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NONLINEAR MODEL IDENTIFICATION
FROM OPERATING RECORDS

Bruce K. Colburn
Alvin R. Schatte

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Two basic response error (RE) model-reference adaptive system (MRAS) identifiers are derived for nonlinear model identification without advance knowledge of the nonlinearities. The methods are developed and shown to be applicable for systems with input and output measurement noise. Design consideration, comparison of linear and nonlinear model fits to a nonlinear plant are given. The applicability of such an approach for the human operator model are discussed.

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NONLINEAR MODEL IDENTIFICATION
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ABSTRACT

Practical considerations are given for the use of MRAS identification techniques for the identification of linear and nonlinear parameters of dynamic systems. The response error ("parallel MRAS") was used to identify single and/or multivalued nonlinear parameters whose forms and "transfer function equivalents" were unknown a priori. All parameter adaptation occurred simultaneously. This paper develops the problem, discusses difficulties unique to the nonlinearity approach, and outlines possible solutions.

NONLINEAR MODEL IDENTIFICATION FROM OPERATING RECORDS

I. INTRODUCTION

System identification has become a refined art and science in the last twenty years [1-5], at least as regards linear system parameter identification. The area of nonlinear identification, however, has been somewhat neglected. As system operation requirements are tightened, efficiency and energy concerns increase, and more complex systems (large-scale systems) are contemplated, the need for detailed accurate knowledge of the full system structure, including nonlinear and time-varying effects, is increased. Some examples of dynamic systems where nonlinear effects are significant include power plants, high-performance aircraft, human operator pilot-models, and transformers.

The objectives of the present work are to develop a model reference adaptive system (MRAS) identifier for estimating parameters of nonlinear pilot models to ascertain if information better than traditional (Kleinman Optimal Control Model, crossover frequency response models) linear models can result. The application of these results would be to identify human dynamic models so

(1) stress lists/danger vehicle maneuvers can be investigated safely by applying the results to the model, not the human

(2) as to use the information in the design of aircraft control systems and for predicting closed-loop flying qualities

(3) as to determine effects of environmental inputs such as temperature, vibration, acceleration, and psychological variables such as motivation, training, and task difficulty

(4) to parameterize changing pilot characteristics during glide-slope to flare-up; air to air tracking in unsteady maneuver, and hybrid (loose/tight) control situations

The concept is shown in Figure I-1.

The control of physical systems using modern control theory has become increasingly important with the advent of the space age and the energy crisis. The need to operate systems at their peak efficiency and obtain the utmost in performance from then requires that the control system have accurate knowledge in the form system models of what the system is doing and what it will do given a set of inputs to it. A special section of control theory known as system identification theory is responsible for developing accurate models of physical systems using the input into the plant and the output of the plant for use by a control engineer in designing control systems and to facilitate greater understanding of the physical system.

System identification can be divided into two major catagories: (1) output tracking, and (2) parametric identification. Output tracking identification develops models for systems which for a given, fixed input will approximate the output of the physical system or plant. Since the model is good only for a particular input, output tracking is of little significance for identification.

Parametric identification is of greater importance since it yields the parameters of the plant. Thus, output tracking occurs for any type of input, yielding a much more versatile and accurate model given the least amount of a priori knowledge of the plant. Parametric system identification consists primarily of three classes: 1) linear, time-invariant, 2) linear, time-varying, and 3) nonlinear, time-invariant identification. The linear, time-invariant identification is best understood and most widely used because such approaches are simple and can be applied easily. However, in many physical systems, major nonlinearities occur (i.e. hysteresis in power systems, saturation in electron devices, etc.) for which linear, time-invariant identifiers may not supply an accurate description. Thus, nonlinear parametric system identification has become a subject of increasing importance as more accurate models become necessary.

Two key approaches exist for systems with one or more nonlinearities. These are 1) when the nonlinear form is known a priori [17] and 2) when the nonlinear form is unknown a priori [7]. The first approach is predicated on knowledge of the nonlinear form in advance (e.g. quadratic curve, symmetrical saturation, etc.) or where the nonlinearity is one in which the parameters to be identified enter linearly (e.g. $y + ay + cy^2 = u$). The second approach is based on a series expansion concept (Taylor series, power series, sine-cosine, etc.) for piecewise continuous curve fittings, when no a priori knowledge of the nonlinearity is available. This latter approach is of the greatest interest, especially in regards to its ability to admit memory nonlinearities (i.e. hysteresis), Figure I-2.

Several different forms and parameterizations exist for modelling nonlinearities in dynamic systems. These include 1) the Hammerstein model [8], 2) Volteira series [9], 3) Uryson models [10], 4) memoryless nonlinearity peicewise series fit, 5) memory nonlinearity piecewise series fit, and 6) known nonlinear model structure. A methodology using forms 4) and 5) which has been developed over the last several years is known as Model Reference Adaptive System (MRAS) Identification [5]. MRAS identification techniques can be broken down into two general catagories: 1) equation error or "series parallel" and 2) response error or "parallel" methods. Block diagrams of these methods are shown in Fig.I-3. The primary differences between these two methods are the way the error is formed and the way stability of the composite system is assured. In the equation error MRAS, the error is formed as a difference between weighted system output states and input states. In the response error formulation, the error is formed as the difference between the system output and the adjustable model output. The equation error system does not guarantee sta-

CONCEPT

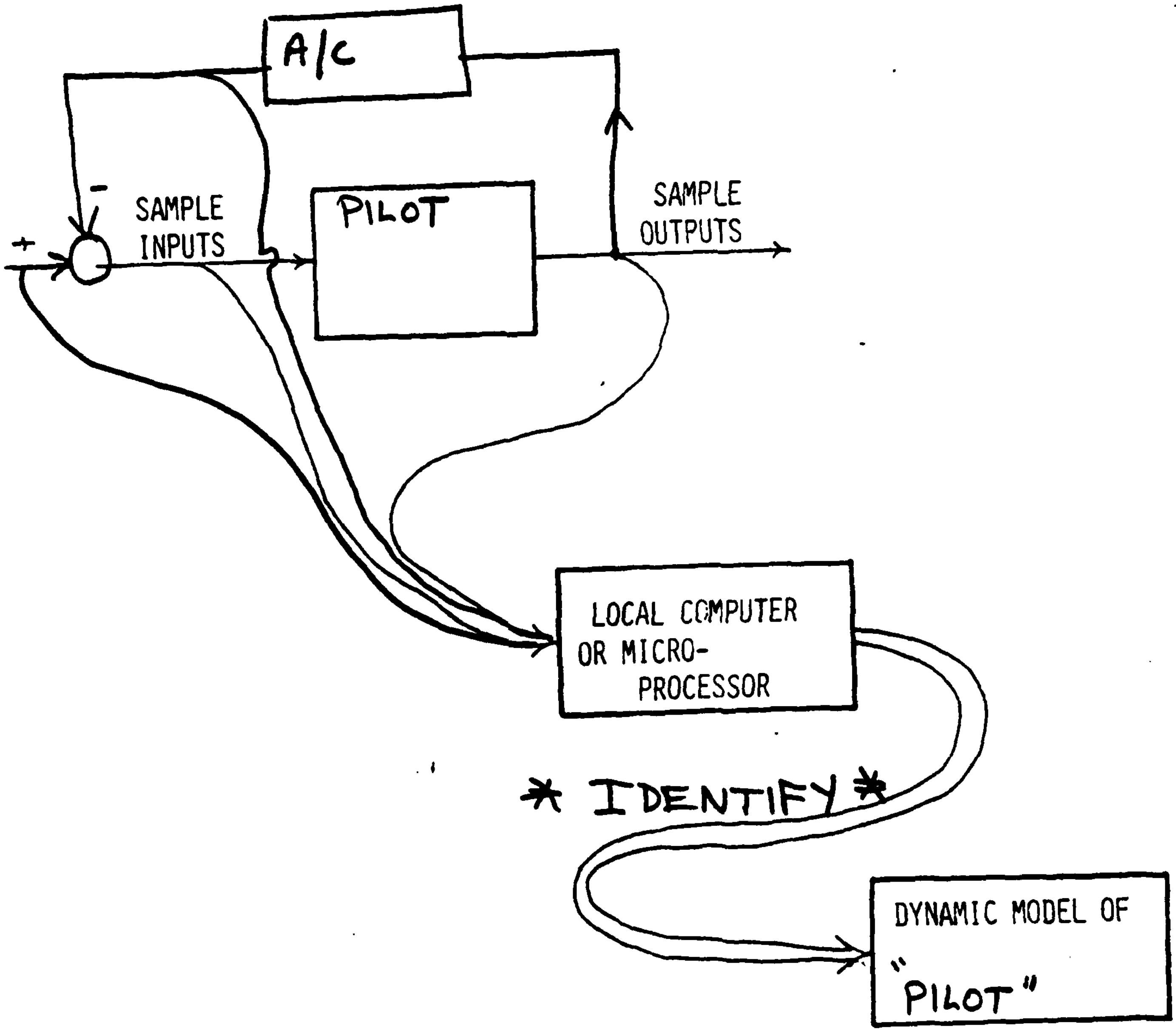


Figure I-1.

NONLINEARITY CHARACTERIZATION

SINGLE-VALUE

$$\hat{f}_h(\hat{x}^h)\Big|_p \; \hat{x}^h = [a_{h1_p} + a_{h2_p}(\hat{x}^h - x_p^h) + a_{k3_p}(\hat{x}^h - \hat{x}_p^h) + \ldots]\, \hat{x}^h$$

MULTI-VALUED

$$F_i(\omega) = A_{i1_p} + A_{i2_p}(\omega - \omega_p) + \ldots.$$

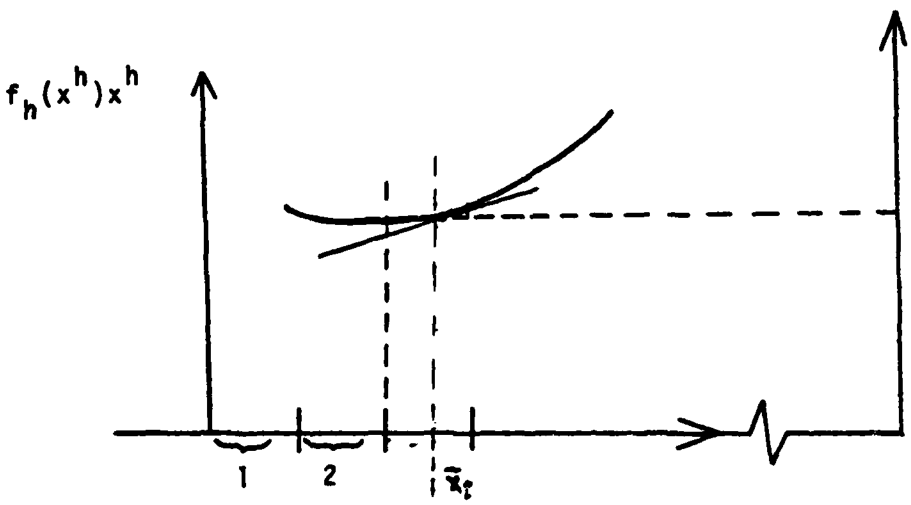
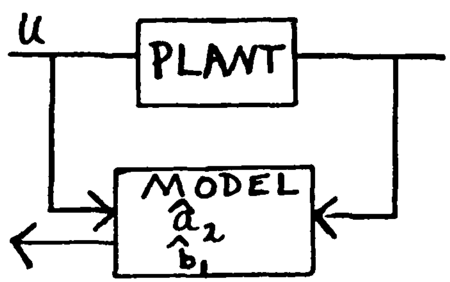


Figure I-2. Example of using a first order Taylor series expansion of f(x) about the point $\bar{x}_i$

TYPES OF MRAS

EQUATION ERROR

"SERIES - PARALLEL"

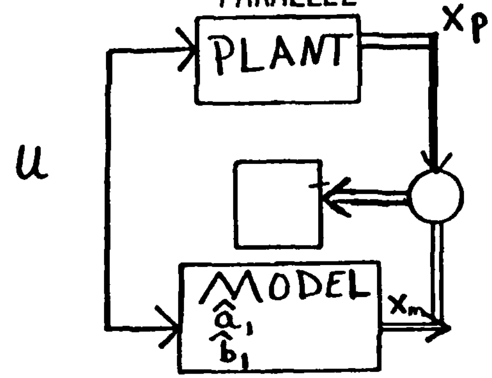


$$E = x_p^{(n)} - \sum_{i=1}^{n} a_i x_p^{(i-1)} - \sum_{i=0}^{m} b_i u^{(i-1)}$$

$$\hat{a}_1 = c_1 \int E \, x_{p_1} \, dt$$

OUTPUT ERROR

"PARALLEL"



$$e = x_m - x_p$$

$$\hat{a}_1 = c_1 \int e \, x_{m_1} \, dt$$

$$\hat{a}_2 = c_2 \int e \, x_{m_2} \, dt$$

Figure I-3

bility of the system while the response error system can guarantee a form of stability. It is this form of stability, known as hyperstability, that has motivated the recent interest in the response error or parallel MRAS that this thesis is concerned with.

Hyperstability theory was developed by Popov [21] as an extension of linear, asymptotic and absolute stability. Its use in linear MRAS identification assured these algorithms were stable in the sense of hyperstability. Thus, the parametric identification of linear dynamic systems under the hyperstability conditions is assured; a very strong statement which non of the previous methods could guarantee. The extension of hyperstability assurance for nonlinear systems, then, is of interest in order to assure exact, nonlinear, parametric identification.

BACKGROUND

Dynamic system identification had its inception in statistics [1,3], and numerous such ad-hoc identification algorithms exist. However, with the resurgence of Lyapanov's stability theory, the field of MRAS was formed [5]. Lyapanov designs produced several algorithms, but these were of limited value. In the early 1960's, though, V. M. Popov developed hyperstability theory which brought about a radical change in MRAS identification. Hyperstable MRAS identification algorithms were developed by Hang [18], Landau , and Johnson [12], amongst others, which accomplished linear, time-invariant identification of dynamic systems. Early systems were constrained by the requirement of the existence of a strictly, positive real function (SPRF), but in 1978, Landau and Johnson removed this condition for hyperstability.

Developments of the response error MRAS technique into nonlinear system parametric identification, however, have been scarce. In 1977, Tomizuka [20] applied hyperstability theory successfully to a very special class of nonlinearities. More recently, Colburn and Schatte [22] have applied hyperstability with limited success to nonlinearities of a more general type.

II. MRAS IDENTIFICATION THEORY

The fundamental concepts and theory underlying MRAS identification to the present will be discussed in this section. The development of nonlinear identification is a straightforward extension of these basic concepts. Two major MRAS methods will be developed. This is followed by an exposé on hyperstability as it is applied to the MRAS identification process, and, concluding, with the proof of the hyperstability of a particularly important linear, MRAS identification technique.

MRAS IDENTIFICATION TECHNIQUES

There exis two primary MRAS identification techniques: (1) the equation error (EE) or series-parallel MRAS, and (2) the response error (RE) or parallel MRAS. The EE method shown in Fig. II-1 was expanded on by Lion [17], and Kudra and Narendra [6]. It was adapted for use with nonlinear systems by Sprague and Kohr [7] and Sehitoglu and Klein [19]. Consider the nonlinear dynamic system:

$$\alpha_n x^{(n)} + f_{n-1}(x^{(e)})x^{(n-1)} + \ldots + f_{i+1}(x^{(h)})x^{(i+1)} + F_i(x^{(i)}) + f_{i-1}(x^{(d)})x^{(i-1)} + \ldots + f_0(x^{(s)})x = u + g_1(u^{(v)})u^{(1)} + \ldots + g_m(u^{(g)})u^{(m)} \tag{2.1}$$

where α_n - constant,

$$(\cdot)^{(i)} = \frac{d^i(\cdot)}{dt^i},$$

u is the input or forcing function, x is the state, and f_k, F_i, and g_m are nonlinear functions of the indicated argument. Equation (2.1) contains two types of nonlinear functions, f and F. Functions of the form f_k and g_k are assumed to be single-valued and piecewise continuous with $f_k(0) = 0$, such as shown in Fig. II-2. The function, F_i, of which only one may occur

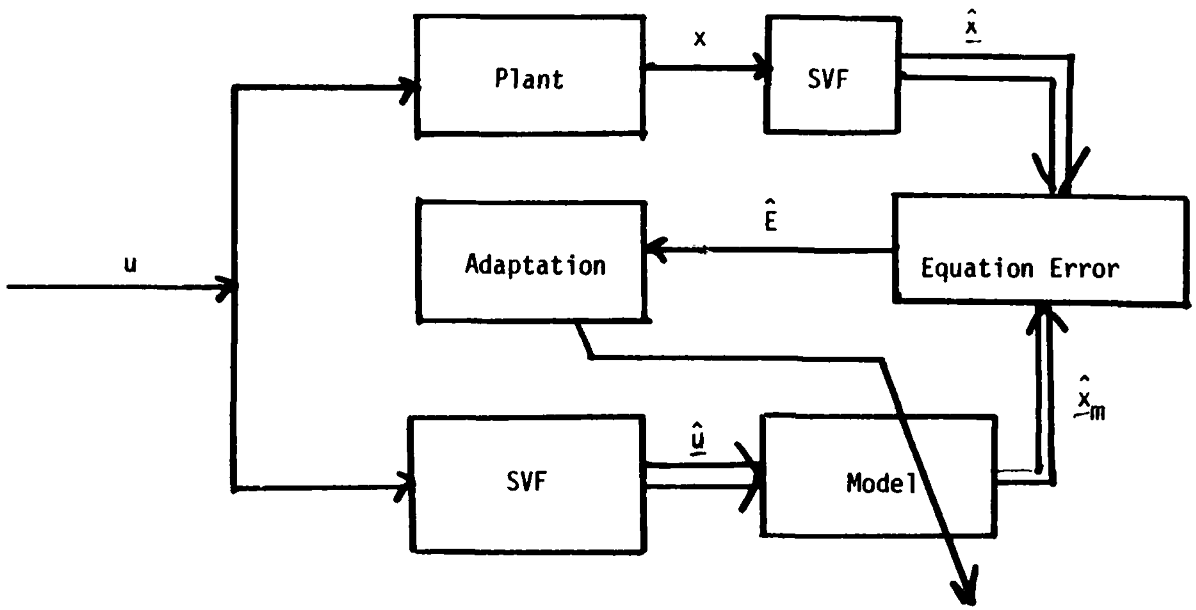


Figure II-1. Equation Error or Series-Parallel MRAS Identification due to uniqueness problems with the constant term associated with the function is assumed to be piecewise continuous and possible multivalued. Also, $F_i(0) \neq 0$ necessarily, see Figure II-3.

Several assumptions on (2.1) must occur:

1) $n > m$, that is, the order n of x must be greater than the order m of u
2) the system input, u, and output, x, must be measurable,
3) the input, u, is selectable and must be sufficiently frequency rich [7] to insure parameter adaptation, and
4) the system may possess at most one multivalued nonlinear function, F.

For the identification process, x and u are filtered by state variable filters (SVF) which develop "pseudo-states" which are approximations, $\hat{x}$ and $\hat{u}$, to the system's actual states, x and u. These "pseudo-states" are then used to form the tracking error or "equation error":

$$\hat{E} = \hat{\alpha}_n \hat{x}^{(n)} + \hat{f}_{n-1}(\hat{x}^{(e)})\hat{x}^{(n-1)} + \ldots + \hat{f}_{i+1}(\hat{x}^{(h)})\hat{x}^{(i+1)} + \hat{F}_i(\hat{x}^{(i)}) + \hat{f}_{i-1}(\hat{x}^{(d)})\hat{x}^{(i-1)} + \ldots + \hat{f}_0(\hat{x}^{(s)})\hat{x} - \hat{u} + \hat{g}_i(\hat{u}^{(v)})\hat{u}^{(i)} + \ldots + \hat{g}_m(\hat{u}^{(g)})\hat{u}^{(m)} \tag{2.2}$$

which is formed from (2.1):

$$\alpha_n x^{(n)} + f_{n-1}(x^{(e)})x^{(n-1)} + \ldots + f_{i+1}(x^{(h)})x^{(i+1)} + F_i(x^i) + f_{i-1}(x^{(d)})x^{(i-1)} + \ldots + f_0(x^{(s)})x - u - g_1(u^{(v)})u^{(1)} - g_m(u^{(g)})u^{(m)} = 0 \tag{2.3}$$

of the actual system and where $(\hat{\ })$ denotes the approximation to the original function.

Exact identification of the above functions entails the estimation of an infinite number of points which describe the curve, f vs. x. This is not practical; therefore, an approximation is sought. Describing each of

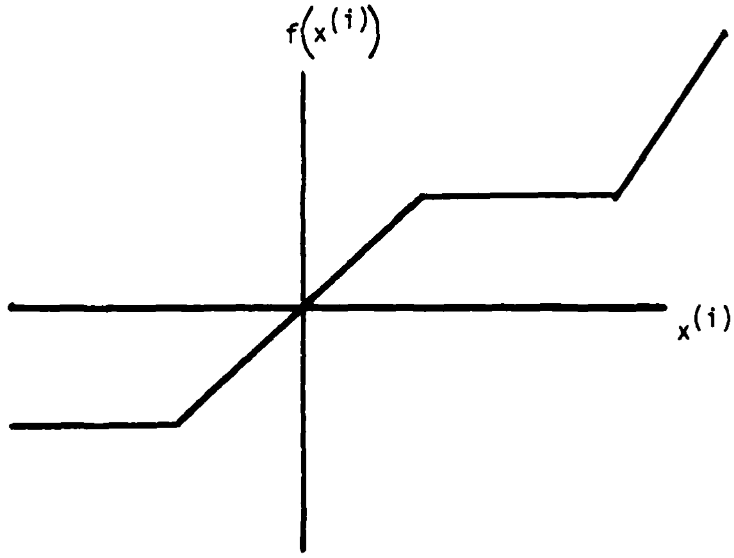


Figure II-2. Example of f_k functions.

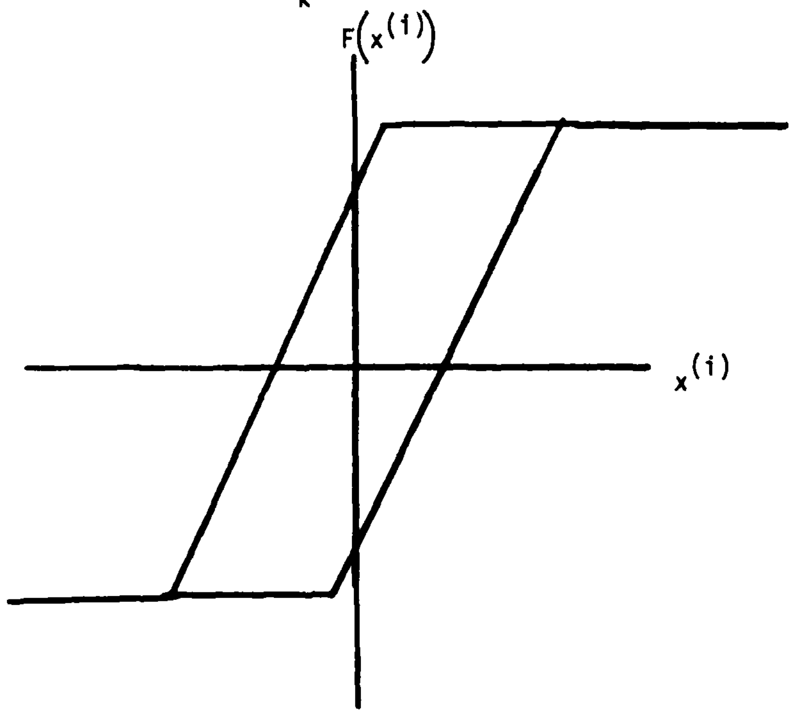


Figure II-3. Example of $F^{(i)}$ functions.

the functions $\hat{f}_i$, $\hat{g}_i$, and $\hat{F}_i$ in (2.2) by a Taylor series expansion in intervals over the expected operating domain, piecewise continuous approximations to the functions can be obtained. For the single-valued functions, $\hat{f}$ and $\hat{g}$, these expansions take the form:

$$\hat{f}_k(\hat{x}^{(h)})\hat{x}^{(k)} = [\hat{a}_{k1p} + \hat{a}_{k2p}(\hat{x}^{(h)} - \bar{x}_{hp}) + \ldots +$$
$$\hat{a}_{krp}(\hat{x}^{(h)} - \bar{x}_{hp})^{r-1} + \ldots]\hat{x}^{(k)}$$

where

k denotes the function being expanded,

r denotes the position in the series expansion

$(\cdot)^{r-1}$ indicates raising the argument to the r-1 st power as opposed to $(\cdot)^{(h)}$ which indicates the h^{th} derivative of $(\cdot)$ with respect to time

p is the interval index

h is the argument index, and

$\bar{x}_{hp}$ is the expansion point in the p^{th} interval.

For the multivalued function, $\hat{F}_i$, the expansion appears as:

$$\hat{F}_i(\hat{x}^{(i)}) = \hat{A}_{i1p} + \hat{A}_{i2p}(\hat{x}^{(i)} - \bar{x}_{ip}) + \ldots +$$
$$\hat{A}_{irp}(\hat{x}^{(i)} - \bar{x}_{ip})^{r-1} + \ldots] \qquad (2.4)$$

where the same notation as above applies. The nonlinear identification process has now been reduced to the estimation of the parameters, $\hat{a}_{krp}$, $\hat{A}_{irp}$, and $\hat{b}_{jrp}$ where the $\hat{b}_{grp}$ are the parameters associated with the expansion of the $\hat{g}_j(\hat{u}^{(r)})\hat{u}^{(j)}$ functions. The error equation now becomes

$$\hat{E} = \hat{\alpha}_n\hat{x}^{(n)} + [\hat{a}_{n-1,1,p} + \hat{a}_{n-1,2,p}(\hat{x}^{(e)} - \bar{x}_{ep}) + \ldots +$$
$$\hat{a}_{n-1,r,p}(\hat{x}^{(e)} - \bar{x}_{ep})^{r-1} + \ldots]\hat{x}^{(n-1)} + \ldots + [\hat{a}_{i+1,1p} +$$
$$\hat{a}_{i+1,2p}(\hat{x}^{(h)} - \bar{x}_{hp}) + \ldots + \hat{a}_{i+1,rp}(\hat{x}^{(h)} - \bar{x}_{hp})^{r-1} + \ldots]\hat{x}^{(i+1)}$$

$$+ [\hat{A}_{i1p} + \hat{A}_{i2p}(\hat{x}^{(i)} - \bar{x}_{ip}) + \dots + \hat{A}_{irp}(\hat{x}^{(i)} - \bar{x}_{ip})^{r-1} + \dots]$$
$$+ [\hat{a}_{i-1,1p} + \hat{a}_{i-1,2p}(\hat{x}^{(d)} - \bar{x}_{dp}) + \dots + \hat{a}_{i-1,rp}(\hat{x}^{(d)} - \bar{x}_{dp})^{r-1}$$
$$+ \dots]\hat{x}^{(i-1)} + \dots + [\hat{a}_{01p} + \hat{a}_{02p}(\hat{x}^{(s)} - \bar{x}_{sp}) + \dots +$$
$$\hat{a}_{0rp}(\hat{x}^{(s)} - \bar{x}_{sp})^{r-1} + \dots]\hat{x} - \hat{u} - [\hat{b}_{11p} + \hat{b}_{12p}(\hat{u}^{(v)} - \bar{u}_{vp}) +$$
$$\dots + \hat{b}_{1rp}(\hat{u}^{(v)} - \bar{u}_{rp})^{r-1} + \dots]\hat{u}^{(1)} - \dots - [\hat{b}_{m1p} +$$
$$\hat{b}_{m2p}(\hat{u}^{(g)} - \bar{u}_{gp}) + \dots + \hat{b}_{mrp}(\hat{u}^{(g)} - \bar{u}_{gp})^{r-1} + \dots]\hat{u}^{(m)} \quad (2.6)$$

The EE method is derived from the minimization of the cost functional

$$J = \frac{1}{2}\hat{E}^2 \quad (2.7)$$

By taking the partial derivatives of (2.6) with respect to the parameters γ (i.e. $\hat{a}_{krp}$), the steepest descent algorithm is obtained:

$$\dot{\gamma} = -K\frac{\partial J}{\partial \gamma} = -K\hat{E}\frac{\partial \hat{E}}{\partial \gamma} \quad (2.8)$$

where K is a positive constant. This yields the parameter adaptation equations:

$$\dot{\hat{\alpha}}_n = -G_n \hat{E}\hat{x}^{(n)}$$
$$\dot{\hat{a}}_{n-1,1p} = -G_{n-1,1p}\hat{E}\hat{x}^{(n-1)}$$
$$\vdots$$
$$\dot{\hat{a}}_{n-1rp} = -G_{n-1,rp}\hat{E}\hat{x}^{(n-1)}(\hat{x}^{(e)} - \bar{x}_{ep})^{r-1}$$
$$\vdots$$
$$\dot{\hat{a}}_{i+1rp} = -G_{i+1,1p}\hat{E}\hat{x}^{(i+1)}$$
$$\vdots$$
$$\dot{\hat{a}}_{i+1rp} = -G_{i+1,rp}\hat{E}\hat{x}^{(i+1)}(\hat{x}^{(h)} - \bar{x}_{rp})^{r-1}$$
$$\dot{\hat{A}}_{i1p} = -G_{i1p}\hat{E}$$
$$\vdots$$

$$
\begin{aligned}
\hat{A}_{irp} &= -G_{irp}\,\hat{E}(\hat{x}^{(i)} - \bar{x}_{ip})^{r-1} \\
\hat{a}_{i-11p} &= -G_{i-11p}\,\hat{E}\,\hat{x}^{(i-1)} \\
&\vdots \\
\hat{a}_{i-1rp} &= -G_{i-1rp}\,\hat{E}\,\hat{x}^{(i-1)}(\hat{x}^{(d)} - \bar{x}_{dp})^{r-1} \\
&\vdots \\
\hat{a}_{01p} &= -G_{01p}\,\hat{E}\,\hat{x}^{(0)} \\
&\vdots \\
\hat{a}_{0rp} &= -G_{0rp}\,\hat{E}\,\hat{x}^{(0)}(\hat{x}^{(s)} - \bar{x}_{sp})^{r-1} \\
\hat{b}_{11p} &= H_{11p}\,\hat{E}\,\hat{u}^{(1)} \\
&\vdots \\
\hat{b}_{1rp} &= H_{1rp}\,\hat{E}\,\hat{u}^{(1)}(\hat{u}^{(v)} - \bar{u}_{vp})^{r-1} \\
&\vdots \\
\hat{b}_{m1p} &= H_{m1p}\,\hat{E}\,\hat{u}^{(m)} \\
&\vdots \\
\hat{b}_{mrp} &= H_{mrp}\,\hat{E}\,\hat{u}^{(m)}(\hat{u}^{(g)} - \bar{u}_{gp})^{r-1} \\
G_{..r} &= 0 \quad r \neq p \\
H_{..t} &= 0 \quad t \neq p
\end{aligned}
\tag{2.9}
$$

The other MRAS approach is known as parallel MRAS or response error (RE), Figure II-4. Consider the following linear dynamic system:

$$x^{(n)} = \sum_{i=0}^{n-1} a_i x^{(i)} + \sum_{i=0}^{m-1} b_i u^{(i)} \tag{2.10}$$

and corresponding model:

$$x_m^{(n)} = \sum_{i=0}^{n-1} \hat{a}_i x_m^{(i)} + \sum_{i=0}^{m-1} \hat{b}_1 u^{(i)} \tag{2.11}$$

Define the tracking error, e, analogous to $\hat{E}$ in (2.2), as

$$e = x - x_m. \tag{2.12}$$

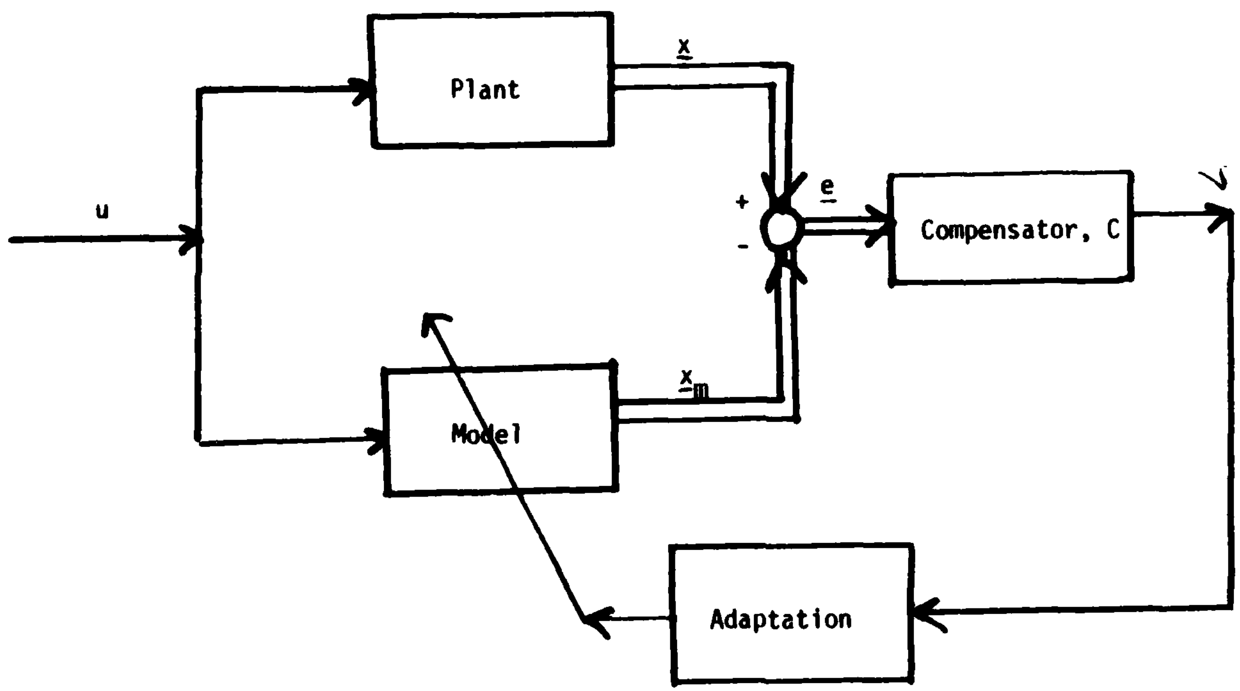


Figure II-4. Parallel or Response Error MRAS Identification

This error is then processed thorugh a compensator, C, to yield the response error, v,:

$$v = e^{(n)} + \sum_{i=0}^{n-1} c_i e^{(i)}. \tag{2.13}$$

Hyperstability theory requires that the transfer function

$$G(s) = \frac{1 + \sum_{i=0}^{n-1} c_i s^i}{1 - \sum_{i=0}^{n-1} a_i s^i} \tag{2.14}$$

be a strictly positive real function (SPRF) for asymptotic hyperstability of the error system (2.13). An SPRF is defined as:

$$\mathrm{Re}\{G(s)\} > 0 \quad \text{for all} \quad \mathrm{Re}\{s\} > 0. \tag{2.15}$$

Hyperstability also yields the following parameter adaptation equations:

$$\begin{aligned} \dot{a}_i &= \alpha_i v x^{(i)} \qquad \forall\ i \in [0,n-1] \\ \dot{b}_i &= \beta_i v u^{(i)} \qquad \forall\ i \in [0,m-1]. \end{aligned} \tag{2.16}$$

Because the derivation of this method is governed by exact stability theory (i.e. hyperstability) parameter identification is guaranteed provided certain conditions (to be discussed in the next section) are met.

Landau [11] and Johnson [12] removed the SPRF requirement by insuring that it always occurred. They did this by adapting the c_i terms in (2.13) by:

$$\dot{c}_i = \gamma_i \, v \, e^{(i)}. \tag{2.17}$$

In doing this, the transfer function, G(s), became unity and, therefore, always an SPRF.

Tomizuka [29] applied hyperstability and RE identification to a class of nonlinear systems of the form:

$$x(t) = \frac{f_{hm}D^m + f_{h(m-1)}D^{m-1} + \ldots + f_{hi}D + f_{h0}}{D^n + a_{n-1}D^{n-1} + \ldots + a_1 D + a_0} n_h(u(t)) \quad (2.18)$$

for $m < n$ and $D = \frac{d}{dt}$, where

$$n_h(u(t)) = n_1(u(t)) + n_2(u(t)) + \ldots + n_\ell(u(t)) \quad (2.19)$$

are known static functions of $u(t)$ and the constants f_{hi}'s and a_i's must be identified. Tomizuka showed that using the parameter adaptation equations:

$$\begin{aligned} \dot{a}_i &= -k_{a_i} \nu D^i x \quad i \in [0,n-1], \; k_{a_i} > 0 \\ \dot{f}_{hj} &= k_{fhj} \nu D^i n_h \quad j \in [0,m], \; h \in [1,\ell], \; k_{fhj} > 0 \end{aligned} \quad (2.20)$$

the transfer function (2.14) remained unchanged and that this algorithm was, therefore, hyperstable.

RE and EE identification methods yield parameter adaptation algorithms similar in structure. However, there are several fundamental differences which exist between the two methods. Foremost is that the RE method is governed by hyperstability theory, while the EE method is not assured to be convergent. The RE method was developed through hyperstability so that $\nu \to 0$ as $t \to \infty$. The EE method uses gradient following techniques to minimize $\hat{E}^2$, with $\hat{E}$ an equation error analogous to ν.

Landau shows that the EE identification yields biased parametric estimates in the presence of measurement noise, while RE yields unbiased results. In the nonlinear case, Colburn and Schatte [21] demonstrated that the RE state estimate (i.e. $x_m^{(i)}$) is much more noise free than the EE state approximations, (i.e. $\hat{x}^{(i)}$), Figure II-5. This is because the EE develops the state information by filtering the noisy input and output while the RE develops state approximations through its own dynamics, giving a more noise free approximation.

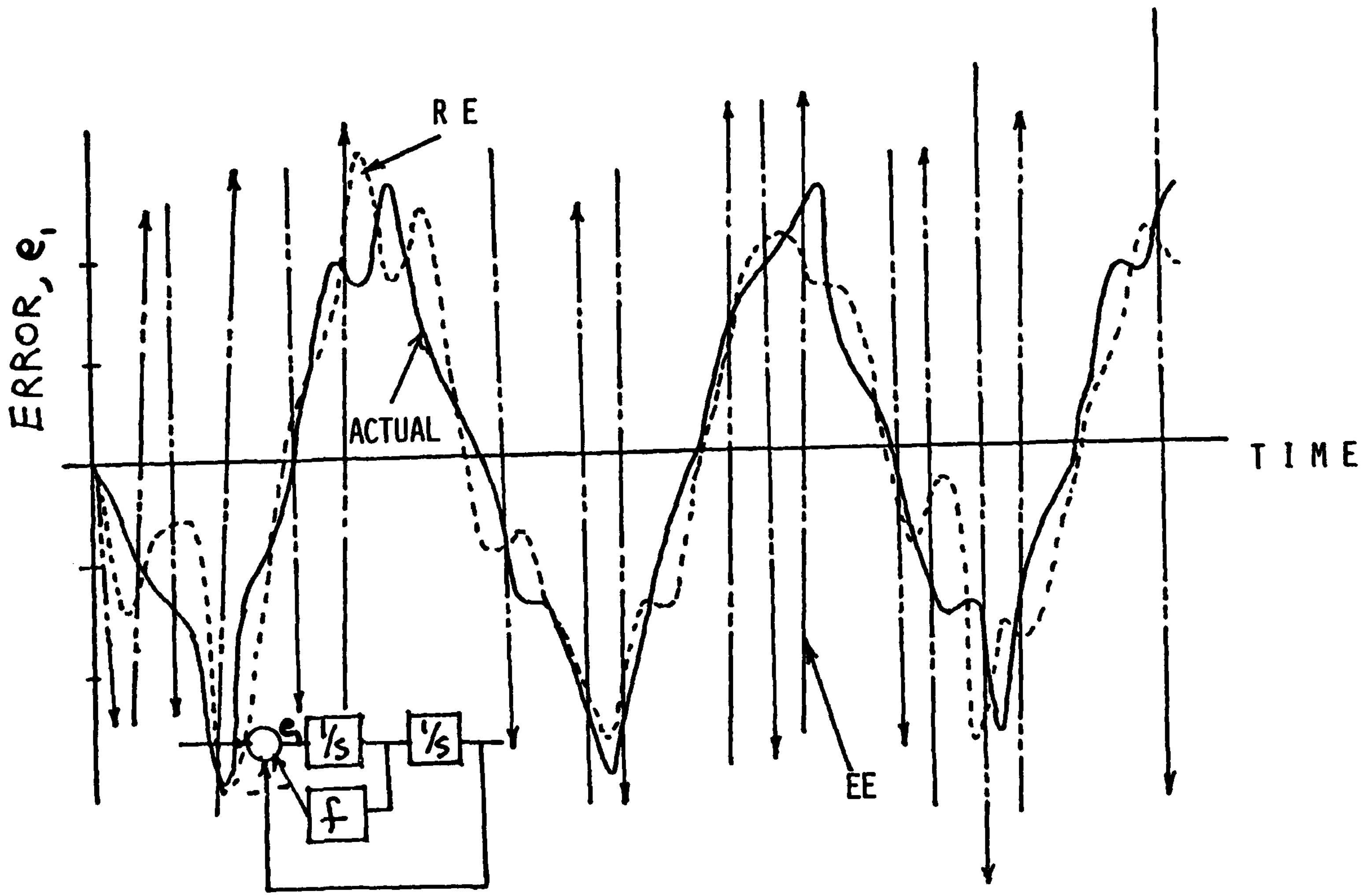


Figure II-5. Comparison of Equation/Response Error.

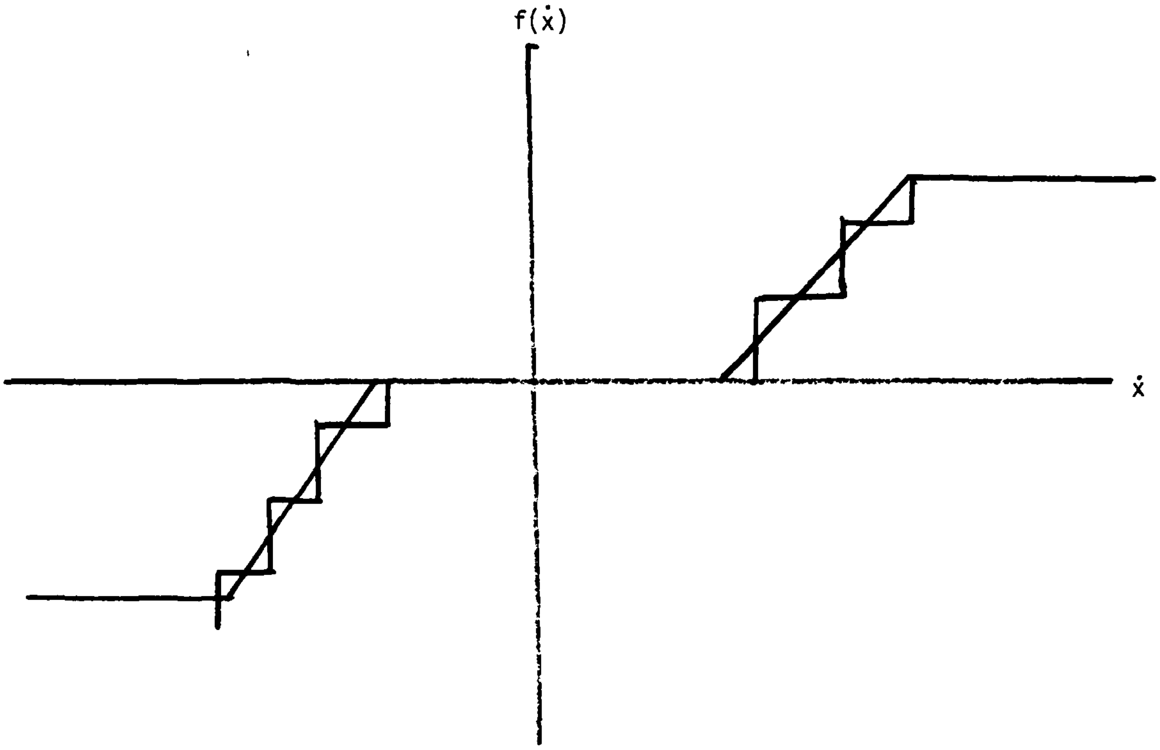


Figure II-6. EE Identification of saturation plus dead-zone.

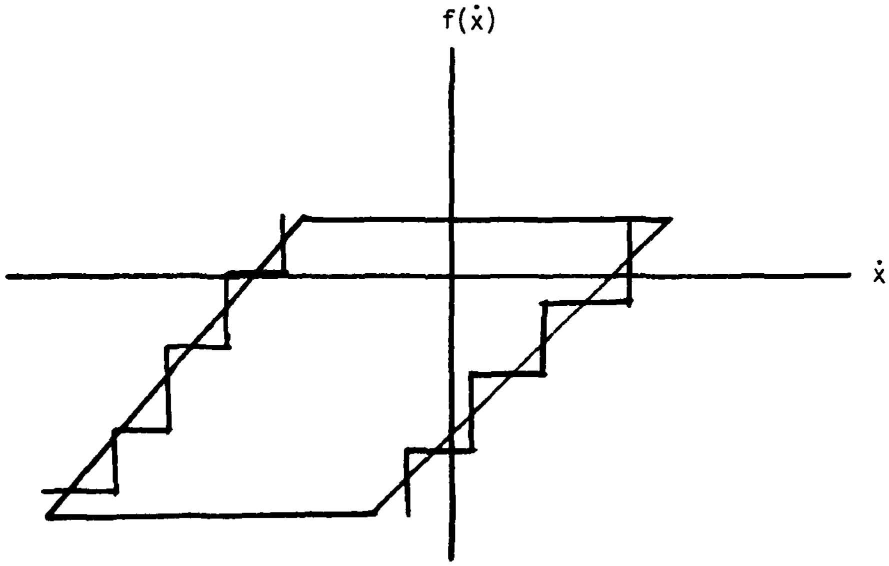


Figure II-7. EE Identification of hysteresis.

In the presence of good (i.e. no noise) state information, though, the EE will identify single-valued and multivalued memory type (e.g. hysteresis) nonlinearities very well, see Fig. II-6,7. This is the chief advantage of the EE method over the RE method because the RE method heretofore, has not been applicable to nonlinear identification.

HYPERSTABILITY CONCEPTS

Hyperstability plays a key note in guaranteeing the stability of the RE identification technique. It is a generalization of linear asymptotic and absolute stability. Consider the system shown in Figure II-8,

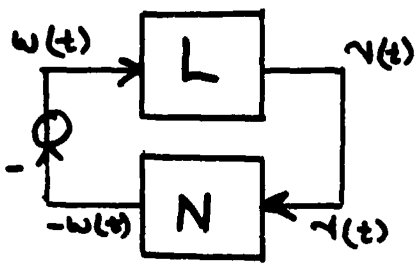


Figure II-8 Basic Hyperstability System

block L is assumed to be composed of linear differential equations. If block N is a linear feedback, then stability bounds may be derived:

$$\lambda_1 < \lambda < \lambda_2 \qquad (2.21)$$

where $-\omega(t) = \lambda v(t)$. This is the linear asymptotic stability case. If block N is nonlinear and obeys:

$$-\omega(t) = \phi(v(t)) \qquad (2.22)$$

$$\phi(v)v > 0 \qquad \forall\ v \neq 0 \qquad (2.23)$$

$$\phi(0) = 0 \qquad (2.24)$$

then the limits on block L and $\phi(v)$ can be obtained. This is the absolute stability problem. Now suppose (2.23) is rewritten

$$\int_0^t - \omega(\tau)\ v(\tau)\ d\tau \geq 0 \quad \forall\ t \geq 0 \qquad (2.25)$$

this, then, is the hyperstability problem. The problem can be stated as *find the conditions on block L given a block N such that condition (2.25)* occurs.

To begin the study of hyperstability, suppose that block L may be described as:

$$\frac{dx}{dt} = Ax + b\omega \tag{2.26}$$

$$v = cx + d\omega \tag{2.27}$$

Several definitions must now be made:

Definition 2.1

Let the hyperstability integral, n, be defined as

$$\eta(0,t) = \int_0^t - \omega(\tau)\, v(\tau)\, d\tau \tag{2.28}$$

Definition 2.2 Minimal Stability

The system (2.26-2.28) is said to posses the property of minimal stability if for any initial condition $x(0) = x_0$ there exists a pair of continuous functions x and w, defined for $t \geq 0$, and satisfying a) equations (2.26) and (2.27), b) the restriction

$$\eta(0,t_1) = \int_0^{t_1} - \omega(\tau)\, v(\tau)\, d\tau \leq 0 \ \forall\ t_1 \geq 0, \tag{2.29}$$

and c) the condition

$$\lim_{t \to \infty} x(t) = 0 \tag{2.30}$$

Definition 2.3 System Transfer Function

The transfer function of system (2.26)-(2.27) is defined as:

$$g(s) = C(sI-A)^{-1}b + d. \tag{2.31}$$

In order to apply hyperstability to a system (2.26)-(2.28) it is necessary that three conditions for this system (2.26)-(2.28) be satisfied.

The first condition is that the system (2.26)-(2.28) be minimally stable in the sense of Def. 2.2. This condition is acceptable since asymptotic stability is of prime importance and, if the system (2.26)-(2.28) is not minimally stable, then asymptotic stability in the presence of an integral restriction of the form (2.28) is impossible. The second condition is that $b \neq 0$. This condition is obviously necessary since the solution of (2.26)-(2.27) is not a function of u and the problem can be solved directly. Finally, the third restriction is that the transfer function of system (2.26)-(2.27) given in (2.31) be not identically equal to zero. This is necessary because if (2.31) was identically zero then $\eta(0,t_1)$ would be identically zero and (2.28) would be meaningless.

Given, that the above conditions are satisfied, hyperstability may now be defined.

Definition 2.4 Hyperstability

System (2.26)-(2.28) is said to be hyperstable if there exist positive constants α, β, γ, and such that the following two properties are satisfied:

1) Property H_s

For every $t \in [t_0,T_0]$ and every solution of the system (2.26)-(2.28) in the interval $[t_0,T_0]$, if for every constant $\beta_0 \geq 0$ for which

$$\eta(t_0,t) \leq \beta_0^2 \quad \forall \ t \in [t_0,T_0] \tag{2.32}$$

then

$$\alpha||x(t)|| \leq \beta_0 + \beta||x(t_0)|| \tag{2.33}$$

for all $t \in [t_0,T_0]$.

2) Property H_p

For every solution of the system (2.26)-(2.28) in the interval $[t_0, T_0]$

$$\eta(t_0,t) \geq -\gamma||x(t_0)|| - \gamma||x(t_0)|| \sup_{t_0\leq\tau\leq t} \alpha||x(\tau)|| \tag{2.34}$$

$\forall\ t \in [t_0,T_0]$.

Simply put, if

$$-\beta_0^2 \leq \eta(0,t) \leq \gamma_0^2 \tag{2.35}$$

then the system (2.26)-(2.28) is hyperstable, and if $\beta_0^2 = 0$, then the system is asymptotically hyperstable [21].

The results of Popov show that if, for a linear system (2.26)-(2.28), the transfer function from $\omega(t)$ to $v(t)$ given by (2.31) is a strictly positive real function (SPRF) and the inequality (2.34) holds then the system (2.26)-(2.28) is asymptotically hyperstable. Willems [26-27] showed that the SPRF insures that

$$\eta(0,t) = \int_0^t - \omega(\tau)\ v(\tau)\ d\tau \leq 0 \quad \forall\ t \geq 0. \tag{2.36}$$

RESPONSE ERROR IDENTIFICATION - APPLICATION OF HYPERSTABILITY

Consider the linear dynamic plant:

$$x^{(n)} = \sum_{i=0}^{n-1} a_i x^{(i)} + \sum_{i=0}^{m-1} b_i u^{(i)} \tag{2.37}$$

where

x - state variable

u - input or forcing function

a_i, b_i - constants

$(\cdot)^{(i)} = \dfrac{d^i(\cdot)}{dt^i}$

and the associated model:

$$x_m^{(n)} = \sum_{i=0}^{n-1} \hat{a}_i x_m^{(i)} + \sum_{i=0}^{m-1} \hat{b}_i u^{(i)} \tag{2.38}$$

Define the tracking error as:

$$e = x - x_m \tag{2.39}$$

and the error dynamics become:

$$e^{(n)} = \sum_{i=0}^{n-1} a_i e^{(i)} + \sum_{i=0}^{n-1} (a_i - \hat{a}_i) x_m^{(i)} + \sum_{i=0}^{m-1} (b_i - \hat{b}_i) u^{(i)} \tag{2.40}$$

Let

$$v(t) = e^{(n)} - \sum_{i=0}^{n-1} \hat{c}_i \, e^{(i)} \tag{2.41}$$

then

$$v(t) = \sum_{i=0}^{n-1} (a_i - \hat{c}_i) e^{(i)} + \sum_{i=0}^{n-1} (a_i - \hat{a}_i) x_m^{(i)} + \sum_{i=0}^{m-1} (b_i - \hat{b}_i) u^{(i)} \tag{2.42}$$

and let

$$\omega(t) = v(t) \tag{2.43}$$

The hyperstability system now is:

$$e^{(n)} = \sum_{i=0}^{n-1} a_i e^{(i)} + \sum_{i=0}^{n-1} (a_i - \hat{a}_i) x_m^{(i)} + \sum_{i=0}^{m-1} (b_i - \hat{b}_i) u^{(i)}$$

$$v(t) = e^{(n)} - \sum_{i=0}^{n-1} \hat{c}_i e^{(i)} \tag{2.44}$$

$$\omega(t) = v(t)$$

where e is defined in (2.39).

<u>Theorem 2.1</u>

If the system (2.44) is minimally stable and the input, u, into the plant and the model satisfies a frequency richness criterion, then the following parameter adaptation equations will yield the hyperstability of system (2.44) and the parameters $\hat{a}_i \to a_i$ and $\hat{b}_i \to b_i$ as $t \to \infty$:

$$\dot{\hat{a}}_i = \alpha_i x_m^{(i)} v, \quad \alpha_i > 0 \quad i \in [0,n-1] \tag{2.45}$$

$$\dot{\hat{b}}_i = \beta_i u^{(i)} v, \quad \beta_i > 0 \quad i \in [0,m-1] \tag{2.46}$$

$$\dot{\hat{c}}_i = \gamma_i e^{(i)} v, \quad \gamma_i > 0 \quad i \in [0,n-1] \tag{2.47}$$

Proof

Since $\omega(t) = v(t)$ the transfer function of system (2.44) is

$$G(s) = 1 \tag{2.48}$$

which is an SPRF, thus

$$\eta(0,t) = \int_0^t - \omega(\tau)\, v(\tau)\, d\tau \leq 0 \quad \forall\, t \geq 0 \tag{2.49}$$

The other bound on the hyperstability integral must now be determined, that is

$$\eta(0,t) = \int_0^t - \omega(\tau)\, v(\tau)\, d\tau \geq -\gamma_0^2 \tag{2.50}$$

Substituting (2.42) into (2.50) one obtains

$$\eta(0,t) = \int_0^t \sum_{i=0}^{n-1} - (a_i - \hat{c}_i) e^{(i)} v d\tau +$$

$$\int_0^t \sum_{i=0}^{n-1} (a_i - \hat{a}_i) x_m^{(i)} v d\tau + \int_0^t \sum_{i=0}^{n-1} - (b_i - \hat{b}_i) u^{(i)} v d\tau \tag{2.51}$$

It is sufficient to look at each of these terms and bound them individually. Thus

$$\int_0^t \sum_{i=0}^{n-1} - (a_i - \hat{c}_i) e^{(i)} v d\tau \geq -\gamma^2{}_\gamma, \tag{2.52}$$

$$\int_0^t \sum_{i=0}^{n-1} - (a_i - \hat{a}_i) x_m^{(i)} v d\tau \geq \gamma^2{}_\alpha, \text{ and} \tag{2.53}$$

$$\int_0^t \sum_{i=0}^{m-1} - (b_i - \hat{b}_i)u^{(i)} v d\tau \geq -\gamma_\beta^2 \ . \tag{2.54}$$

The integration and summation may be interchanged in (2.52)-(2.54) because they are independent of one another, and since the resultant is a sum of integrals, it is sufficient that each of these be bounded, yielding:

$$\int_0^t - (a_i - \hat{c}_i)e^{(i)} v d\tau \geq -\gamma^2{}_{\gamma_i} \ , \ i \ \varepsilon \ [0,n-1] \tag{2.55}$$

$$\int_0^t - (a_i - \hat{a}_i)x_m{}^{(i)} v d\tau \geq -\gamma^2{}_{\alpha_i} \ , \ i \ \varepsilon \ [0,n-1] \tag{2.56}$$

$$\int_0^t - (b_i - \hat{b}_i)u^{(i)} v d\tau \geq -\gamma^2{}_{\beta_i} \ , \ i \ \varepsilon \ [0,m-1] \tag{2.57}$$

Using (2.55), let

$$f_i(t) = -(a_i - \hat{c}_i). \tag{2.58}$$

Differentiating with respect to t yields

$$\dot{f}_i(t) = \dot{\hat{c}}_i \tag{2.59}$$

Define the $\hat{c}_i$ parameter adaptation as

$$\hat{c}_i = \gamma_i e^{(i)} v, \qquad \gamma_i > 0 \tag{2.60}$$

Substituting (2.60) into (2.59) and solving for $f_i(t)$ yields

$$f_i(t) = \gamma_i \int_0^t e^{(i)} v d\tau + f_i(0) \tag{2.61}$$

where $f_i(0) = -(a_i - \hat{c}_i(0))$. Substituting (2.61) into (2.55) yields

$$\gamma_i \int_0^t \left[\int_0^\tau e^{(i)} v d\tau' \right] e^{(i)} v d\tau + f_i(0) \int_0^t e^{(i)} v d\tau \geq$$

$$-\gamma^2{}_{\gamma_i} \tag{2.62}$$

Using Leibnitz's rule:

$$\frac{d}{dt} \int_0^\tau f(\tau',\tau)d\tau' = \int_0^\tau \frac{\partial f}{\partial \tau} \, d\tau' + f(\tau,\tau) \frac{d\tau}{d\tau} \tag{2.63}$$

The first term of (2.62) may be integrated. Let

$$f(\tau',\tau) = e^{(i)}(\tau')v(\tau') \tag{2.64}$$

and use Leibnitz's rule yielding:

$$\frac{\partial f}{\partial \tau} = 0 \text{ and} \tag{2.65}$$

$$f(\tau,\tau) = e^{(i)}(\tau)v(\tau). \tag{2.66}$$

Thus, if

$$F_i(\tau) = \int_0^\tau e^{(i)} v d\tau' \tag{2.67}$$

then from Leibnitz's rule

$$\frac{dF_i}{d\tau} = e^{(i)}(\tau)v(\tau) \tag{2.68}$$

and

$$dF_i = e^{(i)}(\tau)v(\tau)d\tau. \tag{2.69}$$

This substitution, (2.67), puts (2.62) into the form

$$\gamma_i \int_0^t F_i(\tau)dF_i(\tau)d \; + f_i(0) \int_0^t e^{(i)} v d\tau \geq -\gamma^2{}_{\gamma_i} \tag{2.70}$$

which yields,

$$\frac{\gamma_i}{2} [\int_0^t e^{(i)} v d\tau]^2 + f_i(0)\int_0^\tau e^{(i)} v d\tau \geq -\gamma^2{}_{\gamma_i} \tag{2.71}$$

Completing the square in (2.71) yields:

$$\frac{\gamma_i}{2}[\int_0^t e^{(i)} v d\tau + \frac{f_i(0)}{\gamma_i}]^2 - \frac{f_i{}^2(0)}{2\gamma_i} \geq - \frac{f_i{}^2(0)}{2\gamma_i} = \gamma^2{}_{\gamma_i}. \tag{2.72}$$

Thus, the boundedness of (2.55) has been proved with the given parameter adaptation (2.47). Using similar substitutions:

$$g_i(t) = -(a_i - \hat{a}_i) \text{ and} \tag{2.73}$$

$$h_i(t) = -(b_i - \hat{b}_i) \tag{2.74}$$

the boundedness of (2.56) and (2.57) can be proved using the parameter adaptation (2.45) and (2.46), respectively. Since, all the integrals (2.55)-(2.57) are bounded the boundedness of (2.51) is assured and is equal to

$$\gamma_0^2 = \sum_{i=0}^{n-1} \frac{f_i^2(0)}{2\gamma_i} + \sum_{i=0}^{n-1} \frac{g_i^2(0)}{2\alpha_i} + \sum_{i=0}^{m-1} \frac{h_i^2(0)}{2\beta_i} \tag{2.75}$$

or, substituting, for $f_i(0)$, $g_i(0)$, $h_i(0)$

$$\gamma_0^2 = \sum_{i=0}^{n-1} \frac{[\hat{c}_i(0) - a_i]^2}{2\gamma_i} + \sum_{i=0}^{n-1} \frac{[\hat{a}_i(0) - a_i]^2}{2\alpha_i} +$$

$$\sum_{i=0}^{m-1} \frac{[\hat{b}_i(0) - b_i]^2}{2\beta_i} \tag{2.76}$$

and the theorem is proved.

Theorem 2.1 was proved under the assumption of minimal stability of the system (2.44). The following theorem gives the requirements on the plant (2.37) in order for minimal stability to hold.

Theorem 2.2

If the plant (2.37) is asymptotically stable, then the error system (2.44) is minimally stable.

Proof

Minimal stability requires that for any initial condition $e(0) = e_0$, a pair of functions $\omega(t)$ and $e(t)$ exists such that

1) $\eta(0,t) \leq 0$ and

2) $\lim_{t\to\infty} e(t) = 0.$

Define

$$u'(t) = \sum_{i=0}^{n-1} (a_i - \hat{a}_i)x_m^{(i)} + \sum_{i=0}^{m-1} (b_i - \hat{b}_i)u^{(i)} \tag{2.76}$$

then (2.40) becomes

$$e^{(n)} = \sum_{i=0}^{n-1} a_i e^{(i)} + u'(t) \tag{2.77}$$

Let

$$\omega(t) = \sum_{i=0}^{n-1} (a_i - \hat{c}_i)e^{(i)} \tag{2.78}$$

(this can be done by setting $\hat{a}_i = a_i$, $i \in [0,n-1]$ and $\hat{b}_i = b_i$, $i \in [0,m-1]$ in (2.42)-(2.43), and is certainly an allowable $\omega(t)$). By using this $\omega(t)$, $u'(t) = 0$ and (2.77) becomes

$$e^{(n)} = \sum_{i=0}^{n-1} a_i e^{(i)} \tag{2.79}$$

which is asymptotically stable only if the plant is asymptotically stable for any initial condition $x(0) = x_0$. Since the $\omega(t)$ was chosen from a set for which

$$-\gamma_0^2 \leq \eta(0,t) \leq 0 \tag{2.80}$$

then the first condition of minimal stability is maintained and the theorem is proved.

The frequency richness criterion in Theorem 2.1 was discussed by Sprague and Kohr [7]. They showed that for parametric identification to occur when the error was identically equal to zero the number of distinct frequency components of the input, u, to the plant and model is given by

$$N = \frac{n + m + 1}{2}, \tag{2.81}$$

when the parameters to be identified were the a_i, $i \in [0,n-1]$ and b_i, $i \in [0,m-1]$, and the plant was linear. They showed this mathematically for the linear plant and no zero case and inferred it experimentally in the general linear case. For the nonlinear case it was found to be at least as large as (2.81). For the case in which there are also adaptive $\hat{c}_i$ terms the number of distinct frequencies was found to be equal to

$$N = \frac{2n + m + 1}{2} \tag{2.82}$$

using experimental results and empirical inferences.

The development of nonlinear RE identification algorithms is based on the linear algorithm presented here. Its proof is very similar to the proof just presented.

III. HYPERSTABLE NONLINEAR RESPONSE ERROR IDENTIFICATION (HNREI)

The development of linear MRAS identifiers and hyperstability principles along with the nonlinear concepts of the EE method lead to the development of nonlinear RE identification techniques. Heretofore, the extension to nonlinear identification using the RE method has been thwarted by the requirement of the SPRF which is not defined for general nonlinear systems. However, with Landau's [11] and Johnson's [12] papers eliminating the SPRF difficulty through problem reformulation, an approach for HNREI algorithms now exists. In this chapter two different nonlinear identifier formulations will be presented and two nonlinear identification algorithm proved to be hyperstable are given.

Nonlinear Formulations

For the development of HNREI algorithms, two nonlinear formulations were considered: 1) piecewise linear expansions in intervals of the state space and 2) power series expansions over the entire state space. Both of these have distinct advantages and disadvantages. The piecewise linear expansion, Figure III-1, allows the nonlinear function to be divided into a series of intervals in the state space and a linear approximation to the nonlinearity formed. This method has a serious disadvantage, though, when perfect state information is lacking, Figure III-2. It is possible that the identification algorithm could be adapting a parameter in the wrong interval yielding erroneous results. Thus, the degree of accuracy obtainable by the piecewise linear formulation decreases with the loss of perfect state information.

The power series expansion technique, Figure III-3, uses a continuous approach and, hence, does not suffer from the intervalization problem associated with the piecewise linear method. The power series method attempts to approximate the actual nonlinear function as a finite power series:

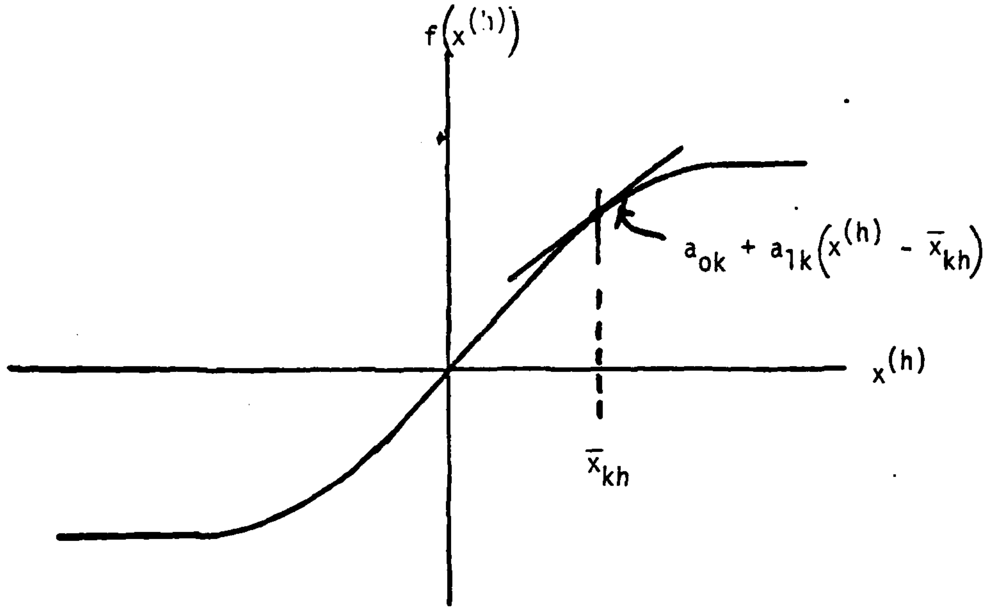


Figure III-1. Piecewise Linear Expansion

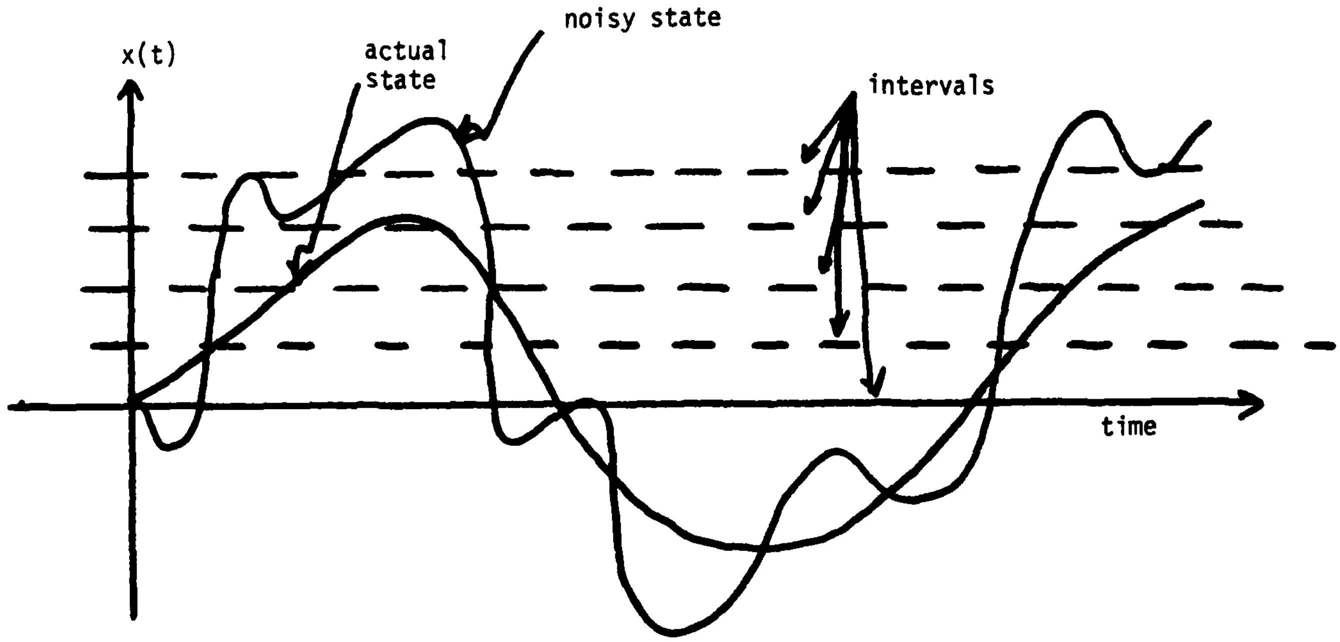


Figure III-2. Interval Selection From States.

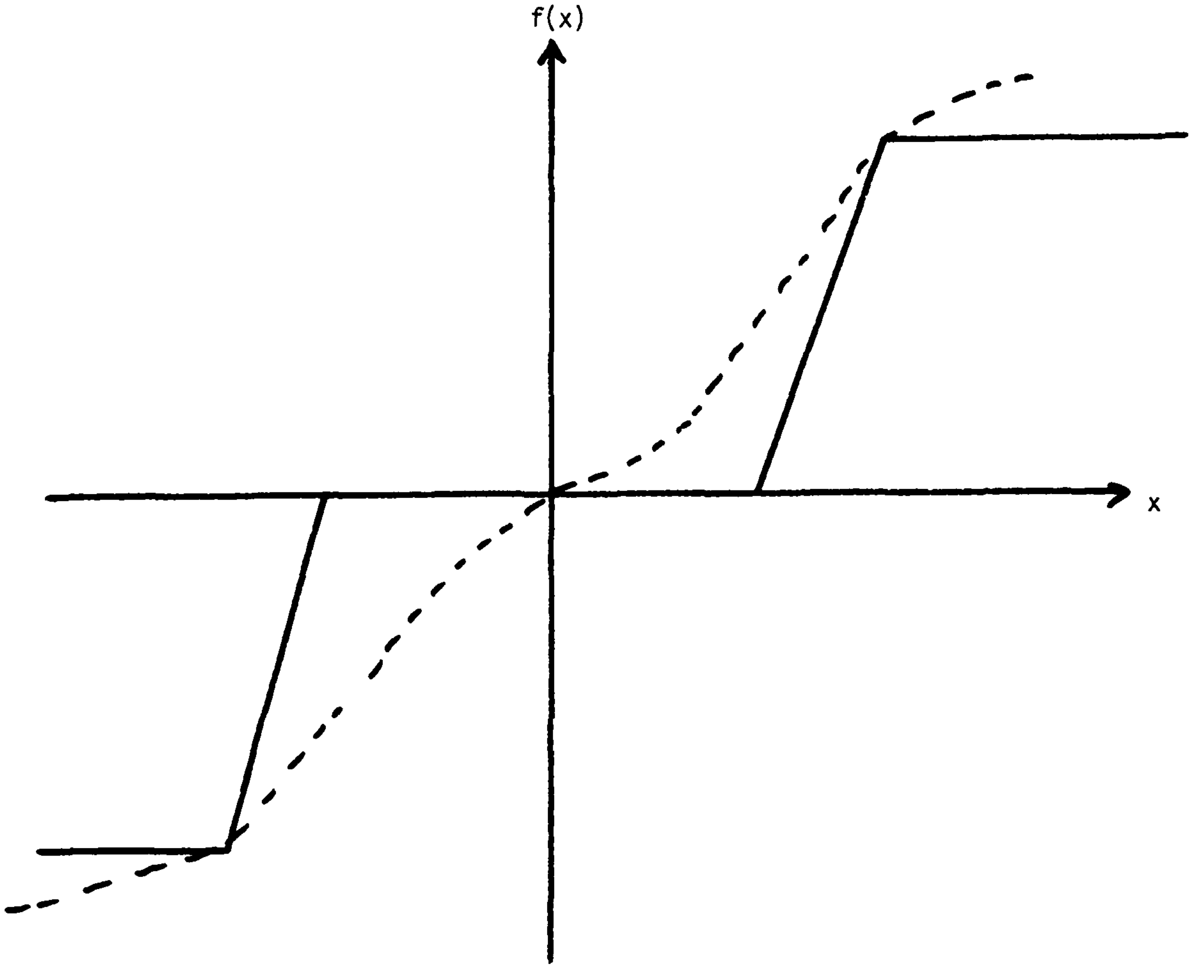


Figure III-3. Power Series Expansion.

$$f(x) = \sum_{i=1}^{N} a_i x^i. \tag{3.1}$$

The principal disadvantage of this method is that for some nonlinearities which can be approximated with a small number of intervals using the piecewise linear expansion may require many terms using a power series expansion. Noisy state measurements also present problems because of the possible high degree of the polynomial and noise biasing.

Nonlinear Identification Algorithms

The nonlinear expansion techniques just discussed were used to develop HNREI. The piecewise linear HNREI required a special interval condition to insure hyperstability while the power series expansion technique required no special conditions.

Piecewise Linear HNREI

Consider the following nonlinear plant:

$$x^{(n)} = \sum_{\substack{i=0 \\ i\neq\ell}}^{n-1} f_i(x^{(i)}) + g_\ell(x^{(\ell)}) + \sum_{i=0}^{m-1} h_i(u^{(i)}) \tag{3.2}$$

with a corresponding model:

$$x_m^{(n)} = \sum_{\substack{i=0 \\ i\neq\ell}}^{n-1} \hat{f}_i(x_m^{(i)}) + \hat{g}_\ell(x_m^{(\ell)}) + \sum_{i=0}^{m-1} \hat{h}_i(u^{(i)}) \tag{3.3}$$

where $(\cdot)^{(i)} = \dfrac{d^i(\cdot)}{dt^i}$, and $\ell \in [0,n-1]$. Let

$$f_i(x^{(i)}) = a_{i1p}x^{(i)} \tag{3.4}$$

$$g_\ell(x^{(\ell)}) = a_{\ell 0p} + a_{\ell 1p}(x^{(\ell)} - \bar{x}_{\ell p}) \tag{3.5}$$

$$h_i(u^{(i)}) = b_{i1p}u^{(i)} \tag{3.6}$$

which have corresponding model nonlinearities

$$\hat{f}_i(x_m^{(i)}) = \hat{a}_{i1p}x_m^{(i)} \tag{3.7}$$

$$\hat{g}_\ell(x_m^{(\ell)}) = \hat{a}_{\ell 0p} + \hat{a}_{\ell 1p}(x_m^{(\ell)} - \bar{x}_{m\ell p}) \tag{3.8}$$

$$\hat{h}_i(u^{(i)}) = \hat{b}_{i1p}u^{(i)} \tag{3.9}$$

where p denotes the p^{th} interval of the state space $x^{(i)}$ or $u^{(i)}$. Thus, the plant and model may be rewritten, respectively, as:

$$x^{(n)} = \sum_{\substack{i=0\\ i\neq\ell}}^{n-1} a_{i1p}x^{(i)} + a_{\ell 0p} + a_{\ell 1p}(x^{(\ell)} - \bar{x}_{\ell p}) + \sum_{i=0}^{m-1} b_{i1p}u^{(i)} \tag{3.10}$$

$$x_m^{(n)} = \sum_{\substack{i=0\\ i\neq\ell}}^{n-1} \hat{a}_{i1p}x_m^{(i)} + \hat{a}_{\ell 0p} + \hat{a}_{\ell 1p}(x_m^{(\ell)} - \bar{x}_{\ell p}) + \sum_{i=0}^{m-1} \hat{b}_{i1p}u^{(i)} \tag{3.11}$$

where the parameters $\hat{b}_{i1p}$, $\hat{a}_{i1p}$, $\hat{a}_{\ell 0p}$, and $\hat{a}_{\ell 1p}$ must be identified in each interval p. Define the tracking error as

$$e = x - x_m \tag{3.12}$$

The error dynamics which are to be made hyperstable are:

$$\begin{aligned} e^{(n)} = {} & \sum_{i=0}^{n-1} a_{i1p}e^{(i)} - a_{\ell 1p}(\bar{x}_{\ell p} - \bar{x}_{m\ell p}) + a_{\ell 0p} - \hat{a}_{\ell 0p} \\ & + (a_{\ell 1p} - \hat{a}_{\ell 1p})(x_m^{(\ell)} - \bar{x}_{m\ell p}) + \sum_{\substack{i=0\\ i\neq\ell}}^{n-1} (a_{i1p} - \hat{a}_{i1p})x_m^{(i)} \\ & + \sum_{i=0}^{m-1} (b_{i1p} - \hat{b}_{i1p})u^{(i)} \end{aligned} \tag{3.13}$$

Let

$$v(t) = e^{(n)} - \sum_{i=0}^{n-1} \hat{c}_{i1p}e^{(i)} \tag{3.14}$$

then

$$v(t) = \sum_{i=0}^{n-1} (a_{i1p} - \hat{c}_{i1p})e^{(i)} - a_{\ell 1p}(\bar{x}_{\ell p} - \bar{x}_{m\ell p}) +$$

$$a_{\ell 0p} - \hat{a}_{\ell 0p} + (a_{\ell 1p} - \hat{a}_{\ell 1p})(x_m^{(\ell)} - \bar{x}_{m\ell p}) +$$

$$\sum_{\substack{i=0 \\ i\neq\ell}}^{m-1} (a_{i1p} - \hat{a}_{i1p})x_m^{(i)} + \sum_{i=0}^{m-1} (b_{i1p} - \hat{b}_{i1p})u^{(i)} \tag{3.15}$$

Define

$$\omega(t) = v(t) \tag{3.16}$$

The hyperstability system is given by (3.13), (3.14) and (3.16).

Theorem 3.1

If the system (3.13)-(3.16) is minimally stable, the input, u, into the plant and the model is sufficiently frequency rich, and the condition $\bar{x}_{\ell p} = \bar{x}_{m\ell p}$ are satisfied, then the following parameter adaptation equations guarantee the hyperstability of system (3.13)-(3.16) and the parametric identification: $\hat{a}_{\ell 0p} \to a_{\ell 0p}$, $\hat{a}_{i1p} \to a_{i1p}$, and $\hat{b}_{i1p} \to b_{i1p}$ as $t \to \infty$:

$$\dot{\hat{a}}_{\ell 0p} = \alpha_{\ell 0p}\, v \qquad \alpha_{\ell 0p} > 0 \tag{3.17}$$

$$\dot{\hat{a}}_{\ell 1p} = \alpha_{\ell 1p}\, v(x_m^{(\ell)} - \bar{x}_{m\ell p}),\ \alpha_{\ell 1p} > 0 \tag{3.18}$$

$$\dot{\hat{a}}_{i1p} = \alpha_{i1p}\, v x_m^{(i)} \quad i \in [0,n-1],\ i \neq \ell,\ \alpha_{i1p} > 0 \tag{3.19}$$

$$\dot{\hat{b}}_{i1p} = \beta_{i1p}\, v u^{(i)},\ \beta_{i1p} > 0,\ i \in [0,m-1] \tag{3.20}$$

$$\dot{\hat{c}}_{i1p} = \gamma_{i1p}\, v e^{(i)},\ \gamma_{i1p} > 0,\ i \in [0,n-1] \tag{3.21}$$

Proof

Since $w(t) = v(t)$,

$$\eta(0,t) = \int_0^t - w(\tau)v(\tau)d\tau \leq 0 \tag{3.22}$$

and, thus, one limit on $\eta(0,t)$ is established. To establish hyperstability the other limit must be satisfied:

$$\eta(0,t) = \int_0^t - w(\tau)v(\tau)d\tau \geq -\gamma_0^2 \tag{3.23}$$

Substituting (3.15) into (3.23) for $w(\tau)$ using (3.16) yields

$$\eta(0,t) = \int_0^t \sum_{i=0}^{n-1} - (a_{i1p} - \hat{c}_{i1p})e^{(i)}vd\tau + \int_0^t a_{\ell 1p} \bullet$$
$$(\overline{x}_{\ell p} - \overline{x}_{m\ell p})vd\tau + \int_0^t - (a_{\ell 0p} - \hat{a}_{\ell 0p})vd\tau + \left[\int_0^t\right.$$
$$\left. - (a_{\ell 0p} - \hat{a}_{\ell 0p})vd\tau\right] + \int_0^t \sum_{i=0}^{m-1} - (b_{i1p} - \hat{b}_{i1p})u^{(i)}vd\tau \geq -\gamma_0^2 \tag{3.24}$$

It is sufficient to bound each of the terms in (3.24) in order to insure the boundedness of (3.24) and by exchanging the integration and summation of terms 1, 5, and 6 and using the same sufficiency argument (3.24) yields:

$$\int_0^t - (a_{i1p} - c_{i1p})e^{(i)}vd\tau \geq \gamma^2{}_{\gamma_{i1p}} \quad i \in [0,n-1] \tag{3.25}$$

$$\int_0^t - (a_{10p} - a_{\ell 0p})vd\tau \geq \gamma^2{}_{\alpha_{\ell 0p}} \tag{3.26}$$

$$\int_0^t - (a_{\ell 1p} - \hat{a}_{\ell 1p})(x_m^{(\ell)} - \overline{x}_{m\ell p})vd\tau \geq \gamma_{\alpha_{\ell 1p}} \tag{3.27}$$

$$\int_0^t - (a_{i1p} - \hat{a}_{i1p})x_m^{(i)}vd\tau \geq -\gamma_{\alpha_{i1p}} \quad i \in [0,n-1],\ i \neq \ell \tag{3.28}$$

$$\int_0^t - (b_{i1p} - \hat{b}_{i1p})u^{(i)}vd\tau \geq -\gamma_{\beta_{i1p}} \quad i \in [0,m-1] \tag{3.29}$$

Using (3.25), let

$$f_{i1p}(t) = -(a_{i1p} - \hat{c}_{i1p}). \tag{3.30}$$

Differentiating (3.30) with respect to time yields:

$$\dot{f}_{i1p}(t) = \dot{c}_{i1p} \tag{3.31}$$

Define the $\hat{c}_{i1p}$ parameter adaptation as:

$$\dot{\hat{c}}_{i1p} = \gamma_{i1p} e^{(i)} v, \quad \gamma_{i1p} > 0 \tag{3.32}$$

Substituting (3.32) into (3.31) and solving for $f_{i1p}(t)$ yields

$$f_{i1p}(t) = \gamma_{i1p} \int_0^t e^{(i)}\, v d\tau + f_{i1p}(0) \tag{3.33}$$

where $f_{i1p}(0) = -(a_{i1p} - \hat{c}_{i1p}(0))$. (3.34)

Subs tituting (3.33) into (3.25) yields

$$\gamma_{i1p} \int_0^t [\int_0^\tau e^{(i)} v d\tau']e^{(i)} v d\tau + f_{i1p} \int_0^t e^{(i)} v d\tau \tag{3.35}$$

By using Leibnitz's rule and completing the square, (3.35) yields:

$$\frac{\gamma_{i1p}}{2}\left[\int_0^t e^{(i)} v d\tau + \frac{f_{i1p}(0)}{\gamma_{i1p}}\right]^2 \geq - \frac{f_{i1p}^2(0)}{2\gamma_{i1p}} = -\gamma^2{}_{\gamma_{i1p}} \tag{3.36}$$

Thus, on the first integral has been obtained using the parameter adaptation (3.21).

Similarly, using (3.26), let

$$g_{\ell 0p}(t) = -(a_{\ell 0p} - \hat{a}_{\ell 0p}). \tag{3.37}$$

Following a development similar to (3.30)-(3.36), (3.26) can be bounded using the parameter adaptation (3.17) obtaining:

$$\frac{\alpha_{\ell 0p}}{2} [\int_0^t v d\tau + \frac{g_{\ell 0p}(0)}{\alpha_{\ell 0p}}]^2 - \frac{g_{\ell 0p}^2(0)}{2\alpha_{\ell 0p}} \geq - \frac{g_{\ell 0p}^2(0)}{2\alpha_{\ell 0p}} = -\gamma^2{}_{\alpha_{\ell 0p}} \tag{3.38}$$

The other integrals (3.27)-(3.29) can be bounded using (3.8)-(3.20), respectively, and obtaining, respectively,

$$\frac{\alpha_{\ell 1p}}{2}\left[\int_0^t (x_m^{(\ell)} - \bar{x}_{m\ell p})v d\tau + \frac{g_{\ell 1p}(0)}{\alpha_{\ell 1p}}\right]^2 - \frac{g^2_{\ell 1p}(0)}{2\alpha_{\ell 1p}}$$

$$\geq - \frac{g^2_{\ell 1p}(0)}{2\alpha_{\ell 1p}} = -\gamma^2_{\alpha_{i1p}} \tag{3.39}$$

$$\frac{\alpha_{i1p}}{2}\left[\int_0^t x_m^{(i)} v d\tau + \frac{g^2_{i1p}(0)}{\alpha_{i1p}}\right]^2 - \frac{g^2_{i1p}(0)}{2\alpha_{i1p}} \geq$$

$$- \frac{g^2_{i1p}(0)}{2\alpha_{i1p}} = -\gamma^2_{\alpha_{i1p}} \tag{3.40}$$

$$\frac{\beta_{i1p}}{2}\left[\int_0^t u^{(i)} v d\tau + \frac{h^2_{i1p}(0)}{\beta_{i1p}}\right]^2 - \frac{h^2_{i1p}(0)}{2\beta_{i1p}} \geq$$

$$- \frac{h^2_{i1p}(0)}{2\beta_{i1p}} = -\gamma^2_{\beta_{i1p}} \tag{3.41}$$

where

$$h_{i1p}(t) = -(b_{i1p} - \hat{b}_{i1p}). \tag{3.42}$$

Thus, all of the integrals (3.25)-(3.29) have been bounded, and, therefore, the integral (3.24) is bounded by $-\gamma_0^2$ where

$$\gamma_0^2 = \sum_{i=0}^{n-1} \frac{f^2_{i1p}(0)}{2\gamma_{i1p}} + \sum_{\substack{i=0\\ i\neq\ell}}^{n-1} \frac{g^2_{i1p}(0)}{2\alpha_{i1p}} + \sum_{i=0}^{m-1} \frac{h^2_{i1p}(0)}{2\beta_{i1p}} +$$

$$\frac{g^2_{\ell 0p}(0)}{2\alpha_{\ell 0p}} + \frac{g^2_{\ell 1p}(0)}{2\alpha_{\ell ip}} \tag{3.43}$$

or, substituting for $f_{i1p}(0)$, $g_{i1p}(0)$, $h_{i1p}(0)$

$$\gamma_0^2 = \sum_{i=0}^{n-1} \frac{(a_{i1p}(0)-a_{i1p})^2}{2\gamma_{i1p}} + \sum_{\substack{i=0\\ i\neq\ell}}^{n-1} \frac{(\hat{a}_{i1p}-a_{i1p})^2}{2\alpha_{i1p}} + \left[\sum_{i=0}^{m-1} \cdot\right.$$

$$\left.\frac{(\hat{b}_{i1p}-b_{i1p})^2}{2\beta_{i1p}}\right] + \frac{(\hat{a}_{\ell 0p}-a_{\ell 0p})^2}{2\alpha_{\ell 0p}} + \frac{(\hat{a}_{\ell 1p}-a_{\ell 1p})^2}{2\alpha_{\ell 1p}} \tag{3.44}$$

Thus,

$$-\gamma_0^2 \leq \eta(0,t) \leq 0 \tag{3.45}$$

and hyperstability is proved.

The conditions for minimal stability of the error system (3.13)-(3.16) are given in the following theorem.

Theorem 3.2

If the plant (3.10) is asymptotically stable, then the error system (3.13)-(3.16) is minimally stable.

Proof

The proof of Theorem 3.2 parallels the proof of Theorem 2.2. Let

$$w(t) = \sum_{i=0}^{n-1} (a_{i1p}-\hat{c}_{i1p})e^{(i)} + a_{\ell 0p}-a_{\ell 1p}(\bar{x}_{\ell p}-\bar{x}_{m\ell p}) \tag{3.46}$$

By using this input, the error system becomes:

$$e^{(n)} = \sum_{\substack{i=0\\ i\neq\ell}}^{n-1} a_{i1p}e^{(i)} + a_{\ell 0p} + a_{\ell 1p}[(x^{(\ell)}-\bar{x}_{\ell p})-$$

$$(x_m^{(\ell)}-\bar{x}_{m\ell p})] \tag{3.47}$$

by setting

$$\hat{a}_{\ell 0p} = 0 \tag{3.48}$$

$$\hat{a}_{\ell 1p} = a_{i1p} \qquad i \in [0,n-1]$$

$$\hat{b}_{i1p} = b_{i1p} \qquad i \in [0,m-1]$$

and

$$u^{(i)}(t) = 0 \qquad i \in [1,m-1]$$

$$x_m^{(i)}(0) = 0 \qquad i \in [0,n-1].$$

Thus, (3.47) becomes

$$x^{(n)} = \sum_{\substack{i=0\\ i\neq\ell}}^{n-1} a_{i1p}x^{(i)} + a_{\ell 0p} + a_{\ell 1p}(x^{(\ell)}-\bar{x}_{\ell p}) \tag{3.49}$$

which must be asymptotically stable in order to insure the minimal stability of (3.13)-(3.16). The final condition of Theorem 3.1,

$$\overline{x}_{\ell p} = \overline{x}_{m\ell p} \tag{3.50}$$

requires substantive a priori knowledge of the plant. This type of knowledge is not readily available and condition (3.50) limits the piecewise linear HNREI.

Power Series HNREI

Consider the following nonlinear plant:

$$x^{(n)} = \sum_{i=0}^{n-1} [\sum_{j=1}^{k_i} a_{ij} x^{(i)j}] + \sum_{i=0}^{m-1} [\sum_{j=1}^{\ell_i} b_{ij} u^{(i)j}] \tag{3.51}$$

where k_i, ℓ_i is a finite integer and $(\cdot)^{(i)j}$ indicates $\frac{d^i(\cdot)}{dt^i}$ raised to the j^{th} power. This plant has a corresponding model:

$$x_m^{(n)} = \sum_{i=0}^{n-1} [\sum_{j=0}^{k_i} \hat{a}_{ij} x_m^{(i)j}] + \sum_{i=0}^{m-1} [\sum_{j=1}^{\ell_i} \hat{b}_{ij} u^{(i)j}] \tag{3.52}$$

Define the tracking error as:

$$e = x - x_m \tag{3.53}$$

and the error dynamics become

$$e^{(n)} = \sum_{i=0}^{n-1} [\sum_{j=1}^{k_i} a_{ij}(x^{(i)j} - x_m^{(i)j})] + \sum_{i=0}^{n-1} [\sum_{j=1}^{k_i} (a_{ij} - \hat{a}_{ij}) x_m^{(i)j}]$$

$$+ \sum_{i=0}^{m-1} [\sum_{j=1}^{\ell_i} (b_{ij} - \hat{b}_{ij}) u^{(i)j}] \tag{3.54}$$

Let the response error be defined,

$$v(t) = e^{(n)} - \sum_{i=0}^{n-1} [\sum_{j=1}^{k_i} \hat{c}_{ij}(x^{(i)j} - x_m^{(i)j}] \tag{3.55}$$

and define

$$w(t) = v(t) \tag{3.56}$$

Thus, the error system to be made hyperstable is (3.54)-(3.56).

Theorem 3.3

If the system (3.54)-(3.56) is minimally stable, and the input, u, into the plant (3.51) and model (3.52) is sufficiently frequency rich, then the following parameter adaptation equations insure the hyperstability of system (3.54)-(3.56) and the parameter convergence

$$\hat{a}_{ij} \to a_{ij},\ \hat{b}_{ij} \to b_{ij} \text{ as } t \to \infty:$$

$$\dot{\hat{a}}_{ij} = \alpha_{ij} x_m^{(i)j} v \qquad i\varepsilon[0,n-1],\ j\varepsilon[1,k_i],\ \alpha_{ij}>0 \tag{3.57}$$

$$\dot{\hat{b}}_{ij} = \beta_{ij} u^{(i)j} v \qquad i\varepsilon[0,m-1],\ j\varepsilon[1,\ell_i],\ \beta_{ij}>0 \tag{3.58}$$

$$\dot{\hat{c}}_{ij} = \gamma_{ij}[x^{(i)j} - x_m^{(i)j}]v \quad i\varepsilon[0,n-1],\ j\varepsilon[1,k_i],\ \gamma_{ij}>0 \tag{3.59}$$

Proof

Since (3.56) holds the upper limit of $\eta(0,t)$ defined as

$$\eta(0,t) = \int_0^t w(\tau)\ v(\tau)\ d\tau \le 0. \tag{3.60}$$

Thus, the other limit for (3.60) must be obtained to prove hyperstability. Substituting (3.55) and (3.56) into (3.60) yields:

$$\eta(0,t) = \int_0^t \sum_{i=0}^{n-1} [\sum_{j=1}^{k_i} - (a_{ij}-\hat{c}_{ij})(x^{(i)j} - x_m^{(i)j}]vd\tau$$

$$+ \int_0^t \sum_{i=0}^{n-1} [\sum_{j=0}^{k_i} - (a_{ij}-\hat{a}_{ij})x_m^{(i)j}]vd\tau + \int_0^t \sum_{i=0}^{\ell_i} -$$

$$(b_{ij}-\hat{b}_{ij})u^{(i)j}]vd\tau \ge -\gamma_0^2 \tag{3.61}$$

By using a sufficiency argument similar to that used in Theorem 2.1 and Theorem 3.1, (3.61) can be written:

$$\int_0^t -(a_{ij}-\hat{c}_{ij})(x^{(i)j}-x_m{}^{(i)j})v\,d\tau \geq -\gamma^2_{\gamma ij},\ i\varepsilon[0,n-1],$$

$$j\varepsilon[1,k_i] \tag{3.62}$$

$$\int_0^t -(a_{ij}-\hat{a}_{ij})x_m{}^{(i)j}v\,d\tau \geq -\gamma^2_{\alpha ij} \quad i\varepsilon[0,n-1],\ j\varepsilon[1,k_i] \tag{3.63}$$

$$\int_0^t -(b_{ij}-\hat{b}_{ij})u^{(i)j}v\,d\tau \geq -\gamma^2_{\beta ij} \quad i\varepsilon[0,m-1],\ j\varepsilon[1,\ell_i] \tag{3.64}$$

Also, following an argument similar to Theorem 2.1 and Theorem 3.1, and defining

$$f_{ij}(t) = -(a_{ij}-\hat{c}_{ij}) \tag{3.65}$$

$$g_{ij}(t) = -(a_{ij}-\hat{a}_{ij}) \tag{3.66}$$

$$h_{ij}(t) = -(b_{ij}-\hat{b}_{ij}) \tag{3.67}$$

the inequalities (3.62)-(3.64) are found to be, respectively,

$$\frac{\gamma_{ij}}{2}\left[\int_0^t (x^{(i)j}-x_m{}^{(i)j})v\,d\tau + \frac{f_{ij}(0)}{\gamma_{ij}}\right]^2 - \frac{f^2{}_{ij}(0)}{2\gamma_{ij}} \geq$$
$$-\frac{f^2{}_{ij}(0)}{2\gamma_{ij}} = -\gamma^2_{\gamma ij} \tag{3.68}$$

$$\frac{\alpha_{ij}}{2}\left[\int_0^t x_m{}^{(i)j}v\,d\tau + \frac{g_{ij}(0)}{\alpha_{ij}}\right]^2 - \frac{g^2{}_{ij}(0)}{2\alpha_{ij}} \geq -\frac{g^2{}_{ij}(0)}{2\alpha_{ij}} =$$

$$-\gamma^2_{\alpha ij} \tag{3.69}$$

$$\frac{\beta_{ij}}{2}\left[\int_0^t u^{(i)j}v\,d\tau + \frac{h_{ij}(0)}{\beta_{ij}}\right]^2 - \frac{h^2{}_{ij}(0)}{2\beta_{ij}} \geq -\frac{h^2{}_{ij}(0)}{2\beta_{ij}} =$$

$$-\gamma^2_{\beta ij} \tag{3.70}$$

which bounds (3.61) by $-\gamma_0^2$ where

$$\gamma_0^2 = \sum_{i=0}^{n-1} \{ \sum_{j=1}^{k_i} [\frac{(\hat{c}_{ij}(0)-a_{ij})^2}{2\gamma_{ij}} + \frac{(\hat{a}_{ij}(0)-a_{ij})^2}{2\alpha_{ij}}]\} + \sum_{i=0}^{m-1} [\sum_{j=1}^{\ell_i} \frac{(\hat{b}_{ij}(0)-b_{ij})^2}{2\beta_{ij}}] \tag{3.71}$$

where the initial conditions

$$f_{ij}(0) = \hat{c}_{ij}(0) - a_{ij} \tag{3.72}$$

$$g_{ij}(0) = \hat{a}_{ij}(0) - a_{ij} \tag{3.73}$$

and

$$h_{ij}(0) = \hat{b}_{ij}(0) - b_{ij} \tag{3.74}$$

have been substituted in (3.68)-(3.70) to obtain (3.71), and hyperstability is proved.

The conditions on the plant (3.51) to insure minimal stability of the error system (3.54)-(3.56) are obtained in the following theorem:

Theorem 3.4

If the plant (3.51) is asymptotically stable, then the error system (3.54)-(3.56) is minimally stable.

Proof

The proof of this theorem parallels that of Theorems 2.2 and 3.2 and consists of finding a $w(t)$ such that

1) $\eta(0,t) \leq 0$ and

2) $\lim_{t\to\infty} e(t) = 0$

for any initial condition $e(0) = e_0$. Let

$$w(t) = \sum_{i=0}^{n-1} [\sum_{j=1}^{k_i} (a_{ij}-\hat{c}_{ij})(x^{(i)j}-x_m^{(i)j})] \tag{3.75}$$

by letting

$$\hat{a}_{ij} = a_{ij} \tag{3.76}$$

$$\hat{b}_{ij} = b_{ij} \tag{3.77}$$

The error equation (3.54) then becomes:

$$e^{(n)} = \sum_{i=0}^{n-1} [\sum_{j=1}^{k_i} a_{ij}(x^{(i)j} - x_m^{(i)j})] \tag{3.78}$$

which has $\lim_{t \to \infty} e(t) = 0$ only if the plant (3.51) is asymptotically stable. Thus, minimal stability is proved.

Thus, two HNREI algorithms have been developed which yield parameter adaptation. This parameter convergence is guaranteed if the plant is asymptotically stable and the input, u, to the plant and model is sufficiently frequency rich. The next section discusses methods of implementing these algorithms and design criteria associated with them.

IV. DESIGN REQUIREMENTS

In the design of nonlinear, parametric identifiers, several design considerations must be made. These considerations can be roughly grouped into two catagories: 1) what type of identifier is to be chosen, and 2) specific nonlinearity considerations. The answer to the first depends on the a priori knowledge of the nonlinear system and the maount of noise in the system. If the functional form of the nonlinearities is known, then a linear, time-invariant identifier might be advised. Similarly, if the nonlinearity forms are unknown and only small (<1%) amounts of noise exist, then the series parallel (equation error) MRAS approach might be best. The criteria for choosing between identifiers is largely a function of experience. However, with unknown structure nonlinearities in the presence of noise, parallel MRAS yields best results.

Fig. IV-1 shows a general block diagram of the nonlinear response error parametric identification algorithm. The algorithm may be broken into two main loops. The inner loop consists of state variable filters (SVF) adjustable model, dynamic compensator (N(s)), and the parameter adaptation. This inner loop presents several considerations for the designer:

1) SVF design
2) Parameter gains
3) PRF and the dynamic compensator design

The SVF's function is to provide "pseudo-derivatives" for use in developing the response error. If $p = \frac{d}{dt}$ then L(p) denotes the SVF and (1) becomes

$$L(p^n x^P) + \dots + L[F_i(p^j x_p) + \dots + L[f_o(p^k x_p)x_p] =$$

$$L[g_o(p^\ell u_p)u_p] + \dots + L[g_m(p^w u_p)u_p] \qquad (4.1)$$

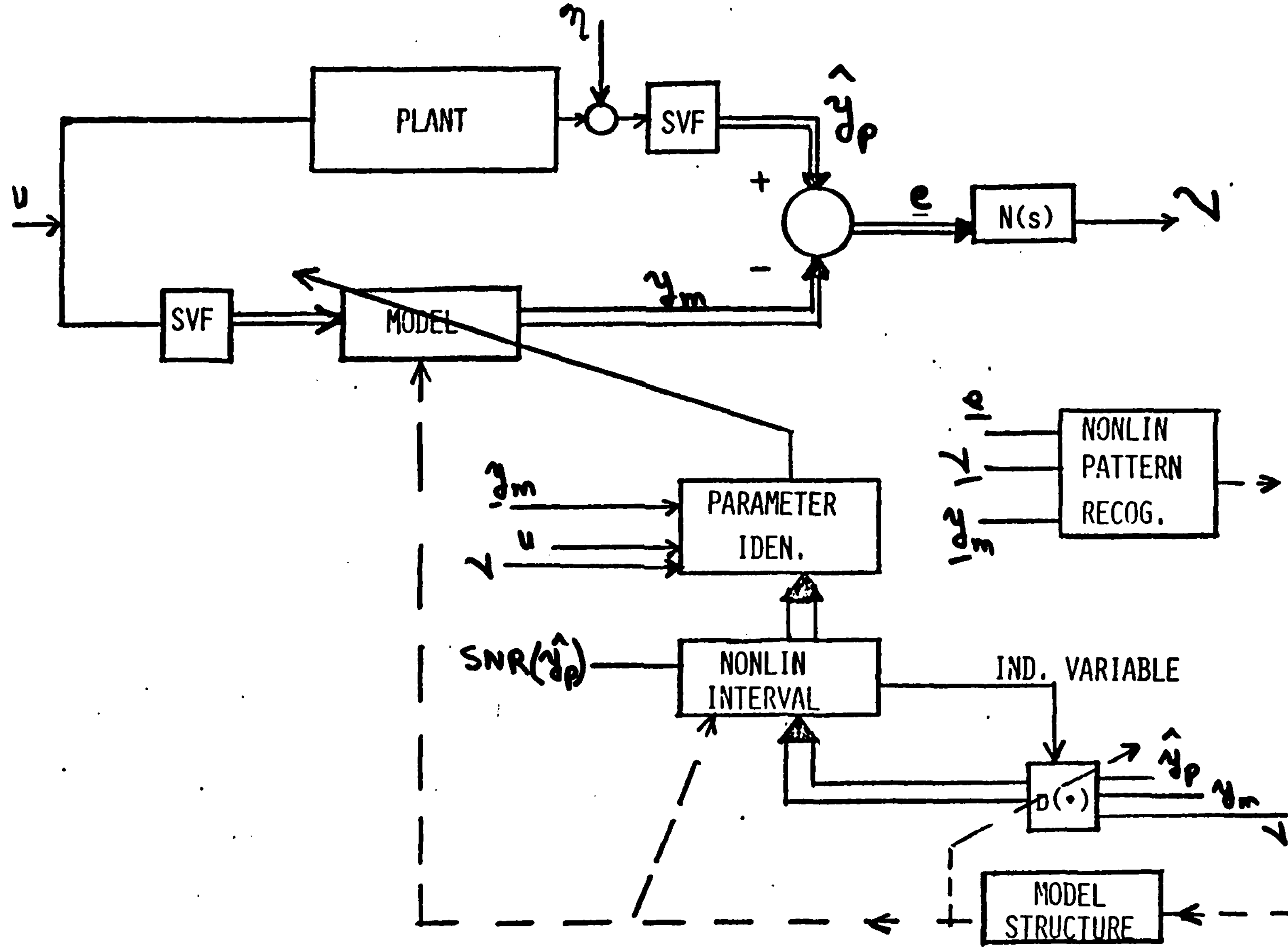

PLANT
SVF
N(s)
SVF
MODEL
PARAMETER IDEN.
NONLIN PATTERN RECOG.
NONLIN INTERVAL
IND. VARIABLE
MODEL STRUCTURE

Figure IV-1

where x_p and u_p are the plant output and input respectively. Sprague and Kohr [7] have shown that if $L(\cdot)$ is a transport operator then $L(\cdot)$ commutes with the nonlinearity to give

$$L(p^n x_p) + \ldots + [F_i\ L(p^j x_p) + \ldots + f_o[L(p^k x_p)]\ x_p =$$

$$g_o[L(p^\ell u_p)]u_p + \ldots + g_m[L(p^w u_p)]u_p \qquad (4.2)$$

allowing the SVF to be moved from the output of the model to its input. This gives compatability between the model states and the corresponding plant "pseudo-states".

Parameter gains should be chosen in such a manner as to give rapid convergence and decrease asymptotically to zero to prevent noise bias. Several algorithms exist [11,12] for the decreasing adaptive gains.

The PRF requirement was addressed earlier. If a priori knowledge of the range of plant parameters exists then a fixed c_i compensator can be developed, saving much computation time and implementation hardware. However, in general, the nonlinear adaptive compensator must be used to obtain the PRF requirement.

The outer loop of the nonlinear identification algorithm consists of the pattern recognition, model structure, and nonlinear interval blocks. Important design consideration occurring in the outer loop are:

1) Pattern recognition block

2) Model structure block
   a) Model order (numerator, denominator degree)
   b) Nonlinearity location
   c) State information

3) Nonlinearity interval block
   a) Interval sizing
   b) Number of intervals

4) Identification time frame

The pattern recognition block must determine if a "possible" nonlinearity has been identified. This block decides when a nonlinearity described by a set of points is Class C^i, $i \geq o$, to qualify as an allowable nonlinearity, Fig. IV-2. Define:

$$\begin{aligned} \Delta^1 f_1 &= p_2 - p_1 \\ \Delta^1 f_2 &= p_3 - p_2 \\ &\vdots \\ \Delta^1 f_{n-1} &= p_m - p_{n-1} \end{aligned} \tag{4.3}$$

the "first difference" fo the set of points.

Then
$$\Delta^2 f_i = \Delta^1 f_{i+1} - \Delta^1 f_i \tag{4.4}$$

and
$$\Delta^i f_j = \Delta^{i-1} f_{j+1} - \Delta^{i-1} f_j \tag{4.5}$$

represent higher differences of the set of points. Now the performance criteria:

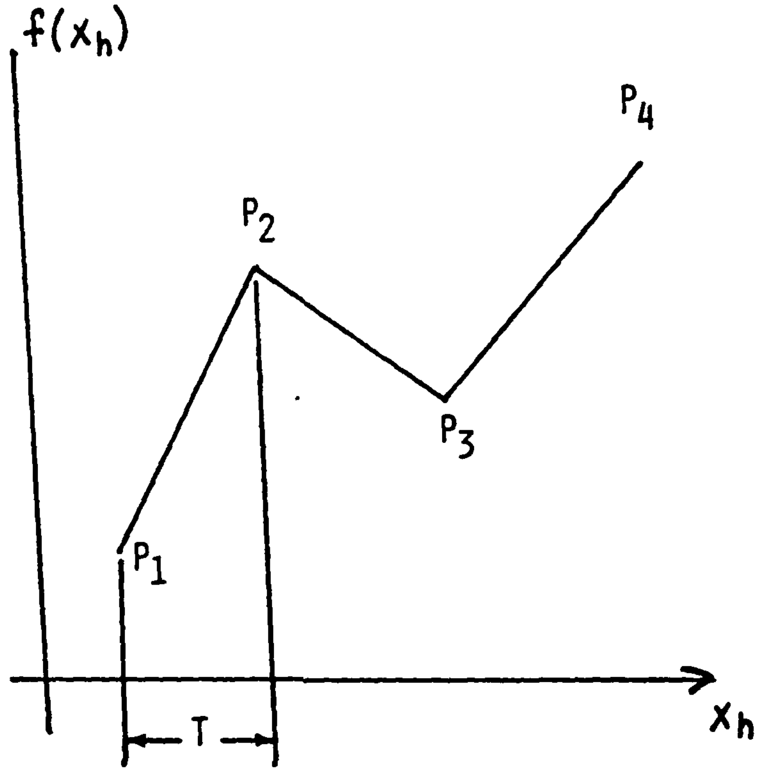
$$J = \frac{1}{N} \sum_{j=1}^{N} \frac{\Delta^i f_{\ell j}}{T_\ell^{\,i}} < \varepsilon \quad \varepsilon \text{ is greater than zero} \tag{4.6}$$

defines the class of functions C^{i-1} where only the $\Delta^i f_{\ell j}$ that differ in sign from the previous, $\Delta^i f_{\ell j-1}$, are used to define the class. N is the number of sign changes, and $T_\ell^{\,i}$ is the width of the interval between the points.

The results of the pattern recognition block feed into the model structure block. In this block almost all of the decisions concerning the model structure and nonlinearity placement are made. This block is nearly always "filled" by the designer himself.

The first piece of information necessary from the model structure block is the model order and preliminary structure. Several model order algorithms exist [13,14,15] to provide possible model structures and information on the number of poles and zeros for the model and any time de-

PATTERN RECOGNITION



$$\triangleq \frac{1}{N} \sum_{J=L}^{N} \left| \frac{\Delta^{(I)} P_{EJ}}{T_J^I} \right| < \epsilon$$

FOR CLASS C^{I-1}

Figure IV-2.

lays.

The placing of any nonlinearities in the afore found model structure is largely a matter of trial and error or intuition as to what states will be affected by the nonlinearity. By minimizing J in (4.6) of the nonlinear pattern recognition block with respect to i an ε can be picked which fixes the class of functions allowable for that particular model state. If the expected plant nonlinearity was not of this class then another model state might be chosen in which to locate a model nonlinearity.

Since both plant and model state information and used to identify possible nonlinearities, it is important that the proper state information is obtained to get the correct answer. A decision must be made whether to use the noisy plant states, themodel states, or a combination of the two. The problem is illustrated in Fig. (IV-3). As can be seen, at t_1 the actual plant state (unmeasurable in general) is in interval 2 while the model state is in interval 3 and the noisy plant state is in interval 4 of the intervalized state space. This problem can be posed as an optimal communications problem where a decision function:

$$\underline{Q} = \underline{f}\ (\hat{\underline{x}}_p) + \underline{g}\ (\underline{x}_m) \tag{4.7}$$

is defined and the functions $\underline{f}$ and $\underline{g}$ are chosen to minimize the probability of selecting the incorrect interval. Q, then, is the independent variable used to determine the interval number of the model's state space.

The nonlinearity interval block decides how the model's state spaces will be broken up in order to identify the nonlinearities (if any) existing in the corresponding plant's state spaces. This problem can be broken into two interrelated parts: 1) interval size and 2) number of intervals. Statistical measurements can be made for a particular input to determine how the Q's of (4.7) range. The interval size can then be cho-

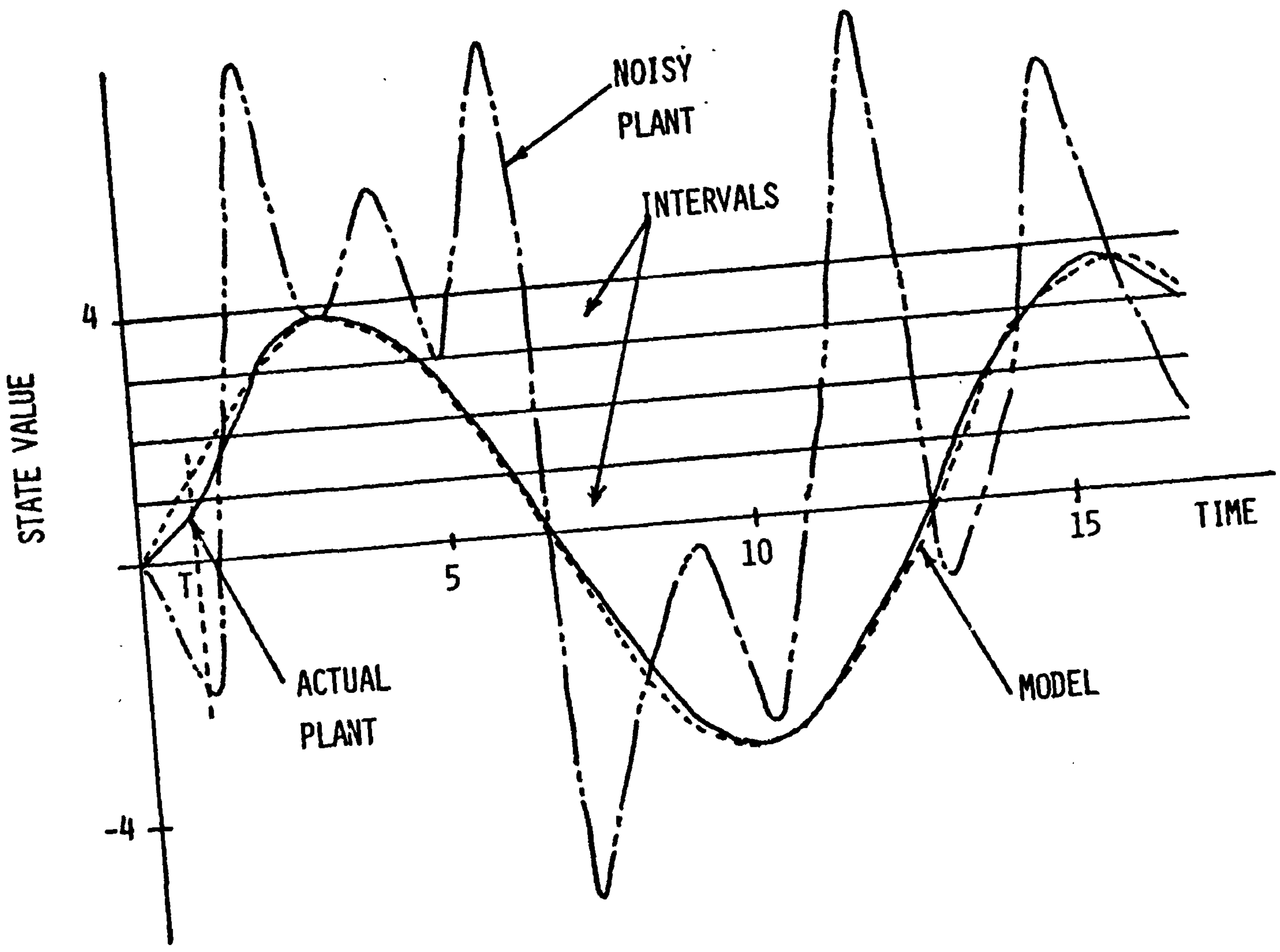


Figure IV-3. Interval Select Problem.

sen to allow for uniform (if any) convergence of the parameters.

The number of intervals presents other problems. As the number of intervals increase, the amount of state space in these intervals decrease. As teh SNR of the independent variables decreases, the probability of an internal misselect increases. Let

$y_{u=o}$ = output with noise present and no input

$y_{u\neq o}$ = output with noise present and nonzero input driving function

$$SNR = \frac{\sum_{i=1}^{N} y_{u=o}(it)^2}{\sum_{i=1}^{N} y_{u=o}(it)^2} - 1 \tag{4.8}$$

As the SNR decreases, the number of intervals the nonlinearity can be broken into and still get an "allowable" (in the sense of (4.6)) fit decreases, (Number of Intervals) $\propto$ SNR (independent variable), where the proportionality constant is designer-selected. As offline test for the proper number of intervals is given using the Theil criterion [16].

$$K = \frac{\sqrt{\sum_{i=1}^{N} \frac{1}{N} e\ (it)^2}}{\sqrt{\sum_{i=1}^{N} \frac{1}{N} x_p\ (it)^2} + \sqrt{\sum_{i=1}^{N} \frac{1}{N} x_m\ (it)^2}} \tag{4.9}$$

As shown in Fig. (IV-4), for a particular example, an optimum number of intervals was found.

The identification time frame is also designer-selected. In general, the longer the time for identification, the more accurate the final answer. However, in practice the amount of time must be weighed against the speed of parameter adaptation and measurement noise levels in order to give an optimum adaptation time.

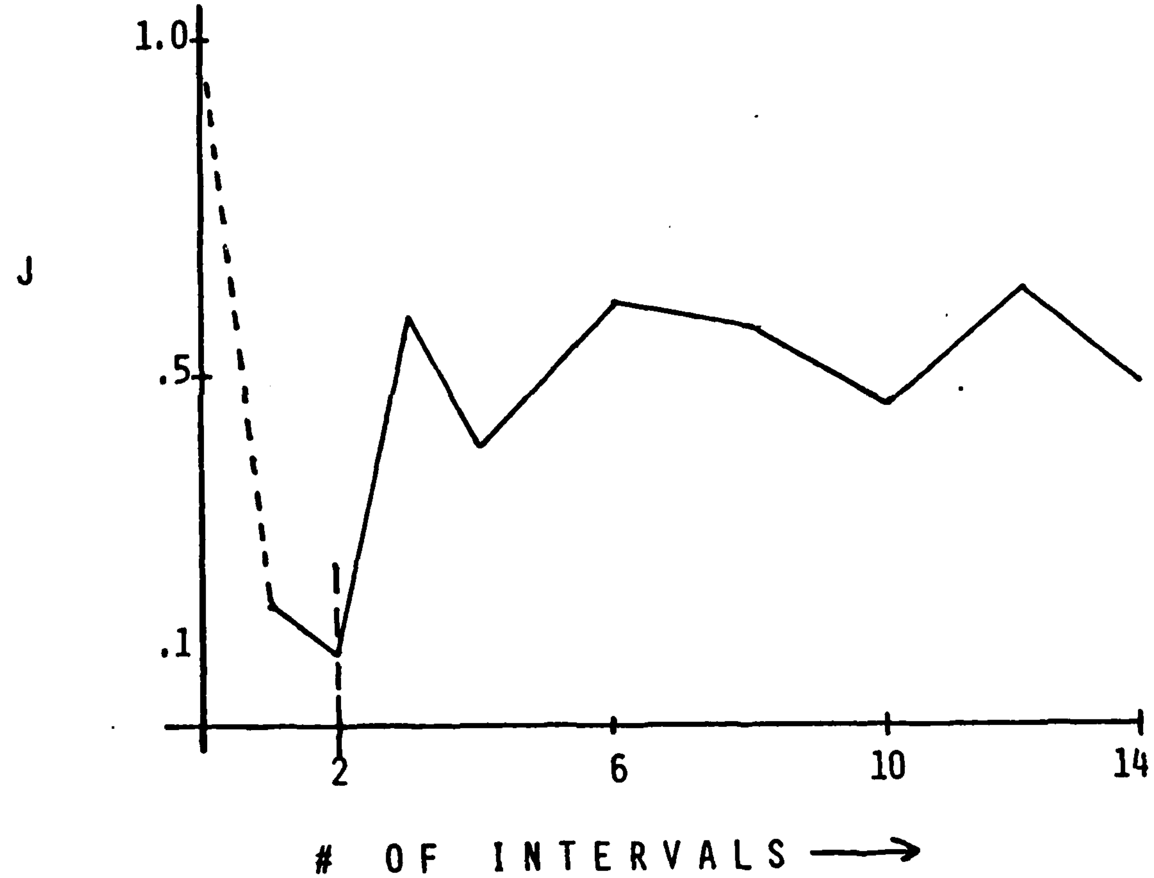


Figure IV-4. Cost Functional J VS. Number of Intervals.

V. RESULTS

The response error MRAS approach was tested on numerous models, since no human pilot operating records were available from FDL at Wright-Patterson AFB, Ohio. The examples illustrate the good parameter tracking possible. It should be emphasized that since no optimal control pilot model formulation was explicitly a part of this study, but instead only the feasibility of MRAS identification for plants with correlated noise, results and their interpretation should be limited to the scope of MRAS nonlinear function and linear parameter estimation.

Example 1 (No Noise)

Plant: $\ddot{x}_p + f(\dot{x})\dot{x} + x = u$

where $f(\dot{x})\dot{x}$ is the saturation function shown in Figure V-1

Model: $\ddot{x}_m + a_{11q}\dot{x}_m + x_m = b_o u$

b_o = constant

a_{11q} = supposed intervalized nonlinearity

Using the interval select nonlinear MRAS approach, with c_i's constant. With SVF poles at -20 and -30, the results are shown in Figure V-1, with noisy measurements

$$x_{avail} = x_p + \eta$$

$$E\{\eta\} = 0$$

$$\sigma = \sqrt{E\{\eta^2\}} = .05x_p$$

Example 2 (Noise)

Plant:

$$\ddot{x}_p = -f(\dot{x}_p) - x_p + u$$

$$y(\text{output}) = x_p + \eta_x$$

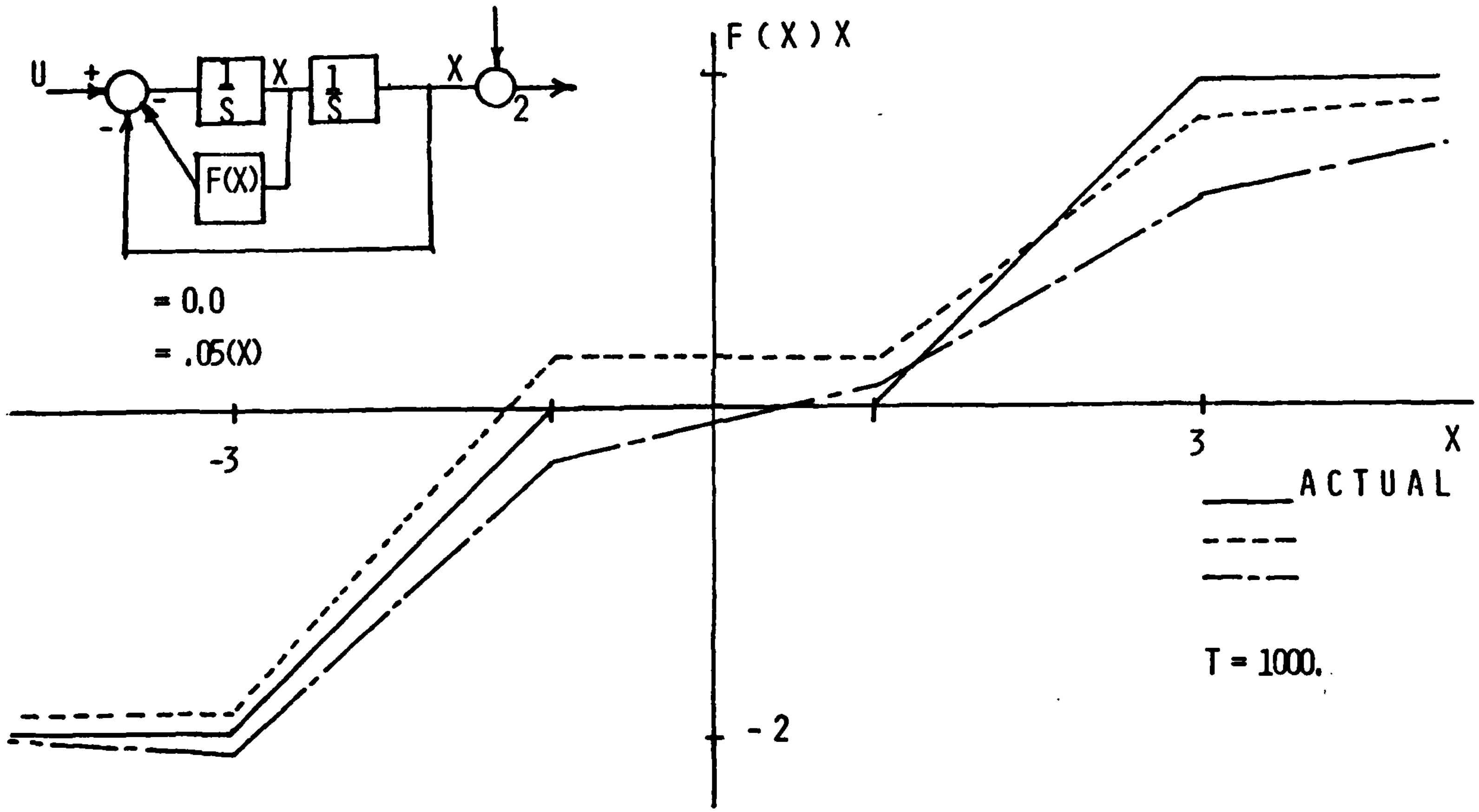


Figure V-1. Response Error Identification.

The available (measured) u is $u' = u + n_u$

$$u = \sum_{i=1}^{\infty} A_i \sin w_i t$$

$$w_i = \{.2, .4, .6, .8, 1, 1.2, 1.4, 1.6\}$$

First, a linear model will be fit to the system and then a nonlinear plant, to compare the improvement in tracking accuracy of the nonlinear version over that of the linear version. The linear version is

$$G_m(s) = \frac{y_m}{u'}(s) = \frac{b_{11}}{s^2 + a_{21}s + a_{11}}$$

and the nonlinear version

$$\ddot{x}_m = u' - f_1(\dot{x}_m) - f_2(x_m)$$

$$y_m = f_3(\dot{x}_m) + f_4(x_m)$$

Each case used $\hat{c}_i$ values varying. The nonlinear case used 5 intervals for each nonlinearity.

The results for the linear case are shown in Figures V-2 through V-13. The nonlinear case is shown in Figures V-14 through V-16.

Results show that the linear model has a poor fit to the (actual) nonlinear plant, because of the forced fit, or lack of degrees of freedom, of the linear model. The nonlinear results, on the other hand, show good model-plant correlation, as can be seen in Figure V-16(b), where a phase plane of y_p vs y_m is plotted for the test input. If the model were a perfect replication, there would be a single line at 45°; only small variations about this line occur.

A complete computer program listing is given in the Appendix of the identifier system for practical nonlinear identification by the interval approach. A flowchart is shown in Figure V-17.

```
N, NS, KOUNT, NI
  2       2       10      100
STATE VARIABLE FILTERS, SVF(I)
 5 000E+02   5 000E+01
T, H, WRITE
 0 000E-01   5 000E-03   2 000E+00
DEGREE OF POWER SERIES EXPANSIONS, K(I)
  1      1      1      0
STATE VECTOR, X(I)
 0 000E-01   0 000E-01   0 000E-01   0 000E-01   0 000E-01   0 000E-01   0 000E-01   0 000E-01   0 000E-01   1 000E+00
1 000E+00   0 000E-01   0 000E-01
GAIN MATRIX, GAIN(I,J)
 1 000E-01
 1 000E-01
 1 000E-01
 1 000E-01
 1 000E-01
*******************PLANT PARAMETERS**************
PLANT ORDER, NUMBER OF INPUT FREQUENCIES
  2       8
AMP(I)          RADIAN FREQUENCY(I)
 1 000E+00       2 000E-01
1 000E+00       4 000E-01
1 000E+00       6 000E-01
1 000E+00       8 000E-01
1 000E+00       1 000E+00
1 000E+00       1 200E+00
1 000E+00       1 400E+00
1 000E+00       1 600E+00
*******************PLANT PARAMETERS**************
***************************NOISE PARAMETERS***************************
INPUT NOISE STANDARD DEVIATION AND CORRELATION, [0 - UNCORRELATED, 1 - CORRELATED WITH INPUT OR OUTPUT]
STANDARD DEVIATION = 5 0000%     CORRELATION =1
OUTPUT NOISE STANDARD DEVIATION AND CORRELATION
STANDARD DEVIATION = 5 0000%     CORRELATION=1
***************************NOISE PARAMETERS***************************
```

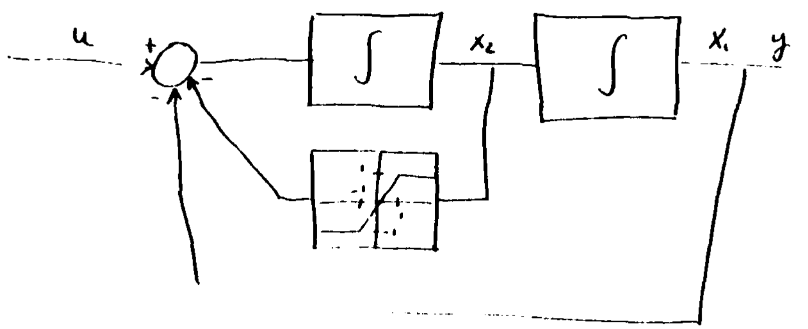


Figure V-2.

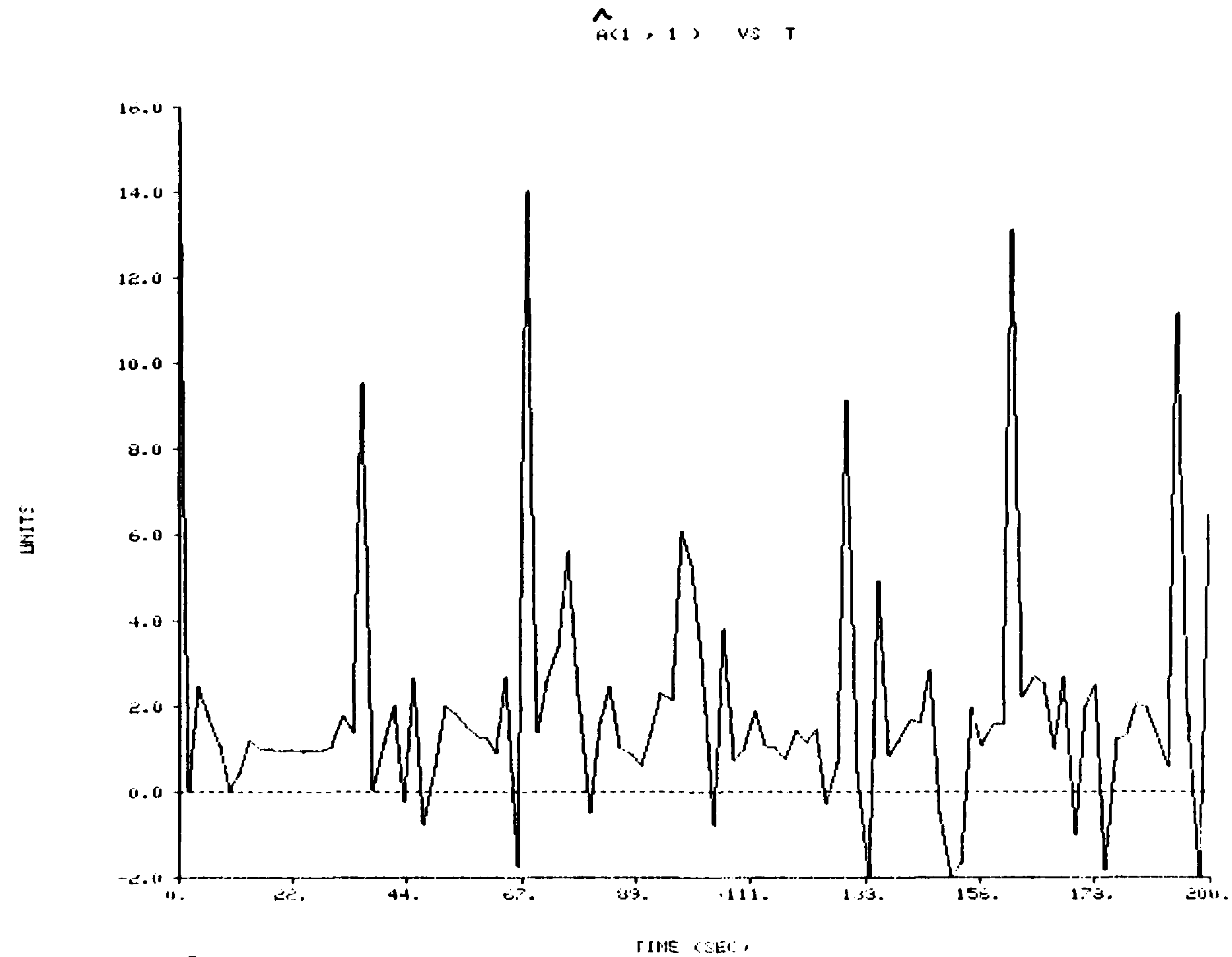

$\hat{A}(1,1)$ VS T
UNITS
16.0
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0.0
-2.0
0.
22.
44.
67.
89.
111.
133.
156.
178.
200.
TIME (SEC)

Figure V-3.

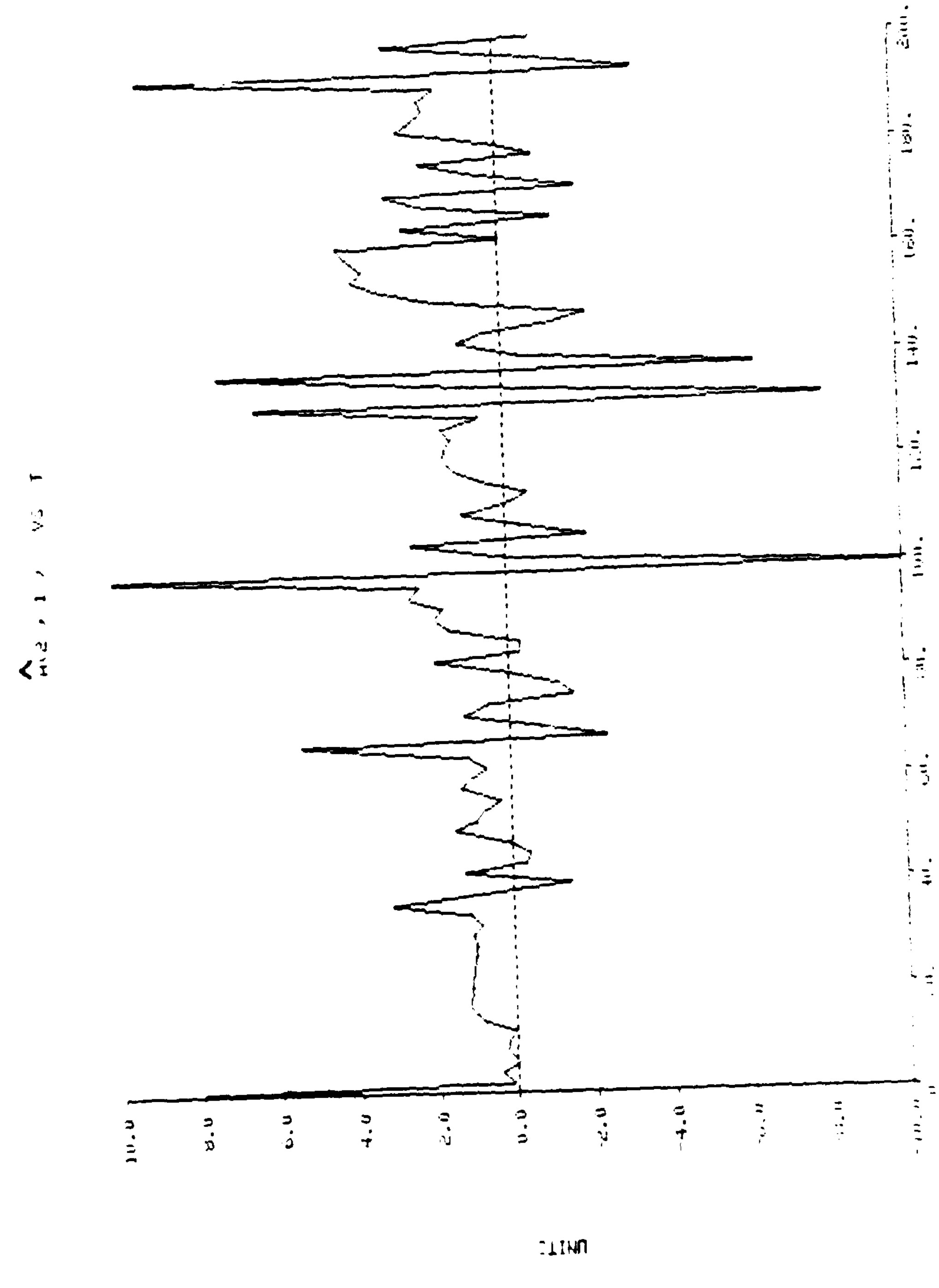


Figure V-4.

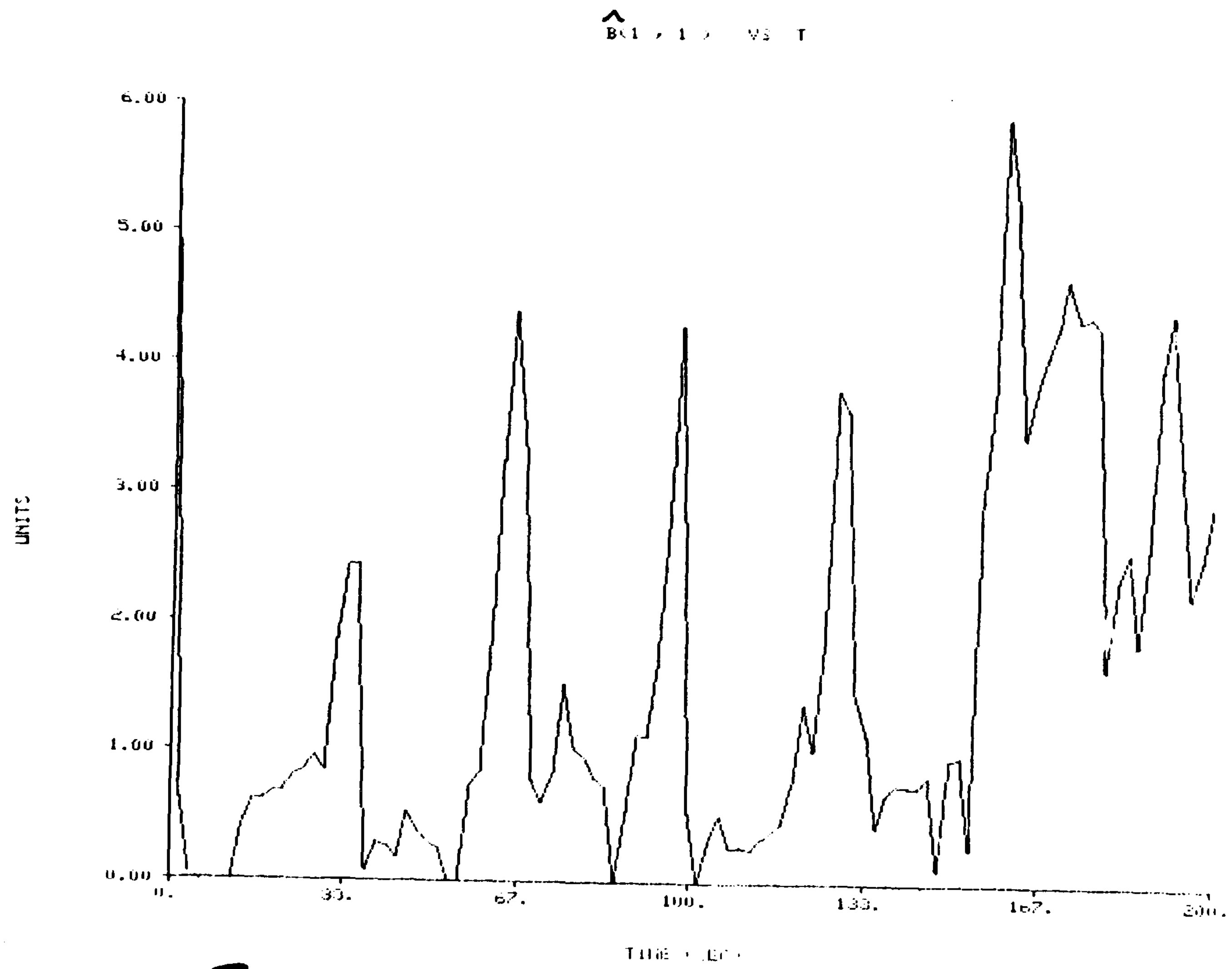


Figure V-5.

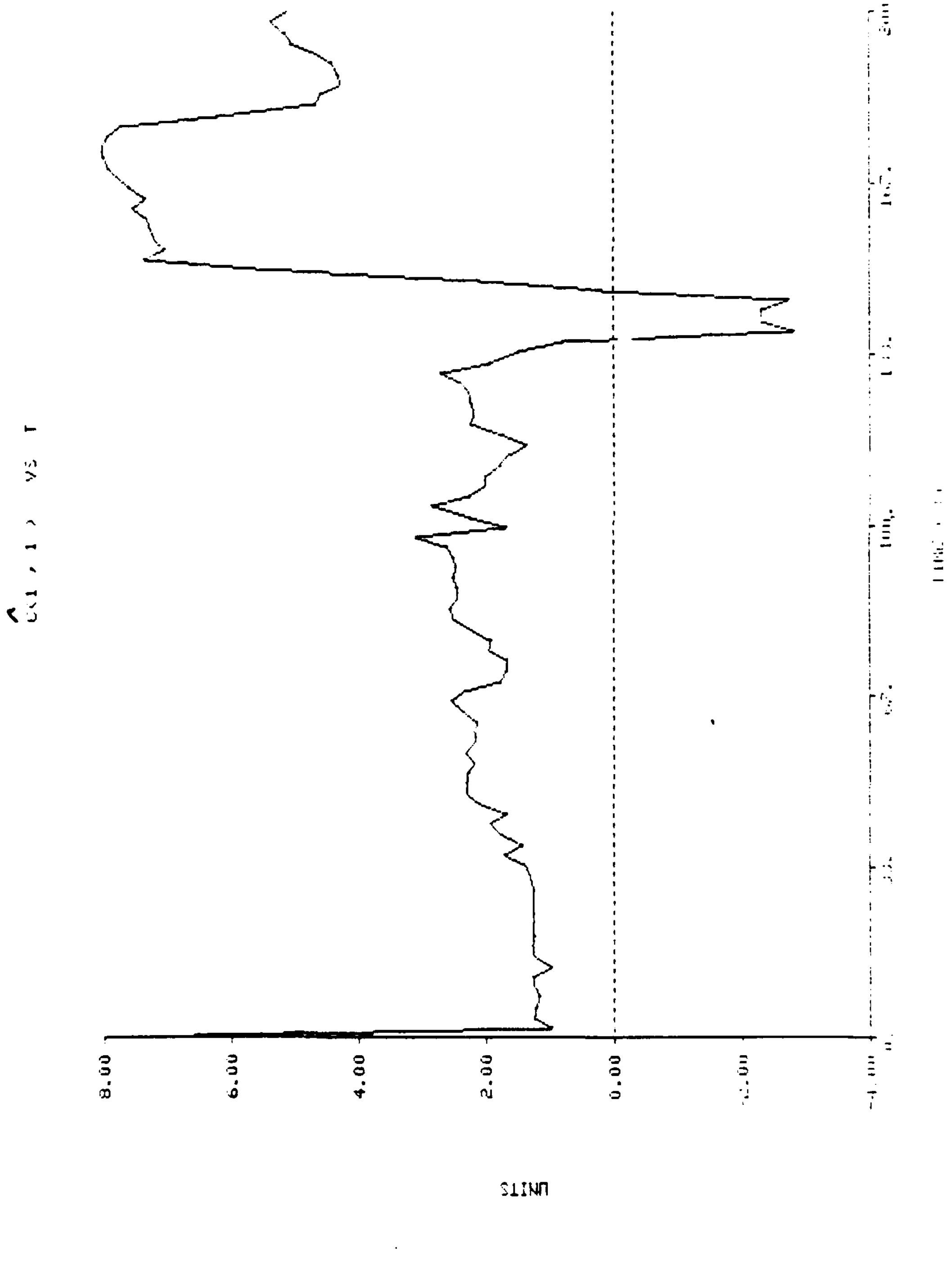


Figure V-6.

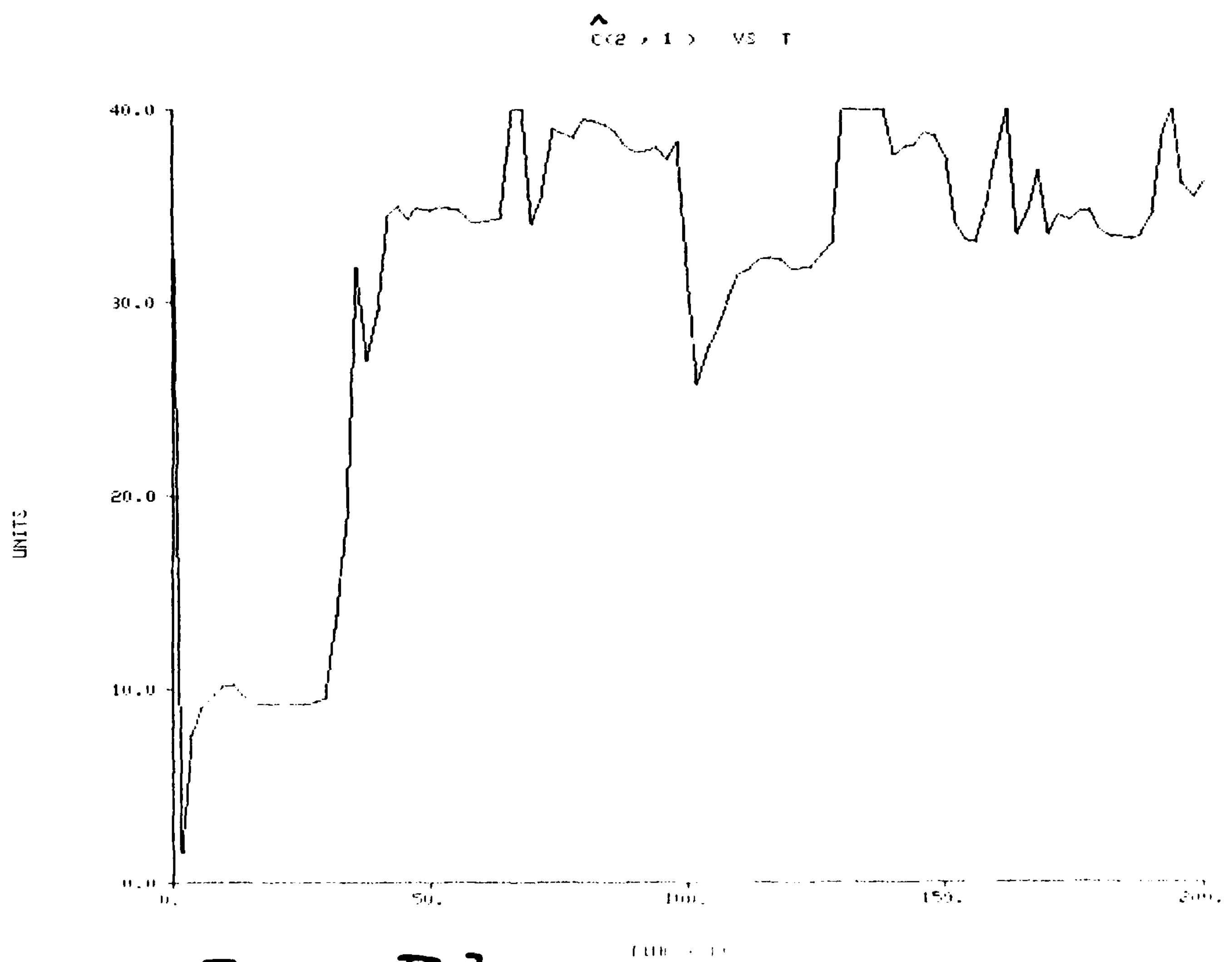


Figure V-7.

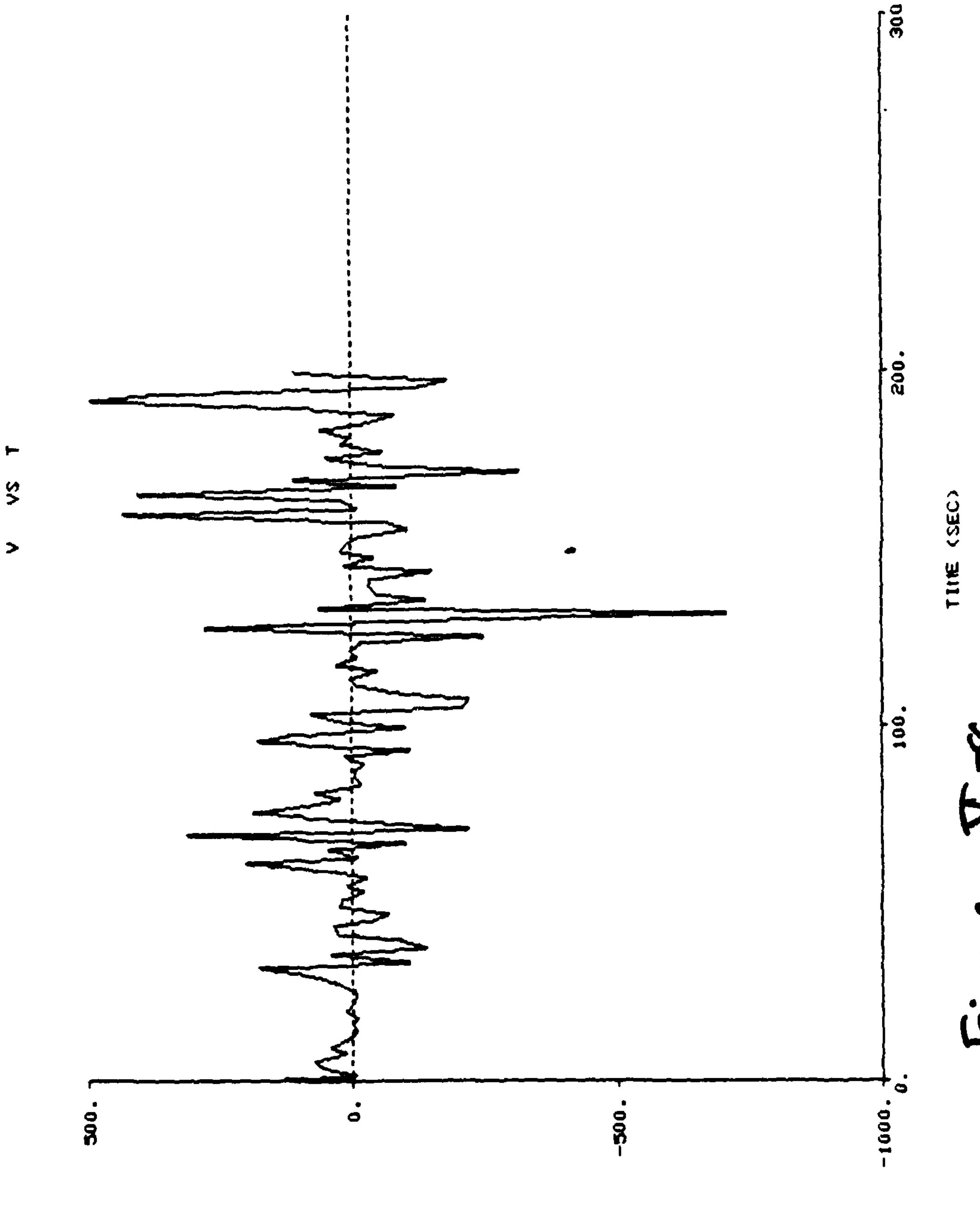


Figure V-8.

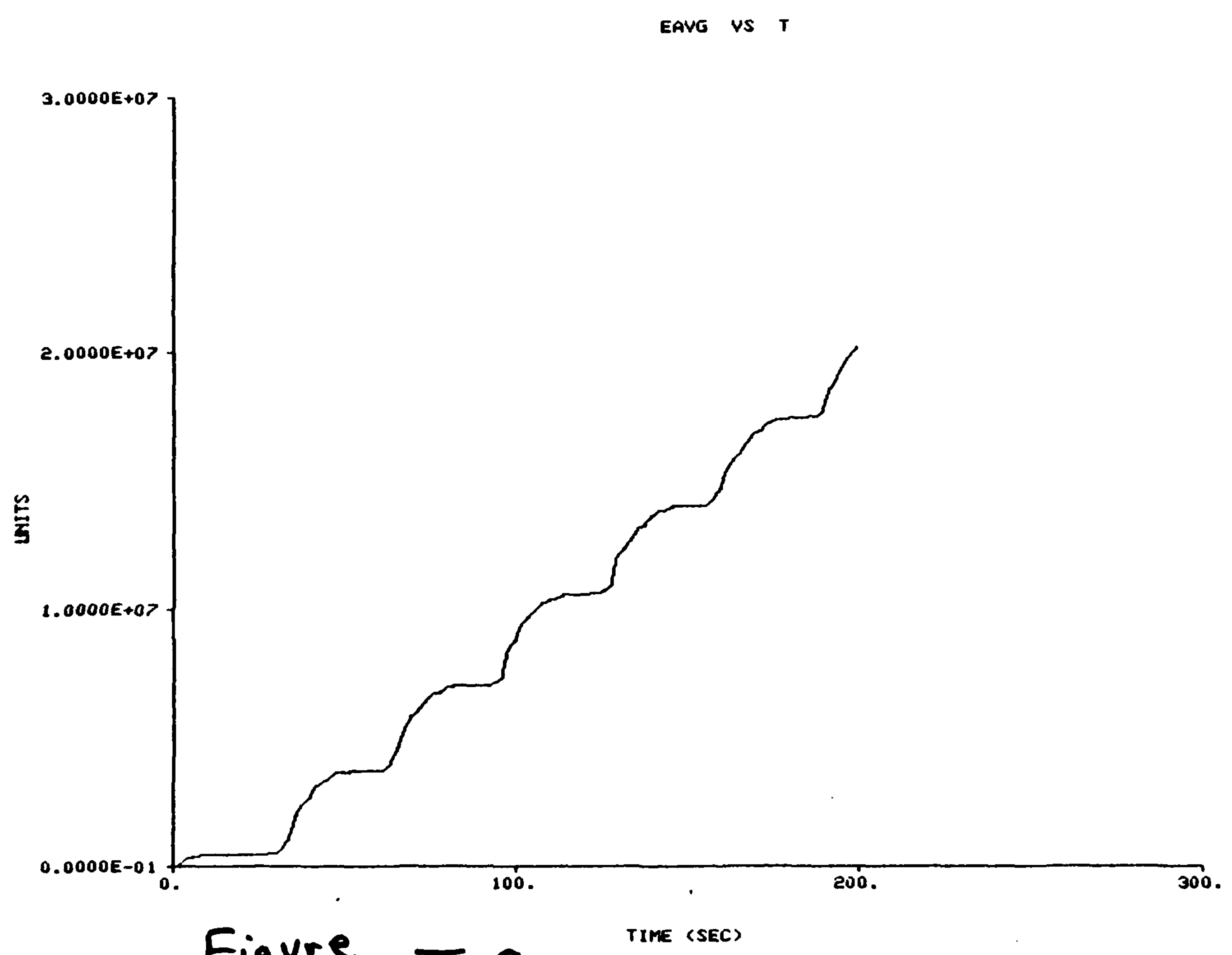

EAVG VS T
3.0000E+07
2.0000E+07
1.0000E+07
0.0000E-01
UNITS
0.
100.
200.
300.
TIME (SEC)

Figure Ⅴ-9.

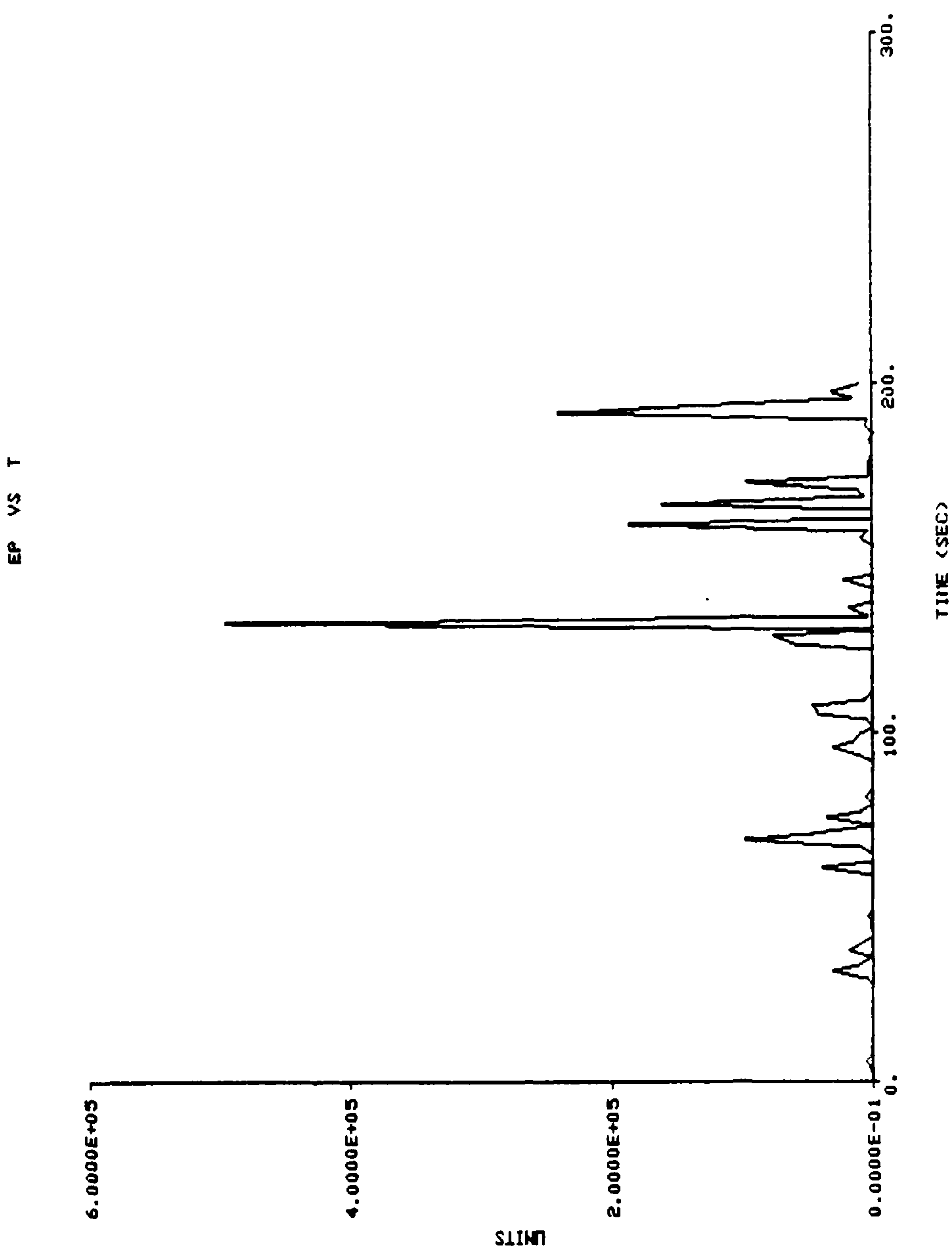

EP VS T
UNITS
6.0000E+05
4.0000E+05
2.0000E+05
0.0000E-01
0.
100.
200.
300.
TIME (SEC)

Figure V-10

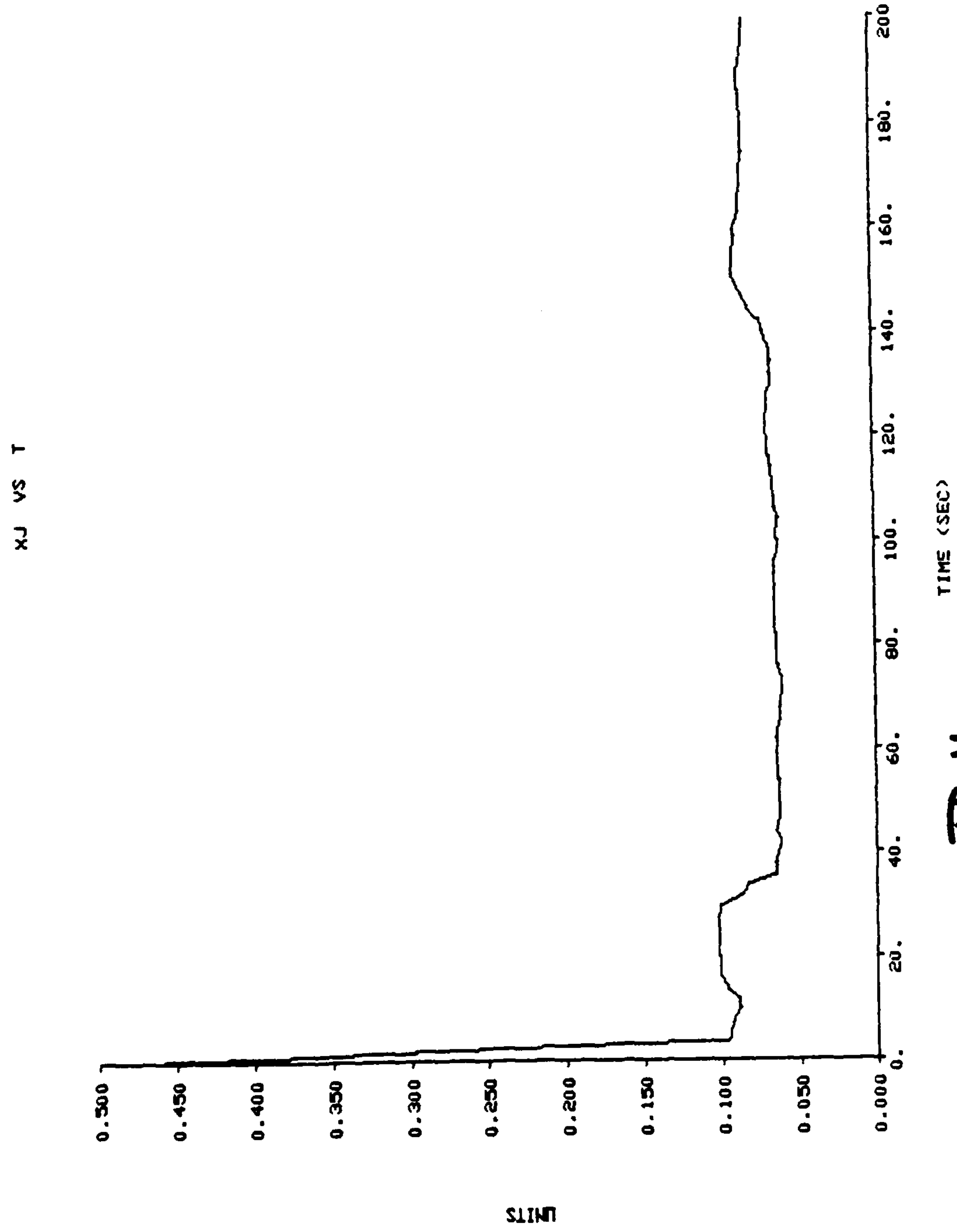

XJ VS T
UNITS
0.500
0.450
0.400
0.350
0.300
0.250
0.200
0.150
0.100
0.050
0.000
0.
20.
40.
60.
80.
100.
120.
140.
160.
180.
200.
TIME (SEC)

Figure V-11

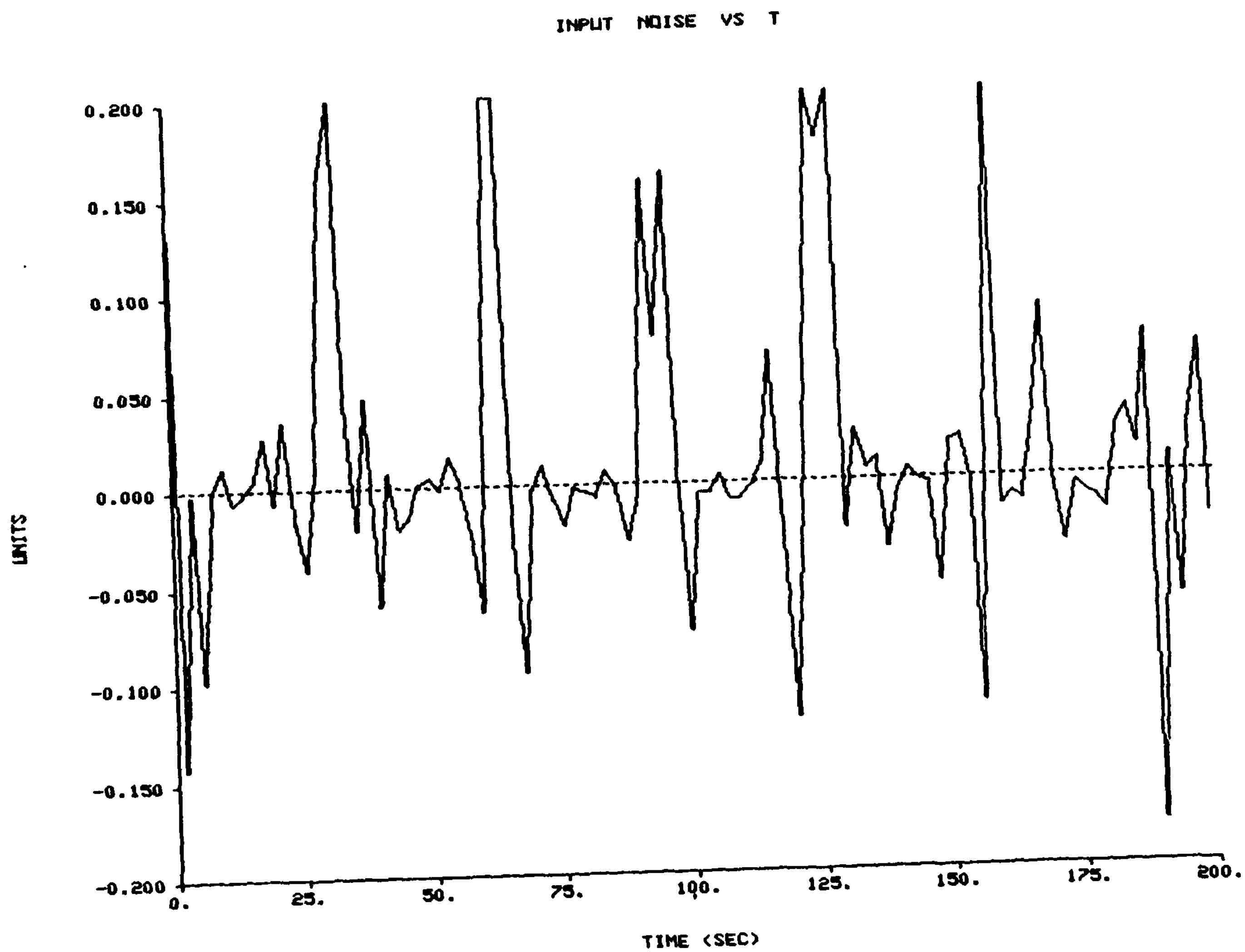

INPUT NOISE VS T
0.200
0.150
0.100
0.050
0.000
-0.050
-0.100
-0.150
-0.200
UNITS
0.
25.
50.
75.
100.
125.
150.
175.
200.
TIME (SEC)

Figure V-12.

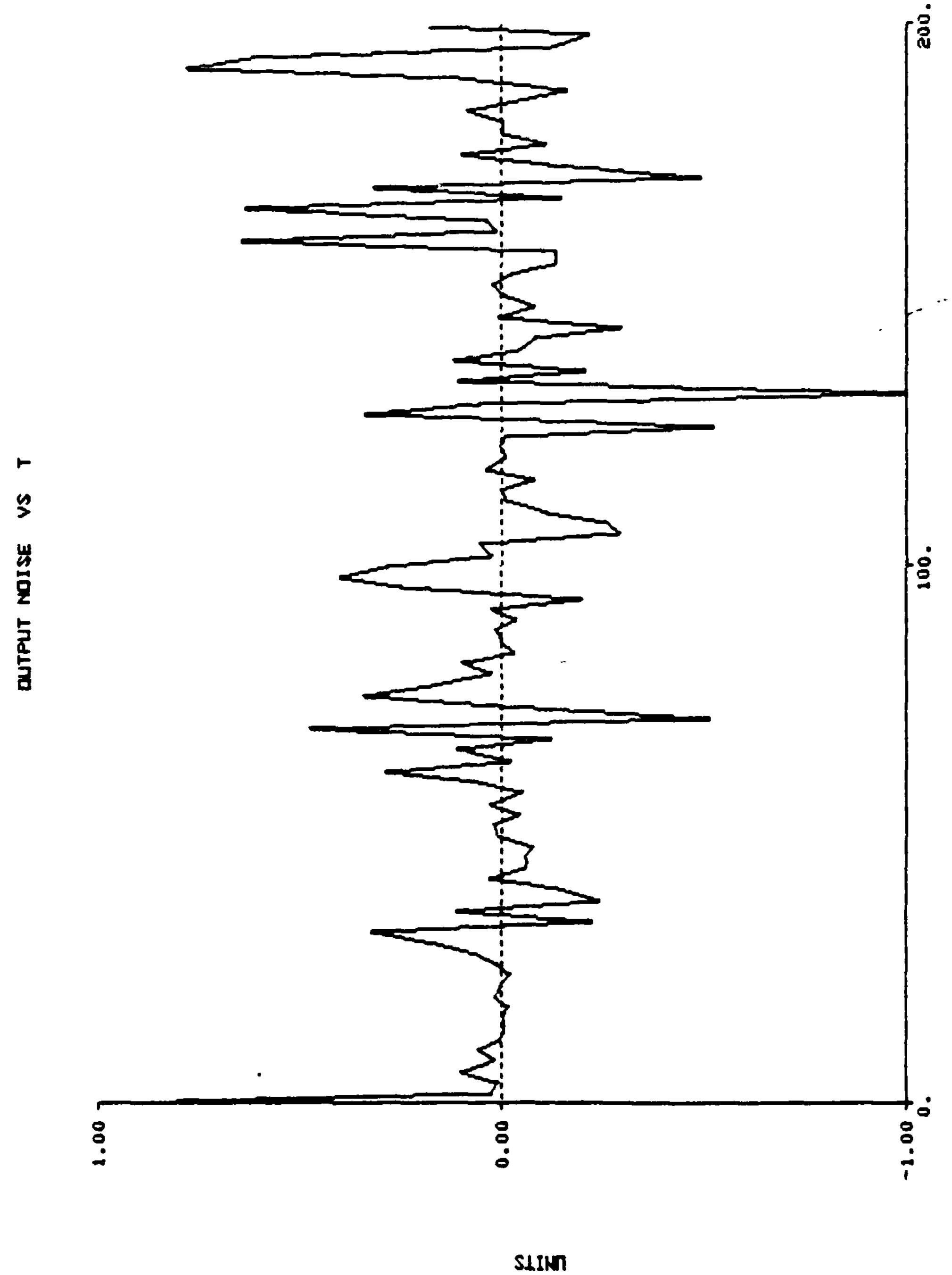


Figure V-13.

```
  2       2       10       1       100       0
  0
COUNTER II =      0
INITIAL STATE VECTOR
 0 000E-01   0.000E-01   0 000E-01   0.000E-01   0.000E-01   0.000E-01   0.000E-01   0.000E-01   0.000E-01   0.000E-01
0 000E-01   0.000E-01   0.000E-01   0.000E-01   0.000E-01   0.000E-01
# INTERVALS  TOP LIMIT      BOTTOM LIMIT
       5       5.000E+00     -5 000E+00
       5       5.000E+00     -5.000E+00
       5       5.000E+00     -5.000E+00
       5       5.000E+00     -5 000E+00
       5       5.000E+00     -5.000E+00
       5       5.000E+00     -5 000E+00
INITIAL PARAMETER VALUES
 0.000E-01   0.000E-01   0.000E-01   0.000E-01   0.000E-01
 0 000E-01   0.000E-01   0.000E-01   0.000E-01   0.000E-01
 0.000E-01   C.000E-01   0.000E-01   0.000E-01   0.000E-01
 0.000E-01   0.000E-01   0.000E-01   0.000E-01   0.000E-01
 1.000E+00   1.000E+00   1.000E+00   1.000E+00   1.000E+00
 1.000E+00   1.000E+00   1.000E+00   1.000E+00   1.000E+00
INITIAL GAIN VALUES
 1.000E-01   1.000E-01   1.000E-01   1.000E-01   1.000E-01
 1 000E-01   1.000E-01   1.000E-01   1.000E-01   1.000E-01
 1 000E-01   1.000E-01   1.000E-01   1.000E-01   1.000E-01
 1.000E-01   1.000E-01   1.000E-01   1.000E-01   1.000E-01
 1 000E-01   1.000E-01   1.000E-01   1.000E-01   1.000E-01
 1.000E-01   1.000E-01   1.000E-01   1.000E-01   1.000E-01
GAIN ADAPTATION VALUES
 1 000E-05   1.000E-05   1.000E-05   1.000E-05   1.000E-05   1.000E-05
STATE VARIABLE FILTER PARAMETERS
 6.000E+02   5.000E+01
T, H, WRITE, ESUM, YPJ, YMJ, EAVG
 0.000E-01   5.000E-02   1.000E+01   0.000E-01   0.000E-01   0.000E-01   0.000E-01
********PLANT PARAMETERS********
PLANT ORDER, # OF INPUT FREQUENCIES
 4       8
AMPLITUDE          RADIAN FREQUENCY OF INPUT SIGNAL
 1.000E+00          2.000E-01
 1.000E+00          4.000E-01
 1.000E+00          8.000E-01
 1.000E+00          1.600E+00
 1.000E+00          3.200E+00
 1.000E+00          6.400E+00
 1.000E+00          1.280E+01
 1 000E+00          2.000E+00
********PLANT PARAMETERS********
*****************************NOISE PARAMETERS*****************************
INPUT NOISE STANDARD DEVIATION AND CORRELATION, [0 - UNCORRELATED , 1 - CORRELATED WITH INPUT OR OUTPUT]
STANDARD DEVIATION =  5.0000%     CORRELATION =1
OUTPUT NOISE STANDARD DEVIATION AND CORRELATION
STANDARD DEVIATION =  5.0000%     CORRELATION =1
CIRCULATING PLANT CORRELATED NOISE
STANDARD DEVIATION =  5 0000%     STATE CORRELATED =1
*****************************NOISE PARAMETERS*****************************
KFLAG =  0     [0 - IDENT & TEST ,1 - TEST ONLY]     TSTOP =   0.000
ISTART =0     [0 - START , 1 - RESTART]
```

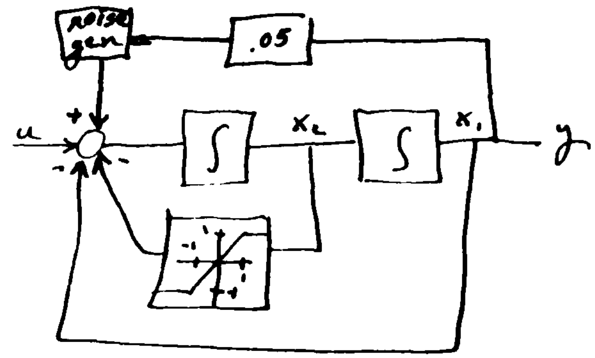


Figure V-14.

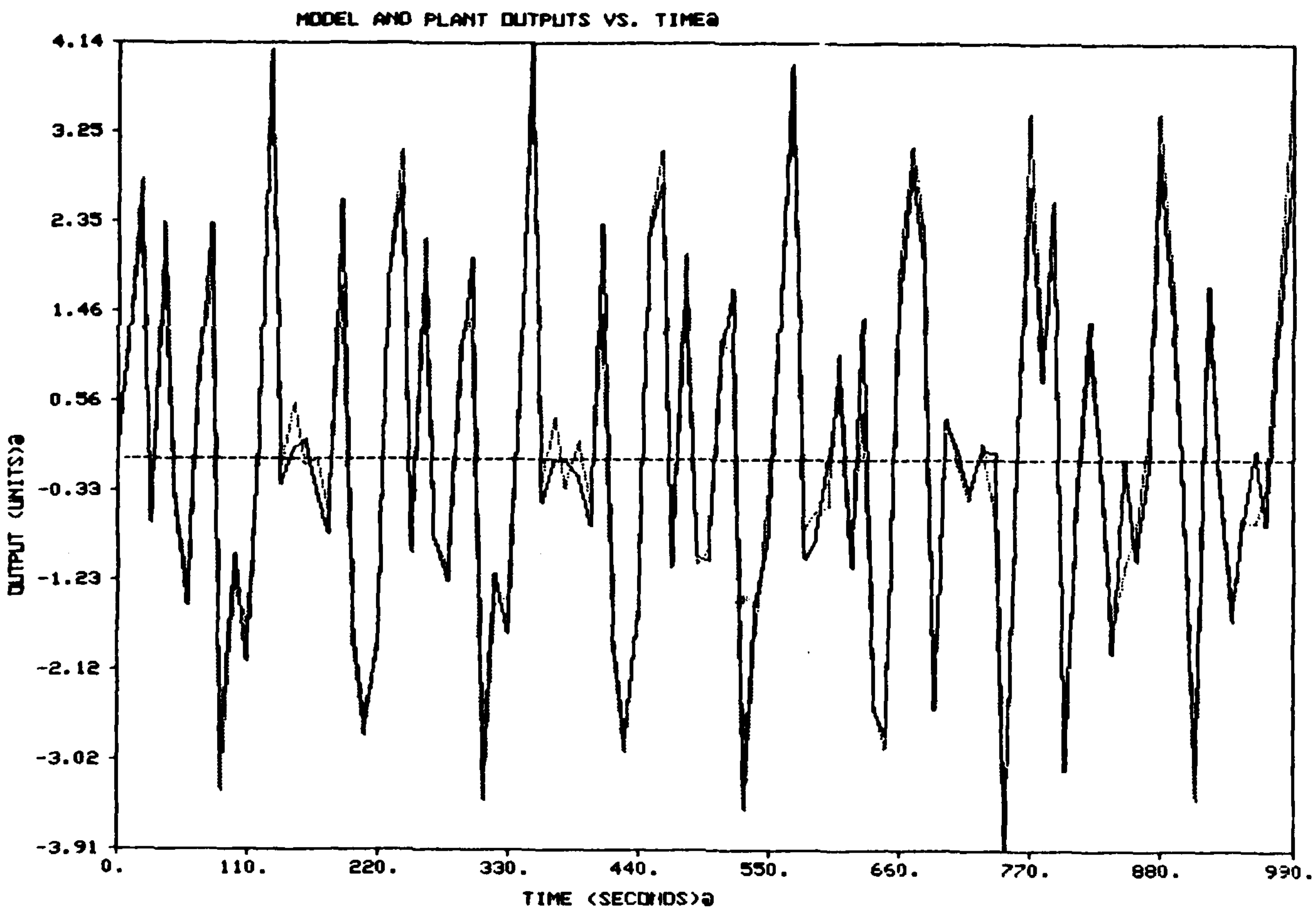

MODEL AND PLANT OUTPUTS VS. TIME@
OUTPUT (UNITS)@
4.14
3.25
2.35
1.46
0.56
-0.33
-1.23
-2.12
-3.02
-3.91
0.
110.
220.
330.
440.
550.
660.
770.
880.
990.
TIME (SECONDS)@

Figure VI-15.

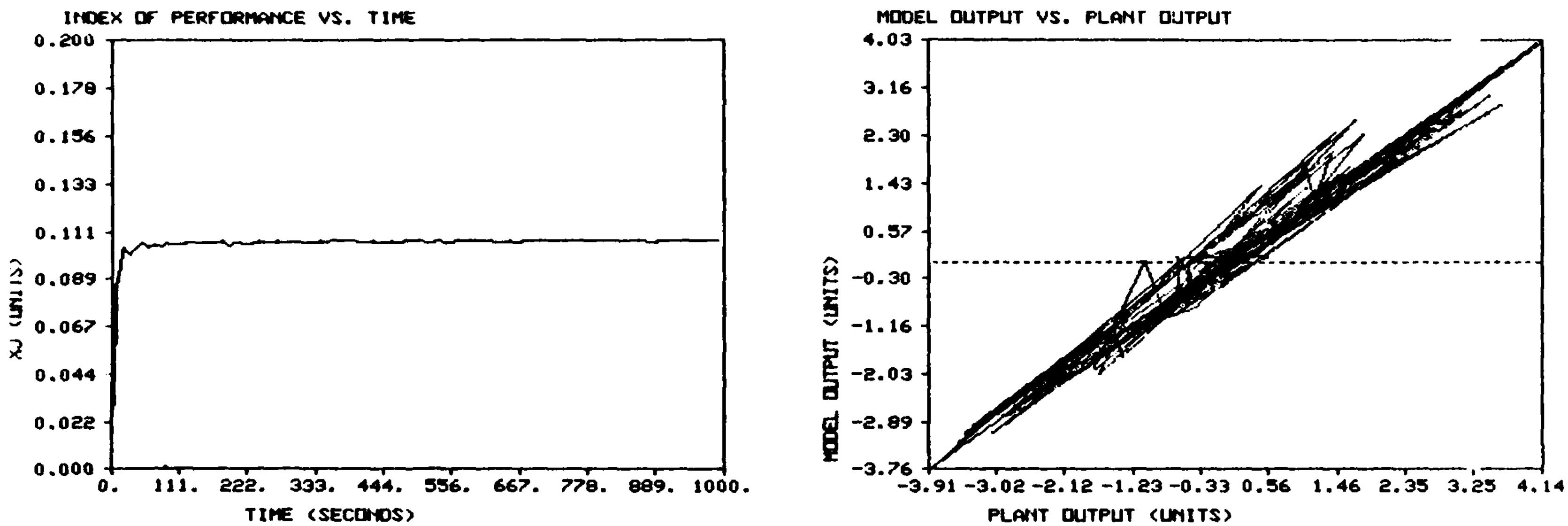

INDEX OF PERFORMANCE VS. TIME
0.200
0.178
0.156
0.133
0.111
0.089
0.067
0.044
0.022
0.000
XJ (UNITS)
0.
111.
222.
333.
444.
556.
667.
778.
889.
1000.
TIME (SECONDS)
MODEL OUTPUT VS. PLANT OUTPUT
4.03
3.16
2.30
1.43
0.57
-0.30
-1.16
-2.03
-2.89
-3.76
MODEL OUTPUT (UNITS)
-3.91 -3.02 -2.12 -1.23 -0.33 0.56 1.46 2.35 3.25 4.14
PLANT OUTPUT (UNITS)

Figure V-16.

NONLINEAR IDENTIFICATION PROCESS

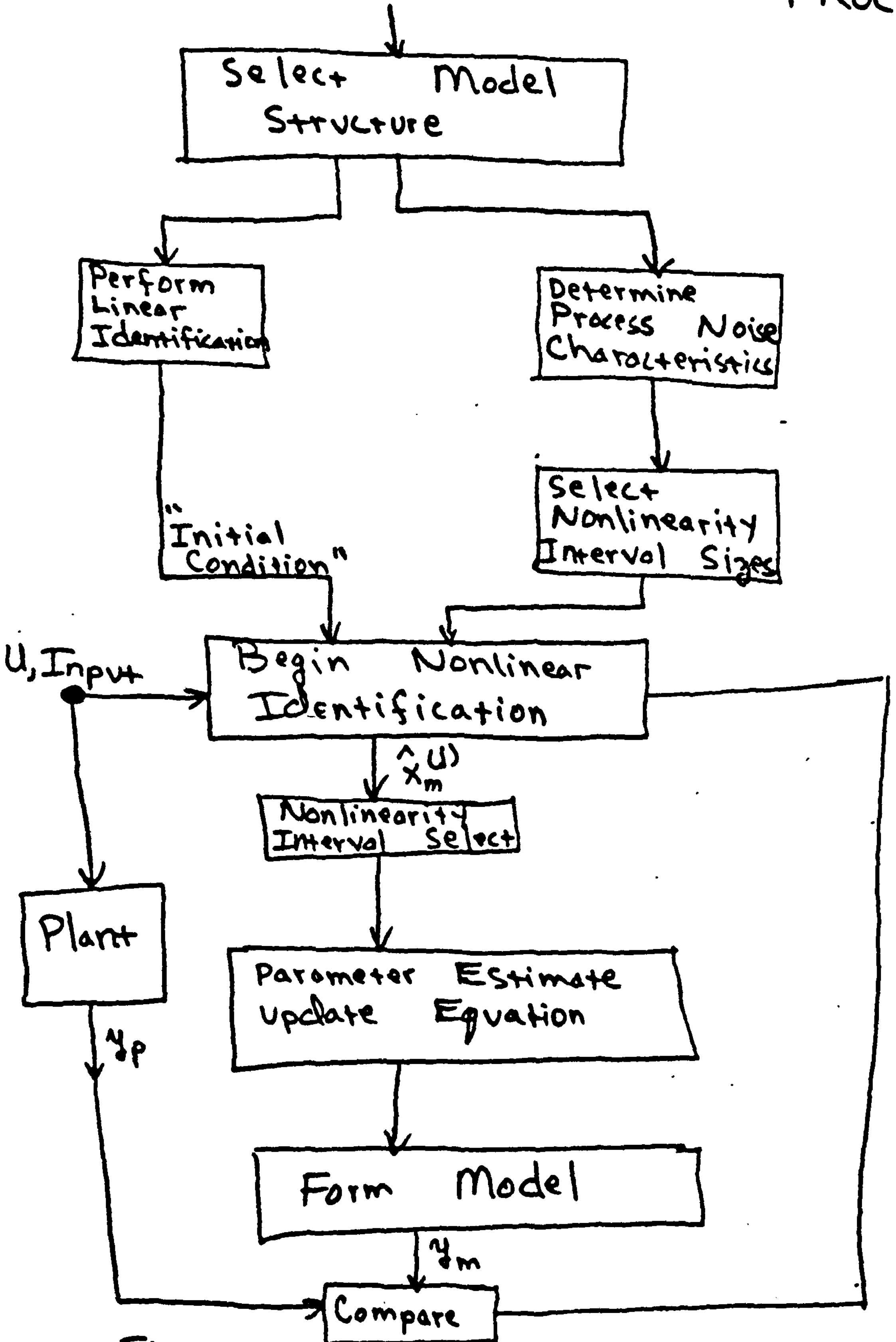


Figure V-17.

VI. SUMMARY AND CONCLUSIONS

The response error MRAS identification can result in meaningful nonlinear identification if set correctly. Chief problems include $\hat{c}_i$ varying by interval, noise biasing effects. This is shown in Figure VI-1 and 2. If one uses fixed c_i terms, the nonlinear intervalized response acts like time varying $\hat{a}_i(t)$ terms for the plant. Even if one uses varying c_i terms, $\hat{c}_i$, there is the sharp time-varying tracking problem as different intervals are entered (Figure VI-2). As shown, in Figure VI-3, there must be vector arrays of $\hat{c}_i$ terms, the vector entries being for different intervals. This means that if there are two $\hat{c}_i$ terms, $\hat{c}_1$ and $\hat{c}_2$, where $\hat{\underline{c}}_1^T = [\ \hat{c}_1^{\,1}, \hat{c}_1^{\,2}, \ldots, \hat{c}_1^{\,p}\ ]$ where there are p intervals to the nonlinearity.

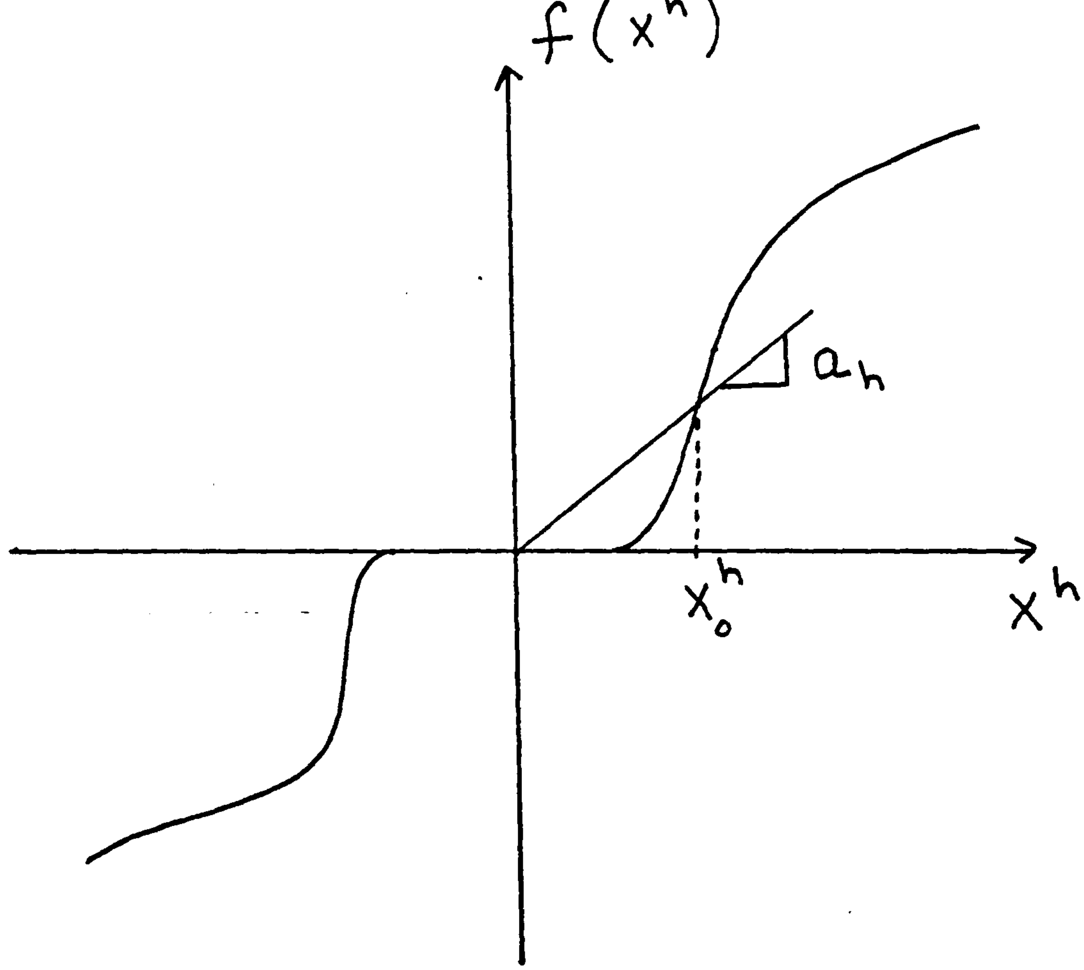
The effect of the number of intervals in the presence of noise has been shown to be a parabola, Figure VI-4. Too few intervals yields an "almost linear" fit, and too many, given noise, yields nonsense due to the interval search problem discussed previously.

To perform total human operator identification, a complete Optimal Control Model (OCM) computer package, plus operator remnant construction, would be needed. Such work was beyond the scope of this project, but could be extended from the present results.

The methods developed here do provide for good nonlinear model identification, _if_ the noise levels are reasonable. Work is underway to minimize the effects of the large correlated noise levels.

PRF PROBLEM

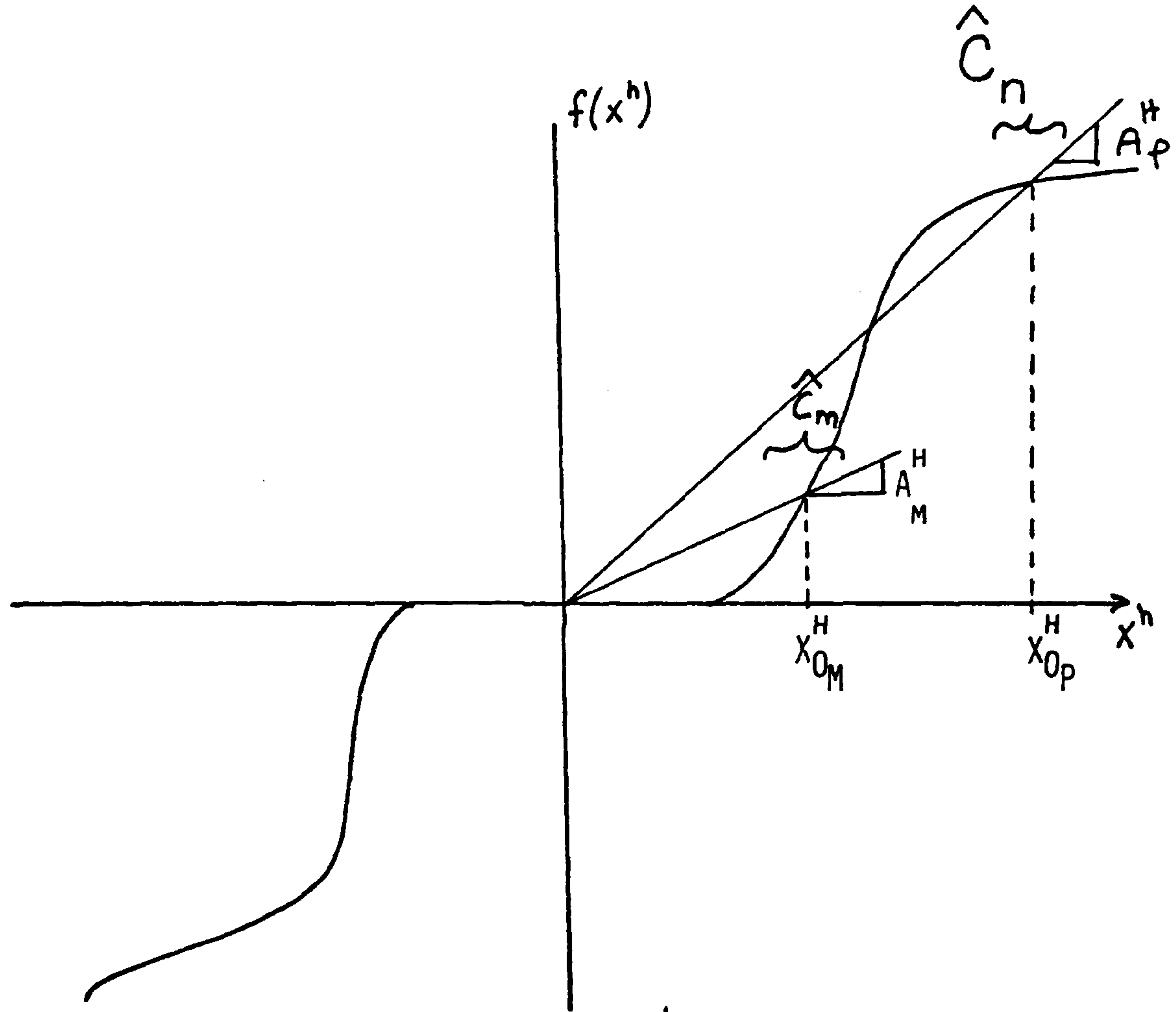
C^I FIXED



$$G(s) = \frac{C_0 + \sum_{i=1}^{n} C_i\, s^i}{\tilde{a}_0 + \sum_{i=1}^{n} \tilde{a}_i\, s^i}$$

Fig. VI-1

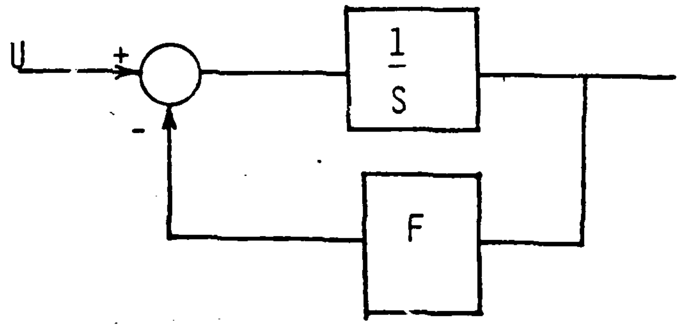
C_I ADJUSTING



$$G(s) = \frac{\hat{c}_0 + \sum_{i=1}^{N} \hat{c}_i S^i}{\tilde{a}_0 + \sum_{i=1}^{N} \tilde{a}_i S^i}$$

Figure VI-2.

SOLUTION



$$\underline{X}\begin{bmatrix} X_o^1 \\ X_o^2 \\ \vdots \\ X_o^N \end{bmatrix} \Longrightarrow \underline{F}\begin{bmatrix} A_1^1 \\ A_1^2 \\ \vdots \\ A_1^N \end{bmatrix} \Longrightarrow \hat{\underline{C}}\begin{bmatrix} \hat{C}_1^1 \\ \hat{C}_1^2 \\ \vdots \\ \hat{C}_1^N \end{bmatrix}$$

Figure VI-3.

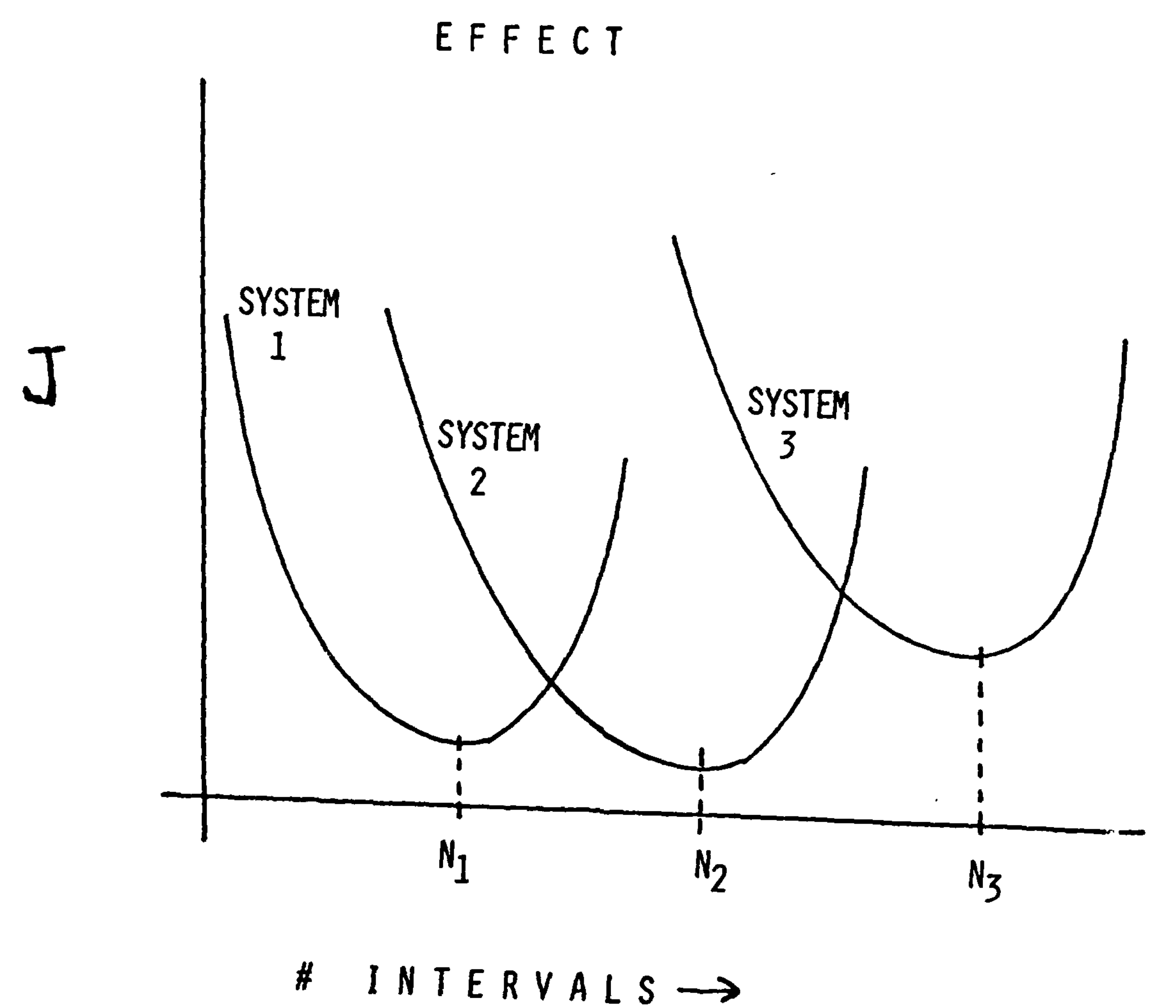


Figure VI-4.

APPENDIX

```
01234567890123456789   ** RSX-11M V3.1 **    25-AUG-80    14:51:33    DK1:[24,1]NONLIN.FTN;6    01234567890123456789
01234567890123456789   ** RSX-11M V3.1 **    25-AUG-80    14:51:33    DK1:[24,1]NONLIN.FTN;6    01234567890123456789
01234567890123456789   ** RSX-11M V3.1 **    25-AUG-80    14:51:33    DK1:[24,1]NONLIN.FTN;6    01234567890123456789


NN      NN     000000     NN      NN  LL             IIIIII     NN      NN
NN      NN     000000     NN      NN  LL             IIIIII     NN      NN
NN      NN   00      00   NN      NN  LL               II       NN      NN
NN      NN   00      00   NN      NN  LL               II       NN      NN
NNNN    NN   00      00   NNNN    NN  LL               II       NNNN    NN
NNNN    NN   00      00   NNNN    NN  LL               II       NNNN    NN
NN  NN  NN   00      00   NN  NN  NN  LL               II       NN  NN  NN
NN  NN  NN   00      00   NN  NN  NN  LL               II       NN  NN  NN
NN    NNNN   00      00   NN    NNNN  LL               II       NN    NNNN
NN    NNNN   00      00   NN    NNNN  LL               II       NN    NNNN
NN      NN   00      00   NN      NN  LL               II       NN      NN        ....
NN      NN   00      00   NN      NN  LL               II       NN      NN        ....
NN      NN     000000     NN      NN  LLLLLLLLLL     IIIIII     NN      NN        ....
NN      NN     000000     NN      NN  LLLLLLLLLL     IIIIII     NN      NN        ....


FFFFFFFFFF   TTTTTTTTTT   NN      NN      ;;;;      666666
FFFFFFFFFF   TTTTTTTTTT   NN      NN      ;;;;      666666
FF               TT       NN      NN      ;;;;    66
FF               TT       NN      NN      ;;;;    66
FF               TT       NNNN    NN              66
FF               TT       NNNN    NN              66
FFFFFFFF         TT       NN  NN  NN      ;;;;    66666666
FFFFFFFF         TT       NN  NN  NN      ;;;;    66666666
FF               TT       NN    NNNN      ;;;;    66      66
FF               TT       NN    NNNN      ;;;;    66      66
FF               TT       NN      NN        ;;    66      66
FF               TT       NN      NN        ;;    66      66
FF               TT       NN      NN      ;;      666666
FF               TT       NN      NN      ;;      666666


01234567890123456789   ** RSX-11M V3.1 **    25-AUG-80    14:51:33    DK1:[24,1]NONLIN.FTN;6    01234567890123456789
01234567890123456789   ** RSX-11M V3.1 **    25-AUG-80    14:51:33    DK1:[24,1]NONLIN.FTN;6    01234567890123456789
01234567890123456789   ** RSX-11M V3.1 **    25-AUG-80    14:51:33    DK1:[24,1]NONLIN.FTN;6    01234567890123456789
```

```
      DIMENSION SVF(10),G(30),NI(30),TX(30),BX(30),INT(30),
     2 AMP(10),W(10),XI(30),A(10),GA(10)
      REAL *8 X(100),F(100),XP(10),FP(10),PARM(30,10),
     1 ESUM,EP,EAVG,V,XJ,YPJ,YMJ,U,UP,Y,YP,
     2 ASUM,BSUM,E(10),GAIN(30,10)
      INTEGER INFILE(20),ERROR(20),PARAM(20),TEST(20),SAVE(20),
     1 HEADER(20)
      COMMON/AD/X1,T1,BT1,N1,I1
      INTEGER P
111   TYPE *,'ENTER INPUT FILE'
      ACCEPT 4003,INFILE
      TYPE *,'ENTER HEADER FILE'
      ACCEPT 4003,HEADER
      TYPE *,'ENTER ERROR FILE'
      ACCEPT 4003,ERROR
      TYPE *,'ENTER PARAMETER FILE'
      ACCEPT 4003,PARAM
      TYPE *,'ENTER TEST FILE'
      ACCEPT 4003,TEST
      TYPE *,'ENTER SAVE FILE'
      ACCEPT 4003,SAVE
      TYPE *,'ARE ALL THE FILES CORRECT [<CR> - YES, 1<CR> - NO] ?'
      ACCEPT 4001,M20
      IF(M20.NE.0)GO TO 111
      CALL ASSIGN(1,INFILE)
      READ(1,1000)N,NS,KK,ISAVE,NT,NN,IPARM
      READ(1,1002)II
      TYPE 430,N,NS,KK,ISAVE,NT,NN,IPARM
      TYPE 430,II
430   FORMAT(1H ,I6)
      IADJ=0
      IF(IPARM.NE.0)IADJ=1
      READ(1,1000)NP,NF
      TYPE 430,NP,NF
      NTOT=N+N
      NS2=NS+NS
      NTOT3=NTOT+N
      NSTATE=NTOT+NTOT+NS2+NP+IADJ
      READ(1,1000)(NI(I),I=1,NTOT3)
      READ(1,1001)(X(I),I=1,NSTATE)
      TYPE 431,(X(I),I=1,NSTATE)
431   FORMAT(1H ,1PE11.4)
      TYPE 430,(NI(I),I=1,NTOT3)
      READ(1,1001)(TX(I),I=1,NTOT3)
      TYPE 431,(TX(I),I=1,NTOT3)
      READ(1,1001)(BX(I),I=1,NTOT3)
      TYPE 431,(BX(I),I=1,NTOT3)
      DO 05 I=1,NTOT3
      P=NI(I)
      READ(1,1001)(PARM(I,J),J=1,P)
      TYPE 431,(PARM(I,J),J=1,P)
05    CONTINUE
      IF(IPARM.EQ.0)GO TO 04
      P=NI(IPARM)
      READ(1,1001)(A(I),I=1,P)
04    DO 06 I=1,NTOT3
      P=NI(I)
      READ(1,1001)(GAIN(I,J),J=1,P)
      TYPE 431,(GAIN(I,J),J=1,P)
06    CONTINUE
      IF(IPARM.EQ.0)GO TO 03
      P=NI(IPARM)
      READ(1,1001)(GA(I),I=1,P)
```

```
      READ(1,1001)(SVF(I),I=1,NS)
      TYPE 431,(SVF(I),I=1,NS)
      READ(1,1001)T,H,WRITE,ESUM,YPJ,YMJ,EAVG
      TYPE 431,T,H,WRITE,ESUM,YPJ,YMJ,EAVG
      READ(1,1001)(AMP(I),W(I),I=1,NF)
      TYPE 431,(AMP(I),W(I),I=1,NF)
      READ(1,1001)STAND1,STAND2,STAND3
      TYPE 431,STAND1,STAND2,STAND3
      READ(1,1000)NSTA1,NSTA2,NSTA3
      TYPE 430,NSTA1,NSTA2,NSTA3
      IF(NSTA1.EQ.0)XSTA1=1.0
      IF(NSTA2.EQ.0)XSTA2=1.0
      READ(1,1000)M1,M2
      TYPE 430,M1,M2
      READ(1,1000)KFLAG
      TYPE 430,KFLAG
      READ(1,1001)TSTOP
      TYPE 431,TSTOP
      READ(1,1000)ISTART
      TYPE 430,ISTART
      CALL CLOSE (1)
      IF(ISTART.NE.0)GO TO 8011
      CALL ASSIGN(1,HEADER)
      WRITE(1,2004)
      WRITE(1,2005)N,NS,KK,ISAVE,NT,NN,IPARM
      WRITE(1,2023)II
      WRITE(1,2006)
      WRITE(1,2007)(X(I),I=1,NSTATE)
      WRITE(1,2008)
      WRITE(1,2009)(NI(I),TX(I),BX(I),I=1,NTOT3)
      WRITE(1,2010)
      DO 07 I=1,NTOT3
      P=NI(I)
      WRITE(1,2007)(PARM(I,J),J=1,P)
07    CONTINUE
      IF(IPARM.EQ.0)GO TO 071
      P=NI(IPARM)
      WRITE(1,2026)(A(I),I=1,P)
071   WRITE(1,2011)
      DO 08 I=1,NTOT3
      P=NI(I)
      WRITE(1,2007)(GAIN(I,J),J=1,P)
08    CONTINUE
      IF(IPARM.EQ.0)GO TO 084
      P=NI(IPARM)
      WRITE(1,2027)(GA(I),I=1,P)
084   WRITE(1,2012)
      WRITE(1,2007)(Q(I),I=1,NTOT3)
      IF(IPARM.NE.0)WRITE(1,2025)QP
      WRITE(1,2013)
      WRITE(1,2007)(SVF(I),I=1,NS)
      WRITE(1,2014)
      WRITE(1,2007)T,H,WRITE,ESUM,YPJ,YMJ,EAVG
      WRITE(1,2022)
      WRITE(1,2015)NP,NF
      WRITE(1,2018)
      DO 083 I=1,NF
      WRITE(1,2019)AMP(I),W(I)
083   CONTINUE
      WRITE(1,2022)
      WRITE(1,2020)
      WRITE(1,2021)STAND1*100.,NSTA1,STAND2*100.,NSTA2,STAND3*100.,
     1 NSTA3
```

```
      WRITE(1,2024)ISTART
      CALL CLOSE (1)
      CALL ASSIGN(1,ERROR)
      CALL CLREF(36)
      WRITE(1,2007)T,V,EAVG,EP,XJ,UNOIS,YNOIS
      CALL CLOSE (1)
      CALL ASSIGN(1,PARAM)
      WRITE(1,2007)T
      IF(IPARM.EQ.0)GO TO 12
      P=NI(IPARM)
      WRITE(1,2007)(A(I),I=1,P)
12    DO 082 I=1,NTOT3
      P=NI(I)
      WRITE(1,2007)(PARM(I,J),J=1,P)
082   CONTINUE
      CALL CLOSE (1)
8011  INDEX=1
      IWRITE=IFIX(WRITE/H)
      UCOR=0
      UNOIS=0.0
      YNOIS=0.0
      XJ=0.0
      KERR=0
      EP=0.0
      V=0.0
C
C     LOCATE THE STATES IN STATE SPACE INTERVALS
C
1     K=N+NS2
      DO 09 I=1,NTOT3
      X1=X(I)  IF(I.GT.NTOT)X1=X(I-NTOT)
      N1=NI(I)
      T1=TX(I)
      BT1=BX(I)
      CALL ATOD
      XI(I)=0.0
      IF(I.EQ.IPARM)XI(I)=XI
      INT(I)=II
      X(K+I)=PARM(I,II)
09    CONTINUE
C                                              I1= INT(IPARM)
C     FORM THE DIFFERENTIAL EQUATIONS X(NSTATE)= A(I1)
C
C     PLANT DIFFERENTIAL EQUATIONS
C
C     INPUT
C
      U=0.0
      DO 21 I=1,NF
      U=U+AMP(I)*SIN(W(I)*T)
21    CONTINUE
C
C     TO INSERT CORRELATED NOISE INTO THE PLANT
C
      IF(STAND3.EQ.0.0)GO TO 11
      XSTAN3=X(NSTA3)*STAND3
      UCOR=GEN(XSTAN3)
11    UC=UCOR+U
C
C     FORM THE SYSTEM
C
      IF(KFLAG.NE.0)UC=X(N+1)
      I1=NSTATE-NP-IADJ
      DO 220 I=1,NP
```

```
      DO 22 I=1,NP
      F(I+I1)=FP(I)
22    CONTINUE
C
C     INSERT MEASUREMENT NOISE ON THE INPUT OF THE PLANT
C
      IF(STAND1.EQ.0.0)GO TO 23
      IF(NSTA1.EQ.0)XSTA1=U
      XSTAN1=STAND1*XSTA1
      UNOIS=GEN(XSTAN1)
23    CONTINUE
      UP=UNOIS+U
C
C     INSERT MEASUREMENT NOISE ON THE OUTPUT OF THE PLANT
C
      IF(STAND2.EQ.0.0)GO TO 24
      IF(NSTA2.NE.0)XSTA2=Y
      XSTAN2=STAND2*XSTA2
      YNOIS=GEN(XSTAN2)
24    YP=YNOIS+Y
C
C             A) MODEL
C
      ASUM=0.0
      BSUM=0.0
      J=N+NS2
      K=J+N
      DO 20 I=1,N
      F(I)=X(I+1)
      ASUM=ASUM-X(J+I)*(X(I)-XI(I))
      BSUM=BSUM+X(K+I)*(X(I+N)-XI(I+N))
20    CONTINUE
      F(N)=ASUM+BSUM
      IF(IPARM.NE.0)F(N)=F(N)-X(NSTATE)
C
C             B) STATE VARIABLE FILTER FOR INPUT AND OUTPUTS
C
      ASUM=0.0
      BSUM=0.0
      K=N+NS
      DO 30 I=1,NS
      SF=SVF(I)
      F(I+N)=X(I+N+1)
      F(K+I)=X(K+I+1)
      BSUM=BSUM-SF*X(I+N)
      ASUM=ASUM-SF*X(K+I)
30    CONTINUE
      SF=SVF(1)
      F(N+NS)=BSUM+SF*UP
      F(N+NS2)=ASUM+SF*YP
      IF(KFLAG.NE.0)GO TO 46
C
C     FORM THE RESPONSE ERROR
C
      V=F(NTOT+NS)-F(N)
      J=N+NS
      L=J+NS+NTOT
      DO 10 I=1,N
      E(I)=X(J+I)-X(I)
      X1=E(I)
      T1=TX(NTOT+I)
      BT1=BX(NTOT+I)
      N1=NI(NTOT+I)
      CALL ATOD
```

```
      X(L+I)=PARM(NTOT+I,I1)
      V=V+E(I)*X(L+I)
10    CONTINUE
C
C     FORM PARAMETER ADAPTAION
C
35    J=N+N62
      K=J+N
      L=K+N
      DO 40 I=1,N
      I3=INT(I)
      I4=INT(N+I)
      I5=INT(NTOT+I)
      F(J+I)=-GAIN(I,I3)*V*X(I)
      F(K+I)=GAIN(N+I,I4)*V*X(N+I)
      F(L+I)=-GAIN(NTOT+I,I5)*V*E(I)
40    CONTINUE
      IF(IPARM.EQ.0)GO TO 41
      I1=INT(IPARM)
      IF(IPARM.GT.N)GA(I1)=-GA(I1)
      F(NSTATE)=-GA(I1)*V
41    CONTINUE
C
C     FORM THE GAIN ADAPTATION
C
      DO 45 I=1,NTOT3
      I1=INT(I)
      GAIN(I,I1)=GAIN(I,I1)/(1+G(I))
45    CONTINUE
      IF(IPARM.NE.0)GA(I1)=GA(I1)/(1+GP)
C
C     FORM THE INSTANTANEOUS AND MEAN SQUARED RESPONSE ERROR
C
      EAVG=EAVG+(V**2)*H
C
C     FORM THE THEIL INDEX OF PERFORMANCE
C
46    ESUM=ESUM+(YP-X(1))**2
      YPJ=YPJ+YP**2
      YMJ=YMJ+X(1)**2
C
C     INTEGRATE THE STATE EQUATIONS
C
50    CALL DERK(NSTATE,X,F,T,H,INDEX)
      IF(INDEX.EQ.2) GO TO 1
      II=II+1
      IF(KFLAG.NE.0)GO TO 80
C
C     REPLACE THE PARAMETERS AND GAINS INTO THEIR MATRICES
C
      K=N+N62
      DO 55 I=1,NTOT3
      I1=INT(I)
      PARM(I,I1)=X(K+I)
55    CONTINUE
      IF(IPARM.EQ.0)GO TO 56
      I1=INT(IPARM)
      A(I1)=X(NSTATE)
56    CONTINUE
      IF(II.GE.30000)II=0
      IF(MOD(II,IWRITE).NE.0)GO TO 1
C
C     CALCUATE THEIL'S INDEX OF PERFORMANCE
```

```
C
C       WRITE OUT THE PARAMETER MATRIX
C
        TYPE 2007, T, EAVG, EP, XJ, F(M1), F(M2)
        CALL ASSIGN(1, ERROR)
        CALL FDBSET(1, 'APPEND')
        WRITE(1, 2007)T, V, EAVG, EP, XJ, UNOIS, YNOIS
        CALL CLOSE (1)
        CALL ASSIGN(1, PARAM)
        CALL FDBSET(1, 'APPEND')
        WRITE(1, 2007)T
        IF(IPARM. EQ. 0)GO TO 59
        P=NI(IPARM)
        WRITE(1, 2007)(A(I), I=1, P)
59      DO 60 I=1, NTOT3
        P=NI(I)
        WRITE(1, 2007)(PARM(I, J), J=1, P)
60      CONTINUE
        CALL CLOSE (1)
        NN=NN+1
        CALL READEF(36, KF)
        IF(KF. NE. 0)GO TO 90
        IF(MOD(NN, ISAVE). EQ. 0)GO TO 200
601     IF(MOD(NN, NT). NE. 0)GO TO 1
C
C       TEST THE MODEL AGAINST THE SYSTEM
C
61      STAND1=0. 0
        STAND2=0. 0
        STAND3=0. 0
        UCOR=0. 0
        UNOIS=0. 0
        YNOIS=0. 0
        KFLAG=1
        ESUM=0. 0
        YPJ=0. 0
        YMJ=0. 0
        XJ=0. 0
        V=0. 0
        NN=0
        DO 70 I=1, NSTATE
        X(I)=0. 0
        F(I)=0. 0
70      CONTINUE
        YP=0. 0
        TSTOP=T
        T=0. 0
        II=0
        TYPE *, 'TEST THE MODEL AGAINST THE SYSTEM'
80      IF(MOD(II, IWRITE). NE. 0)GO TO 1
        XM=FLOAT( I)
        XJ=SQRT(ES M/XM)/(SQRT(YPJ/XM)+SQRT(YMJ/XM))
        TYPE 3001,  , X(1), YP, XJ
        NN=NN+1
        IF(MOD(NN, I AVE). EQ. 0)GO TO 200
81      CALL ASSIG (1, TEST)
        CALL FDBSET(1, 'APPEND')
        WRITE(1, 2007)T, X(1), YP, XJ
        CALL CLOSE (1)
        CALL READEF(36, KF)
        IF(KF NE 0)GO TO 90
        IF(T LT TSTOP)GO TO 1
        TYPE *, 'FINISH MODEL TEST'
```

```
      TYPE *, 'ENTER'
      TYPE *, '1 - STOP IDENT & START TEST'
      TYPE *, '2 - RESTART IDENT'
      TYPE *, '3 - STOP & EXIT PROGRAM'
      TYPE *, '4 - CHANGE GAINS & CONTINUE'
      ACCEPT 4001, ITELL
      GO TO (61,111,180,91), ITELL
91    DO 93 I=1,NTOT3
      P=NI(I)
      DO 92 J=1,P
      TYPE 4002, I, J, GAIN(I,J)
      ACCEPT 4001, M20
      IF(M20 EQ 0)GO TO 92
      TYPE *, 'ENTER GAIN(I,J)='
      ACCEPT 1001, GAIN(I,J)
92    CONTINUE
93    CONTINUE
      GO TO 1
200   CALL ASSIGN(1,SAVE)
      CALL FDBSET(1,'OLD')
      WRITE(1,1000)N,NS,KK,ISAVE,NT,NN,IPARM
      WRITE(1,1002)II
      WRITE(1,1000)NP,NF
      WRITE(1,1000)(NI(I),I=1,NTOT3)
      WRITE(1,1001)(X(I),I=1,NSTATE)
      WRITE(1,1001)(TX(I),I=1,NTOT3)
      WRITE(1,1001)(BX(I),I=1,NTOT3)
      DO 201 I=1,NTOT3
      P=NI(I)
      WRITE(1,1001)(PARM(I,J),J=1,P)
201   CONTINUE
      IF(IPARM EQ 0)GO TO 203
      P=NI(IPARM)
      WRITE(1,1001)(A(I),I=1,P)
203   DO 202 I=1,NTOT3
      P=NI(I)
      WRITE(1,1001)(GAIN(I,J),J=1,P)
202   CONTINUE
      IF(IPARM EQ 0)GO TO 204
      P=NI(IPARM)
      WRITE(1,1001)(GA(I),I=1,P)
204   WRITE(1,1001)(G(I),I=1,NTOT3)
      IF(IPARM NE 0)WRITE(1,1001)GP
      WRITE(1,1001)(SVF(I),I=1,NS)
      WRITE(1,1001)T,H,WRITE,ESUM,YPJ,YMJ,EAVG
      WRITE(1,1001)(AMP(I),W(I),I=1,NF)
      WRITE(1,1001)STAND1,STAND2,STAND3
      WRITE(1,1000)NSTA1,NSTA2,NSTA3
      WRITE(1,1000)M1,M2
      WRITE(1,1000)KFLAG
      WRITE(1,1001)TSTOP
      ISTART=1
      WRITE(1,1000)ISTART
      CALL CLOSE (1)
      IF(KFLAG NE 0)GO TO 81
      GO TO 601
1000  FORMAT(1H ,I3)
1001  FORMAT(1H ,E11 4)
1002  FORMAT(1H ,I6)
2000  FORMAT(1H ,I6,2X,11(1PE9 2,1X))
2001  FORMAT(1H0 ,'STATES')
2002  FORMAT(1H0,'PARAMETERS A K=',I6//)
2003  FORMAT(1H )
2004  FORMAT(1H ,'N, NS, KK, ISAVE, NT, NN, IPARM')
```

```
2007    FORMAT(1H ,10(1PE10.3,2X))
2008    FORMAT(1H ,'# INTERVALS',T15,'TOP LIMIT',T30,'BOTTOM LIMIT')
2009    FORMAT(1H ,5X,I3,T15,1PE10.3,5X,1PE10.3)
2010    FORMAT(1H ,'INITIAL PARAMETER VALUES')
2011    FORMAT(1H ,'INITIAL GAIN VALUES')
2012    FORMAT(1H ,'GAIN ADAPTATION VALUES')
2013    FORMAT(1H ,'STATE VARIABLE FILTER PARAMETERS')
2014    FORMAT(1H ,'T, H, WRITE, ESUM, YPJ, YMJ, EAVG')
2015    FORMAT(1H ,'PLANT ORDER, # OF INPUT FREQUENCIES'/1H ,2(I2,5X))
2018    FORMAT(1H ,'AMPLITUDE',T20,'RADIAN FREQUENCY OF INPUT SIGNAL')
2019    FORMAT(1H ,1PE10.3,T20,1PE10.3)
2020    FORMAT(1H ,'***************************NOISE PARAMETERS*******
        1**********************')
2021    FORMAT(1H ,'INPUT NOISE STANDARD DEVIATION AND CORRELATION,
        1 [0 - UNCORRELATED , 1 - CORRELATED WITH INPUT OR OUTPUT]'/
        2 1H ,'STANDARD DEVIATION =',F8.4,'%',5X,'CORRELATION =',I1/
        3 1H ,'OUTPUT NOISE STANDARD DEVIATION AND CORRELATION'/
        4 1H ,'STANDARD DEVIATION =',F8.4,'%',5X,'CORRELATION =',I1/
        5 1H ,'CIRCULATING PLANT CORRELATED NOISE'/
        6 1H ,'STANDARD DEVIATION =',F8.4,'%',5X,'STATE CORRELATED =',I1)
2022    FORMAT(1H ,'********PLANT PARAMETERS********')
2023    FORMAT(1H ,'COUNTER II =',I6)
2024    FORMAT(1H ,'ISTART =',I1,5X,'[0 - START ; 1 - RESTART]')
2025    FORMAT(1H ,'CONSTANT PARAMETER GAIN ADAPTATION'/
        1 1H ,10(1PE10.3,2X))
2026    FORMAT(1H ,'CONSTANT PARAMETERS'/1H ,10(1PE10.3,2X))
2027    FORMAT(1H ,'CONSTANT PARAMETER GAINS'/1H ,10(1PE10.3,2X))
2030    FORMAT(1H ,'KFLAG = ',I2,5X,'[0 - IDENT & TEST ;1 - TEST ONLY]',
        1 5X,'TSTOP =',F8.3)
3001    FORMAT(1H ,10(1PE10.3,2X))
4001    FORMAT(I1)
4002    FORMAT(1H ,'GAIN(',I2,',',I2,') = ',F9.3)
4003    FORMAT(20A2)
180     STOP
        END
C
C*******************************************************************
C
C       TO LOCATE A VARIABLE X IN AN INTERVAL
C
C*******************************************************************
C
        SUBROUTINE ATOD
        COMMON/AD/X,T,B,N,I
C
        S=(T-B)/N
        I=0
10      I=I+1
        X=X-S
        IF(X.GT.B)GO TO 10
        IF(I.GT.N)I=N
        X=[illegible](I*S)-(S/2.)
        RETURN B
        END
C
C*******************************************************************
C
C       FUNCTION GEN(RMS)
C
C               THIS FUNCTION GENERATES A GAUSSIAN DISTRIBUTED
C               RANDOM NUMBER WITH ZERO MEAN AND STANDARD DEVIATION
C               ,RMS
C
C...................................................................
```

```
      FUNCTION GEN(RMS)
      DATA IX/0/, IY/0/
C
      A=0.0
      DO 1 I=1,12
      CALL RANDU(IX,IY,Y)
1     A=A+Y
      GEN=RMS * (A-6.0)
      RETURN
      END

      SUBROUTINE DERK(M,X,F,T,H,INDEX)
      REAL *8 X(100),F(100),Q(400)
      GO TO (18,19),INDEX
18    KXX=0
      INDEX=2
      DO 35 I=1,M
      J=I+300
35    Q(J)=X(I)
19    KXX=KXX+1
      GO TO (1,2,3,4),KXX
1     DO 5 I=1,M
      Q(I)=F(I)*H
5     X(I)=X(I)+Q(I)*0.5
      T=T+H*0.5
      RETURN
2     DO 6 I=1,M
      J=I+100
      K=I+300
      Q(J)=F(I)*H
6     X(I)=Q(K)+Q(J)*0.5
      RETURN
3     DO 7 I=1,M
      J=I+200
      K=I+300
      Q(J)=F(I)*H
7     X(I)=Q(K)+Q(J)
      T=T+H*0.5
      RETURN
4     DO 8 I=1,M
      J=I+100
      K=I+200
      L=I+300
8     X(I)=Q(L)+((Q(I)+2.0*Q(J)+2.0*Q(K)+F(I)*H)/6.0)
      INDEX=1
      RETURN
      END
```

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