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Department of Ocean Engineering

Cambridge, MA 02139

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A NEW SLENDER-SHIP THEORY

OF WAVE RESISTANCE

by

Francis Noblesse

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A new slender-ship theory of wave resistance is presented. Specifically, a sequence of explicit slender-ship wave-resistance approximations is obtained. These approximations are associated with successive approximations in a slender-ship iterative procedure for solving a new (nonlinear integro-differential) equation for the velocity potential of the flow caused by the ship. The zeroth-, first-, and second-order approximations are given explicitly and examined in some detail.

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A NEW SLENDER-SHIP THEORY OF WAVE RESISTANCE

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ABSTRACT

A new slender-ship theory of wave resistance is presented. Specifically, a sequence of explicit slender-ship wave-resistance approximations is obtained. These approximations are associated with successive approximations in a slender-ship iterative procedure for solving a new (nonlinear integro-differential) equation for the velocity potential of the flow caused by the ship. The zeroth-, first-, and second-order slender-ship approximations are given explicitly and examined in some detail.

The zeroth-order slender-ship wave-resistance approximation, $r^{(0)}$, is obtained by simply taking the (disturbance) potential, ϕ, as the trivial zeroth-order slender-ship approximation $\phi^{(0)} \equiv 0$ in the expression for the Kochin free-wave amplitude function. The classical wave-resistance formulas of Michell (1898) and Hogner (1932) correspond to particular cases of this simple approximation.

The low-speed wave-resistance formulas proposed by Guevel (1974), Baba (1976), Maruo (1977) and Kayo (1978) are essentially equivalent (for most practical purposes) to the first-order slender-ship low-Froude-number approximation, $r^{(1)}_{\ell F}$, which is a particular case of the first-order slender-ship approximation $r^{(1)}$. Specifically, the first-order slender-ship wave-resistance approximation $r^{(1)}$ is obtained by approximating the potential ϕ in the expression for the Kochin function by the first-order slender-ship potential $\phi^{(1)}$, whereas the low-Froude-number approximation $r^{(1)}_{\ell F}$ is associated with the zero-Froude-number limit $\phi^{(1)}_0$ of the potential $\phi^{(1)}$.

A major difference between the first-order slender-ship potential $\phi^{(1)}$ and its zero-Froude-number limit $\phi^{(1)}_0$ resides in the waves that are included in the potential $\phi^{(1)}$ but are ignored in the zero-Froude-number potential $\phi^{(1)}_0$. Results of calculations, due to C.Y. Chen, for the Wigley ship form show that the waves in the potential $\phi^{(1)}$ have a remarkable effect upon the wave resistance; in particular, they cause a large phase shift of the wave-resistance curve towards higher values of the Froude number. Comparison of the first-order slender-ship wave-resistance approximation with experimental results shows fairly good agreement, in considerable improvement with respect to the Guevel-Baba-Maruo-Kayo low-speed theory.

1. Introduction

The problem of predicting the wave resistance of a ship in steady rectilinear motion in a quiescent sea is one of the classical and basic problems in ship hydrodynamics. It is also one of the most difficult. As a matter of fact, in spite of considerable research activities, notably in the past decade, since the famous pioneering study of Michell (1898), discrepancies between values of the wave-resistance coefficient determined experimentally and predicted by various analytical theories and numerical methods often are quite significant and may even be extremely large in some cases, as is clearly apparent from the Proceedings of the recent Workshop on Ship Wave-Resistance Computations (1979).

In this study, a new slender-ship theory of wave-resistance is presented. Specifically, a sequence of explicit slender-ship wave-resistance approximations is obtained. These approximations are associated with successive approximations in a slender-ship iterative procedure for solving a new (nonlinear integro-differential) equation for the velocity potential of the flow caused by the ship. The zeroth-, first-, and second-order slender-ship approximations are given explicitly and examined in some detail.

The zeroth-order slender-ship wave-resistance approximation, $r^{(0)}$, is obtained by simply taking the (disturbance) potential, ϕ, as the trivial zeroth-order slender-ship approximation $\phi^{(0)}\equiv 0$ in the expression for the Kochin free-wave amplitude function. The classical wave-resistance formulas of Michell (1898) and Hogner (1932) correspond to particular cases of the zeroth-order slender-ship approximation $r^{(0)}$. Specifically, the Hogner approximation is obtained by neglecting the waterline integral (of order b^3 in the beam/length ratio b) in comparison to the hull integral (of order b) in the expression for the zeroth-order slender-ship approximation to the Kochin free-wave amplitude function. The Michell approximation, on the other hand, corresponds to the consistent first-order thin-ship approximation to the zeroth-order slender-ship approximation. A simple example is used to demonstrate that neglect of the waterline integral or/and use of the thin-ship approximation may have important effects.

The first-order slender-ship wave-resistance approximation, $r^{(1)}$, is obtained by approximating the potential ϕ in the expression for the Kochin function by the first-order slender-ship potential $\phi^{(1)}$. In the thin-ship limit, the potential $\phi^{(1)}$ becomes the classical Michell potential; more precisely, the Michell potential corresponds to the first-order approximation in a thin-ship expansion of the potential $\phi^{(1)}$. In the zero-Froude-number limit, the potential $\phi^{(1)}$ becomes the potential $\phi_0^{(1)}$,

NOTE

This report is based upon three previous reports entitled:
"Potential Theory of Steady Motion of Ships, Parts 1 & 2",
MIT Dept. of Ocean Engineering Rep. No. 78-4, Sep. 1978, 43 pp.
"Part 3: Wave Resistance" MIT Dept. Ocean Eng. Rep. No. 78-5, Nov. 1978, 26 pp.
"Part 4: Low-Froude-Number Approximations" MIT OE Rep. No. 79-1, May 1979, 40 pp.
A large part of the material included in this report has thus been presented previously. In particular, the new integro-differential equation (5.1) for determining the velocity potential of the flow caused by the ship, upon which the present slender-ship theory is based, was first presented in a manuscript entitled: "The Potential-Flow Problem in the Theory of Steady Motion of a Ship" that was submitted for publication to the Journal of Ship Research in January 1979.

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I am indebted to Mr. Cheng-Yo Chen for his detailed review of the report, and for permitting that figure A be included here. I also wish to thank Professor Louis Landweber and Dr. Som D. Sharma for their continuous support throughout the development of this work.

which is a first-order slender-ship approximation to the zero-Froude-number (double-hull) potential ϕ_0. It is shown in Noblesse and Triantafyllou (1980) that the potential $\phi_0^{(1)}$ provides a fairly good approximation to the potential ϕ_0 for longitudinal motion of a slender body. In particular, the potential $\phi_0^{(1)}$ is proportional to the potential ϕ_0 for ellipsoidal hull forms.

The wave-resistance approximation associated with the use of the zero-Froude-number potential ϕ_0 as an approximation to the potential ϕ in the expression for the Kochin function is referred to as the low-Froude-number approximation, $r_{\ell F}$. This wave-resistance approximation is identical to the (closely-related and essentially-equivalent) low-speed wave-resistance formulas proposed by Guevel, Vaussy, and Kobus (1974), Baba (1976), Maruo (1977), and Kayo (1978). The wave-resistance approximation associated with the first-order slender-ship approximation $\phi_0^{(1)}$ to the zero-Froude-number potential ϕ_0 thus corresponds to a first-order slender-ship approximation, $r_{\ell F}^{(1)}$, to the Guevel-Baba-Maruo-Kayo (GBMK) low-Froude-number approximation $r_{\ell F}$.

The low-Froude-number wave-resistance approximation $r_{\ell F}$, the first-order slender-ship low-Froude-number approximation $r_{\ell F}^{(1)}$, and the zeroth-order slender-ship approximation $r^{(0)}$ (associated with the approximations $\phi=\phi_0$, $\phi=\phi_0^{(1)}$, and $\phi=\phi^{(0)}\equiv 0$, respectively) are compared to one another in the particular case when the "ship" hull is a vertical cylinder with elliptic waterline. For this particular case, the approximations $r^{(0)}$, $r_{\ell F}^{(1)}$, and $r_{\ell F}$, actually are proportional to one another. Specifically, we have $r^{(0)}= r_{\ell F}/(1+b)^2$, and $r_{\ell F}^{(1)}= r_{\ell F}(1+2b)^2/(1+b)^4$, where b is the beam/length ratio of the elliptical cylinder. The relative errors, $\varepsilon^{(0)}$ and $\varepsilon^{(1)}$ say, associated with the approximations $r^{(0)}$ and $r_{\ell F}^{(1)}$, and defined as $\varepsilon^{(0)}=(r_{\ell F}-r^{(0)})/r_{\ell F}$ and $\varepsilon^{(1)}=(r_{\ell F}-r_{\ell F}^{(1)})\,/\,r_{\ell F}$, can then readily be determined in terms of the beam/length ratio b. It may thus be found that we have $\varepsilon^{(0)}= .174$ and $\varepsilon^{(1)}= .016$ for b= .1, and $\varepsilon^{(0)}= .306$ and $\varepsilon^{(1)}= .055$ for b=.2. The first-order slender-ship low-Froude-number approximation $r_{\ell F}^{(1)}$ thus differs from the low-Froude-number approximation $r_{\ell F}$ by only a few percent for a thin vertical elliptical cylinder.

Calculations by Chen and Noblesse (1980) for the case of a vertical cylinder with a waterline in the shape of an ogive have shown that differences between the approximations $r_{\ell F}^{(1)}$ and $r_{\ell F}$ are somewhat larger than for an elliptical cylinder with same beam/length ratio, but remain small, of the order of a few percent for typical values of b. This numerical study also includes results of calculations for ellipsoidal hull forms; these show that the approximations $r_{\ell F}^{(1)}$ and $r_{\ell F}$ are practically indistinguishable for values of the beam/length ratio equal to .1 or .2. Finally, figure A, due to C.Y. Chen[a], for the Wigley ship form[b] show that the wave-resistance curve corresponding to the approximation[c] $r_{\ell F}^{(1)}$ is fairly close to the points determined by averaging the numerical results obtained by several researchers[d] on the basis of the GBMK low-speed theory.

The first-order slender-ship low-Froude-number approximation $r_{\ell F}^{(1)}$, associated with the approximation $\phi=\phi_0^{(1)}$, is a particular case of the first-order slender-ship approximation $r^{(1)}$, which corresponds to the approximation $\phi=\phi^{(1)}$ as was noted previously. A major difference between the potentials $\phi^{(1)}$ and $\phi_0^{(1)}$ resides in the waves that are included in the potential $\phi^{(1)}$ but are ignored in the zero-Froude-number potential $\phi_0^{(1)}$. For the present problem of determining the wave resistance, it naturally seems desirable to retain the waves in the potential $\phi^{(1)}$. As a matter of fact, comparison of the wave-resistance curves associated with the approximation[e] $r^{(1)}$ and the low-Froude-number approximation $r_{\ell F}^{(1)}$ in figure A show that the waves in the potential $\phi^{(1)}$ have a remarkable effect upon the wave resistance. The most important difference between the wave-resistance curves $r^{(1)}$ and $r_{\ell F}^{(1)}$ is a large phase shift. Furthermore, the curve $r^{(1)}$ is slightly lower, in the mean, than the curve $r_{\ell F}^{(1)}$, and the amplitude of the oscillations in the curve $r^{(1)}$ are somewhat less than those in the curve $r_{\ell F}^{(1)}$.

Figure A shows that the first-order slender-ship wave-resistance curve $r^{(1)}$ passes through the five brackets of experimental values[f] at the Froude numbers F= .18, .20, .22, .24, and .266, whereas the curves associated with the low-Froude-number approximation $r_{\ell F}^{(1)}$, the zeroth-order approximation $r^{(0)}$, and the Michell approximation r_M are usually outside (well outside in some cases) these experimental brackets. The points corresponding to the average of numerical results for the GBMK low-speed theory clearly are in phase with the curve $r_{\ell F}^{(1)}$, indeed fairly close to it (somewhat below) as was noted previously, and these points also are usually outside the experimental brackets. The results corresponding to the modification of the GBMK theory by Nakatake, Toshima and Yamazaki (1979), in which a local nonuniform flow transformation is used to incorporate the effect of the nonuniformity of the double-hull flow upon the propagation of waves, are not significantly different from the GBMK results, and in fact are quite close to the curve $r_{\ell F}^{(1)}$ (except for values of the Froude number above .24, where a slight phase shift may be observed).

Also shown in figure A is the wave-resistance curve corresponding to the simplified low-Froude-number asymptotic form of the GBMK theory proposed by Baba (1979). This resistance curve is plausible at sufficiently low values of the Froude number, say for $F \leq .22$, but the curve is well above the experimental brackets at the Froude numbers F=.24 and F=.266. A notable feature of the Baba resistance curve is that its oscillations are considerably less than those of the other curves; it also appears to increase more rapidly as the Froude number increases. Figure A also shows five values of the wave-resistance coefficient obtained by Kitazawa and Kajitani (1979) on the basis of a modified form of the GBMK theory in which the boundary condition at the hull surface is satisfied more accurately. Two of these values are fairly close to the first-order slender-ship curve $r^{(1)}$, but the other three values are much below it; the results corresponding to the two highest Froude numbers are well below the experimental results.

Figure A finally shows the numerical results obtained by Dawson (1979). These results are somewhat below the first-order slender-ship wave-resistance curve $r^{(1)}$, but clearly appear to be in phase with it. Dawson's result at the Froude number .266 is somewhat below the experimental bracket, and the result at F=.20 is on the low side of the experimental bracket. This suggests that Dawson's results may in fact be somewhat low. The first-order slender-ship wave-resistance curve $r^{(1)}$, on the other hand, is slightly on the high side of the experimental brackets, perhaps as a result of the various approximations[e] used for the purpose of simplifying the numerical calculations. The approximation $r^{(1)}$ and the numerical results of Dawson thus appear to be roughly in agreement.

In summary, on the basis of the results shown in figure A, the first-order slender-ship wave-resistance approximation $r^{(1)}$ appears to yield results significantly different from those predicted by the Guevel-Baba-Maruo-Kayo low-speed theory or the several above-mentioned modifications of this theory, and in better agreement with experimental results. As a matter of fact, among the five resistance curves and the four sets of numerical values shown in figure A, only the first-order slender-ship wave-resistance curve $r^{(1)}$ passes through all the five experimental brackets.

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a. I wish to thank Mr. Cheng-Yo Chen for permitting that figure A be included in the present report. This figure is excerpted from Mr. Chen's Ph.D. dissertation, currently in preparation.

b. The Wigley hull is defined by the equation $y=\pm.05(1-4x^2)(1-256z^2)$, where $-.05\le x\le .05$ and $0\ge z\ge -.0625$.

c. The approximation $r^{(1)}_{\ell F}$ shown in figure A does not include the nonlinear free-surface correction integral in expression (6.6) for the Kochin free-wave amplitude function. This free-surface integral was neglected for simplicity.

d. The average values shown in figure A for the wave-resistance coefficient predicted by the Guevel-Baba-Maruo-Kayo low-speed theory have been determined from the numerical results given page 60 of the Proceedings of the Workshop on Ship Wave-Resistance Computations (1979). Specificially, the following values were used:

Froude number:	.16	.18	.20	.22	.24	.266
Kitazawa & Kajitani:	-	-	.55	.41	1.02	.69
Maruo & Suzuki:	.20	.42	.62	.45	1.00	-
Miyata & Kajitani:	-	-	-	-	-	.61
Mori (potential flow):	.20	.47	.59	.41	1.03	.62
Nakatake (Baba):	.22	.48	.57	.40	1.03	.66
Nakatake (Guevel):	.24	.45	.56	.40	-	.65
Yim:	-	-	-	-	1.04	.65
average:	.22	.46	.58	.41	1.02	.65

e. For simplicity, the nonlinear free-surface term $K_o^{(1)}$ in expression (9.3) for the first-order slender-ship approximation to the Kochin free-wave amplitude function was ignored in the approximation $r^{(1)}$ shown in figure A. The near-field potential $\phi_N^{(1)}$ in expression (9.1) for the first-order slender-ship potential $\phi^{(1)}$ was also neglected, as can be justified for low Froude numbers. Finally, calculations were further simplified by replacing the wave potential $\phi_W^{(1)}$ given by equation (9.1b) by its Michell thin-ship approximation, as is approximately justified by the fairly small differences that may be observed in figure A between the zeroth-order slender-ship wave-resistance curve $r^{(0)}$ and the Michell curve r_M.

f. The experimental brackets shown in figure A correspond to the highest and lowest of 11 experimental values consisting of the 9 values given page 52 of the Proceedings of the Workshop on Ship Wave-Resistance Computations (1979) and the 2 values given in Gadd and Hogben (1965). The one negative value given page 52 of the Proceedings of the Workshop for the Froude number .18 was ignored however. Furthermore, the 11 experimental values at the Froude number .266 have been corrected for effects of sinkage and trim by using both multiplicative and additive corrections determined from the numerical results obtained by Gadd (1979), Guevel, Delhommeau, and Cordonnier (1979, 1980), Dawson (1979), and Daube (1980) for the Wigley hull held fixed (no sinkage and trim) and left free to heave and trim.

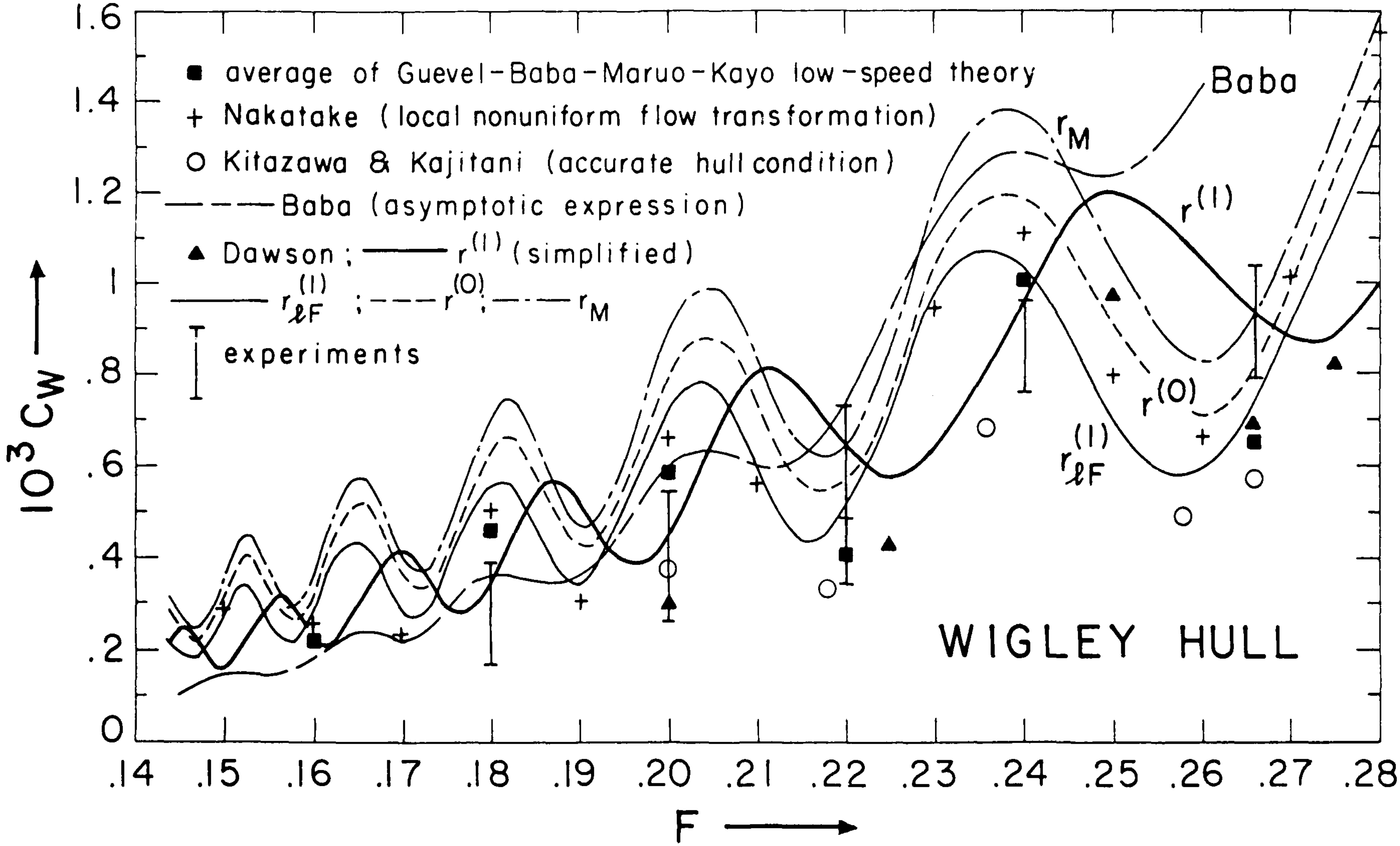

Figure A - Comparison of experimental and numerical results for the Wigley hull

2. Formulation of the problem

The problem examined in this study is that of predicting the wave resistance experienced by a ship, with length L say, in steady rectilinear motion, with speed U, at the free surface of an otherwise quiescent sea of large depth and horizontal extent. Water is supposed to be homogeneous and incompressible, with density ρ. Viscosity effects are ignored, and irrotational flow is assumed. Effects of surface tension, wavebreaking, and spray formation at the ship bow are also neglected.

The flow caused by the ship is rendered independent of time by observing it from a moving system of coordinates attached to the ship. Specifically, the Z axis is chosen vertical and pointing upwards, with the mean sea surface taken as the plane Z=0. The X axis is in the mean sea plane, parallel to the direction of motion of the ship, and pointing towards the bow. The Y axis is in the mean sea plane, orthogonal to the X and Z axes, and oriented so that the X,Y,Z axes form a right-handed Cartesian system of coordinates.

Flow variables are made nondimensional with respect to the length L and the speed U of the ship, and the density of water ρ. The nondimensional coordinates $(x,y,z)\equiv(X,Y,Z)/L$ and velocity potential $\phi\equiv\Phi/UL$, where $\vec{X}$ and Φ are dimensional, thus are defined, and the nondimensional flow velocity is given by $(\partial\phi/\partial x,\partial\phi/\partial y,\partial\phi/\partial z)\equiv(\partial\Phi/\partial X,\partial\Phi/\partial Y,\partial\Phi/\partial Z)/U$.

The velocity potential ϕ satisfies the Laplace equation $\nabla^2\phi=0$ in the flow domain. On the hull surface, the potential ϕ satisfies the condition $\partial\phi/\partial n=n_x$, where n_x is the x component of the unit vector $\vec{n}$ normal to the hull surface and pointing into the water, and $\partial\phi/\partial n\equiv\nabla\phi\cdot\vec{n}$ is the derivative of ϕ in the direction of the vector $\vec{n}$. On the sea surface, given by the equation $z=F^2(\partial\phi/\partial x-|\nabla\phi|^2/2)$, the potential ϕ satisfies the condition

$$\partial\phi/\partial z + F^2(\partial^2\phi/\partial x^2-\partial|\nabla\phi|^2/\partial x+\nabla\phi\cdot\nabla|\nabla\phi|^2/2) = 0,$$

where F is the Froude number defined as $F\equiv U/(gL)^{1/2}$ and $|\nabla\phi|^2\equiv(\partial\phi/\partial x)^2+(\partial\phi/\partial y)^2+(\partial\phi/\partial z)^2$ is the square of the magnitude of the fluid velocity vector $\nabla\phi$.

The position of the sea surface is evidently not known a priori. The sea surface boundary condition will then be transferred to the mean sea surface z=0 by using a Taylor series expansion about the plane z=0, in the usual manner. We may then obtain the mean-sea-surface boundary condition

$$\partial\phi/\partial z + F^2\partial^2\phi/\partial x^2 - F^2 q(\phi) + O(F^4\phi^3) = 0, \tag{2.1}$$

where the term $q(\phi)$ is defined as

$$q(\phi) = \partial|\nabla\phi|^2/\partial x - \nabla\phi\cdot\nabla|\nabla\phi|^2/2 - (\partial\phi/\partial x-|\nabla\phi|^2/2)\partial(\partial\phi/\partial z+F^2\partial^2\phi/\partial x^2)/\partial z, \qquad (2.2)$$

and the term $O(F^4\phi^3)$ represents terms that are at least of fourth power in the Froude number F and of third power in the potential ϕ or its derivatives, and may presumably be neglected.

In accordance with the transfer of the sea-surface boundary condition to the mean sea surface, the Laplace equation will be satisfied in the mean flow domain, d say, bounded upwards by the plane z=0, or more precisely by the portion, σ say, of the plane z=0 outside the ship hull surface; we thus have

$$\nabla^2\phi = 0 \text{ in the mean flow domain d.} \qquad (2.3)$$

The boundary condition on the hull surface will similarly be applied on the mean hull surface, h say, that is the portion of the ship hull located below the plane z=0; this yields

$$\partial\phi/\partial n = n_x \text{ on the mean hull surface h.} \qquad (2.4)$$

A major difficulty of the above problem stems from the sea-surface boundary condition (2.1, 2.2), which is nonlinear. However, the nonlinear term $F^2q(\phi)$ in equation (2.1) is of order $F^2\phi^2$, as may be seen from equation (2.2), and thus can be presumed to be small in comparison with the linear term $\partial\phi/\partial z+F^2\partial^2\phi/\partial x^2$ for slender ship hull forms. The nonlinear term $F^2q(\phi)$ may then be neglected in a first approximation. More generally, this nonlinear term will be incorporated in an iterative manner by expressing the sea-surface boundary condition (2.1) in the form

$$\partial\phi/\partial z + F^2\partial^2\phi/\partial x^2 = F^2q(\phi) \text{ on the mean sea surface } \sigma, \qquad (2.5)$$

and treating the term $F^2q(\phi)$ on the right side as a nonhomogeneous term for the linear condition $\partial\phi/\partial z+F^2\partial^2\phi/\partial x^2=0$.

In summary, the problem of flow caused by steady rectilinear motion of a ship in a calm sea is formulated in this study as the potential-flow problem defined by the Laplace equation (2.3) subject to the boundary conditions (2.4) and (2.5) on the ship hull and on the free surface of the sea, respectively. In addition, the usual "radiation condition", specifying that waves are present only behind the ship, must be imposed for uniqueness of the solution.

3. The Green function

The above-defined boundary-value problem will be solved by formulating an integral equation for the velocity potential ϕ based on the use of a Green function satisfying the linearized sea-surface boundary condition. Specifically, this Green function, which will be denoted by $G(\vec{\xi},\vec{x};F^2)$ or simply by G, is the linearized velocity potential of the flow created at point $\vec{\xi}(\xi,\eta,\zeta\leq 0)$ by a unit outflow at point $\vec{x}(x,y,z\leq 0)$, stemming from a submerged source if $z<0$ or from a flux across the mean sea surface if $z=0$.

The Green function satisfies the radiation condition, namely that there are no waves for $\xi>x$, and the equations

$$\nabla^2 G = \delta(x-\xi)\delta(y-\eta)\delta(z-\zeta) \text{ in } z<0, \tag{3.1a}$$

$$\partial G/\partial z + F^2\partial^2 G/\partial x^2 = 0 \quad \text{on } z = 0, \tag{3.1b}$$

if $\zeta<0$, and

$$\nabla^2 G = 0 \quad \text{in } z<0, \tag{3.2a}$$

$$\partial G/\partial z + F^2\partial^2 G/\partial x^2 = -\delta(x-\xi)\delta(y-\eta) \text{ on } z = 0, \tag{3.2b}$$

if $\zeta=0$. Equations (3.1a,b) are well known. These equations, however, are valid only if ζ is strictly negative, and equations (3.2a,b) are proper in the limiting case $\zeta=0$, as is demonstrated in Noblesse (1981).

The Green function may be expressed in the form

$$4\pi G(\vec{\xi},\vec{x};F^2) = -1/|\vec{\xi}-\vec{x}| + N(\vec{\xi},\vec{x};F^2) + W(\vec{\xi},\vec{x};F^2), \tag{3.3}$$

where the first term on the right side, with $|\vec{\xi}-\vec{x}|\equiv[(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2]^{1/2}$, is the Green function for potential flow in an unbounded fluid, and the functions $N(\vec{\xi},\vec{x};F^2)$ and $W(\vec{\xi},\vec{x};F^2)$ stem from the presence of the sea surface and represent a nonoscillatory near-field disturbance and a wavy disturbance, respectively. The function W, representing the waves behind the singularity at point $\vec{x}$, is given by the integral

$$W = H(x-\xi)(4/F^2)\int_{-\infty}^{\infty} \text{Im}\, \exp[(\zeta+z)(1+t^2)/F^2+i\{(\xi-x)+(\eta-y)t\}(1+t^2)^{\frac{1}{2}}/F^2]dt, \tag{3.4}$$

where $H(x-\xi)$ is the Heaviside unit-step function, which takes the value 1 for $x-\xi>0$ and 0 for $x-\xi<0$, and Im represents the imaginary part.

The function N, representing a nonoscillatory near-field disturbance, is given by the integral

$$N = 1/r' + (2/\pi F^2)\int_{-1}^{1} \text{Im}\, \exp(Z)E_1(Z)dt, \tag{3.5}$$

where

$$r' \equiv [(\xi-x)^2+(\eta-y)^2+(\zeta+z)^2]^{1/2}$$

is the distance between the field point $\vec{\xi}$ and the mirror image of the singularity point $\vec{x}$ with respect to the mean sea plane z=0, Z is the complex function of the real variable t defined by

$$Z \equiv [(\zeta+z)(1-t^2)^{1/2}+(\eta-y)t+i|\xi-x|](1-t^2)^{1/2}/F^2,$$

and $E_1(Z)$ is the exponential integral, which is defined as in Abramowitz and Stegun (1965). For purposes of numerical evaluation, a convenient alternative expression for the near-field disturbance N is

$$N = 1/r' - (2/F^2)[1-(\zeta+z)/(r'+|\xi-x|)] + (2/\pi F^2)\int_{-1}^{1} \mathrm{Im}[\exp(Z)E_1(Z)+\ell n(Z)+\gamma]dt, \quad (3.5a)$$

where γ=0.577...is Euler's constant. Expression (3.5) is well suited for obtaining an ascending series of the near-field disturbance N, useful for evaluating N for small values of r'/F^2. Indeed, the first term in this series is $1/r'$, and the second term is shown in expression (3.5a). The third term in the series is given in Noblesse (1978), where the expressions for ∇N corresponding to expressions (3.5) and (3.5a) may also be found.

An interesting alternative form of expression (3.5) is

$$N = -1/r' + (2/\pi F^2)\int_{-1}^{1} \mathrm{Im}[\exp(Z)E_1(Z)-1/Z]dt. \quad (3.6)$$

However, this expression is not well suited for purposes of numerical evaluation. Neither is expression (3.6) suited for obtaining an asymptotic expansion of the near-field disturbance N for large values of r'/F^2. A complementary integral representation of N suited for obtaining such an asymptotic expansion is

$$N = -1/r' + (2/\pi F^2)\int_{-\infty}^{\infty} \mathrm{Re}[\exp(Z')E_1(Z')-1/Z']dt + (4/F^2)\int_{|\xi-x|/|\eta-y|}^{\infty} \mathrm{Im}\,\exp(Z')dt, \quad (3.6a)$$

where Re represents the real part, and Z' is the complex function defined by

$$Z' \equiv [(\zeta+z)(1+t^2)^{1/2}+i(|\xi-x|-|\eta-y|t)](1+t^2)^{1/2}/F^2.$$

Indeed, an asymptotic expansion for large values of r'/F^2 has been obtained in Noblesse (1975) from expression (3.6a) in the particular case when η=y, for which the last integral on the right side of equation (3.6a) vanishes. The first term in this asymptotic expansion is $(-1/r')$. The four alternative integral representations of the near-field disturbance N given above may be found in Noblesse (1981).

We have

$$N \sim 1/r' \text{ as } r'/F^2 \to 0 \text{ and } N \sim -1/r' \text{ as } r'/F^2 \to \infty . \qquad (3.7a,b)$$

Equation (3.7b) shows that far behind the singularity $\vec{x}$, as $\xi \to -\infty$, the near-field disturbance $-1/|\vec{\xi}-\vec{x}|+N$ in expression (3.3) for the Green function is approximately equal to $-2/(\xi^2+\eta^2+\zeta^2)^{1/2}$. This is negligible in comparison with the wave disturbance W. We may then obtain

$$G \sim (1/\pi F^2)\int_{-\infty}^{\infty} \text{Im} \exp[(\zeta+z)(1+t^2)/F^2+i\{(\xi-x)+(\eta-y)t\}(1+t^2)^{1/2}/F^2]dt \quad \text{as } \xi \to -\infty. \qquad (3.8)$$

This result, which is well known of course, will be used further on in this study (in section 6) for determining the amplitude of the free waves far behind the ship.

4. Basic integral identities for the velocity potential

In this section, basic integral identities for the velocity potential are obtained by applying a classical Green identity to the potential $\phi \equiv \phi(\vec{x})$ and the Green function $G \equiv G(\vec{\xi},\vec{x})$ defined above. This Green identity is

$$\int_{d'}(\phi\nabla^2 G - G\nabla^2\phi)dv = \int_{\sigma'}(\phi\partial G/\partial z - G\partial\phi/\partial z)dxdy + \int_h (G\partial\phi/\partial n - \phi\partial G/\partial n)da$$

$$+ \int_{h_\infty}(\phi\partial G/\partial n - G\partial\phi/\partial n)da, \tag{4.1}$$

where d' is the finite domain bounded by the hull surface h, the mean sea plane z=0, and some surface h_∞ surrounding the hull h, as is shown in figure 1; σ' is the portion of the plane z=0 between the intersection curves c and c_∞ of the hull surface h and the exterior surface h_∞, respectively, with the plane z=0. On the surfaces h and h_∞, we have $\partial\phi/\partial n \equiv \nabla\phi\cdot\vec{n}$ and $\partial G/\partial n \equiv \nabla G\cdot\vec{n}$ where $\vec{n}$ is the unit outward normal vector to h or h_∞, as is shown in figure 1. Furthermore, dv and da represent the differential elements of volume and area at point $\vec{x}$ of the domain d' and the surfaces h or h_∞, respectively.

By expressing the integrand $\phi\partial G/\partial z - G\partial\phi/\partial z$ of the first integral on the right side of equation (4.1) in the form $\phi(\partial G/\partial z + F^2\partial^2 G/\partial x^2) - G(\partial\phi/\partial z + F^2\partial^2\phi/\partial x^2) + F^2\partial(G\partial\phi/\partial x - \phi\partial G/\partial x)/\partial x$, and by using the relation

$$\int_{\sigma'}\partial(G\partial\phi/\partial x - \phi\partial G/\partial x)/\partial x dxdy = \int_c (G\partial\phi/\partial x - \phi\partial G/\partial x)dy + \int_{c_\infty}(G\partial\phi/\partial x - \phi\partial G/\partial x)dy,$$

we may obtain

$$\int_{d'}\phi\nabla^2 G dv - \int_{\sigma'}\phi(\partial G/\partial z + F^2\partial^2 G/\partial x^2)dxdy = \int_{d'}G\nabla^2\phi dv - \int_{\sigma'}G(\partial\phi/\partial z + F^2\partial^2\phi/\partial x^2)dxdy$$

$$+ \int_h (G\partial\phi/\partial n - \phi\partial G/\partial n)da + F^2\int_c (G\partial\phi/\partial x - \phi\partial G/\partial x)dy + I_\infty, \tag{4.2}$$

where I_∞ is defined by

$$I_\infty = \int_{h_\infty}(\phi\partial G/\partial n - G\partial\phi/\partial n)da + F^2\int_{c_\infty}(G\partial\phi/\partial x - \phi\partial G/\partial x)dy.$$

It can be shown that I_∞ vanishes as the exterior surface h_∞ is made ever larger. This term can then be ignored in equation (4.2), and the finite domain d' and region σ' of the mean sea plane may be replaced by the unbounded mean flow domain d and sea surface σ, respectively.

By expressing the potential ϕ in the integrands of the two integrals on the left side of equation (4.2) in the form $\phi = \phi_* + (\phi - \phi_*)$, where $\phi \equiv \phi(\vec{x})$ as was defined previously, and ϕ_* represents the potential at point $\vec{\xi}$, i.e. $\phi_* \equiv \phi(\vec{\xi})$, we may obtain

$$\int_d \phi\nabla^2 G dv - \int_\sigma \phi(\partial G/\partial z + F^2\partial^2 G/\partial x^2)dxdy = C\phi_* + C', \tag{4.3}$$

where C and C' are given by

$$C = \int_d \nabla^2 G dv - \int_\sigma (\partial G/\partial z + F^2 \partial^2 G/\partial x^2) dxdy, \tag{4.4}$$

$$C' = \int_d (\phi-\phi_*) \nabla^2 G dv - \int_\sigma (\phi-\phi_*)(\partial G/\partial z + F^2 \partial^2 G/\partial x^2) dxdy.$$

It may be seen from equations (3.1) and (3.2) that we have $C' \equiv 0$ if $\phi-\phi_* \to 0$ as $\vec{x} \to \vec{\xi}$, that is if the potential is continuous everywhere in the solution domain d and on its boundary σ+h+c, as is assumed here. Use of equation (4.3), with $C' \equiv 0$, in equation (4.2) then yields

$$C\phi_* = \int_d G\nabla^2\phi dv - \int_\sigma G(\partial\phi/\partial z + F^2\partial^2\phi/\partial x^2)dxdy + \int_h (G\partial\phi/\partial n - \phi\partial G/\partial n)da$$
$$+ F^2 \int_c (G\partial\phi/\partial x - \phi\partial G/\partial x)dy, \tag{4.5}$$

where C is given by formula (4.4).

Use of equations (3.1) and (3.2) in expression (4.4) for C shows that we have $C \equiv 1$ if the point $\vec{\xi}$ is in the mean flow domain d or on the mean sea plane σ, but strictly outside the hull surface h, whereas we have $C \equiv 0$ if $\vec{\xi}$ is strictly inside h. We thus have

$$\phi_* = \int_d G\nabla^2\phi dv - \int_\sigma G(\partial\phi/\partial z + F^2\partial^2\phi/\partial x^2)dxdy + \int_h (G\partial\phi/\partial n - \phi\partial G/\partial n)da$$
$$+ F^2 \int_c (G\partial\phi/\partial x - \phi\partial G/\partial x)dy \tag{4.6}$$

for $\vec{\xi}$ in d+σ-h-c, and

$$0 = \int_d G\nabla^2\phi dv - \int_\sigma G(\partial\phi/\partial z + F^2\partial^2\phi/\partial x^2)dxdy + \int_h (G\partial\phi/\partial n - \phi\partial G/\partial n)da$$
$$+ F^2 \int_c (G\partial\phi/\partial x - \phi\partial G/\partial x)dy \tag{4.6'}$$

for $\vec{\xi}$ in $d_i+\sigma_i$-h-c, where d_i and σ_i represent the domain and the portion of the plane z=0, respectively, inside the hull surface h. It can also be seen from equations (3.1) and (3.2) that we have C=1/2 if the point $\vec{\xi}$ is exactly on the hull surface h or its intersection c with the plane z=0, at least for points $\vec{\xi}$ where h+c is smooth; more generally, the value of $4\pi C$(or $2\pi C$) at a point $\vec{\xi}$ of h (or c) is equal to the angle at which d(or σ) is viewed from the point $\vec{\xi}$. We then have

$$\phi_*/2 = \int_d G\nabla^2\phi dv - \int_\sigma G(\partial\phi/\partial z + F^2\partial^2\phi/\partial x^2)dxdy + \int_h (G\partial\phi/\partial n - \phi\partial G/\partial n)da \tag{4.6''}$$
$$+ F^2 \int_c (G\partial\phi/\partial x - \phi\partial G/\partial x)dy$$

for $\vec{\xi}$ exactly on smooth h+c. Equations (4.6) are well known, although the particular case of these equations corresponding to $\nabla^2\phi=0=\partial\phi/\partial z+F^2\partial^2\phi/\partial x^2$ is usually given in the literature, and these equations are usually obtained from the Green identity (4.1) in a manner different from that shown here.

The value of the constant C on the left side of equation (4.5) is discontinuous across the hull surface, C being equal to 1 outside the ship and to 0 inside, as is explicitly indicated in equation (4.6). This discontinuity in the value of C evidently is accompanied by a corresponding discontinuity on the right side of equation (4.5). Specifically, the latter discontinuity stems from the integrals $\int_h \phi \partial G/\partial n da$ and $\int_c \phi \partial G/\partial x dy$ representing potentials due to dipole distributions on the hull h and the waterline c, respectively. An integral identity valid for any point $\vec{\xi}$ — outside, inside, or exactly on the hull surface — can be obtained by eliminating the discontinuity in the value of C in equation (4.5). This can be done by adding the term $C_i \phi_*$ on both sides of equation (4.5), with C_i given by

$$C_i = \int_{d_i} \nabla^2 G dv - \int_{\sigma_i} (\partial G/\partial z + F^2 \partial^2 G/\partial x^2) dxdy. \tag{4.7}$$

Equation (4.5) then becomes

$$I\phi_* = \int_d G\nabla^2 \phi dv - \int_\sigma G(\partial\phi/\partial z + F^2 \partial^2 \phi/\partial x^2) dxdy + \int_h (G\partial\phi/\partial n - \phi \partial G/\partial n) da$$
$$+ F^2 \int_c (G\partial\phi/\partial x - \phi\partial G/\partial x) dy + C_i \phi_* , \tag{4.8}$$

where I is defined as

$$I = \int_{d+d_i} \nabla^2 G dv - \int_{\sigma+\sigma_i} (\partial G/\partial z + F^2 \partial^2 G/\partial x^2) dxdy.$$

Equations (3.1) and (3.2) show that we have $I \equiv 1$ for any point $\vec{\xi}$ in the lower half space $\zeta \leq 0$. By using the divergence theorem

$$\int_{d_i} \nabla^2 G dv = \int_h \partial G/\partial n da + \int_{\sigma_i} \partial G/\partial z dxdy$$

and the relation

$$\int_{\sigma_i} \partial^2 G/\partial x^2 dxdy = -\int_c \partial G/\partial x dy$$

in equation (4.7), we may obtain

$$C_i = \int_h \partial G/\partial n da + F^2 \int_c \partial G/\partial x dy.$$

By substituting the above expression for C_i into equation (4.8), and replacing I by 1, we may finally obtain

$$\phi_* = \int_d G\nabla^2 \phi dv - \int_\sigma G(\partial\phi/\partial z + F^2 \partial^2 \phi/\partial x^2) dxdy + \int_h [G\partial\phi/\partial n - (\phi - \phi_*) \partial G/\partial n] da$$
$$+ F^2 \int_c [G\partial\phi/\partial x - (\phi - \phi_*) \partial G/\partial x] dy . \tag{4.9}$$

The integral identity (4.9) is valid for any point $\vec{\xi}$, whether outside, inside, or exactly on the hull surface h. This new identity thus is essentially equivalent to the set of the three classical identities (4.6), (4.6′), and (4.6′′), which are valid exclusively for $\vec{\xi}$ outside, inside, or on the hull surface h, respectively. As a matter of fact, these three identities can be derived from the identity (4.9) by noting that we have $\int_h \partial G/\partial n da + F^2 \int_c \partial G/\partial x dy = 0$, 1, or 1/2 for $\vec{\xi}$ outside, inside, or on the hull surface h, respectively. This may be shown by writing the above expression in the form

$$\int_h \partial G/\partial n da + \int_{\sigma_i} \partial G/\partial z dx dy - \int_{\sigma_i} \partial G/\partial z dx dy - F^2 \int_{\sigma_i} \partial^2 G/\partial x^2 dx dy =$$
$$\int_{d_i} \nabla^2 G dv - \int_{\sigma_i} (\partial G/\partial z + F^2 \partial^2 G/\partial x^2) dx dy.$$

The above-stated result and the three integral identities (4.6), (4.6′), and (4.6′′) then readily follow from equations (3.1) and (3.2).

An interesting feature of the integral identity (4.9), or of the related identities (4.6), (4.6′) and (4.6′′), is the appearance of a waterline integral for the case of a sea-surface piercing hull (no waterline integral is present in the case of a fully-submerged body). A simple interpretation of the waterline integral may be obtained by considering a sea-surface piercing hull as the "zero-submergence limit" of the slightly-submerged body consisting of the mean hull h closed by a horizontal "lid" ℓ; specifically, the waterline integral can then be shown to stem from the effect of the lid closing the slightly-submerged body $h+\ell$. Details are given in appendix A. The present study is concerned with the problem of potential flow about a ship, that is the "exterior potential-flow problem". Integral identities corresponding to equations (4.6), (4.6′) and (4.6′′) can evidently also be obtained for the "interior potential-flow problem", that is for the potential, ϕ^i say, defined in the interior domain d_i. These integral identities, as well as identities obtained by adding the integral identities for the exterior and interior problems, are listed in Appendix B.

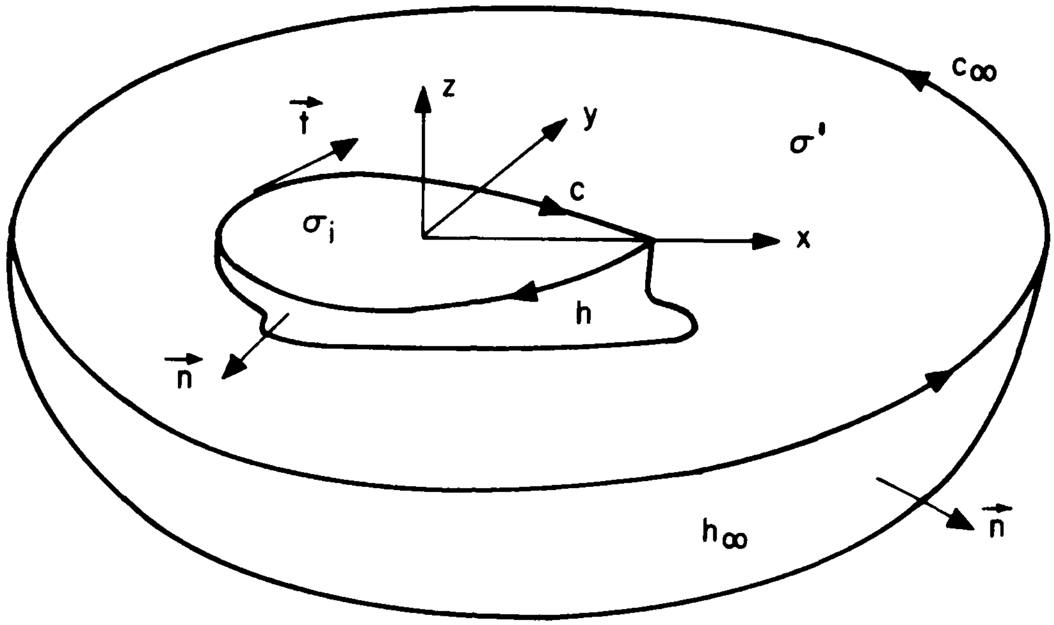

Figure 1—Definition sketch

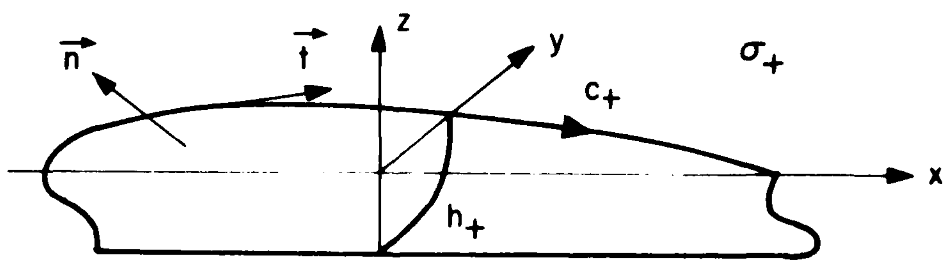

Figure 2—Definition sketch for a single-hull ship with port and starboard symmetry

5. Equation for determining the velocity potential

Use of equations (2.3), (2.4), and (2.5) in the fundamental integral identity (4.9) yields

$$\phi_* = \int_h G n_x da - \int_h (\phi-\phi_*)\partial G/\partial n da + F^2\int_c [G\partial\phi/\partial x-(\phi-\phi_*)\partial G/\partial x]dy - F^2\int_\sigma Gq(\phi)dxdy. \quad (5.1)$$

Equation (5.1) is the basic equation obtained in this study for determining the velocity potential ϕ on the mean hull surface h+c and in the mean fluid domain d+σ. An alternative form of this equation will now be given.

Let n_x, n_y, n_z represent the x, y, z components of the previously-defined unit outward normal vector $\vec{n}$ to the mean hull surface. We also define the unit vector $\vec{t}$ tangent to the mean waterline c, oriented as is shown in figure 1; the x, y, z components of $\vec{t}$ are given by t_x, t_y, 0. If the unit positive vector along the x axis is denoted by $\vec{i}$, we then have

$$\partial\phi/\partial x \equiv \nabla\phi.\vec{i} = (\vec{n}\partial\phi/\partial n+\vec{t}\partial\phi/\partial\ell+\vec{n}\times\vec{t}\partial\phi/\partial d).\vec{i} = n_x\partial\phi/\partial n+t_x\partial\phi/\partial\ell-n_z t_y\partial\phi/\partial d,$$

where $\partial\phi/\partial n$ is the derivative of ϕ in the outward normal direction $\vec{n}$ to h, as was defined previously, ℓ represents the arc length along the mean waterline c (oriented as is shown in figure 1), and $\partial\phi/\partial\ell$ is the derivative of ϕ in the direction of the unit tangent vector $\vec{t}$ to c; finally, $\partial\phi/\partial d$ represents the derivative of ϕ in the direction "d" defined by the unit vector $\vec{n}\times\vec{t}$, which is tangent to the hull surface and pointing downwards. By using equation (2.4), we may then obtain

$$\partial\phi/\partial x = n_x^2 + t_x\partial\phi/\partial\ell - n_z t_y\partial\phi/\partial d.$$

By substituting the above expression for $\partial\phi/\partial x$ into the waterline integral in equation (5.1), and replacing dy by $t_y d\ell$, we may express equation (5.1) in the form

$$\phi(\vec{\xi}) = \varphi(\vec{\xi}) - T(\vec{\xi};\phi). \quad (5.2)$$

The potential $\varphi(\vec{\xi})$ in equation (5.2) is defined as

$$\varphi(\vec{\xi}) = \int_h G n_x da + F^2\int_c G n_x^2 t_y d\ell, \quad (5.2a)$$

and the term $T(\vec{\xi};\phi)$ represents the transform of ϕ defined below. The transform $T(\vec{\xi};\phi)$ may be written as the sum of a linear part, $L(\vec{\xi};\phi)$ say, and the nonlinear part associated with the nonlinear free-surface flux $q(\phi)$. Specifically, we have

$$T(\vec{\xi};\phi) = L(\vec{\xi};\phi) + F^2\int_\sigma Gq(\phi)dxdy, \quad (5.2b)$$

where the linear transform $L(\vec{\xi};\phi)$ is given by

$$L(\vec{\xi};\phi) = \int_h (\phi-\phi_*)\partial G/\partial n da + F^2\int_c [(\phi-\phi_*)\partial G/\partial x - G(t_x\partial\phi/\partial\ell - n_z t_y \partial\phi/\partial d)]t_y d\ell. \qquad (5.2c)$$

The potential $\emptyset(\vec{\xi})$ in equation (5.2) is given explicitly in terms of the Froude number and the geometry (shape) of the hull, as may be seen from equation (5.2a), whereas the term $T(\vec{\xi};\phi)$ is of course not known a priori.

A large class of ships operate at fairly low values of the Froude number. It thus is interesting to briefly examine the limiting case of zero Froude number. Study of this comparatively-simple limiting case will also provide useful knowledge for the case of nonzero Froude number. In the zero-Froude-number limit, the velocity potential ϕ becomes the zero-Froude-number potential, ϕ_0 say. The potential ϕ_0 is the velocity potential of the flow about the hull surface when the sea surface is replaced by the rigid plane z=0, as may be seen from the zero-Froude-number limit of the sea-surface condition (2.5). The potential ϕ_0 thus is also identical to the potential of the flow of an unbounded fluid past the hull and its mirror image with respect to the plane z=0 (the double hull), and indeed is often referred to as the "double-hull potential". Specifically, the zero-Froude-number potential ϕ_0 satisfies the equations

$$\nabla^2\phi_0 = 0 \text{ in } d, \quad \partial\phi_0/\partial n = n_x \text{ on } h, \quad \partial\phi_0/\partial z = 0 \text{ on } \sigma, \qquad (5.3a,b,c)$$

corresponding to equations (2.3), (2.4), and (2.5), respectively.

The equation corresponding to equation (5.2) for determining the zero-Froude-number potential ϕ_0 takes the form

$$\phi_0(\vec{\xi}) = \emptyset_0(\vec{\xi}) - L_0(\vec{\xi};\phi_0), \qquad (5.4)$$

where the potential $\emptyset_0(\vec{\xi})$, corresponding to the potential $\emptyset(\vec{\xi})$ in equation (5.2), is given by

$$\emptyset_0(\vec{\xi}) = \int_h G_0 n_x da, \qquad (5.4a)$$

and the (linear) transform $L_0(\vec{\xi};\phi_0)$, corresponding to the transform $T(\vec{\xi};\phi)$, is defined as

$$L_0(\vec{\xi};\phi_0) = \int_h [\phi_0(\vec{x}) - \phi_0(\vec{\xi})]\partial G_0/\partial n da. \qquad (5.4b)$$

In equations (5.4a,b), G_0 is the zero-Froude-number Green function given by

$$4\pi G_0(\vec{\xi},\vec{x}) = -1/|\vec{\xi}-\vec{x}| - 1/r',$$

where $r' \equiv [(\xi-x)^2 + (\eta-y)^2 + (\zeta+z)^2]^{\frac{1}{2}}$ as was defined below equation (3.5). Equations (5.4a) and (5.4b) can be obtained as the zero-Froude-number limit of equations (5.2a) and (5.2b,c), respectively. Equations (5.4) may also be obtained from equations (5.3a,b,c) in a manner analogous to that used for obtaining equations (5.2) from equations (2.3), (2.4), and (2.5): specifically by applying the Green identity (4.1) to the zero-Froude-number potential ϕ_0 and Green function G_0, as is shown in detail in Noblesse and Triantafyllou (1980).

A particular result of the latter study is relevant to the present problem, namely the fact that we have

$$L_0(\vec{\xi};\phi_0) << \not{\phi}_0(\vec{\xi}) \approx \phi_0(\vec{\xi}), \tag{5.5}$$

for motion of a slender body in the direction of its major dimension (length), like a ship in forward motion. In the special case when the hull surface h is the lower half of an ellipsoid, the potential $\not{\phi}_0(\vec{\xi})$ actually is proportional to the exact potential $\phi_0(\vec{\xi})$, that is we have $\not{\phi}_0(\vec{\xi}) = \lambda\phi_0(\vec{\xi})$, and the value of the constant of proportionality λ is close to unity. For instance, for an ellipsoidal hull form with beam/length and draft/length ratios equal to .15 and .05, respectively, we have $\lambda \simeq .97$; in other words, the slender-ship approximation $\not{\phi}_0(\vec{\xi})$ is smaller than the exact potential $\phi_0(\vec{\xi})$ by about 3%. It is also shown in Noblesse and Triantafyllou (1980) that equation (5.4) may be solved efficiently by using an iterative procedure based on the straightforward recurrence relation

$$\phi_0^{(k+1)}(\vec{\xi}) = \not{\phi}_0(\vec{\xi}) - L_0(\vec{\xi};\phi_0^{(k)}), \tag{5.6}$$

with $k \geq 0$ and the initial (zeroth) approximation $\phi_0^{(0)}$ simply taken as $\phi_0^{(0)} \equiv 0$, so that the potential $\not{\phi}_0(\vec{\xi})$ corresponds to the first approximation $\phi_0^{(1)}$ in the sequence of slender-ship approximations $\phi_0^{(k)}$. For instance, in the special case of the above-defined ellipsoidal hull form, the relative error associated with the second iterative approximation $\phi_0^{(2)}$ is approximately 10^{-3}.

The fact that we have $\not{\phi}(\vec{\xi}) \sim \not{\phi}_0(\vec{\xi})$ in the zero-Froude-number limit and $\not{\phi}_0(\vec{\xi}) \sim \phi_0(\vec{\xi})$ in the slender-ship limit suggests that the velocity potential $\not{\phi}(\vec{\xi})$ defined by equation (5.2a) may provide a useful approximation for common slender ships operating at low Froude number. Furthermore, it is interesting to note that the potential $\not{\phi}(\vec{\xi})$ becomes identical to the classical Michell potential in the thin-ship limit. More precisely, the Michell potential, $\phi_M(\vec{\xi})$ say, given by

$\phi_M(\vec{\xi}) = \int_h G(\vec{\xi},x,y=0,z)n_x da,$

corresponds to the first-order approximation in a thin-ship expansion of the potential $\psi(\vec{\xi})$, as may easily be seen.

In the zero-Froude-number and thin-ship limits examined above, the waterline integral in expression (5.2a) appears to be negligible in comparison with the hull integral. However, the waterline integral may be expected to be significant for a ship form with a blunt bow or/and stern, since we have $|n_x|=1=|t_y|$ at a blunt end. The waterline integral in fact is of utmost importance, as has been demonstrated by Eggers (1980) who showed that the velocity field associated with the hull integral in expression (5.2a) is not continuous along the mean waterline c, and that the waterline integral is precisely needed to render the velocity continuous along c.

The potential $\psi(\vec{\xi})$ defined by equation (5.2a) is satisfactory both in the thin-ship limit and in the zero-Froude-number slender-ship limit, and the velocity field $\nabla\psi$ is everywhere continuous. The potential ψ therefore seems to provide an acceptable slender-ship approximation to the potential ϕ. More generally, the potential ψ may be regarded as the first-order slender-ship approximation $\phi^{(1)}$ in the sequence of slender-ship approximations $\phi^{(k)}$ defined by the recurrence relation

$$\phi^{(k+1)}(\vec{\xi}) = \psi(\vec{\xi}) - T(\vec{\xi};\phi^{(k)}), \tag{5.7}$$

where $T(\vec{\xi};\phi)$ is the transform defined by equations (5.2b,c), and the initial (zeroth) approximation $\phi^{(0)}$ is simply taken as $\phi^{(0)}\equiv 0$.

Equations (5.2) do not require the ship to have port and starboard symmetry; in fact the hull may be a multiple surface, as would be the case for a catamaran or a SWATH ship. In the most common case of a single-hull ship with port and starboard symmetry, equations (5.2) may be simplified somewhat by replacing the mean hull h, waterline c, and sea surface σ by their corresponding positive (port) half h_+, c_+, and σ_+, as is shown in figure 2, and the Green function $G(\vec{\xi},\vec{x})$ by the port- and starboard-symmetry Green function, $\overline{G}(\vec{\xi},\vec{x})$ say, defined as

$$\overline{G}(\vec{\xi},\vec{x}) = G(\vec{\xi},x,y,z) + G(\vec{\xi},x,-y,z).$$

Thus, expression (5.2a) for the first-order slender-ship potential $\phi^{(1)}\equiv\psi$ becomes

$$\phi^{(1)}(\vec{\xi}) = \int_{h_+} \overline{G}n_x da + F^2\int_{c_+} \overline{G}n_x^2 t_y d\ell. \tag{5.8}$$

Numerical evaluation of expressions (5.2a) or (5.8) for the first-order slender-ship potential $\phi^{(1)}$ will be examined further on (in section 9) in this study.

6. The Kochin free-wave amplitude function and the Havelock wave-resistance formula

The wave resistance of a ship can be determined from the wave pattern far behind the ship, as is well known since Havelock (1934). The velocity potential ϕ in the "far field" (at a large distance away from the ship) may be determined in terms of the value of ϕ in the "near field" (on the mean hull surface h+c and on the mean sea surface σ in the vicinity of c) by means of the equation

$$\phi(\vec{\xi}) = \int_h (Gn_x - \phi\partial G/\partial n)da + F^2\int_c [G(n_x^2 + t_x\partial\phi/\partial\ell - n_z t_y \partial\phi/\partial d) - \phi\partial G/\partial x]t_y d\ell - F^2\int_\sigma Gq(\phi)dxdy. \quad (6.1)$$

Equation (6.1) may be obtained from equations (5.2) by replacing the term $\phi-\phi_* = \phi(\vec{x})-\phi(\vec{\xi})$ by ϕ in equation (5.2c) since we have $\phi_*\to 0$ as $(\xi^2+\eta^2+\zeta^2)^{1/2}\to\infty$, and therefore $|\phi_*|<<|\phi|$ for $\vec{\xi}$ in the far field and $\vec{x}$ in the near field. In other words, equation (6.1) corresponds to the far-field limit of equation (5.2). However, it is interesting that equation (6.1) in fact is valid also in the near field. More precisely, equation (6.1) corresponds to equation (4.6) and is valid everywhere in the domain d+τ-h-c, that is for $\vec{\xi}$ strictly outside the hull surface h+c.

If we are only interested in the wave pattern far behind the ship, expression (6.1) for the potential $\phi(\vec{\xi})$ can actually be greatly simplified by replacing the Green function $G(\vec{\xi},\vec{x})$ by the asymptotic approximation (3.8). This yields

$$\phi(\vec{\xi}) \sim (1/\pi)\,\mathrm{Im}\int_{-\infty}^{\infty}\exp[\zeta(1+t^2)/F^2 + i(\xi+\eta t)(1+t^2)^{1/2}/F^2]K(t)dt \text{ as } \xi\to-\infty, \quad (6.2)$$

where the function K(t) is given by

$$K(t) = F^{-2}\int_h (En_x - \phi\partial E/\partial n)da + \int_c [E(n_x^2 + t_x\partial\phi/\partial\ell - n_z t_y\partial\phi/\partial d) - \phi\partial E/\partial x]t_y d\ell - \int_\sigma Eq(\phi)dxdy, \quad (6.3)$$

with the function $E\equiv E(\vec{x};t,F^2)$ defined as

$$E = \exp[F^{-2}(1+t^2)^{1/2}\{(1+t^2)^{1/2}z - i(x+ty)\}]. \quad (6.3a)$$

The equation of the free surface far behind the ship is given by $\zeta=F^2\partial\phi(\xi,\eta,\zeta=0)/\partial\xi$ since the nonlinear terms in the Bernoulli equation may be neglected at a sufficiently large distance away from the ship. By using equation (6.2) we may then obtain

$$\zeta(\xi,\eta) \sim (1/\pi)\,\mathrm{Re}\int_{-\infty}^{\infty}\exp[i(\xi+\eta t)(1+t^2)^{1/2}/F^2]K(t)(1+t^2)^{1/2}dt \text{ as } \xi\to-\infty. \quad (6.4)$$

Equations (6.2) and (6.4) express the potential $\phi(\vec{\xi})$ and the equation of the free surface $\zeta(\xi,\eta)$ far behind the ship in terms of a familiar superposition of elementary plane waves. The function K(t) essentially gives the amplitude of the free wave component at angle $\theta=\tan^{-1}t$ from the x axis; this function will be referred to as the Kochin free-wave amplitude function, as it corresponds to a particular case of the function introduced by Kochin (1936) for determining the drag and lift acting upon a ship in steady rectilinear motion. The nondimensional wave resistance, r say, defined as $r=R/\rho U^2L^2$ where R is the dimensional wave drag, may be directly obtained from the Kochin free-wave amplitude function K(t) by means of the Havelock (1934) formula

$$r \equiv R/\rho U^2L^2 = (1/2\pi)\int_{-\infty}^{\infty}|K(t)|^2(1+t^2)^{1/2}dt. \tag{6.5}$$

This formula may be obtained by performing the change of variable $t=\tan\theta$ in equation (12) page 118 of Eggers, Sharma, and Ward (1967).

Formula (6.3) for the Kochin free-wave amplitude function K(t) may be expressed in the form

$$K(t) = F^{-2}(1+t^2)\int_h \exp\{F^{-2}(1+t^2)z\}E[n_x/(1+t^2)-\{n_z-i(n_x+tn_y)/(1+t^2)^{1/2}\}\phi/F^2]da$$
$$+ \int_c E[n_x^2+i(1+t^2)^{1/2}\phi/F^2+t_x\partial\phi/\partial\ell-n_zt_y\partial\phi/\partial d]t_y d\ell - \int_\sigma Eq(\phi)dxdy, \tag{6.6}$$

where the function $E\equiv E(x,y;t,F^2)$ is given by

$$E = \exp[-iF^{-2}(1+t^2)^{1/2}(x+ty)]. \tag{6.6a}$$

Expression (6.6) for the Kochin function K(t) can be evaluated numerically without difficulty in principle, given the potential ϕ on the mean hull, the tangential velocity components $\partial\phi/\partial\ell$ and $\partial\phi/\partial d$ at the mean waterline c, and the nonlinear free-surface flux $q(\phi)$ on σ. In practice, however, the rapid oscillations of the exponential term E for small values of the Froude number F or/and large values of t may cause a loss of accuracy. Furthermore, some terms in the integrands of the hull and waterline integrals cancel out one another, as is shown below. This requires that comparable methods be used for numerically evaluating the hull and waterline integrals. For instance, it may be useful to partially group the two integrals in the manner shown below, or in some similar fashion.

Let the mean waterline c be defined by the parametric equations

$$x = a(\ell) \text{ and } y = b(\ell), \tag{6.7a,b}$$

where ℓ is the arc length along c as was defined previously. We then have $t_x=a'\equiv da/d\ell$ and $t_y=b'\equiv db/d\ell$. We define the "tangent hull," h^t say, tangent to the mean hull h along the mean waterline c. Specifically, the tangent hull h^t is defined by the equations

$$x = a(\ell) + z\alpha(\ell)\ ,\ y = b(\ell) + z\beta(\ell)\ ,\ 0 \geq z \geq -\delta(\ell)\ , \tag{6.8a,b,c}$$

where $\alpha(\ell)$ and $\beta(\ell)$ are the partial derivatives $\partial x/\partial z$ and $\partial y/\partial z$, respectively, at the waterline c. On the above-defined tangent hull h^t, we have $\vec{n}da=(\partial\vec{x}/\partial z\times\partial\vec{x}/\partial\ell)d\ell dz$. We may then obtain $n_x da=-(b'+z\beta')d\ell dz$, $n_y da=(a'+z\alpha')d\ell dz$, and $n_z da=\{\alpha b'-\beta a'+z(\alpha\beta'-\beta\alpha')\}$ $d\ell dz$. At the waterline c, the unit outward normal vector $\vec{n}$ to h^t is given by

$n_x=-cb'$, $n_y=ca'$, and $n_z=c(\alpha b'-\beta a')$ where $c=1/[1+(\alpha b'-\beta a')^2]^{1/2}$. If the potential ϕ at the point $x=a+z\alpha$, $y=b+z\beta$, z is taken equal to the potential at the corresponding point $x=a$, $y=b$, $z=0$ on c, the hull integral in expression (6.6) can be expressed as the sum of an integral over the tangent hull h^t plus an integral over the hull $h+h^t$. The surface integral over the tangent hull h^t can be partially integrated analytically in the form of a line integral along the mean waterline c, which can be combined with the waterline integral in expression (6.6). We may then express the Kochin function $K(t)$ in the form

$$\begin{aligned}K(t) = \int_c E[&c^2(b')^3+i(1+t^2)^{1/2}b'\phi/F^2+a'b'\partial\phi/\partial\ell+c(b')^2(\beta a'-\alpha b')\partial\phi/\partial d\\ &-\gamma(1-e)\{b'/(1+t^2)+i(b'-ta')\phi/F^2(1+t^2)^{1/2}+(\alpha b'-\beta a')\phi/F^2\}\\ &-\gamma\{\delta e-F^2\gamma(1-e)/(1+t^2)\}\{\beta'/(1+t^2)+i(\beta'-t\alpha')\phi/F^2(1+t^2)^{1/2}+(\alpha\beta'-\beta\alpha')\phi/F^2\}]d\ell\\ + F^{-2}(1+t^2)&\int_{h+h^t}\exp\{F^{-2}(1+t^2)z\}E[n_x/(1+t^2)-\{n_z-i(n_x+tn_y)/(1+t^2)^{1/2}\}\phi/F^2]da\\ - \int_\sigma &Eq(\phi)dxdy,\end{aligned} \tag{6.9}$$

where we have

$$E = \exp[-iF^{-2}(1+t^2)^{1/2}(x+ty)],$$

as was defined previously in equation (6.6a), and

$$c = 1/[1+(\alpha b'-\beta a')^2]^{1/2}, \quad \gamma = [1+i(\alpha+t\beta)/(1+t^2)^{1/2}]/[1+(\alpha+t\beta)^2/(1+t^2)], \text{ and}$$

$$e = \exp[-F^{-2}(1+t^2)\delta\{1-i(\alpha+t\beta)/(1+t^2)^{1/2}\}].$$

In the integral over the tangent hull h^t in expression (6.9), the potential ϕ must be taken equal to the potential at the mean waterline c, as was explained previously; furthermore, the unit normal vector $\vec{n}$ must be taken to be pointing inwards, rather than outwards.

The integrand of the integral over the hull $h+h^t$ in expression (6.9) vanishes as $z\to 0$ since the hull h and the associated tangent hull h^t are tangent along the mean waterline c and have opposite unit normal vectors $\vec{n}$, as was noted above. Furthermore, the exponential factor $\exp\{F^{-2}(1+t^2)z\}$ renders the integrand small for negative values of z, especially if we have $(1+t^2)/F^2 \gg 1$. The modified hull integral over $h+h^t$ in expression (6.9) therefore vanishes as $(1+t^2)d/F^2\to\infty$, where d is of the order of the draft of the ship, and this surface integral can be neglected in comparison with the modified waterline integral in expression (6.9) for sufficiently large values of the parameter $(1+t^2)d/F^2$.

Typical values of the Froude number and of the draft for slow-speed ships may be taken as $F=.15$ and $d=.05$. This yields $d/F^2\approx 2.22$ and $\exp(-d/F^2)\approx .11$, which indicates that the hull integral in expression (6.9) may be neglected in comparison with the waterline integral, as a first approximation. Furthermore, the slopes $\partial x/\partial z \equiv \alpha$ and $\partial y/\partial z \equiv \beta$ of the mean hull h at the waterline c are small for common low-speed ships, so that the last terms (those involving α' and β') in the waterline integral in expression (6.9) may be neglected. Finally, the term $c(b')^2(\beta a'-\alpha b')\partial\phi/\partial d$ also appears to be negligible for common low-speed ships, partly due to the fact that β and α are usually small as was just noted, and that we have $\partial\phi/\partial d \approx -\partial\phi/\partial z = O(F^2)$. Under the above approximations, we may then obtain the following approximate expression for the Kochin function K(t):

$$K(t) \approx \int_c E[\{c^2(b')^2-\gamma(1-e)/(1+t^2)+a'\partial\phi/\partial\ell\}b' \\ +\gamma(1-e)\{i(1+t^2)^{1/2}b'/\gamma(1-e)-i(b'-ta')/(1+t^2)^{1/2}+\beta a'-\alpha b'\}\phi/F^2]d\ell, \quad (6.10)$$

where the nonlinear free-surface correction integral has been ignored. A main recommendation of the above approximate expression for the Kochin function is its relative simplicity. Indeed, equation (6.10) expresses the Kochin function as a single integral on the mean waterline c, and only requires the value of the potential ϕ on c.

The smaller the values of $F^2/d(1+t^2)$, $|\alpha|$ and $|\beta|$, the more accurate the approximate expression (6.10) for the Kochin function K(t) will be. In particular, the tangent hull h^t is identical to the hull h in the case of a hull in the form of a cylinder orthogonal to the plane z=0, and the hull integral in expression (6.9) vanishes exactly if the potential $\phi(\vec{x})$ is taken as the zero-Froude-number potential $\phi_0(x,y)$. By putting $\alpha=0$, $\beta=0$, $c=1$, $\gamma=1$, and $e=0$ in equation (6.10), we may then obtain

$$K(t) = \int_c E[\{(b')^2-1/(1+t^2)+a'\partial\phi/\partial\ell\}b'+it(a'+tb')\phi/F^2(1+t^2)^{1/2}]d\ell. \quad (6.11)$$

The terms $i(1+t^2)^{1/2}t_y\phi/F^2$ and $in_x\phi/F^2(1+t^2)^{1/2}$ in the waterline and the hull integrals, respectively, in expression (6.6) have been combined into the term $it^2b'\phi/F^2(1+t^2)^{1/2}$ in expression (6.11). The fact that the latter term vanishes as $t\to 0$ evidently indicates that the abovementioned terms in the waterline and the hull integrals in expression (6.6) cancel out one another as $t\to 0$. For large values of $(1+t^2)^{1/2}/F^2$ the major contribution to the integral (6.11) stems from points of stationary phase, if any. Points of stationary phase are defined by the relation $dx/d\ell+t\,dy/d\ell = a'+tb'=0$. The term $it(a'+tb')\phi/F^2(1+t^2)^{1/2}$ in equation (6.11), which stems from the terms $i(1+t^2)^{1/2}t_y\phi/F^2$ and $i(n_x+tn_y)\phi/F^2(1+t^2)^{1/2}$ in the waterline and the hull integrals in equation (6.6), thus vanishes at a point of stationary phase. The terms $(b')^3$ and $b'/(1+t^2)$ in expression (6.11), corresponding to the terms $n_x^2 t_y$ and $n_x/(1+t^2)$ in the waterline and the hull integrals in expression (6.6), may also be shown to cancel out at the point of stationary phase $t=-a'/b'$. These cancellations of terms in the waterline integral and in the hull integral in expression (6.6), for $t=0$ and at points of stationary phase, indicate that partial combination of the hull and waterline integrals, in the manner shown above or in some similar fashion, may be useful. The cancellations also demonstrate the importance of the waterline integral.

In the common case of a single-hull ship with port and starboard symmetry, the Kochin free-wave amplitude function $K(t)$ is an even function of t, and the Havelock formula (6.5) becomes

$$r = (1/\pi)\int_0^\infty |K(t)|^2(1+t^2)^{1/2}dt. \tag{6.12}$$

Furthermore, expressions (6.6) and (6.9) for the Kochin function may be expressed in terms of integrals over the positive (port) halves c_+, h_+, σ_+ of the mean waterline c, hull h, and sea surface σ.

The Havelock wave-resistance formulas (6.5) or (6.12), expressions (6.6) or (6.9) for the Kochin free-wave amplitude function $K(t)$, and equation (5.2) for determining the velocity potential ϕ in the "near field" of the ship form a complete set of equations for evaluating the wave resistance of a ship in steady rectilinear motion in a calm sea, given the speed of the ship and the shape and position of its hull. The position of the hull is not known precisely beforehand, due to the sinkage and trim experienced by the ship. Equations for determining the sinkage and trim, which requires evaluation of the hydrodynamic lift and moment exerted upon the ship, must then be added to the above set of equations. However, equations for determining the hydrodynamic lift and moment and the resulting sinkage and trim will not be considered explicitly in this study, which is concerned with the basic hydrodynamical problem of predicting the flow caused by a ship and the ship wave resistance, given its speed and the shape and position of its hull.

Equation (5.2) for determining the velocity potential ϕ may be solved by using an iterative procedure, as was discussed in the previous section. Specifically, a sequence of slender-ship iterative approximations $\phi^{(0)} \equiv 0$, $\phi^{(1)} \equiv \varphi$, $\phi^{(2)}, \ldots$ may be defined, as is indicated by the recurrence relation (5.7). Corresponding sequences of slender-ship approximations $K^{(0)}$, $K^{(1)}, \ldots$ and $r^{(0)}$, $r^{(1)}, \ldots$ to the Kochin free-wave amplitude function K and the wave resistance r can readily be defined, by simply replacing ϕ by $\phi^{(k)}$ and K by $K^{(k)}$, with $k = 0,1,\ldots$, in equations (6.6) and (6.5). The zeroth-, first-, and second-order slender-ship wave-resistance approximations are given and examined in some detail in the following sections. The classical Michell (1898) and Hogner (1932) wave-resistance formulas and the low-Froude-number wave-resistance formulas proposed by Guevel, Vaussy, and Kobus (1974), Baba (1976), Maruo (1977), and Kayo (1978), which can be related to the zeroth- and first-order approximations $r^{(0)}$ and $r^{(1)}$, respectively, will also be considered.

7. The zeroth-order slender-ship wave-resistance approximation

In this section, the zeroth-order slender-ship wave-resistance approximation $r^{(0)}$ corresponding to the zeroth approximation $\phi^{(0)}\equiv 0$ is considered. By putting $\phi\equiv 0$ into expression (6.6), we may obtain the following expression for the zeroth-order slender-ship approximation to the Kochin free-wave amplitude function:

$$K^{(0)}(t) = \int_c E n_x^2 t_y \, d\ell + F^{-2}\int_h \exp\{F^{-2}(1+t^2)z\} E n_x \, da, \tag{7.1}$$

where $E \equiv E(x,y;t,F^2)$ is the exponential function defined by formula (6.6a). An alternative expression for $K^{(0)}(t)$ convenient for computational purposes is given by expression (6.9):

$$K^{(0)}(t) = \int_c E[c^2(b')^2-\gamma(1-e)/(1+t^2)]b' \, d\ell + F^{-2}\int_{h+h^t} \exp\{F^{-2}(1+t^2)z\} E n_x \, da, \tag{7.1a}$$

where h^t is the tangent hull associated to h and defined by equations (6.8), and the functions c, γ, and e are defined below equation (6.9). The waterline integral in formula (7.1a) is expressed in terms of the shape of the mean waterline c, defined by the functions $a(\ell)\equiv x$ and $b(\ell)\equiv y$, the slopes $\alpha(\ell)\equiv\partial x/\partial z$ and $\beta(\ell)\equiv\partial y/\partial z$ of the mean hull h along the waterline c, and the "equivalent-draft function" $\delta(\ell)$ related to the local draft and shape of the framelines of the ship. The hull integral, on the other hand, depends on the precise shape of the "lower hull", at some depth below the mean waterline. This integral is negligible in comparison with the waterline integral for sufficiently large values of $(1+t^2)d/F^2$.

The above-defined zeroth approximation, corresponding to the trivial slender-ship approximation $\phi\equiv 0$, may seem extremely crude. It will however be noted that the approximation (7.1) corresponds to the free waves associated with the first-order slender-ship potential $\phi^{(1)}\equiv\emptyset$ given by equation (5.2a). Further support for the zeroth approximation $K^{(0)}$ is provided by considering expression (6.11) for the Kochin function $K(t)$, obtained in the particular case when $\alpha\to 0$, $\beta\to 0$, and $F^2/d\to 0$, for the limiting case when $t\to 0$, corresponding to the transverse waves in the spectrum of free waves following the ship. By using the relation $(b')^2\equiv 1-(a')^2$ in expression (6.11), we may obtain

$$K(t=0) \sim -\int_c \exp(-ix/F^2)(a'-\partial\phi/\partial\ell)a'b' \, d\ell \quad \text{as} \quad \alpha\to 0,\ \beta\to 0,\ \text{and}\ F^2/d\to 0. \tag{7.2a}$$

In the same limiting case, expression (7.1a) becomes

$$K^{(0)}(t=0) \sim -\int_c \exp(-ix/F^2)(a')^2 b' \, d\ell \quad \text{as} \quad \alpha\to 0,\ \beta\to 0,\ \text{and}\ F^2/d\to 0. \tag{7.2b}$$

For a slender ship, with beam/length and draft/length ratios of order ε say, we have $\partial\phi/\partial\ell=O(\varepsilon^2)<<a'=O(1)$. At the bow and stern of a blunt-ended ship form, both a' and $\partial\phi/\partial\ell$ vanish; however, the result $\partial\phi/\partial\ell<<a'$ remains valid, as may be shown by considering the particular case of flow about an ellipsoid, for instance. It may then be seen from equations (7.2a,b) that for common low-speed ships, for which α, β, and F^2/d are small, we have $K^{(0)}(t)\approx K(t)$ for small values of t. Finally, it is interesting that the classical wave-resistance formulas of Hogner (1932) and Michell (1898) correspond to particular limiting cases of the zeroth-order slender-ship approximation $K^{(0)}$ defined by formula (7.1), as is discussed in some detail below.

For a ship with a fine waterline, that is, if the angle between the mean waterline c and the x axis is sufficiently small, we have $n_x^2|t_y|<<|n_x|<<1$, and the waterline integral in formula (7.1) may be neglected in comparison with the hull integral. The latter integral in fact is identical to the approximation, K_H say, proposed by Hogner (1932), that is we have

$$K_H(t) = F^{-2}\int_h \exp\{F^{-2}(1+t^2)z\}\exp\{-iF^{-2}(1+t^2)^{1/2}(x+ty)\}n_x da. \tag{7.3}$$

The Hogner approximation K_H may thus be obtained as the fine-waterline limit of the zeroth approximation $K^{(0)}$.

In the case of a single-hull ship with mean hull h defined by the equation $y=\pm b(x,z)$, the surface integral in equation (7.3) can be transformed into the double integral

$$K_H(t) = 2\iint_{h_y} \exp[F^{-2}(1+t^2)^{1/2}\{(1+t^2)^{1/2}z-ix\}]\cos[F^{-2}(1+t^2)^{1/2}tb(x,z)]\partial b(x,z)/\partial x dx dz,$$

where h_y is the projection of h onto the ship centerplane y=0. If the ship is thin, that is if b(x,z) is sufficiently small that the term $\cos[F^{-2}(1+t^2)^{1/2}tb(x,z)]$ may be approximated by 1, the Hogner approximation K_H becomes the famous Michell thin-ship approximation, K_M say, first obtained by Michell in 1898 and given by

$$K_M(t) = 2\iint_{h_y} \exp[F^{-2}(1+t^2)^{1/2}\{(1+t^2)^{1/2}z-ix\}]\partial b(x,z)/\partial x dx dz. \tag{7.4}$$

More precisely, the Michell approximation (7.4) corresponds to the first-order approximation in a consistent thin-ship expansion of the zeroth approximation $K^{(0)}$.

If the equation of the mean hull h is expressed in the form $z=-d(x,y)$, the surface integral in equation (7.3) can be transformed into the double integral

$$K_H(t) = \iint_{h_z} \exp\{-F^{-2}(1+t^2)d(x,y)\}\exp\{-iF^{-2}(1+t^2)^{1/2}(x+ty)\}\partial d(x,y)/\partial x dx dy,$$

where h_z is the projection of h onto the mean sea plane z=0. If the ship is flat, that is if d(x,y) is sufficiently small that the term $\exp\{-F^{-2}(1+t^2)d(x,y)\}$ may be approximated by 1, we may obtain the flat-ship approximation, K_H' say, proposed by Hogner (1932) and given by

$$K_H'(t) = \iint_{h_z} \exp\{-iF^{-2}(1+t^2)^{1/2}(x+ty)\}\partial d(x,y)/\partial x dxdy. \tag{7.5}$$

The Hogner approximation (7.3) in fact was obtained by Hogner (1932) as an interpolation formula between the flat-ship approximation (7.5) and the Michell thin-ship approximation (7.4).

If the ship is both thin and flat, that is if the term $\exp\{-iF^{-2}(1+t^2)^{1/2}ty\}$ in formula (7.5) for the Hogner flat-ship approximation and the term $\exp\{F^{-2}(1+t^2)z\}$ in formula (7.4) for the Michell thin-ship approximation may be approximated by 1, both of these approximations yield the Maruo-Tuck-Vossers slender-ship approximation, K_{MTV}, given by

$$K_{MTV} = \int_{stern}^{bow} \exp\{-iF^{-2}(1+t^2)^{1/2}x\}A'(x)dx, \tag{7.6}$$

where $A'(x)\equiv dA(x)/dx$ and A(x) is the cross-sectional area of the mean hull h. The slender-ship approximation (7.6) was obtained by Maruo (1962), Tuck (1964), and Vossers (1962) by using the method of matched asymptotic expansions.

The thin-ship approximation $\exp\{-iF^{-2}(1+t^2)^{1/2}ty\}\sim 1$ and the flat-ship approximation $\exp\{F^{-2}(1+t^2)z\}\sim 1$ are obviously not uniformly valid with respect to t and F. As a matter of fact, differences between the approximations $K^{(0)}$, K_H, K_M, K_H', and K_{MTV} can be quite large. This may be seen clearly in the case of a simple planar hull for which the foregoing approximations can be evaluated analytically. We consider the hull form defined by the equation

$$y=\pm(b/2)(1-2|x|+z/d), \quad \text{where} \quad 0\geq z\geq -d(1-2|x|), \ -1/2\leq x\leq 1/2,$$

and b and d are constants representing the beam/length ratio and the draft/length ratio, respectively. The nondimensional wave resistance $r\equiv R/\rho U^2L^2$ may be expressed in the form

$$r = (16b^2d^2/\pi)\int_0^\infty k^2(t)(1+t^2)^{-1/2}dt,$$

where the function $k(t)\equiv(1+t^2)^{1/2}K(t)/4bd$ is given below. For shortness, the notation

$$\epsilon=(1+t^2)^{1/2}/2F^2, \quad \beta=bt(1+t^2)^{1/2}/2F^2, \quad \delta=d(1+t^2)/F^2,$$

will be used.

The function, k_W say, corresponding to the waterline integral in the zeroth-order approximation (7.1) is given by

$$k_W = 4b^2dF^2(\cos\alpha-\cos\beta)/(1-b^2t^2)(b^2+4d^2+4b^2d^2).$$

The function, k_H, corresponding to the hull integral in expression (7.1) and the Hogner approximation (7.3) is given by

$$[b^2t^2+4d^2(1+t^2)]k_H = b^2t^2[(\sin\beta)/\beta-(\sin\alpha)/\alpha]/(1-b^2t^2) - 4dF^2(1-\cos\alpha)/[1+4d^2(1+t^2)]$$
$$+ 4d^2(1+t^2)[(1-1/\exp\delta)/\delta-(\sin\alpha)/\alpha]/[1+4d^2(1+t^2)] + 4dF^2(\cos\beta-\cos\alpha)/(1-b^2t^2).$$

The function, $k^{(0)}$, corresponding to the zeroth-order approximation (7.1) is given by $k^{(0)}=k_W+k_H$. By adding the above expressions for k_W and k_H, we may then obtain

$$[b^2t^2+4d^2(1+t^2)]k^{(0)} = b^2t^2[(\sin\beta)/\beta-(\sin\alpha)/\alpha]/(1-b^2t^2) - 4dF^2(1-\cos\alpha)/[1+4d^2(1+t^2)]$$
$$+ 4d^2(1+t^2)[(1-1/\exp\delta)/\delta-(\sin\alpha)/\alpha]/[1+4d^2(1+t^2)] + 4dF^2(b^2+4d^2)(\cos\beta-\cos\alpha)/(b^2+4d^2+4b^2d^2$$

The function, k_M, corresponding to the Michell approximation (7.4) is given by

$$k_M = [(1-1/\exp\delta)/\delta-(\sin\alpha)/\alpha+4dF^2(1-\cos\alpha)]/[1+4d^2(1+t^2)],$$

which is identical to the limit b=0 of the functions k_H and $k^{(0)}$. The function, k_H', corresponding to the Hogner flat-ship approximation (7.5) is given by

$$k_H' = [(\sin\beta)/\beta-(\sin\alpha)/\alpha]/(1-b^2t^2),$$

which is identical to the limit d=0 of the functions k_H and $k^{(0)}$. Finally, the function, k_{MTV}, corresponding to the Maruo-Tuck-Vossers approximation (7.6) is given by

$$k_{MTV} = 1-(\sin\alpha)/\alpha,$$

which is identical to the limits b=0 and d=0 of the functions k_H' and k_M, respectively.

Differences between the above functions are most striking and important for large values of t. In the limit $t\to\infty$, the waterline integral k_W and the hull integral (Hogner approximation) k_H become

$$k_W \sim F^2(\cos\beta-\cos\alpha)/(d+b^2/4d+b^2d)t^2 \quad \text{as} \quad t\to\infty,$$

$$k_H \sim F^2[(\cos\alpha)/d-2(b\sin\beta+2d\cos\beta)/(b^2+4d^2)]/b^2t^4 \quad \text{as} \quad t\to\infty.$$

We thus have $k_H \ll k_W$ as $t\to\infty$, and

$$k^{(0)} \sim k_W \quad \text{as} \quad t\to\infty.$$

As $t\to\infty$, the Michell thin-ship approximation k_M and the Hogner flat-ship approximation k_H' become

$k_M \sim F^2(1-\cos\alpha)/dt^2$ as $t \to \infty$,

$k_H' \sim 2F^2(\sin\alpha)/b^2t^3$ as $t \to \infty$.

We thus have $k_H << k_H' << k_M$ in the limit $t\to\infty$. However, the waterline integral k_W and the Michell approximation k_M are of the same order, namely $1/t^2$, as $t\to\infty$. As a matter of fact, the above-given large t asymptotic approximation of the Michell function k_M may be seen to correspond to the thin-ship limit b=0 of the large t approximation of the waterline integral k_W. Finally, the Maruo-Tuck-Vossers approximation k_{MTV} yields

$k_{MTV} \to 1$ as $t \to \infty$,

so that the Havelock wave-resistance integral would in fact be divergent. Indeed, the derivative $A'(x)$ of the ship cross-sectional area $A(x)$ must be continuous and vanish at the bow and stern for the wave resistance to exist. This condition is extremely restrictive, and in fact is not satisfied for usual ship forms. If the beam/length ratio b and the draft/length ratio d are small, of the same order of magnitude ε say, the functions k_W, k_H, $k^{(0)}$, k_M, and k_H' are of the form

$k \sim F^2/\varepsilon^n t^{n+1}$ as $t \to \infty$,

where n=1 for the waterline integral k_W, the zeroth approximation $k^{(0)}$ and the Michell approximation k_M, n=2 for the Hogner flat-ship approximation k_H', and n=3 for the Hogner approximation k_H.

In the limit t=0, corresponding to transverse waves, the Maruo-Tuck-Vossers approximation k_{MTV} and the Hogner flat-ship approximation k_H' are identical, given by

$k_H' = k_{MTV} = 1-2F^2\sin(1/2F^2)$ for $t = 0$.

The Michell approximation k_M and the Hogner approximation k_H are also identical, and given by

$k_H = k_M = F^2[\{1-1/\exp(d/F^2)\}/d-2\sin(1/2F^2)+4d\{1-\cos(1/2F^2)\}]/(1+4d^2)$ for $t = 0$.

This expression for k_H and k_M may be seen to become identical to the expression for k_H' and k_{MTV} in the limit $d\to 0$. However, this limit is obviously not uniform with respect to the Froude number. In particular, we have $k_H \simeq 1$ if $d \ll F^2 \ll 1$, whereas we have $k_H \simeq F^2/d \simeq 0$ if $F^2 \ll d \ll 1$. In the limit t=0, the waterline integral k_W is given by

$$k_W = -4dF^2\{1-\cos(1/2F^2)\}/(1+4d^2/b^2+4d^2) \quad \text{for } t = 0.$$

We thus have $k_W \ll k_H$ for t=0, and

$$k^{(0)} \sim k_H \quad \text{as } t \to 0.$$

The fact that $k_W \cdot \cdot k_H$ as $t \to \infty$, even though $k_W \ll k_H$ for t=0, emphasizes the importance of the waterline integral in expression (7.1) for the zeroth-order slender-ship approximation. It may be interesting to examine in passing the effect of using the thin-ship approximation y=0 in the exponential term E in the expression for the waterline integral, which then becomes

$$\int_c \exp\{-iF^{-2}(1+t^2)^{1/2}x\} n_x^2 t_y \, d\ell .$$

This "thin-ship waterline integral" is of the same type as the Maruo-Tuck-Vossers integral (7.6). We thus may expect the Havelock wave-resistance integral to be divergent in general (except for hull forms having a smooth waterline with cusped ends) if the thin-ship approximation is used in the waterline integral. For instance, for the simple planar hull examined previously, the function, k_W' say, corresponding to the above-defined "thin-ship waterline integral" is given by

$$k_W' = -4b^2dF^2(1-\cos\alpha)/(b^2+4d^2+4b^2d^2),$$

for which the Havelock integral (6.12) is divergent.

The above simple example thus demonstrates that the use of the thin-ship or/and flat-ship approximations may have important effects. In particular, these approximations are clearly not uniform with respect to the Froude number and the integration variable t in the Havelock integral (6.5). Use of such approximations evidently is inherent to the methods of solution based on a systematic thin- or flat-ship perturbation scheme, due to the necessity of transferring the boundary condition at the hull surface to the ship centerplane or waterplane, respectively. It was previously discussed in Noblesse (1976) that this transfer of the hull boundary condition may be overly and unnecessarily restrictive, and may in fact be far more limiting than the linearization of the boundary condition at the sea surface. An essential feature of the present slender-ship theory is precisely to avoid transferring the hull boundary condition, which is imposed on the exact hull surface.

8. The first-order slender-ship low-Froude-number approximation

The next approximation in the sequence of slender-ship wave-resistance approximations is the first-order slender-ship approximation, $r^{(1)}$, obtained by taking the potential ϕ in expression (6.6) for the Kochin free-wave amplitude function K(t) as the first-order slender-ship potential $\phi^{(1)}$ given by equation (5.2a). This first-order slender-ship approximation will be considered in the following section. In the present section, we briefly examine the simpler approximation obtained by taking the potential ϕ as the zero-Froude-number limit $\phi_0^{(1)}$ of the potential $\phi^{(1)}$. The potential $\phi_0^{(1)}$ is given by

$$4\pi\phi_0^{(1)}(\vec{\xi}) = -\int_h [\{(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2\}^{-1/2}+\{(\xi-x)^2+(\eta-y)^2+(\zeta+z)^2\}^{-1/2}] n_x(\vec{x})\, da(\vec{x}), \quad (8.1)$$

as may be seen from equation (5.4a). In the common case of a single-hull ship with port and starboard symmetry, expression (8.1) can readily be expressed as an integral over the positive (port) half h_+ of the mean hull h.

The potential $\phi_0^{(1)}$ may be regarded as a first-order slender-ship approximation to the (exact) zero-Froude-number (double-hull) potential ϕ_0, as was noted previously in section 5 and is specifically indicated in equation (5.6). The low-Froude-number wave-resistance approximation, $r_{\ell F}$ say, associated with the zero-Froude-number potential ϕ_0 is essentially identical to the low-speed wave-resistance formulas proposed by Guevel, Vaussy, and Kobus (1974), Baba (1976), Maruo (1977), and Kayo (1978), as will be shown below. The wave-resistance approximation associated with the first-order slender-ship approximation $\phi_0^{(1)}$ to the zero-Froude-number potential ϕ_0 thus corresponds to a first-order slender-ship approximation, $r_{\ell F}^{(1)}$ say, to the Guevel-Baba-Maruo-Kayo low-Froude-number approximation $r_{\ell F}$.

It is interesting to compare the low-Froude-number wave-resistance approximation $r_{\ell F}$, the first-order slender-ship low-Froude-number approximation $r_{\ell F}^{(1)}$, and the zeroth-order slender-ship approximation $r^{(0)}$, associated with the approximations $\phi=\phi_0$, $\phi=\phi_0^{(1)}$, and $\phi=\phi_0^{(0)}\equiv 0$, respectively. This comparison is particularly simple in the special case when the hull h is a vertical cylinder with elliptic waterline c, and the nonlinear free-surface flux q in expression (6.6) is ignored. Indeed, the wave-resistance approximations $r^{(0)}$, $r_{\ell F}^{(1)}$, and $r_{\ell F}$ can then be shown to be proportional to one another. Specifically, we have

$$r^{(0)} = r_{\ell F}/(1+b)^2, \quad r_{\ell F}^{(1)} = r_{\ell F}(1+2b)^2/(1+b)^4,$$

where b is the beam/length ratio of the elliptical cylinder. The relative errors $\varepsilon^{(0)}=(r_{\ell F}-r^{(0)})/r_{\ell F}$ and $\varepsilon^{(1)}=(r_{\ell F}-r_{\ell F}^{(1)})/r_{\ell F}$ then are given by

$$\varepsilon^{(0)} = 2b(1+b/2)/(1+b)^2, \quad \varepsilon^{(1)} = 2b^2[1+b(1+b/2)/(1+b)]/(1+b)^3.$$

We thus have $\varepsilon^{(0)}=.174$ and $\varepsilon^{(1)}=.016$ for b=.1, and $\varepsilon^{(0)}=.306$ and $\varepsilon^{(1)}=.055$ for b=.2. The first-order slender-ship low-Froude-number approximation $r_{\ell F}^{(1)}$ may then be seen to differ from the low-Froude-number approximation $r_{\ell F}$ by only a few percent for a thin ellipse. More generally, the second-and higher-order slender-ship low-Froude-number wave-resistance approximations $r_{\ell F}^{(k)}$, $k\geq 2$, associated with the sequence of slender-ship iterative approximations $\phi^{(k)}$ defined by the recurrence relation (5.6), can also be shown to be proportional to the approximation $r_{\ell F}$, and rapid convergence of the sequence of approximations $r_{\ell F}^{(k)}$ to $r_{\ell F}$ may be proved. Details will be given elsewhere.

It is noteworthy that the relative error, $\varepsilon^{(1)}$, between the wave-resistance approximations $r_{\ell F}^{(1)}$ and $r_{\ell F}$ is significantly less than the relative error between the corresponding potentials $\phi_0^{(1)}$ and ϕ_0. Indeed, for the present case of an elliptical cylinder we have $\phi_0^{(1)}=\phi_0/(1+b)$, as is shown in Noblesse and Triantafyllou (1980). This yields the relative error $\eta^{(1)}=(\phi_0-\phi_0^{(1)})/\phi_0=b/(1+b)$. We thus have $\eta^{(1)}=.091$ for b=.1 and $\eta^{(1)}=.167$ for b=.2, which may be compared to $\varepsilon^{(1)}=.016$ for b=.1 and $\varepsilon^{(1)}=.055$ for b=.2. This suggests that evaluation of the wave resistance by means of the Havelock and Kochin formulas for determining the energy contained in the waves following the ship may be preferable to pressure integration over the hull. Specifically, use of the first-order approximation $\phi_0^{(1)}$ to the potential ϕ yields a Havelock-Kochin wave-resistance approximation comparable to the wave resistance which may be obtained by hull-integration of the pressure determined from the second-order approximation $\phi_0^{(2)}$.

Calculations by Chen and Noblesse (1980) for the case of a vertical cylinder with a waterline in the shape of an ogive have shown that differences between the approximations $r_{\ell F}^{(1)}$ and $r_{\ell F}$ are somewhat larger than for an elliptical cylinder with same beam/length ratio, but remain small, of the order of a few percent. Furthermore, differences between the approximations $r_{\ell F}^{(1)}$ and $r_{\ell F}$ are significantly smaller for a slender hull than for a thin cylindrical hull with same waterline. This stems from the fact that the potential $\phi_0^{(1)}$ is a better approximation to the potential ϕ_0 for a slender body than for a comparable thin cylinder; for instance, the potential $\phi_0^{(1)}$ is about 13% smaller than the potential ϕ_0 for an elliptical cylinder with beam/length ratio equal to .15, but $\phi_0^{(1)}$ is only about 3% smaller than ϕ_0 for an ellipsoidal hull form with the same value of the beam/length ratio and with draft/length ratio equal to .05. Calculations reported in Chen and Noblesse (1980) in fact show that the approximations $r_{\ell F}^{(1)}$ and $r_{\ell F}$ are practically indistinguishable for ellipsoidal hull forms with beam/length ratios equal to .1 or .2.

The low-Froude-number approximation $K_{\ell F}$ to the Kochin free-wave amplitude function K is defined by equation (6.6) or the equivalent equation (6.9), in which the potential ϕ is taken as the zero-Froude-number potential ϕ_0. An interesting alternative expression for the low-Froude-number Kochin function $K_{\ell F}$ will now be obtained. We start from equation (6.3). By using the hull boundary condition (5.3b), from which follows the relation $n_x^2+t_x\partial\phi_0/\partial\ell-n_zt_y\partial\phi_0/\partial d=\partial\phi_0/\partial x$ as was shown in section 5, equation (6.3) yields

$$K_{\ell F}(t) = F^{-2}\int_h(E\partial\phi_0/\partial n-\phi_0\partial E/\partial n)da + \int_c(E\partial\phi_0/\partial x-\phi_0\partial E/\partial x)t_y d\ell - \int_\sigma Eq(\phi_0)dxdy, \quad (8.2)$$

where $E\equiv E(\vec{x};t,F^2)$ is the exponential function defined by equation (6.3a). Both the potential ϕ_0 and the function E satisfy the Laplace equation in d, and we have $\phi_0\sim 1/r^2$ as $r\equiv(x^2+y^2+z^2)^{1/2}\to\infty$ and E is exponentially small as $z\to-\infty$. We then have the Green identity

$$\int_h(E\partial\phi_0/\partial n-\phi_0\partial E/\partial n)da = \int_\sigma(E\partial\phi_0/\partial z-\phi_0\partial E/\partial z)dxdy = F^2\int_\sigma\phi_0\partial^2E/\partial x^2dxdy,$$

where equation (5.3c) and the relation $\partial E/\partial z+F^2\partial^2E/\partial x^2=0$ on $z=0$ were used. By expressing the term $\phi_0\partial^2E/\partial x^2$ in the form $\partial(\phi_0\partial E/\partial x-E\partial\phi_0/\partial x)/\partial x+E\partial^2\phi_0/\partial x^2$, and using the identity

$$\int_\sigma\partial(\phi_0\partial E/\partial x-E\partial\phi_0/\partial x)/\partial x dxdy = \int_c(\phi_0\partial E/\partial x-E\partial\phi_0/\partial x)dy,$$

we may then obtain

$$F^{-2}\int_h(E\partial\phi_0/\partial n-\phi_0\partial E/\partial n)da + \int_c(E\partial\phi_0/\partial x-\phi_0\partial E/\partial x)t_y d\ell = \int_\sigma E\partial^2\phi_0/\partial x^2dxdy,$$

where the relation $dy\equiv t_y d\ell$ on c was used. Use of this identity in equation (8.2) finally yields

$$K_{\ell F}(t) = \int_\sigma E[\partial^2\phi_0/\partial x^2-q(\phi_0)]dxdy \text{ , with } E = \exp[-iF^{-2}(1+t^2)^{1/2}(x+ty)]. \quad (8.3)$$

If the nonlinear free-surface correction term $q(\phi_0)$ is neglected and the mean sea surface σ is replaced by the entire plane z=0, equation (8.3) becomes identical to the low-speed approximation that was first obtained, in a different manner, by Guevel, Vaussy, and Kobus (1974). It was subsequently noted by Kayo (1978) that integration in Guevel's formula had to be restricted to the portion of the plane z=0 outside the hull, that is the mean sea plane σ, for uniqueness. Expression (8.3) for the low-Froude-number Kochin function $K_{\ell F}$ was previously derived from expression (8.2) by Maruo (1977), in a manner different from that shown here. If the linear term $\partial^2\phi_0/\partial x^2$ and the nonlinear term $q(\phi_0)$ are grouped (as is shown below) expression (8.3) becomes identical to the low-speed approximation obtained by Baba (1976), by using yet a different approach. By making use of equations (5.3a,c), expression (2.2) for the nonlinear correction term $q(\phi_0)$ can be put in the form

$$q(\phi_0)=\partial|\bar{\nabla}\phi_0|^2/\partial x + (\partial\phi_0/\partial x)\bar{\nabla}^2\phi_0 - \bar{\nabla}\cdot|\bar{\nabla}\phi_0|^2\bar{\nabla}\phi_0/2,$$

where $\bar{\nabla}$ is the two-dimensional differential operator $(\partial/\partial x,\partial/\partial y)$. We may then obtain

$$\partial^2\phi_0/\partial x^2-q(\phi_0) = \partial(1-\partial\phi_0/\partial x)(\partial\phi_0/\partial x-|\bar{\nabla}\phi_0|^2/2)/\partial x - \partial(\partial\phi_0/\partial y)(\partial\phi_0/\partial x-|\bar{\nabla}\phi_0|^2/2)/\partial y, \tag{8.4}$$

which is identical to expression (11) in Baba (1976).

In summary, the Guevel-Baba-Maruo-Kayo low-Froude-number wave-resistance approximation $r_{\ell F}$ has been shown to be essentially equivalent (for most practical purposes) to the first-order slender-ship low-Froude-number approximation $r_{\ell F}^{(1)}$. The latter approximation, associated with the potential $\phi_0^{(1)}$, may be regarded as a particular case of the first-order slender-ship approximation $r^{(1)}$, associated with the potential $\phi^{(1)}$; specifically, the potential $\phi_0^{(1)}$ is the zero-Froude-number limit of the first-order potential $\phi^{(1)}$. A major difference between the potentials $\phi^{(1)}$ and $\phi_0^{(1)}$ resides in the waves that are present in the potential $\phi^{(1)}$ but are ignored in the zero-Froude-number potential $\phi_0^{(1)}$. For the present problem of wave resistance, it clearly is appropriate to retain the waves in the potential $\phi^{(1)}$.

9. The first-order slender-ship approximation

The first-order slender-ship wave-resistance approximation $r^{(1)}$ is obtained by taking the potential ϕ in expression (6.6) for the Kochin free-wave amplitude function K(t) as the first-order slender-ship potential $\phi^{(1)} \equiv \mathcal{V}$ given by equation (5.2a), or equation (5.8) in the case of a single-hull ship with port and starboard symmetry. By using expression (3.3) for the Green function, the potential $\phi^{(1)}$ may be expressed in the form

$$\phi^{(1)}(\vec{\xi};F^2) = \phi_0^{(1)}(\vec{\xi}) + \phi_N^{(1)}(\vec{\xi};F^2) + \phi_W^{(1)}(\vec{\xi};F^2) , \qquad (9.1)$$

where $\phi_0^{(1)}(\vec{\xi})$ is the zero-Froude-number potential given by equation (8.1), and the potentials $\phi_N^{(1)}(\vec{\xi};F^2)$ and $\phi_W^{(1)}(\vec{\xi};F^2)$ are defined below. The nonoscillatory near-field potential $\phi_N^{(1)}$ is defined by

$$4\pi\phi_N^{(1)}(\vec{\xi};F^2) = \int_h (N+1/r')n_x da + F^2\int_c (N-1/r')n_x^2 t_y d\ell , \qquad (9.1a)$$

where $N \equiv N(\vec{\xi},\vec{x};F^2)$ is the function given by the integral (3.5) or the alternative integral representations (3.5a) or (3.6a), and $r' \equiv [(\xi-x)^2+(\eta-y)^2+(\zeta+z)^2]^{1/2}$. The wave potential $\phi_W^{(1)}$ in equation (9.1) is given by

$$4\pi\phi_W^{(1)}(\vec{\xi};F^2) = \int_h W n_x da + F^2\int_c W n_x^2 t_y d\ell , \qquad (9.1b)$$

where $W \equiv W(\vec{\xi},\vec{x};F^2)$ is the wave function defined by the integral (3.4).

By using equation (3.4) in equation (9.1b), we may express the wave potential $\phi_W^{(1)}$ in the form

$$\pi\phi_W^{(1)}(\vec{\xi};F^2) = \mathrm{Im}\int_{-\infty}^{\infty}\exp[F^{-2}(1+\tau^2)^{1/2}\{(1+\tau^2)^{1/2}\zeta+i(\xi+\tau\eta)\}]K_\xi^{(0)}(\tau;F^2)d\tau , \qquad (9.2)$$

where the function $K_\xi^{(0)}(\tau;F^2)$ is defined as

$$K_\xi^{(0)}(\tau;F^2) = \int_{c_\xi} E n_x^2 t_y d\ell + F^{-2}\int_{h_\xi} \exp\{F^{-2}(1+\tau^2)z\} E n_x da , \qquad (9.2a)$$

in which c_ξ and h_ξ represent the portions of the mean waterline c and hull h between the ship bow and the plane $x=\xi$, and $E \equiv E(x,y;\tau,F^2)$ is the function given by

$$E(x,y;\tau,F^2) = \exp[-iF^{-2}(1+\tau^2)^{1/2}(x+\tau y)] .$$

Comparison of equations (9.2a) and (7.1) shows that the function $K_\xi^{(0)}(\tau;F^2)$ is closely related to the zeroth-order slender-ship approximation $K^{(0)}(t;F^2)$ to the Kochin free-wave amplitude function.

The nonoscillatory near-field potential $\phi_N^{(1)}$ vanishes as $F \to 0$ (and thus is negligible in comparison with the zero-Froude-number potential $\phi_0^{(1)}$ for sufficiently small values of the Froude number); indeed, we have

$N+1/r' = O[F^2/(r')^2]$ and $F^2(N-1/r') = -2F^2/r' + O[F^4/(r')^2]$ as $F \to 0$.

It must however be noted that these asymptotic approximations are not uniformly valid with respect to r'. As a matter of fact, we have

$N+1/r' = 2/r' - (2/F^2)[1-(z+\zeta)/(r'+|x-\xi|)] + O(r'/F^4)$ as $r' \to 0$, and

$F^2(N-1/r') = -2[1-\zeta/(r'+|x-\xi|)] + O(r'/F^2)$ on c as $r' \to 0$.

The integrand of the waterline integral in expression (9.1a) may thus be seen to be a continuous function. The function N can be evaluated numerically without difficulty by using expressions (3.5) and (3.5a), as is discussed in Noblesse (1978). Indeed, the integrand of the integral in expression (3.5a) is a continuous function for $-1 \leq t \leq 1$ and $0 \leq r' \leq \infty$; the integrand of the integral in expression (3.5) is also continuous for $-1 \leq t \leq 1$ provided r' is not zero (a logarithmic singularity emerges as $r' \to 0$). Expression (3.5a) is well suited for evaluating the function N for small values of r'/F^2, whereas expression (3.5) is better suited for large or moderate values of r'/F^2 (for r'/F^2 greater than about 2.5). In particular, the integral in expression (3.5a) vanishes as $r' \to 0$, whereas the integral in expression (3.5) vanishes as $r' \to \infty$. Furthermore, the three-terms ascending series derived from expression (3.5a) in Noblesse (1978) can be used for sufficiently small values of r'/F^2, and the asymptotic expansion associated with expression (3.6a) and given in Noblesse (1975) can be used for small values of $|y-\eta|/F^2$ and large (or even moderate) values of r'/F^2. For practical purposes, an approximate expression for the function N may actually be sufficient. A fairly-simple algebraic approximation to the function N, obtained by combining the above-mentioned ascending series and asymptotic expansion, will be given elsewhere.

By using equation (9.1) in equation (6.6), we may express the first-order slender-ship approximation to the Kochin free-wave amplitude function in the form

$$K^{(1)} = K^{(0)} + K_0^{(1)} + K_N^{(1)} + K_W^{(1)} - K_\sigma^{(1)} \ , \tag{9.3}$$

where the functions $K^{(0)}$, $K_0^{(1)}$, $K_N^{(1)}$, $K_W^{(1)}$, and $K_\sigma^{(1)}$ are defined below. The function $K^{(0)}$ corresponds to the zeroth-order approximation defined previously by equation (7.1), that is we have

$$K^{(0)}(t;F^2) = \int_c En_x^2 t_y \, d\ell + F^{-2}\int_h \exp\{F^{-2}(1+t^2)z\} En_x \, da. \tag{9.4}$$

The functions $K_0^{(1)}$ and $K_N^{(1)}$ correspond to the use of the approximations $\phi=\phi_0^{(1)}$ and $\phi=\phi_N^{(1)}$, respectively, in the terms in expression (6.6) that depend linearly on ϕ; we thus have

$$K_0^{(1)} + K_N^{(1)} = F^{-2}(1+t^2)^{1/2}\int_c E[i\psi+F^2(t_x\partial\psi/\partial\ell - n_z t_y \partial\psi/\partial d)/(1+t^2)^{1/2}]t_y d\ell$$
$$- F^{-4}(1+t^2)\int_h \exp\{F^{-2}(1+t^2)z\}E\psi[n_z - i(n_x+tn_y)/(1+t^2)^{1/2}]da, \qquad (9.5)$$

where $\psi\equiv\psi(\vec{x};F^2)$ represents the nonoscillatory near-field potential $\phi_0^{(1)} + \phi_N^{(1)}$. In equations (9.4) and (9.5), the function $E\equiv E(x,y;t,F^2)$ is given by

$E(x,y;t,F^2) = \exp[-iF^{-2}(1+t^2)^{1/2}(x+ty)]$.

Equation (9.5), with ψ taken as the wave potential $\phi_W^{(1)}$, also defines the function $K_W^{(1)}$. However, it may be more convenient to express the function $K_W^{(1)}$ in the form

$$K_W^{(1)}(t;F^2) = (1/\pi)\int_{-\infty}^{\infty} k_W^{(1)}(\tau;t;F^2)d\tau \ , \qquad (9.6)$$

which may be obtained by using expression (9.2) for the potential $\phi_W^{(1)}$. The function $k_W^{(1)}(\tau;t,F^2)$ is given by

$$k_W^{(1)}(\tau;t,F^2) = F^{-2}(1+t^2)^{1/2}\int_c E[i\psi+F^2(t_x\partial\psi/\partial\ell - n_z t_y\partial\psi/\partial d)/(1+t^2)^{1/2}]t_y d\ell$$
$$- F^{-4}(1+t^2)\int_h \exp\{F^{-2}(2+\tau^2+t^2)\zeta\}E\psi[n_z - i(n_x+tn_y)/(1+t^2)^{1/2}]da \ , \qquad (9.6a)$$

where the functions $E\equiv E(\xi,\eta;t,F^2)$ and $\psi\equiv\psi(\xi,\eta;\tau,F^2)$ are defined as

$E(\xi,\eta;t,F^2) = \exp[-iF^{-2}(1+t^2)^{1/2}(\xi+t\eta)]$,

$\psi(\xi,\eta;\tau,F^2) = \mathrm{Im}\ \exp[iF^{-2}(1+\tau^2)^{1/2}(\xi+\tau\eta)]\ K_\xi^{(0)}(\tau;F^2)$,

with $K_\xi^{(0)}(\tau;F^2)$ given by equation (9.2a). Finally, the function $K_\sigma^{(1)}$ in equation (9.3) is associated with the integral over the mean sea surface σ in equation (6.6), and is given by

$$K_\sigma^{(1)}(t;F^2) = \int_\sigma \exp\{-iF^{-2}(1+t^2)^{1/2}(x+ty)\}q(\phi^{(1)})dxdy \ , \qquad (9.7)$$

where the nonlinear free-surface flux $q(\phi^{(1)})$ is defined by equation (2.2).

If the mean waterline c, the mean hull h, and the mean sea surface σ in the vicinity of c are subdivided into small rectilinear segments and planar triangular elements, the waterline, hull, and sea-surface integrals can be expressed as sums of analytical functions, corresponding to analytical integration over the elementary rectilinear segments and triangles. The details of this numerical implementation of the first-order slender-ship approximation to practical hull forms will, however, not be examined in this study, which is primarily concerned with the exposition of the theory.

For practical applications, the first-order slender-ship wave-resistance approximation $r^{(1)}$ is regarded as the main result of the present slender-ship theory of wave resistance. However, second-and higher-order approximations can also be defined. Two second-order approximations indeed are briefly examined in the following section.

10. The second-order slender-ship approximation

A second-order slender-ship wave-resistance approximation $r^{(2)}$ can be defined by taking the potential ϕ in expression (6.6) for the Kochin free-wave amplitude function K(t) as the second-order slender-ship potential $\phi^{(2)}$ given by

$$\phi^{(2)}(\vec{\xi}) = \phi^{(1)}(\vec{\xi}) - L(\vec{\xi};\phi^{(1)}) - F^2\int_\sigma Gq(\phi^{(1)})dxdy \ , \tag{10.1}$$

where $L(\vec{\xi};\phi^{(1)})$ is the linear transform of $\phi^{(1)}$ defined by equation (5.2c). An interesting alternative to the above second-order potential may be obtained by seeking a solution of equation (5.2), in which the nonlinear free-surface correction integral will be neglected, in the form $\phi(\vec{\xi}) = k(\vec{\xi})\varphi(\vec{\xi}) \equiv k(\vec{\xi})\phi^{(1)}(\vec{\xi})$, where $k(\vec{\xi})$ is assumed to be a slowly-varying function.

The term $\phi-\phi_* \equiv \phi(\vec{x})-\phi(\vec{\xi}) \equiv k(\vec{x})\varphi(\vec{x})-k(\vec{\xi})\varphi(\vec{\xi}) \equiv k\varphi-k_*\varphi_*$ in equation (5.2c) may be expressed in the form $\phi-\phi_*=(k\varphi-k_*\varphi_*)-k_*(\varphi-\varphi_*)+k_*(\varphi-\varphi_*)$. Similarly, the terms $\partial\phi/\partial\ell \equiv \partial(k\varphi)/\partial\ell$ and $\partial\phi/\partial d \equiv \partial(k\varphi)/\partial d$ may be expressed in the forms $\partial(k\varphi)/\partial\ell-k_*\partial\varphi/\partial\ell+k_*\partial\varphi/\partial\ell$ and $\partial(k\varphi)/\partial d-k_*\partial\varphi/\partial d+k_*\partial\varphi/\partial d$, respectively. The equation

$$\phi(\vec{\xi}) = \varphi(\vec{\xi}) - L(\vec{\xi};\phi) \tag{10.2}$$

obtained by neglecting the nonlinear free-surface correction term in equation (5.2b), can then be expressed in the form

$$k_*[\varphi_*+L(\vec{\xi};\varphi)] = \varphi_* + k_*L(\vec{\xi};\varphi) - L(\vec{\xi};k\varphi).$$

By multiplying this equation by φ_* and using the relations $k_*\varphi_* = \phi_*$ and $k\varphi = \phi$, we may obtain

$$\phi_*[\varphi_*+L(\vec{\xi};\varphi)] = \varphi_*^2 + \phi_*L(\vec{\xi};\varphi) - \varphi_*L(\vec{\xi};\phi), \tag{10.3}$$

which may be regarded as an alternative form of equation (10.2).

If the (Neumann-Kelvin) potential ϕ were actually proportional to the first-order slender-ship potential $\varphi \equiv \phi^{(1)}$, the term $\phi_*\varphi-\varphi_*\phi$ would vanish, and equation (10.3) would yield the solution

$$\phi(\vec{\xi}) = [\phi^{(1)}(\vec{\xi})]^2/[\phi^{(1)}(\vec{\xi})+L(\vec{\xi};\phi^{(1)})]. \tag{10.4}$$

Use of the assumption $|L(\vec{\xi};\phi^{(1)})|<<|\phi^{(1)}(\vec{\xi})|$ in equation (10.4) yields

$$\phi(\vec{\xi}) \simeq \phi^{(1)}(\vec{\xi}) - L(\vec{\xi};\phi^{(1)}).$$

It may then be seen from equation (10.1) that the potential $\phi(\vec{\xi})$ defined by equation (10.4) is approximately equal to the second-order potential $\phi^{(2)}(\vec{\xi})$, if the nonlinear free-surface correction term is ignored. The potential $\overset{\gamma}{\phi}{}^{(2)}$ given by

$$\tilde{\phi}^{(2)}(\vec{\xi}) = [\phi^{(1)}(\vec{\xi})]^2/[\phi^{(1)}(\vec{\xi})+L(\vec{\xi};\phi^{(1)})] - F^2\int_\sigma Gq(\phi^{(1)})dxdy \tag{10.5}$$

thus provides an alternative, computationally-equivalent, second-order slender-ship approximation to the velocity potential ϕ.

In the zero-Froude-number limit, the second-order potential $\phi^{(2)}$ and the modified second-order approximation $\tilde{\phi}^{(2)}$ become the potentials $\phi_0^{(2)}$ and $\tilde{\phi}_0^{(2)}$, respectively, given by

$$\phi_0^{(2)}(\vec{\xi}) = \phi_0^{(1)}(\vec{\xi}) - L_0(\vec{\xi};\phi_0^{(1)}) \ , \tag{10.6a}$$

$$\tilde{\phi}_0^{(2)}(\vec{\xi}) = [\phi_0^{(1)}(\vec{\xi})]^2/[\phi_0^{(1)}(\vec{\xi})+L_0(\vec{\xi};\phi_0^{(1)})] \ , \tag{10.6b}$$

where $\phi_0^{(1)} \equiv \mathcal{\phi}_0$ is the first-order potential given by equation (5.4a) and $L_0(\vec{\xi};\phi_0^{(1)})$ is the linear transform of $\phi_0^{(1)}$ defined by equation (5.4b). It is proved in Noblesse and Triantafyllou (1980) that the modified second-order approximation $\tilde{\phi}_0^{(2)}$ is actually exact in the particular cases of ellipsoidal hull forms. Furthermore, calculations for two-dimensional flows about ogives for various values of the thickness ratio, have shown that the potential $\tilde{\phi}_0^{(2)}$ provides a fairly-accurate approximation to the exact potential ϕ_0 (even for large values of the thickness ratio), superior to the straightforward second-order potential $\phi_0^{(2)}$ (although this approximation is quite satisfactory for values of the thickness ratio equal to .1 and .2). These results for the zero-Froude-number case suggest that the modified second-order slender-ship potential $\tilde{\phi}^{(2)}$ given by equation (10.5) may provide a fairly-accurate approximation to the velocity potential ϕ.

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Appendix A. Interpretation of the waterline integral

Let us consider the closed body consisting of the mean hull surface h closed by a horizontal "lid", ℓ say, slightly submerged a depth δ below the mean sea plane z = 0, so that the "lid" ℓ becomes the "interior waterplane" σ_i in the zero-submergence limit $\delta = 0$. The integral identity corresponding to equation (4.9) for the fully-submerged body h+ℓ is

$$\phi_* = \int_d G\nabla^2\phi dv - \int_{\sigma+\sigma_i} G(\partial\phi/\partial z+F^2\partial^2\phi/\partial x^2)dxdy + \int_{h+\ell}[G\partial\phi/\partial n-(\phi-\phi_*)\partial G/\partial n]da \ , \qquad (A.1)$$

where d is the mean flow domain outside the body h+ℓ. This identity may be expressed in the form

$$\phi_* = \int_d G\nabla^2\phi dv - \int_\sigma G(\partial\phi/\partial z+F^2\partial^2\phi/\partial x^2)dxdy + \int_h[G\partial\phi/\partial n-(\phi-\phi_*)\partial G/\partial n]da + I \ ,$$

where the term I is given by

$$I = -\int_{\sigma_i} G(\partial\phi/\partial z+F^2\partial^2\phi/\partial x^2)dxdy + \int_\ell[G\partial\phi/\partial z-(\phi-\phi_*)\partial G/\partial z]dxdy \ ,$$

since we have $\partial\phi/\partial n = \partial\phi/\partial z$ and $\partial G/\partial n = \partial G/\partial z$ on the horizontal lid ℓ.

By expressing the integrand of the integral over the lid ℓ in the form

$$G(\partial\phi/\partial z+F^2\partial^2\phi/\partial x^2) - (\phi-\phi_*)(\partial G/\partial z+F^2\partial^2 G/\partial x^2) - F^2\partial[G\partial\phi/\partial x-(\phi-\phi_*)\partial G/\partial x]/\partial x \ ,$$

and using the identity

$$\int_\ell \partial[G\partial\phi/\partial x-(\phi-\phi_*)\partial G/\partial x]/\partial x dxdy = -\int_c[G\partial\phi/\partial x-(\phi-\phi_*)\partial G/\partial x]dy \ ,$$

we may then obtain

$$I = -\int_{\sigma_i} G(\partial\phi/\partial z+F^2\partial^2\phi/\partial x^2)dxdy + \int_\ell G(\partial\phi/\partial z+F^2\partial^2\phi/\partial x^2)dxdy$$
$$-\int_\ell(\phi-\phi_*)(\partial G/\partial z+F^2\partial^2 G/\partial x^2)dxdy + F^2\int_c[G\partial\phi/\partial x-(\phi-\phi_*)\partial G/\partial x]dy.$$

In the zero-submergence limit $\delta\to 0$, the first two integrals cancel out one another and the third integral vanishes, so that the above expression reduces to the waterline integral

$$I = F^2\int_c[G\partial\phi/\partial x-(\phi-\phi_*)\partial G/\partial x]dy.$$

The identity (A.1) for the slightly-submerged closed body h+ℓ thus becomes the identity (4.9) for the sea-surface piercing hull h, and the waterline integral in equation (4.9) may be seen to stem from the effect of the lid ℓ closing the slightly-submerged hull h.

Appendix B. Integral identities for the interior problem and for the combined exterior-interior problems

The "interior potential" ϕ^i, defined in the interior domain d_i, can be shown to satisfy the identities

$$\phi^i_* = \int_{d_i} G\nabla^2\phi^i dv - \int_{\sigma_i} G(\partial\phi^i/\partial z+F^2\partial^2\phi^i/\partial x^2)dxdy - \int_h (G\partial\phi^i/\partial n-\phi^i\partial G/\partial n)da - F^2\int_c (G\partial\phi^i/\partial x-\phi^i\partial G/\partial x)dy \tag{B.1a}$$

in the interior domain, that is for $\vec{\xi}$ in $d_i+\sigma_i-h-c$,

$$0 = \int_{d_i} G\nabla^2\phi^i dv - \int_{\sigma_i} G(\partial\phi^i/\partial z+F^2\partial^2\phi^i/\partial x^2)dxdy - \int_h (G\partial\phi^i/\partial n-\phi^i\partial G/\partial n)da - F^2\int_c (G\partial\phi^i/\partial x-\phi^i\partial G/\partial x)dy \tag{B.1b}$$

in the exterior domain, that is for $\vec{\xi}$ in $d+\sigma-h-c$, and

$$\phi^i_*/2 = \int_{d_i} G\nabla^2\phi^i dv - \int_{\sigma_i} G(\partial\phi^i/\partial z+F^2\partial^2\phi^i/\partial x^2)dxdy - \int_h (G\partial\phi^i/\partial n-\phi^i\partial G/\partial n)da - F^2\int_c (G\partial\phi^i/\partial x-\phi^i\partial G/\partial x)dy \tag{B.1c}$$

if $\vec{\xi}$ is exactly on the boundary surface $h+c$ (provided $h+c$ is smooth at the point $\vec{\xi}$).

The integral identity corresponding to equation (4.9) takes the form

$$0 = \int_{d_i} G\nabla^2\phi^i dv - \int_{\sigma_i} G(\partial\phi^i/\partial z+F^2\partial^2\phi^i/\partial x^2)dxdy - \int_h [G\partial\phi^i/\partial n-(\phi^i-\phi^i_*)\partial G/\partial n]da - F^2\int_c [G\partial\phi^i/\partial x-(\phi^i-\phi^i_*)\partial G/\partial x]dy. \tag{B.2}$$

This integral identity, like equation (4.9), is valid for any point $\vec{\xi}$, whether inside, outside, or exactly on the surface $h+c$, and indeed is equivalent to the set of the three classical identities (B.1a,b,c).

If we add the integral identities (4.6) and (B.1b), we may obtain the relation

$$\phi_* = \int_d G\nabla^2\phi dv + \int_{d_i} G\nabla^2\phi^i dv - \int_\sigma G(\partial\phi/\partial z+F^2\partial^2\phi/\partial x^2)dxdy - \int_{\sigma_i} G(\partial\phi^i/\partial z+F^2\partial^2\phi^i/\partial x^2)dxdy + \int_h [G(\partial\phi/\partial n-\partial\phi^i/\partial n)-(\phi-\phi^i)\partial G/\partial n]da + F^2\int_c [G(\partial\phi/\partial x-\partial\phi^i/\partial x)-(\phi-\phi^i)\partial G/\partial x]dy.$$

Addition of the integral identities (4.6′) and (B.1a) yields the same relation, except for the fact that ϕ_* on the left side is replaced by ϕ^i_*. We then have the relation

$$\phi_* = \int_{z<0} G\nabla^2\phi dv - \int_{z=0} G(\partial\phi/\partial z+F^2\partial^2\phi/\partial x^2)dxdy + \int_h [G(\partial\phi/\partial n-\partial\phi^i/\partial n)-(\phi-\phi^i)\partial G/\partial n]da + F^2\int_c [G(\partial\phi/\partial x-\partial\phi^i/\partial x)-(\phi-\phi^i)\partial G/\partial x]dy, \tag{B.3}$$

where ϕ on the left side and in the first two integrals on the right side corresponds to ϕ or ϕ^i for points outside or inside the hull surface $h+c$, respectively. Naturally, the integral relation (B.3) can also be obtained by adding identities (4.9) and (B.2). Indeed, this yields

$$(1-C_i)\phi_* + C_i\phi_*^i = \int_{z<0} G\nabla^2\phi dv - \int_{z=0} G(\partial\phi/\partial z+F^2\partial^2\phi/\partial x^2)dxdy$$
$$+ \int_h [G(\partial\phi/\partial n-\partial\phi^i/\partial n)-(\phi-\phi^i)\partial G/\partial n]da + F^2\int_c [G(\partial\phi/\partial x-\partial\phi^i/\partial x)-(\phi-\phi^i)\partial G/\partial x]dy \ ,$$

where $C_i = \int_h \partial G/\partial n da+F^2\int_c \partial G/\partial x dy$. It was shown below equation (4.9) that we have $C_i=0$ or 1 for $\vec{\xi}$ outside or inside h+c, so that the expression $(1-C_i)\phi_*+C_i\phi_*^i$ is identical to ϕ_* or ϕ_*^i for $\vec{\xi}$ outside or inside h+c, and the above relation is identical to relation (B.3).

Appendix C. Alternative expressions for the Kochin function

It was shown in section 8 that expression (6.6) for the low-Froude-number Kochin free-wave amplitude function $K_{\ell F}$, obtained by taking the potential ϕ as the zero-Froude-number potential ϕ_0 , could be expressed in the Guevel-Baba-Maruo-Kayo form shown in equation (8.3). An expression for the Kochin function similar to expression (8.3) can also be obtained if the potential ϕ in expression (6.3) is approximated by the first-order slender-ship zero-Froude-number potential $\phi_0^{(1)}$, or more generally by any waveless potential, ψ say, that vanishes sufficiently rapidly as $(x^2+y^2)^{1/2} \to \infty$ to permit the use of Green's identity for the potential ψ and the function E in the unbounded exterior domain d.

By writing the expression $n_x^2+t_x\partial\psi/\partial\ell-n_zt_y\partial\psi/\partial d$ in the waterline integral in equation (6.3) in the form $n_x(n_x-\partial\psi/\partial n) + n_x\partial\psi/\partial n+t_x\partial\psi/\partial\ell-n_zt_y\partial\psi/\partial d \equiv n_x(n_x-\partial\psi/\partial n) + \partial\psi/\partial x$, we may obtain

$$\begin{aligned} K(t) = {} & F^{-2}\int_h(E\partial\psi/\partial n-\psi\partial E/\partial n)\,da + \int_c(E\partial\psi/\partial x-\psi\partial E/\partial x)t_y\,d\ell - \int_\sigma Eq(\psi)\,dxdy \\ & + F^{-2}\int_h E(n_x-\partial\psi/\partial n)\,da + \int_c E(n_x-\partial\psi/\partial n)n_xt_y\,d\ell. \end{aligned} \tag{C.1}$$

By applying Green's identity to the functions E and ψ in the mean flow domain d, as was done for deriving equation (8.3) from equation (8.2), we can finally obtain

$$K(t) = F^{-2}\int_h E(n_x-\partial\psi/\partial n)\,da + \int_c E(n_x-\partial\psi/\partial n)n_xt_y\,d\ell + F^{-2}\int_\sigma E[\partial\psi/\partial z+F^2\partial^2\psi/\partial x^2-F^2q(\psi)]\,dxdy. \tag{C.2}$$

If the potential ψ is taken as the zero-Froude-number potential ϕ_0 , we have $\partial\psi/\partial n=n_x$ on h+c and $\partial\psi/\partial z=0$ on σ, and equation (C.2) becomes identical to equation (8.3) for the low-Froude-number approximation $K_{\ell F}$. On the other hand, if ψ is taken as the potential $\phi_0^{(1)}$, equation (C.2) yields the following alternative expression for the first-order slender-ship low-Froude-number approximation $K_{\ell F}^{(1)}$:

$$K_{\ell F}^{(1)}(t) = F^{-2}\int_h E(n_x-\partial\phi_0^{(1)}/\partial n)\,da + \int_c E(n_x-\partial\phi_0^{(1)}/\partial n)n_xt_y\,d\ell + \int_\sigma E[\partial^2\phi_0^{(1)}/\partial x^2-q(\phi_0^{(1)})]\,dxdy. \tag{C.3}$$

Another expression for the Kochin function K(t) may be obtained if the potential ϕ in equation (6.3) is approximated by the first-order potential $\phi^{(1)}$ or its zero-Froude-number limit $\phi_0^{(1)}$, which are defined in the domain d_i inside the hull surface h as well as in the exterior domain d. The potential $\psi\equiv\phi^{(1)}$ or $\phi_0^{(1)}$ is continuous across the hull surface h+c. Equation (C.1) may thus be expressed in the form

$$\begin{aligned} K(t) = {} & F^{-2}\int_{h_i}(E\partial\psi/\partial n-\psi\partial E/\partial n)\,da + \int_{c_i}(E\partial\psi/\partial x-\psi\partial E/\partial x)t_y\,d\ell - \int_\sigma Eq(\psi)\,dxdy \\ & + F^{-2}\int_{h_i}E(n_x-\partial\psi/\partial n)\,da + \int_{c_i}E(n_x-\partial\psi/\partial n)n_xt_y\,d\ell\ , \end{aligned} \tag{C.4}$$

where h_i and c_i represent the interior sides of the mean hull surface h and waterline c. In other words, the derivatives $\partial\psi/\partial n$ and $\partial\psi/\partial x$, which are discontinuous across h+c, are evaluated on the interior side of h+c in equation (C.4), whereas they are evaluated on the exterior side of h+c in equation (C.1).

Both the potential ψ and the exponential function E satisfy the Laplace equation in the interior domain d_i, so that we may use the Green identity

$$\int_{h_i}(E\partial\psi/\partial n-\psi\partial E/\partial n)da = \int_{\sigma_i}(\psi\partial E/\partial z-E\partial\psi/\partial z)dxdy.$$

By using the fact that the function E satisfies the relation $\partial E/\partial z+F^2\partial^2 E/\partial x^2=0$ on σ_i, we may express the integrand $\psi\partial E/\partial z-E\partial\psi/\partial z$ in the form $F^2\partial(E\partial\psi/\partial x-\psi\partial E/\partial x)/\partial x$ $-E(\partial\psi/\partial z+F^2\partial^2\psi/\partial x^2)$. Use of the relation

$$\int_{\sigma_i}\partial(E\partial\psi/\partial x-\psi\partial E/\partial x)/\partial x dxdy = -\int_{c_i}(E\partial\psi/\partial x-\psi\partial E/\partial x)t_y d\ell\ ,$$

then yields

$$\int_{h_i}(E\partial\psi/\partial n-\psi\partial E/\partial n)da = -F^2\int_{c_i}(E\partial\psi/\partial x-\psi\partial E/\partial x)t_y d\ell - \int_{\sigma_i}E(\partial\psi/\partial z+F^2\partial^2\psi/\partial x^2)dxdy.$$

Equation (C.4) may finally be expressed in the form

$$K(t) = F^{-2}\int_{h_i}E(n_x-\partial\psi/\partial n)da + \int_{c_i}E(n_x-\partial\psi/\partial n)n_x t_y d\ell$$
$$- F^{-2}\int_{\sigma_i}E(\partial\psi/\partial z+F^2\partial^2\psi/\partial x^2)dxdy - \int_{\sigma}Eq(\psi)dxdy. \qquad (C.5)$$

If the potential ψ in equation (C.5) is taken as the first-order slender-ship potential $\phi^{(1)}$, we have $\partial\psi/\partial z+F^2\partial^2\psi/\partial x^2= 0$ on σ_i, and equation (C.5) becomes

$$K^{(1)}(t) = F^{-2}\int_{h_i}E(n_x-\partial\phi^{(1)}/\partial n)da + \int_{c_i}E(n_x-\partial\phi^{(1)}/\partial n)n_x t_y d\ell - \int_{\sigma}Eq(\phi^{(1)})dxdy. \qquad (C.6)$$

Equation (C.6) provides an alternative form of the expression for the first-order slender-ship approximation $K^{(1)}$ defined by equations (9.3) through (9.7). If the potential ψ is taken as the zero-Froude-number potential $\phi_0^{(1)}$, equation (C.5) yields

$$K_{\ell F}^{(1)}(t) = F^{-2}\int_{h_i}E(n_x-\partial\phi_0^{(1)}/\partial n)da + \int_{c_i}E(n_x-\partial\phi_0^{(1)}/\partial n)n_x t_y d\ell - \int_{\sigma_i}E\partial^2\phi_0^{(1)}/\partial x^2 dxdy$$
$$- \int_{\sigma}Eq(\phi_0^{(1)})dxdy. \qquad (C.7)$$

An alternative form of expression (C.6) may be obtained by using the relation

$$[\partial\phi^{(1)}/\partial n]_h - [\partial\phi^{(1)}/\partial n]_{h_i} = n_x\ ,$$

where the notation $[\quad]_h$ and $[\quad]_{h_i}$ implies that the term within the brackets, namely $\partial\phi^{(1)}/\partial n$, is evaluated on the exterior and interior sides of the mean hull surface, respectively. This then yields

$$n_x - [\partial\phi^{(1)}/\partial n]_{h_i} = n_x + n_x - [\partial\phi^{(1)}/\partial n]_h\ ,$$

and equation (C.6) may be expressed in the form

$$K^{(1)}(t) = K^{(0)}(t) + F^{-2}\int_h E(n_x-\partial\phi^{(1)}/\partial n)da + \int_c E(n_x-\partial\phi^{(1)}/\partial n)n_x t_y d\ell - \int_{\sigma}Eq(\phi^{(1)})dxdy, (C.$$

where equation (7.1) was used. The following alternative form of expression (C.7):

$$K_{\ell F}^{(1)}(t) = K^{(0)}(t) + F^{-2}\int_h E(n_x-\partial\phi_0^{(1)}/\partial n)da + \int_c E(n_x-\partial\phi_0^{(1)}/\partial n)n_x t_y d\ell - \int_{\sigma_i} E\partial^2\phi_0^{(1)}/\partial x^2 dxdy - \int_\sigma Eq(\phi_0^{(1)})dxdy \qquad (C.9)$$

may be obtained likewise.

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Berkeley, CA 94720

Professor C. C. Mei
Massachusetts Institute of Technology
Department of Civil Engineering
Cambridge, MA 02139

Professor Justin E. Kerwin
Massachusetts Institute of Technology
Department of Ocean Engineering
Cambridge, MA 02139

Professor Phillip Mandel
Massachusetts Institute of Technology
Department of Ocean Engineering
Cambridge, MA 02139

Professor J. Nicholas Newman
Massachusetts Institute of Technology
Department of Ocean Engineering
Room 5-324A
Cambridge, MA 02139

Professor Francis Noblesse
Massachusetts Institute of Technology
Department of Ocean Engineering
Cambridge, MA 02139

Professor Ronald W. Yeung
Massachusetts Institute of Technology
Department of Ocean Engineering
Cambridge, MA 02139

Dr. Robert K. C. Chan
JAYCOR
1401 Camino Del Mar
Del Mar, CA 92014

Mr. Marshall P. Tulin
Hydronautics, Incorporated
7210 Pindell School Road
Laurel, MD 20810

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