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A MODIFIED KOLMOGOROV-SMIRNOV
TEST FOR THE GAMMA AND WEIBULL
DISTRIBUTION WITH UNKNOWN
LOCATION AND SCALE PARAMETERS

THESIS

AFIT/GOR/MA/80D-1

Ramon Cortes
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# A MODIFIED KOLMOGOROV-SMIRNOV TEST FOR THE GAMMA AND WEIBULL DISTRIBUTION WITH UNKNOWN LOCATION AND SCALE PARAMETERS

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by


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## Preface

The purpose of this thesis was to generate tables used for the modified Kolmogorov-Smirnov statistic. These tables are used for testing whether a set of observations is from a Weibull (Gamma) population when the scale and location parameters are not specified but must be estimated from the sample. A brief Monte Carlo investigation is made of the power of the test.

I would like to thank my advisor, Capt. Brian Woodruff, who offered me considerable guidance throughout my thesis project.

I would also like to thank my readers, Lt. Col. James Dunne and Dr. Albert H. Moore, whose advise aided me greatly.

Finally, I wish to acknowledge my gratitude to my classmates for their encouragement when the going got rough.

Ramón Cortés

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## Abstract

The Kolmogorov-Smirnov tables for the Weibull and Gamma distribution were generated in this thesis. These tables are used when testing whether a set of observations are from a Weibull (Gamma) distribution in which the location and scale parameters must be estimated from the sample.

A power investigation of the test against some selected distributions using Monte Carlo techniques was conducted. This procedure shows that the test is reasonably powerful for a number of alternative distributions.

A relationship between the critical values and the shape parameters was investigated for the Weibull and Gamma distributions. No apparent relationship was found for the Gamma distribution. In contrast, an approximate log-linear relationship was found for the Weibull when the shape parameter is between one and four.

# A MODIFIED KOLMOGOROV-SMIRNOV TEST FOR THE GAMMA AND WEIBULL DISTRIBUTION WITH UNKNOWN LOCATION AND SCALE PARAMETERS

## I. Introduction

One of the most important sets of data generated by the Air Force is data dealing with the time to failure of equipment. Failure distributions are of extreme importance to the Air Force since they determine the reliability of almost all mechanical systems in service.

Often these time-to-failure data are especially useful if we can determine to what theoretical distribution they correspond. That is, a test is carried out to determine an agreement between the distribution of a set of sample values and a theoretical distribution. This test is known as the "goodness of fit test." If the frequency distribution of the data compares well with the expected or theoretical distribution, we can then use the theoretical distribution to represent the parent or underlying population. Two very useful theoretical distributions that deal with time-to-failure data are the Gamma and Weibull distributions.

### Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov statistic provides a means of testing whether a set of sample values are from some completely specified continuous theoretical distribution, $F_0(x)$ . Suppose that a population is thought to have some specified cumulative distribution function, say $F_0(x)$ . Then, for any specified value of $x$ , the value of $F_0(x)$ is the proportion of individuals in the population having measurements less than or equal to $x$ . The cumulative step-function of a random sample of $N$ observations is expected to be fairly close to this specified distribution function. If it is not ose enough, this is evidence that the hypothetical distribution is not the correct one.

If $F_0(x)$ is the population cumulative distribution and $S_n(x_i)$ the observed cumulative step-function of a sample (i.e. $S_n(x_i)=\frac{K}{N}$ , where $K$ is the number of observations less than or equal to $x$ ), then the sampling distribution of

$$D = MAX|F_0(x) - S_n(x)| \tag{1}$$

is known, and is independent of $F_0(x)$ , if $F_0(x)$ is continuous. A standard table of the Kolmogorov-Smirnov test (1:425) gives certain critical points of the distribution of $D$ for various samples sizes.

Chi-Square vs. Kolmogorov-Smirnov Test

Another alternative to carry out the tests of goodness of fit, especially useful in the case where population parameters must be estimated, is the Chi-Square test $(\chi^2)$ .

The Kolmogorov-Smirnov test (K-S), however, has three major advantages over the Chi-Square (12:68).

1. The K-S test can be used with small sample sizes, where the validity of the Chi-Square test would be questionable.

2. Often the K-S test appears to be a more powerful test than the Chi-Square test for any sample size.

3. The K-S test will usually require less computation than $\chi^2$. This is especially true when a graphical test is used, for if the hypothesis is rejected, the computation stops at the point of rejection.

The two major advantages the Chi-Square has over the K-S test are (12:68):

1. In cases where parameters must be estimated from the sample, the $\chi^2$ test is easily modified by reducing the number of degrees of freedom. The K-S test has no such known modification, so it is not applicable in such cases.

2. The K-S test cannot be applied to discrete populations, whereas $\chi^2$ can be.

## Problem

The standard tables used for the Kolmogorov-Smirnov test are valid only when testing whether a set of observations are from a completely specified continuous distribution. If one or more parameters must be estimated from the sample then the tables are no longer valid. It has been suggested by (12:68) that if the K-S test is used in this case, the results will be conservative in the sense that the probability of a type I error will be smaller than as given by tables of Kolmogorov-Smirnov statistics. That is, the actual significance level would be lower than that given by the standard tables.

The existence of the probability integral transformation permits us to generate K-S type tables for any partic- distribution, if the parameters to be estimated are parameters of scale or location. David and Johnson (4:182) have shown that these parameters will be completely independent of the distribution of any test statistic based on the cumulative distribution function. The distribution of the test statistic will, in general, depend on the functional form of the distribution of the original variables. Since the K-S type test statistic is one based on the cumulative distribution function, the results of David and Johnson apply in this case.

## Background

Tables for the Kolmogorov-Smirnov test when population parameters are unknown have been generated by Hubert W. Lilliefors at the George Washington University. He generated the tables for the Normal distribution (8:399) and the Exponential distribution (9:387).

## Objectives

This thesis has the following objectives:

1. To generate and present the Kolmogorov-Smirnov tables for the Weibull and Gamma distribution when the scale and location parameters are not specified.

2. To determine the power of the test in selected situations using a Monte Carlo investigation.

3. To investigate the relationship between the critical values and the shape parameter for the Weibull and Gamma distributions.

# II. The Distributions

## Weibull Distribution

Historical Notes. The Weibull distribution is named after the Swedish scientist Waloddi Weibull, who published a paper in 1939 (10:50) giving some of the distribution's uses for an analysis of the breaking strengths of solids. However, it was also known to R.A. Fisher and L.H.C. Tippett who published a paper in 1928.

Application of the Weibull Distribution. In the past ten to fifteen years the Weibull distribution has emerged as the one popular parametric family of failure distributions. Its applicability to a wide variety of failure situations was discussed by Weibull. For example, it has been used to describe vacuum-tube and ball-bearing failures.

The Weibull distribution is closely related to the exponential, but has two additional parameters, the shape parameter and the location parameter. Thus, instead of a single constant failure rate $\lambda$, as in the exponential case, a variety of hazard situations can be treated. For a given Weibull distribution, the failure rate can be either continually increasing, continually decreasing, or else constant. Since many failures encountered in practice, especially those pertaining to nonelectric parts, show an increasing failure rate (due to deterioration or wear) the

Weibull distribution is useful in describing failure pattern of this type.

The Weibull Probability Function. Let the random variable $x$ denote testing or operating time, and let $\eta$ denote the "scale" parameter, $\beta$ the "shape" parameter, and $\gamma$ the "location" parameter. The Weibull probability density function (p.d.f.) is

$$f(x|\eta,\beta,\gamma) = \frac{\beta(x-\gamma)^{\beta-1}}{\eta^{\beta}} \exp\left\{ -\left(\frac{x-\gamma}{\eta}\right)^{\beta}\right\}, \gamma < x \qquad (2)$$

The Weibull distribution function is

$$F(x) = \int_{0}^{x} f(x|\eta,\beta,\gamma)$$

$$= 1 - \exp\left[-\left(\frac{x-\gamma}{\eta}\right)^{\beta}\right], x \geq \gamma \qquad (3)$$

When $\beta=1$ the Weibull distribution specializes to the exponential distribution, and when $\beta=2$ the resulting distribution is known as the Rayleigh distribution. The mean and variance of the Weibull are given as follows:

$$\text{Mean} = \gamma+\eta\Gamma\left(\frac{\beta+1}{\beta}\right) \qquad (4)$$

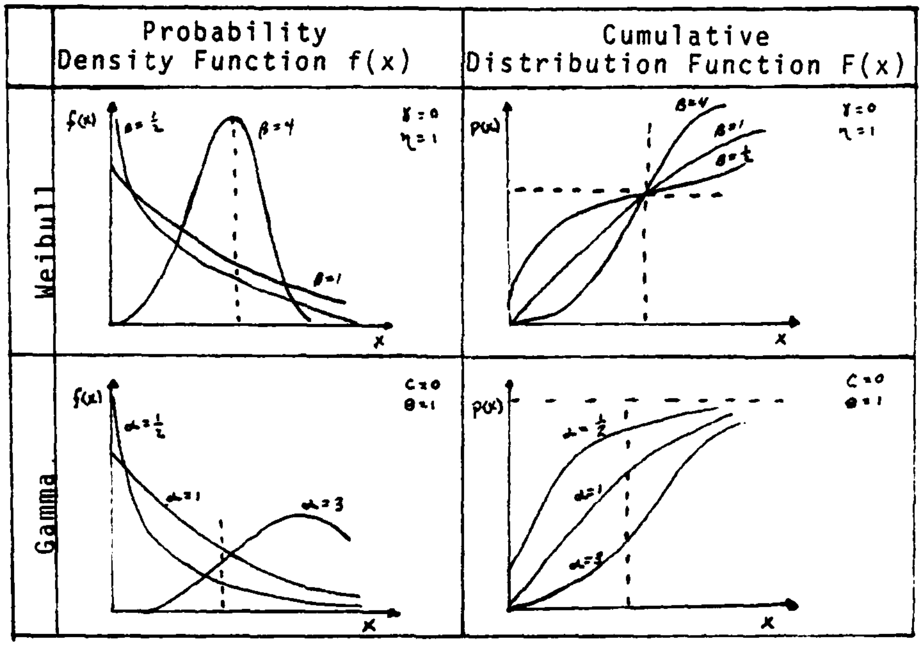


Figure 1. CDF and PDF for Weibull and Gamma Distribution

$$\text{Variance} = \eta^2\left[\Gamma\left(\frac{\beta+2}{\beta}\right)-\Gamma^2\left(\frac{\beta+1}{\beta}\right)\right] \tag{5}$$

A Weibull distribution with $\gamma=0$ and $\eta=1$ is illustrated for different values of the shape parameter $\beta$ in Figure 1.

## Gamma Distribution

Historical Notes. The Gamma distribution is a natural extension of the exponential distribution and has sometimes been considered as a model in life-test problems. It can be derived by considering the time to the $K^{th}$ success in a Poisson process or, equivalently, by considering the K fold convolution (10:55) of an exponential distribution.

The Gamma distribution is the continuous analog of the negative binomial distribution (11:125) which can also be obtained by considering the sum of $K$ variables with a common geometric distribution.

<u>Applications of the Gamma Distribution</u>. The Gamma Distribution has been used widely in queueing systems. For example, it has been used to determine the lengths of time between malfunctions for aircraft engine and the lengths of time between arrivals at a bank, super market, or maintenance checkout queue.

The Gamma distribution is one of the most useful continuous distributions available to the simulation analyst. If the variables from some random phenomenon cannot assume negative values and generally follow a unimodal distribution, then the chances are excellent that a member of the gamma family can adequately model the phenomenon. The Gamma distribution is defined by three parameters, $\theta$, $\alpha$ and $c$, where $\theta$ is the "scale" parameter, $\alpha$ is the "shape" parameter and $c$ is the "location" parameter. The Gamma probability density function (p.d.f.) is

$$f(x|c,\theta,\alpha) = \frac{(x-c)^{\alpha-1}}{\Gamma(\alpha)\theta^{\alpha}} \exp\left(-\frac{(x-c)}{\theta}\right) \qquad (6)$$

$$\theta,\alpha>0 \qquad x\geq c\geq 0$$

The Gamma distribution function is

$$F(x) = \int_0^x f(x|c,\theta,\alpha) \tag{7}$$

This integral does not have a closed form expression. Its values have to be determined by numerical calculations. When $\alpha=1$ , $c=0$ the Gamma density is the exponential density function with an expected value of $\theta$ . If $\alpha$ is an integer value, K , then the Gamma distribution is commonly referred to as an Erlang-K distribution. Furthermore, if $\theta=1$ , then as $\alpha$ becomes large, the Gamma distribution approaches the normal distribution. If we set $\alpha=\nu/2$ and $\theta=2$ (where $\nu$ is the degrees of freedom) we get the Chi-Square distribution. The mean and variance of the Gamma are given as follows:

$$\text{Mean} = \alpha\theta+c \tag{8}$$

$$\text{Variance} = \alpha\theta^2 \tag{9}$$

A Gamma distribution with $c=0$ and $\theta=1$ is illustrated for different values of the shape parameters $\alpha$ in Figure 1.

## III. Methodology

The method that follows was the one used in calculating critical values for the modified D statistic when the scale and location parameter are not specified.

For a fixed sample size $n$, random deviates $x_{(1)}$, $x_{(2)} \cdots x_{(n)}$ were generated from the Weibull and Gamma distributions. Next the random sample $x_{(1)}, x_{(2)} \cdots x_{(n)}$ was used to estimate the scale and location parameter by the method of maximum-likelihood. The resulting estimates of the scale and location parameter and the constant value of the shape parameter are then used to determine $F_0(x)$, the hypothesized distribution function. Finally, $D = \max|F_0(x) - S_n(x)|$ was calculated for the sample size $n$. This procedure was repeated 1000 times, thus generating 1000 independent values of the D statistics. These 1000 values were then ranked, and the 80th, 85th, 90th, 95th and 99th percentiles were found. This entire process was performed for sample sizes from $n=4$ to 30.

After tables for the test procedure were completed, a power comparison was conducted.

What follows is a more detailed description of the steps taken in this procedure. Figure 2 illustrates this procedure.

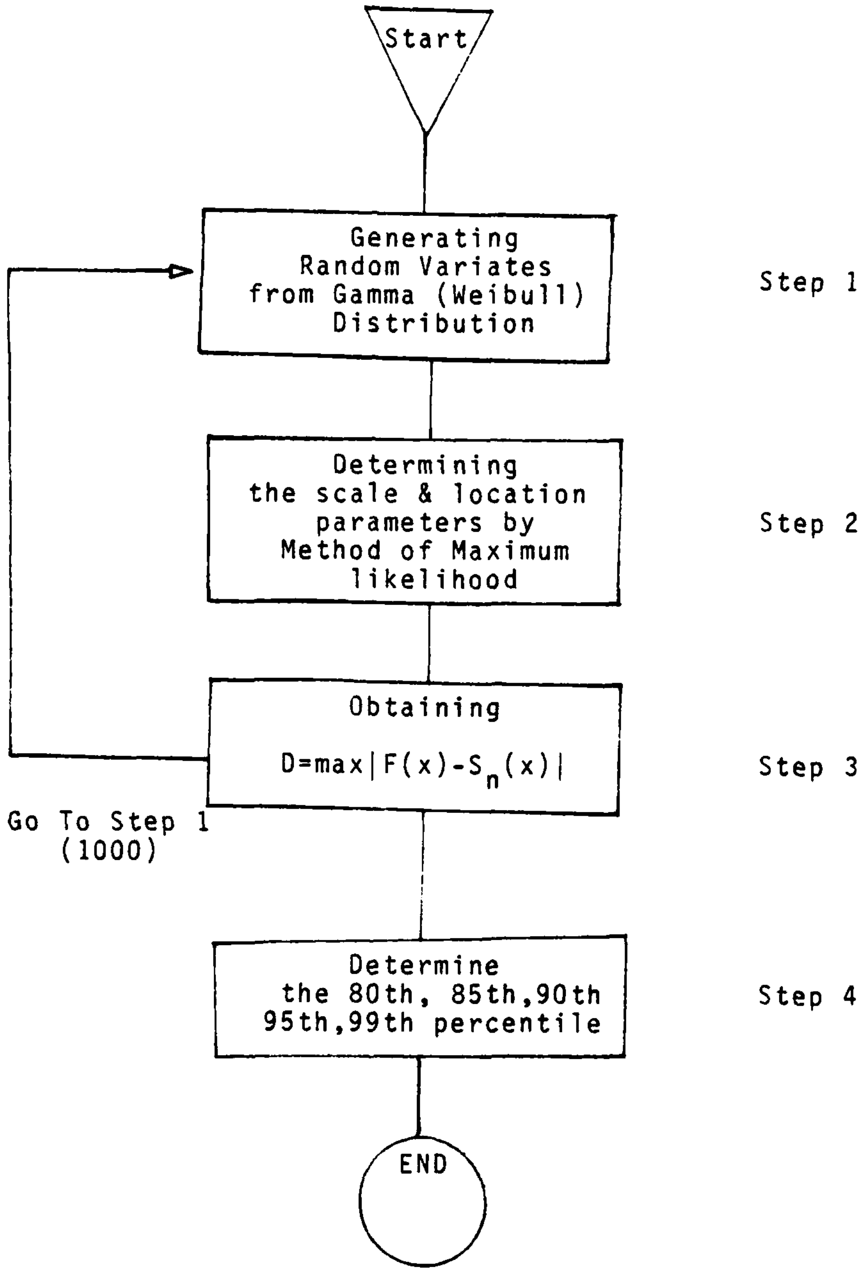


Figure 2. Summary of the Procedure.

## Generation of Random Weibull Deviates

The Weibull pseudo-random deviates were generated by

$$x = |-\ln(u)|^{\frac{1}{\beta}} \tag{10}$$

where $u$ is a pseudo-random deviate from a uniform (0,1) distribution and $\beta$ is the shape parameter. These deviates were obtained on the CDC 6600 computer by using the International Mathematical and Statistics Library (IMSL) subroutine GGWIB (19:306).

## Generation of Random Gamma Deviates

The Gamma pseudo-random deviates were generated using rejection methods by various computational algorithms depending on the value of the shape parameter $\alpha$ . These algorithms are contained in the IMSL subroutine GGAMR (14:1541). This subroutine was used to obtain the Gamma deviates on the CDC 6600 computer.

## Maximum Likelihood Estimation of Weibull and Gamma Parameters

The procedure used to derive the maximum likelihood estimates, $\hat{\eta}$ , $\hat{\beta}$ and $\hat{\gamma}$ , of the Weibull parameters, $\eta$ , $\beta$ and $\gamma$ , and $\hat{\theta}$ , $\hat{\alpha}$ and $\hat{c}$ , of the Gamma parameters, $\theta$ , $\alpha$ and $c$ , was developed by Harter and Moore (7:639). They developed an iterative procedure for censored or uncensored samples for the three parameter Weibull and Gamma distribution. For each of these distributions, the likelihood function is written down, and the three maximum-likelihood equations are obtained. In each case, simultaneous solution of these three equations would yield joint maximum-likelihood estimators for the three parameters. The iterative procedures proposed to solve the equations are applicable in the most general case, in which all three parameters are unknown, and also to special cases in which any one or any two of the parameters are unknown.

Weibull Maximum Likelihood Estimates

The probability density function of the random variable $x$ having a Weibull distribution with location parameter $\gamma \geq 0$, scale parameter $\eta$, and shape parameter $\beta$ is given by

$$f(x|\eta,\beta,\gamma) = \left(\frac{\beta(x-\gamma)^{\beta-1}}{\eta^{\beta}}\right)\exp\left(-\left(\frac{x-\gamma}{\eta}\right)^{\beta}\right)$$

$$\eta,\beta>0,\quad x\geq\gamma>0 \qquad (11)$$

The natural logarithm of the likelihood function of the order statistics $x_1, x_2, \ldots, x_n$, of a sample of size $n$ is given by (7:640)

$$L = \ln n! + n(\ln \beta - \beta \ln \eta) + (\beta-1)\sum_{c=1}^{n} \ln (x_i-\gamma)$$

$$- \sum_{c=1}^{n} (x_1-\gamma)/\eta^{\beta} \qquad (12)$$

The maximum-likelihood equations are obtained by equating to zero the partial derivatives of L with respect to each of the three parameters; these partial derivatives are given by:

$$\frac{\partial L}{\partial \eta} = \frac{-\beta n}{\eta} + \beta \sum_{i=1}^{n} (x_i - \gamma)^{\beta} / \eta^{\beta+1} \tag{13}$$

$$\frac{\partial L}{\partial \beta} = \sum_{i=1}^{n} \ln (x_i - \gamma) - \sum_{i=1}^{n} [(x_i - \gamma)/\eta]^{\beta} \ln [(x_i - \gamma)\eta] \tag{14}$$

$$\frac{\partial L}{\partial \gamma} = (1-\beta) \sum_{i=1}^{n} (x_i - \gamma)^{-1} + \beta \eta^{-\beta} \sum_{i=1}^{n} (x_i - \gamma)^{\beta-1} \tag{15}$$

Gamma Maximum Likelihood Estimates

The probability density function of the random variable $x$ having a Gamma distribution with location parameter $c \geq 0$, scale parameter $\theta$, and shape parameter $\alpha$ is given by

$$f(x,c,\theta,\alpha) = [1/\Gamma(\alpha)\theta] \; [(x-c)/\theta]^{\alpha-1} \cdot \exp[-(x-c)/\theta]$$

$$\theta, \alpha > 0 \qquad x \geq c \geq 0 \tag{16}$$

The natural logarithm of the likelihood function of the order statistics $x_1, x_2, \ldots x_n$ of a sample of size $n$ is given by

$$L = \ln n! - n \ln \Gamma(\alpha) - n\alpha \ln \theta + (\alpha - 1)$$

$$\cdot \sum_{i=1}^{n} \ln (x_i - c) - \sum_{i=1}^{n} (x_i - c)/\theta \qquad (17)$$

The maximum-likelihood equations are obtained by equating to zero the partial derivatives of $L$ with respect to each of the three parameters; these partial derivatives are given by

$$\frac{\partial L}{\partial \theta} = -n\alpha/\theta + \sum_{i=1}^{n} (x_i - c)\theta^2 \qquad (18)$$

$$\frac{\partial L}{\partial \alpha} = -n \ln \theta + \sum_{i=1}^{n} \ln (x_i - c) - n\Gamma'(\alpha)/\Gamma(\alpha) \qquad (19)$$

$$\frac{\partial L}{\partial c} = (1-\alpha) \sum_{i=1}^{n} (x_i - c)^{-1} + n/\theta \qquad (20)$$

where the prime in $\frac{\partial L}{\partial \alpha}$ indicates differentiation with respect to $\alpha$ .

Calculation of Test Statistic D

For the Weibull distribution there was no problem in calculating D in Eq(1) since $F_0(x)$ has a closed form expression Eq.(3). Since the Gamma distribution does not have a closed form expression, an integral calculation had to be done in order to determine $F_0(x)$ before calculating $D$ . This integral calculation was accomplished using the IMSL subroutine MDGAM (16:946) which evaluates the probability that a random variable from a Gamma distribution having shape parameter $\alpha$ with $c=0$ and $\theta=1$ is less than or equal to $x$ .

$$I(x,\alpha) = \int_0^x \frac{e^{-t}t^{\alpha-1}dt}{\Gamma(\alpha)} \tag{21}$$

In order to use this subroutine, the generalized three parameter Gamma density function considered in this thesis had to be transformed to the one parameter Gamma density function used in the subroutine. This transformation was done as follows:

$$F(y) = \int_c^y \frac{(x-c)^{\alpha-1}}{\Gamma(\alpha)\theta^\alpha} e^{-\left(\frac{x-c}{\theta}\right)} dx \tag{22}$$

Let, $z=x-c$ $dz=dx$.

Then, $x=c \rightarrow z=0$

$x=y \rightarrow z=y-c$

and

$$F(y) = \int_0^{y-c} \frac{z^{\alpha-1}}{\Gamma(\alpha)\theta^\alpha} e^{\frac{-z}{\theta}} dz \tag{23}$$

Next, let $t=\frac{z}{\theta}$ , $dt=\frac{dz}{\theta}$

Then $z=0 \rightarrow t=0$

$z=y-c \rightarrow t=\frac{y-c}{\theta}$

and

$$F(y) = \int_0^{\frac{y-c}{\theta}} \frac{t^{\alpha-1}}{\Gamma(\alpha)} e^{-t} \quad dt \tag{24}$$

which is the same integral in Eq(21) with $x=\frac{y-c}{\theta}$ .

## Power Comparison

The power of the test using the modified D statistic was compared for four alternative distributions. These power comparisons were made using Monte Carlo simulation. The procedure is very similar to the procedure used in creating the critical values for the D statistic. Five thousand random samples of size n were generated for each of the four alternative distributions considered. Then the D statistic was calculated and compared to its respective critical value obtained from the table generated. The number of rejections of the two different null hypotheses (that the distribution was from a Weibull or Gamma distribution with known shape parameter) were counted.

## Analysis of Critical Values vs. Shape Parameter

The tables generated in this thesis for the Weibull and Gamma distributions will depend on the value of the shape parameter. For different values of the shape parameter we will have different tables. For this reason we have presented tables for eight different shape parameters. These shape parameters are .5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0 .

Since there is an apparent relationship between the shape parameter and the table's critical values,an investigation of the possibility of finding an expression or equation that indicates the relationship between the shape parameter and the critical values was conducted. Both regression and graphical methods were considered.

## Computer Programs

The computer programs built for generating the Gamma and Weibull distribution were made very flexible. That is they were built to generate tables for any shape parameter input into the programs. The programs used to generate the Gamma and Weibull distributions are presented in Appendix D.

## IV. Use of Tables

In the Kolmogorov Smirnov test a theoretical or assumed distribution $F(x)$, is compared to an observed distribution $S_n(x)$. If the maximum deviation of $|F(x)-S_n(x)|$ exceeds a certain limiting value, $d_{\alpha,n}$ the assumed distribution is rejected.

The steps in applying the modified Kolmogorov Smirnov test when the scale and location parameters are estimated are as follows:

1. Determine the values of the scale and location parameters by one of the methods of parameter estimation.

2. Specify completely (including the shape parameter) the hypothesized distribution $F(x)$.

3. Determine the desired level of significance, $\alpha$, and the desired or planned sample size, $n$. The level of significance, $\alpha$, is the risk of rejecting the hypothesized distribution if it is in fact the true distribution.

4. Using the tables generated in this thesis, select the critical value, $d_{\alpha,n}$.

5. Select a random sample of $n$ items from the population to be tested, and order the observations.

6. Determine the maximum value, $d$, of

$$D = |F_0(x)-S_n(x)|$$

where

$$S_n = i/n \quad \text{when } x_i \le x < x_{i+1}$$

7. If $d > d_{\alpha,n}$ reject the hypothesized distribution. If $d \le d_{\alpha,n}$ the hypothesized distribution cannot be rejected and we say that it is reasonable to assume that the hypothesized distribution is the true distribution.

These steps are illustrated in the two examples which follow.

<u>Example for the Weibull Distribution</u>

A certain engineer is involved with the development of a new turbojet engine. It is important to determine the failure time of this new engine before it is introduced to the market. Six identical engines were tested. The failure time data obtained from these tests were:

.034, .168, .266, .563, 1.344, 3.118 years.

The engineer believes that the failure time of this engine follows a Weibull distribution with shape parameter equal to one and scale and location parameter undetermined. We will now conduct the test developed in this thesis at the 5% level of significance to determine if the engineer's hypothesis is reasonable. That is,

$H_o$: The distribution is Weibull (shape=1)

$H_a$: Another distribution

First we determine the scale and location parameter by

| TABLE I | | | | | |
|---|---|---|---|---|---|
| $i$ | $x_i$ | $F(x_i)$ | $S_n(x_i)$ | $\lvert F(x_i)-S_n(x_i)\rvert$ | $\lvert F(x_i)-S_{n(x_i-1)}\rvert$ |
| 1 | .034 | .036 | .167 | .131 | .036 |
| 2 | .168 | .166 | .334 | .168 | .001 |
| 3 | .266 | .250 | .501 | *.251* | .084 |
| 4 | .563 | .457 | .668 | .211 | .044 |
| 5 | 1.344 | .767 | .835 | .068 | .099 |
| 6 | 3.118 | .966 | 1.000 | .034 | .131 |

Note that $d=\max\lvert F(x)-S_n(x)\rvert$ is .251.

the method of maximum likelihood. The following values were obtained: scale = $\hat{\eta}$ =.923 location = $\hat{\alpha}$ = .0003 Using these estimates and $\beta=1$ , in Eq(3) we can obtain values for the hypothesized distribution. We can then construct Table I. Using the table of critical values for the Weibull distribution for shape=1 we find for $\alpha=.05$ and $n=6$ that $d_{.05,6}$ =.383 . Hence, since .251<.383, we cannot reject the null hypothesis. We conclude that a Weibull distribution with shape parameter equal to one is a reasonable model for the engine failure data.

Example of the Gamma Distribution

Major Smith is the maintenance squadron commander at

Big AFB. During the past six months he has been trying to determine the lengths of time his crew takes to complete a maintenance service for an aircraft engine. This information is vital to him since it will permit him to schedule his crew in a way that will optimize service time. Major Smith, presented this problem to Lt Jones. Lt Jones gathered the following service time data:

.397, .524, .691, .973, 2.548, 2.933 hours.

Lt Jones told Major Smith that he believes that these data follow a Gamma distribution with shape parameter equal to one. Major Smith wants to use the test procedures of this thesis at the 5% level of significance to determine if Lt Jones' hypothesis is reasonable. That is,

$H_o$: The distribution is Gamma (shape=1)

$H_a$: Another distribution

First we determine the scale and location parameter by the method of maximum likelihood. The following values were obtained: scale = $\hat{\theta}$ = 1.009 location = $\hat{c}$ = .341 Using these estimates, and $\alpha$=1, in Eq(24), we can obtain values for the hypothesized distribution function. We can then construct Table II. Using the table of critical values for the Gamma distribution for shape=1 we find for level $\alpha$=.05 and n=6 that $d_{.05,6}$=.383 . Hence, since .220<.383, we cannot reject the null hypothesis. We conclude that a Gamma distribution with shape parameter equal to 1 is a reasonable model for the length of time of a maintenance service.

TABLE II

| $i$ | $x_i$ | $F(x_i)$ | $S_n(x_i)$ | $\lvert F(x_i)-S_n(x_i)\rvert$ | $\lvert F(x_i)-S_{n(x_i-1)}\rvert$ |
|---|---|---|---|---|---|
| 1 | .397 | .054 | .167 | .113 | .054 |
| 2 | .524 | .167 | .334 | .167 | .000 |
| 3 | .691 | .293 | .501 | .208 | .041 |
| 4 | .973 | .466 | .668 | .202 | .035 |
| 5 | 2.548 | .888 | .835 | .053 | *.220* |
| 6 | 2.933 | .923 | 1.000 | .077 | .088 |

Note that $d=\max\lvert F(x)-S_n(x)\rvert$ is .220.

## V. Discussion of Results

The results obtained in this thesis are discussed in this chapter. Results applicable to each objective are presented in the sequence in which the objectives were presented in Chapter I. All tables referenced in this chapter are located in Appendix A and B, and all figures in Appendix C.

### Presentation of the Kolmogorov-Smirnov Tables

The Kolmogorov-Smirnov tables for the Weibull and Gamma distribution when the scale and location parameters are not specified are presented in Appendix A and B respectively. The critical values in these tables were subject to simulation variability. This variability was reflected in the third decimal place of the critical values. Due to this variability the critical values in many tables are not monotonically decreasing functions of sample size when the sample size exceeds 15.

This variability is not unusual for any kind of Monte Carlo simulation. There are several methods which can be used to eliminate this variability. One method is to increase the simulation sample size. In this thesis the critical values were obtained with 1000 samples. If the simulation would have been done with more than 1000 samples, the variability would have been less. For example Table (Ref. Appendix B) was obtained with 5,000 samples and as can be seen the simulation variability has been reduced. The critical values are monotonically decreasing for increasing sample sizes at all levels of significance. All the tables were not generated with 5,000 samples because to do so would require a large amount of computer time. Another method to reduce simulation variability is to run the program with a different seed several times with a relatively small number of samples in each run. Then the critical values are obtained by computing the means of the critical values found on these several runs. For example, the programs used in calculating the tables in this thesis (1000 samples) could have been run five times with a different seed everytime. Then the critical values would have been obtained by computing the mean of the five program runs. Again this procedure was not carried out because it, too, would require a large amount of computer time.

## Validity of Computer Programs

The validity of the computer programs (Appendix D) used to generate the tables presented in this thesis were verified by comparing the critical values obtained with the computer programs with the critical values obtained by Lilliefors in his exponential table (9:387). This comparison is possible because both the Weibull and Gamma distributions become an exponential distribution when the shape parameter is one and the location parameter is zero. When the shape parameter was input to one, the location parameter to zero and the scale parameter set to be estimated in the computer programs the critical values obtained were almost identical to the critical values obtained by Lilliefors in his exponential table. These results can be seen in Table III for sample size $n=10$ .

## Power of the Test

The power of the test was carried out as discussed in Chapter III with only one exception. The Monte Carlo simulation for the power of the test for the Gamma distribution was obtained with 2000 samples of size $n=30$ instead of the 5,000 samples indicated in Chapter III. The reason for this change was the excessive amount of computer time required to do the test with 5,000 samples.

The results obtained are presented in Table IV and V. Table IV represents the probability of rejecting the null hypothesis of a Weibull distribution with shape parameter one using the modified $D$ statistic of this thesis when the sample size is 30. The numbers are the result of Monte Carlo simulation with 5,000 samples for each distribution. Table V represents the probability of rejecting the null hypothesis of a Gamma distribution with shape parameter one using the modified $D$ statistic when the sample size is 30.

| TABLE III | | | | | |
|---|---|---|---|---|---|
| Comparison Between Thesis Program Critical Values and Lilliefors Critical Values | | | | | |
| Level of Significance | .20 | .15 | .10 | .05 | .01 |
| Critical Values Obtain by Lilliefors | .263 | .277 | .295 | .325 | .38 |
| Critical Values Obtain by Thesis Programs | .265 | .279 | .301 | .323 | .381 |

| TABLE IV | | | | | | |
|---|---|---|---|---|---|---|
| Power of the Test for the Weibull | | | | | | |
| Underlying Distribution n = 30 $H_0$: Weibull(shape=1) | Modified K-S test for the Weibull distribution | | | | | |
| | Using critical values from Table 23 | | | | | |
| | $\alpha$=.05 | $\mu_1$ | $\mu_2$ | $\alpha$=.01 | $\mu_1$ | $\mu_2$ |
| Weibull | .05 | .044 | .056 | .01 | .007 | .013 |
| Beta | .98 | .976 | .984 | .94 | .933 | .947 |
| Normal | .96 | .955 | .965 | .86 | .850 | .870 |
| Log Normal | .41 | .396 | .424 | .18 | .169 | .191 |
| Gamma(shape=1) | .05 | .044 | .056 | .01 | .007 | .013 |
| Gamma(shape=2) | .46 | .446 | .474 | .26 | .248 | .272 |
| Weibull(shape=2) | .05 | .044 | .056 | .01 | .007 | .013 |

| TABLE V<br>Power of the Test for the Gamma | | | | | | |
|---|---|---|---|---|---|---|
| Underlying Distribution<br>n = 30<br>$H_0$: Gamma(shape=1) | Modified K-S test for the Gamma distribution | | | | | |
| | Using critical values from Table 8 | | | | | |
| | $\alpha$=.05 | $\mu_1$ | $\mu_2$ | $\alpha$=.01 | $\mu_1$ | $\mu_2$ |
| Gamma | .04 | .031 | .049 | .01 | .006 | .014 |
| Beta | .68 | .660 | .700 | .34 | .319 | .361 |
| Normal | .42 | .398 | .442 | .26 | .241 | .279 |
| Log Normal | .10 | .087 | .113 | .04 | .031 | .049 |
| Weibull(shape=1) | .04 | .031 | .049 | .01 | .006 | .014 |
| Weibull(shape=2) | .01 | .006 | .014 | .001 | .000 | .002 |

The numbers are the result of Monte Carlo simulation with 2,000 samples for each distribution.

In both Tables IV and V, $U_1$ and $U_2$ represent the lower and upper limit of the 95% confidence interval for the probability of rejecting the null hypothesis. This confidence interval is given by:

$$P(\hat{p}-Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}<p<\hat{p}+Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}) = P(U_1<p<U_2)$$

where $\hat{p}$ is the proportion of rejections in the computer runs and $Z_{\alpha/2}$ is the upper $\alpha/2$ cut-off value from a standard normal.

Table IV indicates that when the null hypothesis is true, the test does in fact achieve the claimed significance level. The test has a very high power when used on Beta and Normal distributions, but not so high for the log normal distribution. The results obtained for the Gamma distribution with shape parameter equal to one were the same as those for the Weibull with shape equal to one. This result was expected since both distributions are an exponential distribution under this condition. In contrast, when the test was done with a Gamma distribution with shape parameter equal to two different results were obtained. This demonstrates that the test does reasonably well against the Gamma distribution with shape parameter not equal to one. In contrast, the test is not very powerful against a Weibull with shape parameter equal to two. This suggests that the test may not be able to readily distinguish between different members of the Weibull family.

Table V indicates that when the null hypothesis is true, the test approximates reasonably well the claimed .05 significance level and exactly the .01 significance level. The test has a reasonably good power when used on Beta and Normal distributions, but no so good for the log normal distribution. The results obtained for the Weibull distribution with shape parameter equal to one were the same as those for the Gamma with shape equal to one. This result was expected since both distributions are an exponential distribution under this condition. In contrast, when the test was done with a Weibull distribution with shape parameter equal to two, different results were obtained. Unfortunately, the results obtained for the Weibull distribution with shape parameter equal to two were poor. They indicate that the test could not distinguish between a sample generated from a Gamma distribution with shape parameter equal to one to a sample from a Weibull distribution with a shape parameter equal to two.

In general, the results obtained for the power of the test for the Gamma distribution were poor in comparison to the results obtained for the Weibull distribution even though the null hypothesis in each table was the exponential distribution.

Three possible factors could have caused these different results for the Gamma.

1) The critical values used in the Gamma test were generated with 2,000 samples while the critical values used in the Weibull distribution test were generated with 5,000 samples. This implies that the critical values for the Gamma distribution had more Monte Carlo variability than those for the Weibull distribution.

2) The power of the test for the Gamma distribution was conducted with 2,000 samples while the test for the Weibull distribution was done with 5,000 samples.

3) The cumulative distribution function for the Gamma was determined by an integral calculation. This integral calculation is subject to errors since the values obtained will be limited by some tolerance. For the Weibull distribution such a problem does not exist since its cumulative distribution function has a closed form.

<u>Relationship Between Critical Values and Shape Parameter</u>

As discussed in Chapter III a relationship between the critical values and the shape parameter was investigated. For the Gamma distribution no apparent relationship was found as can be seen in Figures three through 17. Possibly by increasing the number of shape parameters in the investigation a relationship could be found. Another possible explanation for the apparent lack of a relationship is Monte Carlo variability in the critical values. Increasing the number of samples used to generate the critical values for the Gamma distribution may reduce the variability sufficiently to enable a relationship to be found.

For the Weibull distribution an approximate relationship was found when the shape parameter is greater than one as can be seen in Figures 18 through 32. The approximated equation found to represent the relationship between the critical values and the shape parameter (between one and four) is

$$\ln y = a_0 + a_1 x$$

where $a_0$ is a constant, and $a_1$ is the coefficient of the independent variable. The variable x is the independent variable which is the shape parameter, and ln y represents the natural logarithm of the critical value.

This equation was the best expression found to represent the relationship between the critical values and the shape parameter between one and four.

The equations obtained for all levels of significance using three different sample sizes are presented in Table VI.

The $R^2$ value on the table indicates how good the log-linear equation represents the relationship between the critical values and the shape parameter. From these values we can see that with the exception of sample size 15 at level of significance .01 all $R^2$ values are greater than .85 . This indicates that the log-linear equation is a good approximation for the relationship. For example, sample size 15 at level .20 had $R^2$=.95, and sample size 15 at level .15 had $R^2$=.96 . In fact, of the 15 regressions done, 10 had an $R^2$ equal to or greater than .90. From these results it can be concluded that the log-linear equation is a good approximation for the relationship, but not an exact one.

The values presented in Table VI were calculated using the Statistical Package for the Social Sciences (SPSS)(17:11). These equations can be used to obtain approximate critical values for Weibull distributions with any shape parameter between one and four.

TABLE VI

Coefficients and $R^2$ Values for the Relationship between

Critical Values and Shape Parameters

| Level of Significance | | .20 | $R^2$ | .15 | $R^2$ | .10 | $R^2$ | .05 | $R^2$ | .01 | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| n=5 | $a_0$ | -1.057 | .93 | -1.016 | .90 | - .932 | .91 | - .833 | .88 | - .777 | .91 |
| | $a_1$ | - .017 | | - .016 | | - .028 | | - .033 | | - .004 | |
| n=15 | $a_0$ | -1.522 | .95 | -1.469 | .96 | -1.399 | .93 | -1.329 | .90 | -1.148 | .53 |
| | $a_1$ | - .026 | | - .027 | | - .027 | | - .021 | | - .024 | |
| n=30 | $a_0$ | -1.852 | .87 | -1.796 | .89 | -1.715 | .92 | -1.620 | .89 | -1.459 | .94 |
| | $a_1$ | - .020 | | - .023 | | - .028 | | - .023 | | - .022 | |

# VI. Conclusions and Recommendations

## Conclusions

On the basis of the results obtained in this thesis the following conclusions are drawn.

1. The tables generated for the modified Kolmogorov-Smirnov are valid. Monte Carlo studies have shown that the test achieved the claimed significance level when the null hypothesis is true and have good power against several alternative distributions.

2. A log-linear relationship between the critical values and the shape parameter was found for the Weibull distribution when the shape parameter is greater than one. In contrast, there seems to be no simple linear type relationship for the Gamma distribution.

## Recommendations for Further Study

Based on the observations made during this investigation, the following recommendations are proposed for further study:

1) To increase the number of shape parameters used in the investigation of the Weibull (Gamma) distribution to see if a better expression can be obtained for the relationship between critical values and shape parameters.

2) To increase the range of shape parameters investigated

for the Gamma distribution to see if an expression or equation could be found.

3) To increase the range of shape parameters investigated for the Weibull distribution to see if the log-linear relationship between shape parameters and critical values remains valid for shape parameter greater than four.

4) To compare the power of the test between the Chi-square and the modified Kolmogorov-Smirnov test for some selected distributions.

5) To modify the Anderson-Darling and Cramér-von-Mises goodness-of-fit tests in a manner similar to the one presented in this thesis for the K-S test. The result of the three modified tests can then be compared to determine which is most powerful.

6) To conduct the power of the test for the Gamma with 5,000 samples to see if the results could be improved with a reduction in the Monte Carlo variability.

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Appendix A

## Tables for the Gamma Distribution

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)) | | | | |
|---|---|---|---|---|---|
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .389 | .413 | .437 | .469 | .565 |
| 5 | .350 | .367 | .389 | .430 | .497 |
| 6 | .326 | .347 | .371 | .410 | .485 |
| 7 | .297 | .314 | .337 | .368 | .446 |
| 8 | .285 | .301 | .323 | .358 | .427 |
| 9 | .263 | .282 | .304 | .328 | .417 |
| 10 | .248 | .264 | .290 | .318 | .385 |
| 11 | .235 | .251 | .276 | .307 | .367 |
| 12 | .231 | .245 | .270 | .301 | .360 |
| 13 | .221 | .236 | .253 | .280 | .330 |
| 14 | .212 | .228 | .244 | .277 | .352 |
| 15 | .196 | .210 | .234 | .263 | .313 |
| 16 | .195 | .207 | .226 | .259 | .323 |
| 17 | .185 | .196 | .214 | .239 | .286 |
| 18 | .181 | .195 | .215 | .243 | .296 |
| 19 | .175 | .188 | .203 | .234 | .288 |
| 20 | .170 | .185 | .200 | .218 | .275 |
| 21 | .170 | .180 | .201 | .227 | .283 |
| 22 | .162 | .174 | .190 | .216 | .256 |
| 23 | .156 | .167 | .181 | .204 | .256 |
| 24 | .159 | .170 | .183 | .208 | .258 |
| 25 | .152 | .162 | .176 | .200 | .241 |
| 26 | .148 | .159 | .177 | .193 | .247 |
| 27 | .145 | .157 | .170 | .190 | .234 |
| 28 | .141 | .152 | .165 | .184 | .238 |
| 29 | .136 | .147 | .162 | .181 | .220 |
| 30 | .141 | .148 | .161 | .179 | .224 |

Table VII. Shape Parameter Equal to 0.5

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)) | | | | |
|---|---|---|---|---|---|
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .339 | .350 | .399 | .435 | .519 |
| 5 | .330 | .347 | .362 | .394 | .449 |
| 6 | .310 | .327 | .346 | .383 | .456 |
| 7 | .261 | .296 | .324 | .356 | .421 |
| 8 | .271 | .289 | .306 | .336 | .413 |
| 9 | .257 | .274 | .291 | .320 | .390 |
| 10 | .252 | .266 | .286 | .312 | .381 |
| 11 | .235 | .252 | .272 | .295 | .346 |
| 12 | .227 | .240 | .261 | .288 | .354 |
| 13 | .215 | .229 | .243 | .270 | .329 |
| 14 | .208 | .224 | .241 | .263 | .313 |
| 15 | .194 | .207 | .225 | .256 | .304 |
| 16 | .190 | .200 | .216 | .242 | .303 |
| 17 | .184 | .194 | .210 | .238 | .298 |
| 18 | .179 | .187 | .199 | .219 | .266 |
| 19 | .172 | .185 | .200 | .221 | .285 |
| 20 | .167 | .177 | .196 | .220 | .264 |
| 21 | .165 | .176 | .190 | .216 | .263 |
| 22 | .159 | .169 | .185 | .207 | .240 |
| 23 | .161 | .170 | .186 | .204 | .250 |
| 24 | .156 | .166 | .181 | .199 | .246 |
| 25 | .154 | .164 | .176 | .200 | .256 |
| 26 | .147 | .156 | .172 | .190 | .234 |
| 27 | .144 | .154 | .168 | .196 | .232 |
| 28 | .144 | .154 | .169 | .189 | .228 |
| 29 | .142 | .149 | .160 | .179 | .230 |
| 30 | .133 | .146 | .161 | .183 | .218 |

Table VIII. Shape Parameter Equal to 1.0

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)) | | | | |
|---|---|---|---|---|---|
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .383 | .396 | .415 | .449 | .521 |
| 5 | .335 | .356 | .384 | .429 | .516 |
| 6 | .312 | .330 | .354 | .389 | .441 |
| 7 | .297 | .311 | .329 | .357 | .414 |
| 8 | .274 | .284 | .305 | .336 | .391 |
| 9 | .260 | .272 | .292 | .329 | .402 |
| 10 | .245 | .257 | .274 | .303 | .369 |
| 11 | .235 | .250 | .264 | .294 | .348 |
| 12 | .218 | .229 | .246 | .277 | .327 |
| 13 | .213 | .228 | .245 | .268 | .325 |
| 14 | .203 | .215 | .232 | .253 | .308 |
| 15 | .195 | .205 | .219 | .246 | .265 |
| 16 | .190 | .198 | .218 | .239 | .304 |
| 17 | .181 | .193 | .213 | .236 | .270 |
| 18 | .179 | .191 | .203 | .227 | .265 |
| 19 | .173 | .192 | .206 | .226 | .269 |
| 20 | .162 | .172 | .182 | .197 | .236 |
| 21 | .164 | .173 | .185 | .210 | .240 |
| 22 | .155 | .166 | .181 | .198 | .228 |
| 23 | .157 | .165 | .179 | .197 | .232 |
| 24 | .153 | .166 | .176 | .197 | .239 |
| 25 | .150 | .160 | .172 | .188 | .226 |
| 26 | .145 | .156 | .170 | .187 | .216 |
| 27 | .141 | .151 | .162 | .175 | .227 |
| 28 | .139 | .149 | .161 | .177 | .211 |
| 29 | .137 | .145 | .159 | .174 | .216 |
| 30 | .133 | .148 | .159 | .182 | .225 |

Table IX. Shape Parameter Equal to 1.5

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)) | | | | |
|---|---|---|---|---|---|
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .361 | .377 | .401 | .436 | .490 |
| 5 | .334 | .351 | .369 | .407 | .464 |
| 6 | .303 | .316 | .339 | .370 | .425 |
| 7 | .286 | .300 | .324 | .361 | .411 |
| 8 | .267 | .280 | .303 | .338 | .386 |
| 9 | .249 | .264 | .284 | .311 | .357 |
| 10 | .240 | .254 | .272 | .298 | .346 |
| 11 | .234 | .243 | .263 | .288 | .338 |
| 12 | .215 | .229 | .246 | .269 | .317 |
| 13 | .205 | .220 | .236 | .260 | .310 |
| 14 | .195 | .209 | .222 | .244 | .260 |
| 15 | .189 | .199 | .220 | .245 | .290 |
| 16 | .187 | .198 | .213 | .236 | .250 |
| 17 | .179 | .190 | .201 | .226 | .268 |
| 18 | .174 | .185 | .198 | .225 | .268 |
| 19 | .170 | .181 | .195 | .212 | .265 |
| 20 | .169 | .177 | .186 | .205 | .254 |
| 21 | .160 | .170 | .181 | .203 | .235 |
| 22 | .162 | .171 | .184 | .202 | .229 |
| 23 | .154 | .161 | .177 | .195 | .238 |
| 24 | .149 | .157 | .171 | .199 | .230 |
| 25 | .145 | .155 | .168 | .185 | .220 |
| 26 | .144 | .154 | .164 | .185 | .240 |
| 27 | .141 | .149 | .164 | .183 | .225 |
| 28 | .141 | .150 | .164 | .184 | .224 |
| 29 | .139 | .146 | .156 | .173 | .222 |
| 30 | .135 | .142 | .155 | .172 | .205 |

Table X. Shape Parameter Equal to 2.0

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)) | | | | |
|---|---|---|---|---|---|
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .356 | .366 | .391 | .426 | .480 |
| 5 | .335 | .352 | .374 | .405 | .471 |
| 6 | .303 | .315 | .336 | .359 | .399 |
| 7 | .278 | .295 | .309 | .341 | .410 |
| 8 | .264 | .277 | .294 | .321 | .374 |
| 9 | .250 | .264 | .281 | .304 | .352 |
| 10 | .235 | .245 | .261 | .290 | .338 |
| 11 | .221 | .232 | .247 | .270 | .325 |
| 12 | .220 | .232 | .245 | .268 | .313 |
| 13 | .206 | .218 | .235 | .262 | .310 |
| 14 | .196 | .210 | .223 | .241 | .286 |
| 15 | .188 | .199 | .213 | .234 | .290 |
| 16 | .182 | .191 | .203 | .225 | .277 |
| 17 | .176 | .186 | .201 | .221 | .284 |
| 18 | .172 | .183 | .196 | .220 | .257 |
| 19 | .165 | .175 | .189 | .212 | .248 |
| 20 | .163 | .170 | .181 | .200 | .242 |
| 21 | .165 | .175 | .188 | .205 | .250 |
| 22 | .160 | .165 | .184 | .199 | .240 |
| 23 | .154 | .165 | .176 | .193 | .239 |
| 24 | .149 | .158 | .172 | .190 | .231 |
| 25 | .148 | .156 | .168 | .187 | .223 |
| 26 | .143 | .151 | .165 | .183 | .220 |
| 27 | .141 | .150 | .162 | .177 | .213 |
| 28 | .137 | .146 | .159 | .174 | .213 |
| 29 | .140 | .148 | .160 | .174 | .206 |
| 30 | .133 | .144 | .153 | .166 | .212 |

Table XI. Shape Parameter Equal to 2.5

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)) | | | | |
|---|---|---|---|---|---|
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .350 | .366 | .402 | .436 | .491 |
| 5 | .324 | .343 | .361 | .391 | .454 |
| 6 | .301 | .313 | .331 | .362 | .420 |
| 7 | .279 | .296 | .314 | .342 | .407 |
| 8 | .264 | .291 | .299 | .326 | .376 |
| 9 | .239 | .252 | .270 | .296 | .335 |
| 10 | .237 | .249 | .263 | .288 | .346 |
| 11 | .216 | .230 | .248 | .270 | .320 |
| 12 | .213 | .224 | .242 | .267 | .307 |
| 13 | .207 | .218 | .232 | .252 | .286 |
| 14 | .194 | .205 | .220 | .240 | .286 |
| 15 | .188 | .201 | .216 | .235 | .275 |
| 16 | .178 | .199 | .206 | .229 | .280 |
| 17 | .179 | .189 | .206 | .226 | .266 |
| 18 | .174 | .184 | .197 | .218 | .254 |
| 19 | .168 | .178 | .192 | .212 | .259 |
| 20 | .163 | .172 | .183 | .197 | .239 |
| 21 | .158 | .167 | .178 | .201 | .237 |
| 22 | .157 | .164 | .178 | .195 | .237 |
| 23 | .150 | .160 | .170 | .188 | .230 |
| 24 | .151 | .160 | .172 | .191 | .223 |
| 25 | .148 | .158 | .169 | .184 | .215 |
| 26 | .142 | .151 | .161 | .181 | .205 |
| 27 | .139 | .148 | .159 | .177 | .205 |
| 28 | .136 | .145 | .155 | .176 | .202 |
| 29 | .136 | .144 | .153 | .175 | .198 |
| 30 | .130 | .138 | .148 | .167 | .205 |

Table XII. Shape Parameter Equal to 3.0

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR C=MAX(F(X)-S(X)) | | | | |
|---|---|---|---|---|---|
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .352 | .368 | .401 | .440 | .488 |
| 5 | .322 | .338 | .355 | .387 | .459 |
| 6 | .299 | .312 | .329 | .361 | .412 |
| 7 | .277 | .291 | .315 | .341 | .393 |
| 8 | .262 | .276 | .292 | .315 | .373 |
| 9 | .243 | .257 | .273 | .302 | .369 |
| 10 | .239 | .252 | .264 | .289 | .340 |
| 11 | .217 | .229 | .247 | .269 | .324 |
| 12 | .211 | .220 | .234 | .258 | .307 |
| 13 | .202 | .212 | .229 | .252 | .289 |
| 14 | .193 | .204 | .217 | .241 | .283 |
| 15 | .186 | .200 | .213 | .238 | .275 |
| 16 | .182 | .191 | .211 | .230 | .272 |
| 17 | .175 | .187 | .200 | .219 | .265 |
| 18 | .172 | .180 | .196 | .218 | .254 |
| 19 | .165 | .174 | .190 | .209 | .243 |
| 20 | .161 | .171 | .181 | .202 | .231 |
| 21 | .160 | .157 | .180 | .195 | .229 |
| 22 | .157 | .166 | .181 | .200 | .237 |
| 23 | .150 | .161 | .171 | .185 | .217 |
| 24 | .146 | .153 | .165 | .181 | .216 |
| 25 | .143 | .152 | .162 | .180 | .223 |
| 26 | .139 | .146 | .155 | .179 | .215 |
| 27 | .139 | .150 | .159 | .174 | .206 |
| 28 | .138 | .146 | .157 | .173 | .211 |
| 29 | .133 | .140 | .149 | .167 | .212 |
| 30 | .130 | .138 | .147 | .163 | .199 |

Table XIII. Shape Parameter Equal to 3.5

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)) | | | | |
|---|---|---|---|---|---|
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .349 | .366 | .392 | .429 | .482 |
| 5 | .323 | .335 | .357 | .382 | .448 |
| 6 | .296 | .310 | .329 | .356 | .409 |
| 7 | .272 | .287 | .306 | .333 | .385 |
| 8 | .263 | .278 | .292 | .321 | .383 |
| 9 | .237 | .250 | .269 | .296 | .354 |
| 10 | .234 | .247 | .264 | .291 | .342 |
| 11 | .217 | .227 | .242 | .271 | .305 |
| 12 | .210 | .221 | .233 | .256 | .304 |
| 13 | .202 | .213 | .229 | .249 | .292 |
| 14 | .195 | .204 | .216 | .240 | .280 |
| 15 | .186 | .197 | .211 | .230 | .265 |
| 16 | .184 | .193 | .205 | .227 | .272 |
| 17 | .172 | .183 | .196 | .217 | .254 |
| 18 | .171 | .179 | .193 | .218 | .242 |
| 19 | .163 | .172 | .185 | .207 | .250 |
| 20 | .160 | .170 | .192 | .204 | .244 |
| 21 | .156 | .164 | .176 | .197 | .228 |
| 22 | .155 | .164 | .177 | .192 | .225 |
| 23 | .152 | .162 | .173 | .190 | .224 |
| 24 | .147 | .155 | .168 | .190 | .214 |
| 25 | .147 | .156 | .165 | .185 | .216 |
| 26 | .139 | .146 | .157 | .171 | .217 |
| 27 | .137 | .145 | .155 | .171 | .209 |
| 28 | .133 | .141 | .149 | .167 | .202 |
| 29 | .134 | .142 | .153 | .167 | .198 |
| 30 | .132 | .141 | .150 | .164 | .192 |

Table XIV. Shape Parameter Equal to 4.0

# Appendix B

## Tables for the Weibull Distribution

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)) | | | | |
| --- | --- | --- | --- | --- | --- |
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .359 | .380 | .410 | .449 | .546 |
| 5 | .337 | .352 | .368 | .402 | .450 |
| 6 | .316 | .333 | .350 | .383 | .453 |
| 7 | .290 | .309 | .326 | .356 | .434 |
| 8 | .280 | .297 | .314 | .340 | .410 |
| 9 | .269 | .279 | .296 | .323 | .378 |
| 10 | .259 | .270 | .289 | .319 | .377 |
| 11 | .245 | .261 | .282 | .304 | .349 |
| 12 | .236 | .252 | .267 | .290 | .354 |
| 13 | .230 | .241 | .256 | .283 | .340 |
| 14 | .222 | .236 | .250 | .274 | .317 |
| 15 | .210 | .222 | .239 | .265 | .321 |
| 16 | .202 | .214 | .229 | .253 | .295 |
| 17 | .199 | .212 | .229 | .251 | .308 |
| 18 | .194 | .203 | .217 | .235 | .284 |
| 19 | .193 | .203 | .215 | .240 | .286 |
| 20 | .188 | .197 | .209 | .229 | .268 |
| 21 | .181 | .191 | .208 | .225 | .266 |
| 22 | .177 | .187 | .197 | .217 | .253 |
| 23 | .177 | .189 | .199 | .219 | .258 |
| 24 | .172 | .182 | .192 | .214 | .250 |
| 25 | .171 | .181 | .197 | .215 | .259 |
| 26 | .165 | .175 | .183 | .202 | .235 |
| 27 | .162 | .170 | .184 | .203 | .242 |
| 28 | .160 | .169 | .181 | .200 | .231 |
| 29 | .156 | .164 | .174 | .193 | .236 |
| 30 | .154 | .162 | .172 | .194 | .225 |

Table XV. Shape Parameter Equal to 0.5

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)) | | | | |
|---|---|---|---|---|---|
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .339 | .359 | .399 | .435 | .519 |
| 5 | .330 | .347 | .362 | .394 | .449 |
| 6 | .311 | .329 | .347 | .383 | .456 |
| 7 | .285 | .301 | .326 | .357 | .421 |
| 8 | .277 | .293 | .311 | .339 | .413 |
| 9 | .265 | .280 | .296 | .321 | .390 |
| 10 | .257 | .270 | .289 | .313 | .381 |
| 11 | .247 | .261 | .279 | .305 | .346 |
| 12 | .236 | .251 | .268 | .293 | .354 |
| 13 | .229 | .239 | .251 | .277 | .337 |
| 14 | .224 | .235 | .249 | .274 | .319 |
| 15 | .208 | .220 | .239 | .265 | .316 |
| 16 | .201 | .212 | .229 | .252 | .303 |
| 17 | .197 | .209 | .226 | .249 | .304 |
| 18 | .191 | .199 | .211 | .229 | .281 |
| 19 | .193 | .203 | .216 | .237 | .290 |
| 20 | .185 | .196 | .208 | .229 | .267 |
| 21 | .180 | .190 | .206 | .223 | .266 |
| 22 | .176 | .185 | .197 | .214 | .249 |
| 23 | .173 | .185 | .197 | .215 | .250 |
| 24 | .171 | .180 | .192 | .210 | .248 |
| 25 | .171 | .179 | .191 | .214 | .256 |
| 26 | .163 | .175 | .185 | .199 | .236 |
| 27 | .161 | .169 | .183 | .203 | .239 |
| 28 | .159 | .169 | .180 | .200 | .234 |
| 29 | .156 | .163 | .173 | .193 | .230 |
| 30 | .154 | .161 | .175 | .193 | .222 |

Table XVI. Shape Parameter Equal to 1.0

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)) | | | | |
|---|---|---|---|---|---|
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .379 | .392 | .417 | .456 | .552 |
| 5 | .341 | .356 | .382 | .421 | .458 |
| 6 | .329 | .345 | .361 | .389 | .473 |
| 7 | .299 | .315 | .336 | .367 | .432 |
| 8 | .285 | .30[illegible] | .318 | .349 | .413 |
| 9 | .279 | .284 | .301 | .326 | .395 |
| 10 | .252 | .274 | .293 | .319 | .377 |
| 11 | .251 | .264 | .283 | .311 | .345 |
| 12 | .237 | .254 | .274 | .298 | .359 |
| 13 | .229 | .239 | .251 | .281 | .329 |
| 14 | .225 | .235 | .248 | .274 | .326 |
| 15 | .211 | .222 | .239 | .259 | .316 |
| 16 | .203 | .213 | .230 | .253 | .301 |
| 17 | .200 | .212 | .227 | .249 | .302 |
| 18 | .191 | .20[illegible] | .212 | .231 | .269 |
| 19 | .194 | .203 | .215 | .239 | .286 |
| 20 | .185 | .196 | .208 | .229 | .269 |
| 21 | .181 | .189 | .207 | .225 | .265 |
| 22 | .176 | .186 | .196 | .214 | .247 |
| 23 | .174 | .184 | .195 | .214 | .250 |
| 24 | .172 | .182 | .194 | .210 | .249 |
| 25 | .169 | .177 | .190 | .218 | .253 |
| 26 | .165 | .174 | .184 | .202 | .237 |
| 27 | .162 | .170 | .185 | .201 | .2[illegible]1 |
| 28 | .159 | .171 | .182 | .202 | .235 |
| 29 | .155 | .163 | .173 | .186 | .233 |
| 30 | .154 | .162 | .174 | .193 | .226 |

Table XVII. Shape Parameter Equal to 1.5

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)) | | | | |
|---|---|---|---|---|---|
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .364 | .379 | .411 | .450 | .522 |
| 5 | .335 | .349 | .370 | .404 | .456 |
| 6 | .319 | .332 | .351 | .379 | .457 |
| 7 | .287 | .307 | .327 | .359 | .420 |
| 8 | .277 | .295 | .308 | .340 | .403 |
| 9 | .264 | .277 | .293 | .320 | .362 |
| 10 | .255 | .263 | .287 | .310 | .374 |
| 11 | .245 | .256 | .277 | .299 | .344 |
| 12 | .233 | .245 | .266 | .291 | .349 |
| 13 | .223 | .233 | .248 | .273 | .320 |
| 14 | .219 | .229 | .243 | .266 | .317 |
| 15 | .207 | .219 | .233 | .252 | .299 |
| 16 | .198 | .209 | .226 | .250 | .293 |
| 17 | .194 | .203 | .222 | .242 | .298 |
| 18 | .187 | .196 | .207 | .227 | .266 |
| 19 | .190 | .199 | .209 | .231 | .274 |
| 20 | .182 | .193 | .203 | .223 | .263 |
| 21 | .177 | .186 | .202 | .222 | .263 |
| 22 | .172 | .180 | .192 | .211 | .241 |
| 23 | .170 | .179 | .192 | .211 | .249 |
| 24 | .169 | .179 | .189 | .206 | .248 |
| 25 | .166 | .172 | .186 | .211 | .252 |
| 26 | .162 | .170 | .181 | .197 | .232 |
| 27 | .159 | .167 | .181 | .198 | .240 |
| 28 | .156 | .166 | .178 | .195 | .229 |
| 29 | .152 | .160 | .169 | .183 | .230 |
| 30 | .150 | .155 | .170 | .188 | .221 |

Table XVIII. Shape Parameter Equal to 2.0

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)) | | | | |
|---|---|---|---|---|---|
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .355 | .374 | .405 | .447 | .522 |
| 5 | .332 | .346 | .364 | .396 | .456 |
| 6 | .310 | .329 | .345 | .376 | .450 |
| 7 | .285 | .301 | .322 | .354 | .415 |
| 8 | .274 | .291 | .303 | .338 | .403 |
| 9 | .259 | .272 | .288 | .315 | .376 |
| 10 | .252 | .264 | .281 | .309 | .374 |
| 11 | .239 | .253 | .272 | .293 | .341 |
| 12 | .229 | .241 | .263 | .289 | .340 |
| 13 | .220 | .230 | .247 | .269 | .320 |
| 14 | .215 | .226 | .241 | .261 | .312 |
| 15 | .203 | .214 | .229 | .250 | .296 |
| 16 | .197 | .206 | .224 | .247 | .288 |
| 17 | .193 | .205 | .219 | .238 | .294 |
| 18 | .185 | .192 | .205 | .226 | .266 |
| 19 | .187 | .195 | .206 | .223 | .274 |
| 20 | .178 | .183 | .202 | .222 | .261 |
| 21 | .175 | .185 | .199 | .220 | .258 |
| 22 | .170 | .178 | .189 | .208 | .237 |
| 23 | .167 | .177 | .189 | .210 | .249 |
| 24 | .165 | .177 | .186 | .205 | .245 |
| 25 | .164 | .170 | .183 | .206 | .252 |
| 26 | .159 | .168 | .180 | .194 | .230 |
| 27 | .159 | .166 | .177 | .196 | .240 |
| 28 | .154 | .163 | .175 | .194 | .226 |
| 29 | .151 | .157 | .167 | .181 | .230 |
| 30 | .149 | .155 | .166 | .186 | .221 |

Table XIX. Shape Parameter Equal to 2.5

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)) | | | | |
|---|---|---|---|---|---|
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .352 | .370 | .404 | .446 | .522 |
| 5 | .323 | .344 | .361 | .392 | .455 |
| 6 | .307 | .324 | .344 | .372 | .442 |
| 7 | .281 | .297 | .319 | .351 | .414 |
| 8 | .272 | .287 | .301 | .337 | .402 |
| 9 | .257 | .271 | .286 | .312 | .371 |
| 10 | .243 | .262 | .279 | .308 | .374 |
| 11 | .236 | .252 | .268 | .291 | .340 |
| 12 | .227 | .237 | .260 | .288 | .337 |
| 13 | .217 | .227 | .244 | .266 | .320 |
| 14 | .213 | .224 | .238 | .260 | .304 |
| 15 | .201 | .211 | .226 | .248 | .256 |
| 16 | .195 | .204 | .221 | .243 | .262 |
| 17 | .191 | .203 | .217 | .237 | .292 |
| 18 | .183 | .191 | .204 | .223 | .266 |
| 19 | .166 | .194 | .205 | .221 | .274 |
| 20 | .175 | .188 | .200 | .221 | .260 |
| 21 | .174 | .183 | .197 | .219 | .258 |
| 22 | .169 | .177 | .188 | .207 | .236 |
| 23 | .165 | .176 | .188 | .207 | .249 |
| 24 | .164 | .175 | .185 | .202 | .241 |
| 25 | .162 | .169 | .181 | .205 | .252 |
| 26 | .158 | .167 | .178 | .192 | .230 |
| 27 | .157 | .165 | .175 | .194 | .237 |
| 28 | .153 | .161 | .174 | .193 | .225 |
| 29 | .149 | .156 | .166 | .180 | .230 |
| 30 | .147 | .155 | .164 | .133 | .213 |

Table XX. Shape Parameter Equal to 3.0

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)) .20 | .15 | .10 | .05 | .01 |
|---|---|---|---|---|---|
| 4 | .343 | .369 | .402 | .445 | .522 |
| 5 | .329 | .342 | .356 | .387 | .454 |
| 6 | .305 | .322 | .343 | .370 | .436 |
| 7 | .281 | .295 | .319 | .347 | .414 |
| 8 | .263 | .265 | .300 | .337 | .402 |
| 9 | .255 | .269 | .286 | .319 | .367 |
| 10 | .245 | .261 | .278 | .307 | .374 |
| 11 | .234 | .249 | .266 | .288 | .337 |
| 12 | .224 | .236 | .261 | .287 | .337 |
| 13 | .215 | .226 | .242 | .264 | .320 |
| 14 | .211 | .222 | .237 | .260 | .238 |
| 15 | .199 | .211 | .224 | .245 | .296 |
| 16 | .193 | .203 | .219 | .240 | .261 |
| 17 | .183 | .201 | .216 | .236 | .292 |
| 18 | .192 | .189 | .214 | .222 | .263 |
| 19 | .184 | .192 | .203 | .218 | .274 |
| 20 | .174 | .187 | .199 | .221 | .250 |
| 21 | .173 | .181 | .196 | .217 | .258 |
| 22 | .167 | .176 | .188 | .205 | .236 |
| 23 | .165 | .175 | .188 | .206 | .245 |
| 24 | .154 | .173 | .183 | .201 | .241 |
| 25 | .161 | .169 | .186 | .203 | .252 |
| 26 | .157 | .166 | .178 | .192 | .233 |
| 27 | .155 | .165 | .174 | .194 | .237 |
| 28 | .151 | .160 | .173 | .192 | .225 |
| 29 | .147 | .155 | .163 | .179 | .230 |
| 30 | .147 | .154 | .163 | .182 | .214 |

Table XXI. Shape Parameter Equal to 3.5

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR E=MAX(F(X)-S(X)) | | | | |
|---|---|---|---|---|---|
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .347 | .358 | .401 | .439 | .522 |
| 5 | .326 | .341 | .355 | .386 | .454 |
| 6 | .304 | .320 | .342 | .369 | .431 |
| 7 | .279 | .294 | .316 | .344 | .414 |
| 8 | .268 | .283 | .300 | .335 | .402 |
| 9 | .255 | .268 | .285 | .310 | .367 |
| 10 | .245 | .258 | .277 | .306 | .374 |
| 11 | .233 | .248 | .265 | .287 | .337 |
| 12 | .223 | .234 | .259 | .285 | .337 |
| 13 | .215 | .226 | .240 | .262 | .320 |
| 14 | .209 | .221 | .236 | .258 | .298 |
| 15 | .198 | .208 | .223 | .245 | .296 |
| 16 | .191 | .202 | .218 | .239 | .280 |
| 17 | .189 | .201 | .213 | .236 | .290 |
| 18 | .181 | .188 | .204 | .220 | .260 |
| 19 | .183 | .192 | .202 | .218 | .274 |
| 20 | .173 | .185 | .198 | .221 | .260 |
| 21 | .172 | .181 | .195 | .216 | .258 |
| 22 | .166 | .176 | .186 | .202 | .236 |
| 23 | .164 | .174 | .187 | .204 | .244 |
| 24 | .163 | .172 | .183 | .200 | .239 |
| 25 | .160 | .168 | .179 | .201 | .252 |
| 26 | .157 | .166 | .177 | .191 | .229 |
| 27 | .155 | .164 | .173 | .192 | .235 |
| 28 | .150 | .160 | .173 | .191 | .225 |
| 29 | .147 | .154 | .162 | .179 | .227 |
| 30 | .146 | .152 | .162 | .182 | .214 |

Table XXII. Shape Parameter Equal to 4.0

| SAMPLE SIZE N | LEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)) | | | | |
|---|---|---|---|---|---|
| | .20 | .15 | .10 | .05 | .01 |
| 4 | .343 | .364 | .393 | .430 | .496 |
| 5 | .330 | .346 | .366 | .395 | .472 |
| 6 | .310 | .325 | .348 | .383 | .444 |
| 7 | .289 | .306 | .330 | .363 | .418 |
| 8 | .276 | .290 | .310 | .343 | .411 |
| 9 | .261 | .275 | .292 | .320 | .375 |
| 10 | .252 | .266 | .283 | .309 | .365 |
| 11 | .241 | .253 | .271 | .301 | .352 |
| 12 | .234 | .246 | .262 | .291 | .342 |
| 13 | .225 | .235 | .252 | .277 | .324 |
| 14 | .220 | .231 | .246 | .271 | .322 |
| 15 | .210 | .221 | .236 | .263 | .314 |
| 16 | .207 | .218 | .233 | .253 | .303 |
| 17 | .200 | .212 | .226 | .248 | .294 |
| 18 | .195 | .207 | .220 | .242 | .268 |
| 19 | .191 | .201 | .215 | .238 | .262 |
| 20 | .187 | .197 | .211 | .231 | .275 |
| 21 | .181 | .191 | .204 | .227 | .272 |
| 22 | .178 | .187 | .200 | .219 | .256 |
| 23 | .174 | .183 | .195 | .214 | .252 |
| 24 | .171 | .181 | .193 | .211 | .251 |
| 25 | .167 | .177 | .189 | .210 | .251 |
| 26 | .167 | .176 | .187 | .207 | .243 |
| 27 | .162 | .171 | .183 | .200 | .238 |
| 28 | .157 | .166 | .178 | .197 | .238 |
| 29 | .158 | .164 | .175 | .194 | .228 |
| 30 | .154 | .163 | .173 | .190 | .228 |

Table XXIII. Shape Parameter Equal to 1.0 (5,000)

Appendix C

Plots for the Relationship between
Shape Parameters and Critical Values

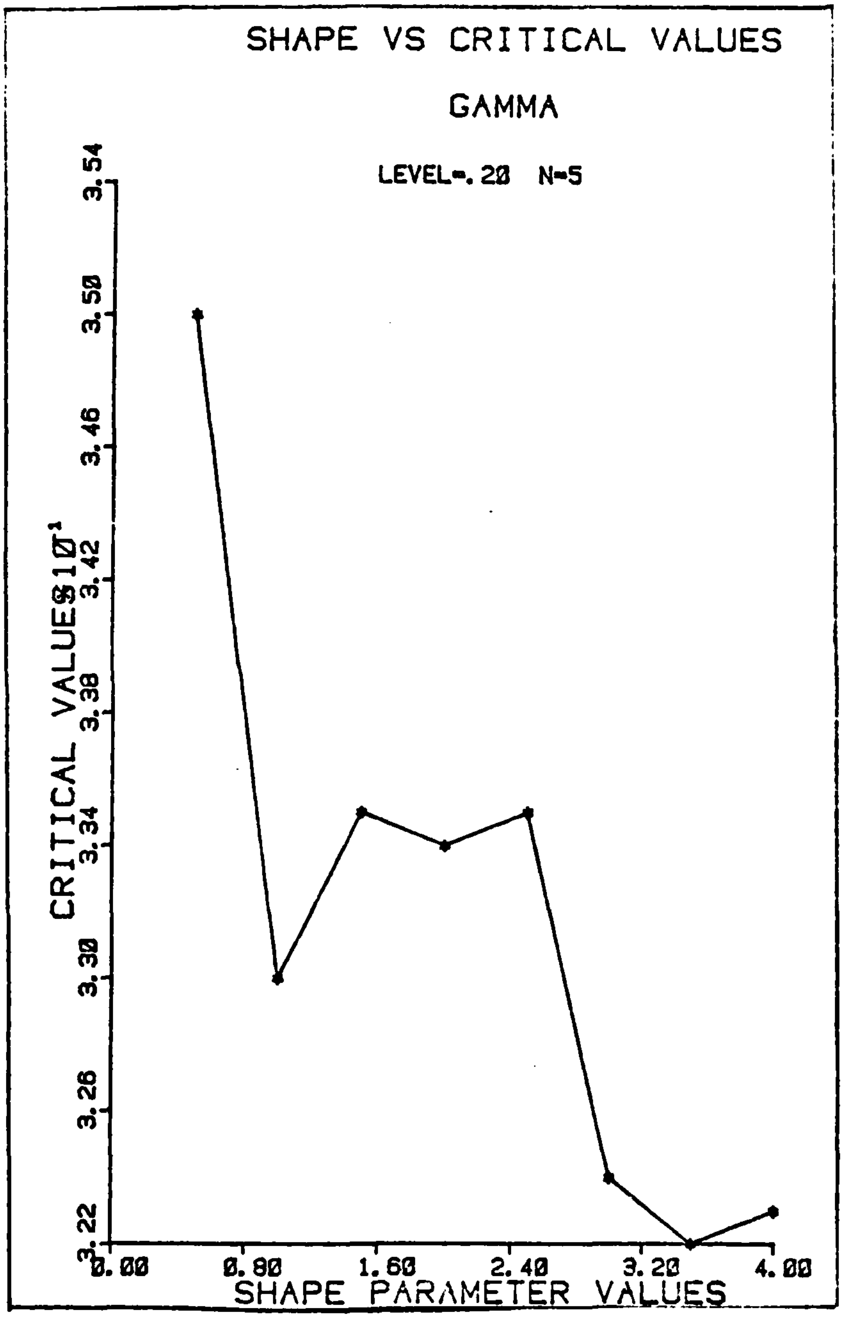


Figure 3. Gamma Level=.20 N=5

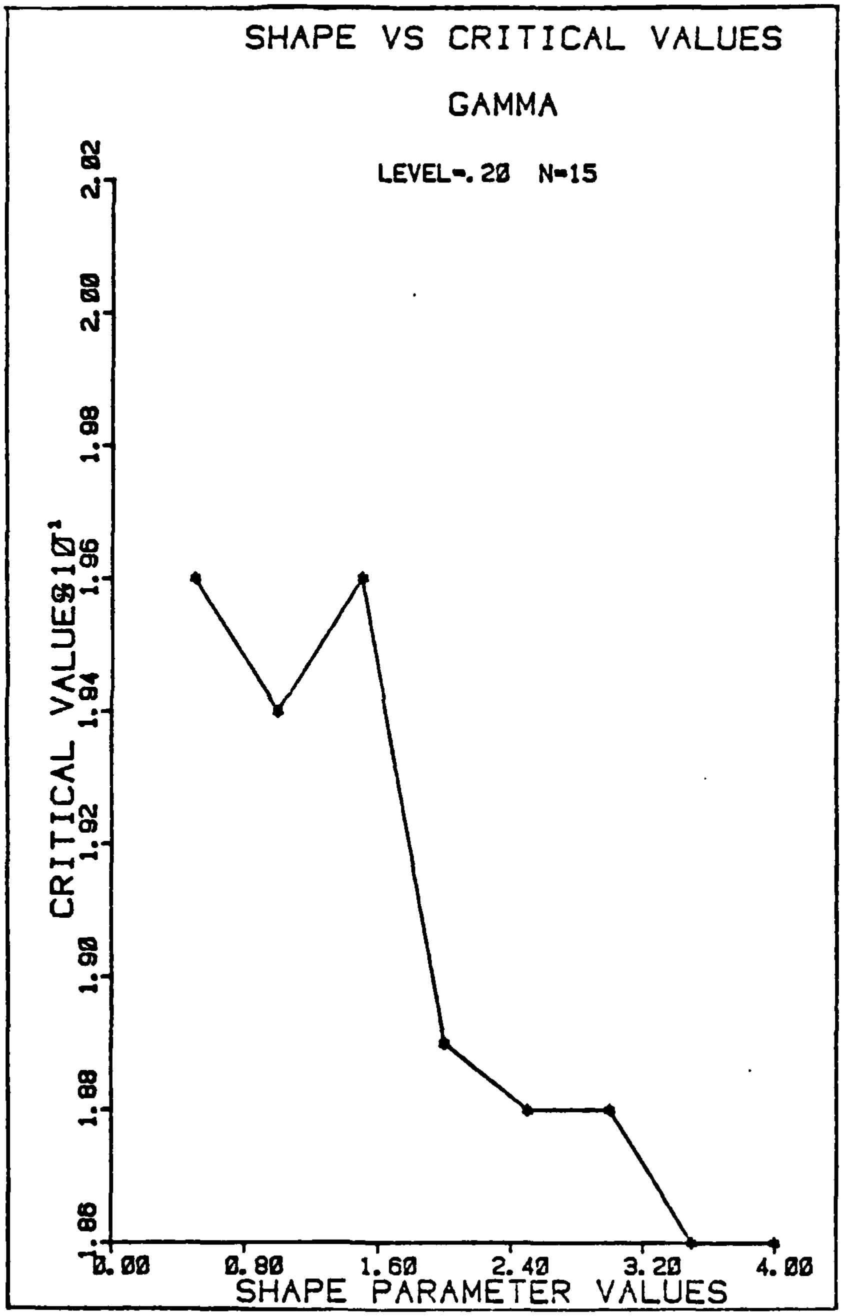


Figure 4. Gamma Level=.20 N=15

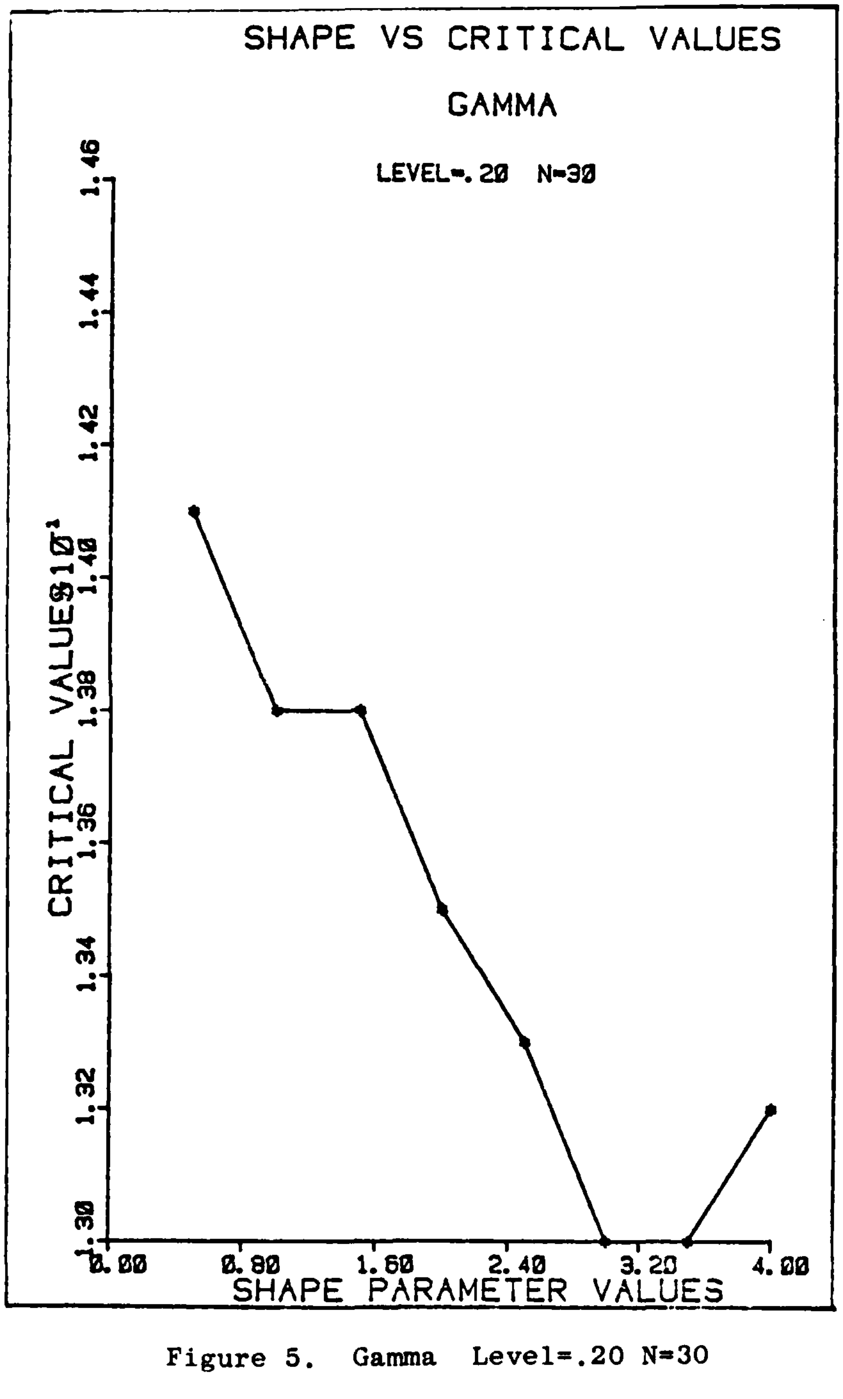


Figure 5. Gamma Level=.20 N=30

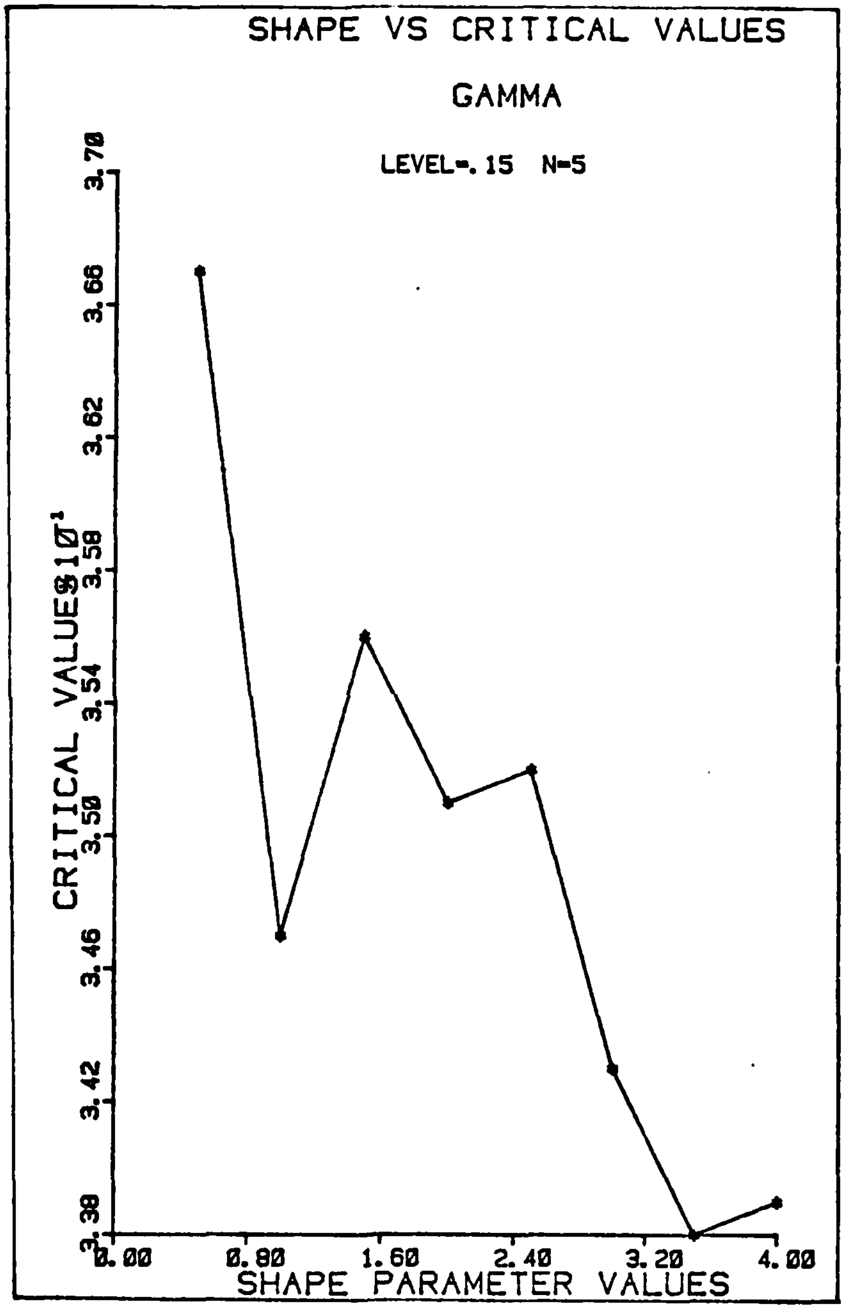


Figure 6. Gamma Level=.15 N=5

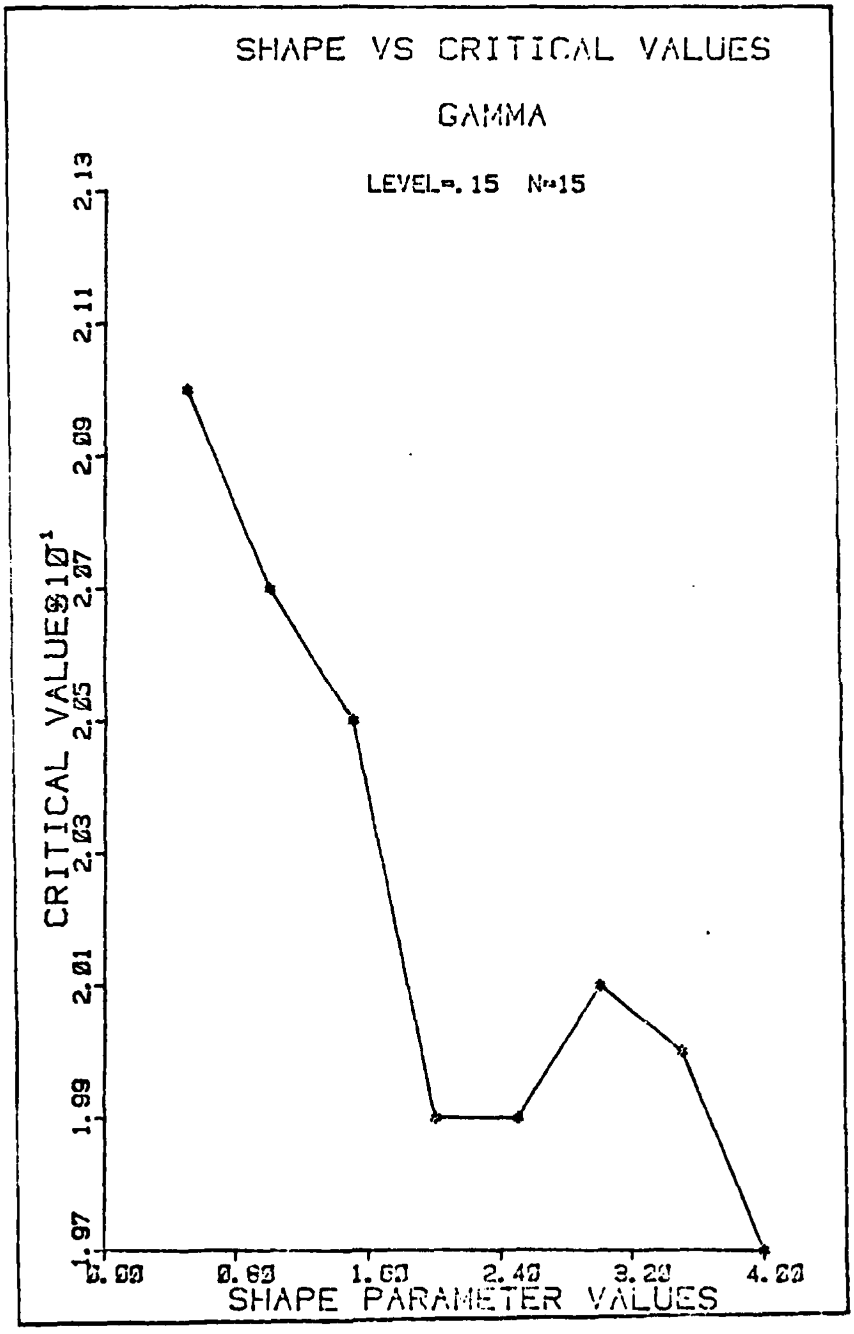


Figure 7. Gamma Level=.15 N=15

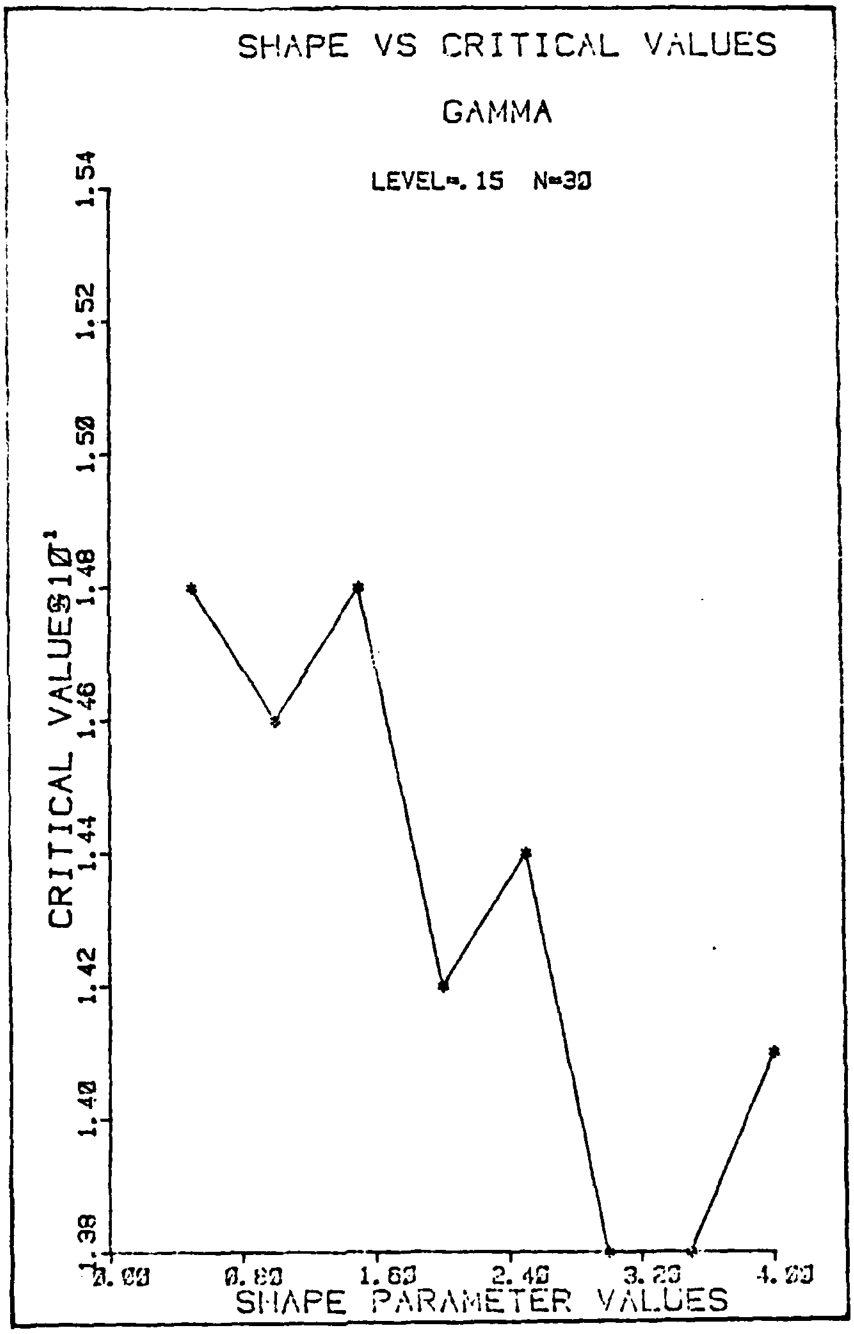


Figure 8. Gamma Level=.15 N=30

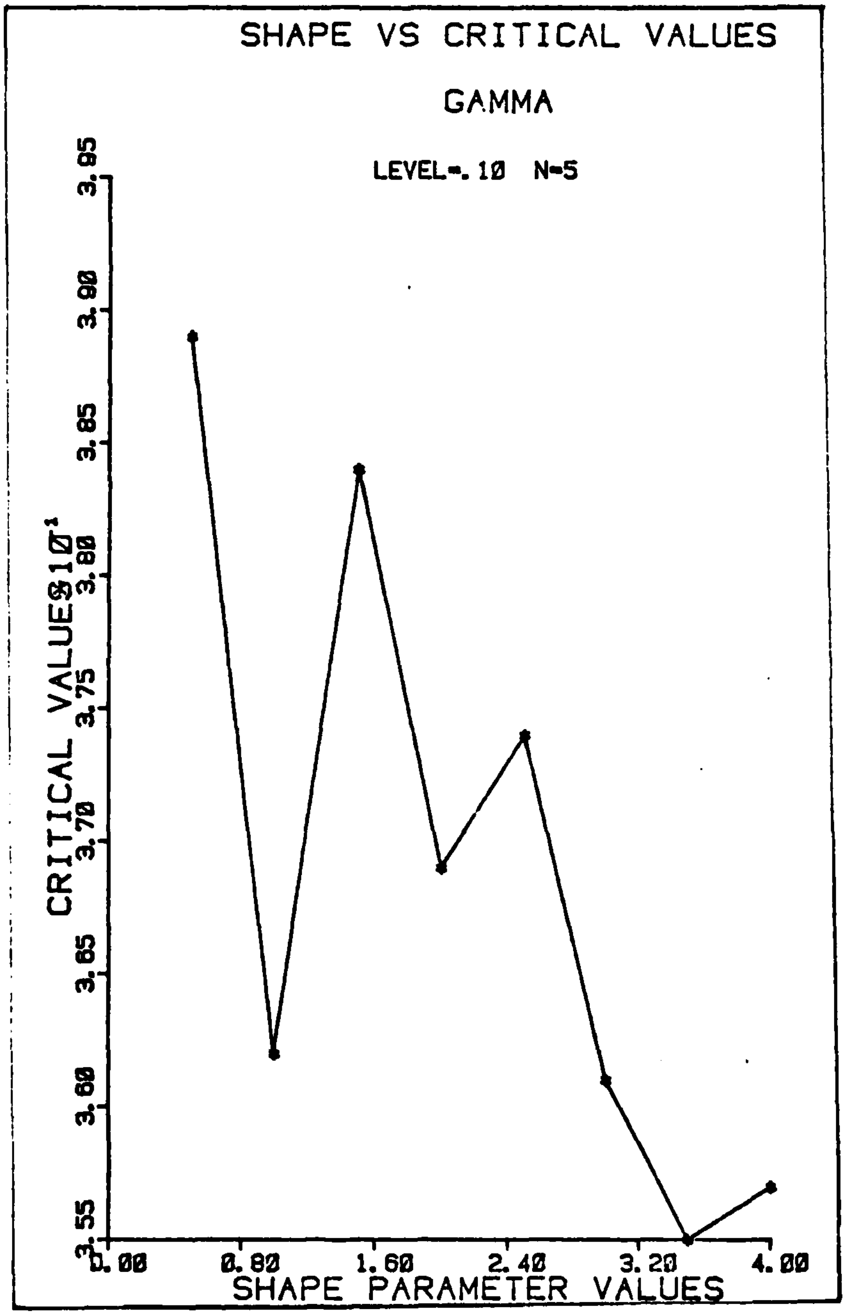


Figure 9. Gamma Level=.10 N=5

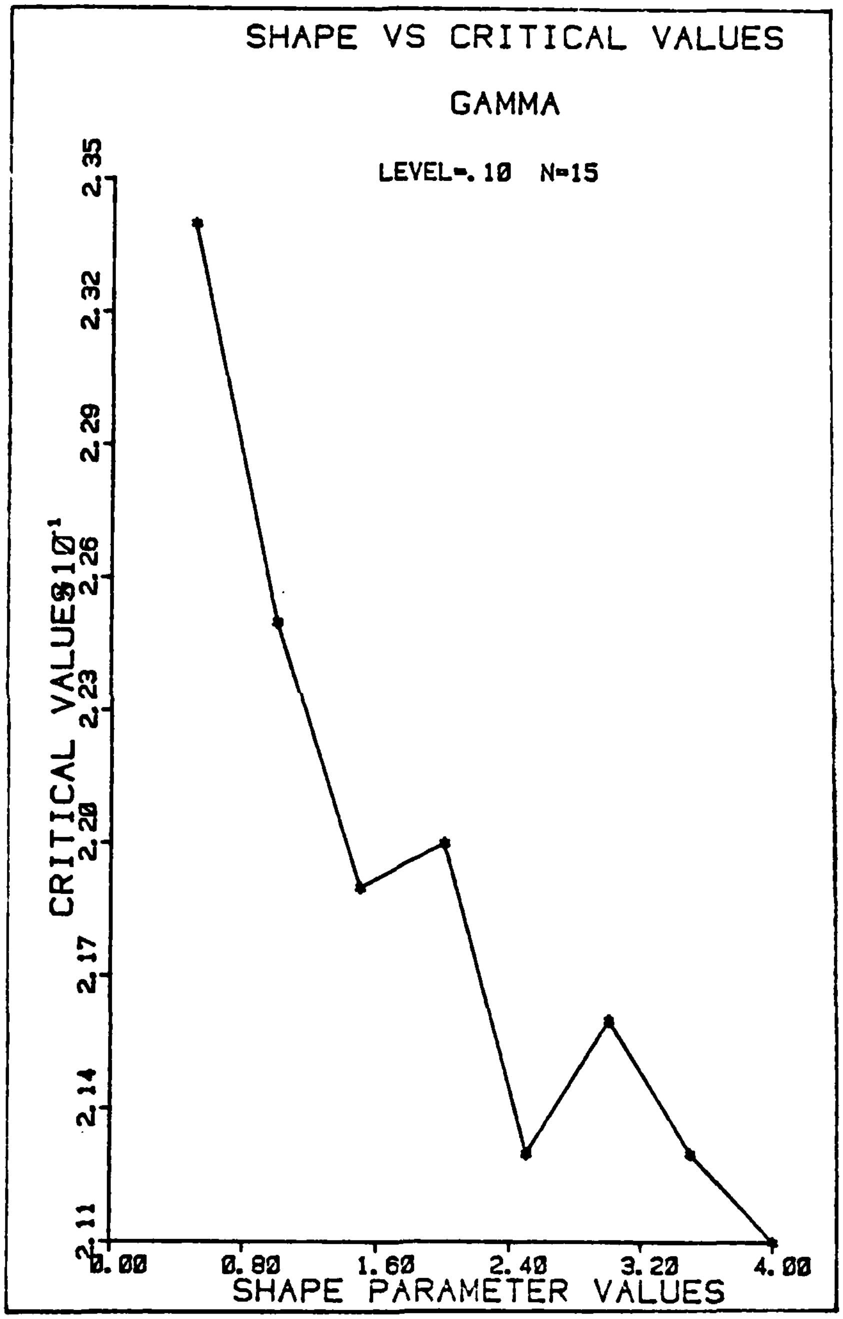


Figure 10. Gamma Level=.10 N=15

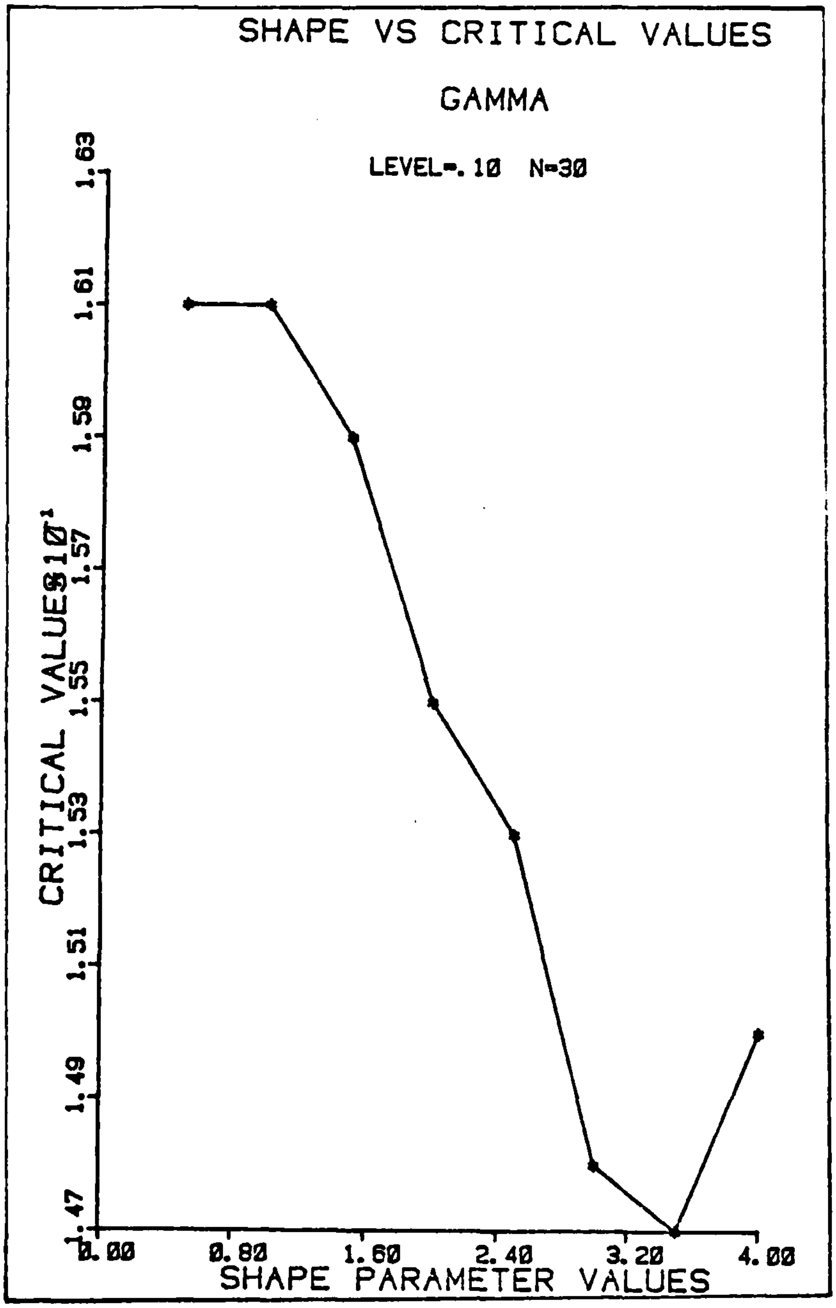


Figure 11. Gamma Level=.10 N=30

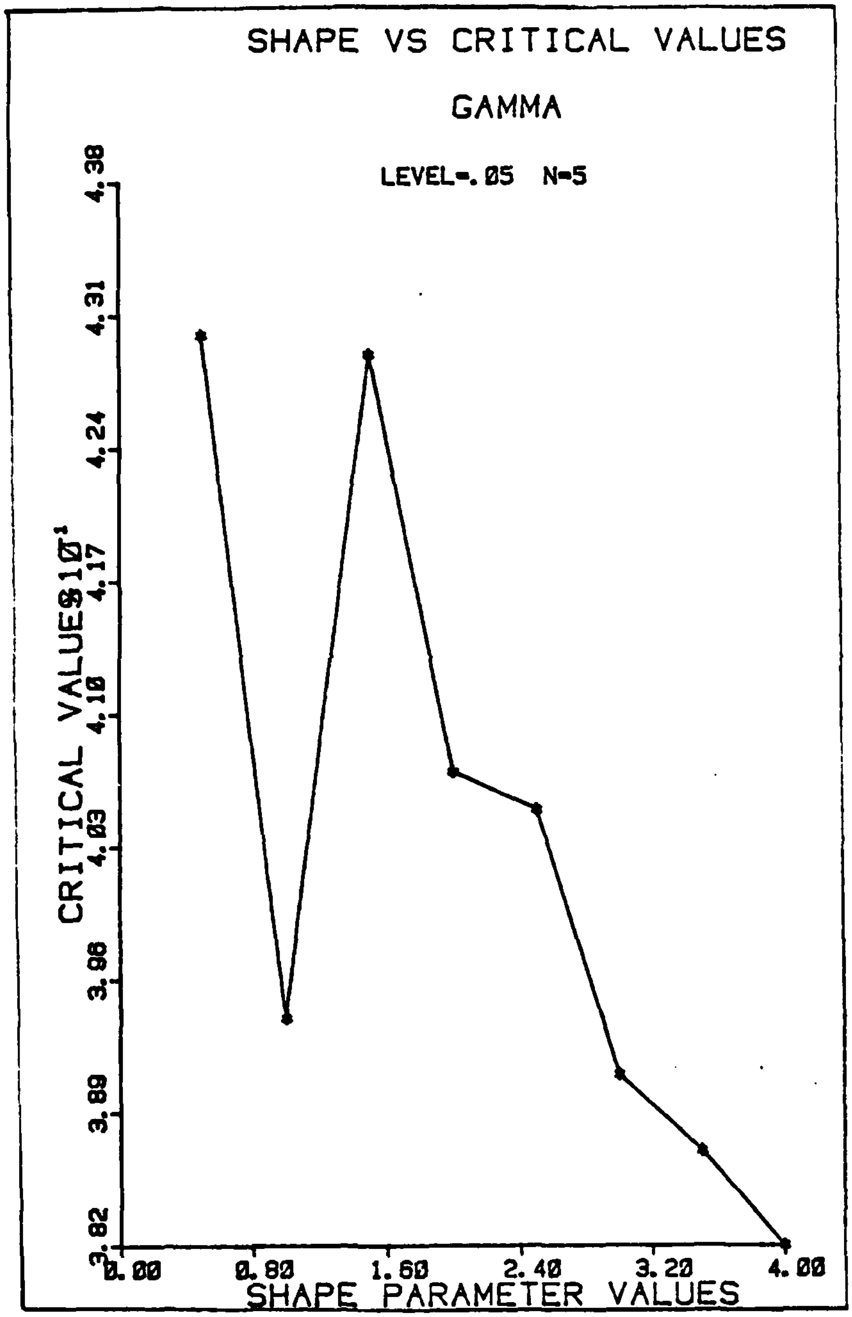


Figure 12. Gamma Level=.05 N=5

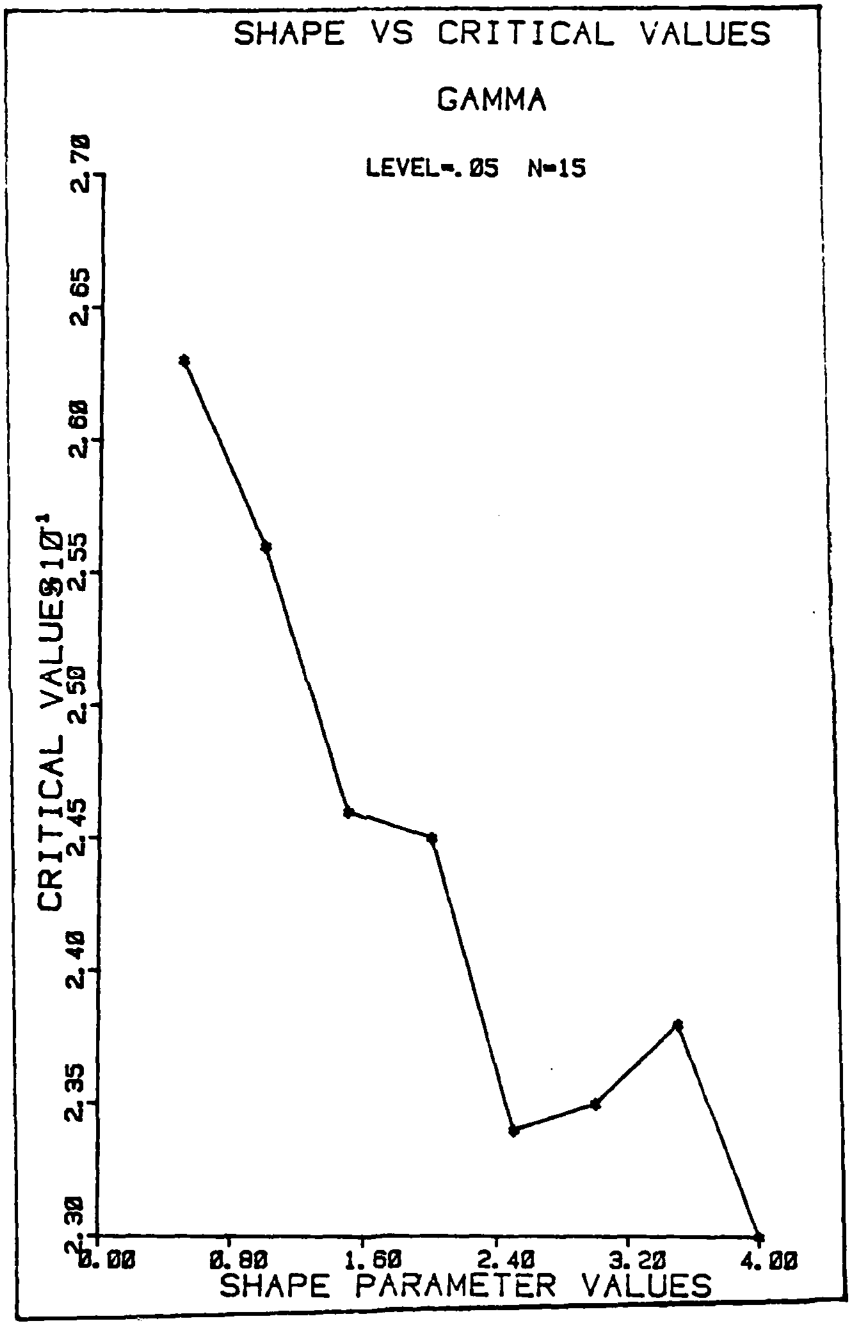


Figure 13. Gamma Level=.05 N=15

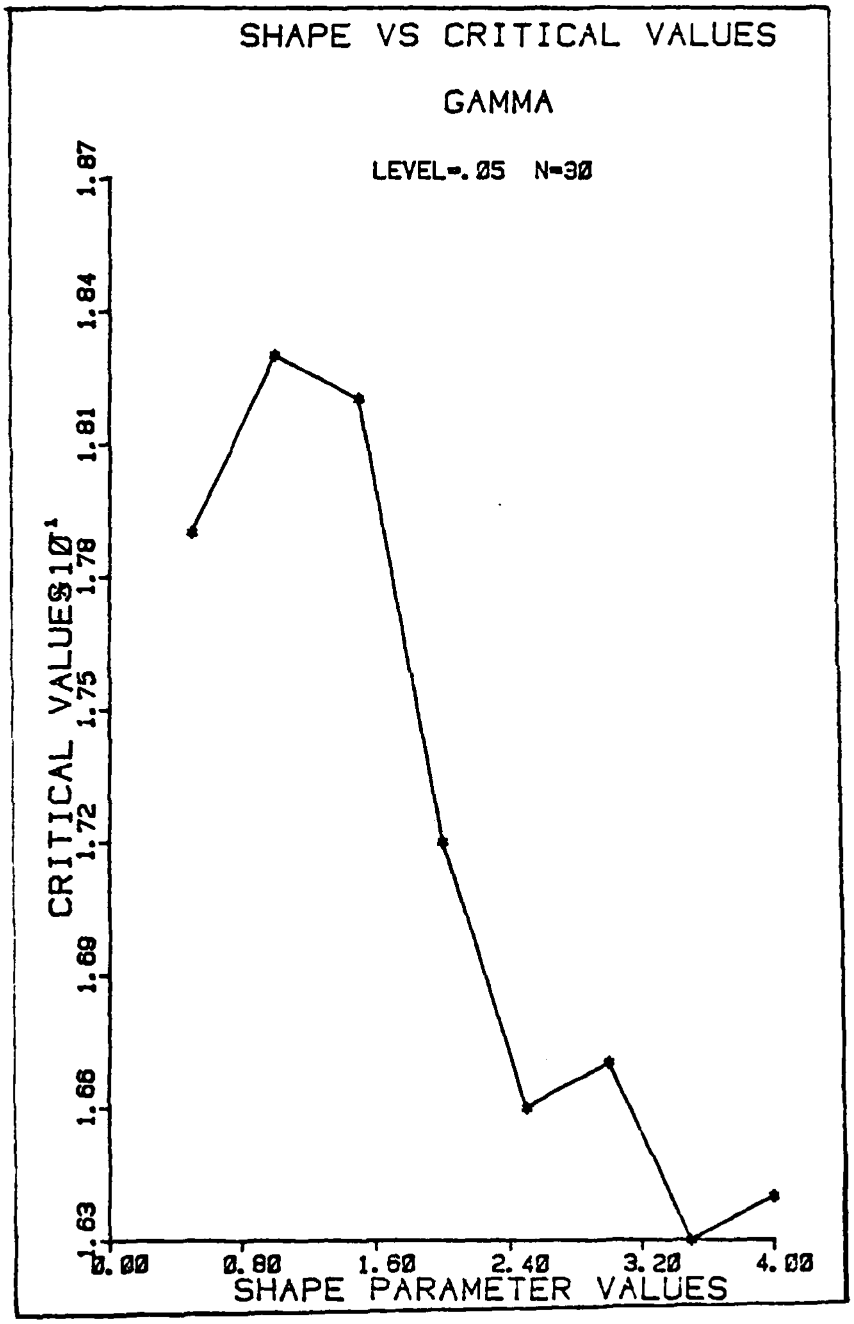


Figure 14. Gamma Level=.05 N=30

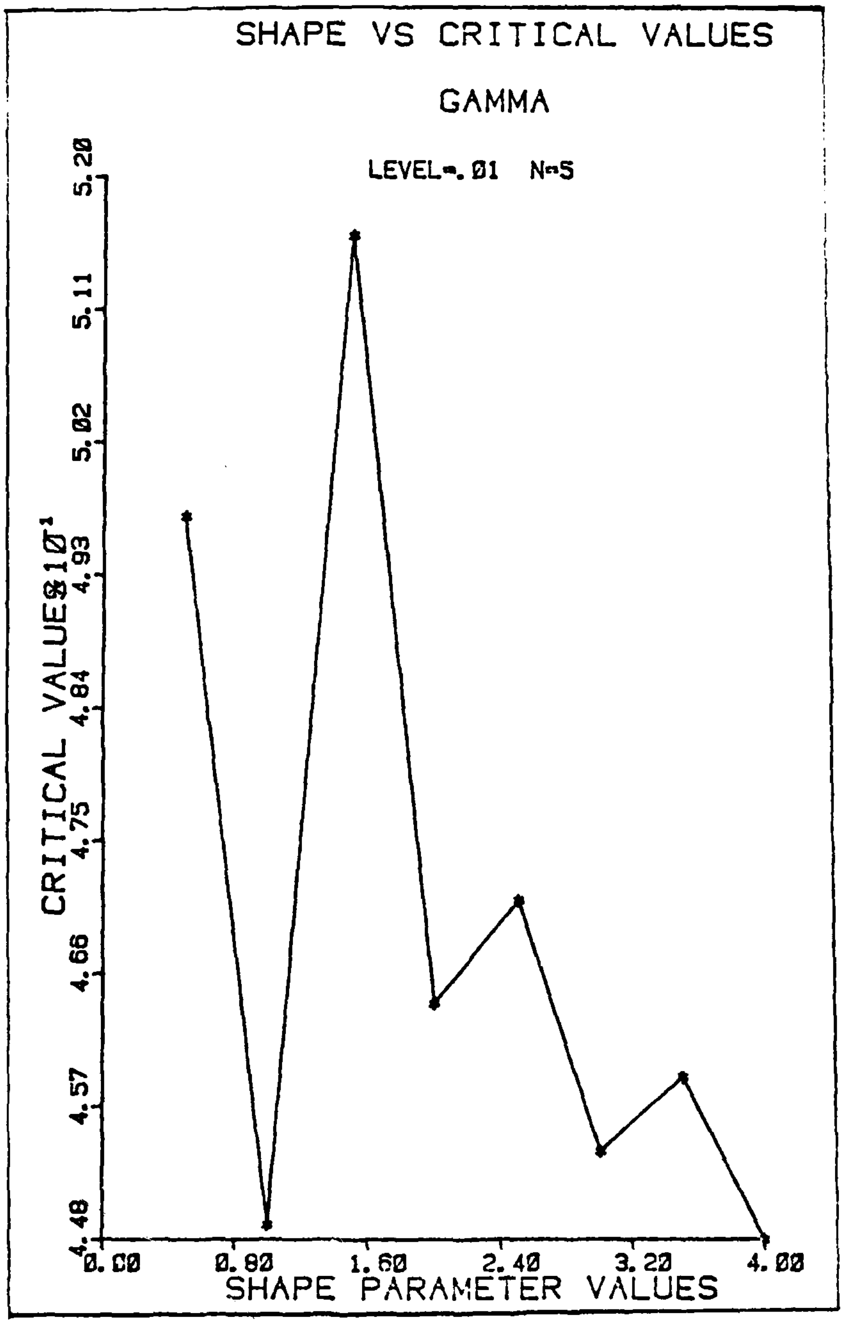


Figure 15. Gamma Level=.01 N=5

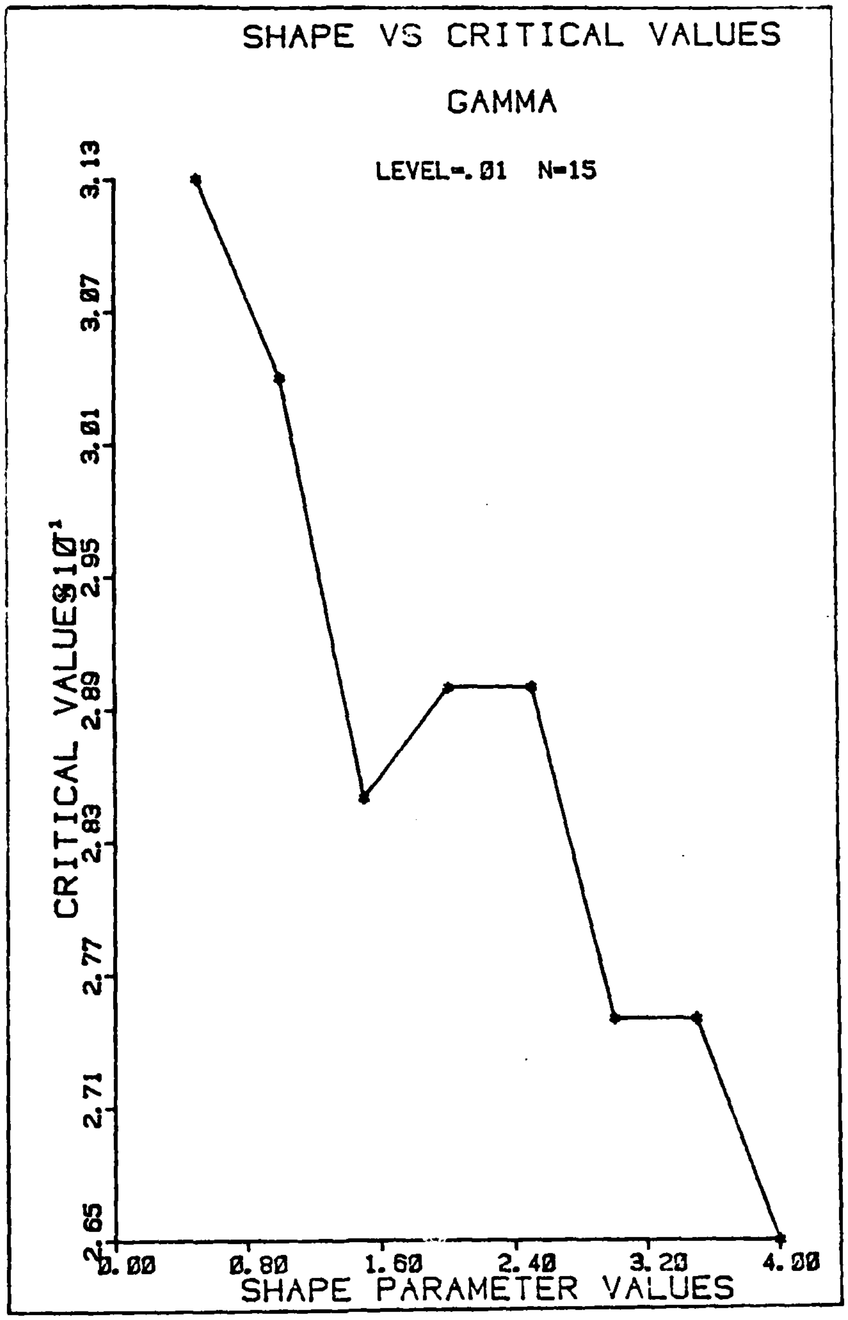


Figure 16. Gamma Level=.01 N=15

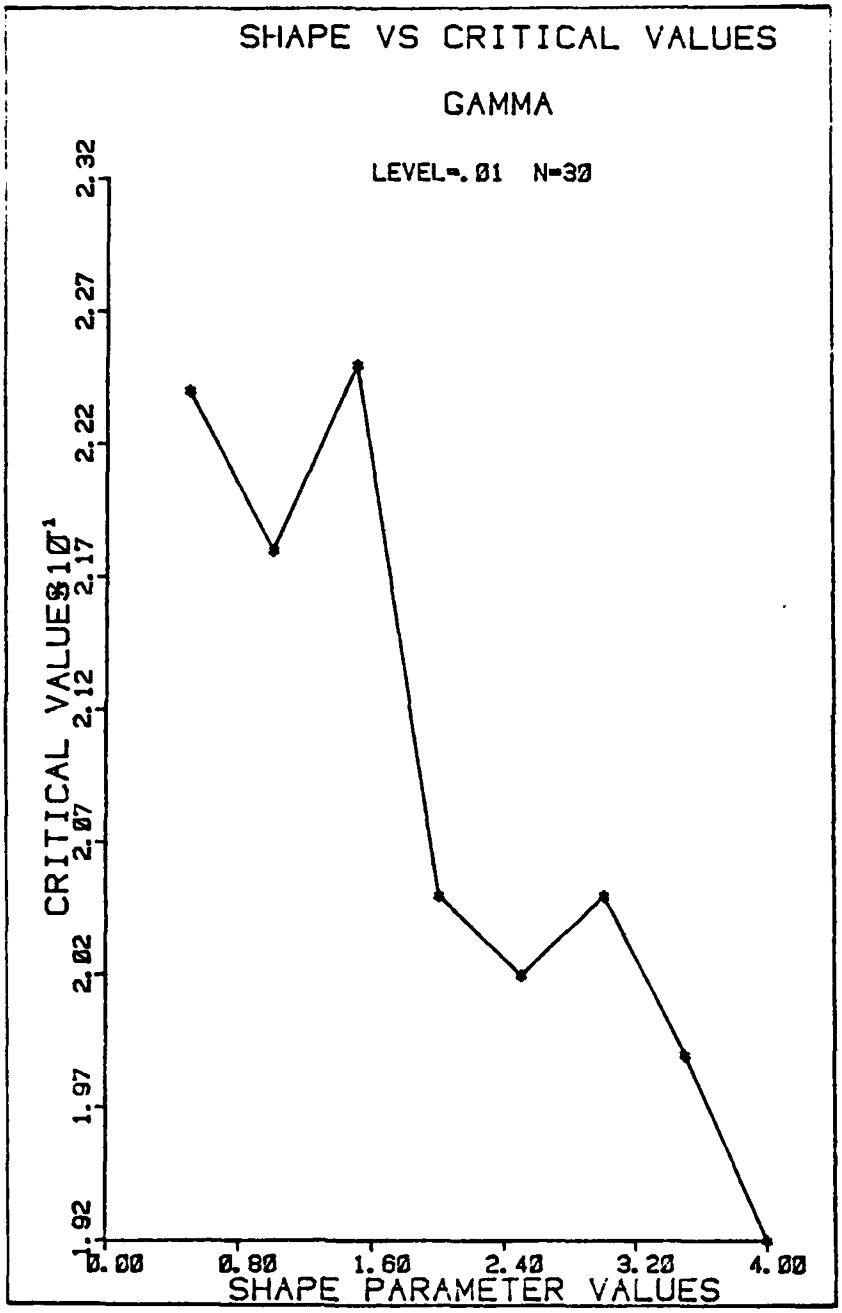


Figure 17. Gamma Level=.01 N=30

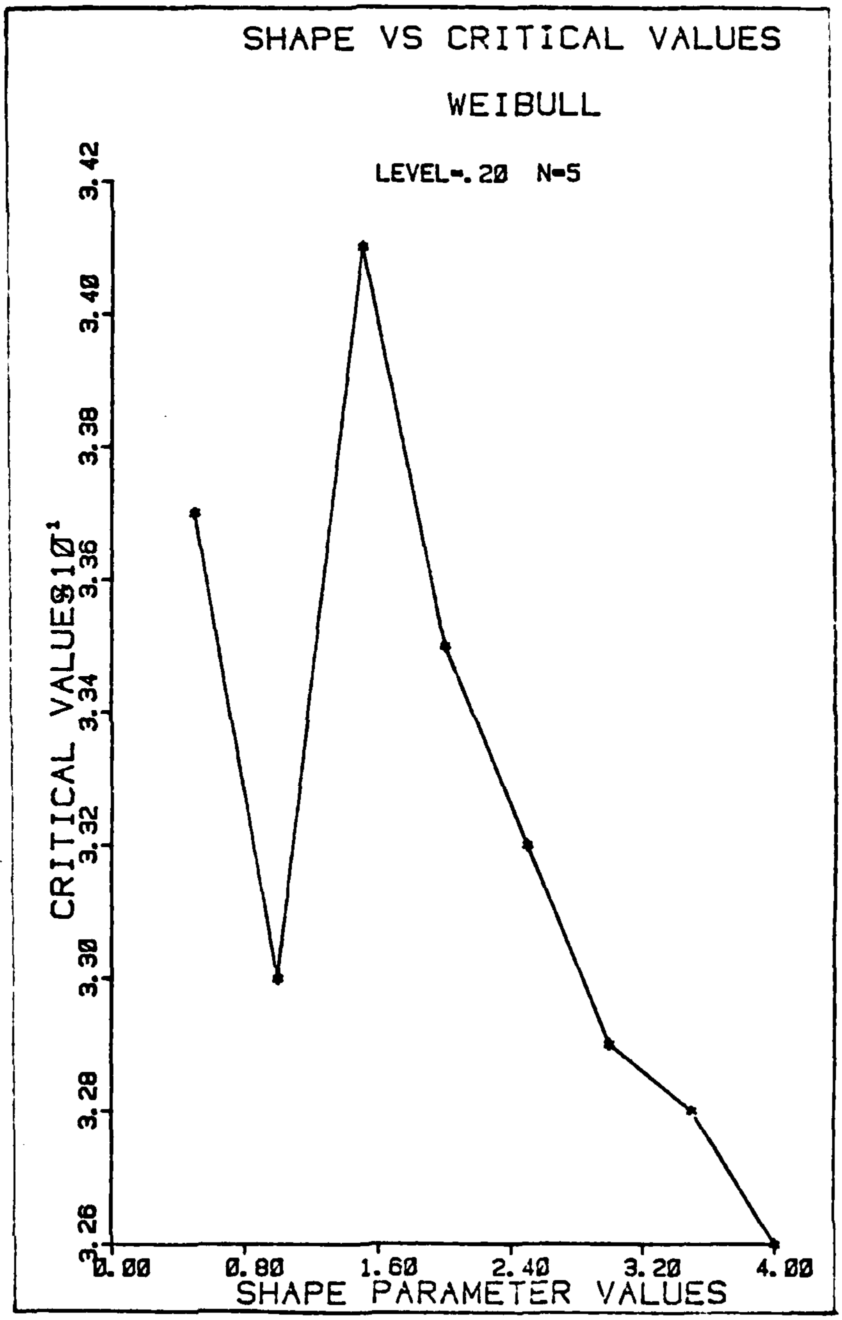


Figure 18. Weibull Level=.20 N=5

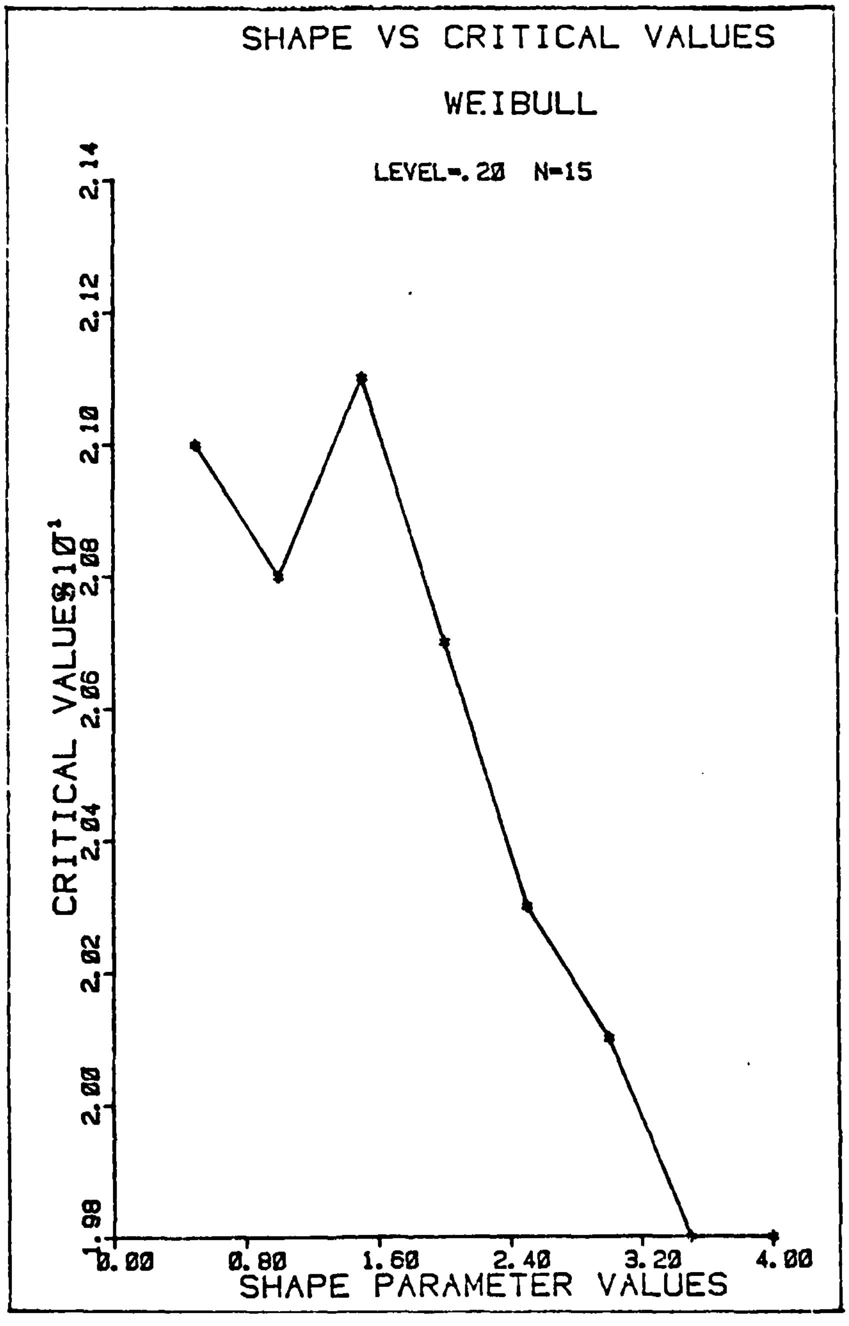


Figure 19. Weibull Level=.20 N=15

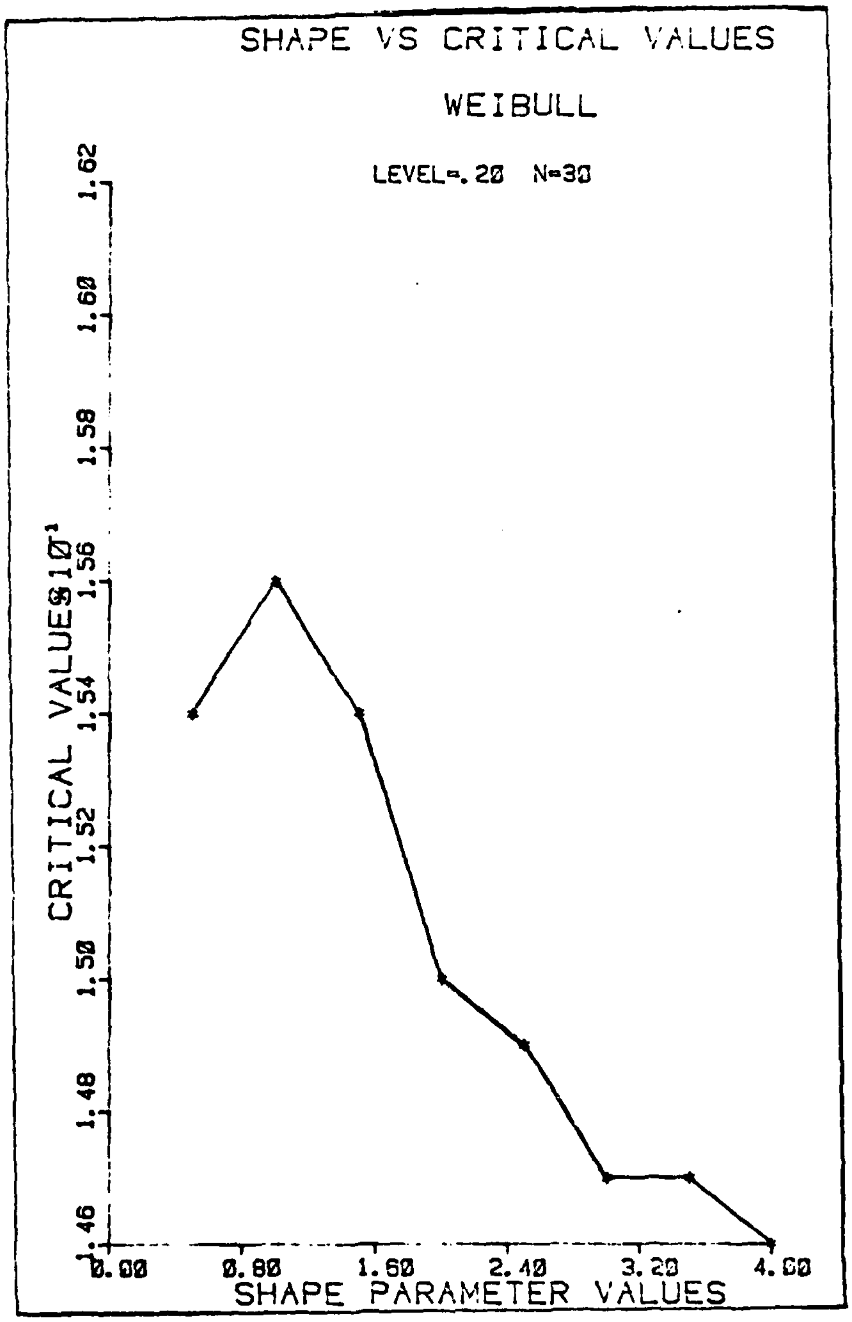


Figure 20. Weibull Level=.20 N=30

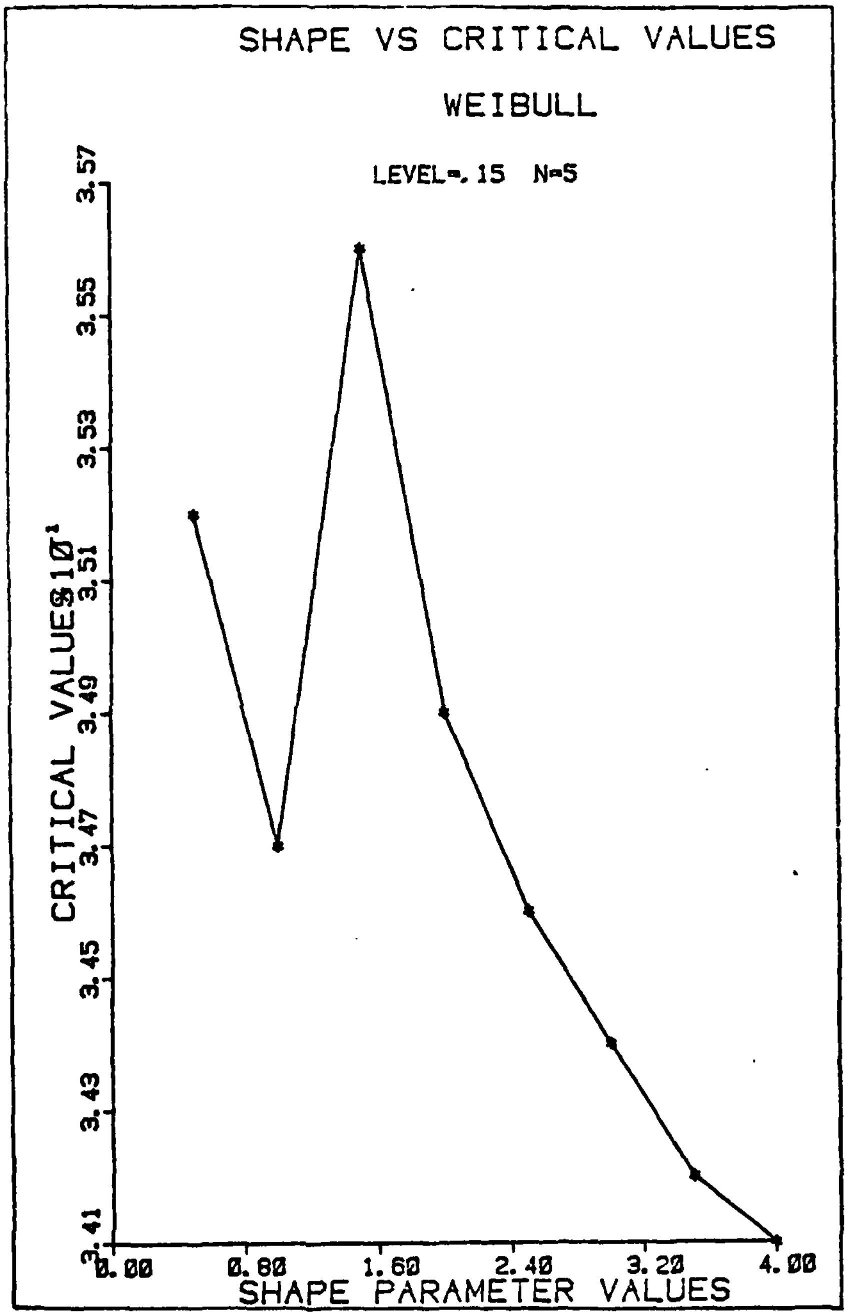


Figure 21. Weibull Level=.15 N=5

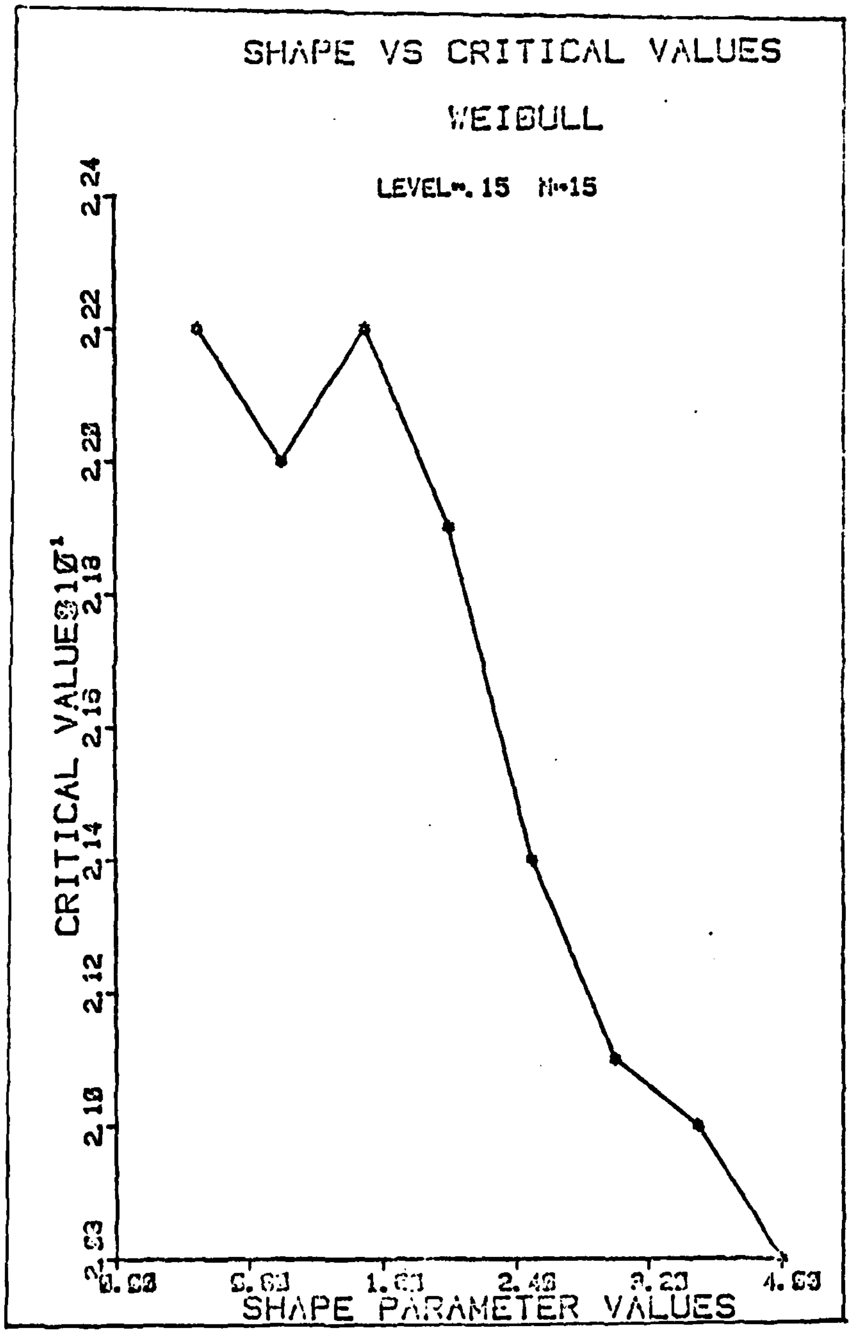


Figure 22. Weibull Level=.15 N=15

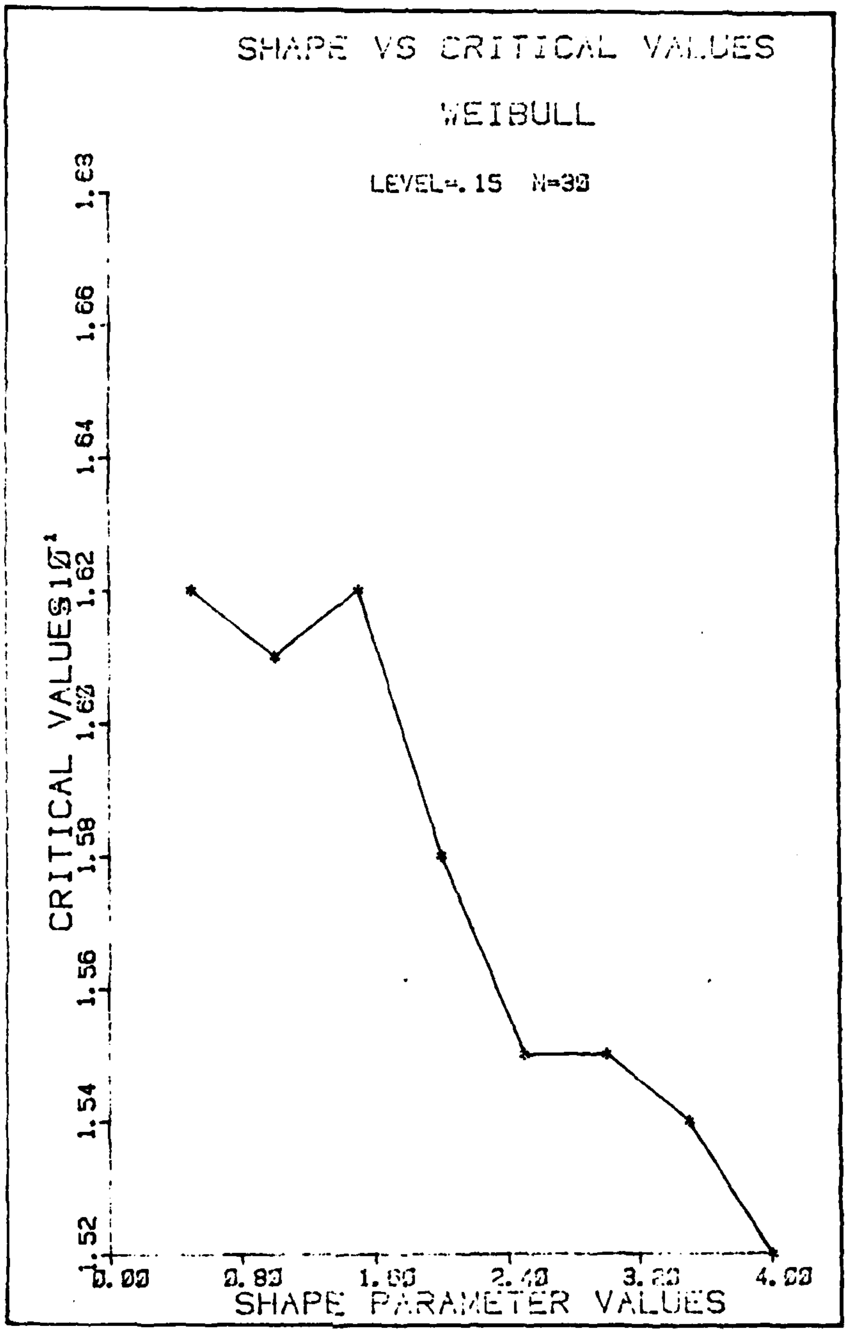


**Figure 23. Weibull Level=.15 N=30**

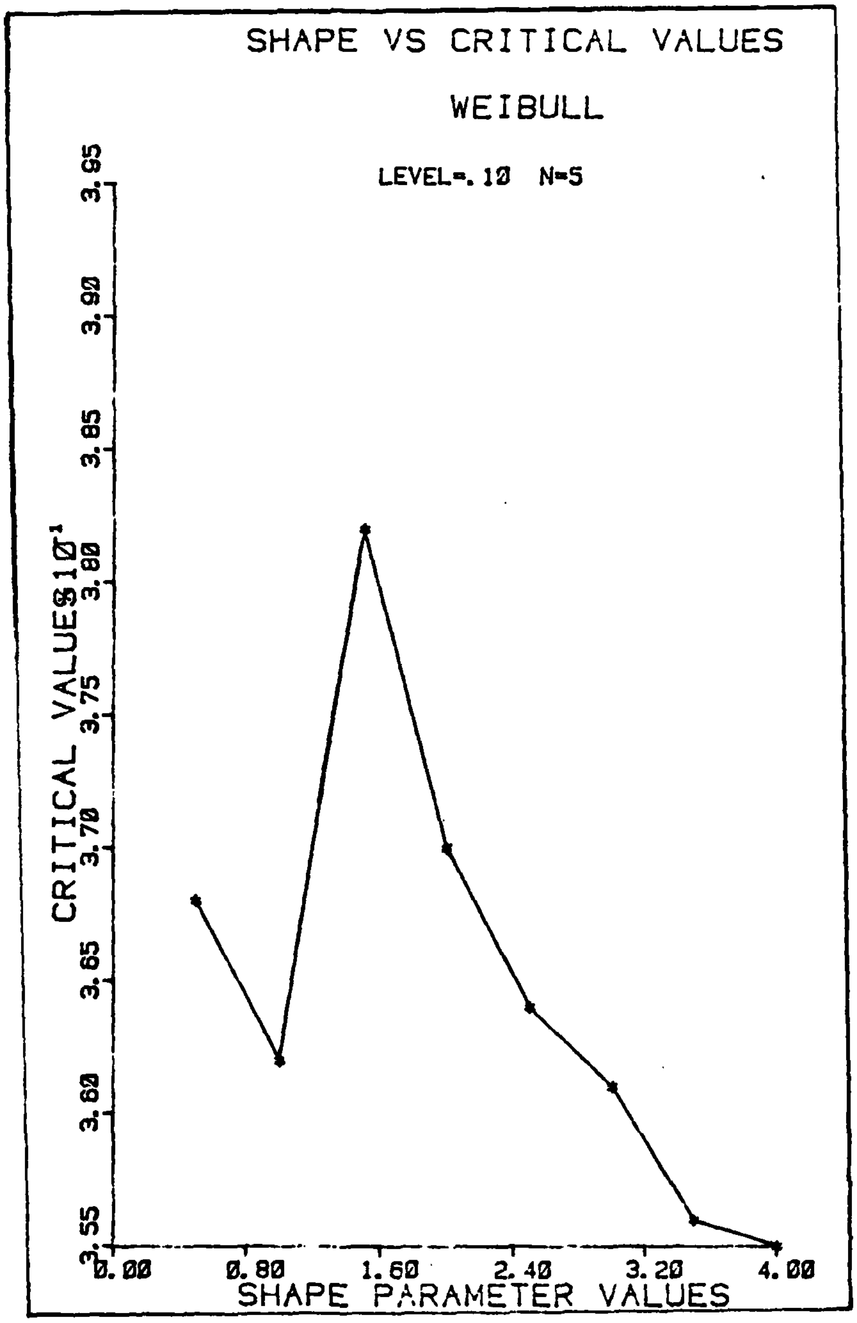


Figure 24. Weibull Level=.10 N=5

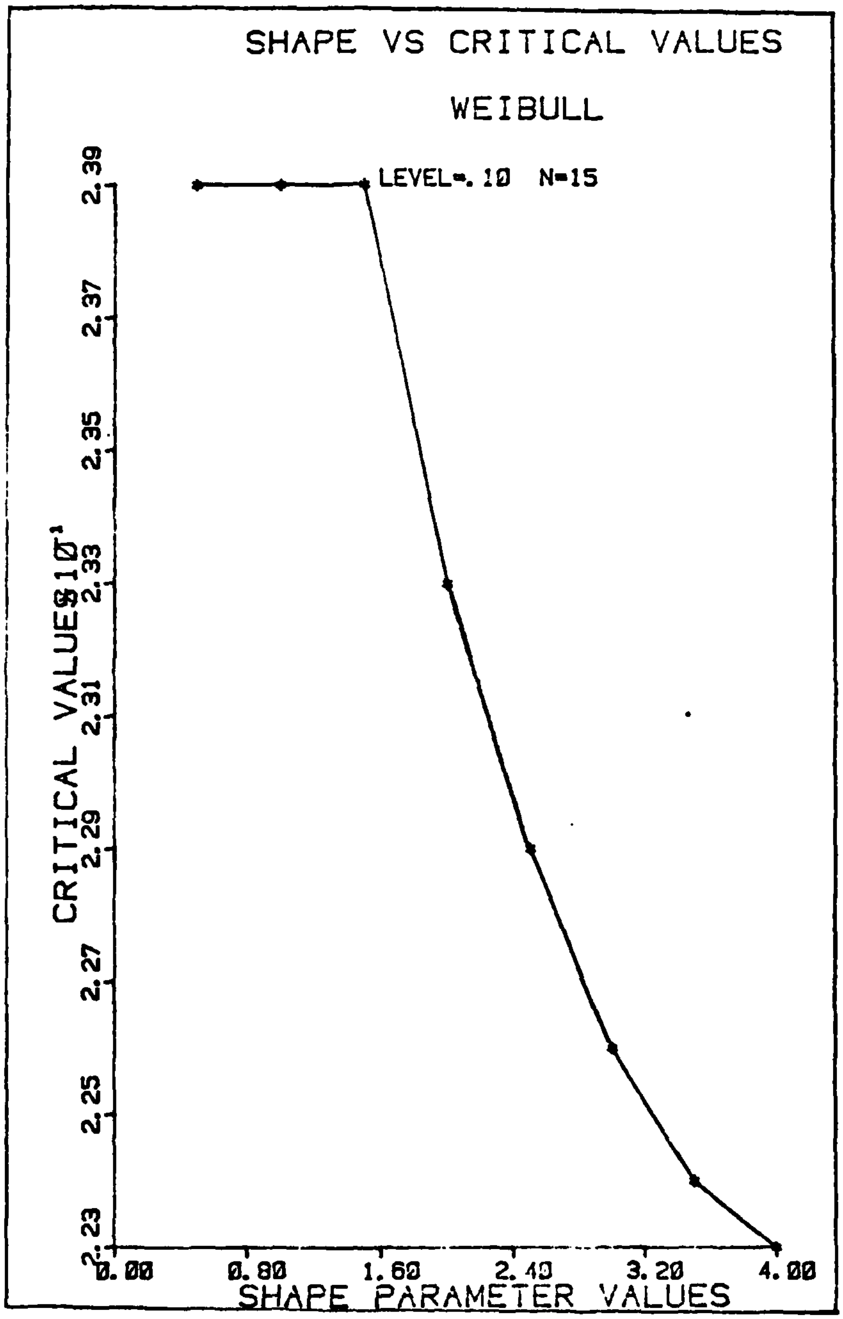


Figure 25. Weibull Level=.10 N=15

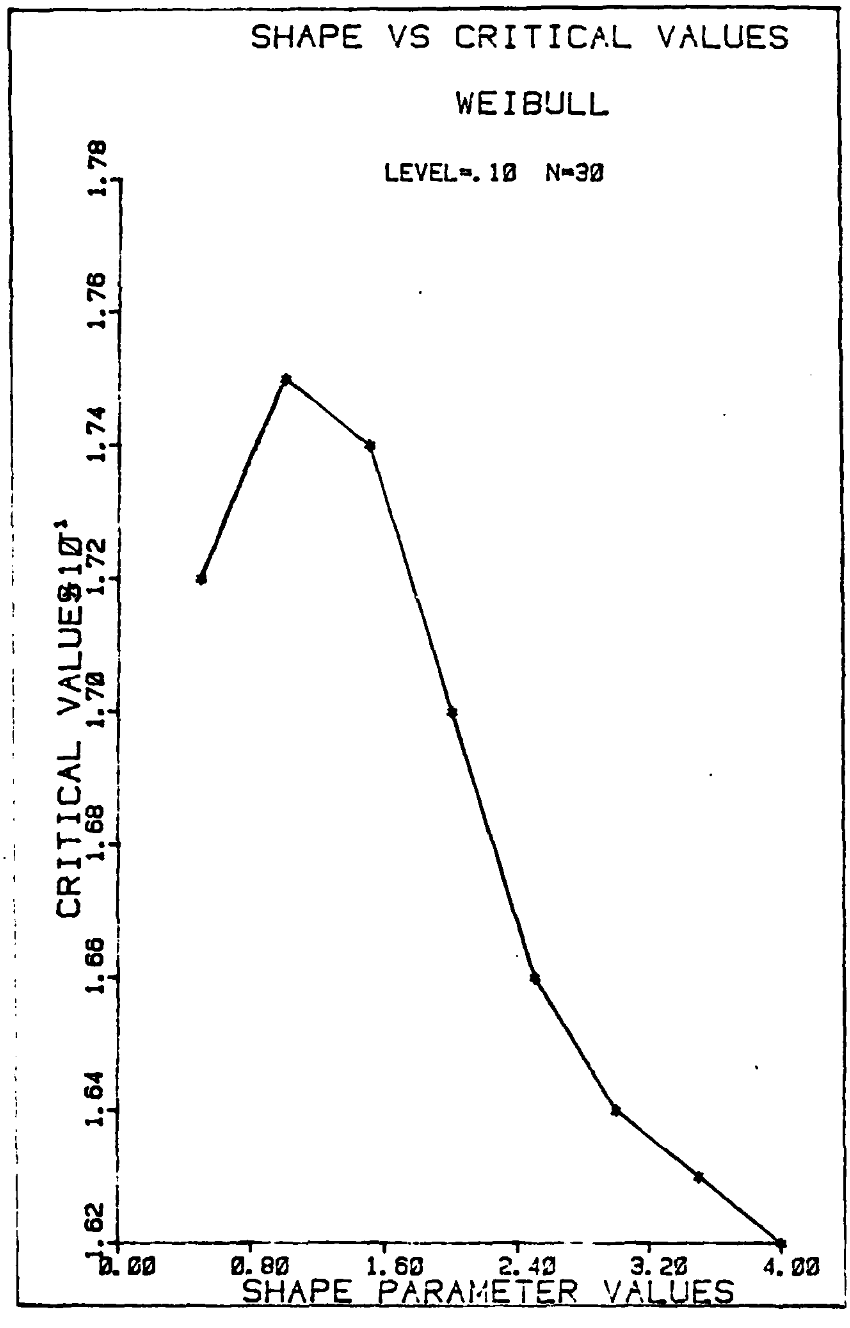


Figure 26. Weibull Level=.10 N=30

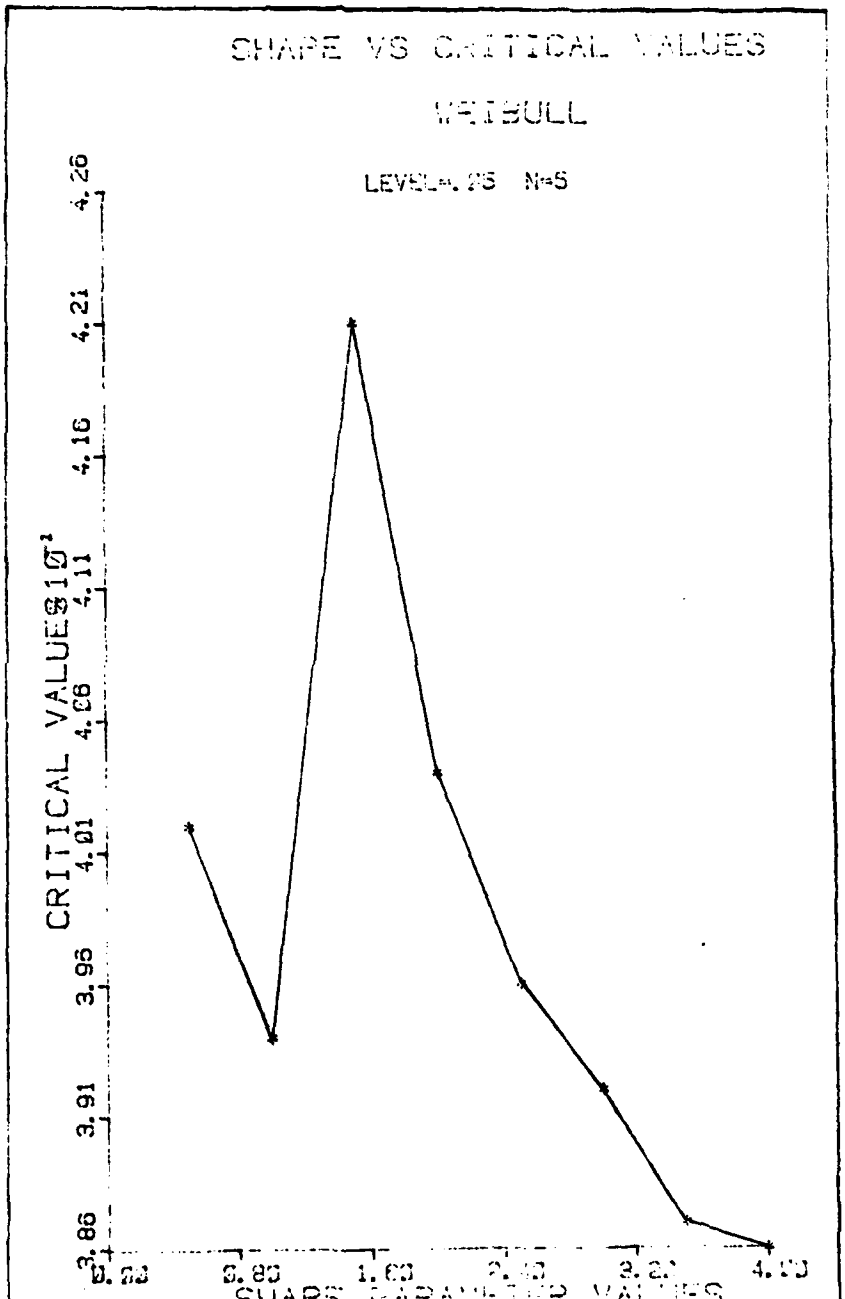


Figure 27. Weibull Level=.05 N=5

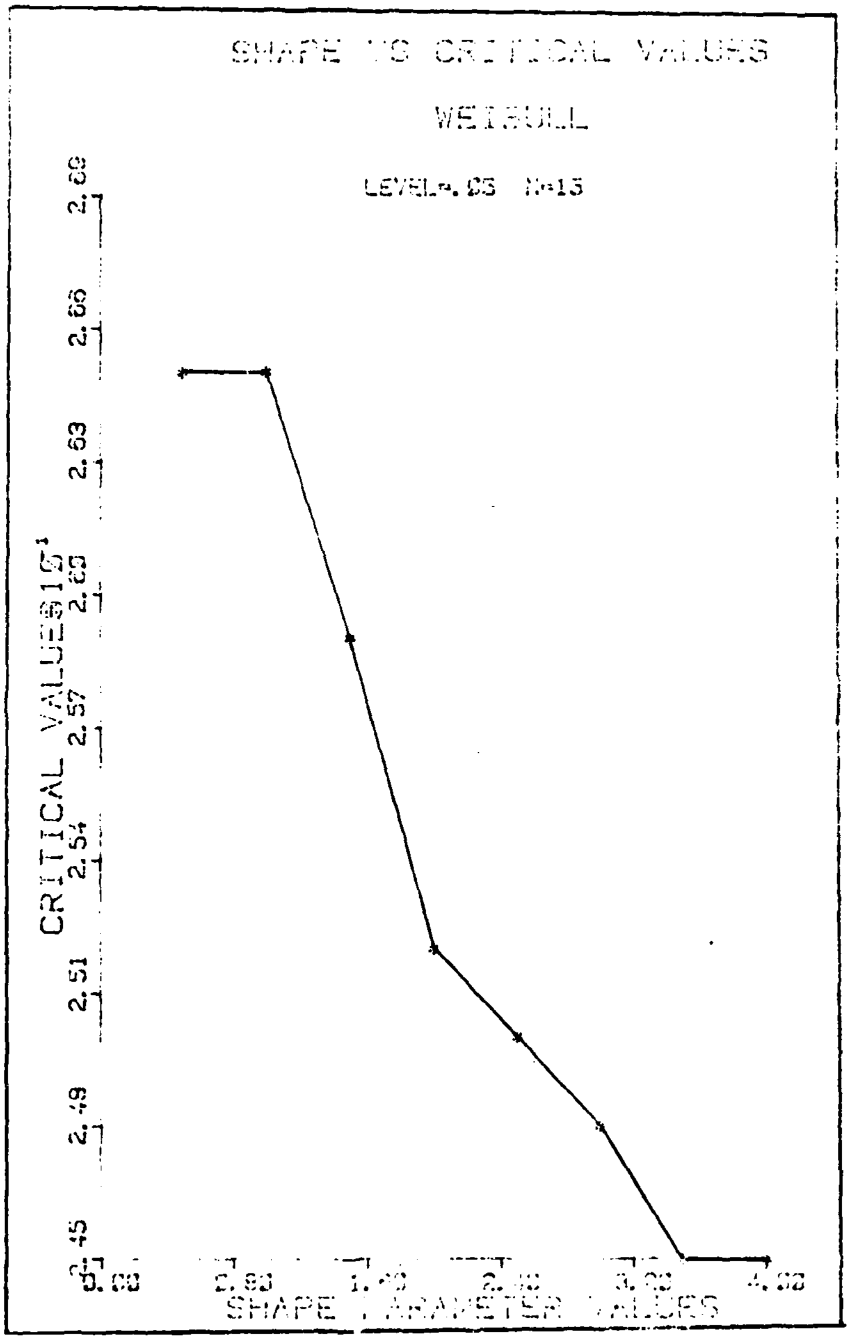


**Figure 28. Weibull Level=.05 N=15**

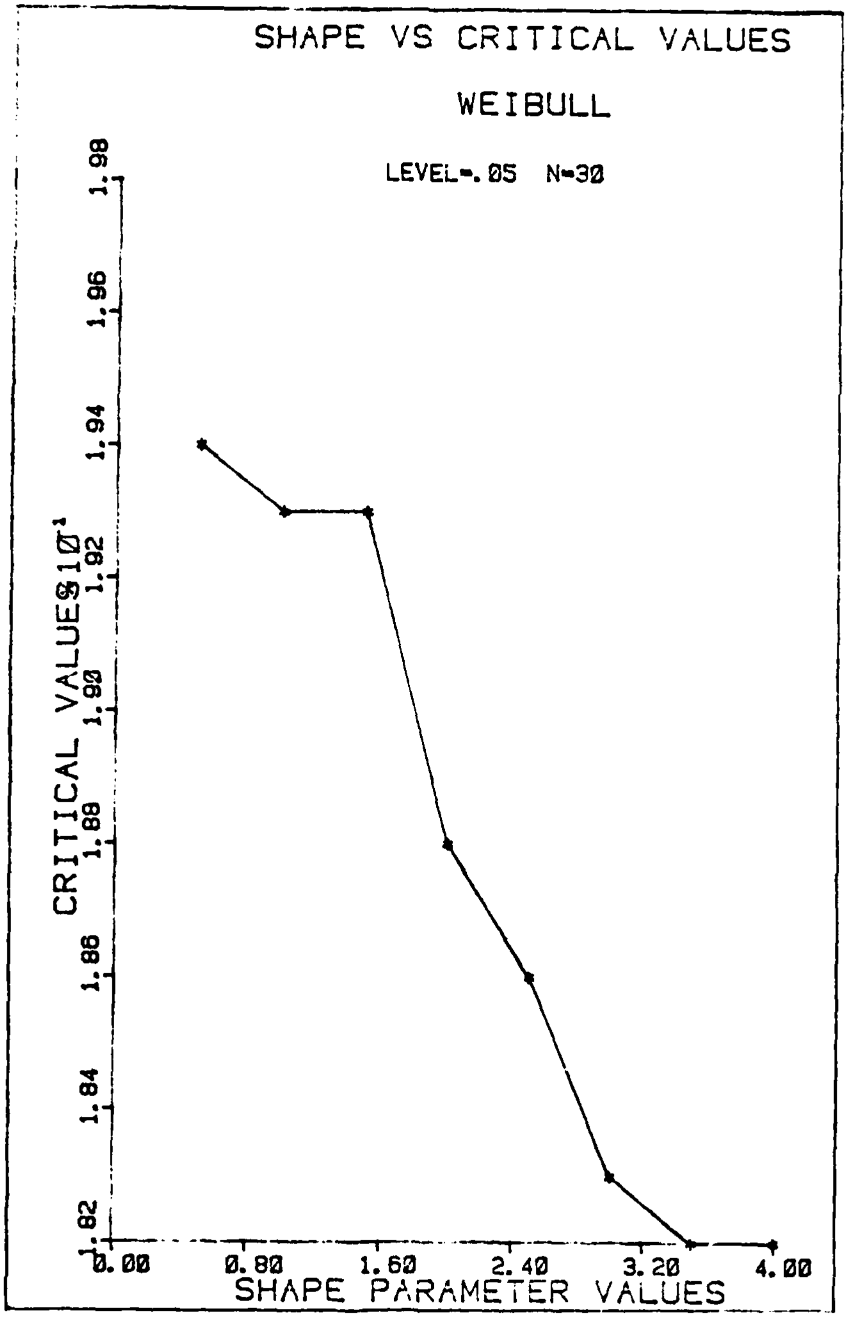


Figure 29. Weibull Level=.05 N=30

AD-A094 732 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOO--ETC F/G 12/1
A MODIFIED KOLMOGOROV-SMIRNOV TEST FOR THE GAMMA AND WEIBULL DI--ETC(U)
DEC 80 R CORTES
UNCLASSIFIED AFIT/GOR/MA/80D-1 NL

2 OF 2
AD
A094732

END
DATE
FILMED
3 81
DTIC

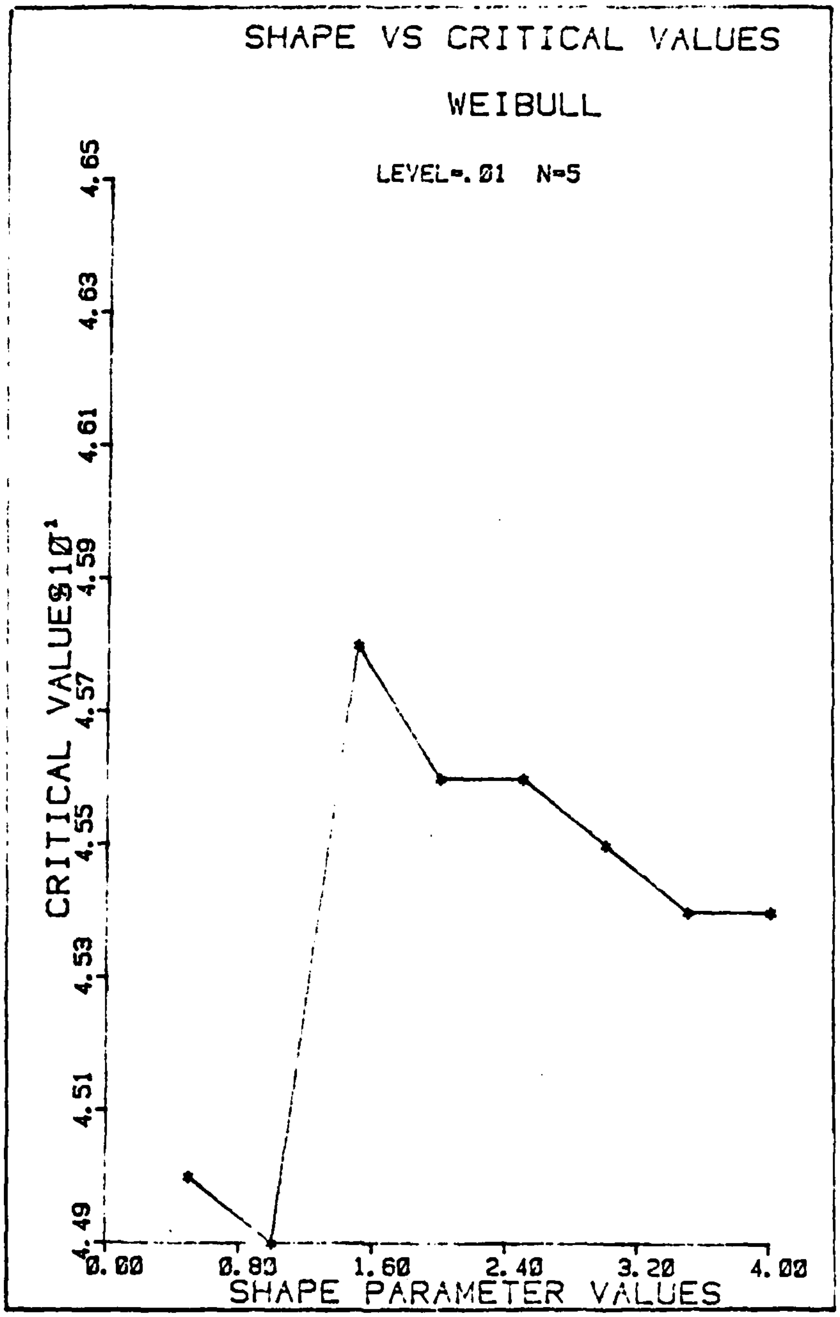


Figure 30. Weibull Level=.01 N=5

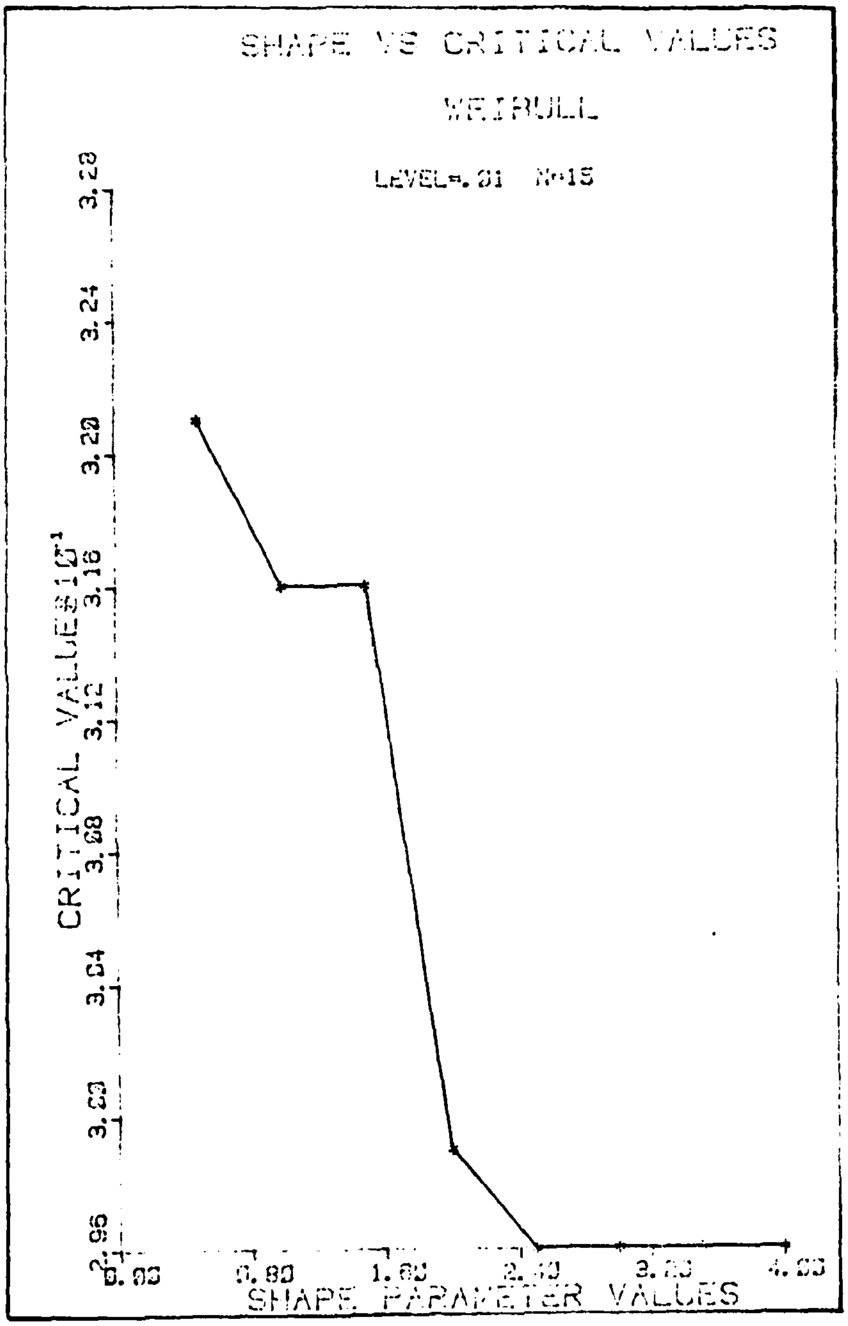


Figure 31. Weibull Level=.01 N=15

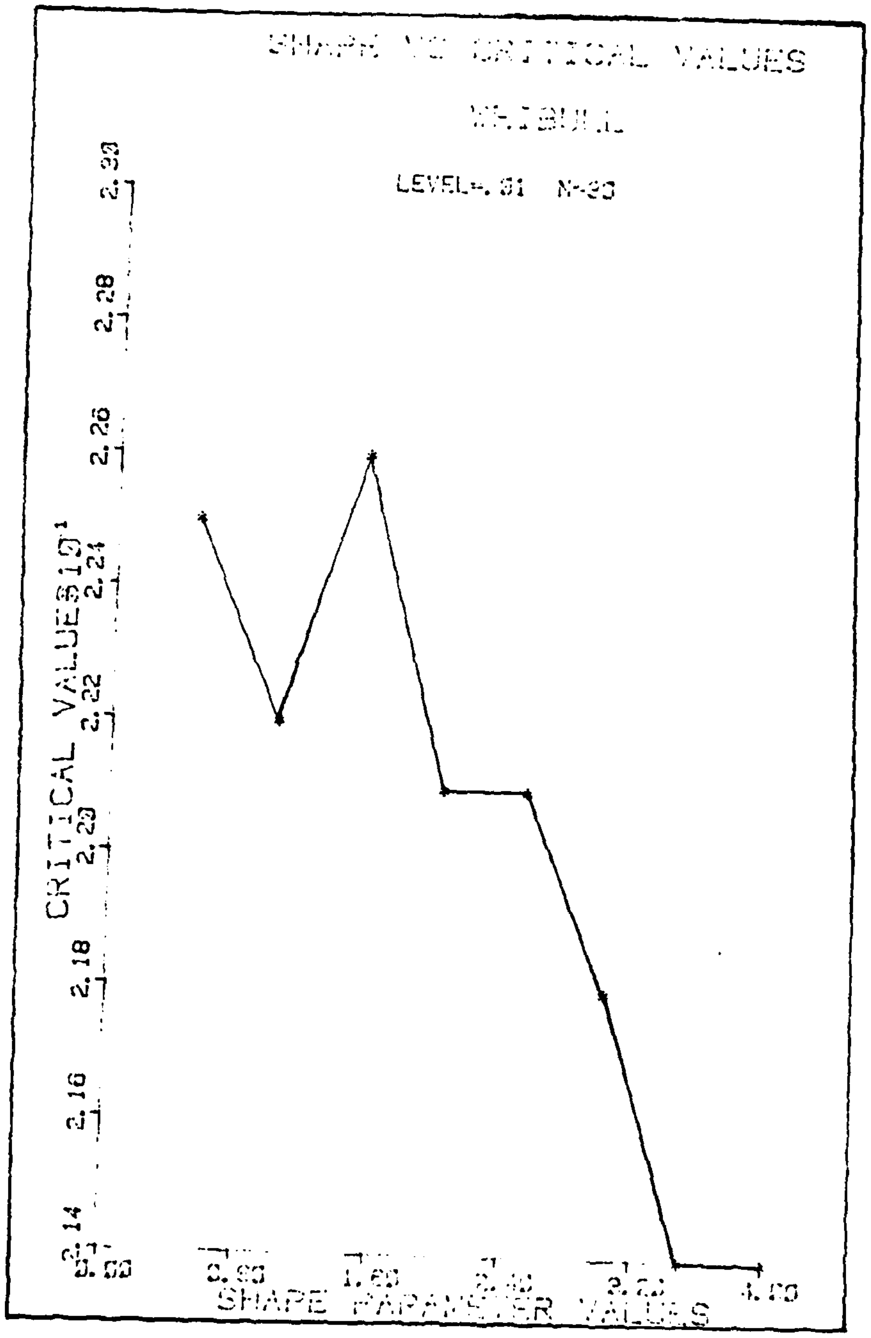


Figure 32. Weibull Level=.01 N=30

Appendix D

Computer Programs

```
RC2,STCS?,T2.,COT',CM13C 3..T?v-+E,CORTES,BOX#4377
ATTACH,IMSL,ID=LIBRARY,SN=ASD.
LIBRARY,IMSL.
FTN.
LGO.
      PROGRAM FIRST(INPUT,OUTPUT,TAPE6=OUTPUT)
C     ****************************************************************
C     ****************************************************************
C     *THIS PROGRAM GENERATES TABLES  FOR THE MODIFIED K-S TEST       *
C     *THE TABLE  GENERATE IS VALID FOR THE WEIBULL DISTRIBUTION      *
C     *N=SAMPLE SIZE, N .LE. 1?.                                       *
C     *SS1=0 IF SCALE PARAMETER THETA IS KNOWN                        *
C     *SS1=1 IF THETA IS TO BE ESTIMATED                              *
C     *SS2=0 IF SHAPE(  K  ) IS KNOWN                                 *
C     *SS2=1 IF ( K ) IS TO BE ESTIMATED                              *
C     *SS3=0 IF LOCATION(C) IS KNOWN                                  *
C     *SS3=1 IF C IS TO BE ESTIMATED                                  *
C     *T1=INITIAL ESTIMATED  OF THETA (OR KNOWN VALUE)                *
C     *C1=INITIAL ESTIMATED OF C (OR KNOWN VALUE)=0. or Z(1)          *
C     *EK1=INITIAL ESTIMATED OF ALPHA (OR KNOWN VALUE)                *
C     ****************************************************************
C     ****************************************************************
      COMMON/RAY/Z(1? ),N
      COMMON/SAND/SS1,SS2,SS3,M,C1,T1,EK1,MR
      DOUBLE PRECISION DSEED
      DIMENSION FX(60),AA(1000)
      DSEED=10000.0D0
      MR=0
      READ*,SS1,SS2,SS3,C1,T1,EK1
      WRITE(6,200)
      DO 100 J=4,?)
      N=J
      M=N
      DO 99 KK=1,1000
      CALL GGWIB(DSEED,EK1,N,Z)
      CALL VSRTA(Z,N)
      CALL WEIBULL(CSJ,TSJ,EKSJ)
      DO 88 L=1,N
      FX(L)=1.-EXP(-((Z(L)-CSJ)/TSJ)**EKSJ)
88    CONTINUE
      TOP=0.0
      BOT=0.0
      XN=N
      DO 500 I=1,N
      RL=I
      IF(RL/XN-FX(I) .GT. TOP)TOP=RL/XN-FX(I)
      IF(FX(I)-(RL-1)/XN .GT. BOT) BOT=FX(I)-(RL-1)/XN
500   CONTINUE
      DIF=TOP
      IF(BOT .GT. DIF) DIF=BOT
      AA(KK)=DIF
99    CONTINUE
      CALL VSRTA(AA,1000)
      WRITE(6,300)N,AA(800),AA(850),AA(900),AA(950),AA(990)
100   CONTINUE
      STOP
200   FORMAT("/",65("-"),/,2X,6HSAMPLE,3X,"I",10X,
     +42HLEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)),
     +/,3X,4HSIZE,4X,"I",?3("-"),/,?X,
     +"",?X,"I",5X,3H.2.,?X,3H.15,10X,3H.10,10X,
     +3H.05,12X,3H.01./,65("-"))
300   FORMAT(/,6X,1I?,4X,"I",?X,1F5.3,4(3X,1F5.3))
      END
      SUBROUTINE WEIBULL(CSJ,TSJ,EKSJ)
C     ****************************************************************
```

```
C      *****************************************************
      DIMENSION T(100),C(55),THETA(55),EK(55),X(55),Y(55)
      COMMON/DAY/Z(100),N
      COMMON/CIND/SS1,SS2,SS3,M,C1,T1,EK1,MR
      CALL IKS)
      EN=N
      DO 2 I=1,N
2     T(I)=Z(I)
      C(1)=C1
      THETA(1)=T1
      EK(1)=EK1
      IF (M) 5,65,32
32    CONTINUE
      EM=M
31    ELNM=0.
      EMR=MR
      MRP=MR+1
33    NM=N-M+1
      DO 34 I=NM,N
      EI=I
34    ELNM=ELNM+ALOG(EI)
      IF (MR) 35,35,74
74    DO 75 I=1,MR
      EI=I
75    ELNM=ELNM-ALOG(EI)
35    DO 20 J=1,55
      IF (J-1) 36,36,37
37    JJ=J-1
      SK=0.
      SL=0.
      DO 6 I=MRP,M
6     SK=SK+(T(I)-C(JJ))**EK(JJ)
      IF (SS1) 7,7,8
7     THETA(J)=THETA(JJ)
      GO TO 9
8     IF (MR) 45,19,20
19    THETA(J)=((SK+(EN-EM)*(T(M)-C(JJ))**EK(JJ))/EM)**(1./EK(JJ))
      GO TO 9
20    X(1)=THETA(JJ)
      LS=0
      DO 21 L=1,55
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-C(JJ))/X(L))**EK(JJ)
      Y(L)=-EK(JJ)*(EM-EMR)/X(L)+EK(JJ)*SK/X(L)**(EK(JJ)+1.)+EK(JJ)*(EN-
     1EM)*(T(M)-C(JJ))**EK(JJ)/X(L)**(EK(JJ)+1.)-EMR*EK(JJ)*ZRK*EXP(-ZRK
     2)/(X(L)*(1.-EXP(-ZRK)))
      IF (Y(L)) 53,73,54
53    LS=LS-1

      IF (LS+L) 55,55,58
54    LS=LS+1
      IF (LS-L) 58,56,58
55    X(LP)=.5*X(L)
      GO TO 61
56    X(LP)=1.5*X(L)
      GO TO 61
58    IF (Y(L)*Y(LL)) 60,73,59
59    LL=LL-1
      GO TO 58
60    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
61    IF (ABS(X(LP)-X(L))-1.E-4) 73,73,21
21    CONTINUE
73    THETA(J)=X(LP)
9     EK(J)=EK(JJ)
10    IF (SS2) 12,12,11
```

```
11    DO 17 I=2RP,1
17    SL=SL+ALOG(T(I)-C(JJ))
      X(1)=EK(J)
      LS=0
      DO 51 L=1,50
      SLK=0.
      DO 18 I=2RP,1
18    SLK=SLK+(ALOG(T(I)-C(JJ))-ALOG(THETA(J)))*(T(I)-C(JJ))**X(L)
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(2RP)-C(JJ))/THETA(J))**X(L)
      Y(L)=(EM-ERP)*(1./X(L)-ALOG(THETA(J)))+SL-SLK/THETA(J)**X(L)+(EN-
     1EM)*(ALOG(THETA(J))-ALOG(T(M)-C(JJ)))*(T(M)-C(JJ))**X(L)/THETA(J)
     2**X(L)+ERR*ZRK*(ALOG(ZRK)/X(L))*EXP(-ZRK)/(1.-EXP(-ZRK))
      IF (Y(L)) 43,52,44
43    LS=LS-1
      IF (LS+L) 47,45,47
44    LS=LS+1
      IF (LS-L) 47,46,47
45    X(LP)=.5*X(L)
      GO TO 50
46    X(LP)=1.5*X(L)
      GO TO 50
47    IF (Y(L)*Y(LL)) 49,52,48
48    LL=LL-1
      GO TO 47
49    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
50    IF (ABS(X(LP)-X(L))-1.E-4) 52,52,51
51    CONTINUE
52    EK(J)=X(LP)
12    C(J)=C(JJ)
62    IF (SS3) 25,23,14
14    IF (1.-EK(J)) 16,78,78
78    IF (SS1+SS2) 57,57,16
16    X(1)=C(J)
      LS=0
      DO 23 L=1,50
      SK1=0.
      SR=0.
      DO 15 I=2RP,1
      SK1=SK1+(T(I)-X(L))**(EK(J)-1.)
15    SR=SR+1./(T(I)-X(L))
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(2RP)-X(L))/THETA(J))**EK(J)
      Y(L)=(1.-EK(J))*SR+EK(J)*(SK1+(EN-EM)*(T(M)-X(L))**(EK(J)-1.))
     1/THETA(J)**EK(J)-ERR*EK(J)*ZRK*EXP(-ZRK)/((T(2RP)-X(L))*(1.-EXP
     2(-ZRK)))
      IF (Y(L)) 39,24,40
39    LS=LS-1
      IF (LS+L) 70,41,70
40    LS=LS+1
      IF (LS-L) 70,42,70
41    X(LP)=.5*X(L)
      GO TO 22
42    X(LP)=.5*X(L)+.5*T(1)
      GO TO 22
70    IF (Y(L)*Y(LL)) 72,24,71
71    LL=LL-1
      GO TO 70
72    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
22    IF(ABS(X(LP)-X(L))-1.E-4) 24,24,23
```

```
23    CONTINUE
24    C(J)=X(LP)
      GO TO 25
57    C(J)=T(1)
25    IF(MF) 6.,38,69
38    DO 53 I=1,M
      IF(C(J)+1.E-4-T(I)) 53,57,57
57    MF=MF+1
53    C(J)=T(1)
58    IF(MF) 6.,69,31
69    SK=0.
      SL=0.
      DO 36 I=MP,M
      SK=SK+(T(I)-C(J))**EK(J)
36    SL=SL+ALOG(T(I)-C(J))
      TK=((T(MP)-C(J))/THETA(J))**EK(J)
      EL=ELN+(EM-EMR)*(ALOG(EK(J))-EK(J)*ALOG(THETA(J)))+(EK(J)-1.)*SL-
     1(SK+(EN-EM)*(T(M)-C(J))**EK(J))/(THETA(J)**EK(J))+EMR*ALOG(1.-EXP
     2(-TK))
      IF(J-3) 30,27,27
27    IF(ABS(C(J)-C(JJ))-1.E-4) 28,28,30
28    IF(ABS(THETA(J)-THETA(JJ))-1.E-4) 29,29,30
29    IF(ABS(EK(J)-EK(JJ))-1.E-4) 4,4,30
30    CONTINUE
4     CONTINUE
      CSJ=C(J)
      TSJ=THETA(J)
      EKSJ=EK(J)
66    RETURN
      END
```

```
RCL.STOSS,T2,1977,GAMMA,T7JL-3,COFTES,30X#4307
ATTACH,IMSL,ID=LIBRARY,SN=ASD.
LIBRARY,IMSL.
FTN.
LGO.
      PROGRAM KSGUIDE(INPUT,OUTPUT,TAPE6=OUTPUT)
C     ****************************************************************
C     ****************************************************************
C     *THIS PROGRAM GENERATES TABLES  FOR THE MODIFIED K-S TEST       *
C     *THE TABLE  GENERATED IS VALID FOR THE GAMMA DISTRIBUTION       *
C     *SAMPLE SIZE N .LE. 100                                         *
C     *SS1=0 IF SCALE PARAMETER THETA IS KNOWN                        *
C     *SS1=1 IF THETA IS TO BE ESTIMATED                              *
C     *SS2=0 IF SHAPE(ALPHA) IS KNOWN                                 *
C     *SS2=1 IF ALPHA IS TO BE ESTIMATED                              *
C     *SS3=0 IF LOCATION(C) IS KNOWN                                  *
C     *SS3=1 IF C IS TO BE ESTIMATED                                  *
C     *T1=INITIAL ESTIMATION  OF THETA (OR KNOWN VALUE)               *
C     *C1=INITIAL ESTIMATED OF C (OR KNOWN VALUE)=0. OR P(1)          *
C     *A1=INITIAL ESTIMATED OF ALPHA (OR KNOWN VALUE)                 *
C     ****************************************************************
C     ****************************************************************
      COMMON/VALUES/P(100)
      COMMON/FXY/T(100)
      COMMON/GAMMA/N,SS1,SS2,SS3,M,C1,T1,A1,MR
      DOUBLE PRECISION DSEED,T,C1,T1,A1
      DIMENSION FX(60),AA(7000),G(50)
      DSEED=10000.000
      MR=
      READ*,SS1,SS2,SS3,C1,T1,A1
      WRITE(6,200)
      DO 100 J=4,30
      N=J
      M=
      DO 803 MM=1,1000
      CALL GGAMR(DSEED,A1,N,G,P)
      CALL VSRTA(P,N)
      DO 3 I=1,N
      T(I)=P(I)
3     CONTINUE
      CALL GAMMA(CSJ,TSJ,ASJ)
      DO 333 L=1,N
      W=(P(L)-CSJ)/TSJ
      XX=ASJ
      CALL MDGAM(W,XX,PROB,IER)
      FX(L)=PROB
333   CONTINUE
      TOP=0.0
      BOT=0.0
      XN=N
      DO 500 I=1,N
      RL=I
      IF(RL/XN-FX(I) .GT. TOP) TOP=RL/XN-FX(I)
      IF(RL/XN-(RL-1)/XN .GT. BOT) BOT=FX(I)-(RL-1)/XN
500   CONTINUE
      DIF=TOP
      IF (BOT .GT. DIF) DIF=BOT
      AA(MM)=DIF
803   CONTINUE
      CALL VSRTA(AA,1000)
      WRITE(6,300)N,AA(800),AA(850),AA(900),AA(950),AA(990)
100   CONTINUE
```

```
      STOP
200   FORMAT("1",63("-"),/,2X,6HSAMPLE,3X,"I",10X,
     *42HLEVEL OF SIGNIFICANCE FOR D=MAX(F(X)-S(X)),
     */,7X,4HSIZE,4X,"I",63("-"),/,1X,
     *"I",3X,"I",7X,3H.2 ,8X,3H.15,10X,3H.1),10X,
     *3H.05,12X,3H.01,/,63("-"))
300   FORMAT(/,4X,1I3,4X,"I",6X,1F5.3,4(8X,1F5.3))
      END
C     ****************************************************************
C     ****************************************************************
      SUBROUTINE GAMMA(CSJ,TSJ,ASJ)
      COMMON/ARY/T(1100)
      COMMON/INA/N,SS1,SS2,SS3,M,C1,T1,A1,MR
      DOUBLE PRECISION T,C,THETA,ALPHA,DLT,DLA,AL,DLC,CE,TH,EM,EN,ELNM
      DOUBLE PRECISION EMP,ET,D2T,DT,D2A,DA,D2C,DC,ENS,GAM,GMA,GAMI,GMAI
      DOUBLE PRECISION GAMI2,DEXP,DABS,DLOG,SL,DG,DGI,DGI2,SR,S1,DGAM
      DOUBLE PRECISION DSAMI,FL,CSJ,TSJ,ASJ,C1,T1,A1,DSEED
      DIMENSION C(1100),THETA(1100),ALPHA(1100)
      DIMENSION DLT(50),DLA(50),AL(50),DLC(50),CE(50),TH(50)
      JI=20
      JH=20
11    FORMAT(I4,3F10.4,2I4)
      C(1)=C1
      THETA(1)=T1
      ALPHA(1)=A1
9     EN=N
      EM=M
86    ELNM=0.D0
      EMP=MR
      MP=MP+1
87    NM=N-M+1
      DO 88 I=NM,N
      EI=I
88    ELNM=ELNM+DLOG(EI)
      IF(MR) 89,89,109
109   DO 110 I=1,MR
      EI=I
110   ELNM=ELNM-DLOG(EI)
89    DO 83 J=1,1100
      IF(J-1) 65,111,111
111   JJ=J-1
      IF(J-JI) 6,139,139
139   IF(J/JH-JJ/JH) 6,6,117
117   J2=J-2
      J3=J-3
      IF(SS1) 119,118,118
118   D2T=THETA(JJ)-2.D0*THETA(J2)+THETA(J3)
      DT=THETA(JJ)-THETA(J2)
      IF(D2T) 135,119,135
135   NT=DABS(DT/D2T)
      GO TO 120
119   NT=999999
120   IF(SS2) 122,122,121
121   D2A=ALPHA(JJ)-2.D0*ALPHA(J2)+ALPHA(J3)
      DA=ALPHA(JJ)-ALPHA(J2)
      IF (D2A)136,122,136
136   NA=DABS(DA/D2A)
      GO TO 123
122   NA=999999
123   IF(SS3) 125,125,124
124   D2C=C(JJ)-2.D0*C(J2)+C(J3)
      DC=C(JJ)-C(J2)
      IF(C(JJ)+5.D-3-T(1)) 140,125,125
140   IF(C(JJ)-5.D-3) 125,125,141
141   IF(D2C) 137,125,137
137   NC=DABS(DC/D2C)
```

```
      GO TO 12
125   N=999999
126   NS=MIN (NT,NA,NC)
      IF(NS) 6,5,142
142   IF(NS-999999) 4,6,5
13    I=NS
      IF(SS1) 127,127,128
127   THETA(J)=THETA(JJ)
      GO TO 12
128   THETA(J)=THETA(JJ)+(DT+.25D0*(ENS+1.D0)*D2T)*ENS
      THETA(J)=DMAX1(THETA(J),1.D-4)
129   IF(SS2) 13 ,13 ,131
13    ALPHA(J)=ALPHA(JJ)
      GO TO 132
131   ALPHA(J)=ALPHA(JJ)+(DA+.25D0*(ENS+1.D0)*D2A)*ENS
      ALPHA(J)=DMAX1(ALPHA(J),1.D-4)
132   IF(SS3) 133,133,134
133   C(J)=C(JJ)
      GO TO 111
134   C(J)=C(JJ)+(DC+.25D0*(ENS+1.D0)*D2C)*ENS
      C(J)=DMAX1(C(J),0.D0)
      C(J)=DMIN1(C(J),T(1))
      IF((1.D0-EMR)*C(J)-T(1)) 112,6,6
6     THETA(J)=THETA(JJ)
      IF(SS1) 13,13,7
7     S1=0.D0
      DO 8 I=MRP,M
8     S1=S1+T(I)-C(JJ)
      IF(N-M+MR) 80,73,74
73    THETA(J)=S1/(EM*ALPHA(JJ))
      GO TO 13
74    GMA=GAM(ALPHA(JJ))
      KS=0
      DO 108 K=1,50
      KK=K-1
      GMAI=GAMI((T(M)-C(JJ))/THETA(J),ALPHA(JJ))
      GMAI2=GAMI((T(MRP)-C(JJ))/THETA(J),ALPHA(JJ))
      DLT(K)=-EM*ALPHA(JJ)/THETA(J)+S1/(THETA(J)**2)+(EN-EM)*(T(M)-C(JJ)
     1)**ALPHA(JJ)*DEXP((C(JJ)-T(M))/THETA(J))/(THETA(J)**(ALPHA(JJ)+1.D
     2O)*(GMA-GMAI))+EMR*ALPHA(JJ)/THETA(J)-EMR*(T(MRP)-C(JJ))**ALPHA(JJ
     3)*DEXP((C(JJ)-T(MRP))/THETA(J))/(THETA(J)**(ALPHA(JJ)+1.D0)*GMAI2)
      TH(K)=THETA(J)
      IF(DLT(K)) 101,13,102
101   KS=KS-1
      IF(KS+K) 105,103,105
102   KS=KS+1
      IF(KS-K) 105,104,105
103   THETA(J)=.5D0*TH(K)
      GO TO 108
104   THETA(J)=1.5D0*TH(K)
      GO TO 108
105   IF(DLT(K)*DLT(KK)) 107,13,106
106   KK=KK-1
      GO TO 105
107   THETA(J)=TH(K)+DLT(K)*(TH(K)-TH(KK))/(DLT(KK)-DLT(K))
      IF(DABS(THETA(J)-TH(K))-1.D-4) 13,13,108
108   CONTINUE
13    ALPHA(J)=ALPHA(JJ)
14    IF(SS2) 44,44,15
15    SL=0.D0
      DO 16 I=MRP,M
16    SL=SL+DLOG(T(I)-C(JJ))
      KS=0
      DO 43 K=1,50
      KK=K-1
      GMA=GAM(ALPHA(J))
```

```
      IF(N-M+MR) 33,7,21
21    GMAI=GAMI((T(M)-C(J))/THETA(J),ALPHA(J))
      GMAI2=GAMI((T(MRP)-C(J))/THETA(J),ALPHA(J))
33    DG=DGAM(ALPHA(J))
76    IF(N-M+MR) 33,77,32
77    DLA(K)=-EM*DLOG(THETA(J))+SL-EN*DG/GMA
      GO TO 78
32    DGI=DGAMI((T(M)-C(J))/THETA(J),ALPHA(J))
      DGI2=DGAMI((T(MRP)-C(J))/THETA(J),ALPHA(J))
38    DLA(K)=-EM*DLOG(THETA(J))+SL-EM*DG/GMA+(EN-EM)*(DG-DGI)/(GMA-GMAI)
     1+EMR*DLOG(THETA(J))+EMR*DGI2/GMAI2
78    AL(K)=ALPHA(J)
      IF(DLA(K)) 39,78,40
39    KS=KS-1
      IF(KS+K) 70,41,70
40    KS=KS+1
      IF(KS-K) 70,42,70
41    ALPHA(J)=.5D0*AL(K)
      GO TO 43
42    ALPHA(J)=1.5D0*AL(K)
      GO TO 43
70    IF(DLA(K)*DLA(KK)) 72,44,71
71    KK=KK-1
      GO TO 70
72    ALPHA(J)=AL(K)+DLA(K)*(AL(K)-AL(KK))/(DLA(KK)-DLA(K))
      IF(DABS(ALPHA(J)-AL(K))-1.D-4) 44,44,43
43    CONTINUE
44    C(J)=C(J)
85    IF(SS3) 112,112,45
45    IF(1.D0-ALPHA(J)) 79,143,143
143   IF(SS1+SS2) 37,37,79
79    IF(N-M) 66,33,46
46    GMA=GAM(ALPHA(J))
83    KS=0
      DO 58 K=1,50
      KK=K-1
      SR=0.D0
      DO 69 I=MRP,M
69    SR = SR + 1.D0/(T(I)-C(J))
      IF(N-M+MR) 66,80,81
80    DLC(K)=(1.D0-ALPHA(J))*SR+EM/THETA(J)
      GO TO 82
81    GMAI2=GAMI((T(MRP)-C(J))/THETA(J),ALPHA(J))
      DLC(K)=(1.D0-ALPHA(J))*SR+(EM-EMR)/THETA(J)+(EN-EM)*(T(M)-C(J))**(
     1ALPHA(J)-1.D0)*DEXP(-(T(M)-C(J))/THETA(J))/(THETA(J)**ALPHA(J)*(GM
     2A-GMAI))-EMR*(T(MRP)-C(J))**(ALPHA(J)-1.D0)*DEXP(-(T(MRP)-C(J))/TH
     3ETA(J))/(THETA(J)**ALPHA(J)*GMAI2)
82    CE(K)=C(J)
51    IF(DLC(K)) 90,112,91
90    KS=KS-1
      IF(KS+K) 54,52,54
91    KS=KS+1
      IF(KS-K) 54,53,54
52    C(J)=.5D0*CE(K)
      GO TO 58
53    C(J)=CE(K)+.5D0*(T(1)-CE(K))
      GO TO 58
54    IF(DLC(K)*DLC(KK)) 57,112,55
55    KK=KK-1
      GO TO 54
57    C(J)=CE(K)+DLC(K)*(CE(K)-CE(KK))/(DLC(KK)-DLC(K))
58    IF(DABS(C(J)-CE(K))-1.D-4) 112,112,56
56    CONTINUE
      GO TO 112
```

```
57     C(J)=T(1)
112    IF(M-) 5 ,113,5
113    DO 11  I=1,M
       IF(C(J)+ . D-4-T(I)) 115,114,114
114    MT=MT+1
115    C(1)=T(1)
116    IF(M-) 6 ,6 ,33
58     S1=0.D0
       SL=0.D0
       DO 92 I=13 ,M
       S1=S1+T(I)-C(J)
92     SL=SL+DLOG(T(I)-C(J))
       GMA=GAM(ALPHA(J))
       IF(N-M+M ) 65,33,95
96     GMAI1=GAMI((T(4)-C(J))/THETA(J),ALPHA(J))
       GMAI2=GAMI((T(MRP)-C(J))/THETA(J),ALPHA(J))
98     EL=ELNM-EM*DLOG(GMA)-EM*ALPHA(J)*DLOG(THETA(J))+(ALPHA(J)-1.D0)*SL
      1  -S1/THETA(J)
       IF(N-M+M ) 55,1  ,99
99     EL=EL+(E -EM)*(DLOG(GMA-GMAI)-DLOG(GMA))
      1+EMR*ALPHA(J)*DLOG(THETA(J))+EMR*DLOG(GMAI2)
101    CSJ=C(J)
       TSJ=THETA(J)
       ASJ=ALP  (J)
59     CONTINUE
       IF(J-2)  3,60,6
60     IF(DABS(C(J)-C(JJ))-1.D-4) 61,61,63
61     IF(DABS(THETA(J)-THETA(JJ))-1.D-4) 62,62,63
62     IF(DABS(ALPHA(J)-ALPHA(JJ))-1.D-4) 4,4,63
63     CONTINUE
4      CONTINUE




66     RETURN
       END
C     ****************** ************************************************
C
       DOUBLE PRECISION FUNCTION GAM(Y)
       DOUBLE PRECISION G,Z,DLOG,DEXP,Y
       Z=Y
       G=0.D0
1      IF(Z-9.D0) 2,2,3
2      G=G-DLOG(Z)
       Z=Z+1.D0
       GO TO 1
3      GAM=G+(Z-.5D0)*DLOG(Z)-Z+.5D0*DLOG(2.D0*3.14159265358979D0)+1.D0/
      1(12.D0*Z)-1.D0/(360.D0*Z**3)+1.D0/(1260.D0*Z**5)-1.D0/(1680.D0*Z**
      27)+1.D0/(1188.D0*Z**9)-691.D0/(360360.D0*Z**11)+1.D0/(156.D0*Z**13
      3)
       GAM=DEXP(GAM)
       RETURN
       END
C
C     ****************** ************************************************
C     ****************** ************************************************
C
       DOUBLE PRECISION FUNCTION DGAM(Y)
       DOUBLE PRECISION DG,Z,Y,DLOG,GAM
       Z=Y
```

```
      DG=0.D0
1     IF (Z-9.D0) 2,2,3
2     DG=DG-1.D0/Z
      Z=Z+1.D0
      GO TO 1
3     DGAM=DG+(Z-.5D0)/Z+DLOG(Z)-1.D0-1.D0/(12.D0*Z**2)+1.D0/(120.D0*Z**
     1      4)-1.D0/(252.D0*Z**6)+1.D0/(240.D0*Z**8)-1.D0/(132.D0*Z**10)
     2      +691.D0/(32760.D0*Z**12)-1.D0/(12.D0*Z**14)
      DGAM=DGAM*GAM(Y)
      RETURN
      END
C
C    ****************************************************************
C    ****************************************************************
C
      DOUBLE PRECISION FUNCTION GAMI(W,Z)
      DOUBLE PRECISION U,W,Z,SU,ELL
      DIMENSION U(50)
C     PRINT*,"I VALUE",W
C     PRINT*,W,Z,SU,ELL
      U(1)=W**Z/Z
      SU=U(1)
      DO 1 L=2,50
      LL=L-1
      ELL=LL
      U(L)=(-U(LL)/ELL)*W*(Z+ELL-1.D0)/(Z+ELL)
1     SU=SU+U(L)
      GAMI=SU
      RETURN
      END
C    ****************************************************************
C    ****************************************************************
C
      DOUBLE PRECISION FUNCTION DGAMI(W,Z)
      DOUBLE PRECISION U,W,V,Z,SU,ELL
      DIMENSION U(50),V(50)
      U(1)=W**Z*DLOG(W)/Z
      V(1)=W**Z/Z**2
      SU=U(1)-V(1)
      DO 1 L=2,50
      LL=L-1
      ELL=LL
      U(L)=(-U(LL)*W/ELL)*(Z+ELL-1.D0)/(Z+ELL)
      V(L)=-V(LL)*W*(Z+ELL-1.D0)**2/((Z+ELL)**2*ELL)
1     SU=SU+U(L)-V(L)
      DGAMI=SU
      RETURN
      END
```

## VITA

Ramón Cortés was born on 23 July 1956 in Caguas, Puerto Rico to Ramón and Isabel Cortés. He graduated from Gautier Benitez High School in 1975 and received a Bachelor of Science degree in Chemistry and Mathematics from the University of Puerto Rico. He was commissioned a Second Lieutenant in the United States Air Force through the AFROTC program on June 13, 1979. Lieutenant Cortes entered the Air Force Institute of Technology Resident School of Engineering in pursuit of a Master's degree in Operation Research in June, 1979.

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The Kolmogorov-Smirnov tables for the Weibull and Gamma distribution were generated in this thesis. These tables are used when testing whether a set of observations are from a Weibull (Gamma) distribution in which the location and scale parameters must be estimated from the sample.

A power investigation of the test against some selected distributions using Monte Carlo techniques was conducted. This procedure

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A relationship between the critical values and the shape parameters was investigated for the Weibull and Gamma distributions. No apparent relationship was found for the Gamma distribution. In contrast, an approximate log-linear relationship was found for the Weibull when the shape parameter is between one and four.

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