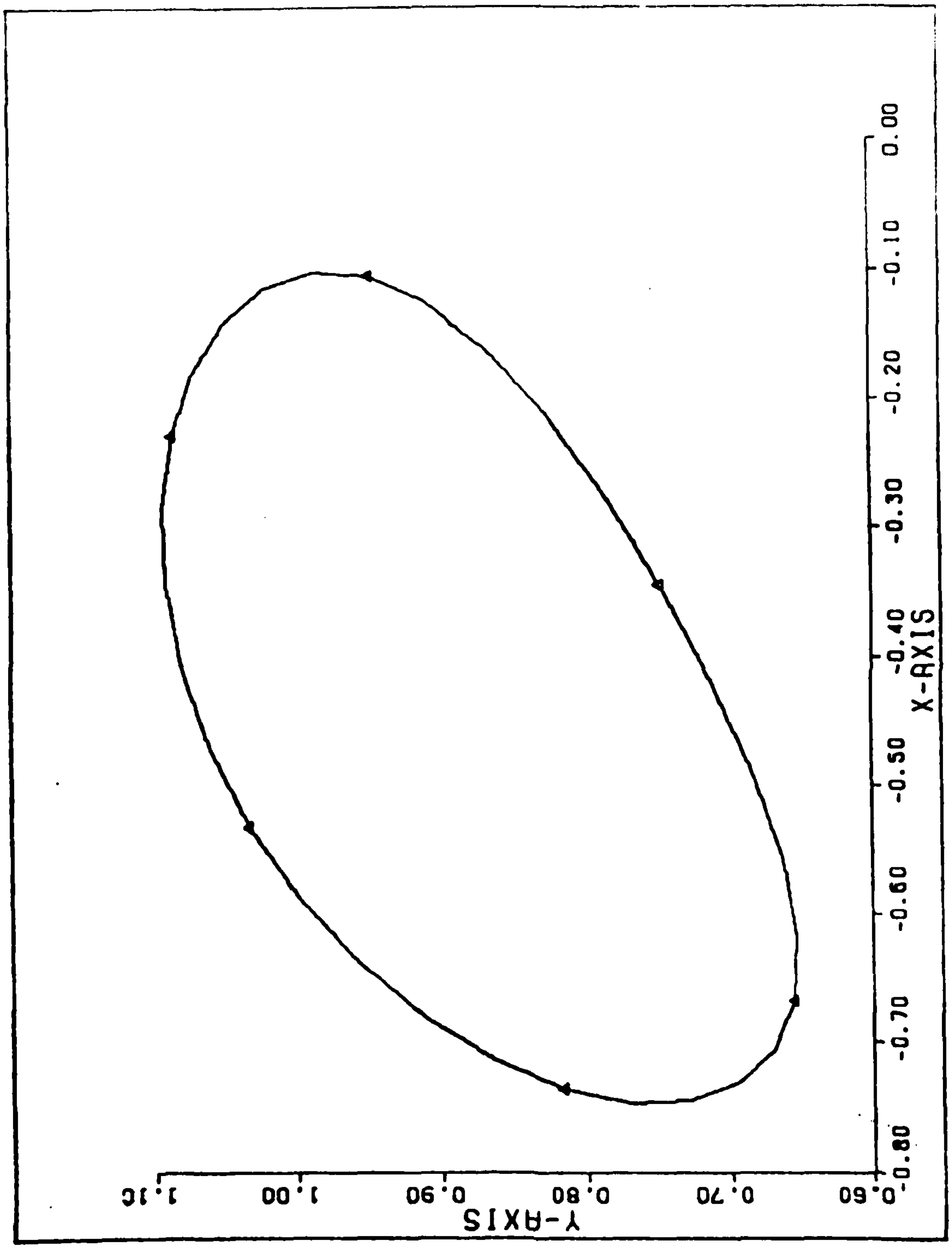

Fig 5. Restricted Wheeler Orbit--One Month

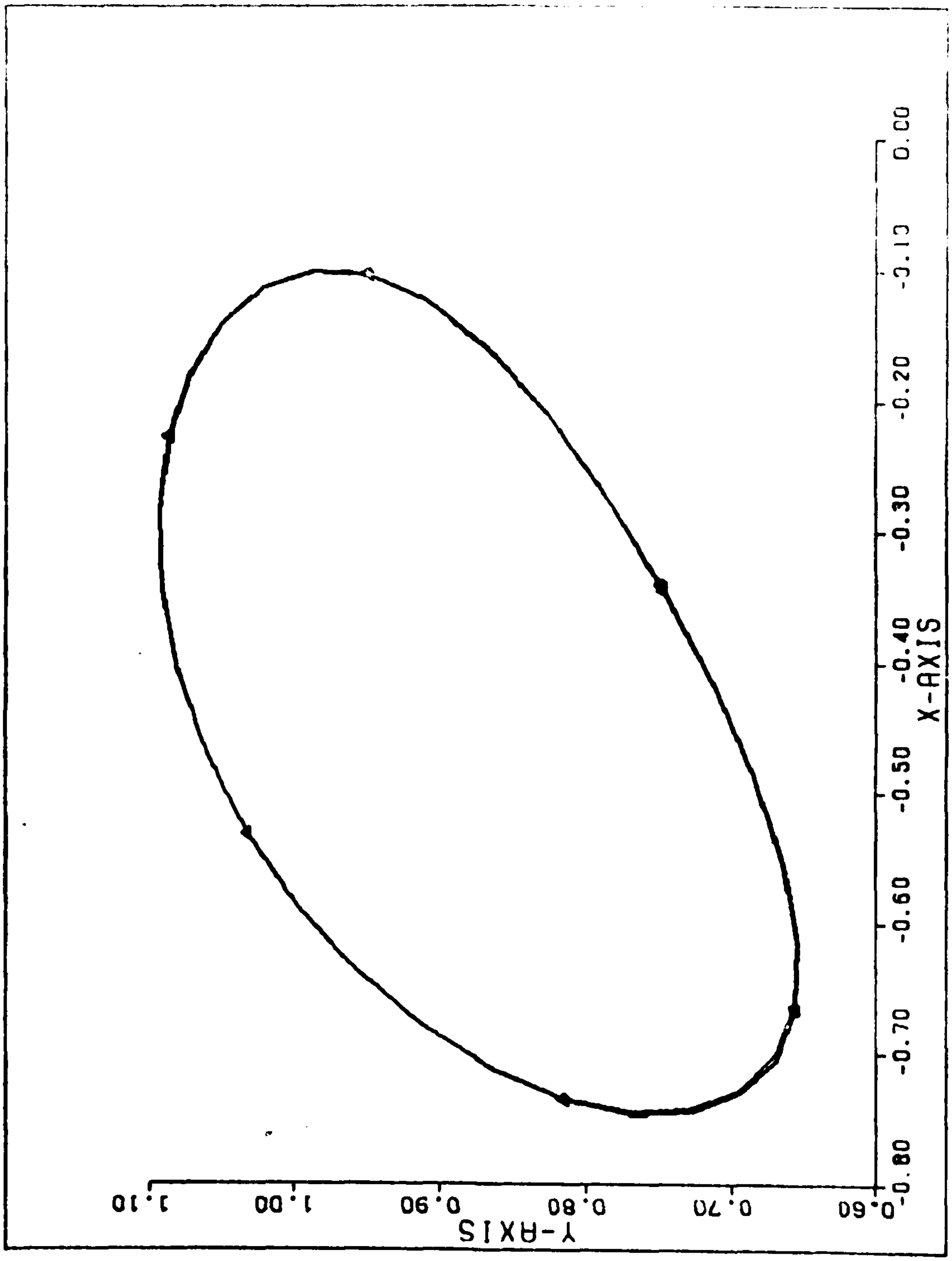

Fig 6. Restricted Wheeler Orbit--Three Months

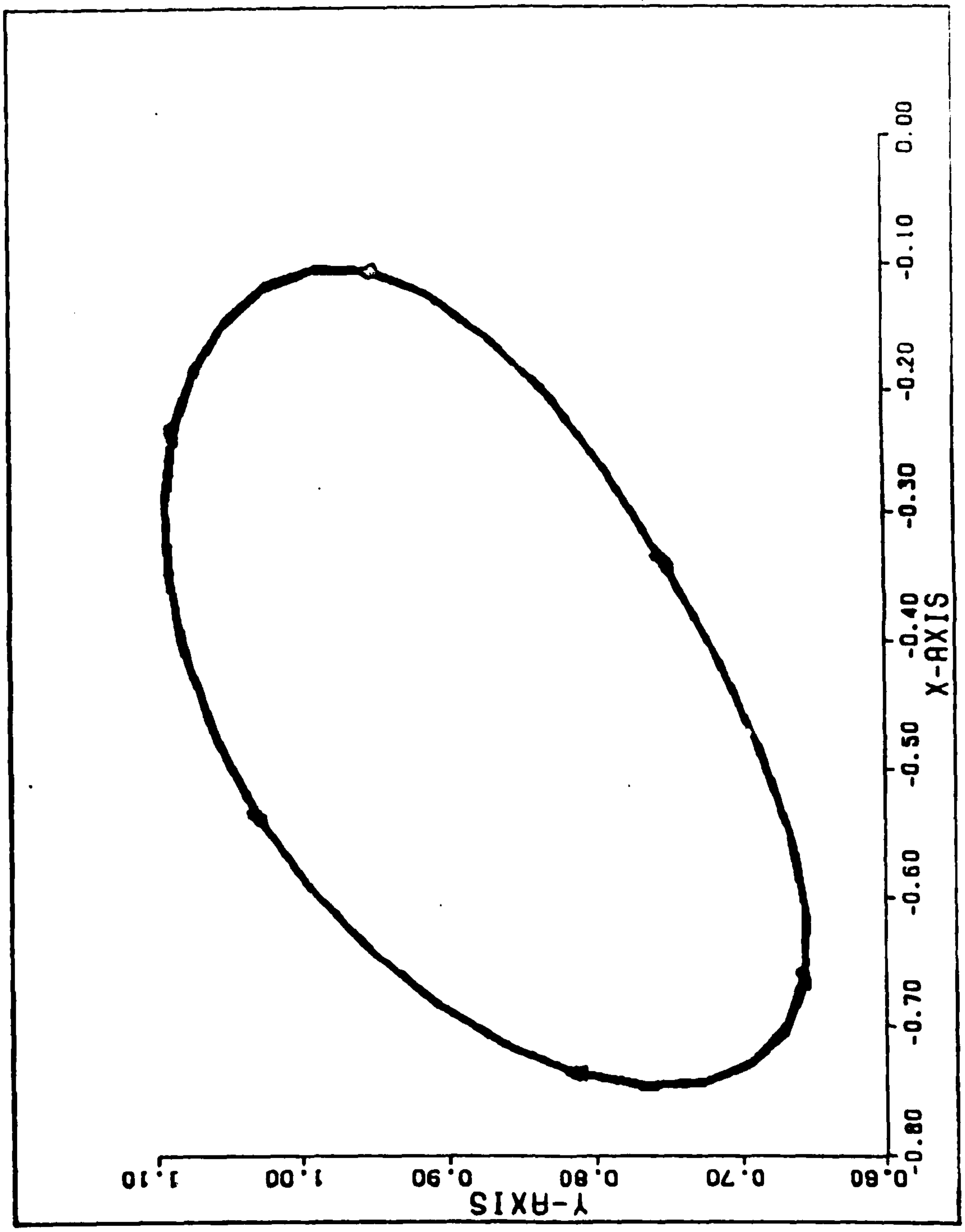

Fig 7. Restricted Wheeler Orbit--Twelve Months

remains on the negative x-axis. This produced data in a format which was easily plotted. Appendix C provides a subroutine which was used in the rotation and Appendix D contains the subroutine used in the plotting of the data. The orbit was also integrated for longer periods and the plots of all the periods are contained in Figs. 5, 6, and 7. The slight disturbance in the orbit as the period increased is due to small errors in the integrator selected.

Four-Body Analysis

The selection of the initial conditions were highly influenced by the periodic orbit developed by Capt Wheeler in Ref 7. If a stable orbit does exist, then the initial conditions of such an orbit should be close to those given by Ref [7:62]. The third dimension of the vector can be related to the moon's vector by giving the satellite the same z and z magnitudes as the moon. The moon and sun's initial conditions are obtained from ephemeris generation techniques discussed earlier. However, Ref [7:62] requires the moon and sun both be on the negative x-axis for those particular initial conditions. This condition occurs once every new moon. Ref [8:3] provides a list of dates of new moon occurences accurate to the nearest minute. The first occurance after 5 Jan 1979 is that of 28.263888 Jan 1979. The equations of motion are then integrated to this date, neglecting the satellite's motion. Once the integration is complete the initial conditions of the satellite are added to the [illegible].

The velocity units are MD/DAY, and the initial velocities are on the order of .1 to .[illegible] MD/DAY. This is equivalent to [illegible] to [illegible]. If stability is expected to have velocity components close to those of the initial conditions, then adding or subtracting delta velocities on the order of 4.5 meters per second or .001 MD/DAY should have the desired effect.

A main program was written which allowed the user to add these delta velocities and specify the length of the integration. The integration period should be variable in length to aid in the search for stability. The length of execution time of the integration requires short integration spans until a likely candidate for stability is found. The program was written for an interactive user which allowed quick observation and interpretation of results. Appendix E contains the main program used in this effort. Appendix F contains the subroutine used in the addition of the delta velocities to the state vector. First, the state vector elements of the satellite are rotated to the rotating frame. Then the delta velocities are added to the rotated elements and these are then transformed back into the nonrotating frame. Once the initial conditions are those which are desired, the integration is executed for the desired time span. The time span is split into time steps which are [illegible] of the lunar synodic [illegible]. The time steps [illegible] [illegible] to [illegible]/30. [illegible] .984 [illegible]. Therefore integrating forward for [illegible] time steps is equivalent to integrating for one lunar synodic month.

The state vector [illegible] 1979 including the satellite elements used in the Wheeler model are contained in Table III. Fig's 8, 9, 10, and 11 are trajectories obtained for periods of one, three, twelve, and sixty synodic months. Adding a delta velocity of .001 MD/DAY to all of the satellite's velocity components of Table III, produces the state vector contained in Table IV. The trajectory plots for one, three, twelve, and sixty synodic months are contained in Fig's 12 through 15. Adding a delta velocity of -.001 MD/DAY to each of the velocity components in Table III, produces the state vector contained in Table V. The resultant trajectory is plotted for the same periods as before and are shown in Fig's 16 through 19.

The question of stability is of paramount importance to the problem. Linear or periodic stability does not exist in the three dimensional model, but stability must be determined and in an easily observable fashion. To that effect, a cross section of the orbit is obtained by slicing through the Lagrangian point, using the yz-plane. For long term stability to occur, the cross sections should fill in separate, but definite, areas on the plane. The reasoning behind this is if an orbit is stable then after a reasonable

Table III. State Vector of 28.263888 Jan 1979

including Wheeler Elements

INITIAL CONDITIONS VECTOR

X(1)= [illegible]

X(2)= [illegible]

X(3)= -.01457307354732

X(4)= 5.433476266665

X(5)= 4.083553482214

X(6)= .000192859160l845

X(7)= .5701945487225

X(8)= -.7312694719864

X(9)= .05406587955571

X(10)= .1940024128416

X(11)= .1502478807353

X(12)= -.01690467092801

X(13)= -.1904858617353

X(14)= -1.082237274618

X(15)= .05406587955571

X(16)= .2008356132464

X(17)= -.059826122177

X(18)= -.01690467092801

ROTATING FRAME COLONY POSITION AND VELOCITY

X= -.7363274298999 XDOT= -.1706730983

Y= .8156863968899 YDOT= -.121592771

Z= .05406587955571 ZDOT= -.01690467092801

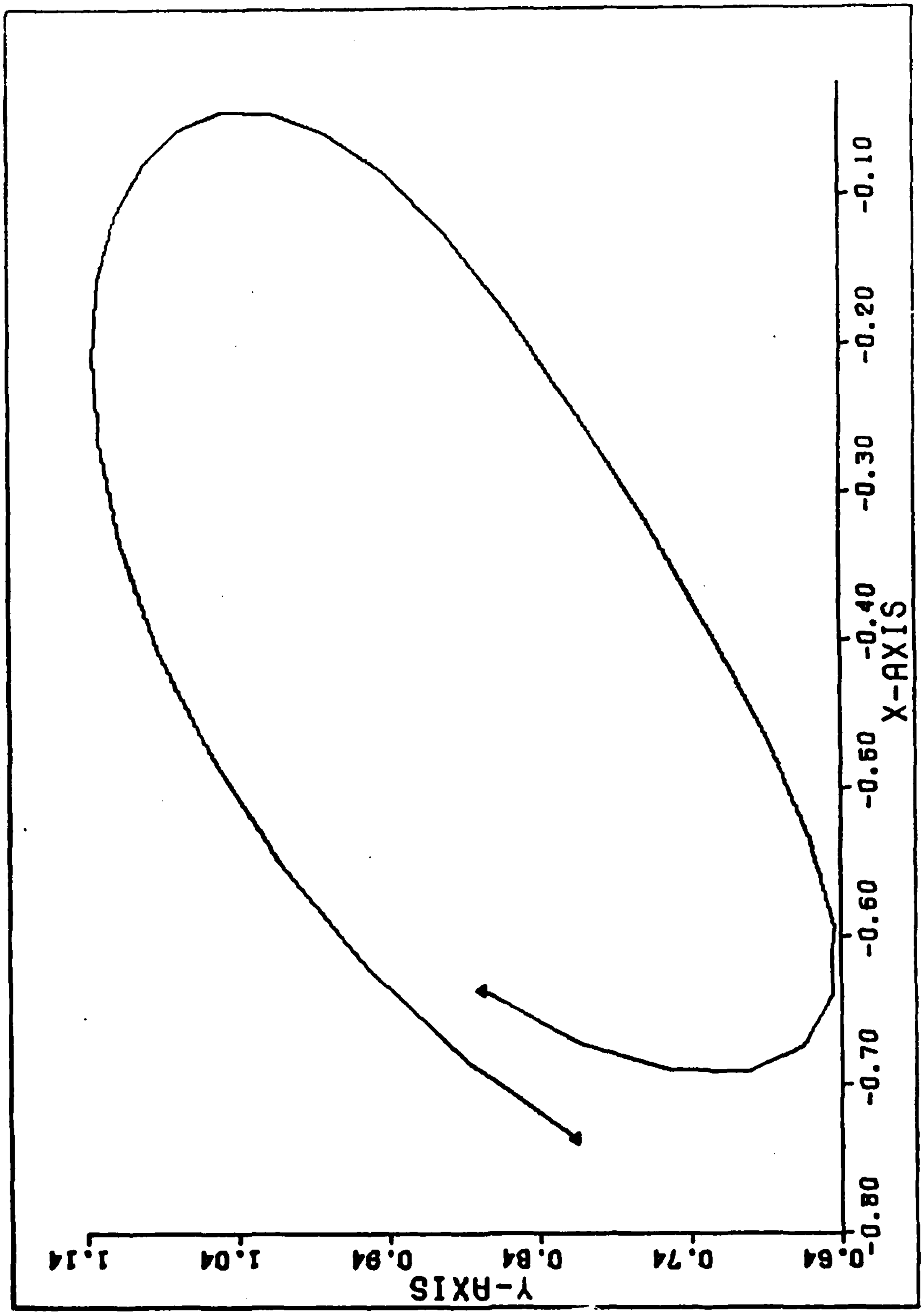

Fig 8. Table III Wheeler Orbit--One Month

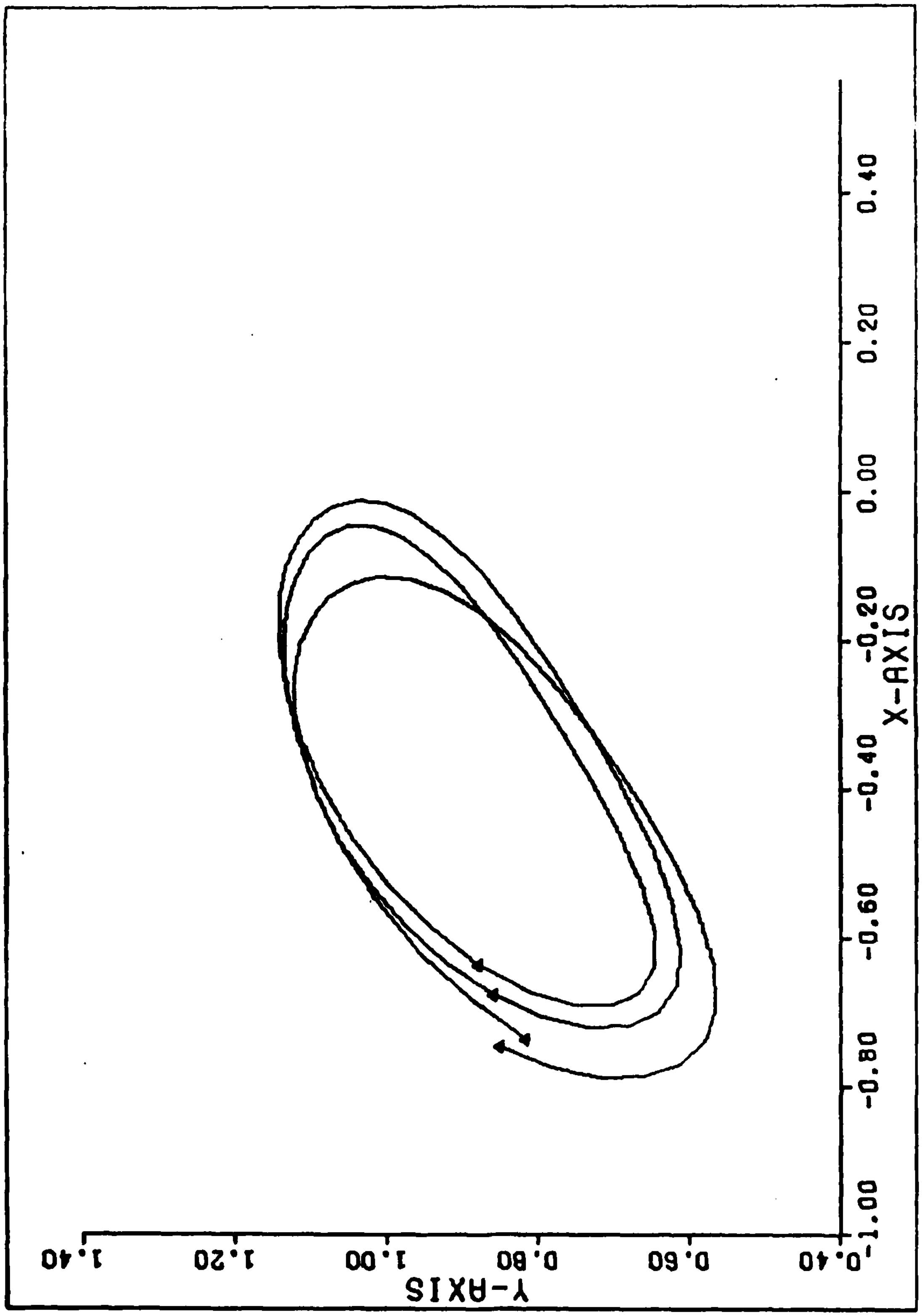

Fig 9. Table III Wheeler Orbit--Three Months

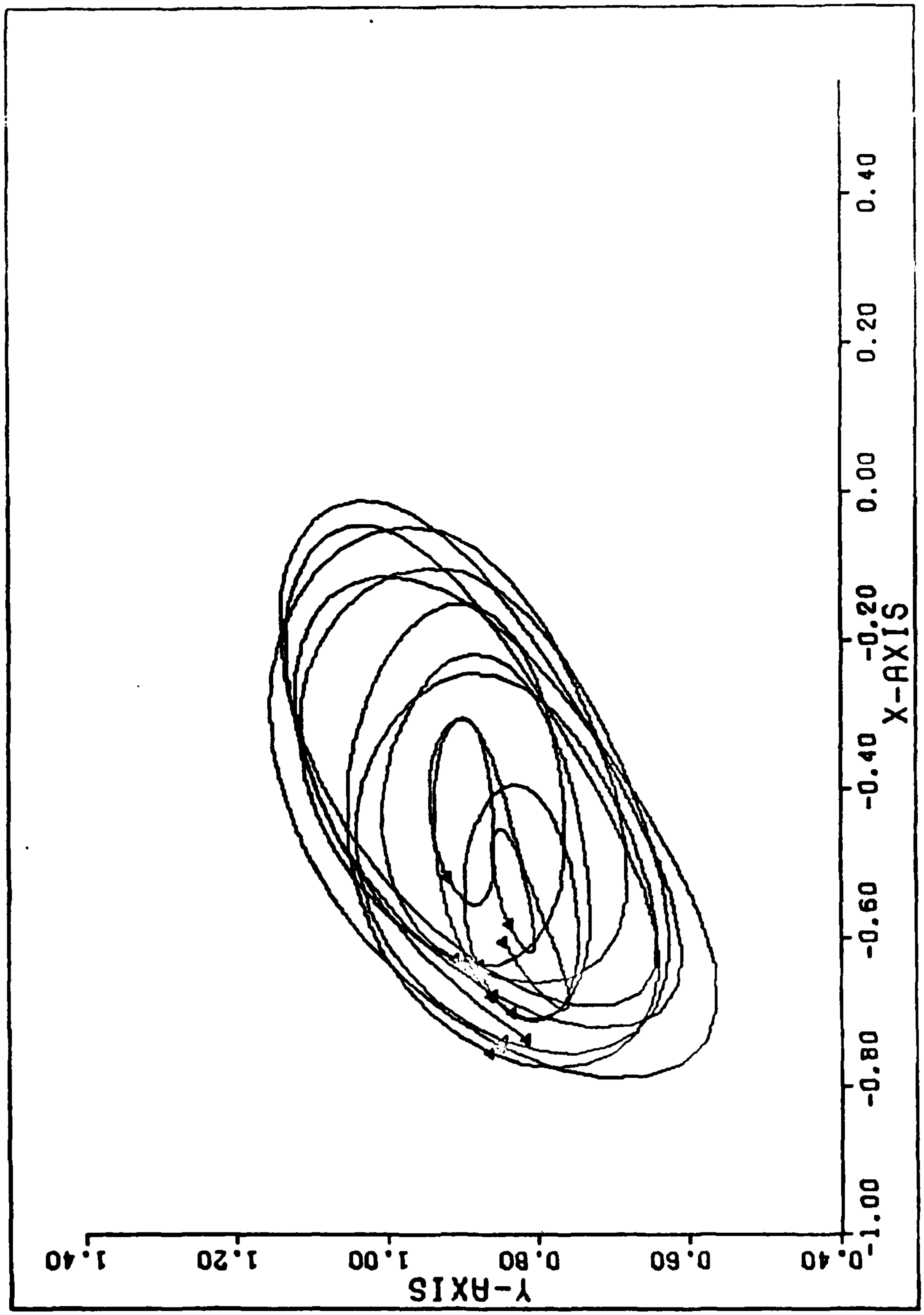

Fig 10. Table III Wheeler Orbit--Twelve Months

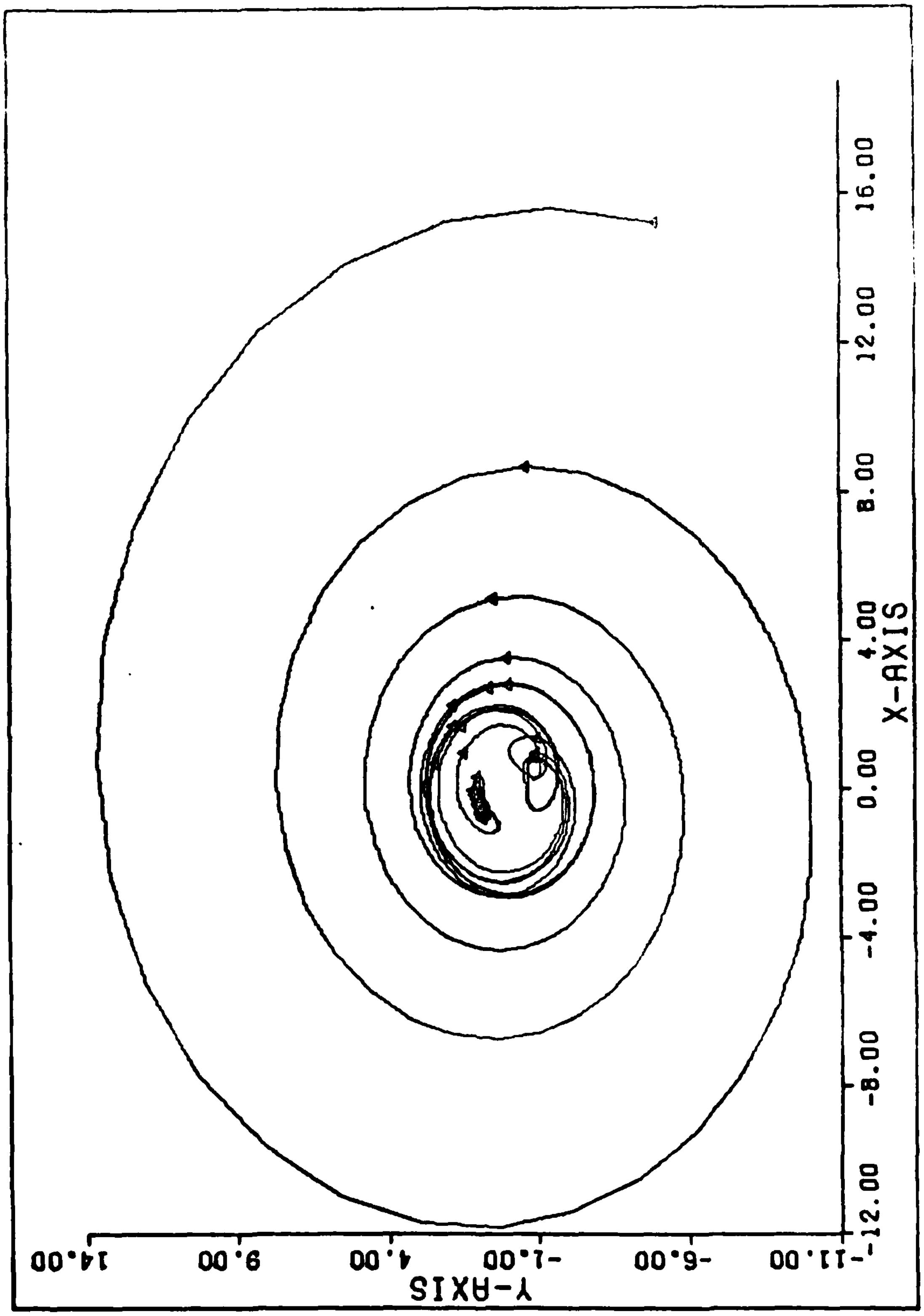

Fig 11. Table III Wheeler Orbit--Sixty Months

Table IV. State Vector of 28.263888 Jan 1979

with Wheeler Elements [illegible]

INITIAL CONDITIONS VECTOR

X(1)= 232 [illegible]

X(2)= [illegible]

X(3)= .0149730 [illegible]

X(4)= 5.43347626665

X(5)= 4.0835534822 14

X(6)= .0001928591601845

X(7)= .5701945487225

X(8)= -.7312694719864

X(9)= .05406587955571

X(10)= .1940024128416

X(11)= .1502478807353

X(12)= -.01690467092801

X(13)= -.1904858617353

X(14)= -1.082237274618

X(15)= .05406587955571

X(16)= .1994321080191

X(17)= -.05965241818428

X(18)= -.01590467092801

ROTATING FRAME COLONY POSITION AND VELOCITY

X= -.7363274298999 XDOT= -.1696730983

Y= .8156863968899 YDOT= -.120592771

Z= .05406587955571 ZDOT= -.01590467092801

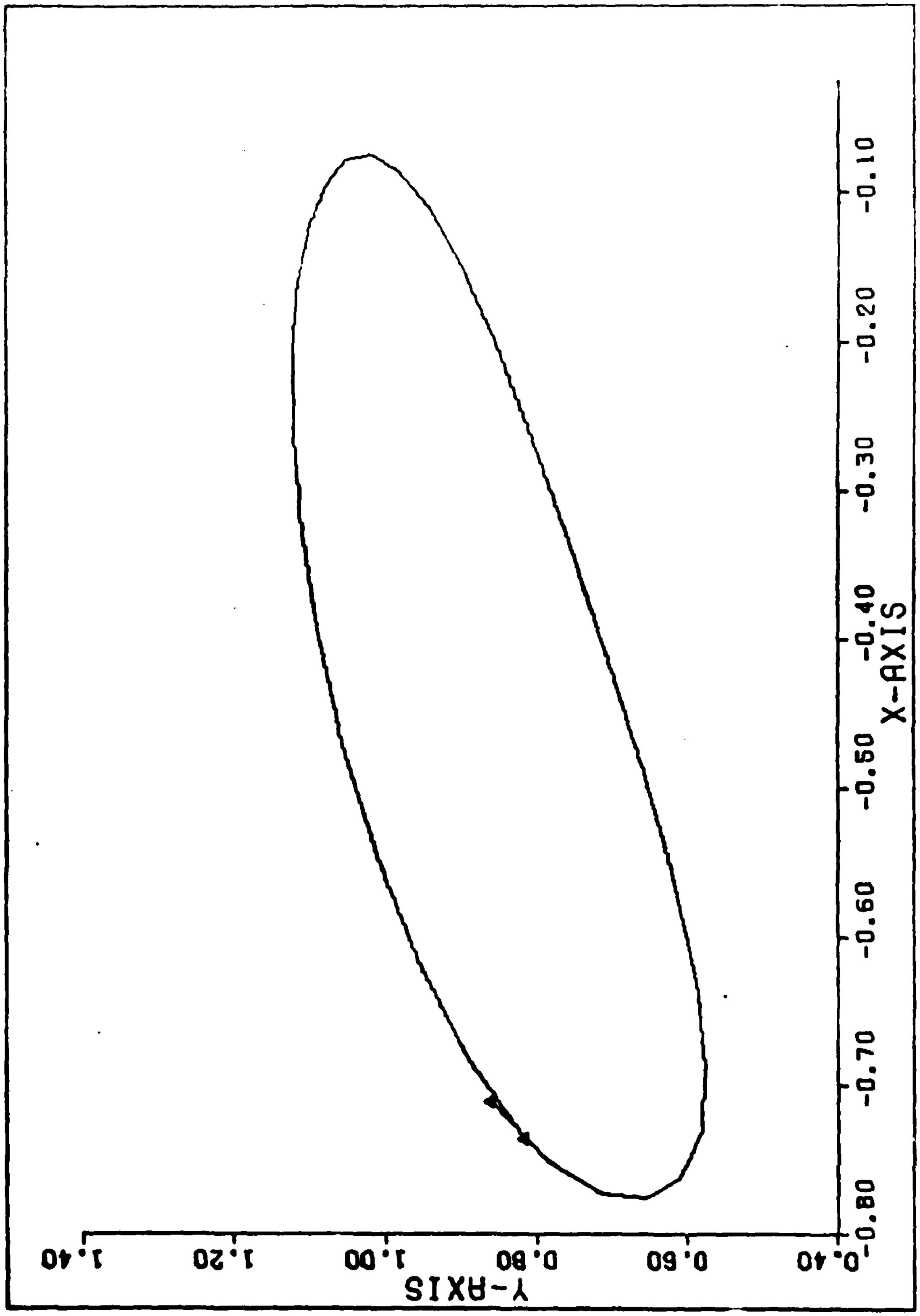

Fig 12. Table IV Modified Wheeler Orbit--One Month

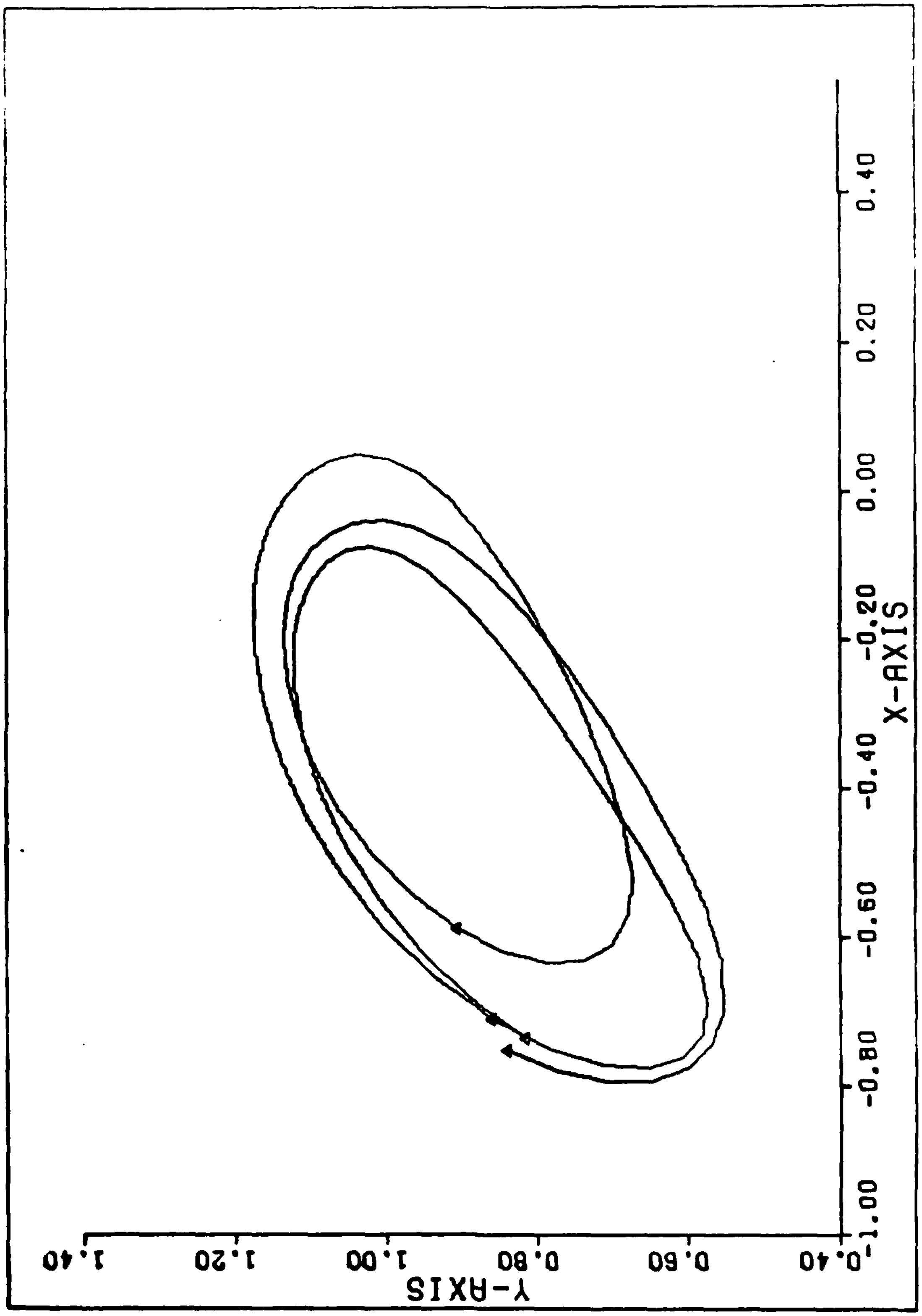

Fig 13. Table IV Modified Wheeler Orbit--Three Months

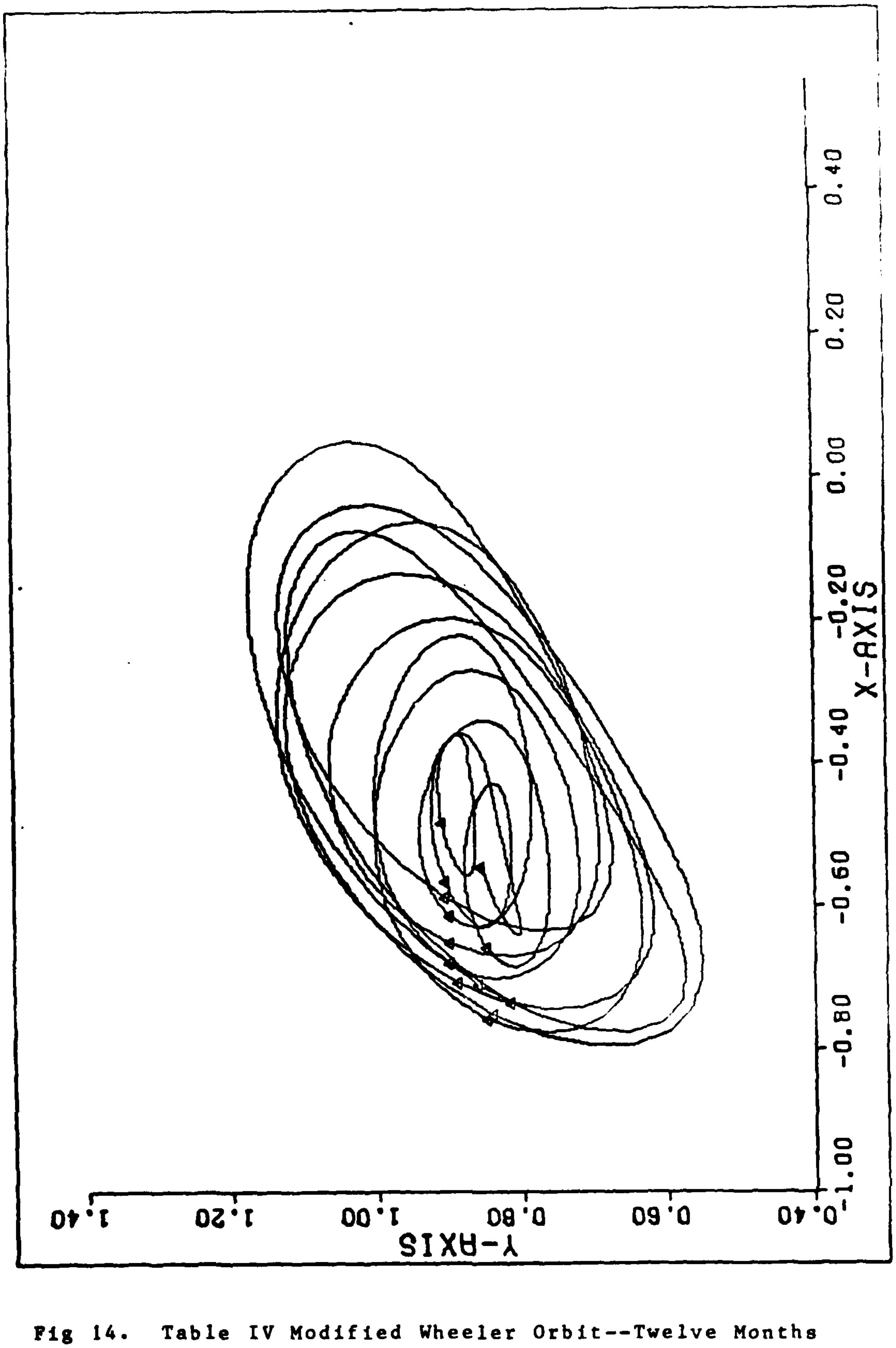

Fig 14. Table IV Modified Wheeler Orbit--Twelve Months

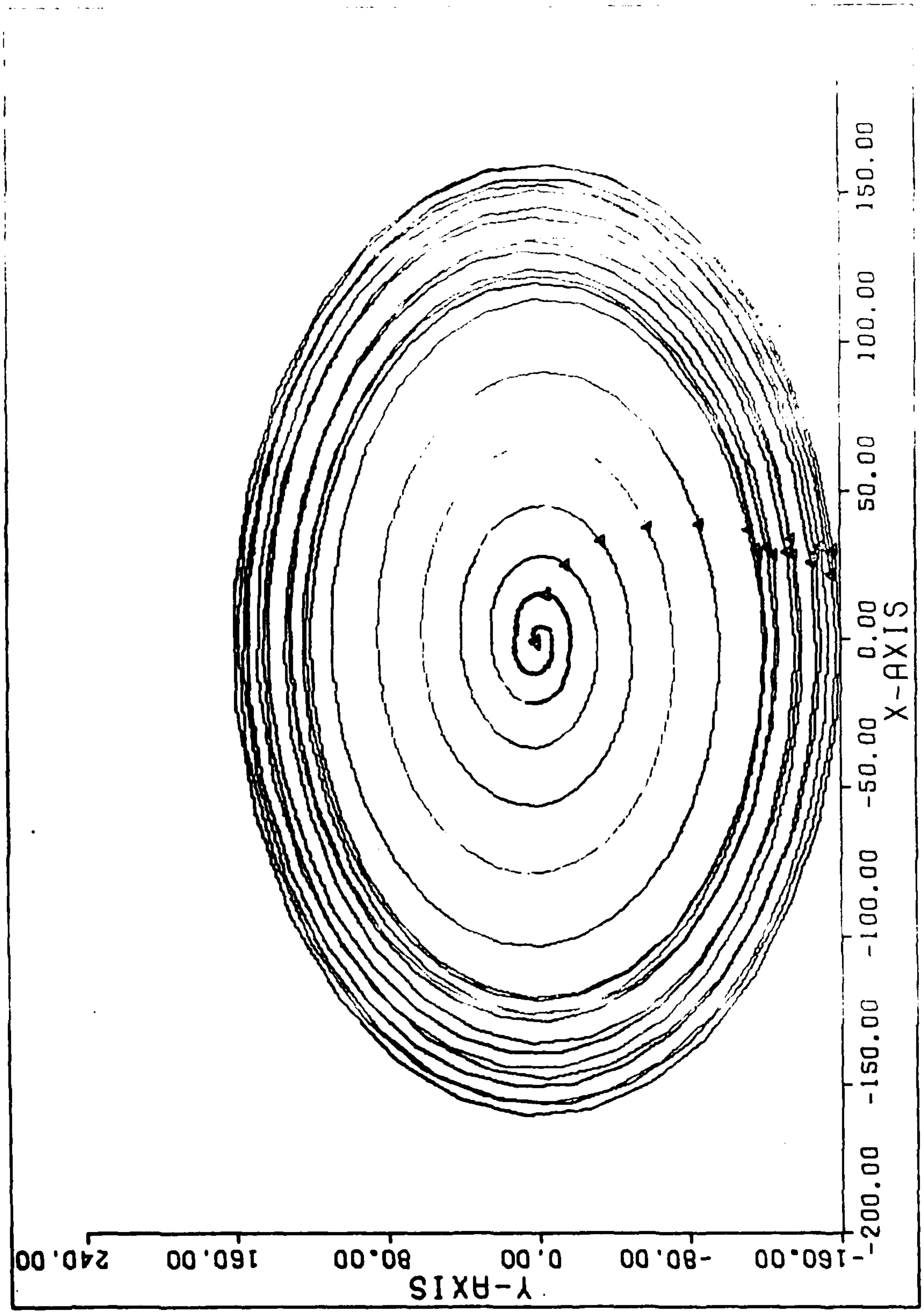

Fig 15. Table IV Modified Wheeler Orbit--Sixty Months

Table V. State Vector of 28.26[illegible] Jan 1979 with

Wheeler Elements at [illegible]

[illegible]

X(1)= [illegible]

X(2)= [illegible]

X(3)= [illegible]

X(4)= 5.4[illegible]

X(5)= 4.083553482214

X(6)= .0001928591601845

X(7)= .5701945487225

X(8)= -.7312694719864

X(9)= .05406587955571

X(10)= .1940024128416

X(11)= .1502478807353

X(12)= -.01690467092801

X(13)= -.1904858617353

X(14)= -1.082237274618

X(15)= .05406587955571

X(16)= .2022391184736

X(17)= -.05999982616972

X(18)= -.01790467092801

ROTATING FRAME COLONY POSITION AND VELOCITY

X= -.7363274298999 XDOT= -.1716730983

Y= .8156863968899 YDOT= -.122592771

Z= .05406587955571 ZDOT= -.01790467092801

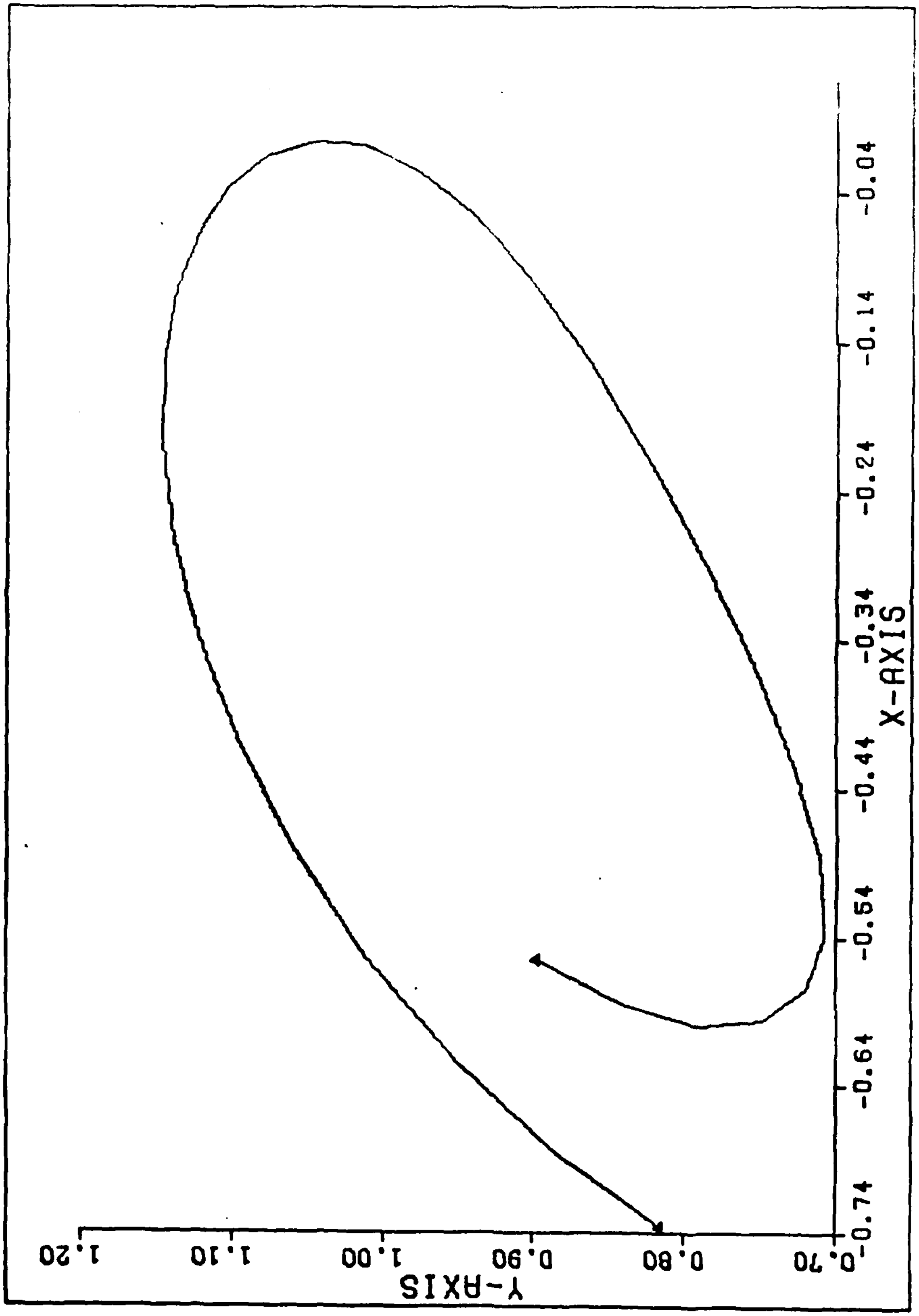

Fig 16. Table V Modified Wheeler Orbit--One Month

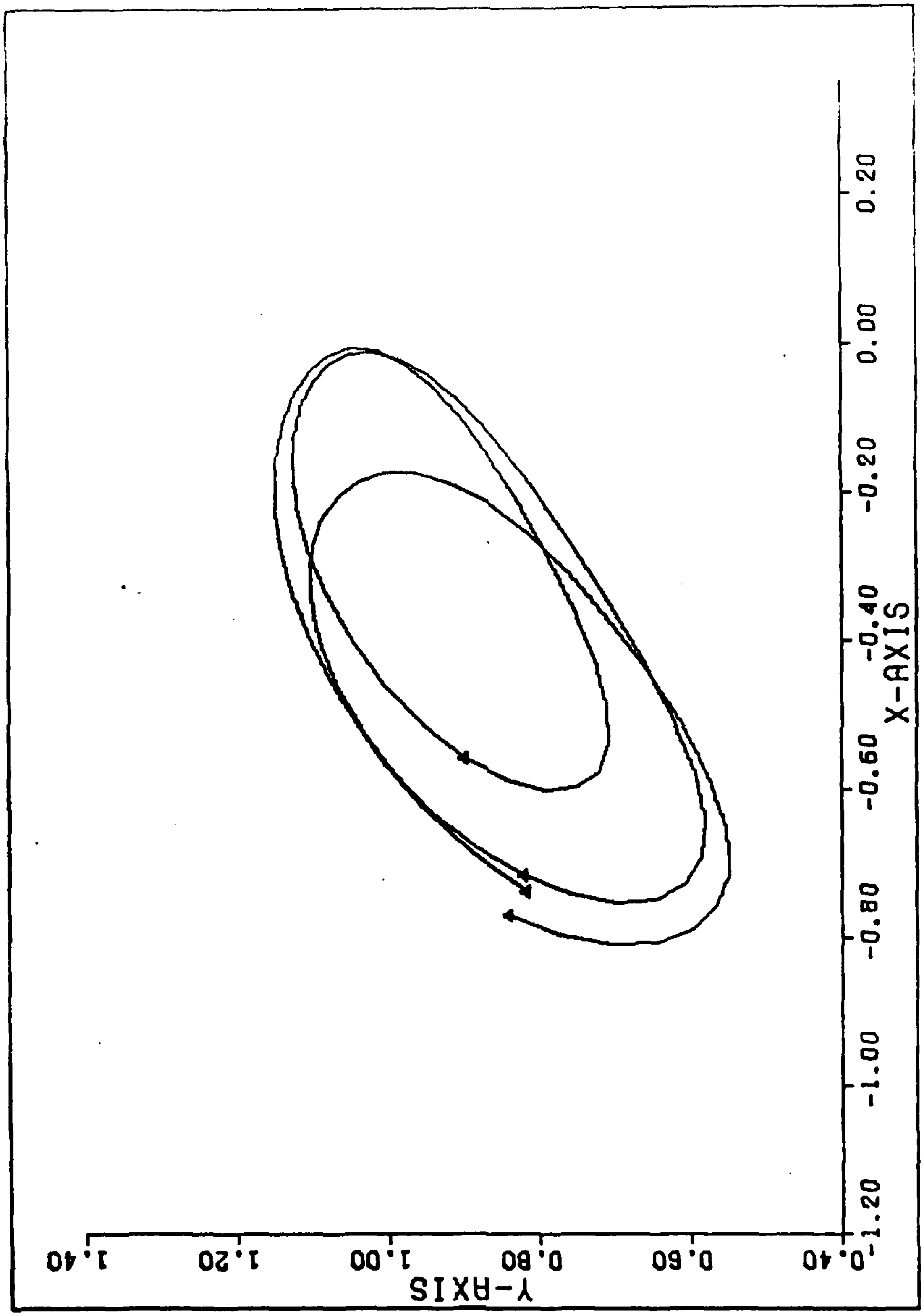

Fig 17. Table V Modified Wheeler Orbit--Three Months

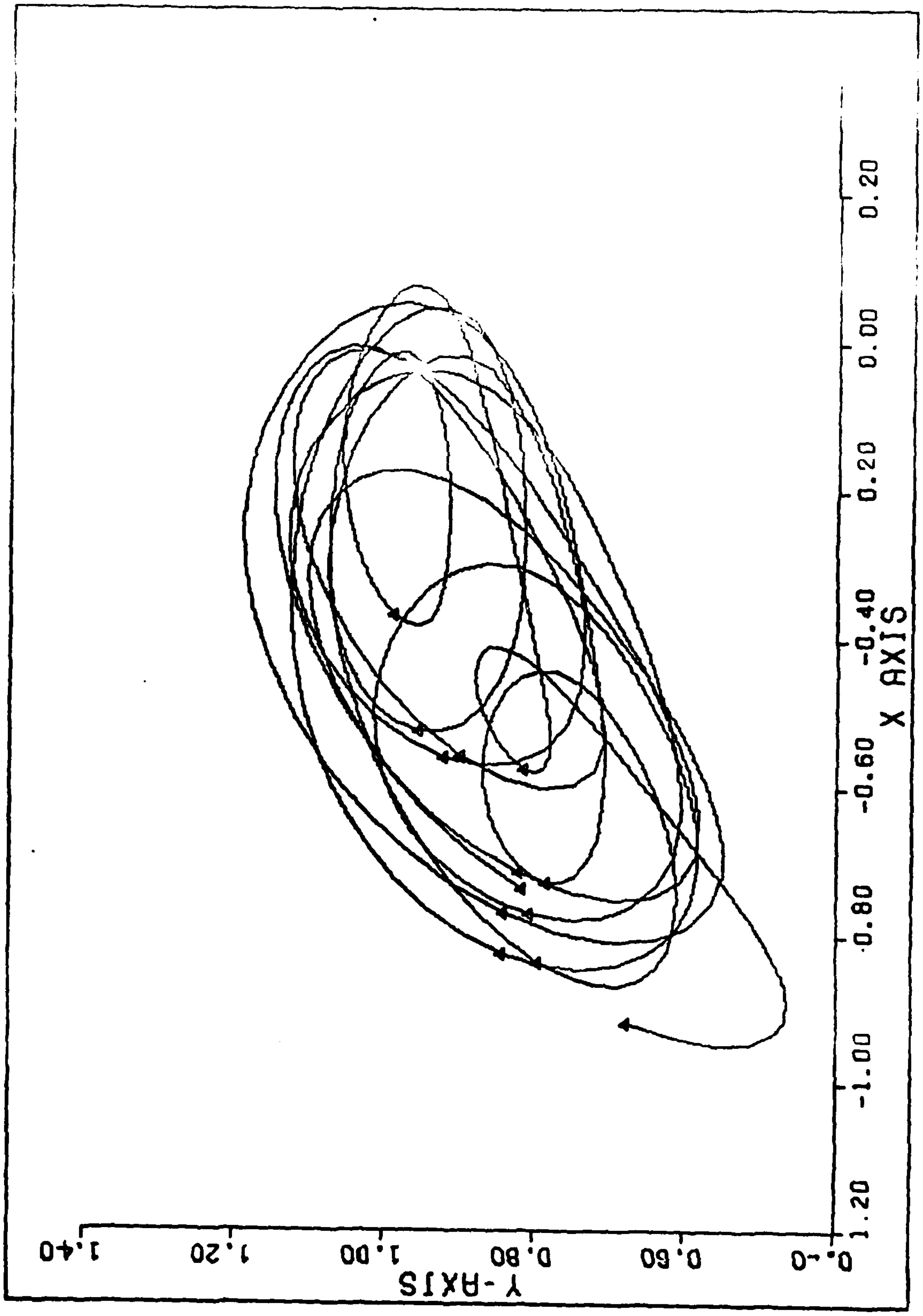

Fig 18. Table V Modified Wheeler Orbit--Twelve Months

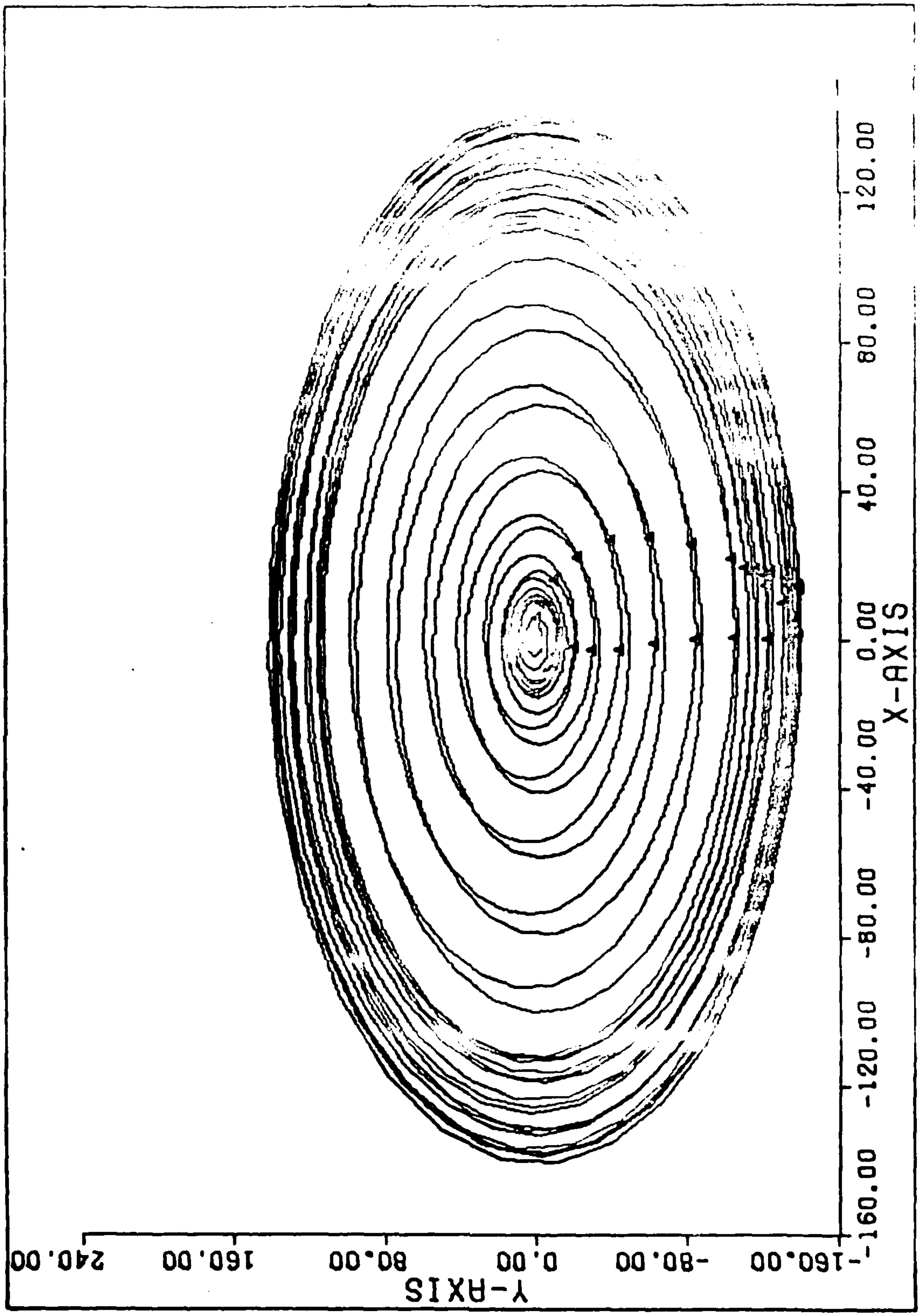

Fig 19. Table V Modified Wheeler Orbit--Sixty Months

[illegible], the orbit [illegible] out a finite volume of [illegible] volume then the orbit would [illegible], [illegible] in this volume, for a longer period, then the orbit is stable for that period. The subroutine in Appendix C which is used for [illegible] trajectory [illegible] orbit in Fig 11 is analyzed in this manner and is presented in Fig 20. Similiarly, the orbits from Fig's 13 and 19 are analyzed and their cross sections are presented in Fig's 21 and 22 respectively. An analysis of the plots show the orbits to be unstable for all three cases.

The search was continued in the manner described above until a likely candidate was found. The plots for the system are contained in Fig's 23 through 30. Fig 30 is the plot of the cross section for a period of fifty years. The cross sections fill only the definite areas inscribed and do not fall out these areas. This leads to the conclusion that this particular state vector produces a stable trajectory for the required time.

The state vector contained in Table VI is not coincident with a new moon. Indeed, it is 5.01 days prior to the new moon, but an important point is the initial position is precisely the initial position of the Wheeler orbit. The velocities differ only in the third decimal place. The state vector is integrated forward to the new moon to find the set of initial conditions required for a stable orbit. This

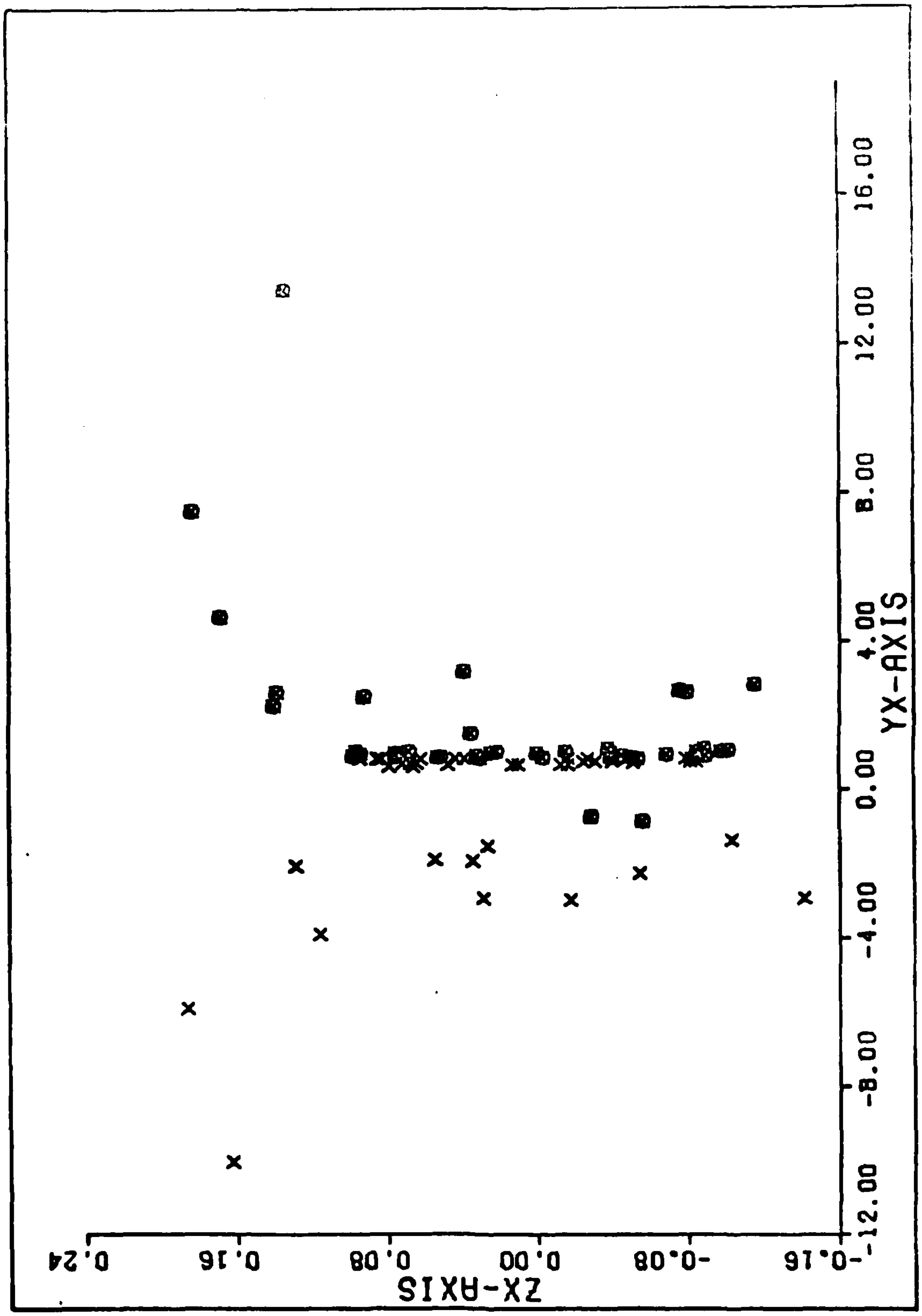

Fig 20. Cross Section of Orbit of Fig 11

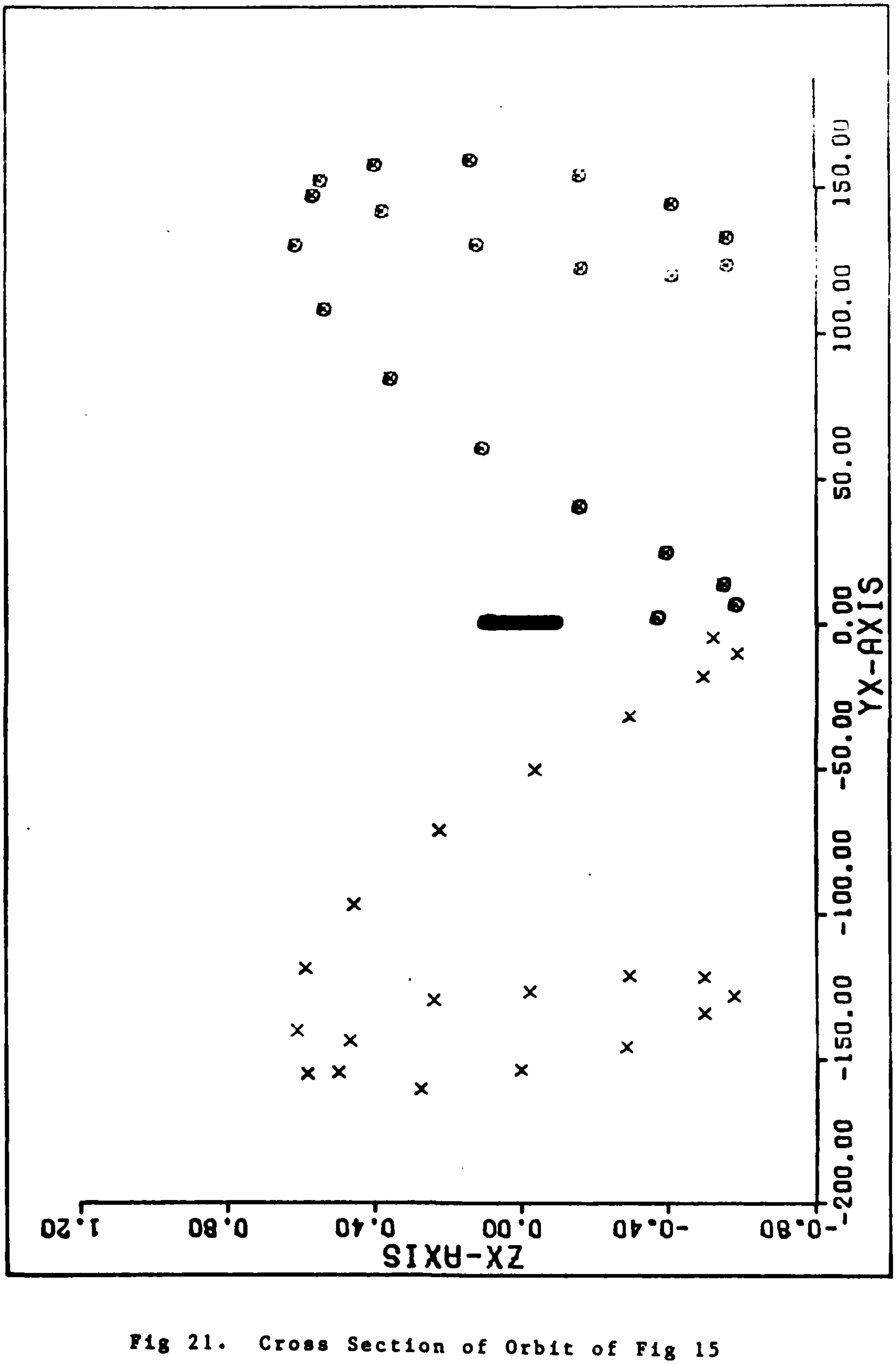

Fig 21. Cross Section of Orbit of Fig 15

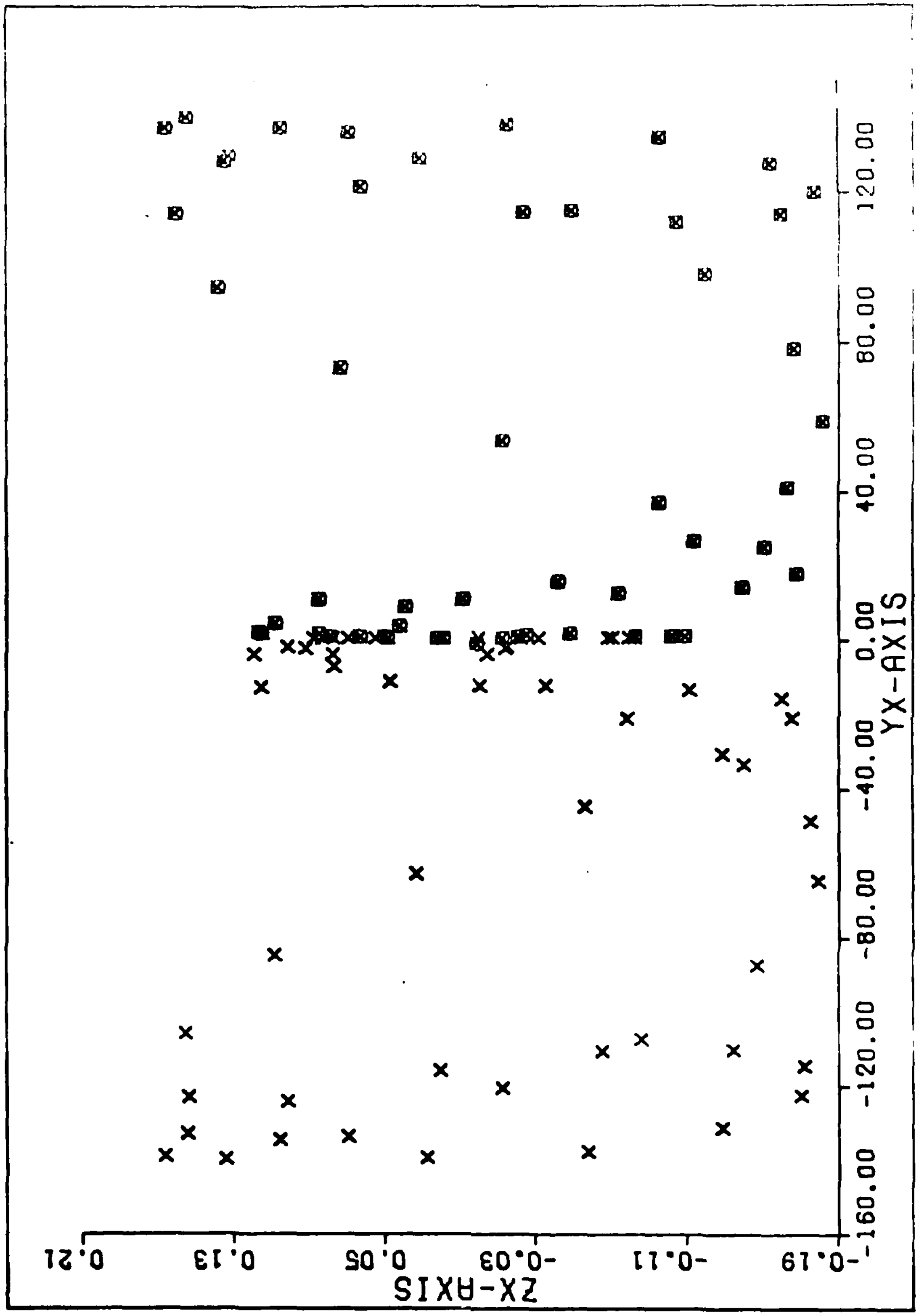

Fig 22. Cross Section of Orbit of Fig 19

Table VI. [illegible] State Vector [illegible] 1979 Including [illegible]

[illegible]

INITIAL [illegible]

X(1)= [illegible]

X(2)= [illegible]324.0192[illegible]

X(3)= -.01548148446983

X(4)= 5.772936900634

X(5)= 5.594798567918

X(6)= .0001694978992515

X(7)= -.5624053120187

X(8)= -.7970584471

X(9)= .08042470202791

X(10)= .1985222851766

X(11)= -.120556823555

X(12)= .007137337355667

X(13)= -1.090992584716

X(14)= -.1313680409769

X(15)= .07847315514708

X(16)= .01046712105918

X(17)= -.2064773418006

X(18)= .01826280010529

ROTATING FRAME COLONY POSITION AND VELOCITY

X= -.7363274299 XDOT= -.162673098

Y= .81568639689 YDOT= -.127592776

Z= .07847315514708 ZDOT= .01826280010529

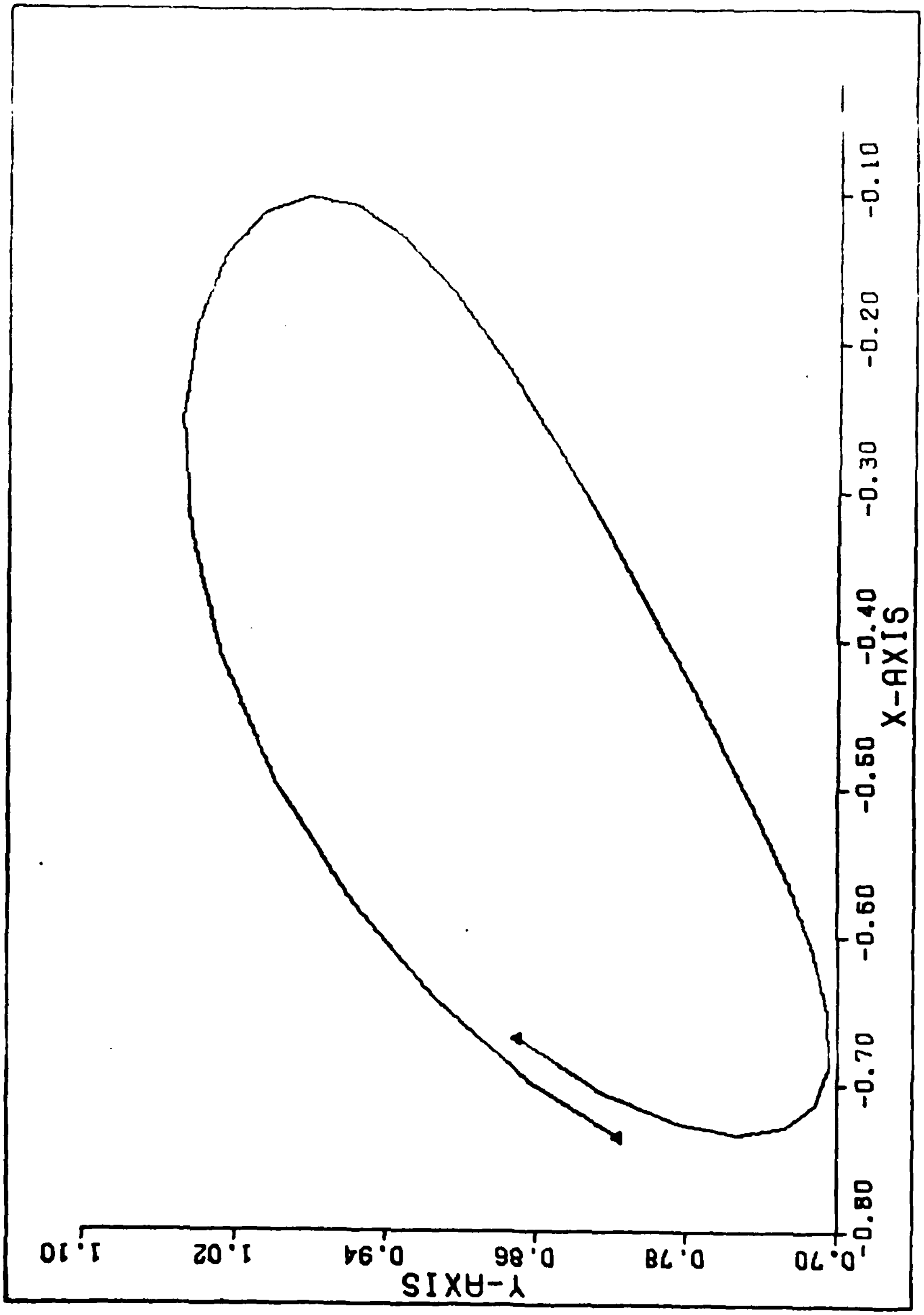

Fig 23. Table VI Stable Orbit Candidate--One Month

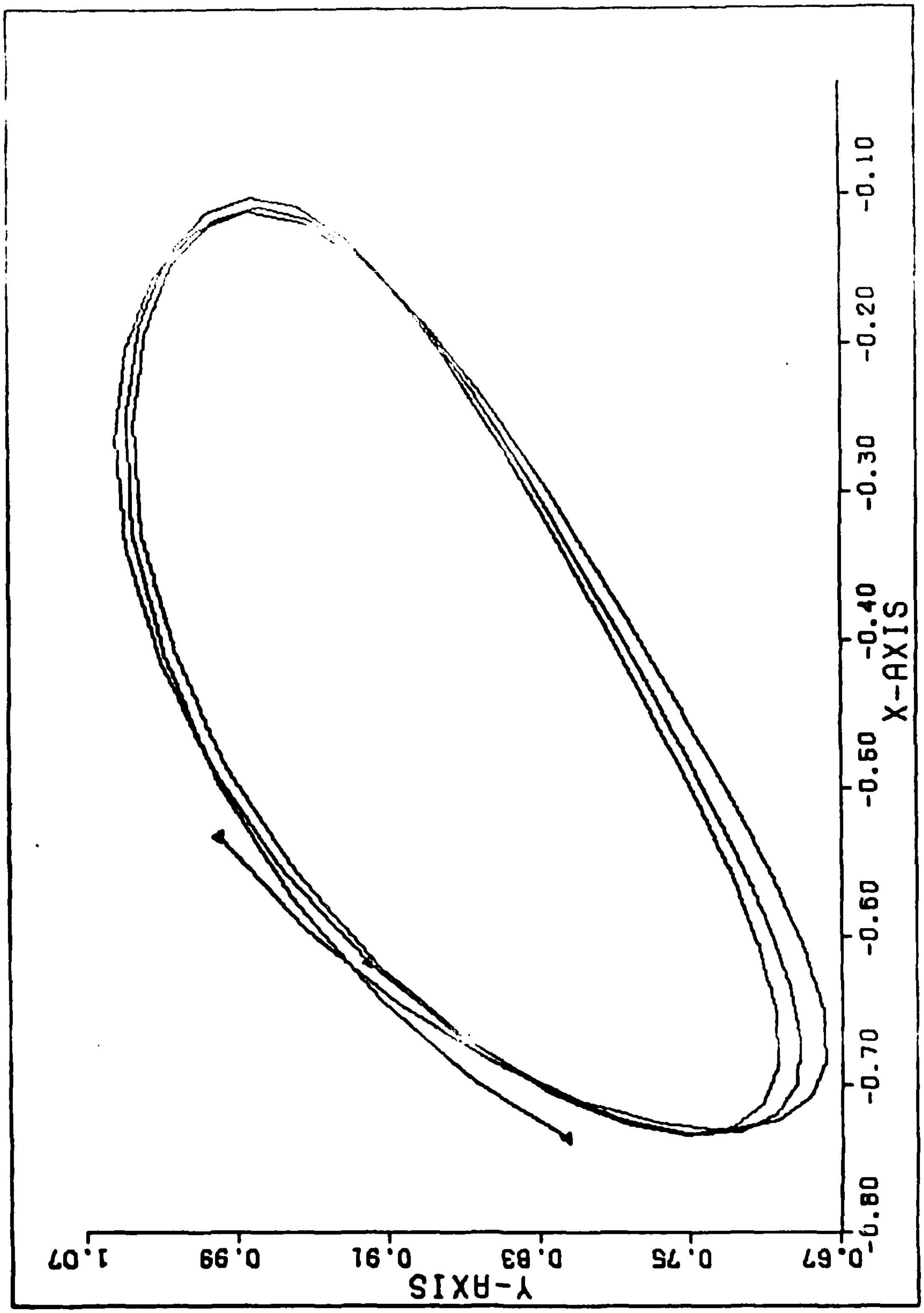

Fig 24. Table VI Stable Orbit Candidate--Three Months

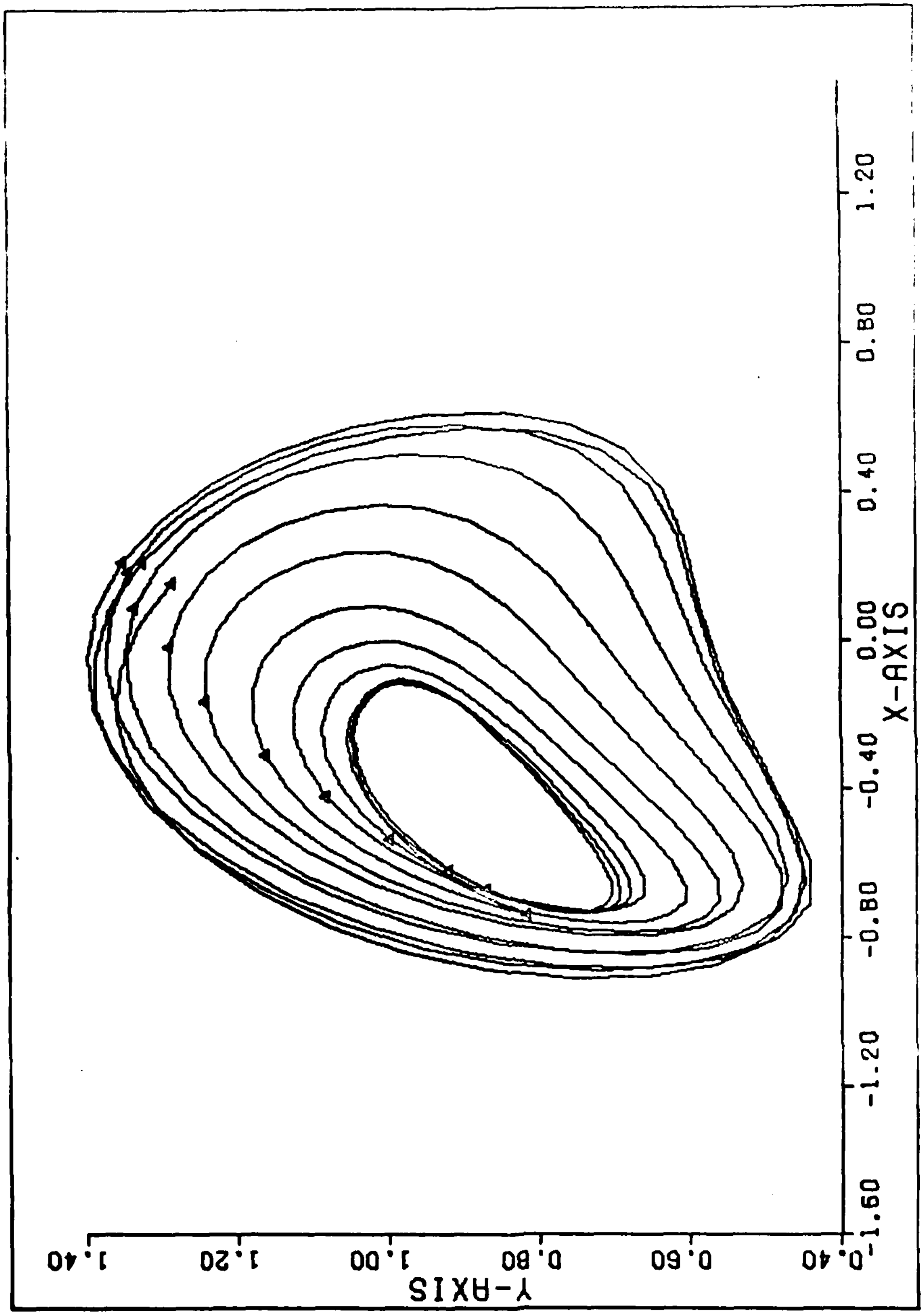

Fig 25. Table VI Stable Orbit Candidate--Twelve Months

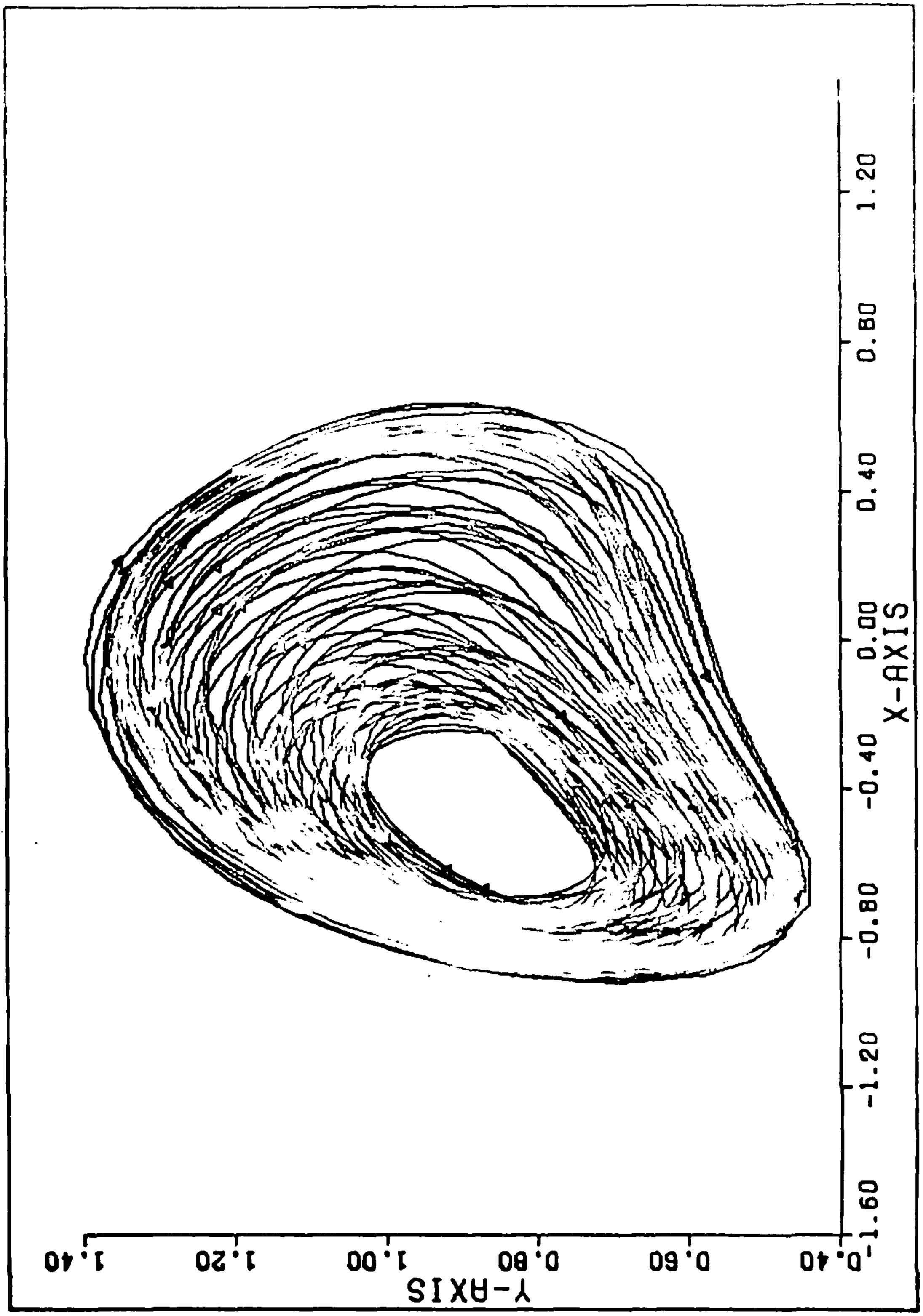

Fig 26. Table VI Stable Orbit Candidate--Sixty Months

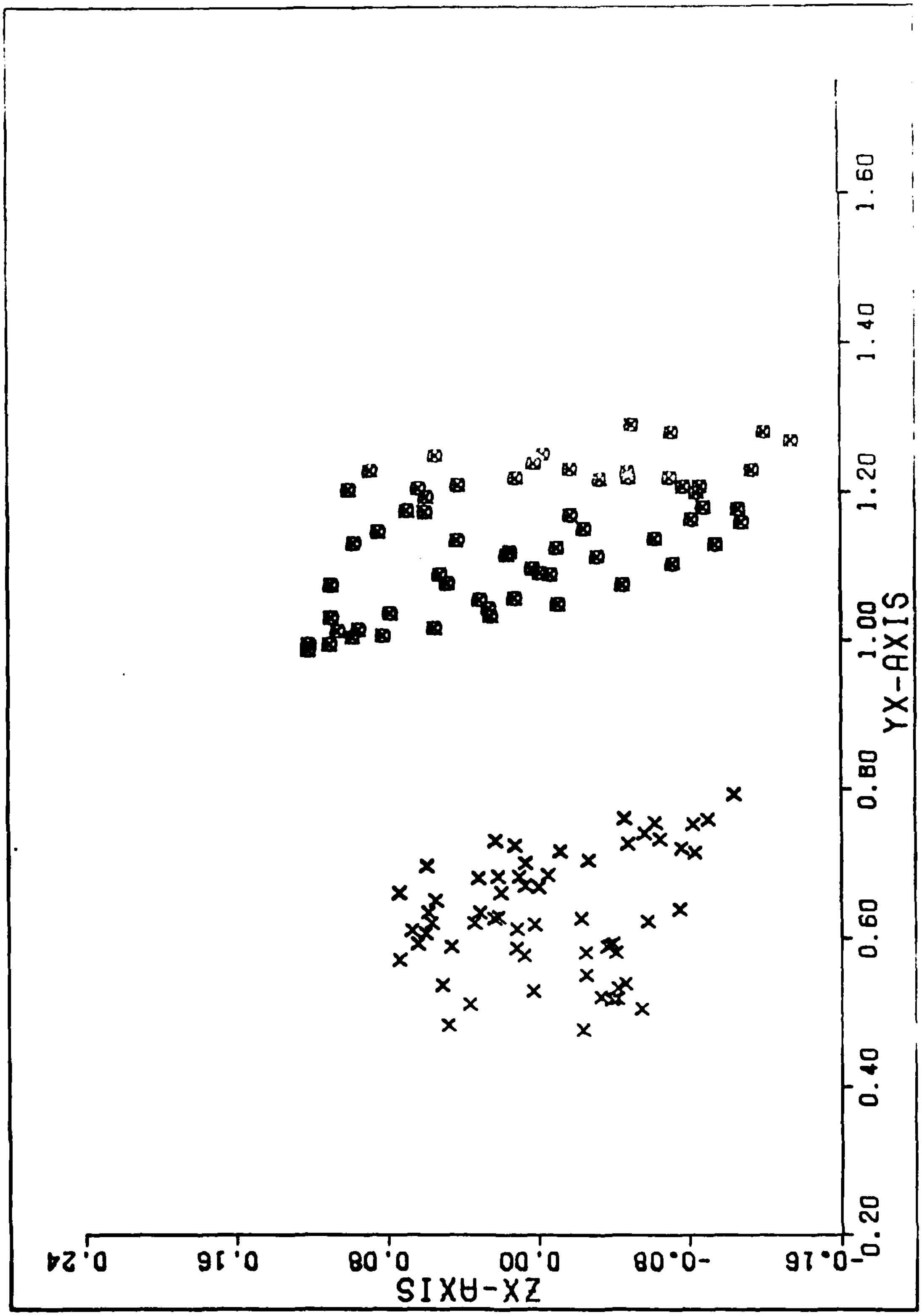

Fig 27. Cross Section of Orbit of Fig 26

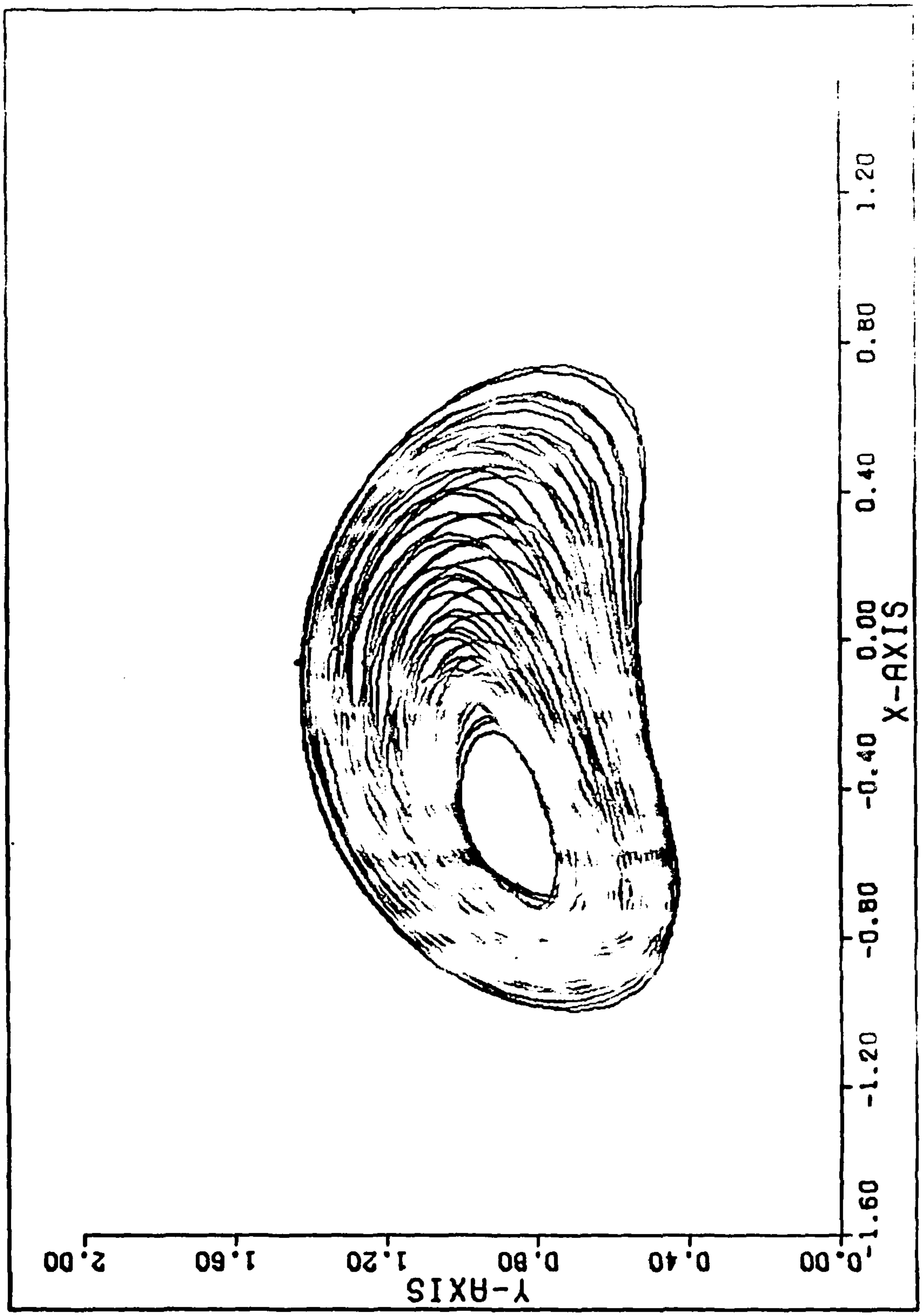

Fig 28. Stable Orbit Candidate-- 60-120 Months

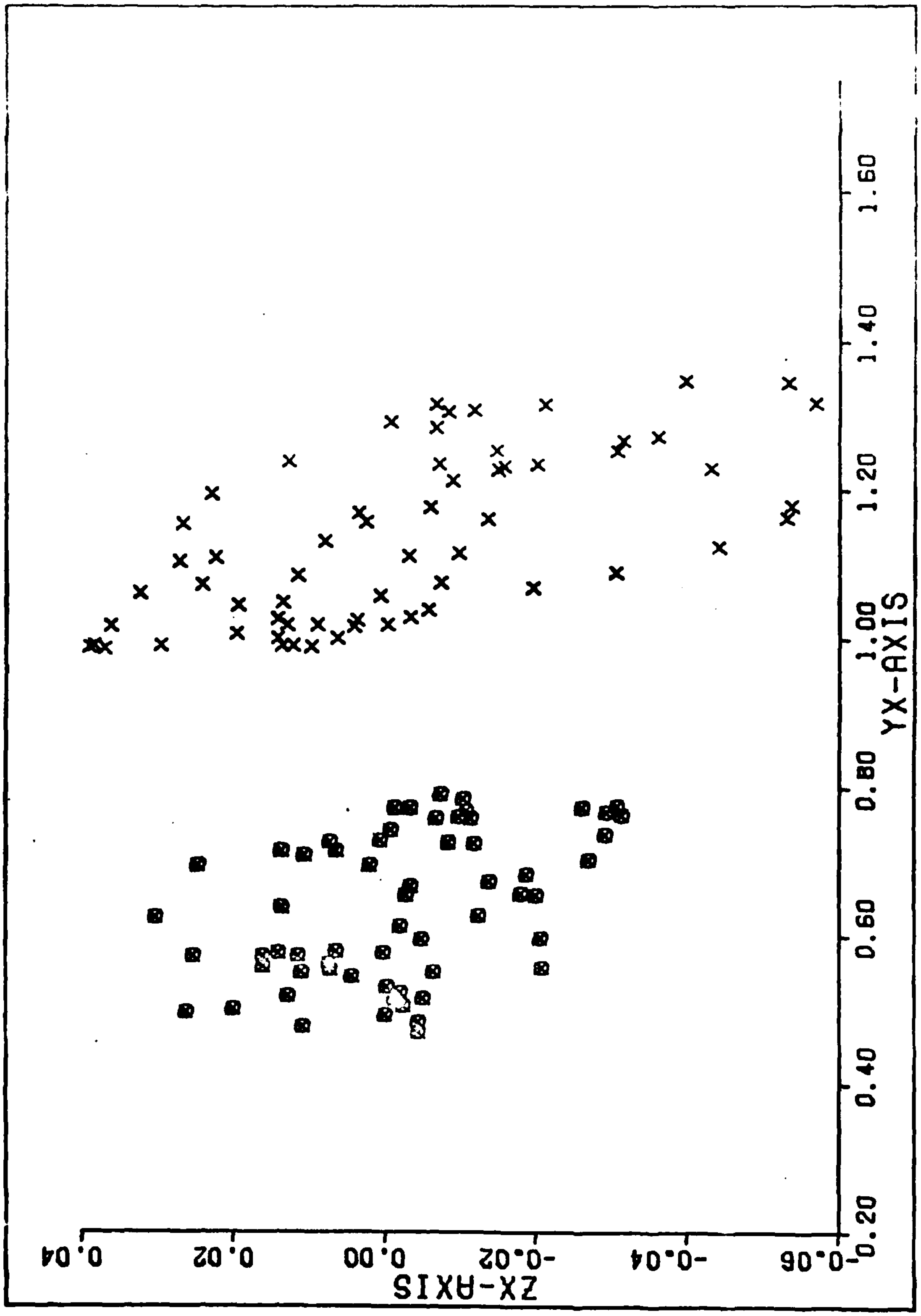

Fig 29. Cross Section of Orbit of Fig 28

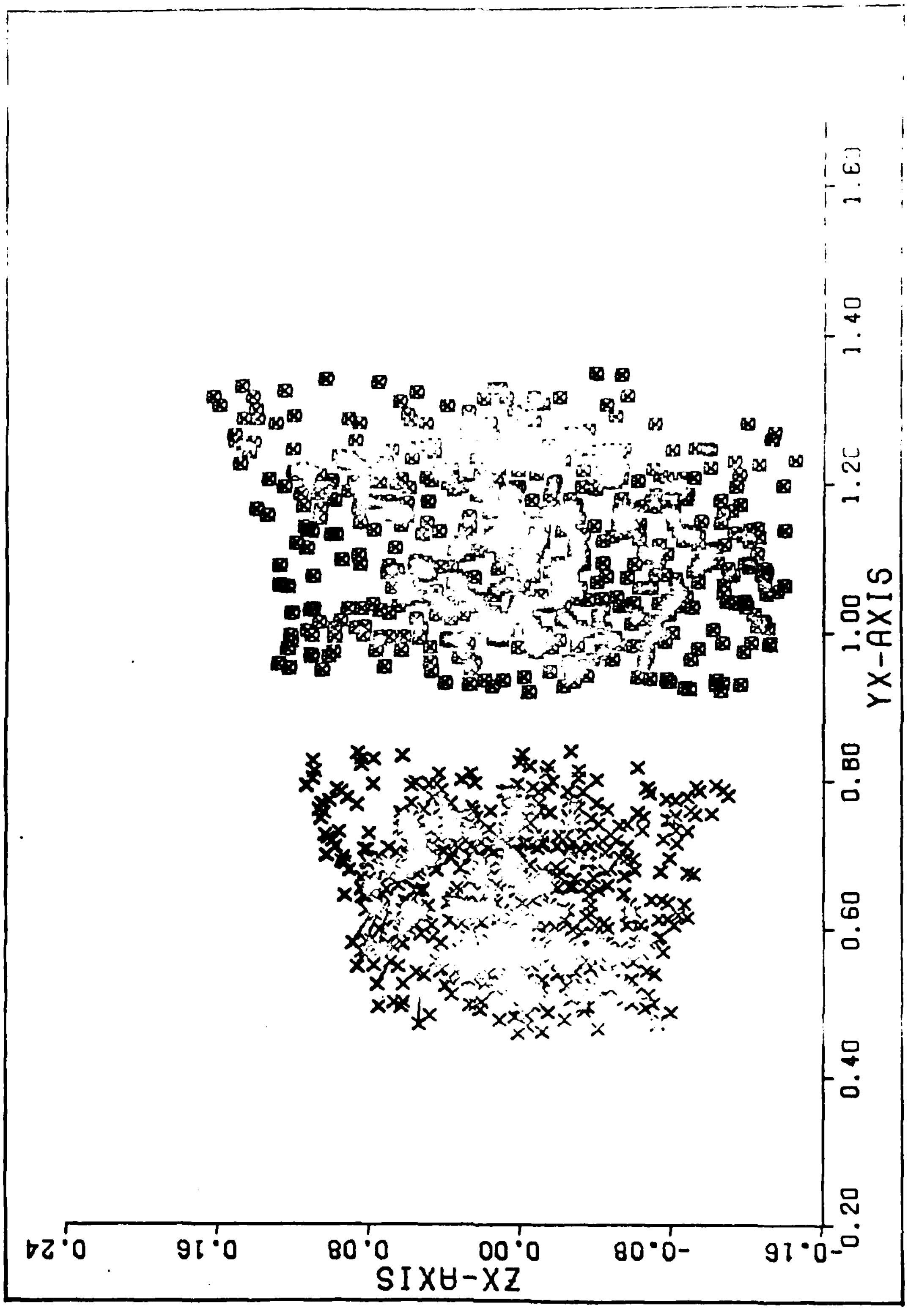

Fig 30. Cross Section of Stable Orbit-- 600+ Months

state vector [illegible] Table VII.

[illegible] If the state vector [illegible] forward for approximately [illegible] days, [illegible] positions and velocities [illegible] Table [illegible] VIII contains the state vector of 23.0 Jan [illegible] with the stable orbit elements.The subsequent plots and cross sections are contained in Fig's 31 through 39. These plots are essentially the same as the plots that are produced from Table VI, and the conclusion can be reached that the state vector is at least marginally stable. Further investigation of this state vector does indeed show marginal stability for the system, but the results are inseparable from those produced from Table VI, and are therefore not reproduced here.

Finally, Ref [3:54] refers to a 180° out of phase orbit as proposed by Ref 6. To search for this orbit the state vector from Table VI was integrated forward for fifteen time steps, or half the period. The satellite's position and velocity vector is then recorded for use in the system analysis. The state vector of Table VI is then combined with this position and velocity vector and tested for stability. The vector produces a similiar orbit to that produced in Table VI and is reproduced in Table IX. The orbits and their cross section for a period of sixty months are produced in

Table VII. State Vector of 22.263988 Jan 1979 Including

Stable Orbit [illegible]

INITIAL CONDITIONS VECTOR

X(1)= 232.4126015398

X(2)= [illegible]

X(3)= -.0185730735423[illegible]

X(4)= 5.433426266665

X(5)= 4.083553482214

X(6)= .0001928591601845

X(7)= .5701945487225

X(8)= -.7312694719864

X(9)= .05406587955571

X(10)= .1940024128416

X(11)= .1502478807353

X(12)= -.01690467092801

X(13)= -.5646495963434

X(14)= -.9507939452589

X(15)= .1214735632235

X(16)= .1815202065839

X(17)= -.09474670225174

X(18)= -.002393534008842

ROTATING FRAME COLONY POSITION AND VELOCITY

X= -.4025971030472 XDOT= -.1863345732500

Y= 1.029929058532 YDOT= -.08488786601906

Z= .1214735632235 ZDOT= -.002393534008842

Table VIII. Sensitivity Test State Vector

INITIAL CONDITIONS VECTOR

X(1)= [illegible]

X(2)= [illegible]

X(3)= -.0155242696305

X(4)= 5.788942315224

X(5)= 3.56928947062

X(6)= .0001682791094172

X(7)= -.6116525585634

X(8)= -.7651139719598

X(9)= .07847315515033

X(10)= .1901730539485

X(11)= -.1315409385366

X(12)= .008262800093982

X(13)= -1.096901223061

X(14)= -.06580341132929

X(15)= .07847315514708

X(16)= -.0019155744420186

X(17)= -.2067336060609

X(18)= .01826280010529

ROTATING FRAME COLONY POSITION AND VELOCITY

X= -.7363274299 XDOT= -.162673098

Y= .81568639689 YDOT= -.127592776

Z= .07847315514708 ZDOT= .01826280010529

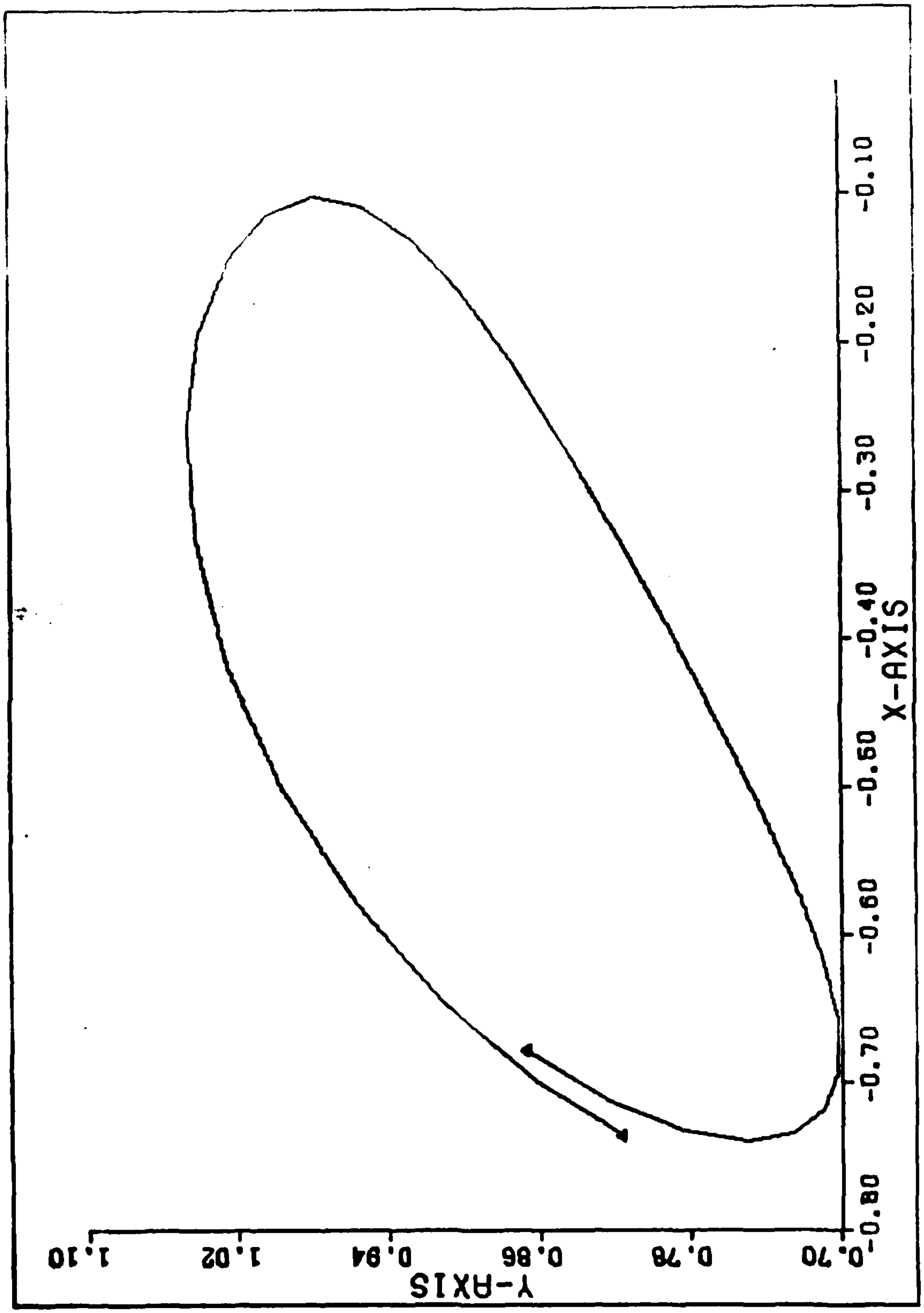

Fig 31. Table VIII Sensitivity Orbit--One Month

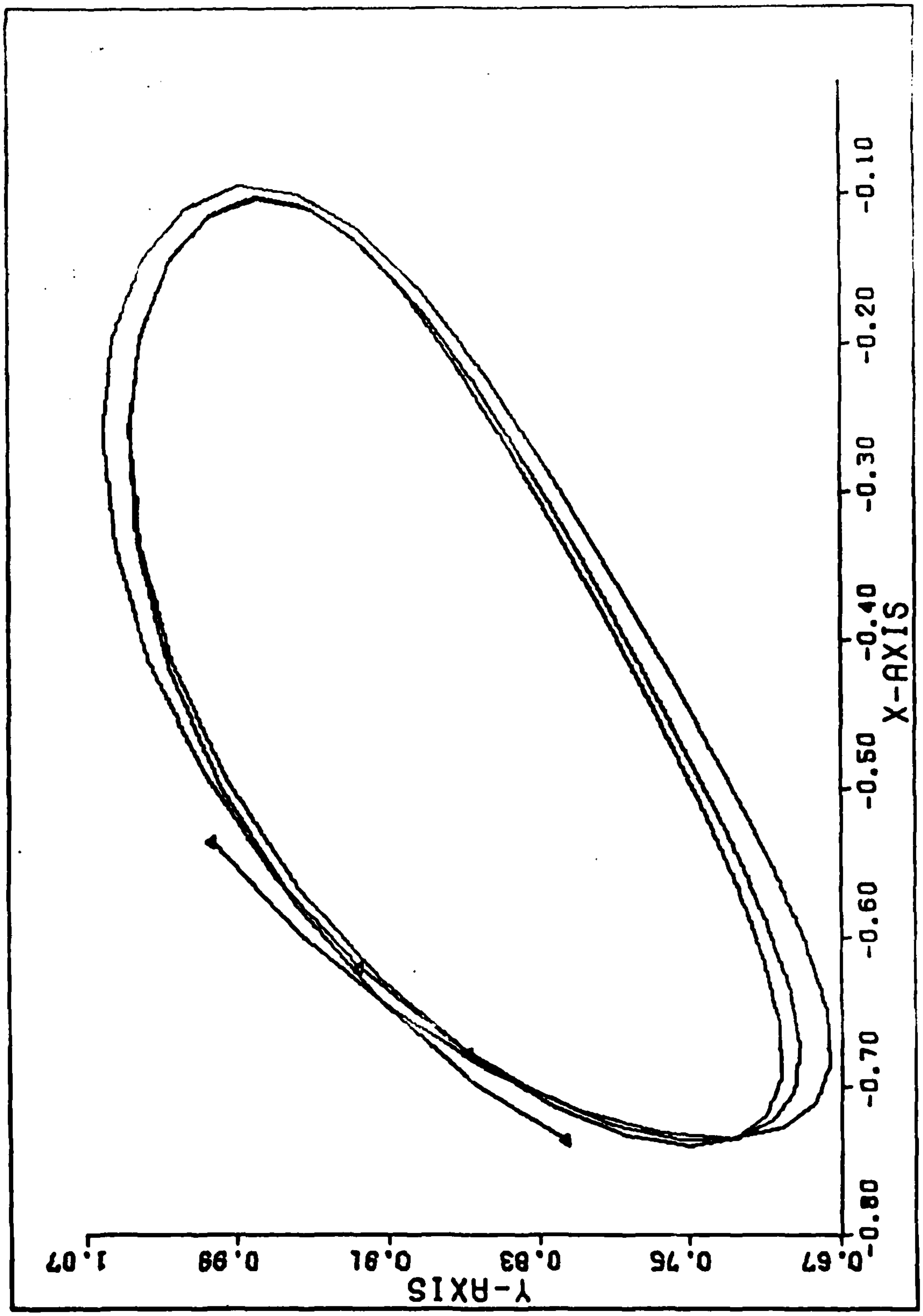

Fig 32. Table VIII Sensitivity Orbit--Three Months

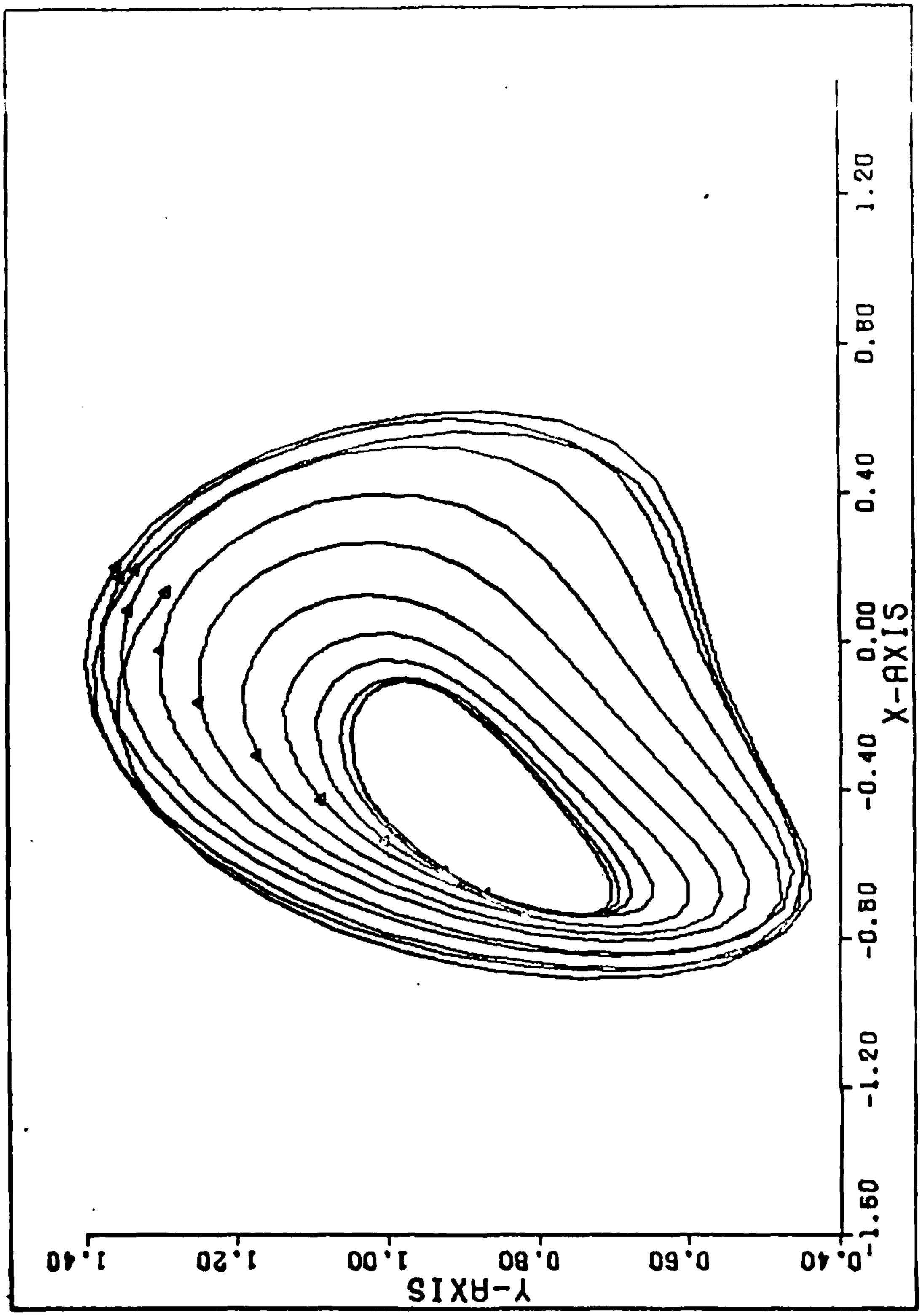

Fig 33. Table VIII Sensitivity Orbit--Twelve Months

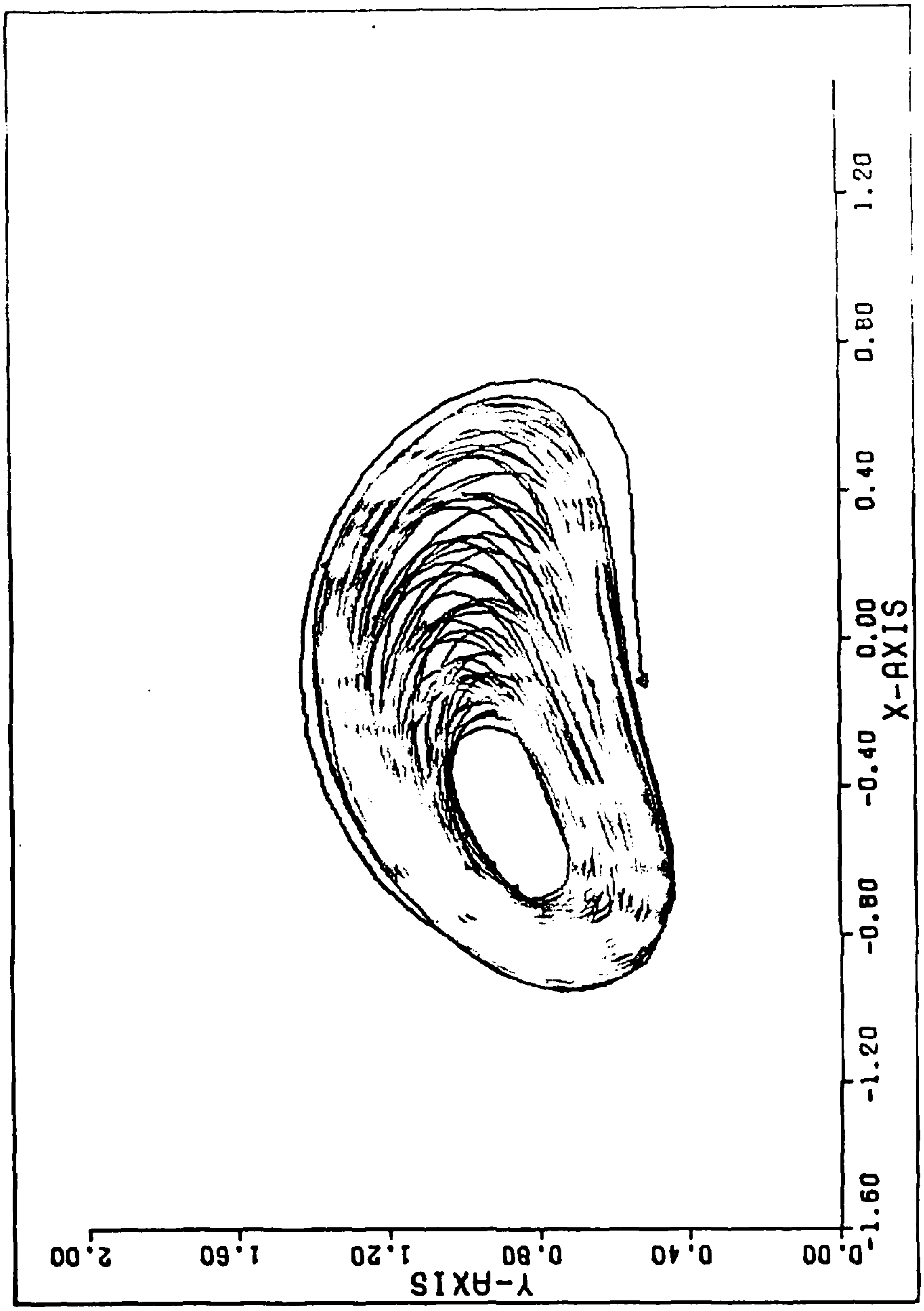

Fig 34. Table VIII Sensitivity Orbit--Sixty Months

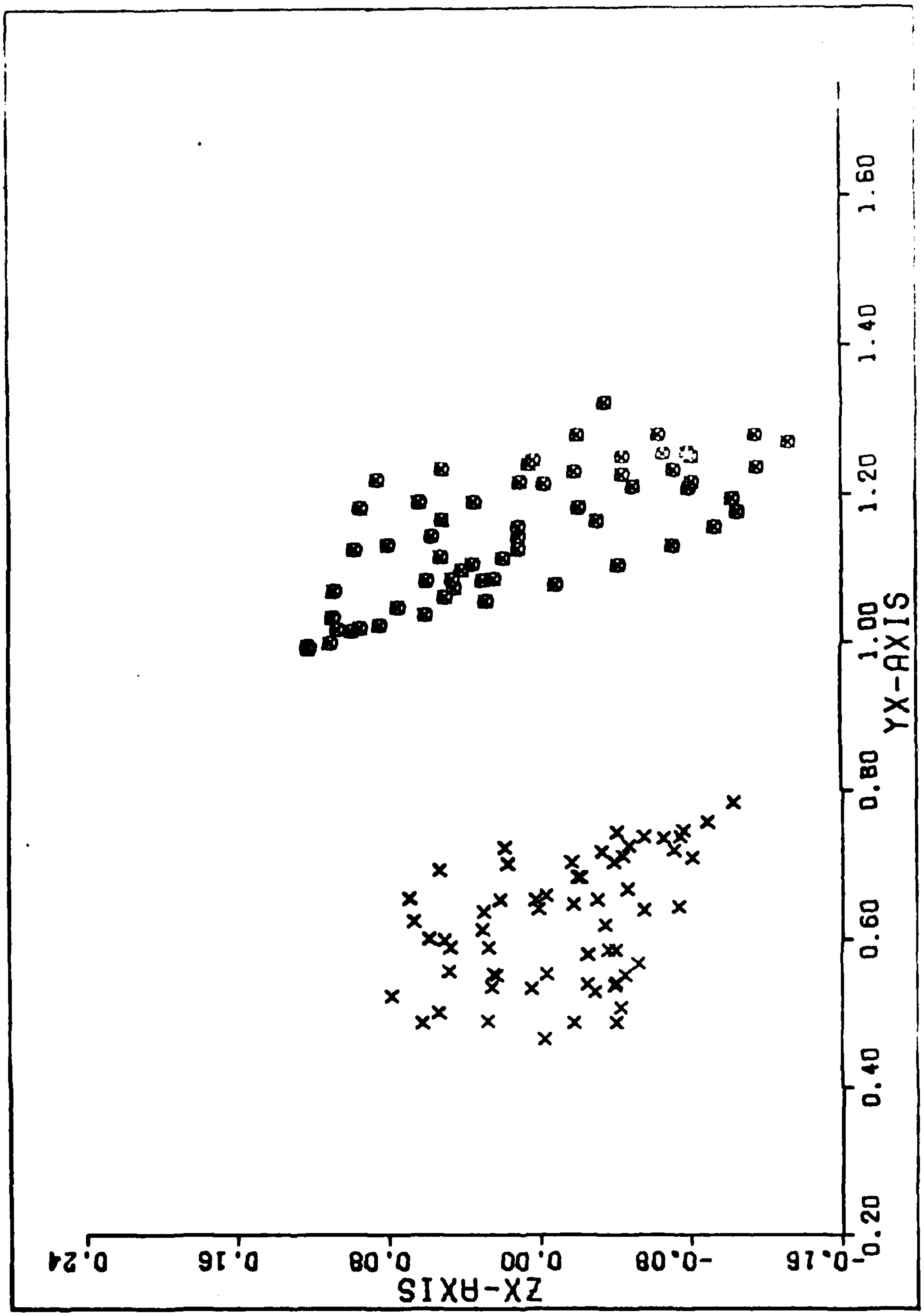

Fig 35. Cross Section of Orbit of Fig 34

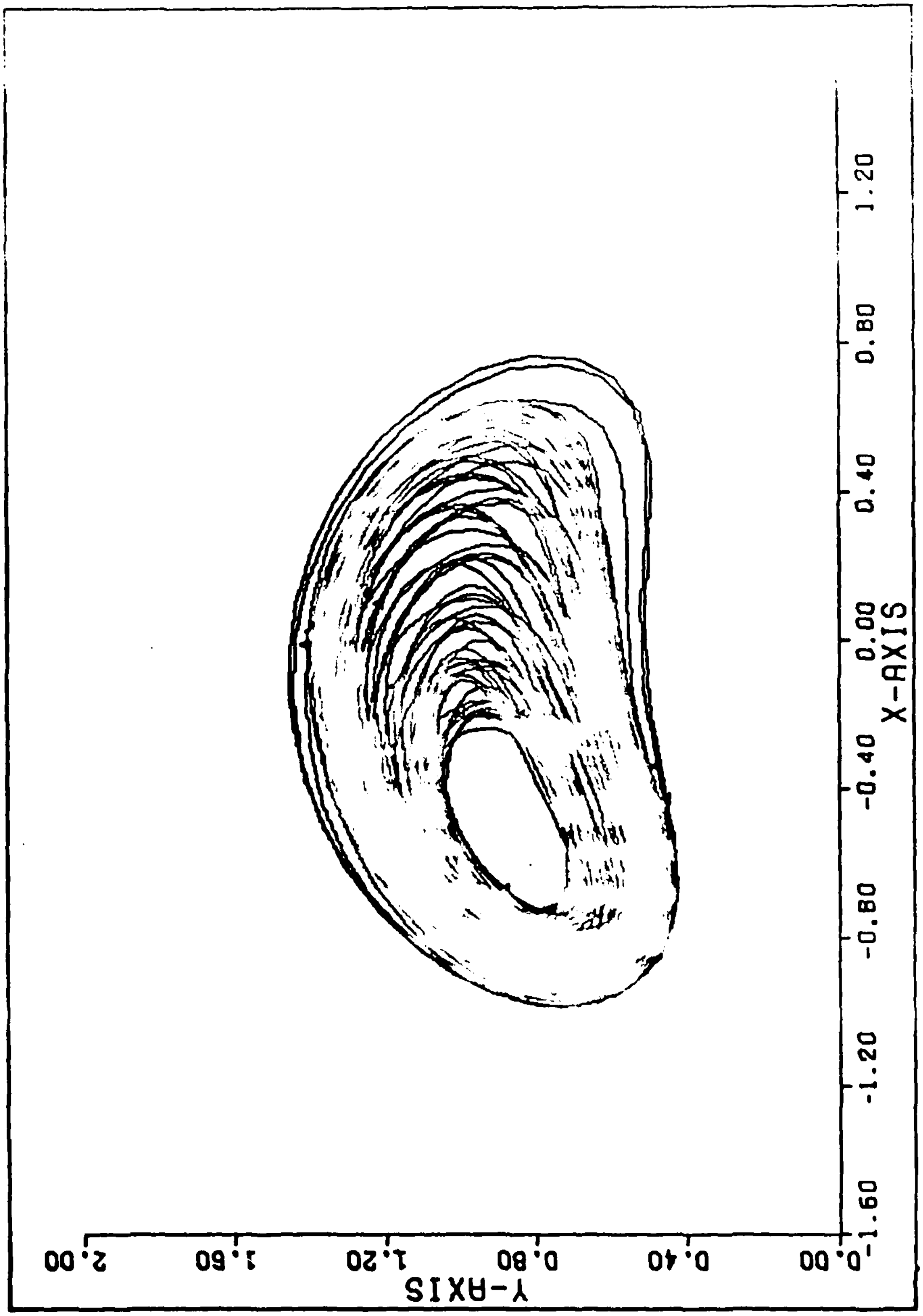

Fig 36. Sensitivity Orbit 60-120 Months

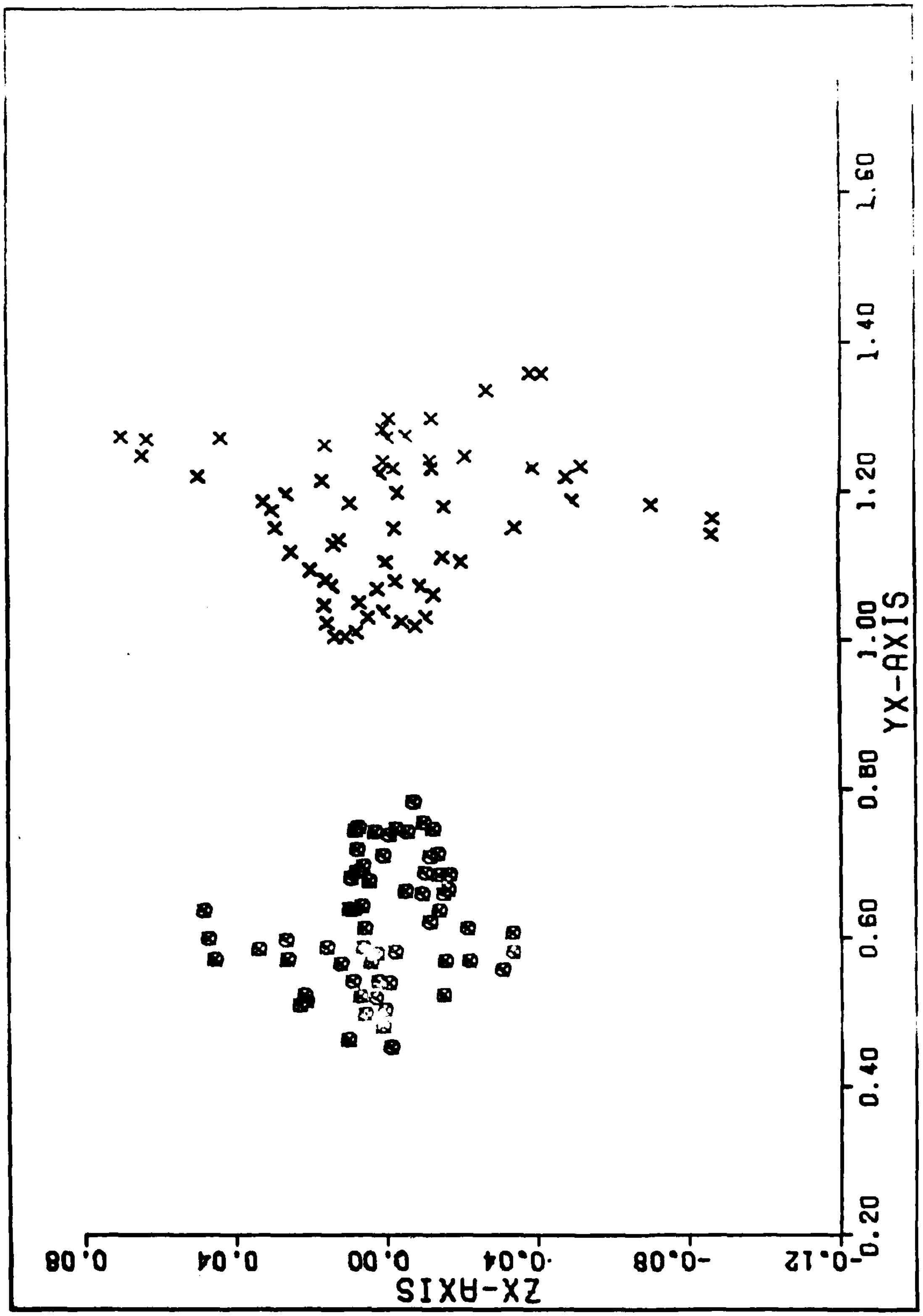

Fig 37. Cross Section of Orbit of Fig 36

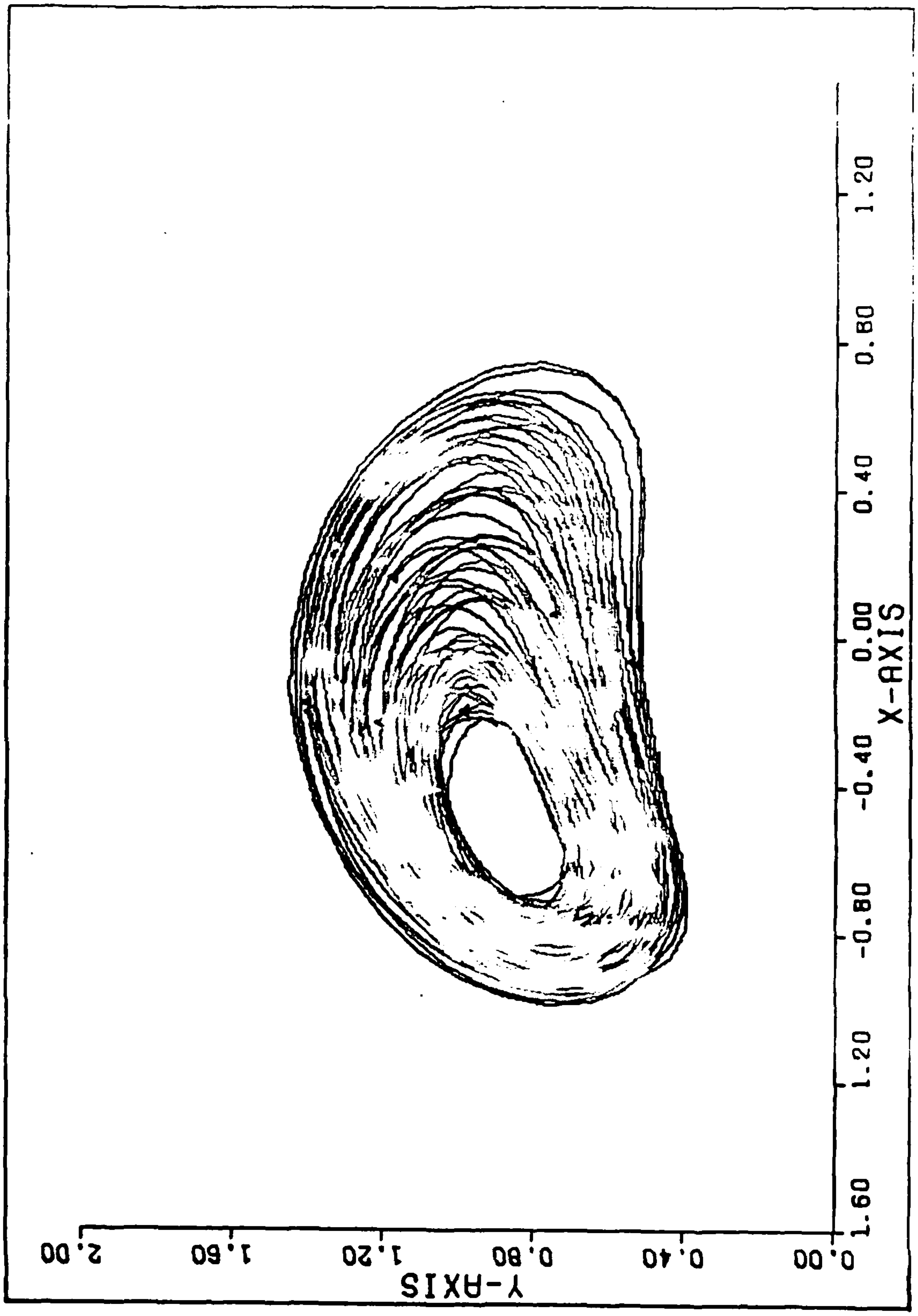

Fig 38. Sensitivity Orbit 120-180 Months

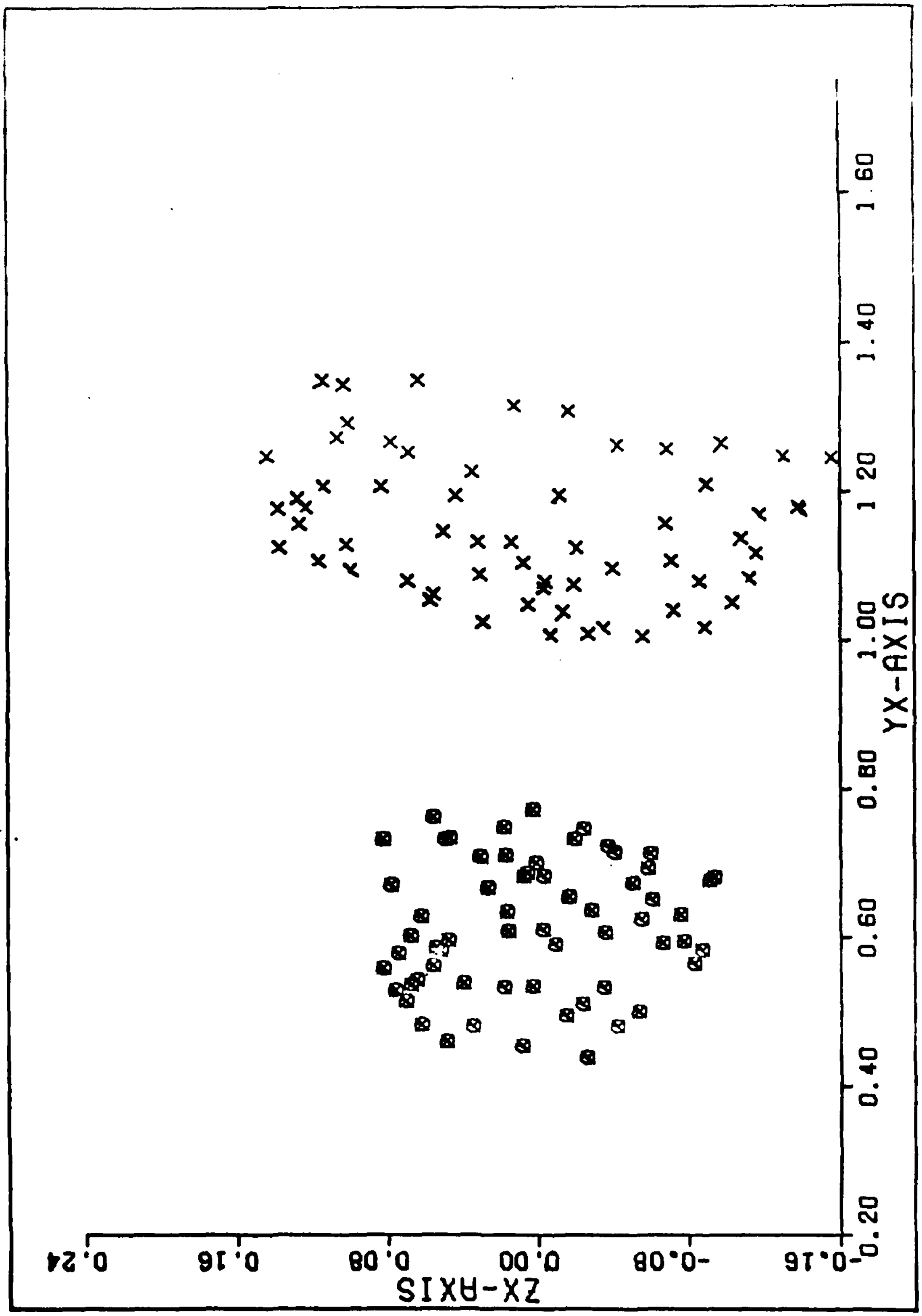

Fig 39. Cross Section of Orbit of Fig 38

Table IX. State Vector of 180° Out of Phase Orbit

INITIAL CONDITIONS VECTOR

X(1)= 204.3194374608

X(2)= -324.0192778463

X(3)= -.01548148447034

X(4)= 5.772936900824

X(5)= 3.594798567616

X(6)= .000169497899237

X(7)= -.5624053126209

X(8)= -.79705844467333

X(9)= .08042470200617

X(10)= .198522285084

X(11)= -.120556823687

X(12)= .007137337369057

X(13)= -.8200582334128

X(14)= .4868110858079

X(15)= .0785

X(16)= -.07662306948368

X(17)= -.2249784474187

X(18)= .0213

ROTATING FRAME COLONY POSITION AND VELOCITY

X= -.0750263 XDOT= -.228

Y= .950711099 YDOT= -.0670999

Z= .0785 ZDOT= .0213

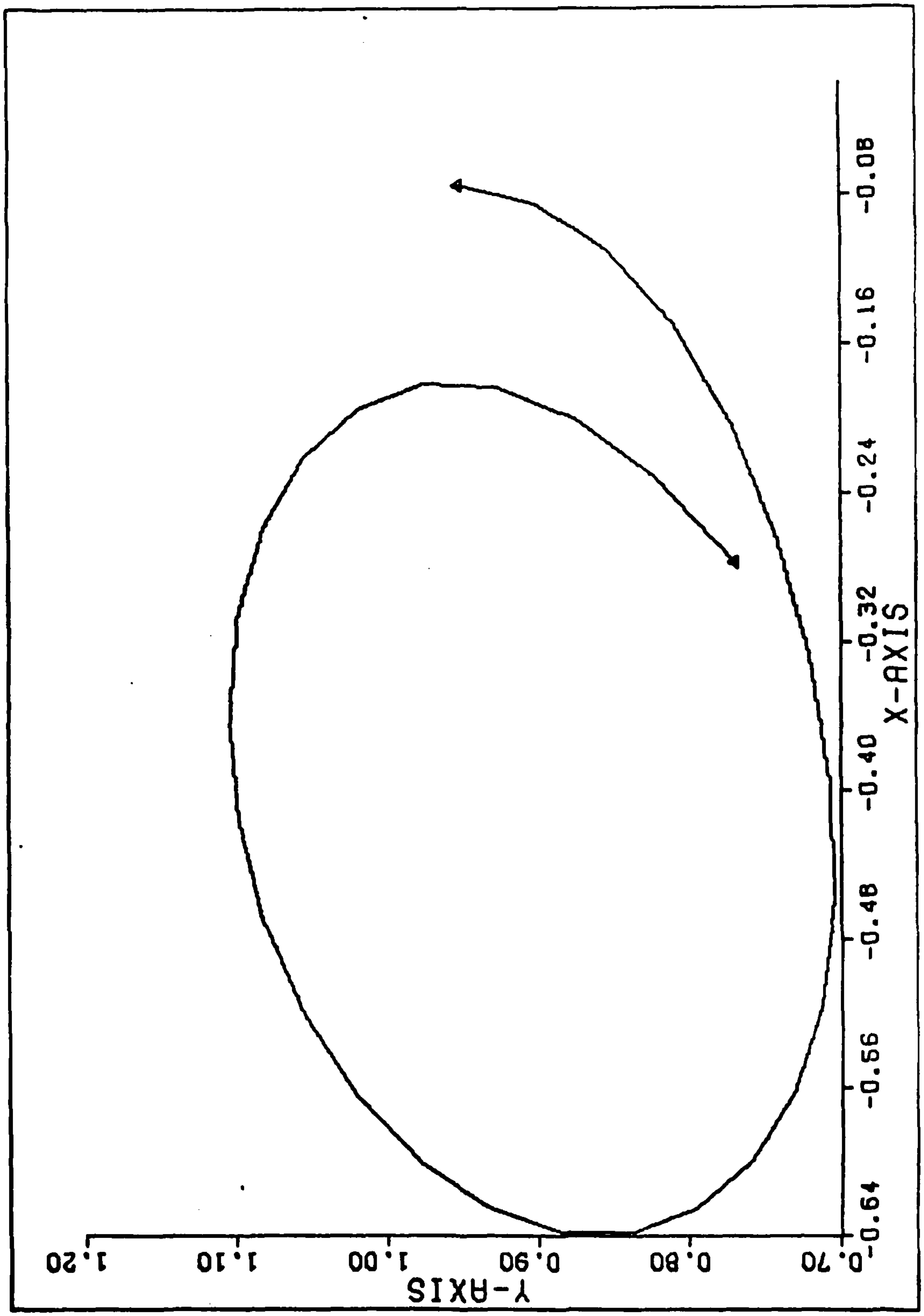

Fig 40. 180° Out of Phase Orbit--One Month

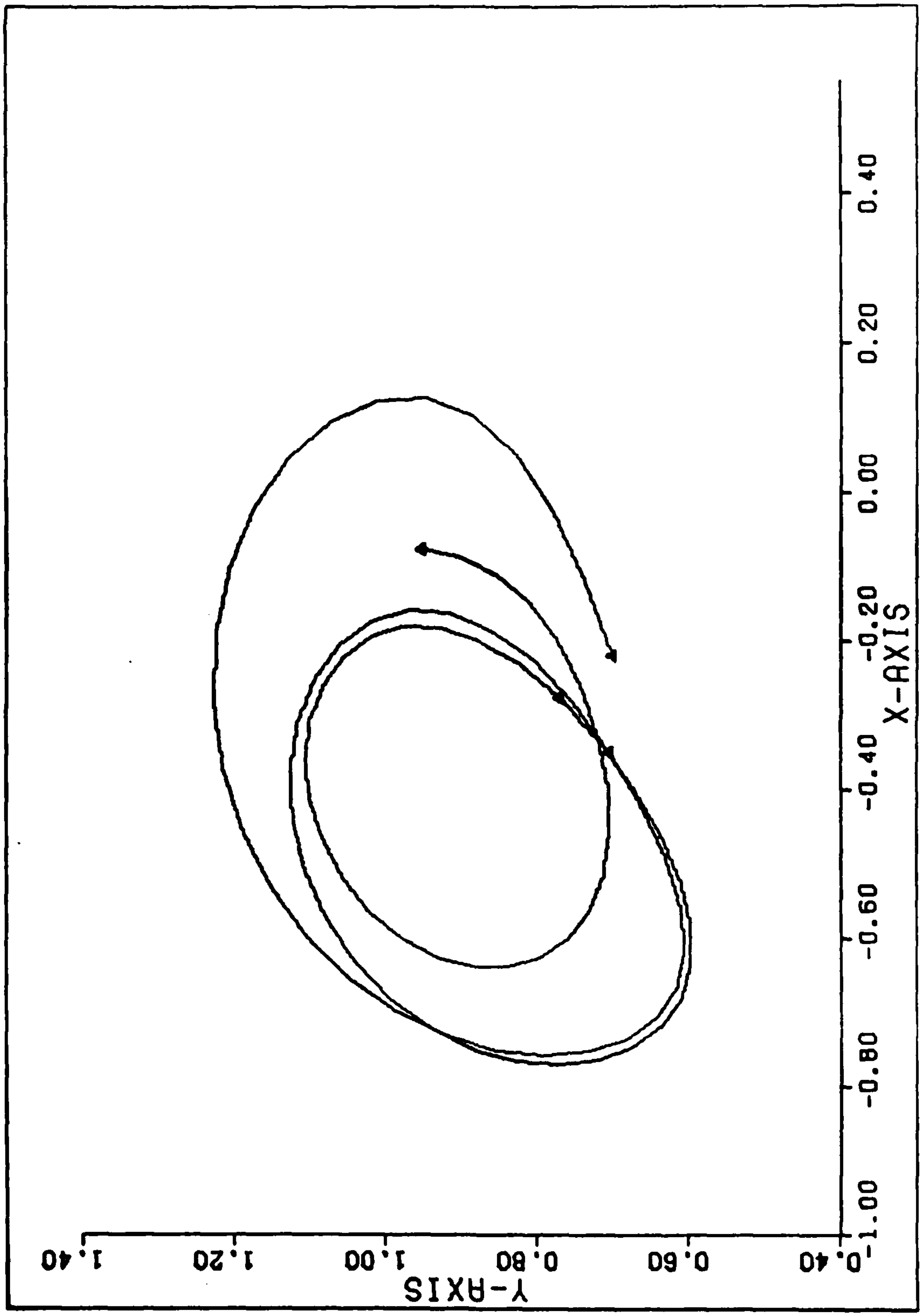

Fig 41. 180° Out of Phase Orbit--Three Months

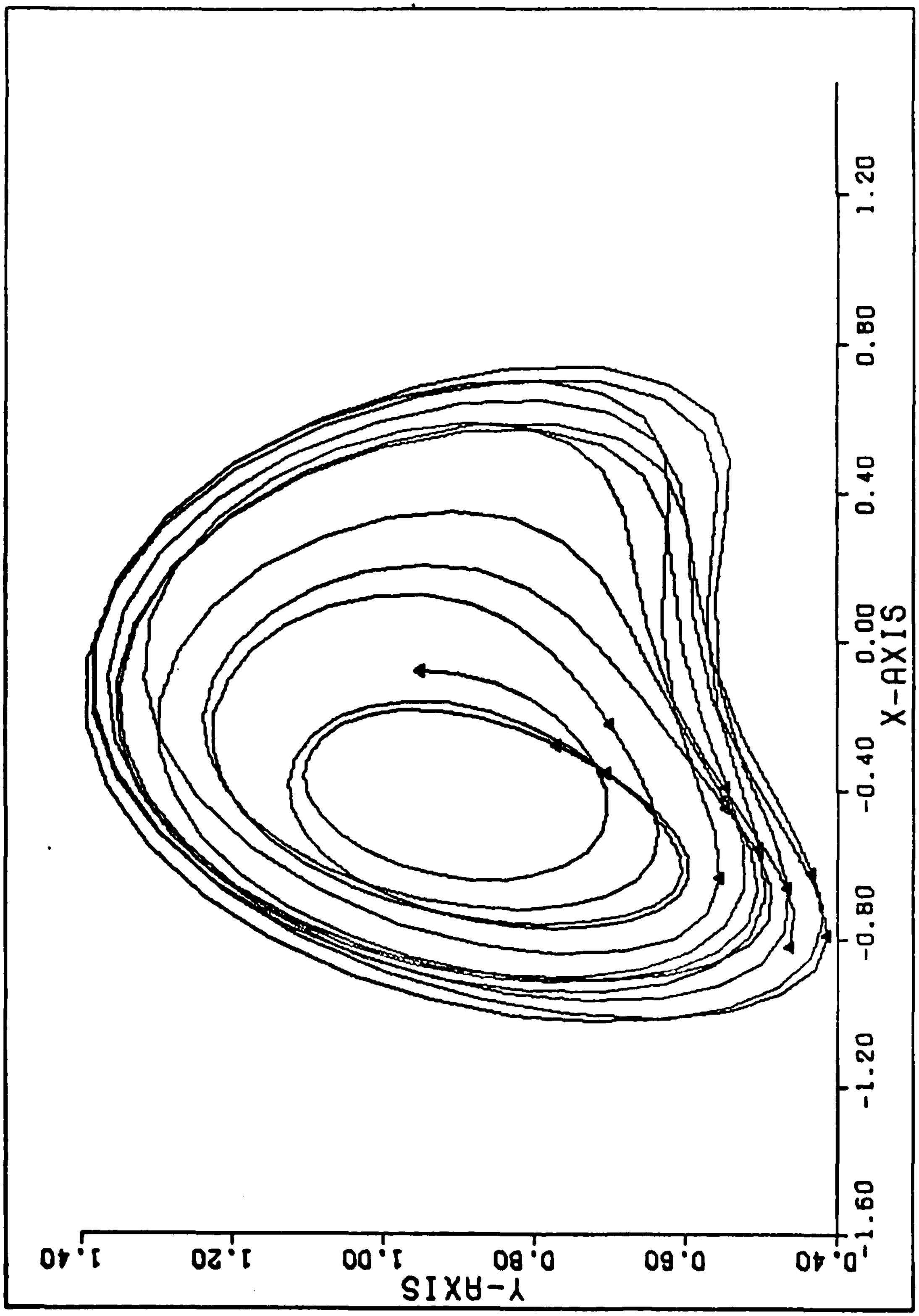

Fig 42. 180° Out of Phase Orbit--Twelve Months

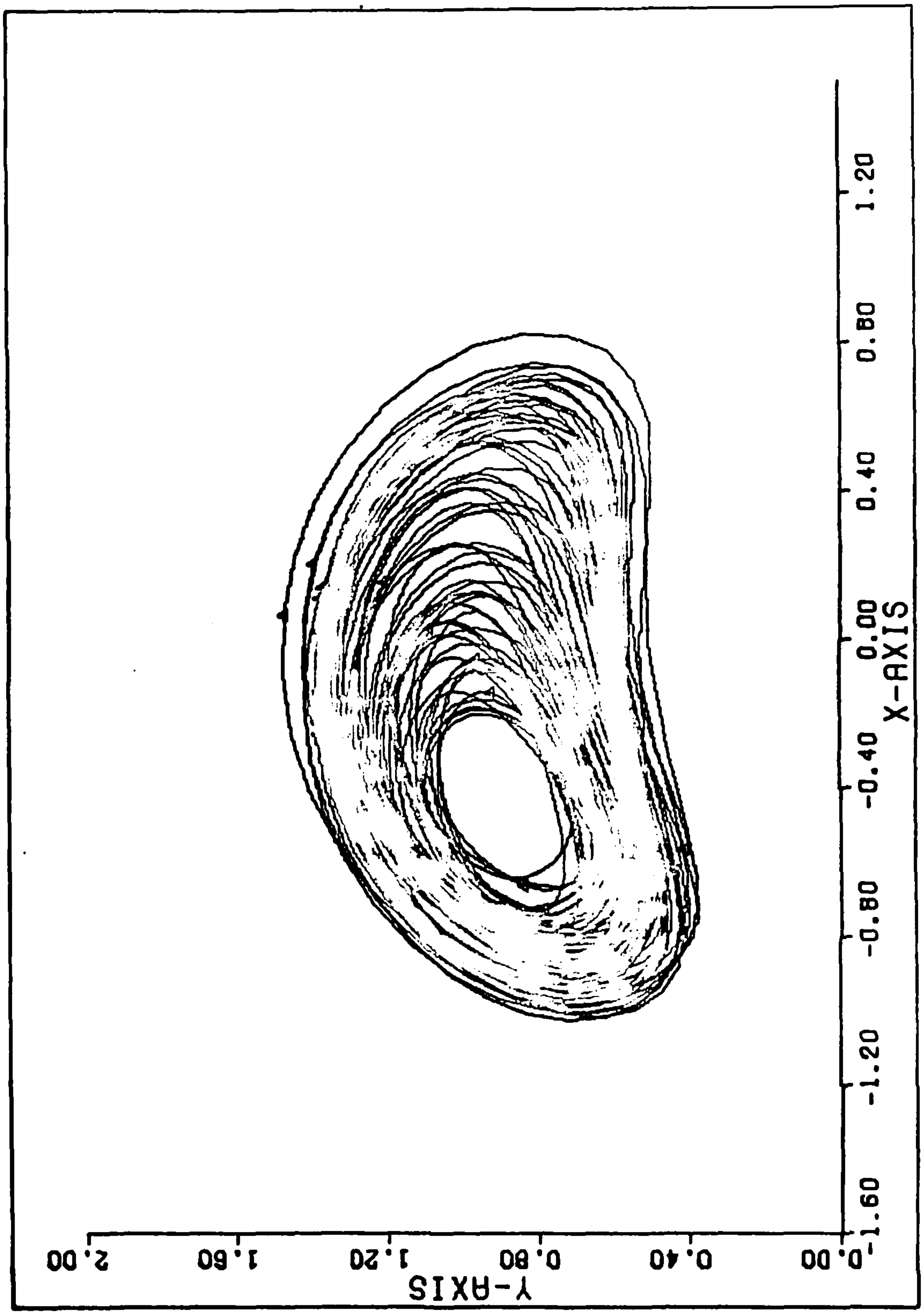

Fig 43. 180° Out of Phase Orbit--Sixty Months

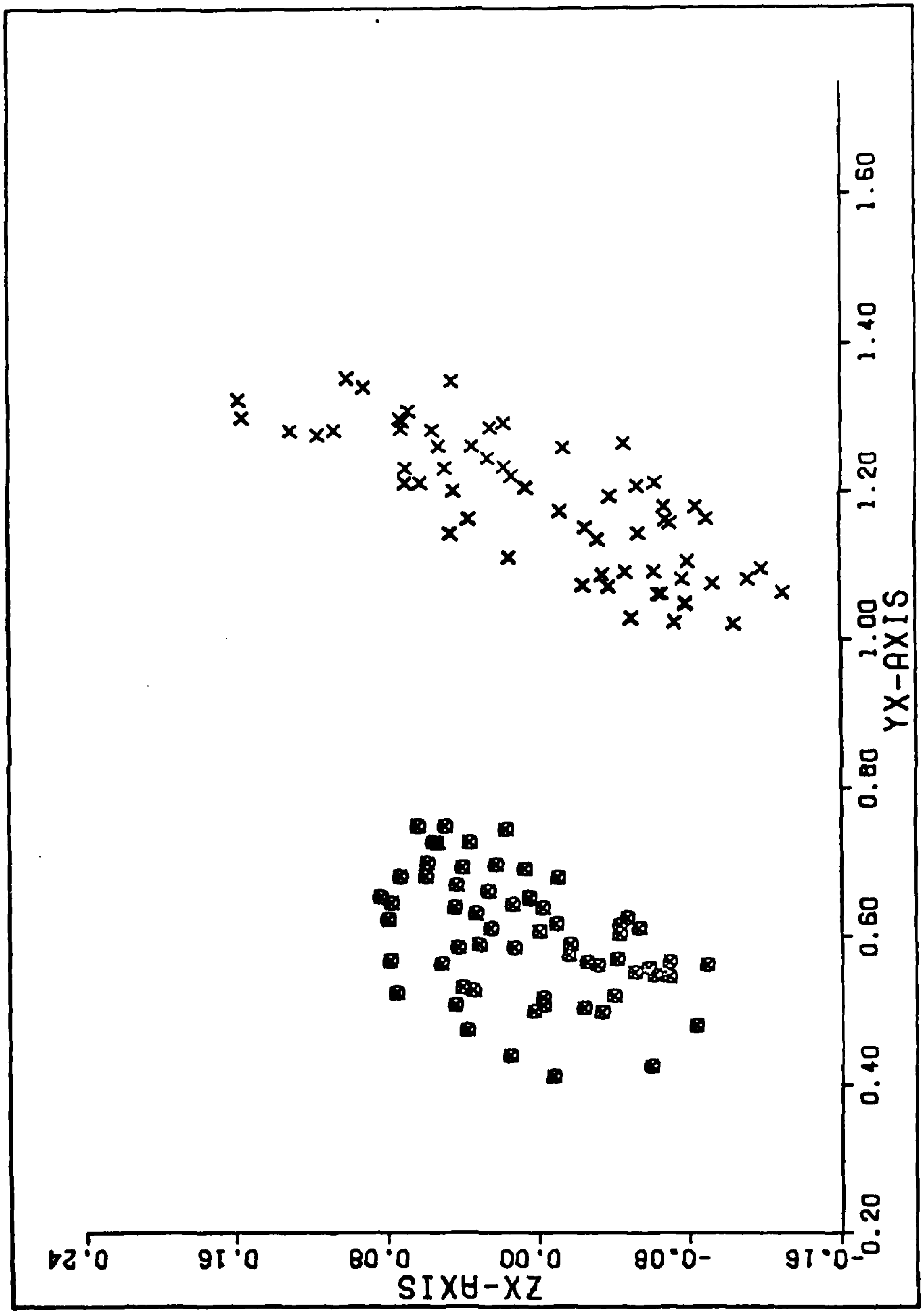

Fig 44. Cross Section of Orbit of Fig 43

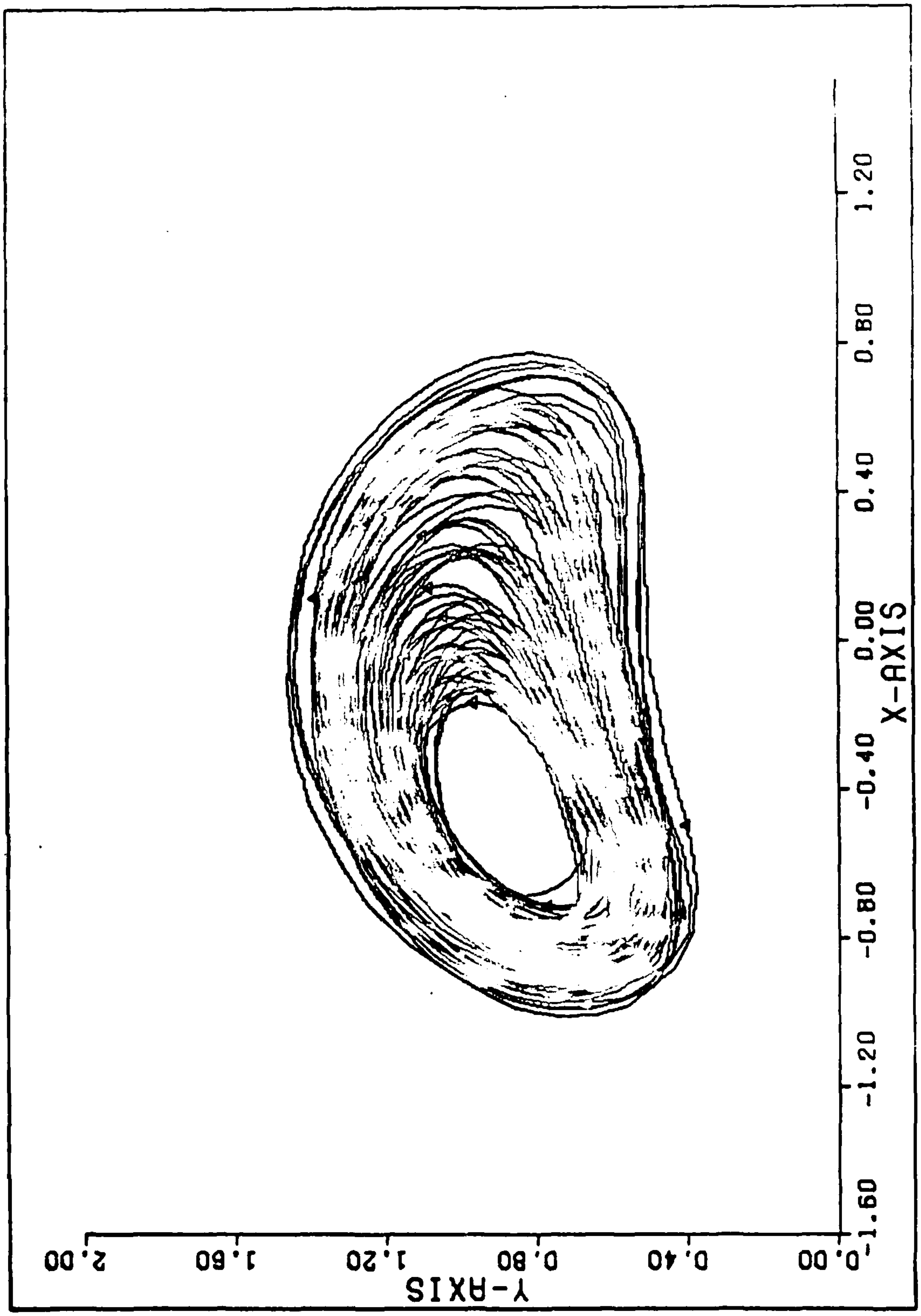

Fig 45. 180° Out of Phase Orbit 60-120 Months

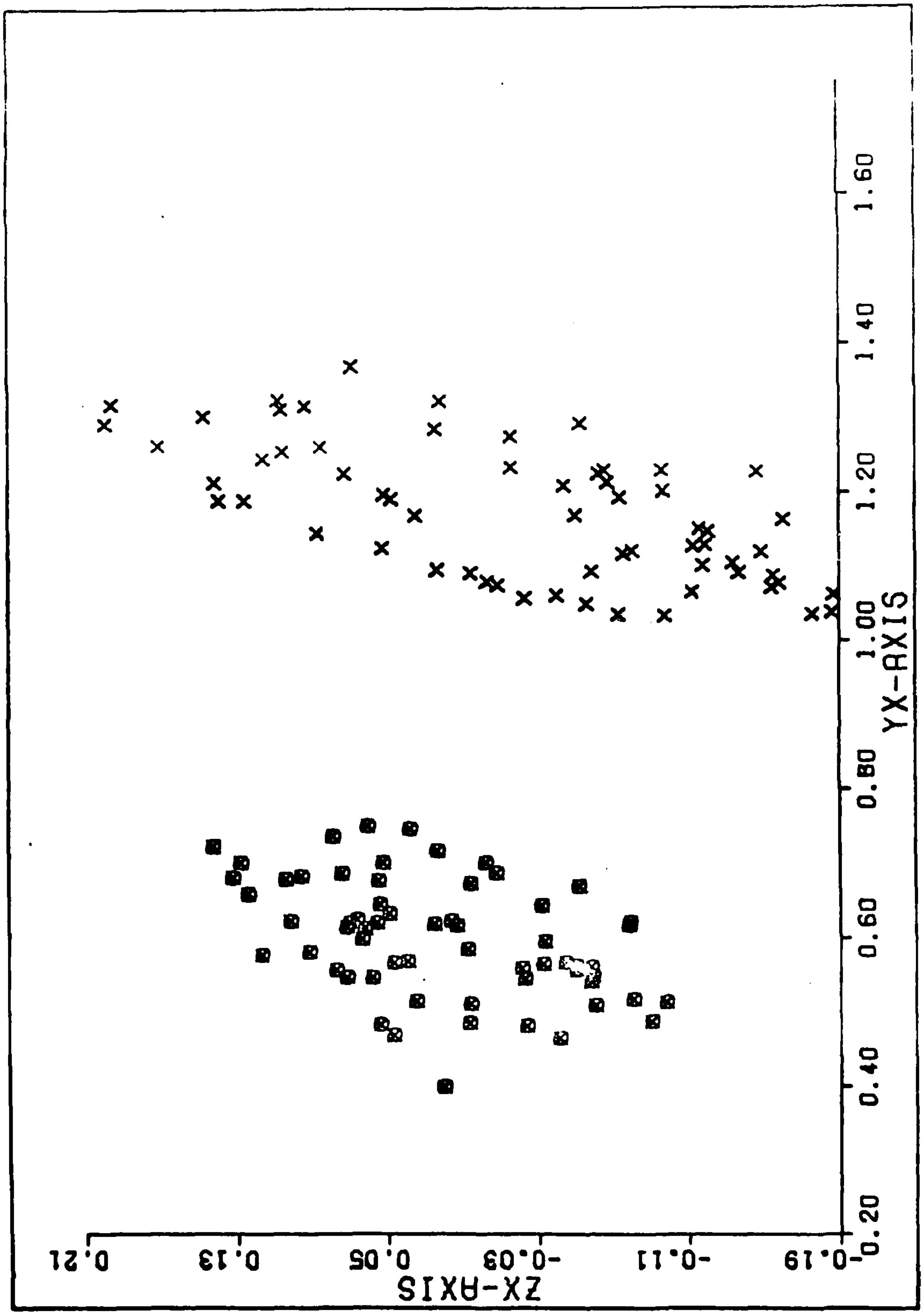

Fig 46. Cross Section of Orbit of Fig 45

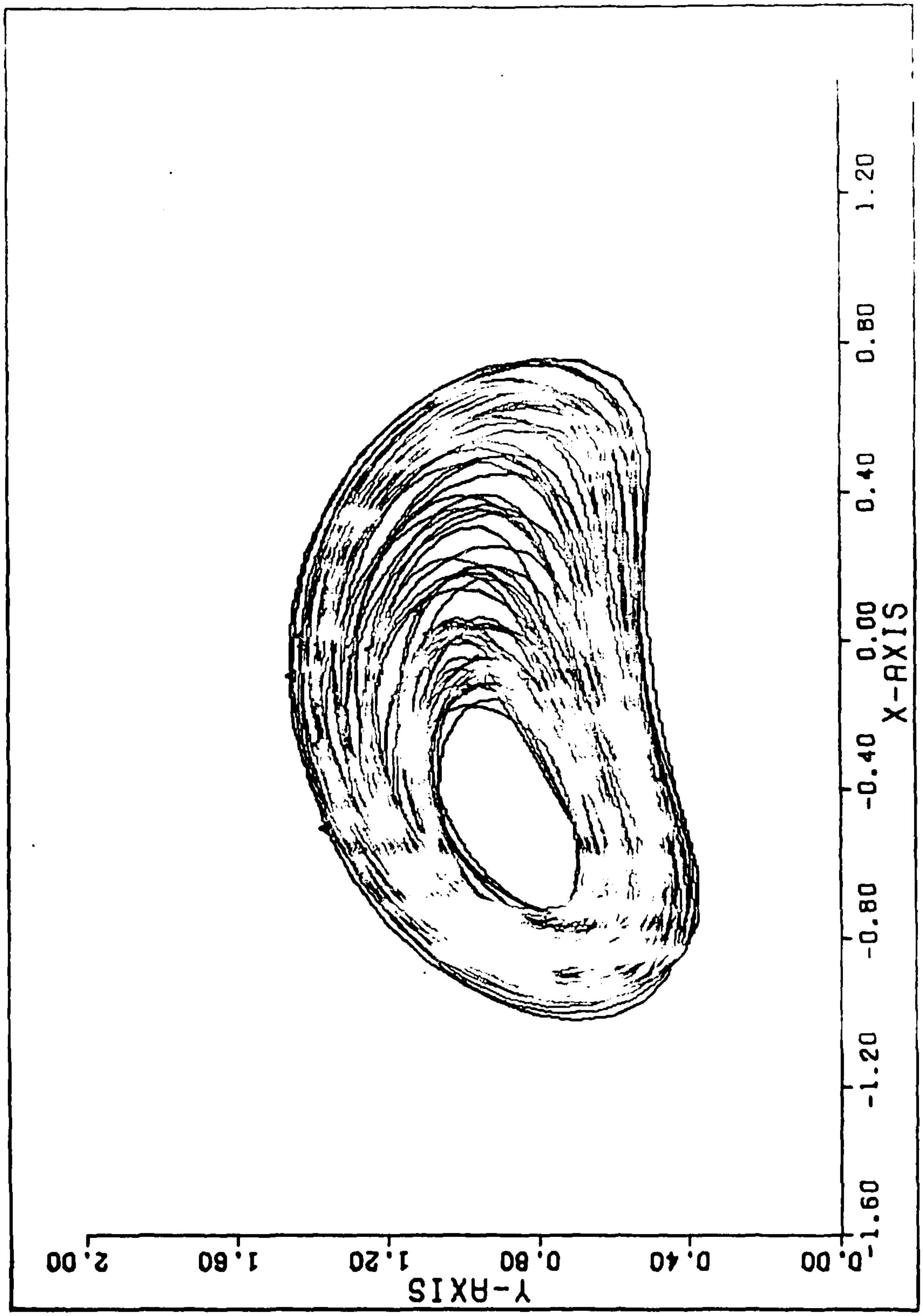

Fig 47. 180° Out of Phase Orbit 120-180 Months

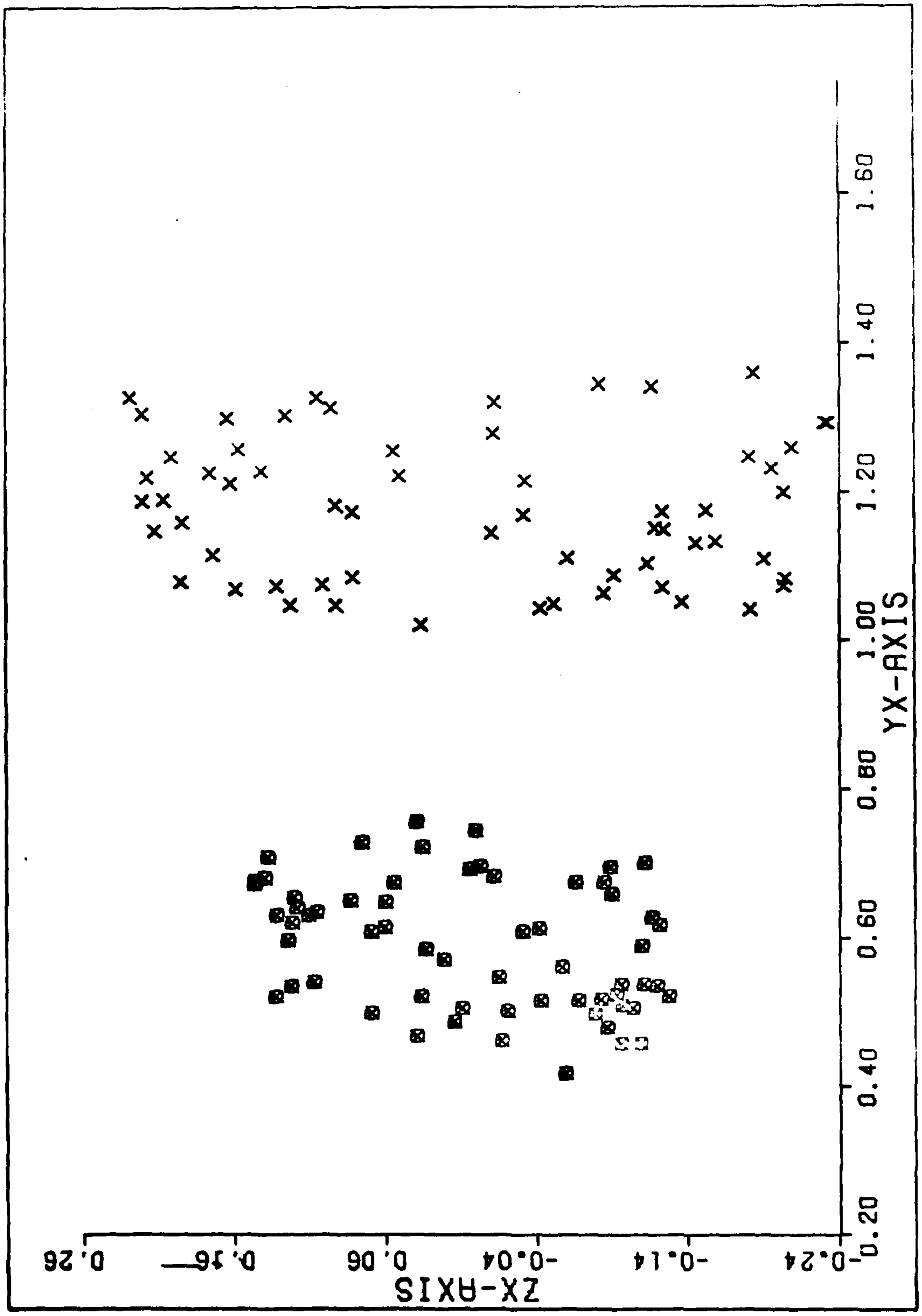

Fig 48. Cross Section of Orbit of Fig 47

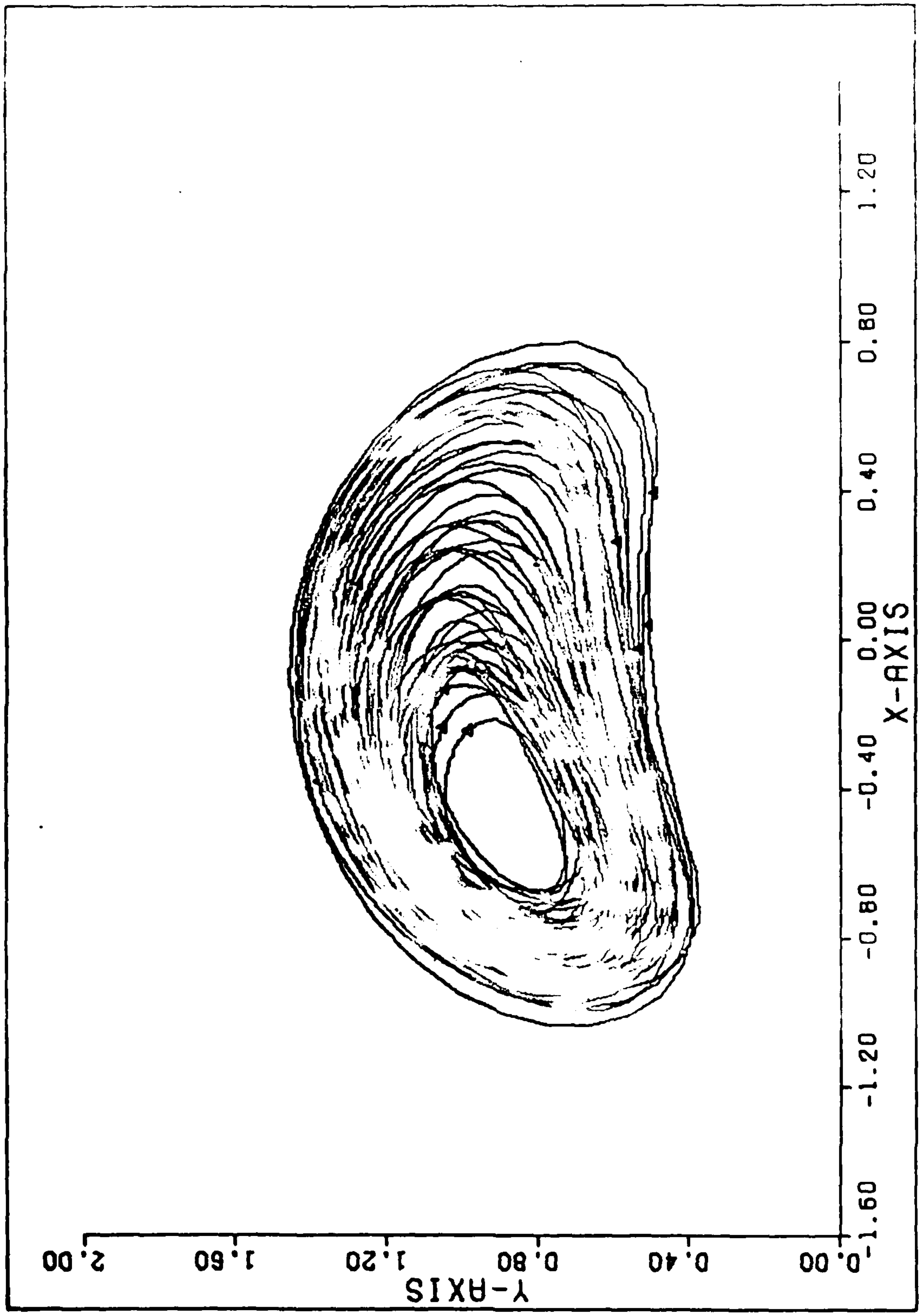

Fig 49. 180° Out of Phase Orbit 180-228 Months

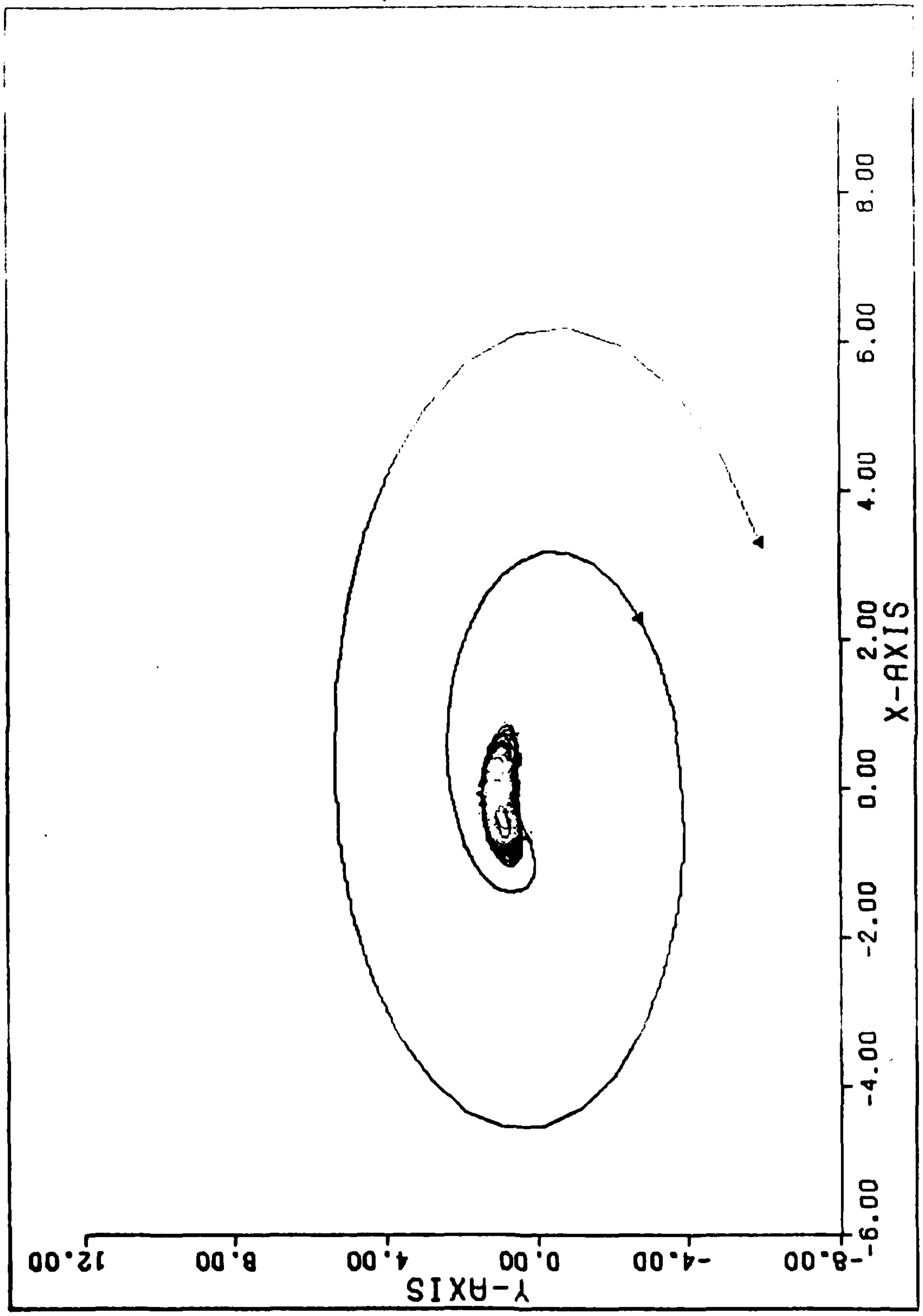

Fig 50. 180° Out of Phase Orbit 180-240 Months

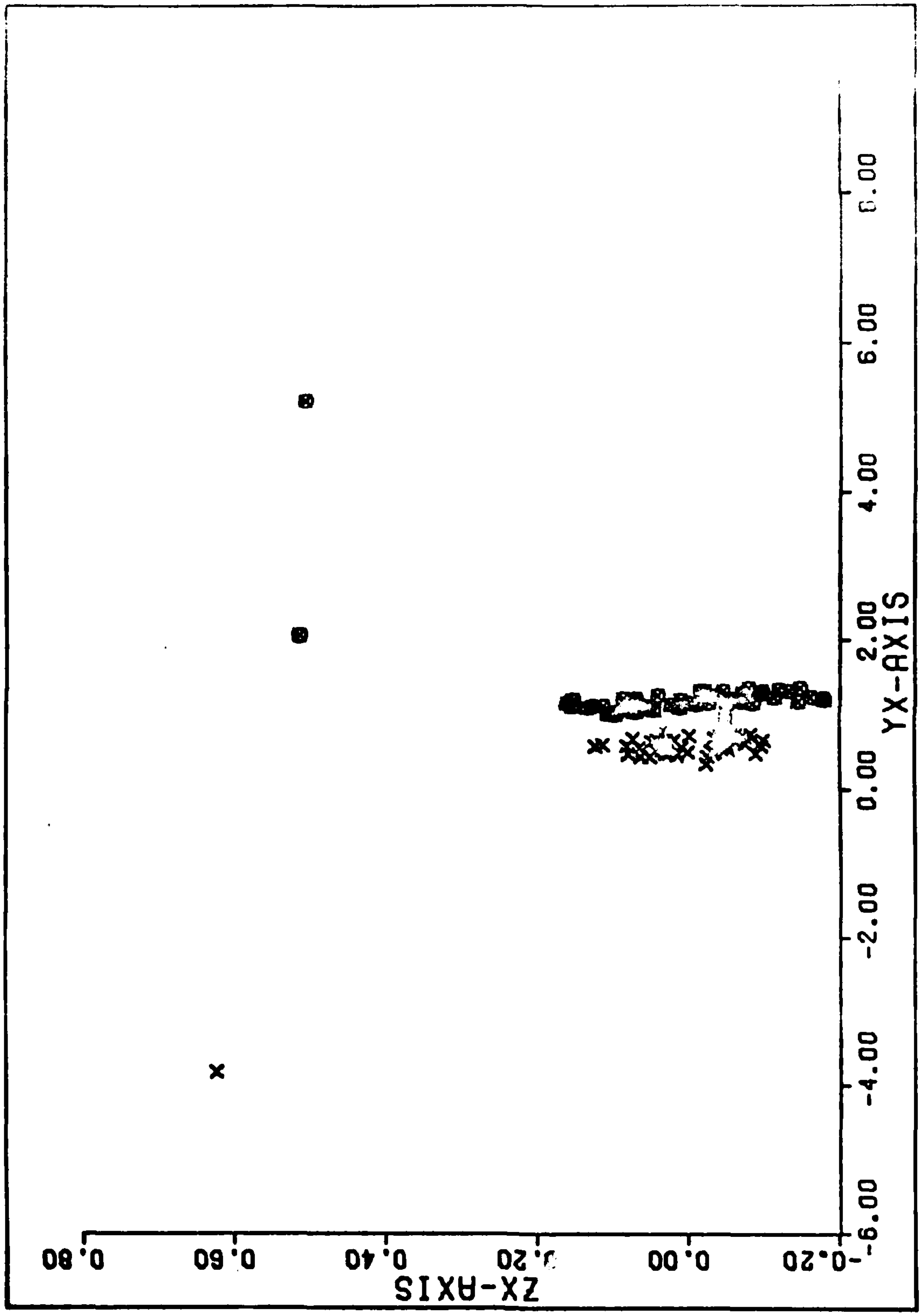

Fig 51. Cross Section of Orbit of Fig 50

[illegible] perturbations, the [illegible] orbits and their [illegible]. An [illegible] only marginally stable.

Conclusions and Recommendations

The primary conclusion of the report is stable orbits do exist in the three dimensional system. These orbits are stable for at least fifty years and allow planners to select vehicles whose lifetimes compare with this period of stability. Additionally, The stable orbits are nominally insensitive to injection error and can be maintained with a small initial correction to the actual stable orbit. The addition of a marginally stable orbit 180° out of phase with the stable orbit increases the coverage available to remote sensing equipment stationed in the Lagrangian vicinities. Also, the [illegible] to twenty years, which is a sufficient period for an unmanned satellite. The use of a controller would probably insure the stability of that particular orbit for an additional ten years.

AD-A094 773 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOO--ETC F/G 22/3
THREE DIMENSIONAL ORBITAL STABILITY ABOUT THE EARTH-MOON EQUILA--ETC(U)
DEC 80 H A TILTON
UNCLASSIFIED AFIT/GA/AA/80D-4 NL

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The state vector producing the stable orbit was found completely by accident. The date of the new moon was misread and the state vector produced was [illegible] days short of the actual new moon. This was a very fortunate error, because the stable orbit was found almost immediately. The state vector in Table VII has a position vector differing in magnitude by approximately .5 MD, or 200,000 kilometers. In hindsight, the problem has become much more rigorous than initially thought. The stable orbit would not have been found if the error had not occurred. Further, from an analysis of the pertinent figures, if a different ephemeris was selected, the orbit would not have been found either. Wheeler's initial conditions are only approximately matched in the first three months, and then again only after a period in excess of five years.

The error and ephemeris selection were tested to define the uniqueness of the stable orbit. The state vector of Table VI was integrated forward for a random length of time, where the Wheeler conditions were inserted in the prospect of reproducing the stable orbit. While not conclusive, the results indicated that the conditions were unique for a short time period, i.e. five years. The fact that a stable orbit using the Wheeler conditions at a new moon was not discovered, does not discount the possibility of one existing. The fact that the orbits are very ephemeris oriented should allow for the discovery of one using the correct ephemeris. The search for such an orbit was not continued because the stable orbit had already been found.

Further research in this area should include the use of controllers on all orbits described, to increase their periods of stability [illegible] orbits [illegible], [illegible] orbit using the Wheeler initial conditions, should be continued to prove or disprove the possibility of a stable orbit existing with a starting point as a new moon. Finally, the orbits described should be extended to the end of their stability in a search of the stable lifetime. The orbit in Table VI was actually integrated in excess of sixty years and showed no sign of decaying into instability.

[illegible]

1. [illegible] Many-Body [illegible]," [illegible] Part II, [illegible]

2. [illegible]

3. Beekman, W.D., "Application and Comparison of Stable Periodic Orbits in the Vicinity of Lagrange [illegible] L4 and L5 to a Four-Body Truth Model," Unpublished Thesis, Dayton: Air Force Institute of Technology, 197[illegible].

4. Brouwer, D. and Clemence, C.M., Methods of Celestial Mechanics, New York: Academic Press, 1961.

5. Heppenheimer, T.A., "Steps Toward Space Colonization: Colony Location and Transfer Trajectories," Journal of Spacecraft and Rockets, 15:305-12, (1975).

6. Kolenkiewicz, R. and Carpenter, L., "Stable Periodic Orbits About the Sun-Perturbed Earth-Moon Triangular Points," AIAA Journal, 6:1302, (1968).

7. Wheeler, J.E., "Determination of Periodic Orbits and Their Stability in the Very Restricted Four-Body Model in the Vicinity of the Lagrangian Points L4 and L5," Unpublished Thesis, Dayton: Air Force Institute of Technology, Dec 1978.

8. The American Ephemeris and Nautical Almanac, Washington D.C.: U.S. Government Printing Office, 197[illegible].

Appendix A

Subroutine Containing the Equations of Motion

```
      SUBROUTINE F(T,X,DX,NEQN)
      COMMON/DFIL/GSUN,MDA,MDE,MS,MU,GE,PI
      DIMENSION X(18),DX(18)
      REAL MDE,MS,MDA,MU
      RS=SQRT((ABS(X(1)))**2+(ABS(X(2)))**2+(ABS(X(3)))**2)
      RM=SQRT((ABS(X(7)))**2+(ABS(X(8)))**2+(ABS(X(9)))**2)
      RC=SQRT((ABS(X(13)))**2+(ABS(X(14)))**2+(ABS(X(15)))**2)
      RCS=SQRT((ABS(X(13)-X(1)))**2+(ABS(X(14)-X(2)))**2+(ABS(X(1
     &5)-X(3)))**2)
      RCM=SQRT((ABS(X(13)-X(7)))**2+(ABS(X(14)-X(8)))**2+(ABS(X(1
     &5)-X(9)))**2)
      RMS=SQRT((ABS(X(7)-X(1)))**2+(ABS(X(8)-X(2)))**2+(ABS(X(9)-
     &X(3)))**2)
      DX(1)=X(4)
      DX(2)=X(5)
      DX(3)=X(6)
      DX(4)=-GSUN*X(1)/RS**3
      DX(5)=-GSUN*X(2)/RS**3
      DX(6)=-GSUN*X(3)/RS**3
      DX(7)=X(10)
      DX(8)=X(11)
      DX(9)=X(12)
      DX(10)=GE*(-X(7)/RM**3-MS*((X(7)-X(1))/RMS**3+X(1)/RS**3))
      DX(11)=GE*(-X(8)/RM**3-MS*((X(8)-X(2))/RMS**3+X(2)/RS**3))
      DX(12)=GE*(-X(9)/RM**3-MS*(X(9)-X(3))/RMS**3+X(3)/RS**3)
      DX(13)=X(16)
      DX(14)=X(17)
      DX(15)=X(18)
      DX(16)=GE*(-(1-MU)*X(13)/RC**3-MS*((X(13)-X(1))/RCS**3
     &+X(1)/RS**3)-MU*((X(13)-X(7))/RCM**3+X(7)/RM**3))
      DX(17)=GE*(-(1-MU)*X(14)/RC**3-MS*((X(14)-X(2))/RCS**3
     &+X(2)/RS**3)-MU*((X(14)-X(8))/RCM**3+X(8)/RM**3))
      DX(18)=GE*(-(1-MU)*X(15)/RC**3-MS*((X(15)-X(3))/RCS**3
     &+X(3)/RS**3)-MU*((X(15)-X(9))/RCM**3+X(9)/RM**3))
      RETURN
      END
```

Appendix B

Subroutine For Determining the Velocity Correction Vector

```
      SUBROUTINE ENT(F,TOUT,I1,I2,A1,A2,A3,X)
      COMMON /DFIL/GSUN,MDA,MDE,MS,MU,GE,PI
      DIMENSION X(18),XC(18),XD(18)
      DIMENSION R1(3),AC(3)
      DIMENSION A(3,3),B(3),C(3,3)
      DIMENSION WKAREA(800),IWORK(30)
      REAL MS,MU,MDE,MDA
      REAL LAT ,LON
      DO 10 KK=1,3
      DO 1 I=1,18
      XC(I)=X(I)
      XD(I)=0.
1     CONTINUE
      IF(I1.GT.1)GOTO9
      AC(1)=A1
      AC(2)=A2
      AC(3)=A3
9     IF(I1.LT.7)GOTO11
      AC(1)=A1*COS(A2)*COS(A3)
      AC(2)=A1*SIN(A2)*COS(A3)
      AC(3)=A1*SIN(A3)
11    NEQN=18
      ERR=1E-9
      IFLAG=1
      T=0.
      CALL ODE(F,NEQN,XC,T,TOUT,ERR,ERR,IFLAG,WKAREA,IWORK)
      DO 2 I=1,18
      XD(I)=XC(I)
2     CONTINUE
      I3=I1+2
      DO 3 I=I1,I3
      R1(I-(I1-1))= (XC(I)-AC(I-(I1-1)))
3     B(I-(I1-1))=XC(I)
      DELTAV=1E-4
      DO 4 I=1,3
      DO 8 K=1,18
8     XC(K)=X(K)
      XC(I+(I2-1))=XC(I+(I2-1))+DELTAV
      IFLAG=1
      T=0.
      CALL ODE(F,NEQN,XC,T,TOUT,ERR,ERR,IFLAG,WKAREA,IWORK)
      DO 5 J=1,3
      A(J,I)=XC(J+(I1-1))-XD(J+(I1-1))
      A(J,I)=A(J,I)/DELTAV
5     CONTINUE
4     CONTINUE
      IDGT=7
      N=3
      N1=1
      CALL LINV2F(A,N,N,C,IDGT,WKAREA,IER)
      CALL VMULFF(C,R1,N,N,N1,N,N,B,N,IER)
      DO 7 I=1,3
      X(I+(I2-1))=X(I+(I2-1))-B(I)
```

Appendix C

Subroutine Used to Transform to the Rotating Frame

```
      SUBROUTINE COLROT(I,N,T,X,PI)
      DIMENSION X(18)
      THETA=ATAN2(X(8),X(7))+PI
      IF(THETA.LT.0)THETA=THETA+2.*PI
      A=X(13)*COS(THETA)+X(14)*SIN(THETA)
      B=-X(13)*SIN(THETA)+X(14)*COS(THETA)
      C=X(15)
      AD=X(16)*COS(THETA)+X(17)*SIN(THETA)
      BD=-X(16)*SIN(THETA)+X(17)*COS(THETA)
      IF(I.NE.N)GOTO1
      PRINT*,' '
      PRINT*,' '
      PRINT*,'ROTATING FRAME COLONY POSITION AND VELOCITY'
      PRINT*,' '
      PRINT*,'X= ',A,' XDOT= ',AD
      PRINT*,' '
      PRINT*,'Y= ',B,' YDOT= ',BD
      PRINT*,' '
      PRINT*,'Z= ',C,' ZDOT= ',X(18)
      PRINT*,' '
1     WRITE(6) T,A,B,C
      RETURN
      END
```

Appendix D

Plotting Subroutine

```
      A=X(I)
      B=X(I+1)
      IF((A-XL)*(B-XL).GE.0)GO TO 4
      XIN=ABS(X(I)-X(I+1))
      DX=XL-A
      DX=DX/XIN
      YIN=Y(I+1)-Y(I)
      DY=DX*YIN
      L=L+1
      YX(L)=Y(I)+DY
      ZIN=Z(I+1)-Z(I)
      DZ=DX*ZIN
      ZX(L)=Z(I)+DZ
4     CONTINUE
      CALL PLOT(0.,0.,-3)
      PRINT*,'NUMBER OF PLOT?'
      READ 15,HFD
      CALL SYMBOL(0.,0.,.25,HFD,0.,2)
15    FORMAT(1A2)
      CALL PLOT(.5,.5,3)
      CALL PLOT(9.25,.5,2)
      CALL PLOT(9.25,6.5,2)
      CALL PLOT(.5,6.5,2)
      CALL PLOT(.5,.5,2)
      CALL PLOT(1.,1.,-3)
      CALL SCALE(YX,7.75,L,1)
      CALL SCALE(ZX,5.,L,1)
      CALL AXIS(0.,0.,7HYX-AXIS,-7,7.75,0.,YX(L+1),YX(L+2))
      CALL AXIS(0.,0.,7HZX-AXIS,7,5.,90.,ZX(L+1),ZX(L+2))
      CALL LINE(YX,ZX,L,1,-2,1)
      CALL LINE(YX,ZX,L,1,-1,4)
10    CONTINUE
      PRINT*,'HOW MANY TO PLOT?'
      READ*,N1
      DO 11 II=1,N1
      XX(II)=X(II)
      YY(II)=Y(II)
11    CONTINUE
      CALL PLOT(15.,0.,-3)
      CALL PLOT(-.5,-.5,3)
      CALL PLOT(8.25,-.5,2)
      CALL PLOT(8.25,5.5,2)
      CALL PLOT(-.5,5.5,2)
      CALL PLOT(-.5,-.5,2)
      CALL SCALE(XX,7.75,N1,1)
      CALL SCALE(YY,5.,N1,1)
      CALL AXIS(0.,0.,6HX-AXIS,-6,7.75,0.,XX(N1+1),XX(N1+2))
      CALL AXIS(0.,0.,6HY-AXIS,6,5.,90.,YY(N1+1),YY(N1+2))
      CALL LINE(XX,YY,N1,1,30,2)
```

Appendix E

Main Interactive Program

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5,TAPE6,TAPE7,TAPE8)
      DIMENSION X(18),WKAREA(800),IWORK(30)
      COMMON /DFIL/GSUN,MDA,MDE,MS,MU,GE,PI
      REAL MS,MU,MDA,MDE
      REAL LAT,LON
      EXTERNAL F
      DATA GSUN,MDA,MDE,MS,MU,GE/1.7442438E4,2.5695187E-3,
     &60.268165,328912.,.0121506683,5.2386349E-2/
      PI=ACOS(-1.)
C
C     INITIAL POSITION VECTOR
C
      PRINT*,"#3 EQUINOX OF 1950.0"
      REWIND 5
      READ(5) T,X
      REWIND 6
      NEQN=18
      T=0.
      I=1
      N=1
      DELTAT=29.530589/30.
      TOUT=DELTAT
      ERR=1E-9
      PRINT*,'INPUT NUMBER OF DAYS TO INTEGRATE---1874 OR LESS'
      READ*,NN
      CALL COLROT(I,N,T,X,PI)
      REWIND 6
      CALL COLLOC(T,X,PI)
      PRINT*,' '
      CALL COLROT(I,N,T,X,PI)
      NN=NN+1
      DO 2 I=2,NN
      IFLAG=1
1     CALL ODE(F,NEQN,X,T,TOUT,ERR,ERR,IFLAG,WKAREA,IWORK)
      T=TOUT
      TOUT=TOUT+DELTAT
      CALL COLROT(I,NN,T,X,PI)
2     CONTINUE
      REWIND 7
      WRITE(7)T,X
      STOP
      END
```

Appendix F

Subroutine Used to Locate the Orbit

```
      SUBROUTINE COLLOC(T,X,PI)
      DIMENSION X(18)
      THETA=ATAN2(X(8),X(7))+PI
      IF (THETA.LT.0) THETA=THETA+2.*PI
      PRINT*,'INPUT NEW INITIAL CONDITIONS? 1=YES,0=NO'
      READ*, N
      IF(N.EQ.1)GOTO1
      PRINT*,'INPUT DELTAS TO INITIAL CONDITIONS?'
      READ*, N
      IF(N.EQ.0) GOTO2
      PRINT*,'INPUT DELTA VALUES'
      READ*, A,B,C,AD,BD,CD
      A1=X(13)*COS(THETA)+X(14)*SIN(THETA)
      B1=-X(13)*SIN(THETA)+X(14)*COS(THETA)
      AD1=X(16)*COS(THETA)+X(17)*SIN(THETA)
      BD1=-X(16)*SIN(THETA)+X(17)*COS(THETA)
      A=A+A1
      B=B+B1
      C=X(15)+C
      AD=AD+AD1
      BD=BD+BD1
      CD=CD+X(18)
      GOTO4
1     PRINT*,'INPUT NEW INITIAL CONDITIONS'
      READ*, A,B,C,AD,BD,CD
4     CONTINUE
      X(13)=A*COS(THETA)-B*SIN(THETA)
      X(14)=A*SIN(THETA)+B*COS(THETA)
      X(15)=C
      X(16)=AD*COS(THETA)-BD*SIN(THETA)
      X(17)=AD*SIN(THETA)+BD*COS(THETA)
      X(18)=CD
2     PRINT*,'DO YOU WANT THE CONDITIONS ON PFILE?'
      PRINT*,' '
      READ*,N
      REWIND 8
      IF(N.EQ.1)WRITE(8)T,X
      PRINT*,' '
      PRINT*,'              INITIAL CONDITIONS VECTOR'
      DO 3 I=1,18
      PRINT*,' '
      PRINT*,'              X(',I,')= ',X(I)
3     CONTINUE
      RETURN
      END
```

Vita

[illegible]

Hampshire, graduating from Salem High School in June of [illegible]. In February of 1968, Capt. Tilton attended basic training at Lackland AFB, Texas. While enlisted he attended various technical training schools and was assigned to Nellis AFB, Nevada, Clark AB, Phillipine Islands, and Takhli RTAFB, Thailand, through July 1973. During this period, he attained sufficient college credit to be accepted to AFIT's Airman's Education and Commissioning Program and attended the University of Texas at Austin. He graduated in 1975 with a BS in Aerospace Engineering. Following Officer Training School, Captain Tilton was assigned to Sunnyvale, AFS, California where he performed duties as a Satellite Operations Director.

Captain Tilton married Theresa [illegible] in August of 1970. They have one son, William, and currently reside in Dayton, Ohio. Following graduation from AFIT School of Engineering, Captain Tilton will be assigned to SAC Hqs., Offutt AFB, Neb. where he will work in the field of satellite survivability.

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18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Stability of the Earth-Moon-Sun-L4 System
Four Body Stability
Four Body Problem
L4
L5

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A search for a stable three dimensional orbit for a satellite about L4 is performed. A proposed two dimensional very restricted orbit is used to supply the initial conditions required for the search. An ephemeris of high accuracy is generated from a specific date and time using actual positions for the sun and moon. The generated sun and moon position and velocity vectors are used in the integration of the system's equations of motion. A stable orbit is found and is tested for its length of stability. The orbit is found to have a

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20. stable lifetime in excess of six hundred lunar synodic months. The sensitivity of the orbit to the sun's and moon's position is tested and [illegible] slightly sensitive for an error in position of one quarter day. Finally, a predicted 180° out of phase orbit is found and is determined to be only marginally stable.

DATE
ILME