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NAVAL UNDERSEA RESEARCH AND DEVELOPMENT CENTER

A TRANSFER MATRIX FOR MODULARLY DRIVEN TRANSDUCER ARRAYS

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Sensor and Fire Control Department

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A TRANSFER MATRIX FOR MODULARLY-DRIVEN ARRAYS

By H. H. Ding and J. A. Ward

INTRODUCTION

In the course of working with certain sonar transmitting array problems, we must determine the transfer matrix relating element head velocities to amplifier input signals. We require this matrix to predict element head velocities as a function of a given distribution of amplifier input signals and vice-versa without resorting to scale-model or prototype array studies.

For the purposes of this memo, we define this transfer matrix in the following manner for an array of N elements:

$$[S_i] \stackrel{d}{=} [K_{ij}]\ [v_j] \qquad (i,j = 1,2,3,\ldots, r,\ldots,N)$$

where: S_r = the signal into the amplifier driving element r.

v_r = the head velocity at element r.

$[K_{ij}]$ = the N x N transfer matrix

The purpose of this memo is to present the development of this transfer matrix for several types of modularly-driven transducer arrays. The simplest type of modularly-driven array is that with only one element per module. The development of $[K_{ij}]$ for this type of array is presented first to prepare the way for more general developments for several types of modularly-driven arrays.

We would like to emphasize at this point that the transfer matrix used in this memo may be computed utilizing data from available computer programs. No significant modification of existing programs is required; rather a new program must be written which would use several available programs in effecting final computation of the $[K_{ij}]$ transfer matrix. However, for large arrays, the equations developed here may not be economically feasible without further simplification and development of shortcuts by a capable computer programmer.

$[K_{ij}]$ FOR THE SIMPLE CASE

For this development, we will assume identical elements and identical amplifiers in addition to assuming one element per amplifier. For the rth element of an N-element array, the following diagram provides the notation used here:

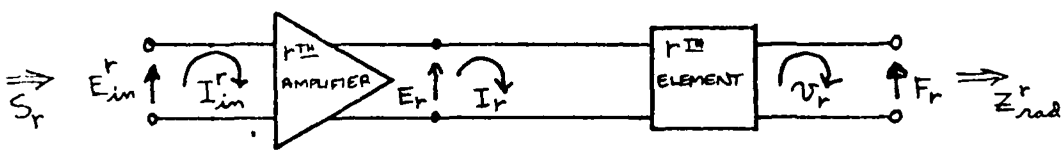

Figure 1: The rth Module

Knowledge of array geometry, medium propagation constants, etc., allows computation of mutual coupling impedances for some array shapes. For our N-element array:

$$[F_i] \overset{d}{=} [Z_{ij}]\ [v_j] \qquad (i,j = 1,2,3,\ldots,r,\ldots,N)$$

where: F_r = force at the head of the rth element.

v_r = velocity at the head of the rth element.

$[Z_{ij}]$ = N x N mutual impedance matrix.

and, obviously, $\left(\frac{1}{v_r}\right)(F_r) = \left(\frac{1}{v_r}\right)\left(\sum_j Z_{rj} v_j\right)$

The A-matrices used to describe the amplifier and the transducer element are defined as follows:

$$\begin{bmatrix} E^r_{in} \\ I^r_{in} \end{bmatrix} \stackrel{d}{=} \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} E_r \\ I_r \end{bmatrix} \qquad \text{(amplifier)}$$

$$\begin{bmatrix} E_r \\ I_r \end{bmatrix} \stackrel{d}{=} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} F_r \\ v_r \end{bmatrix} \qquad \text{(transducer)}$$

Rewriting the transducer equation in another form:

$$E_r = AF_r + Bv_r$$

$$I_r = CF_r + Dv_r$$

$$E_r = A\left(\frac{F_r}{v_r} + \frac{B}{A}\right) v_r$$

$$I_r = C\left(\frac{F_r}{v_r} + \frac{D}{C}\right) v_r$$

By means of the fact that:

$$Z_{ec} \stackrel{d}{=} \frac{B}{A}$$

$$Z_{ic} \stackrel{d}{=} \frac{D}{C}$$

$$\frac{F_r}{v_r} = \sum_j Z_{rj} \frac{v_j}{v_r} \quad (j = 1,2,\ldots,N)$$

$$E_r = A\left(\sum_j Z_{rj} v_j + Z_{ec} v_r\right)$$

$$I_r = C\left(\sum_j Z_{rj} v_j + Z_{ic} v_r\right) \quad (j = 1,2,3,\ldots, N)$$

Since the elements are identical, we may write

$$[E_i] = A\ [Z^E_{ij}]\ [v_j]$$

$$[I_i] = C\ [Z^I_{ij}]\ [v_j] \qquad (i,j = 1,2,3,\ldots,r,\ldots,N)$$

for the entire array, where:

$$Z^E_{ij} \stackrel{d}{=} Z^I_{ij} \stackrel{d}{=} Z_{ij} \qquad (i \neq j)$$

$$Z^E_{ij} \stackrel{d}{=} Z_{ij} + Z_{ec} \qquad (i = j)$$

$$Z^I_{ij} \stackrel{d}{=} Z_{ij} + Z_{ic} \qquad (i = j)$$

We may now determine $[K_{ij}]$ for two types of amplifier drive.

If: $S_r = E^r_{in}$; $\quad [K_{ij}] \stackrel{d}{=} [K^E_{ij}]$ (voltage drive)

If: $S_r = I^r_{in}$; $\quad [K_{ij}] \stackrel{d}{=} [K^I_{ij}]$ (current drive)

For voltage drive:

$$[S_i] = [E^i_{in}] = \alpha\ [E_i] + \beta\ [I_i]$$

$$[S_i] = (\alpha A\ [Z^E_{ij}] + \beta C\ [Z^I_{ij}])\ [v_j]$$

$$[S_i] \stackrel{d}{=} [K^E_{ij}]\ [v_j]$$

Thus: $$[K^E_{ij}] = \alpha A\ [Z^E_{ij}] + \beta C\ [Z^I_{ij}] \qquad (i,j = 1,2,3,\ldots,r,\ldots,N)$$

Note that for an amplifier that behaves as a perfect *voltage* source ($\beta = 0$), $[K^E_{ij}]$ reduces to $\alpha A\ [Z^E_{ij}]$.

For current drive:

$$[S_i] = [I^i_{in}] = \gamma\ [E_i] + \delta\ [I_i]$$

$$[S_i] = (\gamma A\ [Z^E_{ij}] + \delta C\ [Z^I_{ij}])\ [v_j]$$

$$[S_i] \stackrel{d}{=} [K^I_{ij}]\ [v_j]$$

Thus: $[K^I_{ij}] = \gamma A \, [Z^E_{ij}] + \delta C \, [Z^I_{ij}]$ $\qquad (i,j = 1,2,3,\ldots,r,\ldots,N)$

Again, note that for an amplifier that behaves as a perfect *current* source ($\gamma = 0$), $[K^I_{ij}]$ reduces to $\delta C \, [Z^I_{ij}]$

Once $[K^E_{ij}]$ or $[K^I_{ij}]$ has been determined, it is a straightforward matter to determine $[S_i]$ given $[v_j]$ or to determine $[v_j]$ given $[S_i]$, since

$$[v_j] = [K_{ij}]^{-1} \, [S_i]$$

and $$[S_i] = [K_{ij}] \; [v_j] \qquad (i,j = 1,2,3,\ldots, N)$$

Also, inversion of $[K_{ij}]$ should be aided by the symmetry existent *for this simple case.*

We can expand on this a little by removing the assumption that all elements are identical and all amplifiers are identical. If we do this, the following definitions become necessary, for the rth element and the rth amplifier:

$$[A^A_r] \stackrel{d}{=} \begin{bmatrix} \alpha_r & \beta_r \\ \gamma_r & \delta_r \end{bmatrix}$$

$$[A_r] \stackrel{d}{=} \begin{bmatrix} A_r & B_r \\ C_r & D_r \end{bmatrix}$$

$$Z^r_{ec} \stackrel{d}{=} \frac{B_r}{A_r}$$

$$Z^r_{ic} \stackrel{d}{=} \frac{D_r}{C_r}$$

Thus; $[Z^E_{ij}] \stackrel{d}{=} [Z_{ij}] + \mathrm{diag}\,(Z^i_{ec})$

where $$\mathrm{diag}\,(Z^i_{ec}) \stackrel{d}{=} \begin{bmatrix} Z^1_{ec} & 0 & \cdots & 0 \\ 0 & Z^2_{ec} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Z^N_{ec} \end{bmatrix}$$

and: $[Z^I_{ij}] \stackrel{d}{=} [Z_{ij}] + \text{diag}\ (Z^i_{ic})$

Finally; $[K^E_{ij}] = \text{diag}\ (\alpha_i)\ \text{diag}\ (A_i)\ \ ([Z_{ij}] + \text{diag}\ (Z^i_{ec})\ )$

$$+ \text{diag}\ (\beta_i)\ \text{diag}\ (C_i)\ \ ([Z_{ij}] + \text{diag}\ (Z^i_{ic})\ )$$

or $[K^E_{ij}] = \text{diag}\ (\alpha_i A_i + \beta_i C_i)\ [Z_{ij}] + \text{diag}\ (\alpha_i A_i Z^i_{ec} + \beta_i C_i Z^i_{ic})$

Similarly: $[K^I_{ij}] = \text{diag}\ (\gamma_i A_i + \delta_i C_i)\ [Z_{ij}] + \text{diag}\ (\gamma_i A_i Z^i_{ec} + \delta_i C_i Z^i_{ic})$

This form readily shows that the off-diagonal terms of $[K_{ij}]$ become unimportant compared to the on-diagonal terms when either Z_{ec} or Z_{ic} is made large enough. One further simplification can be shown by assuming the amplifier to be a perfect voltage source (or current source):

Voltage source: $\beta = 0$

$$[K^E_{ij}] = \text{diag}\ (\alpha_i A_i)\ [Z_{ij}] + \text{diag}\ (\alpha_I B_i)$$

Current source: $\gamma = 0$

$$[K^I_{ij}] = \text{diag}\ (\delta_i C_i)\ [Z_{ij}] + \text{diag}\ (\delta_i D_i)$$

Thus, if Z^i_{ec} is made large ($B_i >> A_i$), the diagonal terms of $[K^E_{ij}]$ take the approximate form:

$$\alpha_i\ (A_i Z_{ii} + B_i) = \alpha_i A_i\ (Z_{ii} + Z^i_{ec})$$

where $Z^i_{ec} >> Z_{ij}$

Similarly for $[K^I_{ij}]$; diagonal terms take the form:

$$\delta_i (C_i Z_{ii} + D_i) = \delta_i C_i \; (Z_{ii} + Z^i_{ic})$$

where $\quad D_i >> C_i \quad$ and $\quad Z^i_{ic} >> Z_{ii}$.

A MORE GENERAL CASE (PARALLELED ELEMENTS)

Consider an N-element transmitter array consisting of G modules, each driven by a single amplifier. The transducer elements within a single module are connected electrically in parallel and are Q in number (Q can vary from module to module). The following diagram shows the amplifier and elements of the gth module along with the notation used in this section:

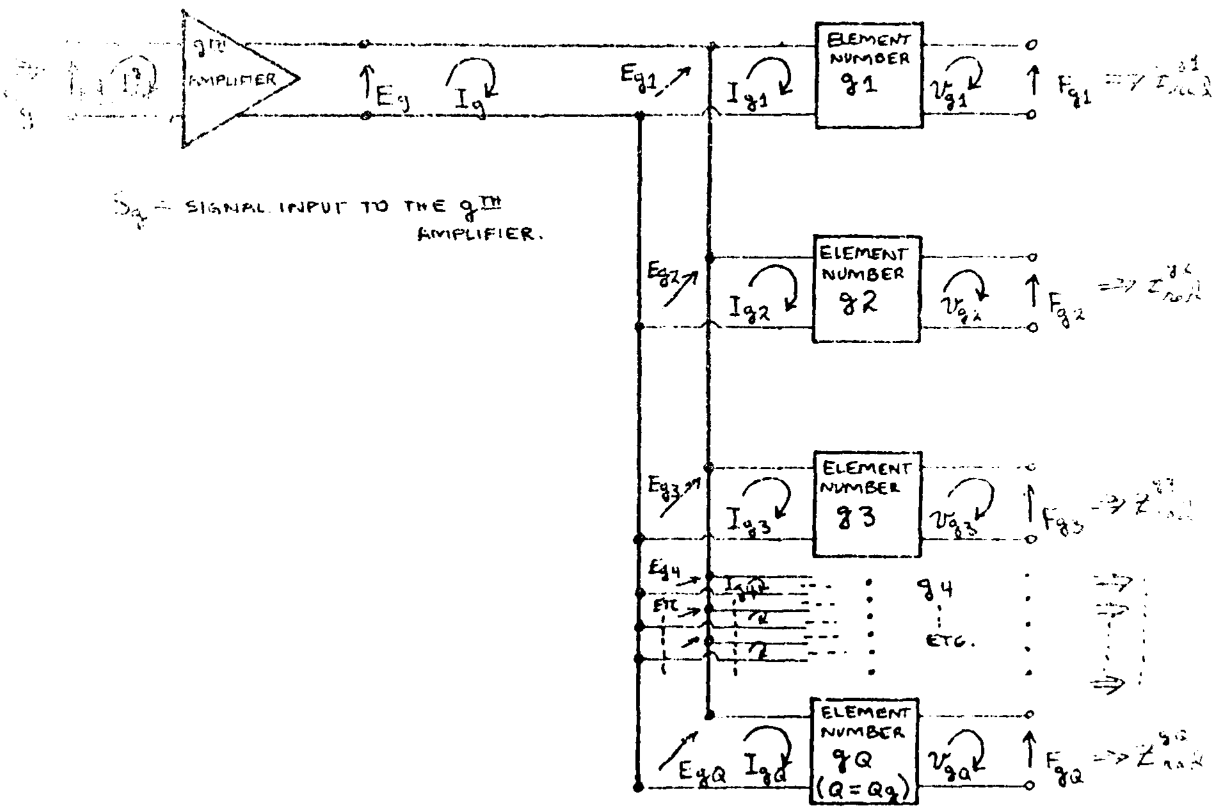

Figure 2: The gth Module Containing Q_g Elements

Note that the number of elements is not constrained, nor are they necessarily identical. For this general development, we will not assume identical amplifiers nor will we assume identical elements.

To avoid possible snarls in notation in the subsequent development, we choose to state our subscripting convention at this point.

$(i,j = 1,2,3,\ldots,r,\ldots, N)$ N = total array elements.

$(k,m = 1,2,3,\ldots,g,\ldots, G)$ G = total array modules.

$(n = 1,2,3,\ldots,q,\ldots, Q_k)$ Q_k = total elements in kth module.

$(p = 1,2,3,\ldots,q,\ldots,Q_m)$ Q_m = total elements in mth module.

If q or Q appear in a double subscript (e.g.: gq or gQ) it will be understood that q or Q refer to elements within the module denoted by the other subscript (e.g.: in gq, q is the qth element of the gth module and in gQ, Q is understood to be Q_g).

Thus, we can now write $[F_i] = [Z_{ij}]\,[v_j]$ in a form which names the modules as well as the elements and which is more suited to our needs here. This form is as follows:

$$[F_i] = \begin{bmatrix} [F_n]_1 \\ [F_n]_2 \\ \vdots \\ [F_n]_G \end{bmatrix} = \Big[[F_n]_k\Big]$$

and

$$[v_j] = \begin{bmatrix} [v_p]_1 \\ [v_p]_2 \\ \vdots \\ [v_p]_G \end{bmatrix} = \Big[[v_p]_m\Big]$$

Thus:

$$\begin{bmatrix}[F_n]_k\end{bmatrix} = \begin{bmatrix}[Z_{np}]_{km}\end{bmatrix} \begin{bmatrix}[v_p]_m\end{bmatrix}$$

where $[Z_{np}]$ is a $Q_k \times Q_m$ rectangular matrix in general and where $[Z_{np}]_{km}$ is an N x N square matrix.

Moreover, our convention can be extended to allow representation of single column vector components and single square matrix elements in the following manner:

F_{gq} is the force at the qth element of the gth module.

Z_{ghqs} is the mutual impedance relating the force at element q of module g to the velocity at element s of module h.

v_{hs} is the velocity at element s of module h.

Note that the ordering of subscripts is consistent with:

$$[F_n]_g = \begin{bmatrix} F_{g1} \\ F_{g2} \\ \vdots \\ F_{gQ} \end{bmatrix} \qquad (Q = Q_g)$$

and

$$[Z_{np}]_{gh} = \begin{bmatrix} Z_{gh11} & Z_{gh12} & \cdots & Z_{gh1Q} \\ Z_{gh21} & Z_{gh22} & \cdots & Z_{gh2Q} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{ghQ1} & Z_{ghQ2} & \cdots & Z_{ghQQ} \end{bmatrix} \qquad (Q = Q_g \text{ or } Q_h \text{ depending on order})$$

With these statements in mind, we write (for the qth element of the gth module):

$$\begin{bmatrix} E^g_{in} \\ I^g_{in} \end{bmatrix} = \begin{bmatrix} \alpha_g & \beta_g \\ \gamma_g & \delta_g \end{bmatrix} \begin{bmatrix} E_g \\ I_g \end{bmatrix} \qquad (g\text{th amplifier})$$

$$\begin{bmatrix} E_{gq} \\ I_{gq} \end{bmatrix} = \begin{bmatrix} A_{gq} & B_{gq} \\ C_{gq} & D_{gq} \end{bmatrix} \begin{bmatrix} F_{gq} \\ v_{gq} \end{bmatrix} \quad (q\text{th element})$$

Because the elements are connected in parallel within a single module, we write (for the gth module):

$$\begin{bmatrix} E_{g} \\ I_{g} \end{bmatrix} = \begin{bmatrix} E_{g1} \\ \sum_n I_{gn} \end{bmatrix} = \begin{bmatrix} E_{g2} \\ \sum_n I_{gn} \end{bmatrix} = \ldots = \begin{bmatrix} E_{gQ} \\ \sum_n I_{gn} \end{bmatrix}$$

As shown previously, the mutual and self impedances are written as follows:

$$\left[[F_n]_k \right] = \left[[Z_{np}]_{km} \right] \left[[v_p]_m \right]$$

or, for short: $[F_{kn}] = [Z_{kmnp}]\,[v_{mp}]$

and this again leads to: $\left(\frac{1}{v_{gq}}\right)(F_{gq}) = \left(\frac{1}{v_{gq}}\right)\left(\sum_{m,p} Z_{gmqp} v_{mp}\right)$

Rewriting the transducer equation for the qth element of the gth module gives:

$$E_{gq} = A_{gq}\, F_{gq} + B_{gq}\, v_{gq}$$
$$I_{gq} = C_{gq}\, F_{gq} + D_{gq}\, v_{gq}$$

$$E_{gq} = A_{gq}\left(\frac{F_{gq}}{v_{gq}} + Z_{ec}^{gq}\right) v_{gq}$$

$$I_{gq} = C_{gq}\left(\frac{F_{gq}}{v_{gq}} + Z_{i}^{gq}\right) v_{gq}$$

where: $Z^{gq}_{ec} \stackrel{d}{=} \frac{B_{gq}}{A_{gq}}$

$Z^{gq}_{ic} \stackrel{d}{=} \frac{D_{gq}}{C_{gq}}$

Finally: $E_{gq} = A_{gq}(\sum_{m,p} Z_{gmqp} \, v_{mp} + Z^{gq}_{ec} v_{gq})$

$I_{gq} = C_{gq}(\sum_{m,p} Z_{gmqp} v_{mp} + Z^{gq}_{ic} v_{gq})$

We may now write (for the entire array of G modules):

$[E_{kn}] = \text{diag } (A_{kn}) \; [Z^{e}_{kmnp}] \; [v_{mp}]$

$[I_{kn}] = \text{diag } (C_{kn}) \; [Z^{i}_{kmnp}] \; [v_{mp}]$

where: $\text{diag } (A_{kn}) \stackrel{d}{=} \text{diag}\Big(\text{diag } (An)_k \Big)$

$[Z^{e}_{kmnp}] \stackrel{d}{=} \Big[[Z^{e}_{np}]_{km} \Big]$

and where: $Z^{e}_{kmnp} = Z^{i}_{kmnp} = Z_{kmnp}$ Iff($k \neq m$ or $n \neq p$)

$Z^{e}_{kmnp} = Z_{kmnp} + Z^{kn}_{ec}$ Iff ($k = m$ and $n = p$)

$Z^{i}_{kmnp} = Z_{kmnp} + Z^{kn}_{ic}$ Iff ($k = m$ and $n = p$)

Finally, we can determine the transfer matrix, $\Big[[K_{np}]_{km} \Big]$, for both the instance where $S = E_{in}$ and where $S = I_{in}$.

If: $S_i = E^{i}_{in}$; $[K_{kmnp}] \stackrel{d}{=} [K^{e}_{kmnp}]$

$$[S_i^e] = \text{diag}\ (\alpha_i)\ [E_i] + \text{diag}\ (\beta_i)\ [I_i]$$

$$\text{or: } [S_i^e] = \text{diag}\ (\alpha_i)\ [E_i] + \text{diag}\ (\beta_i)\ [\sum_n I_{kn}]$$

Since we wish to determine the relationship between the head velocities of the elements and the signal inputs of their associated amplifiers, we rewrite the above equation in a form which facilitates the desired development.

$$[S_n^e]_k = \text{diag}\ (\alpha_{kn}) \left[[E_n]_k\right] + \text{diag}\ (\beta_{kn}) \left[[\sum_n I_n]_k\right]$$

where $[S_n^e]$, $[E_n]$, and $[\sum_n I_n]$ are column vectors having Q_k identical components.

By substituting where appropriate:

$$[S_{kn}^e] = \Big(\text{diag}\ (\alpha_{kn})\ \text{diag}\ (A_{kn})\ [Z_{kmnp}^e] + \text{diag}\ (\beta_{kn})\ \text{diag}\ (C_{kn}) \left[[\sum_n Z_{np}^i]_{km}\right]\Big) \times [v_{mp}]$$

$$\text{Thus: } [S_{kn}^e] = [K_{kmnp}^e]\ [v_{mp}]$$

or, more formally:

$$\left[[S_n^e]_k\right] = \left[[K_{np}^e]_{km}\right] \left[[v_p]_m\right]$$

$$\text{where: } \left[[K_{np}^e]_{km}\right] = \text{diag}\left(\text{diag}(\alpha n)_k\right) \text{diag}\left(\text{diag}(An)_k\right) \left[[Z_{np}^e]_{km}\right]$$

$$+ \text{diag}\left(\text{diag}(\beta_n)_k\right) \text{diag}\left(\text{diag}(Cn)_k\right) \left[[\sum_n Z_{np}^i]_{km}\right]$$

If: $S_i = I^i_{in}$: $[K_{kmnp}] \stackrel{d}{=} [K^i_{kmnp}]$

By the same procedures: $$\Big[[S^i_n]_k\Big] = \Big[[K^i_{np}]_{km}\Big]\Big[[v_p]_m\Big]$$

where: $$\Big[[K^i_{np}]_{km}\Big] = \mathrm{diag}\Big(\mathrm{diag}\,(\gamma_n A_n)_k\Big)\Big[[Z^e_{np}]_{km}\Big]$$
$$+ \mathrm{diag}\Big(\mathrm{diag}\,(\delta_n C_n)_k\Big)\Big[[\Sigma_n Z^i_{np}]_{km}\Big]$$

The General Modular Array with Series Elements

The development in this case is nearly identical in principle to that presented in the previous section. The results will be presented here without comment.

① $$\Big[[S^e_n]_k\Big] = \Big[[K^e_{np}]_{km}\Big]\Big[[v_p]_m\Big] ;$$

$$\Big[[K^e_{np}]_{km}\Big] = \mathrm{diag}\Big(\mathrm{diag}\,(\alpha_n A_n)_k\Big)\Big[[\Sigma_n Z^e_{np}]_{km}\Big]$$
$$+ \mathrm{diag}\Big(\mathrm{diag}\,(\beta_n C_n)_k\Big)\Big[[Z^i_{np}]_{km}\Big] ;$$

② $$\Big[[S^i_n]_k\Big] = \Big[[K^i_{np}]_{km}\Big]\Big[[v_p]_m\Big] ;$$

$$\Big[[K_{np}]_{km}\Big] = \mathrm{diag}\Big(\mathrm{diag}\,(\gamma_n A_n)_k\Big)\Big[[\Sigma_n Z^e_{np}]_{km}\Big]$$
$$+ \mathrm{diag}\Big(\mathrm{diag}\,(\delta_n C_n)_k\Big)\Big[[Z^i_{np}]_{km}\Big] .$$

More General Modular Arrays

If an array has several modules wherein some elements are connected electrically in parallel and others are connected in series, the transfer matrix for the array may be developed in a manner similar to the developments presented in this memo. We do not include such a development here as we feel that it would be of very limited usefulness at the present time.

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