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ADVANCED TECHNIQUES FOR BLACK BOX MODELING (EFFECT OF SIGNAL QUANTIZATION; MULTIRATE SAMPLING OF WIDEBAND SYSTEMS)

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Approximation problem		Rational model

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The pencil-of- function method is a black box modeling method. The research described here deals with the issues of (1) effect of quantized input output signals and (2) the modeling of wideband systems. The development shows that the pencil-of-functions method together with certain statistical corrections and/or the use of multirate sampling lead to enhanced transfer function models. Also, it is shown that LF, MF and HF test results, when employed in conjunction with the procedure

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described for adjoining smallband transfer functions, yield successful wideband identification.

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PREFACE

This effort was conducted by University of South Florida under the sponsorship of the Rome Air Development Center Post-Doctoral Program for Rome Air Development Center. Mr. John F. Spina RADC/RBCT was the task project engineer and provided overall technical direction and guidance.

The RADC Post-Doctoral Program is a cooperative venture between RADC and some sixty-five universities eligible to participate in the program. Syracuse University (Department of Electrical Engineering), Purdue University (School of Electrical Engineering), Georgia Institute of Technology (School of Electrical Engineering), and State University of New York at Buffalo (Department of Electrical Engineering) act as prime contractor schools with other schools participating via sub-contracts with prime schools. The U. S. Air Force Academy (Department of Electrical Engineering), Air Force Institute of Technology (Department of Electrical Engineering), and the Naval Post Graduate School (Department of Electrical Engineering) also participate in the program.

The Post-Doctoral Program provides an opportunity for faculty at participating universities to spend up to one year full time on exploratory development and problem-solving efforts with the post-doctorals splitting their time between the customer location and their educational institutions. The program is totally customer-funded with current projects being undertaken for Rome Air Development Center (RADC), Space and Missile Systems Organization (SAMSO), Aeronautical System Division (ASD), Electronics Systems Division (ESD), Air Force Avionics Laboratory (AFAL), Foreign Technology Division (FTD), Air Force Weapons Laboratory (AFWL), Armament Development and Test Center (ADTC), Air Force Communications Service (AFCS), Aerospace Defense Command (ADC), HW USAF, Defense Communications Agency (DCA), Navy, Army, Aerospace Medical Division (AMD), and Federal Aviation Administration (FAA).

Further information about the RADC-Doctoral Program can be obtained from Mr. Jacob Scherer, RADC/RBC, Griffis AFB, NY, 13441, telephone Autovon 587-2543, Commercial (315) 330-2543.

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EVALUATION

The research described here deals with the issues of (1) the effect on black box identification accuracy of quantized (noisy) input/output data, and (2) the modeling of wideband systems by frequency partitioning and the use of multirate sampling within the sub bands. The development presented here shows that the pencil-of-function method together with selected statistical corrections on the contaminated data and/or the use of multirate sampling leads to enhanced transfer function identification. The enhancement is quantitatively described in terms of normalized mean square errors between the "true" transfer function, the identified transfer function without statistical corrections and the identified transfer function with the statistical corrections.

John F. Spina
JOHN F. SPINA
Project Engineer

ADVANCED TECHNIQUES FOR BLACK-BOX MODELING

1. INTRODUCTION

The pencil-of-functions method is a black-box modeling method [1]-[2]. Given an input, output response pair of a system under test, the algorithm leads to a comprehensive description of the system in the form of a transfer function. Although the method was originally developed for use upon linear networks, its applicability has been extended by Weiner and Ewen [3]-[4] to nonlinear Volterra models. The method has been implemented in a FORTRAN program and is available from RADC together with necessary user instructions[5]. The research described here deals with the important issues of signal quantization during analog-to-digital conversion, and the black-box modeling of wideband systems.

1. Quantization

Practical analog-to-digital (A/D) converters employ small word lengths, typically 8 to 16 bits, and, as a rule, one can trade word length for higher conversion speed, cost remaining fixed. Unfortunately, small word lengths lead to degradation in the accuracy of the identified transfer function [6]. It is shown here that the statistical properties of the quantization error can be exploited to improve the accuracy and reliability of the identified parameters. The study thus demonstrates that higher speed implementations and/or additional cost benefits may be achieved for the pencil-of-functions method than have heretofore been realized.

2. Wideband Identification

Communication systems utilize many wideband circuits, for example, amplifiers for spread-spectrum signals. Black-box modeling, or identification, of these circuits poses both a theoretical and a practical challenge. A multirate sampling approach to identification of wideband transmittances is discussed. It permits determination of the transfer function of a four-to-five decade bandwidth system from simple transient tests. Clever selection of sampling rates and test inputs reduces the wideband problem into three, simpler smallband problems. The smallband transfer functions are identified via the pencil-of-functions method and then adjoined, systematically, to construct the wideband transfer function estimate.

The report is structured as follows. Section 2 describes the pencil-of-functions method in brief. Theoretical details are omitted, for they can be found elsewhere [1], [2]. The description is included here for convenience of the reader, and also to emphasize the discrete-time version of the method. A computer program for conversion from s to z domain transfer functions is given in Appendix A. Section 3 presents the study on improvement of quantization-caused degradation, through a statistical approach. The key to this turns out to be the determinant of the Gram matrix of the integrated signals. A computer program, "GQUANT", developed for the particular case of impulse response modeling, is given in Appendix B. Section 4 discusses the results of the study on wideband systems. Included are equations and tables for ready selection by the test engineer of inputs and sampling rates for the LF, MF and HF band transient tests. These pulse inputs have been selected after careful study and are considered both effective and laboratory realizable. A computer program, "USPEC", which generates the amplitude spectra of the recommended pulses is given in Appendix C.

2. PENCIL-OF-FUNCTIONS METHOD

Recorded input, output responses of a network can be integrated to yield a family of signals, called measurement signals. Application of the pencil-of-functions theorem [1] to this family yields, in a closed form, the identified parameters of the network function. The procedure for this black-box modeling method is described below. Although proofs are omitted, the usefulness of the technique will be demonstrated with examples. Discrete-time signals are chosen for the presentation here, because of inherent computational advantages, although such signals must often be obtained by sampling a continuous-time system.

2.1 SIMULTANEOUS NUMERATOR AND DENOMINATOR DETERMINATION

Identification Problem

Given the input-output observations

$$\{u(k)\},\ \{y(k)\}, \qquad k=0,1,..,K \tag{1}$$

arising from a physical system (see Fig. 1) believed to be linear, and of finite order, it is desired to find a system model

$$H(z) = \frac{b_1 z^{-1} + \ldots\ldots + b_n z^{-n}}{1 + a_1 z^{-1} + \ldots\ldots + a_n z^{-n}} \tag{2}$$

$$= \sum_{i=0}^{n} \frac{d_i z^{-1}}{1 - c_i z^{-1}} \tag{3}$$

which best fits the observations, in some sense (see Fig. 2). A solution can be obtained by use of the pencil-of-functions theorem as discussed below.

For convenience denote sequences $\{u(k)\}$ and $\{y(k)\}$ simply as u and y, respectively. Also, denote the inner-product of two sequences as

$$< x,y > \overset{\text{def}}{=} \sum_{k=0}^{K} x(k)\ y(k) \tag{4}$$

Measurement Sequences

From the given sequences y and u we form the following set of sequences, called measurement sequences:

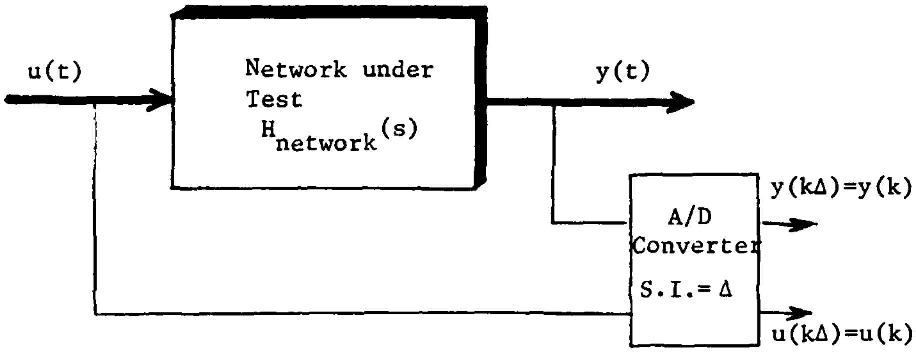


Fig. 1. Response-pair from system under test

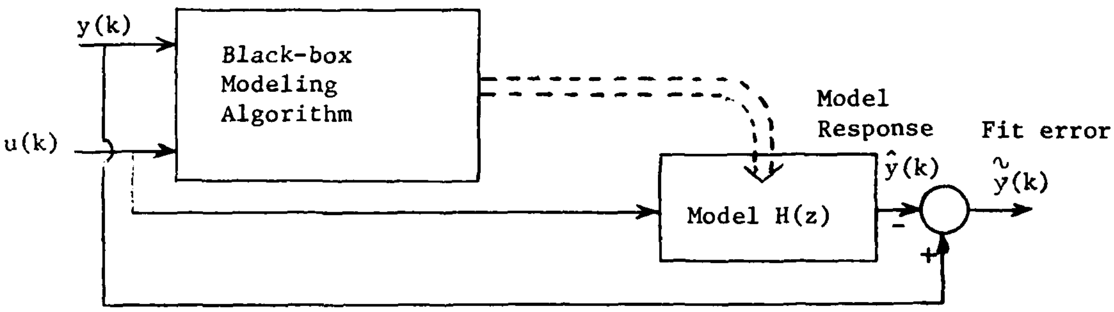


Fig. 2. Identification problem: Find H(z) so that
$\hat{y}(k)$ is close to y(k)

$$
\begin{aligned}
y_1(k) &= y(k) \\
y_2(k) &= y_1(0)+y_1(1)+\ldots+y_1(k) \\
&\;\;\vdots \\
y_{n+1}(k) &= y_n(0)+y_n(1)+\ldots+y_n(k)
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
u_1(k) &= u(k-1) \\
u_2(k) &= u_1(0)+u_1(1)\;\ldots\;+u_1(k) \\
&\;\;\vdots \\
u_{n+1}(k) &= u_n(0)+u_n(1)+\ldots\;+u_n(k)
\end{aligned}
\tag{6}
$$

where n is the order of the model desired. That is, n is the degree of the network function H.

Note that these sequences represent repeated discrete integrations of the observed signals y(k) and u(k), respectively, i.e.,

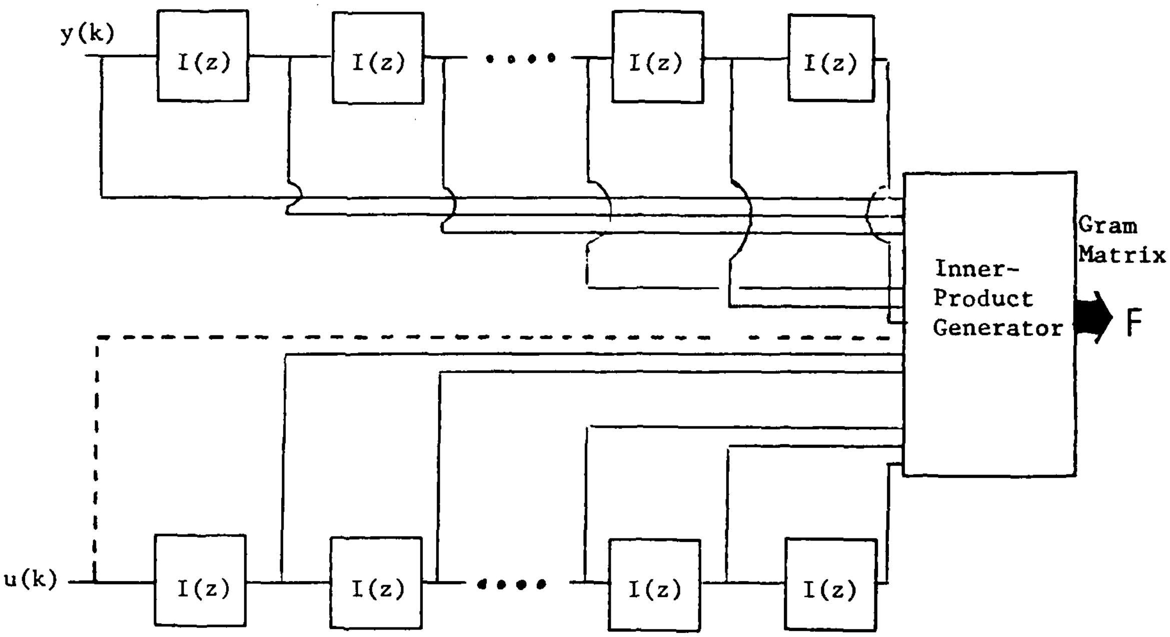
$$y_{j+1}(k) = \sum_{\ell=0}^{k} y_j(\ell) \qquad j=1, \ldots, n \tag{7}$$

$$u_{j+1}(k) = \sum_{\ell=0}^{k} u_j(\ell) \qquad j=1, \ldots, n \tag{8}$$

Equivalently, $y_{j+1}(k)$ is obtained by passing $y_j(k)$ through the filter $I(z) = z/(z-1)$ as shown in Fig. 3. Likewise, $u_{j+1}(k)$ is obtained by passing $u_j(k)$ through the discrete integrator $I(z)$.

Gram Matrix

Next form the following inner-product matrix



Digital integrator $I(z) = \frac{z}{z - 1}$

Fig. 3. Generation of integrated signals and the Gram matrix (u(k) included in the inner-product generator if model is required to have direct transmission)

$$F = \begin{bmatrix} \langle y_1,y_1\rangle & \cdots & \langle y_1,y_N\rangle & \langle y_1,u_2\rangle & \cdots & \langle y_1,u_N\rangle \\ \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ \langle y_N,y_1\rangle & \cdots & \langle y_N,y_N\rangle & \langle y_N,u_2\rangle & \cdots & \langle y_N,u_N\rangle \\ \langle u_2,y_1\rangle & \cdots & \langle u_2,y_N\rangle & \langle u_2,u_2\rangle & \cdots & \langle u_2,u_N\rangle \\ \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ \langle u_N,y_1\rangle & \cdots & \langle u_N,y_N\rangle & \langle u_N,u_2\rangle & \cdots & \langle u_N,u_N\rangle \end{bmatrix} \tag{9}$$

where we have used the notation $N = n+1$ for convenience. This $(N+n) \times (N+n)$ dimensional matrix is the Gram matrix [10] of the $(N+n)$ dimensional vector sequence

$$\{\underline{f}(k)\}, \qquad k = 0,1,\ldots, K \tag{10}$$

where[1]

$$\underline{f}(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \\ \cdot \\ \cdot \\ y_N(k) \\ u_2(k) \\ \cdot \\ \cdot \\ u_N(k) \end{bmatrix} \tag{11}$$

To state this observation formally, we have

$$F = \sum_{k=0}^{K} \underline{f}(k)\underline{f}^T(k) \tag{12}$$

[1]The entry $u_1(k)$ is omitted in $\underline{f}(k)$, and therefore in the formation of the gram matrix F, whenever direct transmission in the model is absent (that is when the coefficient b_o in the function $H(z)$ is constrained to be zero).

Diagonal Cofactors

Denote the diagonal cofactors of F as D_i:

$$D_i = i,i \text{ cofactor of } F \tag{13}$$

Recall that the i,i cofactor of a square matrix is the determinant of the matrix after deleting the ith row and the ith column.

Parameters of the Network Function

The parameters of the network function are given by the square-roots of D_i up to a multiplicative constant. That is

$$\left[\sum_{i=1}^{N} \sqrt{D_i}\,(1 - z^{-1})^{i-1} \right] Y(z) = \left[\sum_{i=1}^{n} \sqrt{D_{N+i}}\, z^{-1}(1 - z^{-1})^{i-1} \right] U(z) \tag{14}$$

which can be normalized, by dividing by $D = \sqrt{D_1} + \ldots + \sqrt{D_N}$, so that the leading coefficient becomes unity. Clearly the computed transfer function becomes

$$H(z) = \frac{z^{-1}\left[\sum_{i=1}^{n} \sqrt{D_{N+i}}(1-z^{-1})^{i-1} \right] /D}{\left[\sum_{i=1}^{N} \sqrt{D_i}\,(1-z^{-1})^{i-1} \right] /D} \tag{15}$$

REMARKS

- Note that the first measurement signal is the network output itself, $y_1 = y$. Next follow its successive integrations. Each of these signals can be expressed directly in terms of y(k). Indeed, if we let $I(z) = z/(z-1)$, $Y_{j+1}(z) = I^j(z)\,Y(z)$ so that $y_{j+1}(k) = i_j(k) \circledast y(k)$ where $i_j(k)$ is the inverse transform of $I^j(z)$ and $\circledast$ denotes discrete-time convolution.
- The dimensionality of the measurement vector $\underline{f}(k)$ is $2n+1 = N+n$ when the direct transmission term b_o is constrained to be zero. If the network does have direct transmission, $u_1(k) = u(k)$ should be included in the vector $\underline{f}$ so that its dimensionality, as well as that of the corresponding Gram matrix F, becomes $2n+2 = 2N$. The right hand side of equation (14) modifies slightly as follows

$$\left[\sum_{i=1}^{N} \sqrt{D_i}\,(1 - z^{-1})^{i-1} \right] Y(z) = \left[\sum_{i=1}^{N} \sqrt{D_{N+i}}\,(1 - z^{-1})^{i-1} \right] U(z) \tag{16}$$

The counterpart of equation (15) follows from (16) and is therefore not given here.

To illustrate the steps of the method, a simple example is given next. (The reader, unfamiliar with the pencil-of-functions method, may wish to work the details with pencil and paper; others may skip this example.)

Example 1

Consider the setup of Fig. 4 where $u_1(k)$ denotes the input signal and $y_1(k)$ the output. The network is known to have direct transmission and of first order (i.e., the s-domain transfer function is of the type $(d_1 s + d_0)/(s + c_0)$). The measurements are made every 1 ms for 5 samples, k = 0,1,...,4.

Unit pulse input

Suppose the following signals are generated as a result of a unit pulse input (only y_1 and u_1 may have been recorded in real time):

$y_1(k)$	1.0	1.2	0.96	0.768	0.6144
$y_2(k)$	1.0	2.2	3.16	3.928	4.5424
$u_1(k)$	1	0	0	0	0
$u_2(k)$	1	1	1	1	1

The Gram matrix of the signals y_1, y_2, u_1 and u_2 is

$$F = \begin{bmatrix} 4.3289 & 12.4811 & 1.0 & 4.5424 \\ 12.4811 & 51.8881 & 1.0 & 14.8304 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 4.5424 & 14.8304 & 1.0 & 5.0 \end{bmatrix}$$

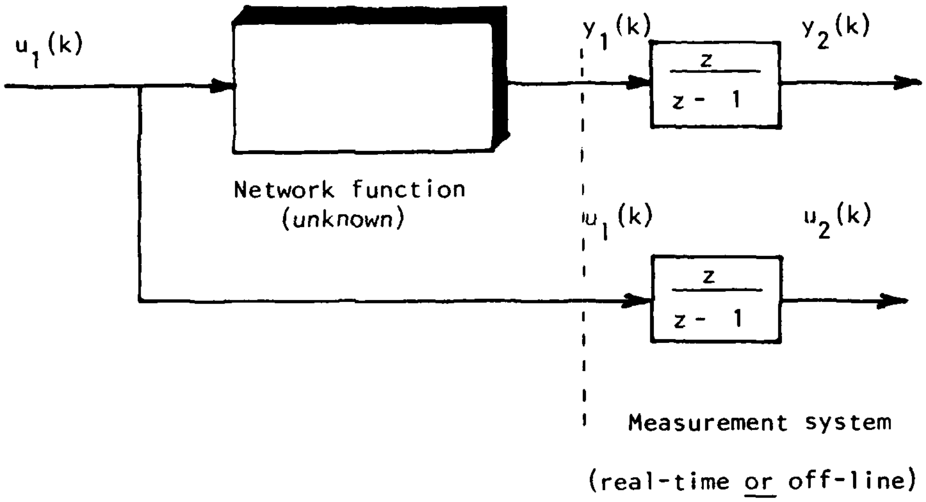
which yields the following square-roots of the diagonal cofactors.

$$\sqrt{D_1} = 3.5032 \qquad \sqrt{D_2} = 0.87581 \qquad \sqrt{D_3} = 1.7516 \qquad \sqrt{D_4} = -6.1307$$

Note that the signs of these square-roots are chosen in direct correspondence with the signs of the cofactors of the first row of F [1]. Now, substitution into (16) and division by $(D_1 + D_2)$ leads to the equation

$$(1 - 0.8\, z^{-1})\, Y(z) = (1 + 0.4\, z^{-1})\, U(z)$$

Clearly, the true parameters have been recovered.



$$\text{Unknown network function} = \frac{1 + 0.4z^{-1}}{1 - 0.8z^{-1}}$$

Fig. 4. A first order test system

Results of computer simulation on a fourth order network function are presented next.

Example 2.

The network function considered is

$$H(s) = \frac{[s^2 + 0.31(10^6)s + 0.003(10)^{12}]}{[s^4 + 0.804(10^6)s^3 + 1.4481(10^{12})s^2 + 0.009686(10^{18})s + 0.007056(10^{24})\ ]}$$

$$= \frac{[s + 10^4][s + 0.3(10^6)]}{[s^2 + 0.004(10^6)s + 0.0049(10^{12})]\ [s^2 + 0.8(10)^6 s + 1.44(10^{12}]}$$

s-poles: $(-0.002 \pm j\ 0.0699714)(10^6)$

$(-0.400 \pm j\ 1.131371\ )(10^6)$

It was converted to a digital equivalent form (using the programs STOZ in Appendix A and pole-zero $z = e^{s\Delta}$ transform [5], [8]) for computer simulation. With a sampling interval $\Delta = 0.5$ μs the z-domain transfer function turns out to be

$$H(z) = \frac{2.00z^{-2} - 3.7114409z^{-3} + 1.7128304z^{-4}}{1 - 3.379158z^{-1} + 4.428628z^{-2} - 2.718099z^{-3} + 0.6689807z^{-4}}$$

The system was excited by a $\pm$ square 5 μs pulse (see Fig. 5a). The model identified by the proposed method is

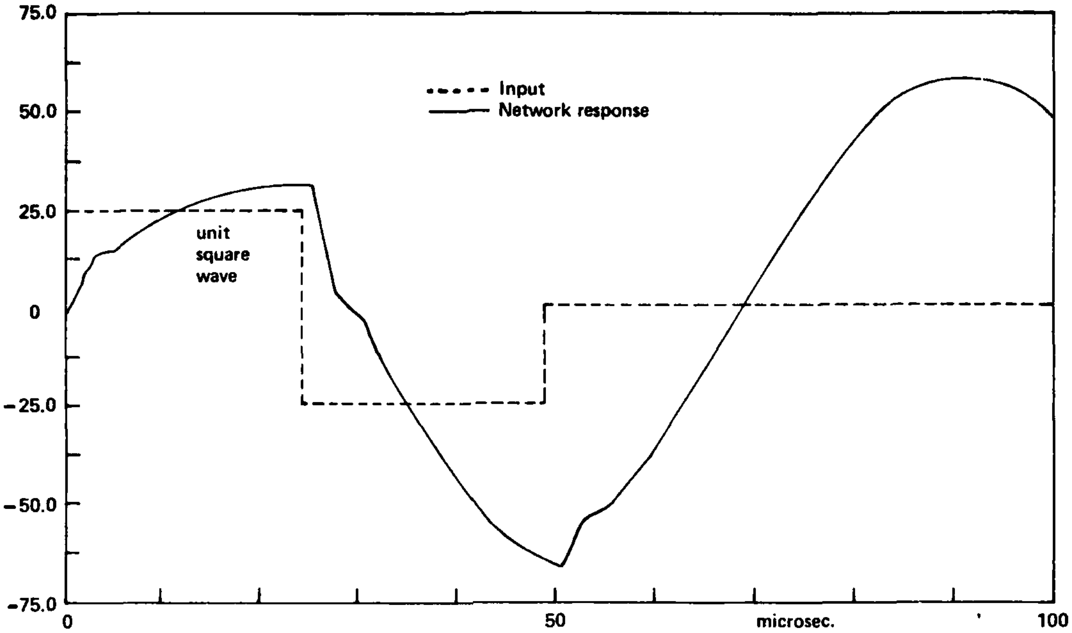
$$\hat{H}(z) = \frac{2.00z^{-2} - 3.71150z^{-3} + 1.7128z^{-4}}{1 - 3.3792z^{-1} + 4.4286z^{-2} - 2.7181z^{-3} + 0.66898z^{-4}}$$

s-poles: $(-0.002 \pm j\ 0.0699714)(10^6)$

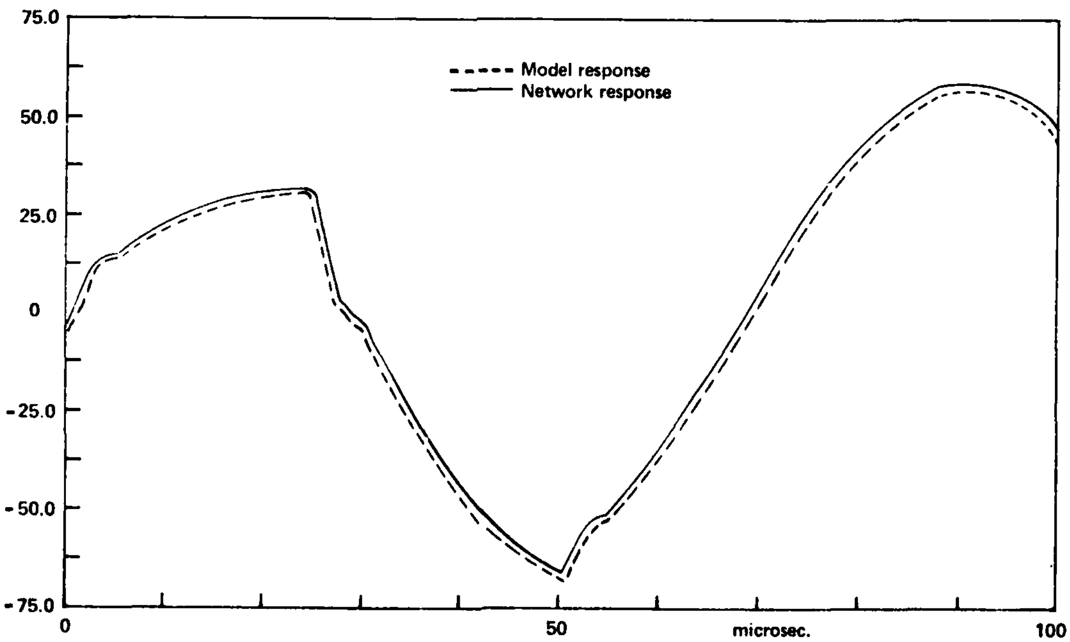
$(-0.399 \pm j\ 1.131373\ )(10^6)$

Using the inverse of the pole-zero transform, the s-domain transfer function can be obtained. The poles turn out as shown above.

The response of the model and the actual network response are compared in Fig. 5b.



(a) input and response



(b) model response and actual response

Fig. 5 IDENTIFICATION OF A FOURTH ORDER SYSTEM
(data uncorrupted by noise)

REMARKS

- When the network under test is of order n, i.e., when the model order is equal to the intrinsic order of the network, the rank of the matrix F equals its dimensionality minus one.

- The matrix F is positive semi-definite.

- In actual application the matrix F will be formed from quantized versions of signals y and u. Call this corrupted matrix as G. It will be shown that $E\{G\}=F+\sigma^2 P$, where P denotes the correlation matrix of unit noise and E denotes the statistical expectation operator. It will be shown in Section III that $E\{G\}$ has full rank (equal to the dimensionality of F).

As seen earlier, the pencil of functions method uses the square-roots of the diagonal cofactors of F. A very important advantage of the method is the following.

"Since F is positive semi-definite (G positive definite with unit probability), its diagonal cofactors are non-negative (strictly positive). Hence, there is a built-in check and stopping point when, due to computational errors or wrong choice of model order, one or more of these cofactors turns out to be negative."

The computations involve finding the cofactors of a 2n + 1 or 2n + 2 dimensional matrix. For the special case of impulse response modeling the calculation of denominator and numerator coefficients can be decoupled, so that computations involve only an n + 1 dimensional matrix. This will be discussed next.

2.2 DECOUPLED PROCEDURE FOR MODELING IMPULSE RESPONSES

Consider that y(k) is the impulse response of a network and that a suitable K has been selected such that $y(k) \simeq 0$ for $k>K$. We define the reverse-time integrated signals as follows [2], [11]

$$\begin{aligned} y_1(k) &= y(k) \\ y_2(k) &= y_1(k)+\dots+y_1(K), \\ &\;\;\vdots \\ y_N(k) &= y_n(k)+\dots+y_n(K), \end{aligned} \tag{17}$$

(Recall, N=n+1). Let F be defined as

$$F = \begin{bmatrix} \langle y_1,y_1\rangle & \cdots & \langle y_1,y_N\rangle \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \langle y_N,y_1\rangle & \cdots & \langle y_N,y_N\rangle \end{bmatrix}, \quad \langle y_i,y_j\rangle = \sum_{k=1}^{K} y_i(k)y_1(k) \tag{18}$$

or, equivalently,

$$F = \sum_{k=1}^{K} \underline{f}(k)\underline{f}^T(k) \tag{19}$$

where $\underline{f}^T(k) = [y_1(k)\ y_2(k)\ \ldots .\ y_N(k)]$. Then, it can be shown that the denominator polynomial is given by

$$A(z) = z^{-n}[\sum_{i=1}^{N} \sqrt{D_i}\ (z-1)^{N-i}]/\sqrt{D_1} \tag{20}$$

where D_i denotes the ith diagonal cofactor of the matrix F. Note the positive powers of z on the right hand side. Further, the numerator coefficients are obtained as:

$$\begin{bmatrix} b_o \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{bmatrix} = - \begin{bmatrix} p_{11} & \cdots & p_{1N} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ p_{N1} & \cdots & p_{NN} \end{bmatrix}^{-1} \begin{bmatrix} q_1 \\ \cdot \\ \cdot \\ \cdot \\ q_N \end{bmatrix}, \tag{21a}$$

$$p_{ij} = \langle w(k+1-i),\ w(k+1-j)\rangle \tag{21b}$$

$$q_i = \langle y(k),\ w(k+1-i)\rangle \tag{21c}$$

where w(k) is the impulse response (i.e., inverse z-transform) of 1/A(z) and inner products are summed from k=0 to K.

If the network is known to have no direct transmission, i.e., b_o is suspected to be zero, then N should be replaced by n on the right hand side, b_o by b_1 and in forming the inner products w(k+1-i) should be replaced by w(k-i) (likewise, w(k+1-j) should be replaced by w(k-j)).

Three examples will be presented next. The first is a simple, paper-pencil type example; it considers the same impulse response as did Example 1 (page 9) but with a long record length. The final example is interesting because it deals with an impulse response which, theoretically, requires an infinite order system (of type 1)) for exact reproduction; a fifth order model is computed by the pencil-of-functions method which yields a fractional energy error of 0.0359. In the first two examples the true transfer function is recovered by the modeling technique, i.e., the fractional energy error is zero.

Notation -

$\hat{y}(k\Delta)$ or $\hat{y}(k)$	Model response
$\tilde{y}(k\Delta)$ or $\tilde{y}(k)$	Model response error $y(k)-\hat{y}(k)$
$S = \sum_{k=0}^{K} y^2(k)$	Response energy
$\varepsilon = \sum_{k=0}^{K} \tilde{y}^2(k)$	Error energy
$\nu = \varepsilon / S$	Fractional energy error, or simply fractional error, or normalized mean square error
$\eta = 100(1-\nu)$	Per cent modeling efficiency

Example 3

Given the left hand side of $y_1(k) = 1.5(0.8)^k - 0.5\delta_{k0}$, we find for K=40

$$F = \begin{bmatrix} 4 & 20 \\ 20 & 100 \end{bmatrix} \qquad \text{(inner products are from } k=1 \text{ to } 40)$$

Then $D_1 = 100$ and $D_2 = 4$. Equation (20) yields

$$A(z) = z^{-1}(10z-8)/10 = 1 - 0.8z^{-1}$$

Equation (21), in turn becomes

$$\frac{1}{0.36}\begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \frac{1}{0.36}\begin{bmatrix} 1.5 - 0.18 \\ 1.2 \end{bmatrix}$$

which produces $B(z) = (1 + 0.4z^{-1})$. The model has been identified perfectly with zero fractional error.

<u>Example 4</u>

A fourth order network is known to have zero direct transmission ($b_0=0$). The numerical data of its impulse response,

$$y(t) = 10\ e^{-2t}\ \mathrm{Sin}(2t) - 2\ e^{-0.5t}\ \mathrm{Sin}(4t)$$

is recorded at uniformly sampled intervals of $\Delta = 0.2$ sec. For K=150 (which signifies a long record; K = 30 sec), we find

$$F = \begin{bmatrix} 4.51403 & 2.38856 & 0.779798 & 0.114119 & -0.05724740 \\ & 2.09775 & 0.953032 & 0.253410 & 0.00074006 \\ & & 0.501610 & 0.157312 & 0.01499790 \\ & & & 0.057232 & 0.00960080 \\ & & & & 0.00352616 \end{bmatrix}$$

$$\det F = 0.54\mathrm{E}{-14}$$

Note - All summations have employed a multiplication factor Δ, for scaling purposes, both in forming the integrated signals and in forming the inner-products. However, to undo the effect of this scaling, the ith diagonal cofactor has to be multiplied with Δ^{2i} to yield D_i. The entire process will be called Δ-scaling

The values of $\sqrt{D_i/D_1}$ are

1	1.50410	1.33762	0.58517	0.11959

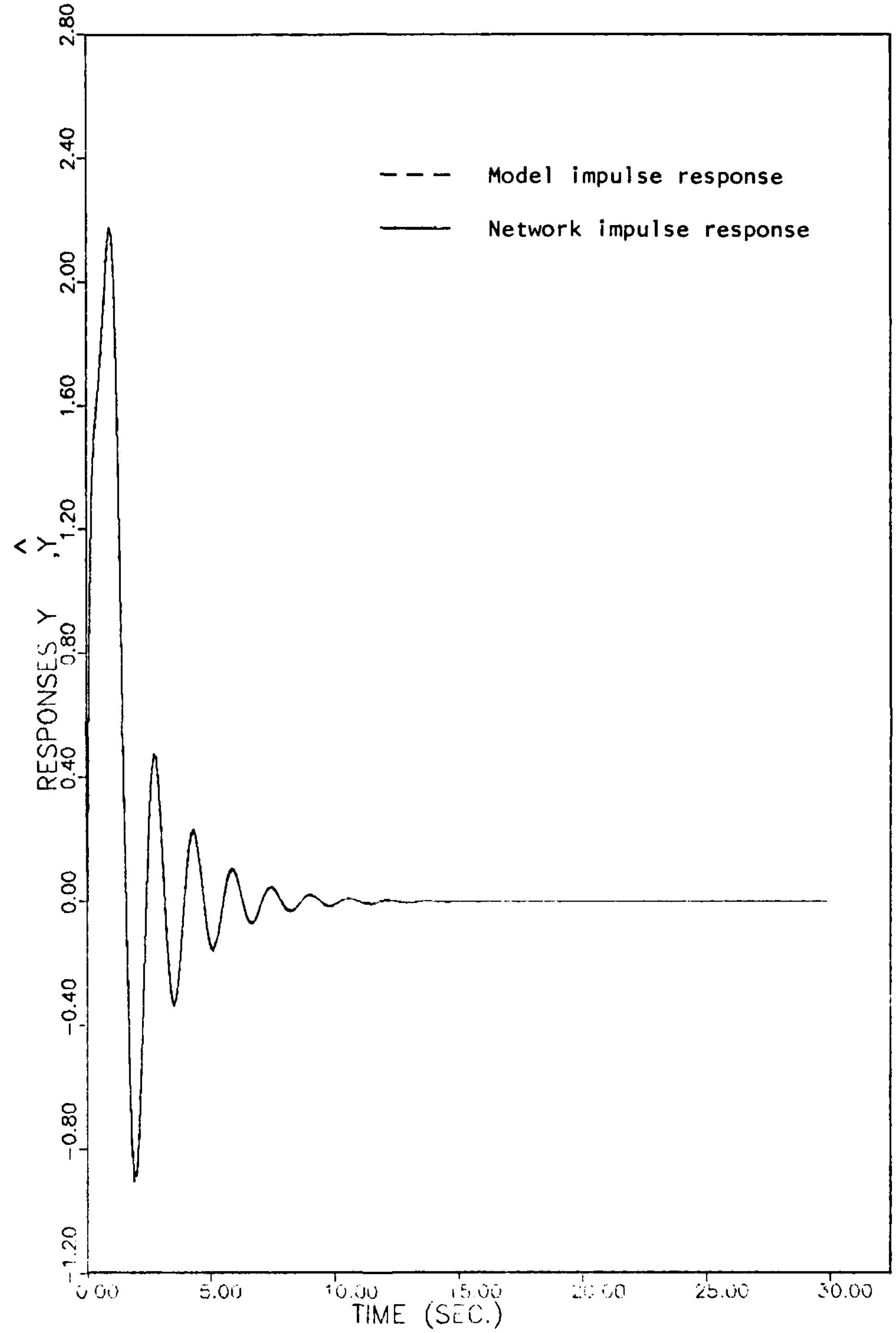


Fig. 6. Comparison of network and model impulse responses

Equations (20) and (21) yield the following denominator and numerator coefficients.

Denominator	1	-2.49588	2.82521	-1.5760	0.36786
Numerator	0	-1.31238	1.68950	-1.55568	0.00158

The fractional energy error turns out to be ν= 0.1E-6. As seen from Fig. 6 the model response $\hat{y}$ is indistinguishable from the true response y.

Example 5

Here we consider a problem in approximation. This terminology, rather than identification, is appropriate since the square pulse

$$y(t) = \begin{cases} 1 & \text{for } 0 < t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

cannot be exactly reproduced as the impulse response of a finite order linear system. A fifth order model is desired whose impulse response approximates this signal. Using Δ=0.05 sec. and setting y(0)=0, y(k)=1 for k=1,..,20, y(k)=0 for k=21,..,40 the following Gram matrix is obtained.

$$F = \begin{bmatrix} 1.0 & 0.525000 & 0.192500 & 0.0553438 & 0.01328250 & 0.002767190 \\ & 0.358750 & 0.146781 & 0.0448284 & 0.01117940 & 0.002391640 \\ & & 0.0635731 & 0.0201175 & 0.00514064 & 0.001119480 \\ & & & 0.0065189 & 0.00169444 & 0.000373780 \\ & & & & 0.00044604 & 0.000099363 \\ & & & & & 0.000022306 \end{bmatrix}$$

det F = 0.344E-26 (note - Δ scaling is employed)

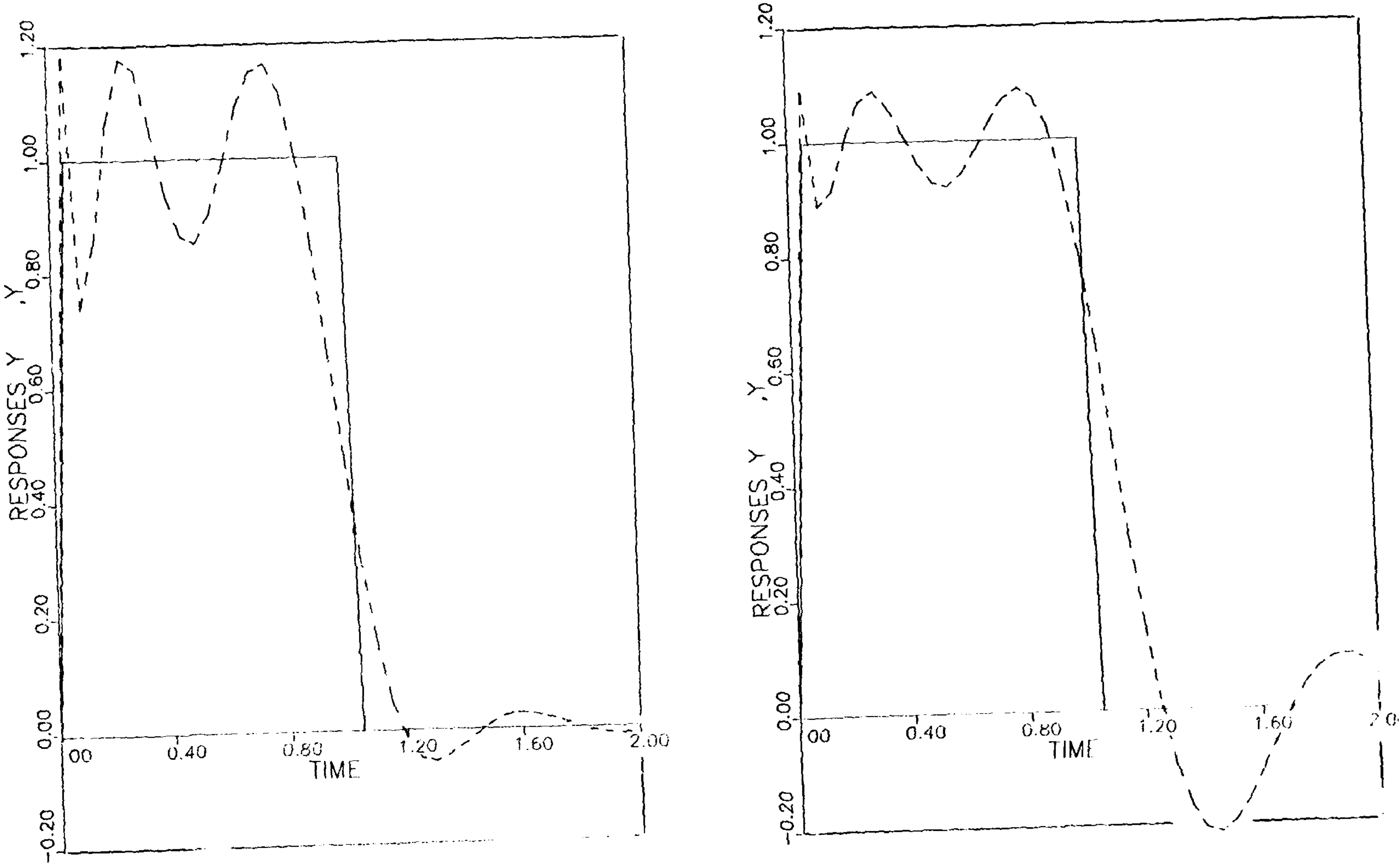
The values of $\sqrt{D_i/D_1}$ are

1 1.1458 0.67865 0.22900 0.042415 0.003383

Equations (20) and (21) yield the following z transfer function coefficients.

Denom.	1	-3.854184	6.095388	-4.93206	2.037106	-0.0342867
Numer.	0	-1.179760	3.806755	-5.19377	3.53997	-1.039771

The fractional energy error turns out to be ν=0.0359 with a corresponding modeling efficiency of 96.4%. The model response $\hat{y}$ is compared with



(a)

(b)

Fig. 7. Comparison of ideal and model impulse responses. Correction upon Gram matrix applied for (b).

the desired, ideal, response y in Fig. 7a.

A brute force application of the correction procedure given in the next section (Section 3) results in the model response shown in Fig. 7b. Of course better approximations can only be obtained with higher order models.

3. QUANTIZATION ERROR: IMPROVEMENT OF ESTIMATES (PENCIL-OF-FUNCTIONS METHOD)

Practical analog-to-digital (A/D) converters employ small word lengths, typically 8 to 16, and as a result incur quantization error in the representation of the signal. This, in turn, causes degradation in the accuracy of the identified transfer function [6]. It will be shown in this section that the statistical properties of the quantization error can be exploited to improve the accuracy of the parameter estimates. A computer program "GQUANT" incorporating the technique developed is given in Appendix B.

The principle of analog-to-digital conversion is explained well in references [6], [9]. For our purposes certain essential properties are most pertinent. If b bits are used (including the sign bit) and XMSB is the analog value of the most significant bit (next to the sign bit), then the following observations and properties hold.

(a) The step size equals

$$\delta = \frac{XMSB}{2^{b-2}} \tag{22}$$

(b) For an input y to the A/D convertor the analog value of the output is

$$x = y + e \tag{23}$$

where $|e| < \frac{\delta}{2}$ for roundoff and $|e| < \delta$ for truncation.

(c) If the signal excursions during each sampling time-interval Δ are large compared to δ, then

$$x(k) = y(k) + e(k) \qquad y(k) \overset{\text{def}}{=} y(k\Delta) \tag{24}$$

where e(k) is an independent sequence of random variables having a uniform distribution over one step size δ. In case of roundoff, this distribution is centered at zero, so that the random variable e(k) has a zero mean and a variance [7].

$$\mathrm{Var}\{e(k)\} = \frac{\delta^2}{12} \tag{25}$$

In the ensuing discussion we will assume the A/D converter employs roundoff.

(d) Under the assumptions in (c) above, the error sequence e(k) is uncorrelated with the parent sequence y(k)

Simulation shows that neither of the properties (c) or (d) strictly hold in practice. However, we will use these properties exercising caution where necessary.

For definiteness we will discuss in detail the correction technique for impulse response modeling method of subsection 2.2. Parallel formulas are applicable to the simultaneous denominator and numerator modeling procedure of subsection 2.1, but will not be given here. Recall that the poles of the model are obtained from the Gram matrix of the signal y and its successive integrations. We therefore begin with the analysis and correction of the quantized Gram matrix.

3.1 GRAM MATRIX OF THE QUANTIZED SIGNAL

We will use the model of equation (24) for the quantized signal

$$x(k) = y(k) + e(k)$$

where $\{E\, e(k)\} = 0$, $E\{e(k)\, e(\ell)\} = 0$ and $E\{y(k)\, e(\ell)\} = 0$ for all k and ℓ. For the reversed time integrated signals we have

$$x_i(k) = y_i(k) + e_i(k) \tag{26}$$

where $e_i(k)$ *are derived from* $e(k)$ *through equations analogous to* (17), i.e.,

$$e_{i+1}(k) = e_i(k) + e_i(k+1) + \ldots + e_i(K) \tag{27}$$

Define also the vector sequences

$$\underline{g}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ \cdot \\ \cdot \\ x_N(k) \end{bmatrix}, \qquad \underline{p}(k) = \begin{bmatrix} e_1(k) \\ e_2(k) \\ \cdot \\ \cdot \\ e_N(k) \end{bmatrix}, \quad k=0,\ldots,K \tag{28}$$

Then the Gram matrix of the quantized signal can be written as

$$G = \sum_{k=0}^{K} \underline{g}(k)\, \underline{g}^T(k)$$

$$= \sum_{k=0}^{K} [\underline{f}(k)\, \underline{f}^T(k) + \underline{f}(k)\, \underline{p}^T(k) + \underline{p}(k)\, \underline{f}^T(k) + \underline{p}(k)\underline{p}^T(k)] \tag{29}$$

Observation 1

$$E\{G\} = \sum_{k=0}^{K} \underline{f}(k)\underline{f}^T(k) + E\{\sum_{k=0}^{K} \underline{p}(k)\underline{p}^T(k)\}$$

$$= F + \sigma^2 P \tag{30}$$

where P is the unit noise covariance matrix defined below. Further, if properties (c) and (d) strictly hold then

$$\sigma^2 = \frac{\delta^2}{12}$$

Observation 2

The unit noise covariance matrix is given by

$$P = E\{\sum_{k=0}^{K} \underline{p}(k)\ \underline{p}^T(k)\} \tag{31}$$

where $\underline{p}(k) = [e_1(k)\ e_2(k)\ ..\ e_N(k)]^T$ as before, but $e_1(k) = e(k)$ is taken to be a zero mean, unit variance, uncorrelated sequence.

Remark

If properties (c) and (d) do not strictly hold, then the value of σ^2 (and possibly the definition of P) should be modified. We will estimate σ^2 so as to satisfy the following criterion.

Jain's Identification Criterion

Consistent with the noise and bias models the estimated Gram matrix should achieve a minimum of the determinant.

Whatever method is used to choose the estimated Gram matrix, care should be taken to make sure that its determinant remains nonnegative, since the determinant of the true Gram matrix is nonnegative (see page 13). An approach to estimation of the Gram matrix is presented in subsection 3.4. First, however, we discuss the computation of the unit noise covariance matrix.

3.2 UNIT NOISE COVARIANCE MATRIX

Examination of the sequences

$e_1(k) = e(k)$

$e_2(k) = e(k) + \ldots + e(K)$

$e_3(k) = e(k) + \ldots + (K+1-k)e(K)$

$e_4(k) = e(k) + \ldots + (K+1-k)^2 e(K)$

leads to the general formula

$$e_{i+1}(k) = \sum_{\ell=0}^{K-k} \ell^{i-1} e(k-\ell) \tag{33}$$

We then have (using the definitions in (28) and (31))

$$\underline{p}(k) = \sum_{\ell=0}^{K-k} \begin{bmatrix} \delta_\ell \\ 1 \\ \ell \\ \cdot \\ \cdot \\ \ell^{n-1} \end{bmatrix} e(k-\ell) \tag{34a}$$

$$= \sum_{\ell=0}^{K-k} \underline{r}(\ell)\, e(k-\ell) \tag{34b}$$

where $\underline{r}^T(\ell) = [\delta_{\ell 0} \;\; 1 \;\; \ldots \;\; \ell^{n-1}]$; $\delta_{\ell 0}$ is the unit pulse sequence. Then

$$P = E\{ \sum_{k=0}^{K} \sum_{\ell=0}^{K-k} \sum_{m=0}^{K-k} \underline{r}(\ell)\, \underline{r}^T(m) \;\; e(k-\ell)\, e(k-m)\}$$

and, since e is a zero mean, unit variance, uncorrelated sequence,

$$P = \sum_{k=0}^{K} \sum_{\ell=0}^{K-k} \underline{r}(\ell)\, \underline{r}^T(\ell)$$

$$= \sum_{k=0}^{K} (K-k+1)\, \underline{r}(k)\, \underline{r}^T(k) \tag{35}$$

Note that P is determined entirely by the integers N and K, the dimensionality of F (recall N=n+1) and the length of the observed sequences, respectively. Clearly, P can be precomputed and stored.

3.3 ESTIMATION OF QUANTIZATION ERROR VARIANCE

The discussion in subsection 3.1, specifically equation (30), leads us to estimate F as

$$\hat{F} = G - \sigma^2 P \tag{36}$$

where σ^2 will be chosen so as to minimize the determinant of $\hat{F}$.

One possible approach to this minimization is developed here. We use the fact that the rank of the true Gram matrix F is n, i.e., its determinent is zero. Rewriting (36)

$$\hat{F} = G - \sigma^2 P$$

we set the determinant of both sides to zero. If the quantization error is small, we can approximate the determinant of the right hand side by the first two terms of the determinant expansion theorem. Thus

$$|\hat{F}| = 0 \simeq G - \sigma^2 \ \Sigma \ \det[G,P]_i \qquad (37)$$

where the notation $[G,P]_i$ means the matrix obtained by replacing the ith column of G by the ith column of P.

Then

$$\hat{\sigma}^2 = \frac{|G|}{\Sigma \det[G,P]_i} \qquad (38)$$

and, of course,

$$\hat{F} = G - \hat{\sigma}^2 P \qquad (39)$$

Note that formula (39) can also be applied recursively, by replacing G in (39) with the last estimate of F. An exit must be made when the determinant of the estimated matrix ceases to reduce further (or begins to increase).

3.4 SIMULATION EXAMPLES

As stated earlier, a FORTRAN IV computer program "GQUANT" has been developed for simulation and modeling of quantized impulse responses. A rational model of the type given in equation (1) is produced, except that b_0 is constrained to zero; i.e., the network is assumed to have no direct transmission. (Slight modification in the computation of numerator coefficients enables this constraint to be removed.) Equivalent s-domain description can be obtained through appropriate z to s transformation. Salient features of the program are the following.

It can be used in either a simulation mode (IRESP=1 or 2) or in network-response-data entry mode (IRESP=0)

Model can be obtained for unquantized signal (ISPN= -1, IFIX =-1, NFIX immaterial) when in the simulation mode, or actual response-data when in data entry mode.

Model can be obtained for the quantized signal (ISPN=1) without any statistical correction (IFIX= -1). Intended for use in simulation mode. Model can be obtained for the quantized signal (ISIM=1 or 2 and ISPN=1, or ISIM=0 and ISPN=0) with statistical correction (IFIX=1); the use of IBIAS=1 performs a bias correction in addition to statistical noise correction.

Two examples are given below, one in which a second order network response is simulated and another in which a fourth order response is simulated. Thus both examples pertain to simulated impulse responses.

$H_{ideal}(z)$	True transfer function of the network.
$H(z)$	Transfer function obtained by application of pencil-of-functions method upon unquantized signal. Note that $H(z)$ need not be equal to $H_{ideal}(z)$; among the reasons for this are computation errors, and the use of $K \neq \infty$.
$H_{quant}(z)$	Transfer function obtained by from the quantized signal. (no correction is applied)
$\hat{H}(z)$	Transfer function obtained from the quantized signal; one or more iterations of statistical correction for quantization errors are used.
$\hat{\bar{H}}(z)$	Transfer function obtained from the quantized signal; in addition to statistical correction for quantization errors, correction is also applied for possible bias in the data. It should be mentioned that the usefulness of bias correction arises both because the quantization errors in particular record of data may not be zero-mean, and also because $K \neq \infty$ may produce an apparent bias in data.
NDIG	Length of binary word $b_{NDIG} \cdots b_2 b_1$ (note b_{NDIG} is the sign bit, b_{NDIG-1} the most significant bit, ..., and b_1 the least significant bit; also, we have employed a mid-tread type of quantizer in simulation)
XMSB	The analog weight (or value) of the most significant bit.

Example 6

A second order network with zero direct transmission is simulated. Its impulse response

$$y(t) = 2e^{-2t} - 2e^{-0.5t}$$

is sampled uniformly at intervals $\Delta = 0.2$ sec. apart. The coefficients of the transfer function $H_{ideal}(z)$ are

Denominator

1	-1.575157	0.606530

Numerator

0	-0.469035	0

Without quantization the modeling program yields (using ISPN = -1, IFIX = -1, NFIX immaterial) the following results:

H(z) (using ISPN = -1, IFIX = -1, NFIX immaterial) $\nu = 0.6E-8$

Denominator

1	-1.575180	0.606551

Numerator

0	0.469050	-0.000041

Experiment 1

For XMSB = 5.0 Volts and NDIG = 10, the program yields the following results

$H_{quant}(z)$ (using ISPN = 1, IFIX = -1, NFIX immaterial) $\nu = 0.84E-3$

Denominator

1	-1.635921	0.662757

Numerator

0	0.513516	-0.114095

H(z) (using ISPN = 1, IFIX = 1, NFIX = 3; includes bias correction) $\nu=0.62E-3$

Denominator

1	-1.628129	0.655569

Numerator

0	0.506869	-0.098299

The impulse responses of $H_{quant}(z)$ and $\hat{\overline{H}}(z)$ are compared with that of $H_{ideal}(z)$ in Fig. 8a and 8b. (The quantized signal used in determining these transfer functions is shown in Fig. 8c.) Although the improvement through statistical correction is hard to discern from these figures, the fractional energy error clearly points to a slight improvement.

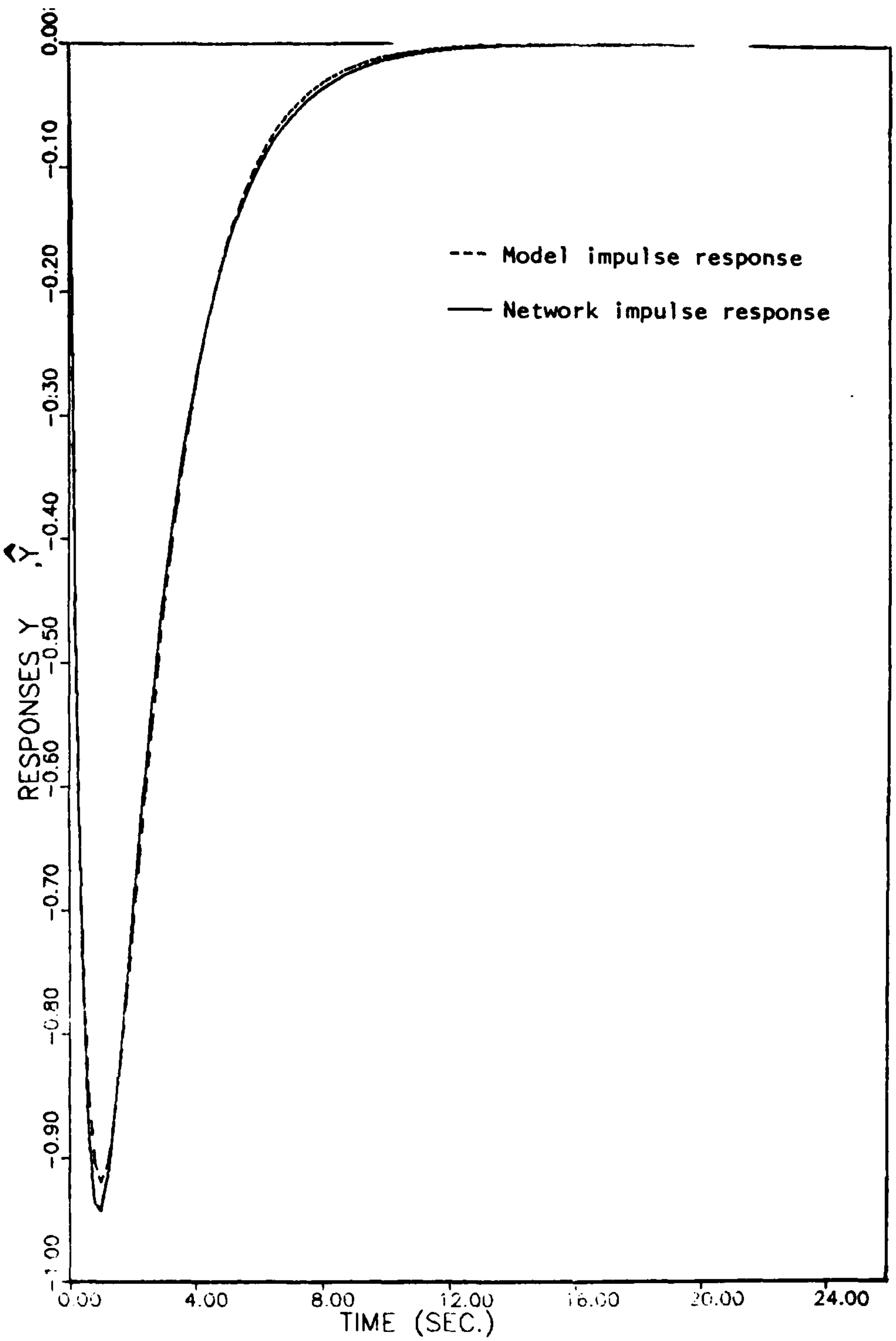


Fig. 8a Comparison of network and model impulse responses.
Model is obtained from quantized data.
(No statistical correction is applied)

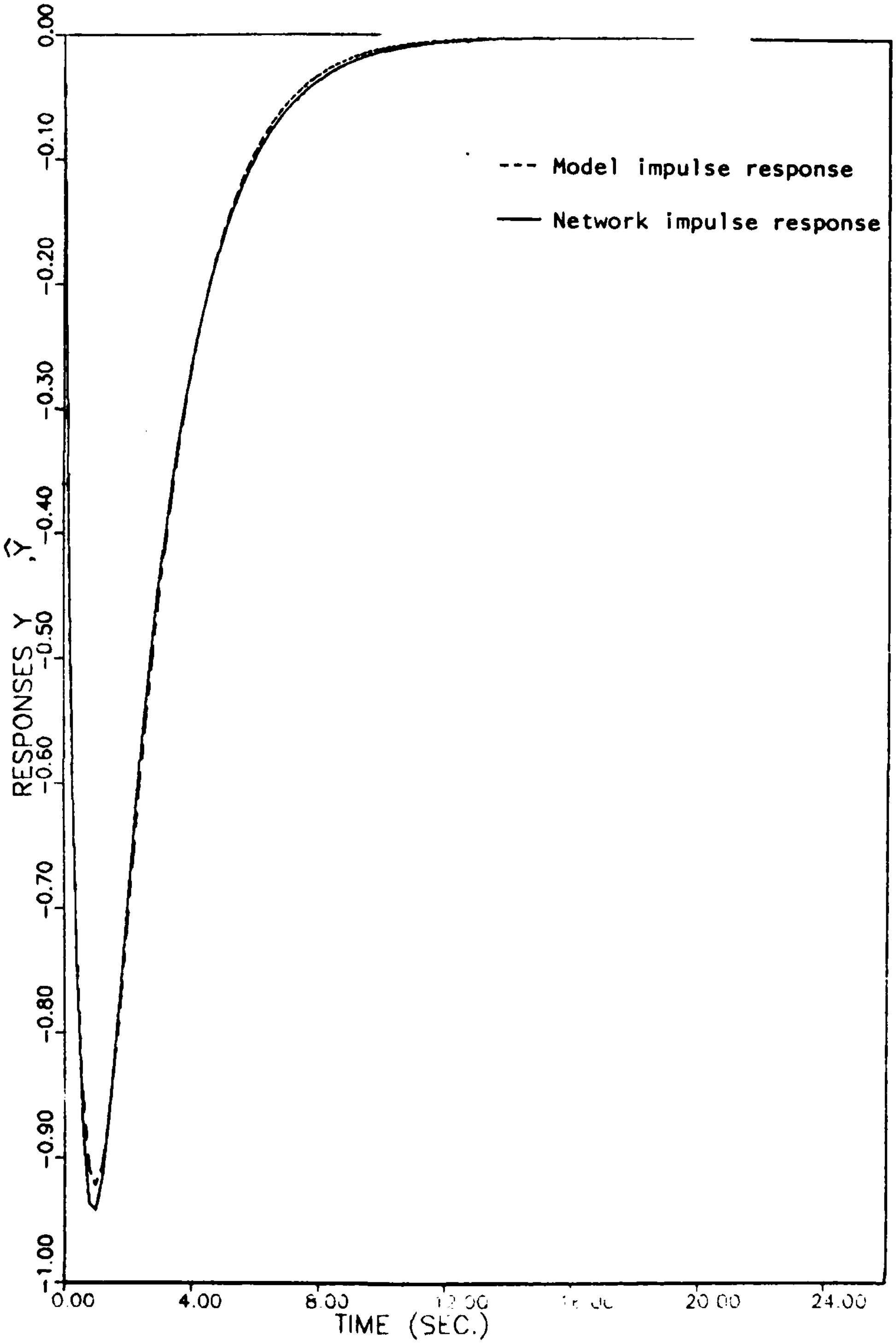


Fig. 8b Comparison of network and model impulse responses. Model is obtained from quantized data; statistical correction is applied.

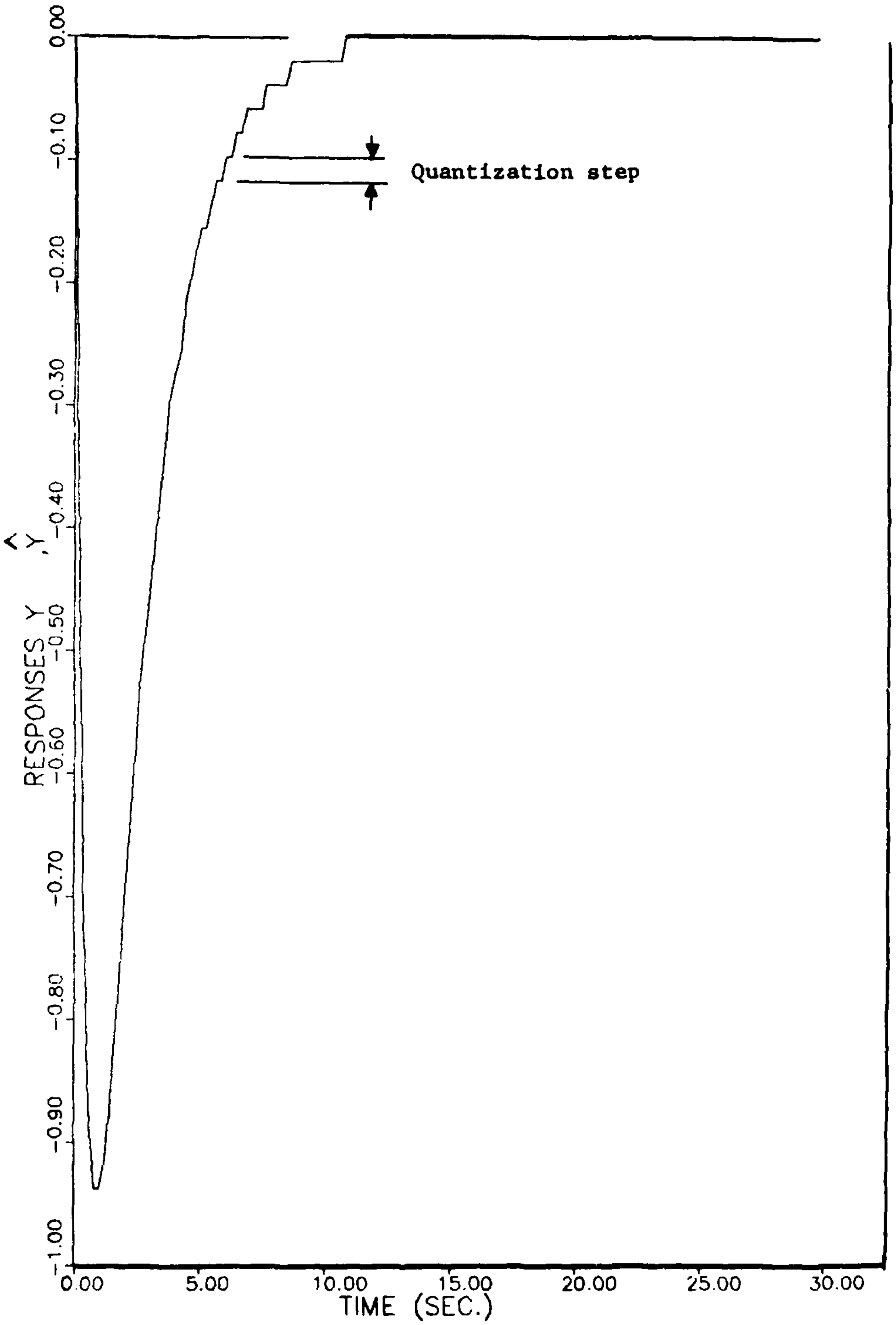


Fig. 8c. Quantized signal used for the determination of model transfer function (see Fig. 8a and 8b for model response)

A more impressive improvement is achieved in the next experiment.

Experiment 2

For XMSB = 5.0 Volts and NDIG = 7 , the program yields the following results

$H_{quant}(z)$ (using ISPN = 1, IFIX = -1, NFIX immaterial) $\nu = 0.0061$

Denominator

1	-1.721641	0.743261

Numerator

0	0.571452	-0.263324

$\hat{H}(z)$ (using ISPN = 1, IFIX = 1, NFIX = 1; IBIAS = 0) $\nu = 0.0047$

Denominator

1	-1.703728	0.726890

Numerator

0	0.550597	-0.219990

$\bar{\hat{H}}(z)$ (using ISPN = 1, IFIX = 1, NFIX = 1; IBIAS = 1) $\nu = 0.0042$

Denominator

1	-1.696989	0.720718

Numerator

0	0.543307	-0.204371

Example 7

A fourth order network with zero direct transmission is simulated. The impulse response

$$y(t) = 10\ e^{-2t}\ \mathrm{Sin}(2t) - 2\ e^{-0.5t}\ \mathrm{Sin}(4t)$$

is sampled uniformly at intervals $\Delta = 0.2$ sec. apart. The coefficients of the transfer function $H_{ideal}(z)$ are

Denominator

1	-2.495629	2.824925	-1.577498	0.367879

Numerator

0	1.312168	-1.688152	1.553863	0

Without quantization the modeling program yields (using ISPN = -1, IFIX = -1, NFIX immaterial) the following results:

$H(z)$ (using ISPN = -1, IFIX = -1, NFIX immaterial) $\nu = 0.1E-6$

Denominator

1	-2.495883	2.825209	-1.577598	0.367863

Numerator

0	-1.312376	1.689499	-1.555676	0.001578

For XMSB = 5.0 Volts and NDIG = 10, the program yields the following results

$H_{quant}(z)$ (using ISPN = 1, IFIX = -1, NFIX immaterial) $\nu = 0.076$

Denominator

1	-3.003323	3.619004	-2.057970	0.455114

Numerator

0	-1.290046	2.315347	-1.991812	0.834767

H(z) (using ISPN = 1, IFIX = 1, NFIX = 1; IBIAS = 0) $\nu = 0.045$

Denominator

1	-2.930222	3.483747	-1.979050	0.440495

Numerator

0	-1.368196	2.599335	-2.567285	1.185892

H(z) (using ISPN = 1, IFIX = 1, NFIX = 1; IBIAS = 1) $\nu = 0.040$

Denominator

1	-2.910943	3.447190	-1.956858	0.436115

Numerator

0	-1.393888	2.690009	-2.731991	1.280967

Clearly, a reduction in energy error has been achieved via statistical correction.

Remarks

The application of the statistical correction was predicated upon several assumptions. Experiments show that these assumptions are not satisfactorily met. The following comments therefore arise.

- The quantization error process e(k) is not white. It might be useful in future work to model this error process as a first order process and estimate the corner frequency of this process together with its intensity.

- The correlation between the quantization error e(k) and the input signal y(k) is not zero. This may be ameliorated by the use of a well known technique [13] namely the addition and, after quantization, the subtraction of a dither signal[2]. This is shown in Fig. 9 . The application of this

[2]Pseudo-random binary signals are often used as dither signals.

technique to our problem, and the extent of improvement achieved [14], remain subjects of future investigation.

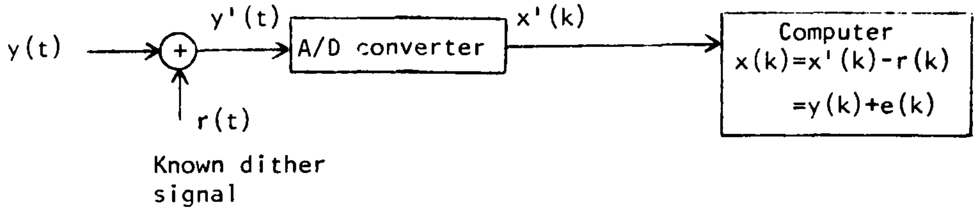


Fig. 9 Use of dither signal to decorrelate y(k) and e(k)

- In estimating the intensity (variance) of noise via equation (36) only the first two terms of the determinant expansion were retained. Perhaps three terms, i.e., constant linear and quadratic, should be retained in order to get a more accurate estimate of σ^2. However, we feel that the benefit of this step would be realized only after the steps 1 and 2 stated above have been taken.

4. WIDEBAND IDENTIFICATION

Determining the transfer function of a network from its observed input-output responses represents the inverse of the analysis problem. Interest in this problem arises from the frequent need for a relatively simple mathematical description of the system so that behavior for other anticipated inputs may be predicted up to acceptable accuracies. However, the identification of wideband networks presents some unique difficulties. Consider, for example, a network whose frequencies of interest range from f_o Hz to $(10^5)f_o$ Hz. To identify the corner frequencies at the low end, one would require an observation record of length $T \approx 1/f_o$ sec. On the other hand, in order to avoid aliasing effects the sampling rate must be chosen in excess of $2(10^5)f_o$, say $f_s = (10^6)f_o$. A million samples of data for both input and output are thus produced. Apart from the difficulties of storing this staggering amount of data and the impracticability of processing them, serious numerical difficulties also arise from this simple minded approach to identification; for instance, the low frequency poles cannot be represented in z-domain accurately even with a 64-bit computer word. A possible remedy is to break the problem into two or three smallband[3] problems. The network dynamics can be identified for each of these, and this information can be used to estimate the wideband transfer function.

A multirate sampling approach to identification of wideband transmittances is presented in this section. It permits efficient

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[3] A frequency band of less than two decades will be termed as smallband.

determination of the transfer function of a four-to-five decade bandwidth system from transient tests. Clever selection of sampling rates and exciting inputs reduces the wideband problem into three, simpler smallband problems. Each smallband problem encompasses only one-to-two decades of bandwidth. The three transfer functions $H_L(s)$, $H_M(s)$, and $H_H(s)$ are easily identified via the pencil-of-functions method, and then adjoined to build the wideband transfer function estimate $H(s)$. The technique is demonstrated by simple illustrative examples and a realistic RF amplifier example.

Frequency regions (sub-bands)

The concept of small band descriptions begins by splitting the wideband region into three regions. As shown in Fig. 10, these regions may be termed as low-frequency band, medium frequency band, and the high-frequency band; in short, LF, MF, and HF.[4] These regions may be chosen covering approximately equal ranges on the logarithmic scale. Denote the band edges as f_o, f_1, f_2 and f_3, and the respective mid-region frequencies as f_L, f_M, and f_H. The latter may be -- although not necessarily, chosen as the geometric means of the band edge frequencies.

By design, the following inequalities hold

$$0.1f_L < f_o < f_L < f_1 < 10f_L \quad (40a)$$

$$0.1f_M < f_1 < f_M < f_2 < 10f_M \quad (40b)$$

$$0.1f_H < f_2 < f_H < f_3 < 10f_H \quad (40c)$$

[4] In some cases where prior knowledge of the approximate frequency characteristic of the network is available, it may be more appropriate to choose the regions as LF, LMF and HF, or as LF, MHF and HF.

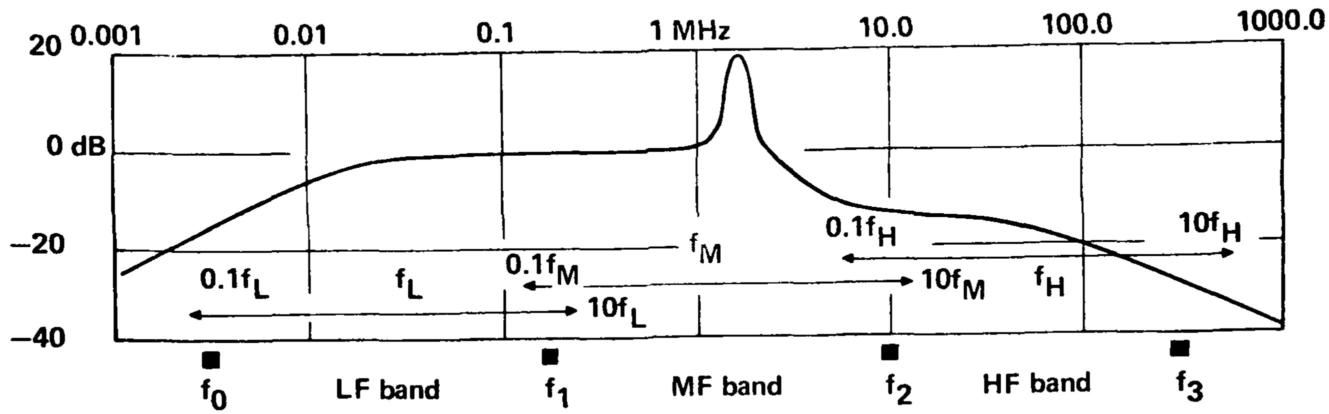

Fig. 10 Definition of LF, MF and HF frequency bands

Sampling Rates

For the three small band problems the sampling intervals are chosen as[5]

$$\Delta_L = 1/100f_L \quad (41a)$$

$$\Delta_M = 1/100f_M \quad (41b)$$

$$\Delta_H = 1/100f_H \quad (41c)$$

The sampling rates are of course the reciprocals of these numbers. By this choice -- and in view of (40), the sampling rates become at least ten times the highest frequency of interest in the respective bands. If the system input are selected so as to excite frequencies only in one of the bands, then by using the prescribed sampling rate aliasing effects would be avoided.

Now, if K, the number of samples used for the identification procedure, is taken as 1000, the length of the record would be 2π times the longest time constant of the band under consideration (for example, with this choice of K for the LF case, $T_L = 1000\Delta_L = \frac{10}{f_L} \approx \frac{1}{f_o} = 2\pi\tau_o$). Such record lengths are considered adequate for practical identification of low edge corner frequencies, and storing and processing 1000 samples of data is well within today's minicomputer capability.

Inputs

The key to the conversion of the wideband problem to three small band problems is the careful selection of inputs which excite

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[5] These are conservative values in anticipation of 500 to 2000 data points. Larger values up to five times, and accordingly fewer data points, may be used with some caution.

frequencies essentially limited to one of the bands. At first this would seem to pose no real difficulty, for we can choose a narrowband signal for the test. However, a little thought would reveal that testing with very narrowband signals would be in direct conflict with the basic philosophy of system identification, which is broadband modeling with transient tests. Therefore a judicious compromise must be made between these conflicting requirements.

The following inputs are suggested as a rough guide. Experimentation and experience leads to a much richer variety of signals which meet the above compromise strategy. Two different considerations have been kept in mind in the selection of these inputs: the spectral requirement stated above and, equally important, easy realizability in the laboratory.

a) LF Input -

For the low frequency band the input selected is a triangular pulse, either a full cycle $TR_{+,-}(t)$ or[6] a half cycle $TR_{+}(t)$ (see Fig. 11). In either case, the total duration of the pulse is taken to be $T_L/2$ and the pulse is followed by zero input for the remainder of the time, i.e., from $T_L/2$ to T_L. The magnitude spectra of these inputs can be shown to be

$$|TR_{+,-}(f)| = \frac{T_L}{4\Delta_L} \left| \frac{\text{Sin}\pi f T_L/8}{\pi f T_L/8} \right|^2 |\text{Sin}2\pi f T_L/8| \tag{42}$$

$$|TR_{+}(f)| = \frac{T_L}{4\Delta_L} \left| \frac{\text{Sin}\pi f T_L/4}{\pi f T_L/4} \right|^2 \tag{43}$$

where, keeping (15a) in mind, $T_L = K/100f_L$.

Figure 11. Input waveforms for LF tests.

[6] For networks which pass d.c., $TR_{+,-}(t)$, i.e., a full cycle triangular pulse, is recommended; this reduces the predominance of a d.c component in the network response.

Unit peak values for the pulses have been assumed. The amplitude spectra are tabulated[7] in Tables 1 and 2

TABLE 1

Magnitude Spectra of +,- triangular pulse ($\wedge\!\!\vee\!-$)

f/f_L	$\lvert TR_{+,-}(f) \rvert$ K=20	K=200	K=1000	K=2000
0.01	-56.1 dB	-36.1 dB	-22.1 dB	-16.1 dB
0.1	-36.1	-16.1	-3.5	-1.8
0.5	-22.1	-3.5	-16.1	-29.8
1.0	-16.1	-1.8	-29.8	$-\infty$
2.0	-10.3	$-\infty$	$-\infty$	$-\infty$
10.0	-1.8	$-\infty$	$-\infty$	$-\infty$
max in band	-1.8	-1.8	-1.8	-1.8

Note: zero dB corresponds to a magnitude of $2b/\Delta_L = K/4$

TABLE 2

Magnitude spectra of a + triangular pulse ($\wedge\!\!-$)

f/f_L	$\lvert TR_{+}(f) \rvert$ K=20	K=200	K=1000	K=2000
0.01	-0.0 dB	-0.0 dB	-0.0 dB	-0.1 dB
0.1	-0.0	-0.1	-1.8	-7.8
0.5	-0.0	-1.8	-29.8	-35.8
1.0	-0.1	-7.8	-35.8	$-\infty$
2.0	-0.3	$-\infty$	$-\infty$	$-\infty$
10.0	-7.8	$-\infty$	$-\infty$	$-\infty$
max in band	0.0	-0.1	-1.8	-7.8

Note: zero dB corresponds to a magnitude of $2b/\Delta_L = K/4$

[7] Minus infinity is used whenever the spectral amplitude is less than -200 dB below reference level.

It is clear from Tables 1 and 2 that the spectra of these pulse inputs diminishes to -30 dB or more (below in-band maxima) at the LF-MF boundary, provided N is chosen greater than or equal to 200[8]. This insures that the frequencies in the MF region are not excited by application of these inputs. A possible exception is the case where there is a sharp resonant peak in the MF band, particularly at the LF-MF boundary. However, the presence of such a peak is generally known before hand; such a resonant component in the output can be filtered before performing identification on the LF test data.

MF Input -

For the medium frequency band the input selected is an oscillatory pulse, modulated either by a decaying exponential OEX(t) or by a diminishing one-quarter-cycle triangular wave OTR(t). In either case, the frequency of oscillation is taken to be f_M, the center frequency of the band. The duration of the oscillation is taken to be $T_M/2$ (see Fig. 12), followed by zero input for the remainder of the time, i.e., from $T_M/2$ to T_M. In presenting the spectral analysis below it is assumed that the on-set of the pulse begins with the maxima of the oscillation, i.e., the pulse is triggered at its maximum value. Thus $u(t) = m(t) \text{ Cos } 2\pi f_M t$ where m(t) denotes the modulating envelope. The spectra of these inputs can be shown to be as follows:

$$|OEX(f)| = \frac{1}{2} \left| M(f+f_M) + M(f-f_M) \right| , \tag{44}$$

$$M(f) = \frac{1}{(a+j\omega)} \left[1 - e^{-(a+j\omega)T_M/2} \right]$$

$$|OTR(f)| = \frac{1}{2} \left| M(f+f_M) + M(f-f_M) \right| ,$$

$$M(f) = \frac{1}{j\omega} \left[1 - \frac{\text{Sin}\pi T_M f/2}{\pi T_M f/2} (\text{Cos}\pi T_M f/2 - j\text{Sin}\pi T_M f/2) \right] \tag{45}$$

[8]If the sampling interval were chosen five times the value suggested in (41a) the magnitude spectrum diminishes to -30 dB at the LF-MF boundary even for N=20.

where $\omega=2\pi f$, 'a' is the inverse time-constant associated with the exponential decay and, keeping (41b) in mind, $T_M=K/100f_M$. Unit peak values have been assumed.

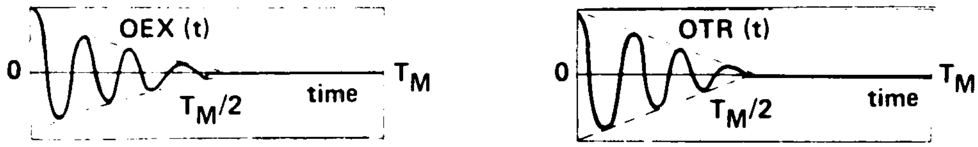


Figure 12. Input waveforms for MF test.

In order to delineate the spectral characteristics of the input OEX(t), three different values of 'a' will be considered: a=0, $a=2/T_M$, and $a=4/T_M$. The corresponding waveshapes will be denoted as $OEX_0(t)$, $OEX_1(t)$ and $OEX_2(t)$, respectively. The amplitude spectra of $OEX_0(t)$, $OEX_1(t)$, $OEX_2(t)$ and OTR(t) are tabulated in Tables 3 to 6 respectively.

TABLE 3

Magnitude Spectra of an Oscillatory pulse

	$\lvert OEX_0\ (f)\rvert$			
f/f_M	K = 20	K = 200	K = 1000	K = 2000
0.010	0.1 dB	-74.0 dB	-74.0 dB	-74.1 dB
0.100	0.1	-34.0	-37.8	$-\infty$
0.500	0.1	-7.4	-21.4	$-\infty$
0.909	0.0	-0.5	-3.6	-20.5
0.990	0.0	-0.0	-0.1	-0.2
1.000	0.0	0.0	0.0	0.0
1.010	-0.0	0.0	-0.0	-0.1
1.100	-0.0	0.3	-3.5	$-\infty$
2.000	-0.4	$-\infty$	$-\infty$	$-\infty$
10.000	-29.1	$-\infty$	$-\infty$	$-\infty$

Note: zero dB corresponds to the resonant peak at f_M. Minus infinity is used whenever the spectral amplitude is less than -200dB below reference level.

TABLE 4

Magnitude spectra of an Oscillatory exp. pulse

f/f_M	$\lvert OEX_1(f)\rvert$ K = 20	K = 200	K = 1000	K = 2000
0.010	-1.7 dB	-26.2 dB	-53.1 dB	-63.1 dB
0.100	-1.6	-23.4	-36.7	-49.7
0.500	-1.1	-6.9	-20.7	-33.5
0.909	-0.2	-0.5	-3.4	-14.6
0.990	-0.0	-0.0	-0.1	-0.2
1.000	0.0	0.0	0.0	0.0
1.010	0.0	0.0	0.0	-0.1
1.100	0.2	0.3	-3.2	-15.7
2.000	1.2	-13.6	-27.4	-33.5
10.000	-12.9	-29.9	-43.8	-49.9

Note: zero dB corresponds to the resonant peak at f_M

TABLE 5

Magnitude spectra of an Oscillatory exp. pulse

f/f_M	$\lvert OEX_2(f)\rvert$ K = 20	K = 200	K = 1000	K = 2000
0.010	-0.0 dB	-15.0 dB	-41.7 dB	-53.2 dB
0.100	-0.0	-14.2	-34.0	-43.4
0.500	-0.0	-5.6	-19.1	-27.4
0.909	-0.0	-0.4	-2.9	-9.8
0.990	-0.0	-0.0	-0.1	-0.2
1.000	0.0	0.0	0.0	0.0
1.010	0.0	0.0	0.0	-0.1
1.100	0.0	0.2	-2.6	-10.0
2.000	-0.1	-8.1	-21.4	-27.4
10.000	-9.3	-24.1	-37.8	-43.8

Note: zero dB corresponds to the resonant peak at f_M

TABLE 6

Magnitude Spectra of an Oscillatory triangular pulse

f/f_M	|OTR(f)| K = 20	K = 200	K = 1000	K = 2000
0.010	0.1 dB	-38.0 dB	-51.9 dB	-58.1 dB
0.100	0.1	-18.0	-36.3	-43.8
0.500	0.1	-4.3	-21.2	-27.4
0.909	0.0	-0.3	-2.3	-8.7
0.990	0.0	-0.0	-0.0	-0.1
1.000	0.0	0.0	0.0	0.0
1.010	-0.0	0.0	0.0	-0 1
1.100	-0.0	0.2	-2.1	-9.5
2.000	-0.3	-7.6	-21.4	-27.4
10.000	-9.3	-23.9	-37.8	-43.8

Note: zero dB corresponds to the resonant peak at f_M

It is clear from Tables 3 and 4 that the spectra of pulses $OEX_0(t)$ and $OEX_1(t)$ diminish to -25 dB or more at the Mf-LF and MF-HF boundaries provided N is chosen greater than or equal to 200[9]. This insures that the frequencies in the LF and HF regions are not excited significantly by application of these inputs. Tables 5 and 6 show that OEX_2 and OTR spectra diminish only to -15 dB at these boundaries; these pulses are useful in the initial stages of testing, or when the network's corner frequencies are spread over the band.

HF Input -

The inputs used for the MF band are equally useful for the HF band with T_M replaced by T_H. Tables 3 to 6 also hold with f_M replaced by f_H.

[9] If the sampling interval is chosen five times the value suggested in (41b), the magnitude spectrum diminishes to -30 dB at the boundaries even for N=20.

Small band Identification

Input-output data obtained from smallband tests can be analyzed by use of the Fortran program IGRAM [5] and the s-domain smallband transfer functions obtained therefrom. The program, however, requires that the transfer function order (degree of the denominator polynomial) be specified. If the order of a smallband transfer function is known from circuit considerations, then the identification is performed for this order and for at least one order higher and lower. For example, if the LF band behavior is expected to be of order 4, then identification should be performed for n=3, 4 and 5. The lowest order model yielding satisfactory fractional error (see page 15) should be accepted as the model for that smallband. If, on the other hand, the smallband order is not known, then an upward modeling strategy must be adopted. Starting from an initial order, a low guess, increasingly higher orders are attempted until the fractional error in identification turns out to be acceptably small.

Thus, the smallband transfer functions $H_L(s)$, $H_M(s)$ and $H_H(s)$ become available. From these the overlapping critical frequencies, or ideally speaking common critical frequencies, are carefully isolated. This isolation of common critical frequencies is useful in the next, and final, step in wideband identification.

Adjoined Wideband Transfer Function

The transfer functions obtained from the smallband tests must be adjoined to form the overall wideband transfer function. For convenience we will drop the hat (carat) on the identified TFs, smallband or wideband. The reader must, however, bear this in mind.

In the notation to follow we will use C to denote gain constant; and H with suitable subscripts to denote transfer functions, which are assumed to be in the Bode canonical form

$$C \; \frac{s^{\ell}(1 + s/z_1) \; \ldots . \; (1 + s/z_m)}{(1 + s/p_1) \; \ldots . \; (1 + s/p_n)} \qquad (\ell = 0,\ \text{positive or negative integer}) \tag{46}$$

The first subscript (on H) refers to the test from which the transfer function is obtained; the second subscript, if any, denotes the band with which the critical frequencies (poles and/or zeroes) are shared. For example, $H_{MH}(s)$ denotes the part of $H_M(s)$ whose critical frequencies are shared with H (s). Ideally speaking, of course, $H_{MH}(s) = H_{HM}(s)$.

Thus we have

$$H_L(s) = C'_L \; H_{LL}(s) \; H_{LM}(s) \tag{47a}$$

$$H_M(s) = C'_M \; H_{ML}(s) \; H_{MM}(s) \; H_{MH}(s) \tag{47b}$$

$$H_H(s) = C'_H \; H_{HM}(s) \; H_{HH}(s) \tag{47c}$$

Critical Frequency Adjustment -

In practice the overlapping critical frequencies in two small-band tests will not turn out to be identical through the corresponding identifications. For example,a critical frequency s'_i (pole, or zero) common to $H_M(s)$ and $H_H(s)$, may be identified as s_{i-} in the MF identification and as s_{i+} in the HF identification. We will adjust both of them to a common value given by their geometric mean

$$s_i = \sqrt{s_{i-} \; s_{i+}} \tag{48}$$

Assume that this process has been performed on H_{LM} and H_{ML}, and likewise on H_{MH} and H_{HM}. In order to avoid unduly complicated notation we will let the original symbols denote these adjusted transfer functions, so that now

$$H_{LM}(s) = H_{ML}(s) \tag{49a}$$

$$H_{MH}(s) = H_{HM}(s) \tag{49b}$$

Other adjustments include setting s_i to 0 when it turns out to be well below $2\pi f_-$, where f_- denotes the left boundary of the frequency band, but is known to be zero from circuit considerations; care should be taken in this case to let $s_i(1 + s/s_i)$ to s. Thus, in the canonical form of (46) the gain should be divided by s_i when the term $(1 + s/s_i)$ is replaced by s. Another case of adjustment occurs when s_i turns out to be much larger than $2\pi f_+$, where f_+ denotes the right boundary of the frequency band; it is then useful to set the term $(1 + s/s_i)$ to just 1. These two types of adjustments occur in Example 8 on page 47.

Gain Adjustment

To obtain equalization at the boundaries of the frequency bands the gains are adjusted as follows.

$$C_L = C_L' \left|\frac{H_M(\lambda_1)H_H(\lambda_2)}{H_L(\lambda_1)H_M(\lambda_2)}\right|^{1/2} \tag{50a}$$

$$C_M = C_M' \left|\frac{H_L(\lambda_1)H_H(\lambda_2)}{H_M(\lambda_1)H_M(\lambda_2)}\right|^{1/2} \tag{50b}$$

$$C_H = C_H' \left|\frac{H_L(\lambda_1)H_M(\lambda_2)}{H_M(\lambda_1)H_H(\lambda_2)}\right|^{1/2} \tag{50c}$$

where[10]

$$\lambda_1 = j2\pi f_1$$

$$\lambda_2 = j2\pi f_2$$

Recall that f_1 is the boundary between the LF and MF regions, and f_2 is the boundary between the MF and HF regions (see Fig. 10, page 36).

As stated earlier, the purpose of this gain adjustment is to minimize gain discontinuity at the boundaries. However, phase mismatch may still exist at these boundaries for the redefined smallband transfer functions. These transfer functions are

$$H_L(s) = C_L\ H_{LL}(s)H_{LM}(s) \tag{51a}$$

$$H_M(s) = C_M\ H_{ML}(s)H_{MM}(s)H_{MH}(s) \tag{51b}$$

$$H_H(s) = C_H\ H_{HM}(s)H_{HH}(s) \tag{51c}$$

Wideband Transfer Function -

The wideband transfer function is taken to be

$$H(s) = C\ H_{LL}(s)H_{LM}(s)H_{MM}(s)H_{HM}(s)H_{HH}(s) \tag{52}$$

---

[10] For certain wideband networks only two smallbands, LF and HF (with boundary frequencies f_o, f_1 and f_2), might be necessary. In such cases only (50a) and (50c) are needed with H_M deleted and $\lambda_1=\lambda_2=j2\pi f_1$; likewise, only (51a) and (51c) are needed wherein the subscript M is replaced by the subscript H.

where the constant C is selected to match the gain of one of the smallband transfer functions at a chosen frequency, perhaps $H_M(s)$ at the midband frequency f_M.

Example 8 -

Consider an a.c. coupled network believed to have frequencies of interest from 0.02 MHz to 50 MHz, thus encompassing 3.3 decades. The parameters of the system are given in Table 7. In this case it is adequate to break up the frequency region into two smallbands as follows:

$$\text{LF} \qquad f_o = 0.02 \text{ MHz to } f_1 = 1.0 \text{ MHz} \qquad f_L = 0.1 \text{ MHz} \tag{53a}$$

$$\text{HF} \qquad f_1 = 1.0 \text{ MHz to } f_2 = 50.0 \text{ MHz} \qquad f_H = 10.0 \text{ MHz} \tag{53b}$$

The inequalities in (40) are clearly satisfied. From (41) and Tables 1 and 3 it appears reasonable to choose[11]

$$\Delta_L = 0.1\ \mu s \qquad K_L = 200 \qquad T_L = 20\ \mu s$$

$$\Delta_H = 0.001\ \mu s \qquad K_H = 200 \qquad T_H = 0.2\ \mu s$$

and the inputs $u_L(t) = TR_{+,-}(t)$, $u_H(t) = 0EX_0(t)$. We, however, select

$$\Delta_L = 0.05\ \mu s \qquad K_L = 200 \qquad T_L = 10\ \mu s \tag{54a}$$

$$\Delta_H = 0.002\ \mu s \qquad K_H = 100 \qquad T_H = 0.2\ \mu s$$

and define the test pulses explicitly as follows

$$u_L(t) = \begin{cases} \text{One complete cycle triangular wave over 0 to 5 sec.} \\ \text{0 level over 5 to 10 sec.} \end{cases} \tag{55a}$$

$$u_H(t) = \begin{cases} \text{Cos}\left(\frac{2\pi}{0.05} t\right), & 0 \le t \le 0.1 \\ 0 & 0.1 < t \le 0.2 \end{cases} \tag{55b}$$

To simulate the LF test response, the system function $H_{true}(z)$ corresponding to $\Delta_L' = 0.005$ μsec. is excited by $u_L(k)$ of (55a); the response is then resampled at 1/10th rate (i.e., every 10th output sample is picked up). In a laboratory test this artifice of using a high sampling rate H(z) to preserve the integrity of the network response, and then resampling the output, is of course not necessary. The network output can be sampled directly at the desired rate $1/\Delta_L$.

---

[11] Recall, Δ denotes sampling interval, K the total number of samples and T the total duration of the test. Of course $T = K\Delta$.

Table 7

A Wideband System: Example 8

$$H_{true}(s) = \frac{(10^6)s}{(s+10^6)(s+10^8)}$$

$\Delta_L' = 0.005\mu s$

$$H_{true,\Delta_L'}(z) = \frac{0.003924z^{-1}(1-z^{-1})}{1-1.601543z^{-1}+0.603505z^{-2}}$$

$\Delta_H = 0.002\mu s$

$$H_{true,\Delta_H}(z) = \frac{0.0018108z^{-1}(1-z^{-1})}{1-1.8167327z^{-1}+0.81709493z^{-2}}$$

The results of identification from IGRAM using the method of Subsection 2.1 are given below.

LF Test -

Both first and second order identifications were performed. Since the first order model (predicted response) has a fit error of $\nu = 0.8E-4$, it is decided the LF behavior is first order. For n=1 the Gram matrix, the square-roots of the diagonal cofactors, the z-domain model and the corresponding s-domain model (using pulse-invariant conversion [5], pages 80-82) are given below

$$F = \begin{bmatrix} 0.00074285 & 0.00037153 & -0.075250 & 5.1001 \\ & 1.03620000 & -5.198800 & -354.5100 \\ & & 33.336000 & 1266.7000 \\ & & & 346090.00 \end{bmatrix}$$

det F = 0.128

The values of $\sqrt{D_i/D_1}$ are

1	0.050249	0.010103	-0.7329E-6

$$\hat{H}_L(z) = \frac{0.0096187(1-z^{-1})}{1-0.95215z^{-1}}$$

$$\hat{H}_L(s) = \frac{0.0101(s-0.14(10^4))}{(s+0.981(10^6))} \simeq \frac{1.0297(10^{-8})s}{(s/0.981(10^6)+1)} \quad \text{(Frequency adjusted)}$$

HF test -

As in the LF case, here also a first order model is found adequate producing a fit error (fractional energy error) ν = 0.40E-4. For n=1 the Gram matrix, the square-roots of the diagonal cofactors, the z-domain model and the corresponding z-domain model (using pulse-invariant conversion) are given below.

$$F = \begin{bmatrix} 0.00090304 & 0.00045161 & -0.073612 & -0.52301 \\ & 0.01842400 & 0.449400 & -1.61760 \\ & & 25.000000 & 12.50000 \\ & & & 410.38000 \end{bmatrix}$$

$$\det F = 0.22E-4$$

The values of $\sqrt{D_i/D_1}$ are

1	0.22817	-0.0023153	0.0022408

$$\hat{H}_H(z) = \frac{-0.00006 + 0.00188517z^{-1}}{1 - 0.814217z^{-1}}$$

$$\hat{H}_H(s) = \frac{0.0023s + 1.009(10^6)}{s + 102.76(10^6)} \simeq \frac{0.00982}{s/1.028(10^8) + 1} \qquad \text{(Frequency adjusted)}$$

Gain Adjustment -

At $s = j2\pi f_1$, where f_1=1MHz, the gains of the LF and HF transfer functions turn out to be 0.00998 and 0.00981 respectively. The adjusted gain constants (using (50) and (51)) are $C_L=1.021(10^{-8})$ and $C_H=0.00990$.

Wideband Transfer Function -

The wideband transfer function is

$$H(s) = C \frac{s}{(s/0.981(10^6) + 1)(s/1.028(10^8) + 1)}$$

where $C=1.023(10^{-8})$ is obtained from gain matching at $s=j2\pi f_1$; f_1=1MHz.

Comparison -

The Bode plots of $\hat{H}(s)$ and $H_{true}(s)$ (of Table 7) are compared in Fig. 13. It appears that satisfactory wideband identification has been achieved.

Remark

The procedure of adjoining the smallband transfer functions can of course be programmed.

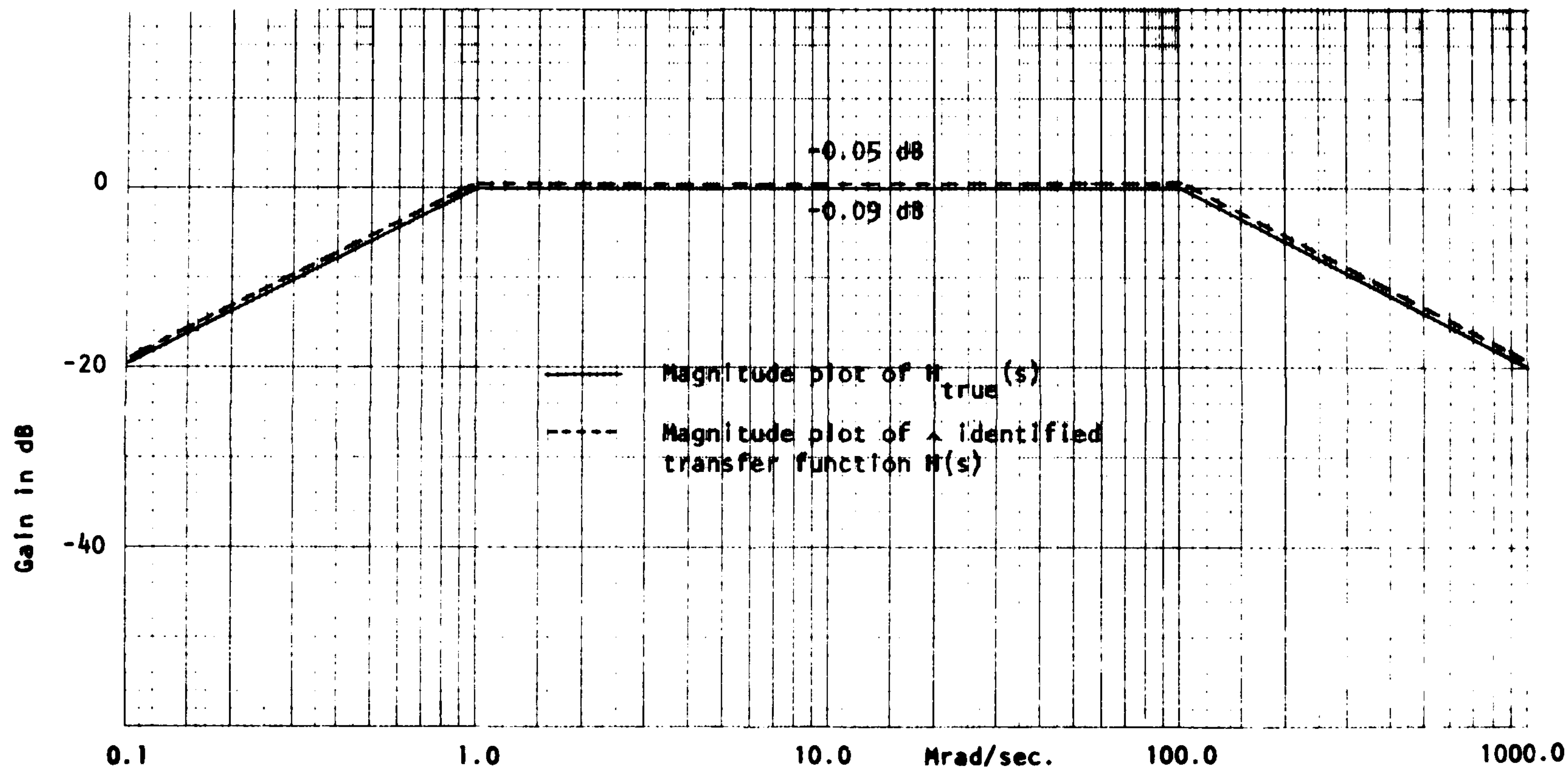


Fig. 13 Comparison of the magnitude (Bode) plots of the identified transfer function and the network function of a wideband system

Example 9

As a second example of wideband identification consider the RF amplifier (Fig. 14) of reference [5]. The frequency regions are

LF	f_o=0.002 MHz to f_1=0.1 MHz	f_L=0.01 MHz
MF	f_1=0.1 MHz to f_2=10 MHz	f_M=1.0 MHz
HF	f_2=10 MHz to f_3=1000 MHz	f_H=100 MHz

The smallband transfer functions, identified from LF MF HF tests through IGRAM [5] are

$$\hat{H}_L(s) = -20.125 \frac{(s-0.0015(10^6))(s+0.0012(10^6))}{(s+0.034(10^6))(s+0.075(10^6))}$$

$$\simeq -0.7892(10^{-8}) \frac{s^2}{[s/0.034(10^6)+1][s/0.075(10^6)+1]}$$

$$\hat{H}_M(s) = -520(10^6) \frac{1}{(s+24.92(10^6))}$$

$$-20.55 \frac{1}{(s/25.31(10^6)+1)}$$

$$H_H = 2.79(10^7) \frac{(s-19060(10^6))}{(s+25.7(10^6))(s+1140(10^6))}$$

$$18.432 \frac{(-s/19606(10^6)+1)}{[s/25.31(10^6)+1][s/1140(10^6)+1]}$$

The second step of each of the above is obtained after frequency adjustment as outlined on page 45.

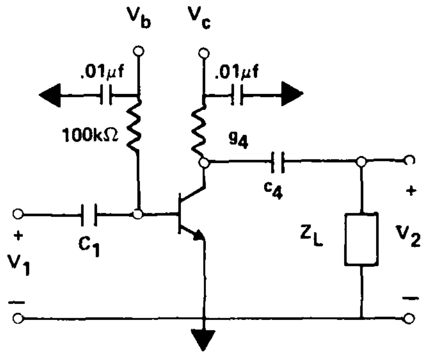
Gain Adjustment -

At f_1=0.1 MHz the gains of the LF and MF transfer functions turn out to be 19.9536 and 20.5437 respectively. At f_2=10 MHz the gains of the MF and HF transfer functions are computed to be 7.6776 and 6.8759 respectively. The adjusted gain constants (using (50) and (51)) are $C_L=(0.9602)C_L'=0.7578$, $C_M=(0.9327)C_M'=19.17$, and $C_H=(1.0414)C_H'=19.19$.

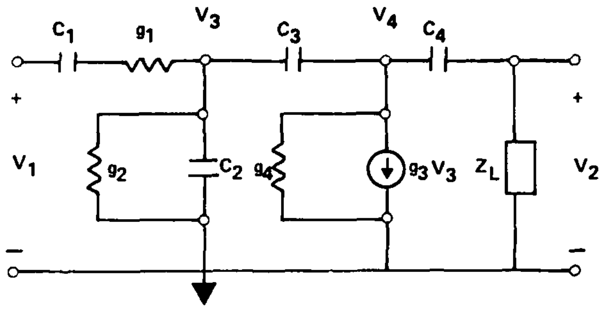
Wideband Transfer Function -

The transfer function of the network is estimated as

$$\hat{H}(s) = -C \frac{s^2[-s/19060(10^6)+1]}{[s/0.034(10^6)+1][s/0.075(10^6)+1][s/25.31(10^6)+1][s/1140(10^6)+1]}$$



(a) Schematic



(b) Equivalent circuit

$C_1=0.01\ \mu F$ $\quad C_3=5\ pF$ $\quad g_1=4$ mmho $\quad g_3=40$ mmho $\quad Z_L=1$ Kohm (real)

$C_2=50\ pF$ $\quad C_4=0.01\ \mu F$ $\quad g_2=1$ mmho $\quad g_4=0.5$ mmho

$$H(s) = \frac{8(10^7)\ s^2(s-8000(10^6))}{(s+0.033(10^6))(s+0.08(10^6))(s+25.2(10^6))(s+1205.1(10^6))}$$

$$= \frac{-\ 7.983(10^{-9})[s/8000.0(10^6)+1]}{[s/0.033(10^6)+1][s/0.08(10^6)+1][s/25.2(10^6)+1][s/1205.1(10^6)+1]}$$

Fig. 14 A common-emitter wideband amplifier where $C=8.0596(10^{-9})$ is obtained from gain matching with $\hat{H}_M(s)$ at $s=j2\pi f_M$, $f_M=1$ MHz.

Comparison -

The Bode plots of $\hat{H}(s)$ and $H_{true}(s)$ (of Fig. 14) are compared in Fig. 15. It appears that satisfactory wideband identification has been achieved from smallband time domain tests.

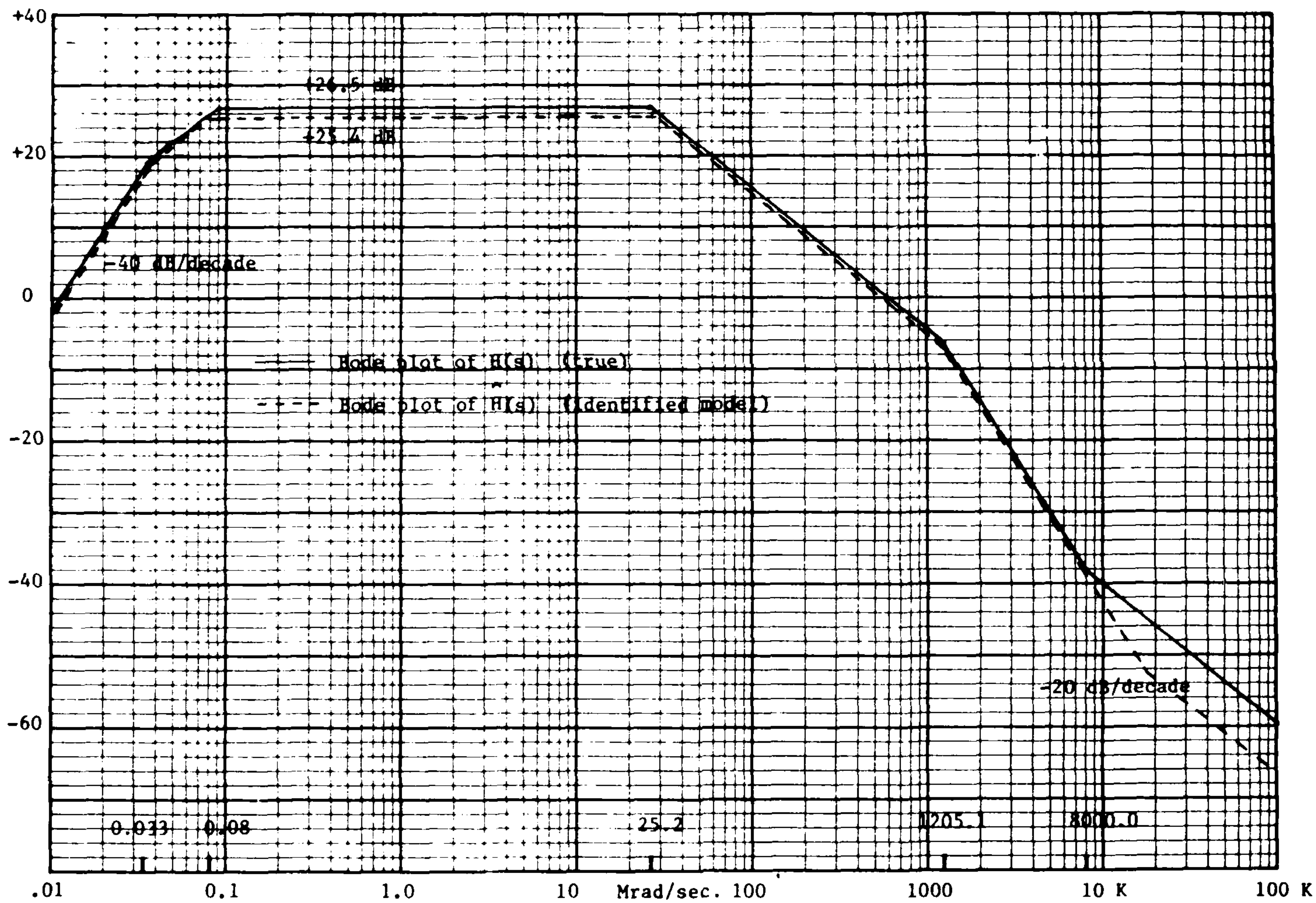


Fig. 15 Comparison of the magnitude (Bode) plots of the identified transfer function and the network function of an RF amplifier

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APPENDIX A

LISTING OF

PROGRAM

STOZ

```
C     ****************
C
C         STOZ
C
C     ****************
C
C
C
C        GIVEN THE CONTINUOUS DESCRIPTION, PROGRAM COMPUTES THE
C        EQUIVALENT DISCRETE DOMAIN DESCRIPTION OF A LINEAR
C        DYNAMIC SYSTEM
C
C        STOZ GENERATES H(Z) AND THE CORRESPONDING DIFFERENCE
C        EQUATION FROM THE TRANSFER FUNCTION H(S)
C
C        THE INPUT ARRAYS A AND B ARE FILLED ACCORDING TO THE
C        DIFFERENTIAL EQUATION
C              B(1)*Y(T)+B(2)*D(1,Y(T))+...+B(N+1)*D(N,Y(T))
C                 -A(1)*U(T)-A(2)*D(1,U(T))-...-A(N+1)*D(N,U(T)) = 0
C        WHERE D(M,F(T)) = THE MTH TIME DERIVATIVE OF FUNCTION, F
C
C        B(N+1) MUST EQUAL 1
C
C        RETURNS ARRAYS A AND B CONTAINING THE EQUIVALENT DISCRETE
C        DESCRIPTION STORED ACCORDING TO THE DIFFERENCE EQUATION
C              B(1)*Y(K)+B(2)*Y(K-1)+...+B(N+1)*Y(K-N)
C                 -A(1)*U(K)-A(2)*U(K-1)-...-A(N+1)*U(K-N) = 0
C
C        B(1) ALWAYS EQUALS 1
C
C        THE POLES OF THE CONTINUOUS DOMAIN MUST BE DISTINCT AND
C        NON-ZERO FOR THE TRANSFORMATION TO BE VALID
C        ----------------------------------------
C
C        DATA CARD SET PREPARATION
C
C        N = ORDER OF SYSTEM
C        N (MAXIMUM) = ONE LESS THAN THE DIMENSION SUBSCRIPT
C
C        IMTHD = 0 FOR THE IMPULSE INVARIANT DESCRIPTION .
C              = 1 FOR THE PULSE INVARIANT DESCRIPTION .
C              = 2 FOR THE TRAPEZOIDAL INVARIANT DESCRIPTION .
C              = 3 FOR THE LOGRITHMIC TRANSFORM DESCRIPTION .
C
C        IPOLZ = 1 IF POLES AND ZEROES ARE READ
C                 MUST BE COMPLEX (REAL,IMAGINARY)
C                 NEGATIVES OF POLES AND ZEROS READ: IE,
C                 FOR A POLZ OF (S+2), INPUT +2.0     +0.0
C                 (2F10.0 PER POLZ, 4 POLZS PER CARD)
C
C              = 0 IF DENOMINATOR AND NUMERATOR ARE READ
C                 IN POLYNOMIAL FORM.
C                 COEFFICIENTS ORDERED FROM LOW TO HIGH DEGREE,
C                 DENOMINATOR INFORMATION ALWAYS READ FIRST.
C                 HIGHEST ORDER DENOMINATOR COEFF MUST BE 1.0
C
C              NOTE:  WHEN POLE-ZERO DATA IS ENTERED A GAIN CARD
C                     MUST FOLLOW THE LAST POLE CARD.
C                     IF THERE ARE NO ZEROS, USE BLANK CARDS IN
C                     THE NORMAL ZEROS POSITIONS.
C
C        DELTA = SAMPLING INTERVAL
```

```
C
C
C         START EACH DATA CARD SET WITH A DESCRIPTION CARD,
C           CONTAINING UP TO 51 CHARACTERS COLS 2-52
C         FIRST DATA CARD CONTAINS*
C           N,IMTHD,IPOLZ,IN 3F5 FORMAT, PLUS DELTA IN F10.0 FORMAT
C         SECOND GROUP OF DATA CARDS IS N POLES OR N+1 DENOMINATOR
C           COEFFICIENTS.  AFTER LAST POLE CARD, USE A GAIN CARD.
C         LAST GROUP OF DATA CARDS IS N ZEROS (OR BLANKS).
C           OR N+1 NUMERATOR COEFFICIENTS (BLANKS FOR ZERO COEFFS)
C         THE DATA FORMAT FOR EACH OF THE SYSTEM PARAMETER CARDS
C           IS 8F10.0
C
C         AS MANY SETS OF DATA CARDS MAY BE RUN AS DESIRED
C         -----------------------------------------------------
C
C      STOZ MAIN PROGRAM
C
       REAL B(20),A(20),RR(20),RI(20),DELTA,TEMP(20)
       COMPLEX CR(20),CA(20),CB(20),CAA(20),CA1(20),
      1 TEM(20),CON1,CON2,CONT
       DIMENSION TITLE(70)
C
C      READ TITLE AND FIRST DATA CARD
C
100    READ(5,920)TITLE
       IF(EOF(5).NE.0)GO TO 5999
       WRITE(6,910)
910    FORMAT(6(/))
       WRITE(6,920)TITLE
       READ(5,920)TITLE
920    FORMAT(70A1)
       WRITE(6,930)
930    FORMAT(/,1X,71(*+*))
       READ(5,940)N,IMTHD,IPOLZ,DELTA
940    FORMAT(3I5,F10.0)
       WRITE(6,950)N,IMTHD,IPOLZ,DELTA
950    FORMAT(//3X,*N =*,I4,5X,*IMTHD =*,I4,5X,*IPOLZ =*,I4,5X,
      1 *DELTA =*,G17.10,//)
       NP1=N+1
       NP2=N+2
       NPNP1=N+N+1
       NPNP2=N+N+2
       IF(IPOLZ.NE.1) GO TO 300
C
C      READ POLES AND ZEROS
C
       READ(5,960)(CR(I),I=1,N)
960    FORMAT(8F10.0)
       CALL POLCON(CR,TEM,0,N)
       DO109 I=1,NP1
109    B(I)=TEM(I)
C
C      READ GAIN CARD
       READ(5,960) RK
C
C      READ ZEROS
       READ(5,960) (CA(I),I=1,N)
       CALL POLCON(CA,TEM,0,N)
       DO 209 I=1,NP1
209    A(I)=TEM(I)*RK
       GO TO 310
C
C      READ DENOMINATOR AND NUMERATOR COEFFICIENTS
```

```
C
300   READ(5,960) (B(I),I=1,NP1)
      READ(5,960) (A(I),I=1,NP1)
310   CONTINUE
C
C     PRINT DENOMINATOR AND NUMERATOR COEFFICIENTS
C
      WRITE(6,970)
970   FORMAT(* S-DOMAIN DENOMINATOR*)
      CALL PRVEC(B,NP1)
      WRITE(6,980)
980   FORMAT(* S-DOMAIN NUMERATOR*)
      CALL PRVEC(A,NP1)
C
C     DETERMINE ORDER OF NUMERATOR
C
      NN=N
      DO 309 I=1,NP1
      II=NP1+1-I
      IF(A(II).NE.0.0) GO TO 400
309   NN=NN-1
400   CONTINUE
      WRITE(6,990) NN
990   FORMAT(* ORDER OF S-DOMAIN NUMERATOR =*,I5,//)
      IF(NN.LT.0) GO TO 5029
      NNP1=NN+1
C
C     FACTOR DENOMINATOR TO FIND POLES
C
      IF(IPOLZ.NE.0) GO TO 500
      CALL POLRT(B,TEMP,N,RR,RI,IER)
      DO 409 I=1,N
409   CR(I)=CMPLX(RR(I),RI(I))
500   WRITE(6,991)
991   FORMAT(* POLES OF THE S-DOMAIN*)
      CALL PRCVEC(CR,N)
C
      IF(IMTHD.NE.3) GO TO 1500
C
C     LOGRITHMIC TRANSFORM
C
1000  CONTINUE
      WRITE(6,9100)
9100  FORMAT(/,* LOGRITHMIC TRANSFORM*)
C
C     WORK ON NUMERATOR
C
      IF(NN.EQ.0) GO TO 1030
      IF(IPOLZ.NE.0) GO TO 1010
      CALL POLRT(A,TEMP,NN,RR,RI,IER)
      DO 1009 I=1,NN
1009  CA(I)=CMPLX(RR(I),RI(I))
1010  WRITE(6,9200)
9200  FORMAT(* ZEROS IN  S DOMAIN*)
      CALL PRCVEC(CA,NN)
      DO 1029 I=1,NN
1029  CA(I)=CEXP(CA(I)*DELTA)
      IF(NN.EQ.N) GO TO 1100
1030  DO 1039 I=NNP1,NP1
      CAA(I)=0.000
1039  CA(I)=0.0
1100  CONTINUE
C
C     NOW THE FIRST NN ENTRIES OF CA CONTAIN THE
```

```
C     Z-DOMAIN ZEROS OF THE TRANSFER FUNCTION, WHILE THE
C     REMAINING ENTRIES ARE ZEROED OUT.
C
C     WORK ON DENOMINATOR
C
      DO 1129 I=1,N
1129  CR(I)=CEXP(CR(I)*DELTA)
C
C     NOW CR CONTAINS THE N Z-DOMAIN POLES
C
C     FORM NUMERATOR AND DENOMINATOR
C     Z-DOMAIN POLYNOMIALS
C
      IF(NN.EQ.0) CAA(1)=1.0
      IF(NN.NE.0) CALL PCSTZ(CA,CAA,C,NN)
      CALL PCSTZ(CR,CB,0,N)
C
C     NOW CB CONTAINS THE N+1 Z-DOMAIN DENOMINATOR COEFFICIENTS,
C     AND CAA CONTAINS THE NN+1 NUMERATOR COEFFICIENTS.
C
C     ADJUST DC GAIN CONSTANT
C
      A1=A(1)/B(1)
      A2=1.0
      DO 1209 I=1,NN
1209  A2=A2+CAA(I+1)
      B2=1.0
      DO 1219 I=1,N
1219  B2=B2+CB(I+1)
      FAC=A1*B2/A2
      DO 1229 I=1,NNP1
1229  CAA(I)=CAA(I)*FAC
C
C      NOW CAA CONTAINS THE ADJUSTED Z-DOMAIN NUMERATOR COEFFICIENTS
C      AND FAC CONTAINS THE GAIN FACTOR USED FOR THE ADJUSTMENT
C
      GO TO 5000
C
1500  CONTINUE
C
C     NON-LOGRITHMIC TRANSFORMATIONS
C
C     ADJUST FOR DIRECT TRANSMISSION
C     THIS ROUTINE REQUIRES THAT B(NP1) = 1.0
C
      CONT=(0.0,0.0)
      IF(NN.LT.N) GO TO 1510
      CONT=A(NP1)
      DO1509 I=1,N
1509  A(I)=A(I)-CONT*B(I)
C
C     FIND NUMERATOR CONSTANTS FOR PARTIAL FRACTION EXPANSION
C
1510  DO1529 I=1,N
      CON1=1.0
      CON2=0.0
      DO1519 J=1,N
      CON2=CON2*CR(I)+A(N-J+1)
      IF(I-J)1512,1519,1512
1512  CON1=CON1*(CR(I)-CR(J))
1519  CONTINUE
1529  CA(I)=CON2/CON1
      WRITE(6,9300)
9300  FORMAT(* NUMERATOR CONSTANTS OF THE FACTORIZED H(S)*)
```

```
      CALL PRCVEC(CA,N)
C
C     CONVERT THE FIRST CRDER PARTIAL FRACTIONS TO Z DOMAIN
C
      NMTHD=IMTHD+1
      GO TO (2000,3000,4000), NMTHD
C
C     IMPULSE INVARIANT
C
2000  DO2009I=1,N
      CA(I)=CA(I)*DELTA
2009  CR(I)=CEXP(CR(I)*DELTA)
      GO TO 4500
C
C     PULSE INVARIANT
C
3000  DO3009 I=1,N
      CON1=CEXP(CR(I)*DELTA)
      CA(I)=CA(I)*(CON1-1.0)/CR(I)
3009  CR(I)=CON1
      GO TO 4500
C
C     TRAPEZCIDAL INVARIANT
C
4000  ICHECK=2
      DO4009 I=1,N
      CON1=CEXP(CR(I)*DELTA)
      CON2=CA(I)/(CR(I)*CR(I)*DELTA*CON1)
      CONT=CONT+CON2*((1.0-CR(I)*DELTA)*CON1-1.0)
      CA(I)=CON2*(1.0-CON1)*(1.0-CON1)
4009  CR(I)=CON1
      GO TO 4500
C
C     CONSTRUCT THE Z DOMAIN DENOMINATOR
C     AND NUMERATOR POLYNOMIALS
C
4500  CONTINUE
      CALL PCSTZ(CR,CB,0,N)
      DO 4509 I=1,N
4509  CAA(I)=0.0D00
      DO 4519 K=1,N
      CALL PCSTZ(CR,CA1,K,N)
      DO 4519 J=1,N
4519  CAA(J)=CAA(J)+CA1(J)*CA(K)
      CAA(NP1)=0.0
C ******
      IF(IMTHD.NE.1)GO TO 4521
      DO 4520 I=1,N
      II=NP1-I
4520  CAA(II+1)=CAA(II)
      CAA(1)=0.0
4521  CONTINUE
C
C             ADJUST FOR DIRECT TRANSMISSION
C
C     DTXC=(0.0,0.0)
C     IF(NN.NE.N) CONT=DTXC
C     CAA(NP1)=CONT*CB(NP1)
      DO 4529 I=1,N
4529  CAA(I)=CAA(I)+CONT*CB(I)
C
C             SHIFT NUMERATOR TO COMPLETE PULSE INVARIANT TRANSFORM
C             WHEN NUMERATOR HAS LOWER ORDER THAN DENOMINATOR
C
```

```
      CONTINUE
C
C     --------------------------------------------------------------
C
C
C     PRINT THE TRANSFORMED COEFFICIENTS
C
5000  CONTINUE
C
      WRITE(6,9510)
9510  FORMAT(* POLES IN THE Z DOMAIN*)
      CALL PRCVEC(CR,N)
      WRITE(6,9520) FAC
9520  FORMAT(* GAIN FACTOR USED =*,E14.7,//)
      WRITE(6,9530)
9530  FORMAT(* ZEROS IN THE Z DOMAIN*)
      CALL PRCVEC(CA,NN)
      WRITE(6,9540)
9540  FORMAT(* Z-DOMAIN DENOMINATOR*)
      CALL PRCVEC(CB,NP1)
      WRITE(6,9550)
9550  FORMAT(* Z-DOMAIN NUMERATOR*)
      CALL PRCVEC(CAA,NP1)
      DO 5019 I=1,NP1
      B(I)=CB(I)
5019  A(I)=CAA(I)
      GO TO 130
C
5029  WRITE(6,9560)
9560  FORMAT(/,1X,*NUMERATOR ORDER LESS THAN ZERO*,//)
5999  STOP
      END
      SUBROUTINE PCSTZ(C,R2,K,N)
C
C        PCSTZ CONSTRUCTS A Z-DOMAIN POLYNOMIAL COEFFICIENT ARRAY
C     FROM AN ARRAY OF ITS ROOTS.
C
      DIMENSION C(1),R2(1)
      COMPLEX C,R2
      NP1=N+1
      DO1I=2,NP1
1     R2(I)=0.0
      R2(1)=1.0
      DO3I=1,N
      IF(I-K)6,3,6
6     DO4JJ=1,I
      J=I-JJ+1
4     R2(J+1)=R2(J+1)-C(I)*R2(J)
3     CONTINUE
      RETURN
      END
      SUBROUTINE PRCVEC(A,N)
C
C     PRCVEC PRINTS A COMPLEX VECTOR
C        C      A COMPLEX NUMBER OF THE FORM A+JB IS PRINTED ( A, B J)
C
      DIMENSION A(1)
      COMPLEX A
      IF(N.EQ.0) GO TO 100
      WRITE(6,920)
      WRITE(6,910)(A(I),I=1,N)
910   FORMAT(1X,1H(,F22.15,1H,,F22.15,3H J))
100   WRITE(6,920)
920   FORMAT(2(/))
```

```
      RETURN
      END
      SUBROUTINE PRVEC(A,N)
C
C     THIS SUBROUTINE OUTPUTS A SINGLE DIMENSIONED ARRAY
      DIMENSION A(1)
C
      WRITE(6,1)(A(I),I=1,N)
1     FORMAT(1X,10F13.5)
31    FORMAT(/)
      RETURN
      END
      SUBROUTINE POLCON(C,R2,K,N)
C
C     A POLYNOMIAL CONSTRUCTION PROGRAM NEEDED FOR ZTOS
C
      DIMENSION C(1),R2(1)
      COMPLEX C,R2,COMP
      REAL DC(2)
      EQUIVALENCE (COMP,DC)
      NP1=N+1
      DO10I=2,NP1
10    R2(I)=0.0
      R2(1)=1.0
      DO4I=1,N
      COMP=C(I)
      IF(I.EQ.K.OR.(DC(1).EQ.0.0.AND.DC(2).EQ.0.0))GO TO 4
      DO2JJ=1,I
      J=I-JJ+1
2     R2(J+1)=R2(J+1)*C(I)+R2(J)
      R2(1)=R2(1)*C(I)
4     CONTINUE
      RETURN
      END
      SUBROUTINE POLRT(XCOF,COF,M,ROOTR,ROOTI,IER)
C
C          COMPUTES THE REAL AND COMPLEX ROOTS OF A REAL POLYNOMIAL
C
C       DESCRIPTION OF PARAMETERS
C          XCOF -VECTOR OF M+1 COEFFICIENTS OF THE POLYNOMIAL
C                ORDERED FROM SMALLEST TO LARGEST POWER
C          COF  -WORKING VECTOR OF LENGTH M+1
C          M    -ORDER OF POLYNOMIAL
C          ROOTR-RESULTANT VECTOR OF LENGTH M CONTAINING REAL ROOTS
C                OF THE POLYNOMIAL
C          ROOTI-RESULTANT VECTOR OF LENGTH M CONTAINING THE
C                CORRESPONDING IMAGINARY ROOTS OF THE POLYNOMIAL
C          IER  -ERROR CODE WHERE
C                IER=0  NO ERROR
C                IER=1  M LESS THAN ONE
C                IER=2  M GREATER THAN 36
C                IER=3  UNABLE TO DETERMINE ROOT WITH 500 INTERATIONS
C                       ON 5 STARTING VALUES
C                IER=4  HIGH ORDER COEFFICIENT IS ZERO
C
      DIMENSION XCOF(1),COF(1),ROOTR(1),ROOTI(1)
C
C
C
C          LIMITED TO 36TH ORDER POLYNOMIAL OR LESS.
C          FLOATING POINT OVERFLOW MAY OCCUR FOR HIGH ORDER
C          POLYNOMIALS BUT WILL NOT AFFECT THE ACCURACY OF THE RESULT
C
C       METHOD
```

```
C           NEWTON-RAPHSON ITERATIVE TECHNIQUE.  THE FINAL ITERATIONS
C           ON EACH ROOT ARE PERFORMED USING THE ORIGINAL POLYNOMIAL
C           RATHER THAN THE REDUCED POLYNOMIAL TO AVOID ACCUMULATED
C           ERRORS IN THE REDUCED POLYNOMIAL.
C
      IFIT=0
      N=M
      IER=0
      IF(XCOF(N+1))10,25,10
   10 IF(N) 15,15,32
C
C        SET ERROR CODE TO 1
C
   15 IER=1
   20 IF(IER)201,201,200
200   WRITE(6,203)IER
203   FORMAT(1X,*ERROR CALLED FROM POLRT, IER = *,I3)
201   RETURN
C
C        SET ERROR CODE TO 4
C
   25 IER=4
      GO TO 20
C
C        SET ERROR CODE TO 2
C
   30 IER=2
      GO TO 20
   32 IF(N-36) 35,35,30
   35 NX=N
      NXX=N+1
      N2=1
      KJ1 = N+1
      DO 40 L=1,KJ1
      MT=KJ1-L+1
   40 COF(MT)=XCOF(L)
C
C        SET INITIAL VALUES
C
   45 XO=.00500101
      YO=0.01000101
C
C        ZERO INITIAL VALUE COUNTER
C
      IN=0
   50 X=XO
C
C        INCREMENT INITIAL VALUES AND COUNTER
C
      XO=-10.0*YO
      YO=-10.0*X
C
C        SET X AND Y TO CURRENT VALUE
C
      X=XO
      Y=YO
      IN=IN+1
      GO TO 59
   55 IFIT=1
      XPR=X
      YPR=Y
C
C        EVALUATE POLYNOMIAL AND DERIVATIVES
C
```

```
   59 ICT=0
   60 UX=0.0
      UY=0.0
      V =0.0
      YT=0.0
      XT=1.0
      U=COF(N+1)
      IF(U) 65,130,65
   65 DO 70 I=1,N
      L =N-I+1
      TEMP=COF(L)
      XT2=X*XT-Y*YT
      YT2=X*YT+Y*XT
      U=U+TEMP*XT2
      V=V+TEMP*YT2
      FI=I
      UX=UX+FI*XT*TEMP
      UY=UY-FI*YT*TEMP
      XT=XT2
   70 YT=YT2
      SUMSQ=UX*UX+UY*UY
      IF(SUMSQ) 75,110,75
   75 DX=(V*UY-U*UX)/SUMSQ
      X=X+DX
      DY=-(U*UY+V*UX)/SUMSQ
      Y=Y+DY
78    IF(ABS(DY)+ABS(DX)-1.0E-10)100,80,80
C
C         STEP ITERATION COUNTER
C
   80 ICT=ICT+1
      IF(ICT-500) 60,85,85
   85 IF(IFIT)100,90,100
   90 IF(IN-5) 50,95,95
C
C         SET ERROR CODE TO 3
C
   95 IER=3
      GO TO 20
  100 DO 105 L=1,NXX
      MT=KJ1-L+1
      TEMP=XCOF(MT)
      XCOF(MT)=COF(L)
  105 COF(L)=TEMP
      ITEMP=N
      N=NX
      NX=ITEMP
      IF(IFIT) 120,55,120
  110 IF(IFIT) 115,50,115
  115 X=XPR
      Y=YPR
  120 IFIT=0
122   IF(ABS(Y)-1.0E-8*ABS(X))135,125,125
  125 ALPHA=X+X
      SUMSQ=X*X+Y*Y
      N=N-2
      GO TO 140
  130 X=0.0
      NX=NX-1
      NXX=NXX-1
  135 Y=0.0
      SUMSQ=0.0
      ALPHA=X
      N=N-1
```

```
140 COF(2)=COF(2)+ALPHA*COF(1)
145 DO 150 L=2,N
150 COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)
155 ROOTI(N2)=Y
    ROOTR(N2)=X
    N2=N2+1
    IF(SUMSQ) 160,165,160
160 Y=-Y
    SUMSQ=0.0
    GO TO 155
165 IF(N) 20,20,45
    END
```

```
C       ******************************
C
C                         PROGRAM "GQUANT"
C                         IMPULSE-RESPONSE MODELING
C                         BY PENCIL-OF-FUNCTIONS METHOD
C                         DEC 1979 (FOR RADC)
C
C      PROGRAM "GQUANT" USES CHARACTERISTICS OF QUANTIZATION ERROR
C                         IN PENCIL-OF-FUNCTIONS METHOD TO PRODUCE
C                         IMPROVED TRANSFER FUNCTION.
C                         MODELS IMPULSE-RESPONSE OF CHANNEL/NETWORK.
C                         CAN BE USED IN SIMULATION MODE (BIN. OR DEC.)
C                         OR ON EXPERIMENTALLY RECORDED RESPONSES.
C
C                          ++++++++++++
C
C      ******************************
       DIMENSION F(500),U(500),UU(500),X(500,8),G(8,8),AM(8,8)
       DIMENSION GN(8,8),GEST(8,8),GCUM(8,8),E(8,8),EN(8,8)
       DIMENSION V(16),VV(16),AMP(8),SR(8),SI(8),SPH(8)
       DIMENSION TITLE(70),IBUF(512)
       DOUBLE PRECISION DT,AD,BD,ERROR
       COMMON /DA0/ISPN,XMSB,DELTA,SIG2,DT,DI,BIAS,IBIAS
       COMMON /DA1/FBAR,EBAR,FESUM,FESUM
       COMMON /IO/IR,ILT,IPR,IRCUND,IPLT
       REWIND5
       MAXPL=500
       MAX=8
       MAX2=2*MAX
       IR=5
       ILT=6
       ISKIP=0
       NSTRT=2
       CALL VEQUAT(MAXPL,U,F,0,10)
       CALL VEQUAT(MAXPL,UU,F,0,11)
       CALL VEQUAT(MAX2,V,VV,0,0)
       WRITE(ILT,2)
       READ(IR,8) (TITLE(I),I=1,70)
       WRITE(ILT,18) (TITLE(I),I=1,70)
       READ(IR,8) (TITLE(I),I=1,70)
       READ(IR,8) (TITLE(I),I=1,70)
       READ(IR,4)NPT,IRAD,NDIG,N,ISIM,NCOMP,IPLT,NNPT,
      +XMSB,DT,BIAS
       NP1=N+1
       NP2=NP1+1
       NP3=N+3
       NPNP2=N+N+2
       NPNP1=N+N+1
       IF(NNPT.EQ.0)NNPT=NPT
       IF(DT.EQ.0.0)DT=1.0
       IF(ISIM.EQ.0)READ(IR,180)(F(K),K=1,NNPT)
       IF(ISIM.EQ.0)GO TO 61
       IF(ISIM.EQ.1)READ(IR,180)(V(I),I=1,NPNP2)
       IF(ISIM.EQ.1)CALL RESPON(F,U,N,V,VV,NNPT)
       IF(ISIM.EQ.1)GO TO 61
       DO 60 I=1,NCOMP
       READ(IR,5)AMP(I),SR(I),SI(I),SPH(I)
 60    WRITE(ILT,11)I,AMP(I),SR(I),SI(I),SPH(I)
       CALL SIGNAL(F,NNPT,AMP,SR,SI,SPH,DT,NCOMP)
 61    CONTINUE
       IF(IPLT.GE.2)CALL PLOTS(IBUF,512,9)
 1111  READ(IR,8)(TITLE(I),I=1,70)
       IF(EOF(IR).NE.0)GO TO 998
```

```
      WRITE(ILT,3)
      WRITE(ILT,18) (TITLE(I),I=1,70)
      READ(IR,4)IPR,IREM,ISPN,IFIX,NFIX,IBIAS,IYY,IZZ,GI
      IROUND=0
C     ROUNDOFF OPTION
C
410   CONTINUE
      IF(IROUND.NE.0)CALL QUANTZ(F,X,NDIG,IRAD,NPT,MAXPL)
88    CONTINUE
      IF(IROUND.NE.0)GO TO 99
      DO 30 K=1,NPT
30    X(K,1)=F(K)+BIAS
99    CONTINUE
      IF(NP1.GT.1)CALL INGRAT(X,NPT,NP1,MAXPL,-1)
C
C     COMPUTE GRAM MATRIX
C
      NPP=NP1
      IF(IBIAS.NE.0)NPP=NP2
      DO 44 I=1,NPP
      DO 44 J=1,NPP
      AD=0.0
      IF(ISPN.EQ.0.AND.IROUND.EQ.0) GO TO 43
      DO 42 K=NSTRT,NPT
42    AD=AD+X(K,I)*X(K,J)
      GN(I,J)=AD*DT
      GDUM(I,J)=GN(I,J)
43    CONTINUE
      IF(IROUND.EQ.0)G(I,J)=GN(I,J)
44    CONTINUE
      IF(ISPN.NE.0.OR.IROUND.NE.0)
     1CALL GKRDCT(GN,E,DET,V,NPP,NPP,MAX,1)
      IF(IROUND.EQ.0)WRITE(ILT,171)DET
      IF(IROUND.EQ.1)WRITE(ILT,172)DET
      IF(IPR.GE.1)CALL PRTMAT(GN,NPP,NPP,MAX,-1)
C     WRITE(ILT,1)
      IRD=IROUND
      IF(IROUND.EQ.0)IROUND=IROUND+1
      IF(IRD.EQ.0.AND.ISPN.NE.-1)GO TO 410
      IF(IFIX.EQ.-1)GO TO 203
C
C
C     ESTIMATE OF  **  G
C
156   CALL BUILDZ(AM,V,NP1,NPT,MAX,NFIX)
C     -----NP1 REPLACED BY NPP NEXT 3 CARDS--
      CALL FIX(GDUM,AM,GEST,E,V,NPP,NPP,SIG2,MAX,IFIX)
      IF(IFIX.EQ.1)WRITE(ILT,482)SIG2
      CALL GKRDCT(GEST,E,DET,V,NPP,NPP,MAX,1)
      WRITE(ILT,162)DET
      IF(IPR.GE.1)CALL PRTMAT(GEST,NP1,NP1,MAX,0)
      DO 154 I=1,NP1
      DO 154 J=1,NP1
154   GDUM(I,J)=GEST(I,J)
      NFIX=NFIX-1
      IF(NFIX.GE.1)GO TO 156
      ISKIP=1
      IROUND=0
C
C     CALCULATE ERROR MATRIX
C
150   IF(IPR.LE.2.OR.ISKIP.EQ.0)GO TO 151
      DO 32 I=1,NP1
      DO 32 J=1,NP1
```

```
      E(I,J)=G(I,J)-GEST(I,J)
32    EN(I,J)=G(I,J)-GN(I,J)
      WRITE(ILT,161)
      CALL PRTMAT(E,NP1,NP1,MAX,-1)
      WRITE(ILT,163)
      CALL PRTMAT(EN,NP1,NP1,MAX,-1)
151   CONTINUE
C
C     DETERMINE NUMERATOR
C
203   CONTINUE
      IF(ISPN.EQ.0)GO TO 998
      CALL VEQUAT(NP1,V(NP2),VV,0,10)
      CALL RESPON(X(1,1),U,N,V,VV,NPT)
      CALL INGRAT(X,NPT,NP1-IREM,MAXPL,2)
C     CHANGES MADE HEREON FOR E(3)=0
      L=N-IREM
      IF(IBIAS.NE.0)L=N-IREM+1
      LP1=L+1
      LP2=L+2
      IF(IBIAS.NE.0)CALL VEQUAT(NPT,X(1,LP1),U,0,11)
      CALL VEQUAT(NPT,X(1,LP2),F,0,1)
      CALL VEQUAT(NPT,X(1,LP2),BIAS,0,3)
      DO 216 I=1,L
      DO 216 J=1,LP1
      G(I,J)=0.
      DO 215 K=1,NPT
215   G(I,J)=G(I,J)+X(K,I+1)*X(K,J+1)
216   G(I,J)=G(I,J)*DT
C205   CALL PRTMAT(G,L,LP1,MAX,205)
      CALL GKRDCT(G,E,DET,VV,L,L,MAX,0)
C207   CALL PRTMAT(E,L,L,MAX,207)
      CALL VEQUAT(NP1,VV,AMP,0,0)
      DO 219 I=1,L
      DO 219 J=1,L
219   VV(I)=VV(I)+E(I,J)*G(J,LP1)/DET
      FMEAN=0.0
      IF(IBIAS.NE.0)FMEAN=VV(L)
      V(NP2)=0.0
      CALL VEQUAT(N,V(NP3),VV,0,1)
211   WRITE(ILT,303)
      WRITE(ILT,210)(V(I),I=1,NP1)
      WRITE(ILT,210)(V(I),I=NP2,NFNP2)
      IF(IBIAS.NE.0)WRITE(ILT,305)FMEAN
      CALL RESPON(X(1,2),U,N,V,VV,NNPT)
      ERROR=0.0
      FFSUM=0.0
      DO 213 K=1,NNPT
      FFSUM=FFSUM+F(K)*F(K)
      X(K,3)=F(K)-X(K,2)
      X(K,1)=F(K)
213   ERROR=ERROR+X(K,3)*X(K,3)
      FFSUM=FFSUM*DT
      ERROR=ERROR*DT
      RATIO=ERROR/FFSUM
      WRITE(ILT,304)ERROR,FFSUM,RATIO
      IF(IPR.GE.2)WRITE(ILT,110)(F(K),K=1,NNPT)
      WRITE(ILT,1)
      IF(IPR.GE.2)WRITE(ILT,110)(X(K,2),K=1,NNPT)
      WRITE(ILT,1)
      IF(IPP.GE.2)WRITE(ILT,110)(X(K,3),K=1,NNPT)
C
      T0=0.0
      IF(IPLT.GE.2)CALL PLOP(NNPT,2,X,MAXPL,T0,DT,1HY,1HT,IBUF)
```

```
C     FORMAT  STATEMENTS
C
5     FCRMAT(5F10.3)
4     FORMAT(I4,6I2,I4,4F10.0)
8     FORMAT(70A1)
18    FORMAT(2X,70A1)
903   FCRMAT(10(5X,F5.0))
904   FCRMAT(10(5X,I5))
11    FORMAT(2X,I2,* AMP=*,F8.2,*  S=*,F10.4,*  + J*,F10.4,
     1*  PHASE=*,F10.4)
140   FCRMAT(10X,8HG MATRIX)
160   FCRMAT(10X,8HP MATRIX)
161   FCRMAT(10 ,12HGTRUE - GEST)
163   FCRMAT(10X,14HGTRUE - GNOISY)
162   FORMAT(10X,11HGEST MATRIX,*  (DET=*,G13.6,*)*)
210   FCRMAT(2X,5(2X,G13.6))
110   FCRMAT(20(1X,F5.2))
C110  FORMAT(2X,10(2X,F10.5))
178   FORMAT(10X,14HNOISY X MATRIX)
179   FCRMAT(10X,8HX MATRIX)
171   FCRMAT(10X,16HTRUE GRAM MATRIX,* (DET=*,G13.6,*)*)
172   FORMAT(10X,17HNOISY GRAM MATRIX,* (DET=*,G13.6,*)*)
180   FORMAT(5F10.0)
303   FORMAT(2X, *EST NUM/DENOM VECTOR*)
304   FCRMAT(/, X,*ERROR=*,G13.6,*EESUM=*,G13.6,*RATIO=*,G13.6,/)
305   FORMAT(2X,*ESTIMATED MEAN=*,G13.5)
482   FORMAT(*  ESTIMATED NOISE VAR=*,G12.5)
1     FORMAT(/)
2     FCRMAT(1H1)
3     FORMAT(////)
C
C     ISIM=0 FOR MODELING ACTUAL RESPONSE DATA
C           1 OR 2 FOR SIMULATION (1*M(7)) (2*SUMS OF EXP AND OSC)
C     NDIG=BIN BITS(INC SN-BIT), OR DECIMAL MANTISSA, ROUNDOFF IN BOTH
C     IRAD=2 FOR BINARY, 10 FOR DECIMAL
C     NP1=INTEGRATED FUNCTIONS, THE FIRST IS DATA
C     IPR=0 FOR MINIMAL PRINTING, OTHERWISE 1 OR 2
C     ISPN=0 IF ANALYSIS OF R.OFF ERROR SIGNAL ONLY,1 FOR ROUNDED SIG
C          -1 IF ANALYSIS OF TRUE (UNROUNDED) SIGNAL ONLY
C     NCOMP= COMPONENTS (A*EXP(SR T) * SIN(SI T)) TYPE
C     INT=1 (OR 0) FOR FORWARD INTEGRATION, -1 FOR REVERSE
C     DT=SAMPLING INT   XMSB=WEIGHT OF MSB (R(N-1))
C     NPT=DATA PTS.,  NNPT=POINTS ON PLOT,  N=DEGREE OF MODEL
C     IFIX=-1 FOR NO CORRECTION, 1
C     NFIX=1 FOR QZ CORREC., 2 FOR QZ QZ CORREC
C           3 FOR BIAS AND QZ QZ CORRECTION
C     IFIX=0  IF GEST=GN-AM, 1 IF NOISE VAR TO BE ESTIMATED
C          -1 IF NO CORRECTION IS TO BE APPLIED (ISPN MUST BE 1)
C
      GO TO 1111
998   CONTINUE
      CALL PLOT(0.,0.,999)
      STOP
      END
      SUBROUTINE SIGNAL(F,NPT,AMP,SR,SI,SPH,DT,NCOMP)
C     ------------------------------------------------
      DIMENSION F(1),AMP(1),SR(1),SI(1),SPH(1)
      COMMON /IO/IR,ILT,IPR,IRCUND
      DOUBLE PRECISION A,B,C,X
      DO 12 K=1,NPT
12    F(K)=0.0
      DO 20 I=1,NCOMP
      A=SR(I)*DT
      B=SI(I)*DT
```

```
      C=SPH(I)
      DO 15 KK=1,NPT
      K=KK-1
      X=AMP(I)
      IF(A.NE.0.0)X=X*DEXP(A*K)
      IF(B.NE.0.0)X=X*DSIN(B*K+C)
15    F(KK)=X+F(KK)
20    CONTINUE
C
      IF(IPR.LT.2)GO TO 30
      WRITE(ILT,9)
      WRITE(ILT,6)(F(K),K=1,NPT)
      WRITE(ILT,1)
30    CONTINUE
1     FORMAT(/)
6     FORMAT(20(1X,F5.2))
9     FORMAT(10X,* F SIGNAL*)
      RETURN
      END
      SUBROUTINE QUANTZ(F,X,NDIG,IRAD,NPT,NDIM)
C     -----------------------------------------------
C     PERFORMS BINARY OR DECIMAL QUANTIZATION
      DIMENSION F(1),X(NDIM,1)
      DOUBLE PRECISION CT,AC,BC
      COMMON /DA0/ISPN,XMSB,DELTA,SIG2,DT,QI,BIAS,IBIAS
      COMMON /DA1/FBAR,EBAR,FESUM,EESUM
      COMMON /IO/IR,ILT,IPR,IROUND
C
C9
      FBAR=0.
      EBAR=0.
      FESUM=0.
      EESUM=0.
C
C     BINARY QUANTIZATION
C     WORD=SN BIT,MSB,...,LSB  MAX NEG=-2*XMSB
      IF(IRAD.NE.2)GO TO 551
      IF(XMSB.EQ.0.0)XMSB=5.0
      NDIG1=NDIG-1
      DELTA=(2.0*XMSB)/(2.0**NDIG1)
      DEL=DELTA/2.0
      SIG2=DELTA*DELTA/12.0
      WRITE(ILT,489)DELTA,SIG2
      DO 81 K=1,NPT
      XLEV=0.0
      SN=-1.0
      XX=F(K)+BIAS
      IF(XX.GE.0.0)SN=1.0
      XX=SN*XX
      DD=2.0*XMSB
      XQ=2.0*XMSB-DELTA
      DO 82 I=1,NDIG
      DD=DD/2.0
      DIF=ABS(XX-XLEV)
      IF(DIF.LE.DEL)XQ=XLEV
      XTEM=XLEV-DD
      IF(XX.GE.XLEV)XTEM=XLEV+DD
      XLEV=XTEM
C     WRITE(ILT,210)XX,DD,DIF,XLEV,XQ
82    CONTINUE
      IF(XX.LT.DEL)XQ=0.0
      X(K,1)=SN*XQ
81    CONTINUE
      GO TO 711
```

```
C
C42
C46    DECIMAL QUANTIZATION
551    CONTINUE
       IF(IRAD.NE.10)GO TO 711
       AAA=10.0**NDIG
       DELTA=1.0/AAA
       SIG2=DELTA*DELTA/12.0
       WRITE(ILT,489)DELTA,SIG2
       DO 91 K=1,NPT
       X(K,1)=0.0
       FB=F(K)+BIAS
       XTEM=ABS(FB)
       SN=1.0
       IF(FB.LT.0.0)SN=-1.0
       XTEM=XTEM*AAA
       XTEM=XTEM+0.5
       IX=XTEM
       XTEM=IX
       XTEM=XTEM/AAA
       X(K,1)=SN*XTEM
91     CONTINUE
C
C      SSQ VALUES
C
711    CONTINUE
       DO 211 K=1,NPT
       FB=F(K)+BIAS
       X(K,2)=X(K,1)-FB
       FBAR=FBAR+FB
       EBAR=EBAR+X(K,2)
       EESUM=EESUM+X(K,2)*X(K,2)
       FESUM=FESUM+FB*X(K,2)
       IF(ISPN.EQ.0)X(K,1)=X(K,2)
211    CONTINUE
       EESUM=EESUM*DT
       FESUM=2.0*FESUM*DT
       FBAR=FBAR/NPT
       EBAR=EBAR/NPT
       WRITE(ILT,482)FBAR,EBAR,FESUM,EESUM
       IF(IPR.LE.2) GO TO 411
       WRITE(ILT.8)
       WRITE(ILT,110)(X(K,1),K=1,NPT)
       IF(ISPN.EQ.0)GO TO 411
       WRITE(ILT.18)
       WRITE(ILT,115)(X(K,2),K=1,NPT)
       WRITE(ILT,1)
411    CONTINUE
999    CONTINUE
C      FORMAT  STATEMENTS
C
8      FORMAT(10X,*ROUNDED F+BIAS SIGNAL*)
18     FORMAT(10X,*ROUNDOFF ERROR E*)
210    FORMAT(2X,5(2X,G11.4))
110    FORMAT(20(1X,F5.2))
115    FORMAT(1X,20(1X,F5.3))
178    FORMAT(10X,14HNOISY X MATRIX)
179    FORMAT(10X,8HX MATRIX)
482    FORMAT(2X,5HFBAR=,E11.4,6H EBAR=,E11.4,5H FE2=,E11.4,4H EE=,E11.4)
489    FORMAT(2X,6HDELTA=,F10.3,5H SIG=,E12.4)
1      FORMAT(/)
       RETURN
       END
       SUBROUTINE CORUPT(F,X,NDIG,IRAD,NPT,NDIM)
```

```
C      ----------------------------------------------
C      ADDS NOIS
       DIMENSION F(1),X(NDIM,1)
       DOUBLE PRECISION DT,AD,RC
       COMMON /DA0/ISPN,XMSB,DELTA,SIG2,DT,QI,BIAS,IBIAS
       COMMON /DA1/FBAR,EBAR,FESUM,EESUM
       COMMON /IO/IR,ILT,IPR,IRCLND
C
C9
       FBAR=0.
       EBAR=0.
       FESUM=0.
       EESUM=0.
C
       NDIG1=NDIG-1
       DELTA=(2.0*XMSB)/(2.0**NDIG1)
       SIG2=DELTA*DELTA/12.0
       WRITE(ILT,489)DELTA,SIG2
C
C
       IS=2458169
       IS2=397665
       SIGMA=SQRT(SIG2)
       CALL NRML(NPT,1,1,0.,SIGMA,IS,IS2,X(1,2),0)
       DO 26 K=1,NPT
26     X(K,1)=F(K)+BIAS+X(K,2)
C
       DO 211 K=1,NPT
       FB=F(K)+BIAS
       FBAR=FBAR+FB
       EBAR=EBAR+X(K,2)
       EESUM=EESUM+X(K,2)*X(K,2)
       FESUM=FESUM+FB*X(K,2)
       IF(ISPN.EQ.0)X(K,1)=X(K,2)
211    CONTINUE
       EESUM=EESUM*DT
       FESUM=2.0*FESUM*DT
       FBAR=FBAR/NPT
       EBAR=EBAR/NPT
       WRITE(ILT,482)FBAR,EBAR,FESUM,EESUM
       IF(IPR.LE.2) GO TO 411
       WRITE(ILT,8)
       WRITE(ILT,110)(X(K,1),K=1,NPT)
       IF(ISPN.EQ.0)GO TO 411
       WRITE(ILT,18)
       WRITE(ILT,115)(X(K,2),K=1,NPT)
       WRITE(ILT,1)
411    CONTINUE
999    CONTINUE
C      FORMAT  STATEMENTS
C
8      FORMAT(10X,16HROUNDED F SIGNAL)
18     FORMAT(10X,16HROUNDOFF ERROR E)
210    FORMAT(2X,5(2X,G11.4))
110    FORMAT(20(1X,F5.2))
115    FORMAT(1X,20(1X,F5.3))
482    FORMAT(2X,5HFBAR=,E11.4,6H EBAR=,E11.4,5H FE2=,E11.4,4H EE=,E11.4)
489    FORMAT(2X,6HDELTA=,F10.3,5H SIG=,E12.4)
1      FORMAT(/)
       RETURN
       END
       SUBROUTINE INGRAT(X,NPT,NP1,NDIM,INT)
C      ----------------------------------------------
       DIMENSION X(NDIM,1)
```

```
      DOUBLE PRECISION CT,SC,BC
      COMMON /D-0/ISPN,XMSB,DELTA,SIG2,DT,QI,BIAS,IBIAS
      COMMON /IO/IR,ILT,IPR,IROUND
C     GENERATE INTEGRATEC SIGNALS FROM CATA IN X(K,1)
C     INT=1 OR 0 FOR FORWARD INT., -1 FCR REVERSE
C     INT=2 FOR UNIT DELAYS (X(K,I+1)=X(K-1,I))
      N=NP1-1
      NP2=NP1+1
      IOPT=INT+2
      GO TO(51,11,11,91),IOPT
C     FORWARD INTEGRATICN
11    CONTINUE
      DO 40 J=1,N
      JJ=J+1
      X(1,JJ)=X(1,1)
      DO 40 K=2,NPT
      K1=K-1
      X(K,JJ)=X(K1,JJ)+X(K,J)
40    CONTINUE
      GO TO 70
C     REVERSE INTEGRATION
51    CONTINUE
      DO 60 J=1,N
      JJ=J+1
      X(NPT,JJ)=X(NPT,1)
C     X(NPT,JJ)=0.C
      BD=X(NPT,JJ)
      DO 60 KK=2,NPT
      K=NPT+1-KK
      K1=K+1
      BD=BD+QI*X(K,J)
      X(K,JJ)=BD
60    CONTINUE
      IF(IBIAS.EQ.0)GO TO 62
      IPWR=IBIA -1
      DO 61 KK=1,NPT
      TIME=DT*KK
      K=NPT+1-KK
61    X(K,NP2)=TIME**IPWR
62    CONTINUE
      GO TO 70
C     GENERATE UNIT DELAYS
91    CONTINUE
      DO 93 I=2,NP1
      I1=I-1
      X(1,I)=0.0
      DO 93 K=2,NPT
      K1=K-1
93    X(K,I)=X(K1,I1)
      GO TO 81
70    CONTINUE
      SC=1.C
      DO 80 I=2,NP1
      SC=SC*DT
      DO 80 K=1,NPT
80    X(K,I)=SC*X(K,I)
81    CONTINUE
      IF(IPR.LT.4)GO TO 99
      IF(IROUND.EQ.1)WRITE(ILT,178)
      IF(IROUND.EQ.0)WRITE(ILT,179)
      DO 180 I=1,NP2
180   WRITE(ILT,110)(X(K,I),K=1,NPT)
C     WRITE(ILT,1)
99    CONTINUE
```

```
C
C110    FORMAT(4(1X,F12.6))
110    FORMAT(20(1X,F5.2))
178    FORMAT(10 ,14HNOISY X MATRIX)
179    FORMAT(10X,8HX MATRIX)
1      FORMAT(/)
       RETURN
       END
       SUBROUTINE FIX(G,F,C,D,X,N,NC,SIG,NDIM,IFIX)
C      ---------------------------------------
C
C      ESTIMATE NOISE INTENSITY SIG (ASSUME WHITE NOISE)
C      CORRECT NOISY MATRIX= C
C      (P) DENOTES NOISE MATRIX FOR UNIT NOISE
C      NC IS THE NONZERO SUBMATRIX OF F =COV OF NOISE
C
       DIMENSION G(NDIM,1),P(NDIM,1),C(NDIM,1),D(NDIM,1),X(1)
       IF(IFIX.EQ.0)GO TO 51
       JCT=0
       SIG=0.0
3      SUMDET=0.0
       CALL GKRDCT(G,D,GDET,X,0,N,NDIM,0)
       JCT=JCT+1
       IF(JCT.EQ.1)DETG=GDET
       DO 5 J=1,NC
       DO 7 II=1,N
       DO 7 JJ=1,N
       C(II,JJ)=G(II,JJ)
       IF(JJ.EQ.J)C(II,JJ)=P(II,JJ)
7      CONTINUE
       CALL GKRDCT(C,D,DET,X,N,N,NDIM,0)
       SUMDET=SUMDET+DET
5      CONTINUE
       SI=GDET/SUMDET
51     CONTINUE
       DO 9 I=1,N
       DO 9 J=1,N
9      C(I,J)=G(I,J)-SI*P(I,J)
       IF(IFIX.EQ.0)GO TO 11
       CALL GKRDCT(C,D,CDET,X,0,N,NDIM,0)
       IF(CDET.LT.0.0)SI=SI/2.0
       IF(CDET.LT.0.0)GO TO 51
       IF(JCT.GE.5)GO TO 11
       IF(CDET/DETG.GT.0.1)CALL MEQUAT(N,N,G,C,NDIM,1)
       SIG=SIG+SI
       IF(CDET/DETG.GT.0.1)GO TO 3
11     RETURN
       END
       SUBROUTINE BUILDR(A,X,N,MAX)
C      ------------------------------------
C      CONVERSION MATRIX: REVERSE INTEGRATION -- I.R. MODELING
       DIMENSION A(MAX,1),X(1),Y(20)
       DOUBLE PRECISION CT,Y
       COMMON /DAO/ISPN,XMSB,DELTA,SIG2,CT,OI,BIAS,IBIAS
       COMMON /IO/IR,ILT,IPR,IRCUND
       NM1=N-1
       DO 11 I=1,N
       Y(I)=0.0
       DO 11 J=1,N
11     A(I,J)=0.0
       A(N,N)=1.0
C
       DO 20 JJ=1,NM1
       J=N-JJ
```

```
      DO 15 KK=1,2
      K=KK-1
      DO 15 I=J,NM1
15    A(I+K,J)=A(I+K,J)+A(I+1,J+1)*(1.0-K-K)
20    CONTINUE
      QQ=1.0
      DO 22 J=2,N
      QQ=QQ*QI
      DO 22 I=J,N
22    A(I,J)=QQ*A(I,J)
C
      DO 25 I=1,N
      IF(IPR.GE.3)WRITE(ILT,5)(A(I,JJ),JJ=1,N)
      DO 25 J=1,N
25    Y(I)=Y(I)+A(I,J)*X(J)
      DO 28 I=1,N
28    X(I)=Y(I)/Y(1)
      IF(IPR.GE.3)WRITE(6,7)(X(I),I=1,N)
5     FORMAT(2X,10G12.5)
7     FORMAT(*  ESTIMATED PARAMETER VECTOR*,/,10G13.6)
      RETURN
      END
      SUBROUTINE GKROCT(X,Y,DET,XLAMDA,NN,N,MAX,ICHT)
C     ---------------------------------------------------------------
      DIMENSION XLAMDA(1)
      DIMENSION X(MAX,1),Y(MAX,1)
      DOUBLE PRECISION A,B,C,D,E
      INTEGER NUM(2,20)
      DOUBLE PRECISION DT,SC,AC,BC
      COMMON /DA0/ISPN,XMSR,DELTA,SIG2,DT,QI,BIAS,IBIAS
      COMMON /IO/IR,ILT,IPR,IRCUND
      IGKR=1
      IF(N.NE.1)GO TO 3
      Y(1,1)=1.
      DET=X(1,1)
      GO TO 61
3     CONTINUE
      DO 6 I=1,N
      DO 6 J=1,N
6     Y(J,I)=X(J,I)
      A=1.0
      DO 43 I=1,N
      B=0.0
      L=I
      M=I
C
C          FIND LARGEST ENTRY A(L,M) IN THELOWER DIAGONAL SUBMATRIX
C
      DO 18 J=I,N
      DO 18 K=I,N
      IF(ABS(Y(K,J)).LE.DABS(B))GO TO 18
      B=ABS(Y(K,J))
      L=K
      M=J
18    CONTINUE
C
C         INTERCHANGE ROWS
C
      IF(L.EQ.I)GO TO 24
      DO 23 J=1,N
      C=Y(L,J)
      Y(L,J)=Y(I,J)
23    Y(I,J)=C
C
```

```
C       INTERCHANGE COLUMNS
C
24      IF(M.EQ.I)GO TO 29
        DO 28 J=1,N
        C=Y(J,M)
        Y(J,M)=Y(J,I)
28      Y(J,I)=C
29      NUM(1,I)=L
        NUM(2,I)=M
        B=Y(I,I)
        Y(I,I)=A
        DO 42 J=1,N
        IF(J.EQ.I)GO TO 42
        C=-Y(I,J)
        Y(I,J)=0.0
        DO 41 K=1,N
        D=Y(K,I)*C
        E=Y(K,J)*B+D
        IF(DABS(E).LT.1.0D-10*DABS(D))E=0.0
41      Y(K,J)=E/A
42      CONTINUE
43      A=B
C
C       RESTORE COLUMNS
C
        DO 58 I=2,N
        J=N+1-I
        K=NUM(2,J)
        IF(K.EQ.J)GO TO 52
        DO 51 L=1,N
        C=Y(K,L)
        Y(K,L)=Y(J,L)
51      Y(J,L)=C
52      K=NUM(1,J)
C
C       RESTORE ROWS
C
        IF(K.EQ.J)GO TO 58
        DO 57 L=1,N
        C=Y(L,K)
        Y(L,K)=Y(L,J)
57      Y(L,J)=C
58      CONTINUE
        DET=A
61      CONTINUE
        IF(ICPT.NE.1)GO TO 1000
        IF(Y(1,1).LT.0.0)GO TO 1000
C       .....
        SC=1.0
        DO 200 I=2,N
        SC=SC*DT
        IF(IGKR.EQ.0)XLAMDA(I)=Y(I,1)/Y(1,1)
        IF(IGKR.EQ.0)GO TO 200
        A=Y(I,I)
        IF(Y(I,I).LT.0.0)A=0.0
        IF(IGKR.EQ.2)A=ABS(Y(I,I))
        XLAMDA(I)=DSQRT(A/Y(1,1))
C       IF(Y(I,1).LT.0.0)XLAMDA(I)=-XLAMDA(I)
        XLAMDA(I)=SC*XLAMDA(I)
200     CONTINUE
        XLAMDA(1)=1.0D0
        IF(IPR.GE.1)WRITE(6,106)(XLAMDA(I),I=1,N)
        NPP=N
        IF(IBIAS.NE.0)NPP=N-1
```

```
      CALL BUILDR(Y,XLAMDA,NPP,MAX)
106   FORMAT(5X,*SYNTHETIC PARAMETER VECTOR*,/,10G12.5)
1000  CONTINUE
      RETURN
      END
      FUNCTION COMB(N,M)
C     CALCULATE  COMBINATION M OUT OF N
      IF(N.LE.0)GO TO 99
      L=1
      LD=1
      IF(M.EQ.0)GO TO 8
      MN1=N-M+1
      DO 5 I=MN1,N
5     L=L*I
      DO 7 I=1,M
7     LD=LD*I
8     COMB=L/LD
99    RETURN
      END
      SUBROUTINE BUILDZ(Z,R,NP1,NPT,NDIM,NFIX)
C     -----------------------------------
      DIMENSION Z(NDIM,1),R(1)
      DOUBLE PRECISION DT
      COMMON /D10/ISPN,XMSR,DELTA,SIG2,DT,QI,BIAS,IBIAS
      COMMON /IO/IR,ILT,IPR,IRCUND
      TIME=DT*NPT
      ICPT=NFIX+1
      GO TO(201,101,101,201),ICPT
101   SC=DT
      DO 24 I=1,NP1
      Z(I,1)=SC*NPT
      IF(I.GE.2)Z(I,2)=DT*SC*COMB(NPT-1+I,I)
24    SC=SC*DT
      DO 166 J=3,NP1
      DO 166 I=J,NP1
      AB=TIME**(I+J-2)
      AC=1.0/(I+J-3) - 1.0/(I+J-2)
166   Z(I,J)=AB*AC*DT
      Z(3,3)=Z(3,2)*DT*(NPT+1.0)/2.0
      WRITE(ILT,160)
      GO TO 301
201   CONTINUE
      DO 210 J=1,NP1
      DO 210 I=J,NP1
      IF(I.EQ.1)Z(1,1)=TIME
210   Z(I,J)=(TIME**(I+J-1))/(I+J-1)
      WRITE(ILT,161)
301   CONTINUE
      DO 174 I=1,NP1
      DO 168 J=I,NP1
168   Z(I,J)=Z(J,I)
      IF(IPR.GE.3)WRITE(ILT,220)(Z(I,J),J=1,NP1)
174   CONTINUE
160   FORMAT(10X,* QUANT. NOISE *)
161   FORMAT(10X,*BIAS EFFECT*)
220   FORMAT(2X,5(2X,G13.6))
      RETURN
      END
      SUBROUTINE PRTMAT(A,M,N,NDIM,LOC)
C     ------------------------
C
C     PRINTS A MATRIX, AND AN INTEGER (PERHAPS A LOCATION) IF LOC.GE.1
C
      DIMENSION A(NDIM,1)
```

```
      IF(LOC.GE.1)WRITE(6,5)LOC
      DO 31 I=1,M
31    WRITE(6,18)(A(I,J),J=1,N)
5     FORMAT(*    LOCATION/INTEGER=*,I5)
18    FORMAT(2X,10G13.6)
      RETURN
      END
      SUBROUTINE RESPON(X,V,N,GAMMA,XLAMDA,MP1)
C     ------------------------------------------------------------------
      DIMENSION X(1),V(1),GAMMA(1),XLAMDA(1)
      DOUBLE PRECISION XSAV,A0,B0
      NM1=N-1
      NP1=N+1
      NPNP1=N+N+1
      NPNP2=N+N+2
      DO 19 I=1,NPNP1
19    XLAMDA(I)=0.0
      XSAV=0.0
      DO 20 K=1,MP1
      IF(N.EQ.1)GO TO 25
      DO 21 I=1,NM1
      J=NP1-I
21    XLAMDA(J)=XLAMDA(J-1)
25    CONTINUE
      DO 22 I=1,N
      J=NPNP2-I
22    XLAMDA(J)=XLAMDA(J-1)
      XLAMDA(1)=XSAV
      XLAMDA(NP1)=V(K)
      XSAV=0.0
      DO 23 I=1,NPNP1
23    XSAV=XSAV-GAMMA(I+1)*XLAMDA(I)
      IF(DABS(XSAV).GE.1.0E10)XSAV=0.0
20    X(K)=XSAV
      RETURN
      END
      SUBROUTINE VEQUAT(NPT,Y,X,NPUL,IOPT)
C     --------------------------
C     IOPT=0   SET  Y  TO ZERO
C          1 OR 2  SET  Y=X   (PRINT IF 2)
C          3  SET Y=Y+ CONST X
C          9  SET  Y  TO ZERO
C          10 SET  Y=IMPULSE
C          11 SET  Y=STEP
      DIMENSION X(1),Y(1)
      IF(IOPT.EQ.0)IOPT=9
      DO 33 K=1,NPT
      IF(IOPT.EQ.1.OR.IOPT.EQ.2)Y(K)=X(K)
      IF(IOPT.EQ.3)Y(K)=Y(K)+X(1)
      IF(IOPT.GE.9)Y(K)=0.0
      IF(IOPT.EQ.11)Y(K)=1.0
33    CONTINUE
      IF(IOPT.EQ.2)WRITE(6,6)(Y(K),K=1,NPT)
6     FORMAT(2X,10G12.5)
      IF(IOPT.EQ.10)Y(1)=1.0
      RETURN
      END
      SUBROUTINE MEQUAT(M,N,B,A,NDIM,IOPT)
C     --------------------------------
C     IOPT=0   SET B TO ZERO
C          1      B EQUAL TO A
C          10     B TO IDENTITY
      DIMENSION A(NDIM,1),B(NDIM,1)
      DO 33 I=1,M
```

```
      DO 33 J=1,N
      IF(IOPT.NE.1)B(I,J)=0.0
      IF(IOPT.EQ.10.AND.I.EQ.J)B(I,J)=1.0
      IF(IOPT.EQ.1)B(I,J)=A(I,J)
33    CONTINUE
      RETURN
      END
      SUBROUTINE PLOP(NPT,NF,Y,NDIM,T0,DT,LABEL,INDEP,IBUF)
C     ----------------------------------------------
C     NPT=NUMB OF TIME PTS  (WARNING: NDIM SHOULD BE.GE.NPT+2)
C     NF=NUMBER OF FUNS
C     Y(K, ) DATA ARRAY OF DIMENSION NDIM,NF
C     T0=INITIAL TIME, DT=TIME INCREMENT
C     LABEL, INDEP = TITLES FOR Y AND X AXES
      DIMENSION Y(NDIM,NF),YY(2),LABEL(1),INDEP(1)
      DIMENSION X(512),IBUF(512)
      COMMON /IO/IR,ILT,IPP,IRCLND,IPLT
      M=NF*NDIM
      M1=M+1
      M2=M+2
      NPT1=NPT+1
      NPT2=NPT+2
      X(1)=T0
      DO 9 K=2,NPT
9     X(K)=X(K-1)+DT
      DO 8 I=1,NF
      DO 8 K=NPT1,NDIM
8     Y(K,I)=Y(NPT,I)
C
C     INITIALIZE(LIQ. INK, 12IN.PAPER)     MAX.LENGTH=60IN
      CALL PLOTMX(60.0)
C     SET ORIGIN
      CALL PLOT(0.,-.5,3)
      CALL FACT R(5.0/6.5)
C     BEGIN PLOTTING
      CALL SCALE(X,6.5,NPT,1)
      CALL SCALE(Y(1,1),10.0,M,1)
      CALL AXIS(0.,0.,11HTIME (SEC.),
     *-16,6.5,0.,X(NPT1),X(NPT2))
      CALL AXIS(0.,0.,16HRESPONSES Y    ,Y,
     *16,10.,90.,Y(M1),Y(M2))
      WRITE(6,6)X(NPT1),X(NPT2)
      WRITE(6,7)Y(M1),Y(M2)
6     FORMAT(1X,*T0,DIV (6.5 DIV)*,4(1X,F7.3))
7     FORMAT(1X,*Y0,DIV (10 DIV)*,4(1X,F7.3))
      DO 10 I=1,NF
      Y(NPT1,I)=Y(M1)
      Y(NPT2,I)=Y(M2)
      IF(I.EQ.1.OR.IPLT.EQ.3)CALL LINE(X,Y(1,I),NPT,1,I-1,I)
      IF(I.EQ.2.AND.IPLT.EQ.2)CALL DASHLN(X,Y(1,2),NPT,1)
10    CONTINUE
      CALL PLOT(10.,0.,-3)
      RETURN
      END
```

APPENDIX C

LISTING OF

PROGRAM

USPEC

```
C
C      ****************************
C
C
C          USPEC
C
C
C
C      ****************************
C      "USPEC"   FOR RADC
C      EVALUATES THE MAGNITUDE SPECTRUM
C      OF THE FOLLOWING PULSE INPUTS (SEE SECTION 4
C      OF THE REPORT):
C
C      1. TR+,-(T) ONE CYCLE TRI WAVE, FOLLOWED BY ZERO LEVEL
C      2. TR+(T)   +VE HALF CYCLE TRI WVAE, FOLLOWED BY ZERO LEVEL
C      3. OEX0(T)  OSCILLATORY PULSE (NO DECAY), FOLLOWED BY ZERO LEVEL
C      4. OEX1(T)  SAME AS 3, WITH EXP. DECAYING ENV. - ONE TIME CONST
C      5. OEX1(T)  SAME AS 3, WITH EXP. DECAYING ENV. - TWO TIME CONST
C      6. OTR(T)   OSCILLATORY PULSE WITH TRI ENV.,FOLLOWED BY ZERO LEVEL
C
C      FOR PULSES 3 TO 6 THE OSCILLATION IS A COSINE WAVE
C
C
       DIMENSION N(10),F(20),FR(20),X(20,10)
       DATA N(1)/20/,N(2)/100/,N(3)/200/,N(4)/1000/,N(5)/2000/
       DATA F(1)/.01/,F(2)/.1/,F(3)/.5/,F(4)/.9091/,F(5)/.990099/
       DATA F(6)/.999/,F(7)/1.001/,F(8)/1.1/,F(9)/1.5/,F(10)/2.0/
       DATA F(11)/3.0/,F(12)/10./,F(13)/11./
       PI=4.0*ATAN(1.0)
       PI2=2.0*PI
       NN=5
       NF=13
       IS1=1
       IS2=6
       NF1=0
       DO 30 J=1,NF
       IF(F(J).LE.1.0)NF1=NF1+1
30     CONTINUE
         ITEST=0
         IF(ITEST.NE.1)GO TO 29
         N(1)=100
         F(1)=0.1
         IS1=3
         IS2=IS1
         NN=1
         NFF=1
         IPR=3
29     CONTINUE
       IF(ITEST.NE.1)IPP=0
       DO 100 ISIG=IS1,IS2
       NFF=NF
       IF(ISIG.LE.2)NFF=NF-NF1
       DO 95 I=1,NN
       DO 94 J=1,NFF
       FR(J)=F(J)-1.0
       IF(ISIG.EQ.6.AND.F(J).LE.0.001)FR(J)=-0.9
       IF(ISIG.GE.3.OR.ITEST.EQ.1)GO TO 31
       FR(J)=F(J+NF1)-1.0
31     CONTINUE
       GO TO (41,41,51,51,51,61,71),ISIG
41     CONTINUE
       BF=(0.125D-2)*N(I)
```

```
      IF(IPR.GE.3)WRITE(6,6)BF
      BF=BF*FR(J)
      IF(IPR.GE.3)WRITE(6,6)BF
      THETA=PI*BF
      IF(ISIG.EQ.2)THETA=2.0*THETA
      IF(IPR.GE.2)WRITE(6,7)THETA
      XX=1.0
      IF(THETA.NE.0)XX=SIN(THETA)/THETA
      IF(IPR.GE.2)WRITE(6,8)XX
      XY=XX*XX
      IF(IPR.GE.3)WRITE(6,9)XY
      IF(ISIG.EQ.1)D=SIN(2.0*THETA)
      IF(ISIG.EQ.1)XX=XX*ABS(D)
      GO TO 93
 51   CONTINUE
      A=0.0
      B=0.025*N(I)
      IF(ISIG.EQ.4)A=200.0/N(I)
      IF(ISIG.EQ.5)A=400.0/N(I)
      EAB=EXP(-A*B)
      IF(IPR.GE.2)WRITE(6,13)A,B,EAB
      XR=0.0
      XI=0.0
      DO 57 KF=1,2
      FF=FR(J)+2.0*(KF-1)
      W=PI2*FF
      C=COS(W*B)
      S=SIN(W*B)
      IF(IPR.GE.2)WRITE(6,14)W,C,S
      YR=1.0-EAB*C
      YI=-EAB*S
      ZR=A*YR+W*YI
      ZI=A*YI-W*YR
      D=A*A+W*W
      XR=XR+ZR/D
      XI=XI+ZI/D
 57   CONTINUE
      XX=SQRT(XR*XR+XI*XI)
      IF(IPR.GE.2)WRITE(6,15)XR,XI,XX
      IF(J.EQ.NF1)XC=XX
      GO TO 93
 61   CONTINUE
      B=0.025*N(I)
      IF(IPR.GE.2)WRITE(6,23)B
      XR=0.0
      XI=0.0
      DO 67 KF=1,2
      FF=FR(J)+2.0*(KF-1)
      W=PI2*FF
      THETA=PI*FF*B
      D=1.0
      IF(THETA.NE.0.0)D=SIN(THETA)/THETA
      C=COS(THETA)
      S=SIN(THETA)
      IF(IPR.GE.2)WRITE(6,24)W,THETA,D,C,S
      YR=1.0-D*C
      YI=D*S
      XR=XR+YI/W
      XI=XI-YR/W
 67   CONTINUE
      XX=SQRT(XR*XR+XI*XI)
      IF(IPR.GE.2)WRITE(6,15)XR,XI,XX
      IF(J.EQ.NF1)XC=XX
      GO TO 93
```

```
71     CONTINUE
93     CONTINUE
       X(J,I)=XX
94     CONTINUE
       DO 96 J=1,NFF
       XX=X(J,I)
       IF(ISIG.GT.3.AND.NFF.GE.NF1)XX=X(J,I)/XC
       X(J,I)=-200
       IF(XX.GE.1.0E-10)X(J,I)=20.0*ALOG10(XX)
96     CONTINUE
95     CONTINUE
       WRITE(6,11)ISIG,(N(I),I=1,NN)
C      WRITE(6,1)
       DO 97 I=1,NFF
       FRQ=FR(I)
       IF(ISIG.GT.3)FRQ=FR(I)+1.0
97     WRITE(6,6)FRQ,(X(I,J),J=1,NN)
       WRITE(6,1)
100    CONTINUE
C
C
1      FORMAT(/)
6      FORMAT(2X,F12.3,9F12.1)
7      FORMAT(2X,*THETA=*,F13.6)
8      FORMAT(2X,*Y=SIN( )/( )=*,F13.6)
9      FORMAT(2X,*Y,Y=*,F13.6)
11     FORMAT(10X,*MAG. IN DB FOR DIFFERENT FREQ,NPT*,*  ISIG=*,
      +I1,//,7X,*F/FO/*,10(7X,I4,1X))
12     FORMAT(14X,(4X,I4,4X))
13     FORMAT(2X,*A=*,F10.5,*  B=*,F10.5,*  EAB=*,F10.5)
14     FORMAT(2X,*W=*,F10.5,*  COS=*,F10.5,*  SIN=*,F10.5)
15     FORMAT(2X,* XX REAL=*,F10.5,*    XX IMAG=*,3F10.5)
23     FORMAT(2X,*B=*,F10.5)
24     FORMAT(2X,*W=*,F10.5,*  THETA=*,F10.5,*  SIN( )/( )*,F10.5,
      +/,2X,*COS=*,F10.5,*  SIN=*,F10.5)
       STOP
       END
```

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